

Formal Semantics - Cameron Sabiston

1. (a) (10 pts) $Y := 20 ; X := 1 + Y$

$\langle Y := 20, \sigma \rangle \rightarrow \sigma(Y)$

$\langle X := 1 + Y, \{Y \rightarrow 20\} \rangle \rightarrow$

$\langle X := 1 + 20, \{Y \rightarrow 20\} \rangle \rightarrow$

$\langle X := 21, \{Y \rightarrow 20\} \rangle \rightarrow$

$\{X \rightarrow 21, Y \rightarrow 20\}$

(b) (15 pts) $Y := 20 ; \text{if } (Y < 10) \text{ then } X := 3 \text{ else } X := 4$

$\langle Y := 20, \sigma \rangle \rightarrow \sigma(Y)$

$\langle \text{if } (Y < 10) \text{ then } X := 3 \text{ else } X := 4, \{Y \rightarrow 20\} \rangle \rightarrow$

$\langle \text{if } (20 < 10) \text{ then } X := 3 \text{ else } X := 4, \{Y \rightarrow 20\} \rangle \rightarrow$

$\langle \text{if False then } X := 3 \text{ else } X := 4, \{Y \rightarrow 20\} \rangle \rightarrow$

$\langle X := 4, \{Y = 20\} \rangle \rightarrow$

$\{X \rightarrow 4, Y \rightarrow 20\}$

(c) (15 pts) $X := 1; \text{while } (\text{not } (X < 1)) \text{ do } X := X - 1$

$\langle X := 1, \sigma \rangle \rightarrow \sigma(X)$

$\langle \text{while } (\text{not}(x < 1)) \text{ do } X := X - 1, \{X \rightarrow 1\} \rangle \rightarrow$

$\langle \text{if not } (X < 1) \text{ then } (X = X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 1\} \rangle \rightarrow$

$\langle \text{if not } (1 < 1) \text{ then } (X = X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 1\} \rangle \rightarrow$

$\langle \text{if not } (\text{False}) \text{ then } (X := X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 1\} \rangle \rightarrow$

$\langle X := X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1 \rangle \rightarrow$

$\langle X := 1 - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1 \rangle \rightarrow$

$\langle X := 0; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1 \rangle \rightarrow$

$\langle \text{while } (\text{not}(x < 1)) \text{ do } X := X - 1, \{X \rightarrow 0\} \rangle \rightarrow$

$\langle \text{if not}(x < 1) \text{ then } (X := X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 0\} \rangle \rightarrow$

$\langle \text{if not}(0 < 1) \text{ then } (X := X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 0\} \rangle \rightarrow$

$\langle \text{if not}(\text{True}) \text{ then } (X := X - 1; \text{while } (\text{not}(X < 1)) \text{ do } X := X - 1) \text{ else skip}, \{X \rightarrow 0\} \rangle \rightarrow$

$\{X \rightarrow 0\}$

(d) (20 pts) $X := (\text{let } Y = 15 \text{ in } Y + 3)$

$\langle X := (\text{let } Y := 15 \text{ in } Y + 3), \sigma \rangle \rightarrow \sigma(\text{let } Y := 15 \text{ in } Y + 3)$

$\langle X := (\text{let } Y := 15 \text{ in } Y + 3) \rangle \rightarrow$

$\langle X := ((15)(Y + 3)) \rangle \rightarrow$

$\langle X := 18 \rangle \rightarrow$

$\{X \rightarrow 18\}$

(e) (20 pts) $X := (\text{fun } Y \rightarrow Y + 10) 15$

$\langle X := ((\text{fun } Y \rightarrow Y + 10) 15), \sigma \rangle \rightarrow \sigma((\text{fun } Y \rightarrow Y + 10) 15) \rightarrow$

$\langle X := ((\text{fun } Y \rightarrow Y + 10) 15) \rangle \rightarrow$

$\langle X := ((15)(Y + 10)) \rangle \rightarrow$

$\langle X := 25 \rangle \rightarrow$

$\{X \rightarrow 25\}$

2. (a) (15 pts) $X := 3; Y := X + 5$

$\langle X := 3, \sigma \rangle \Downarrow \sigma(X)$

$\langle Y := X + 5, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle Y := 3 + 5, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle Y := 8, \{X \rightarrow 3\} \rangle \Downarrow$

$\{X := 3, Y := 8\}$

(b) (15 pts) $X := 3; \text{if } (X < 3) \text{ then } Y := 4 \text{ else } Y := 5 - X$

$\langle X := 3, \sigma \rangle \Downarrow \sigma(X)$

$\langle \text{if } (X < 3) \text{ then } Y := 4 \text{ else } Y := 5 - X, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle \text{if } (3 < 3) \text{ then } Y := 4 \text{ else } Y := 5 - X, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle \text{if } (\text{False}) \text{ then } Y := 4 \text{ else } Y := 5 - X, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle Y := 5 - X, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle Y := 5 - 3, \{X \rightarrow 3\} \rangle \Downarrow$

$\langle Y := 2, \{X \rightarrow 3\} \rangle \Downarrow$

$\{X := 3, Y := 2\}$