Dynamical Systems Homework

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For this homework I decided to recreate a dynamical system that describes Romeo and Juliet's love and hate dynamics [Sprott, 2004]. The two-dimensional system has four parameters and is described by the following equations:

$$\frac{dR}{dt} = a * R + b * J$$
$$\frac{dJ}{dt} = c * J + d * R$$

The four parameters determine each lover's "romantic style". On the one hand, the parameters a and c indicate how much each lovers' own feelings guide their behavior. On the other hand, the parameters b and d indicate how much each lovers' feelings are guided by the feelings of the other. I found the latter parameters' description quite awkward psychologically as the author explained them. So, I thought about them as representing each lovers' beliefs about how the other feels about them. Positive values would indicate accuracy in their beliefs, and negative values would indicate inaccuracy in their beliefs.

I tested different values of the parameters to see how the dynamics of the system were affected. Figure 1 shows the phase portrait of the system for positive values of all the parameters ($a = 0.5$, $b = 0.2$, $c = 0.7$, $d = 0.5$). Under this parameter combination, the system exhibits the existence of a repeller. Every set of initial conditions moves away from 0, and towards positive or negative infinity depending on the specific values of the initial conditions.

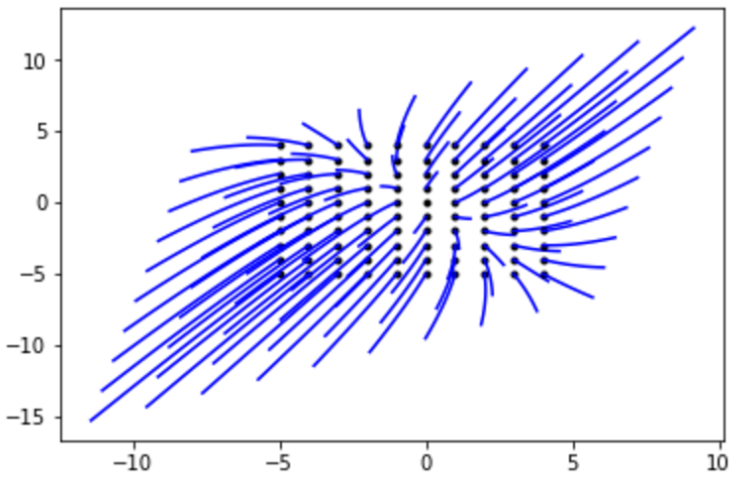

Figure 1: Phase portrait of the system when $a = 0.5$, $b = 0.2$, $c = 0.7$, and $d = 0.5$.

Figure 2 shows the phase portrait of the system when Romeo's parameters are $a = -0.6$ and $b = -2$, and Juliet's parameters are $c = 0.7$ $d = 0.8$. Under this parameter combination, the system oscillates around 0 showing a set of trajectories that seems to suggest the existence of a limit cycle attractor.

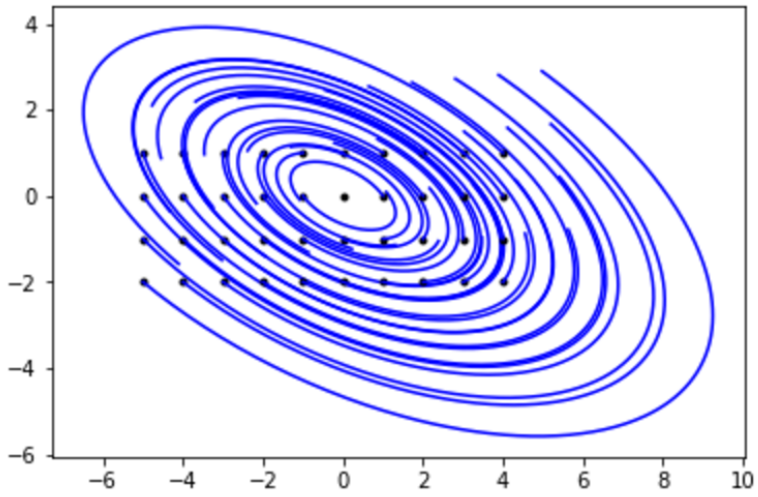

Figure 2: Phase portrait of the system when $a = -0.6$, $b = -2$, $c = 0.7$, and $d = 0.8$.

Finally, Figure 3 shows the phase portrait of the system when all the parameter values are negative ($a = -0.4$, $b = -0.3$, $c = -0.1$, and $d = -0.1$). In this case, the trajectories of the system all seem to evolve towards 0 regardless of the initial conditions. This indicates the existence of an attractor in the system.

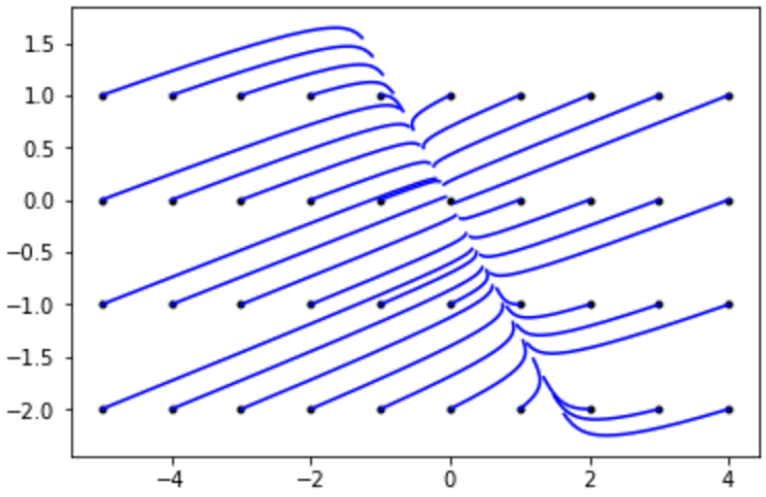

Figure 3: Phase portrait of the system when $a = -0.4$, $b = -0.3$, $c = -0.1$, and $d = -0.1$.

References

[Sprott, 2004] Sprott, J. (2004). Dynamical models of love. *Nonlinear Dynamics, Psychology, and Life Sciences*, 8(3):303–314.