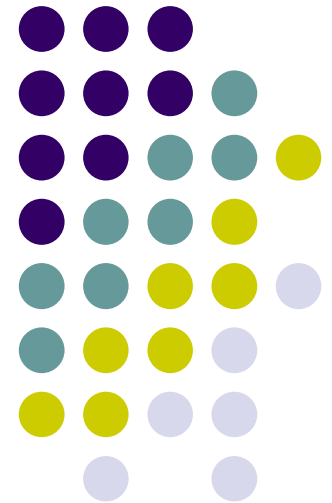


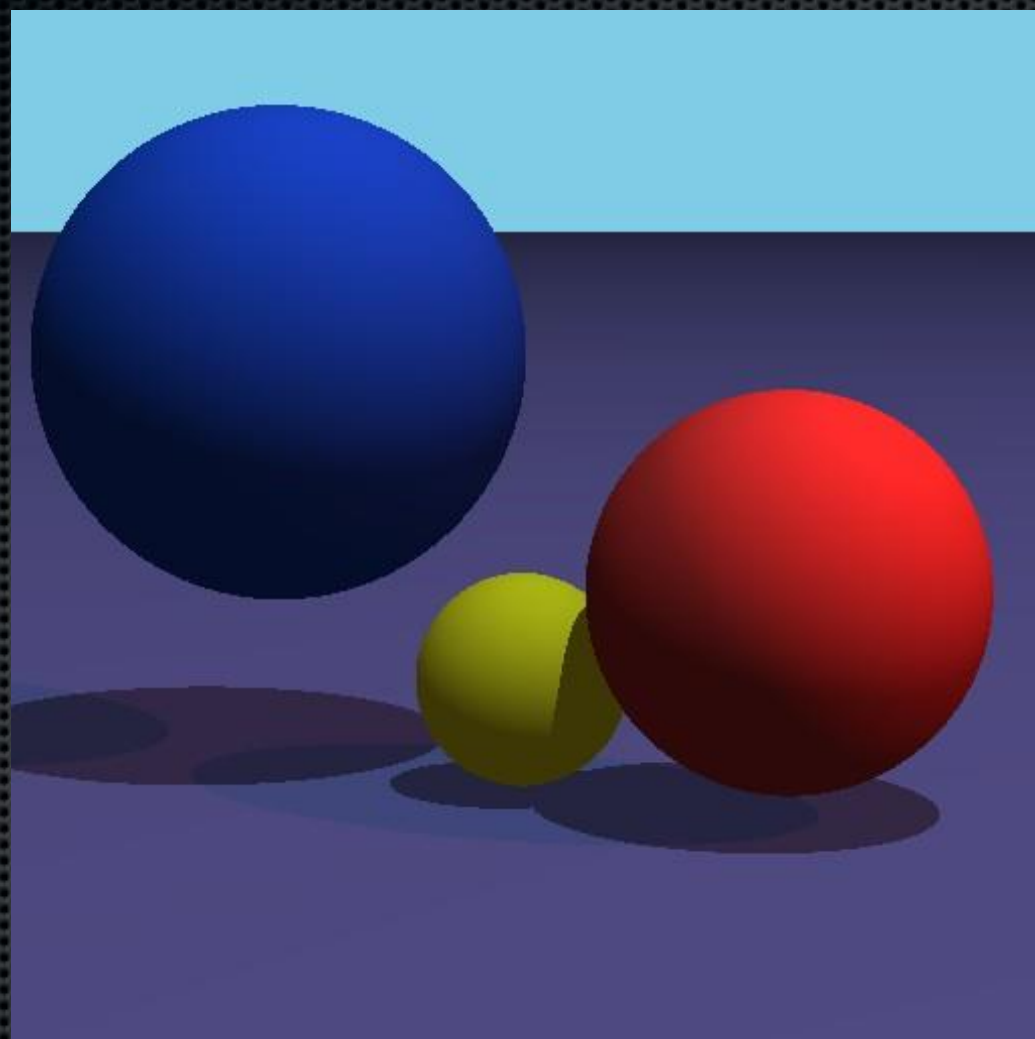
Ray Tracing

Joshua Cuneo

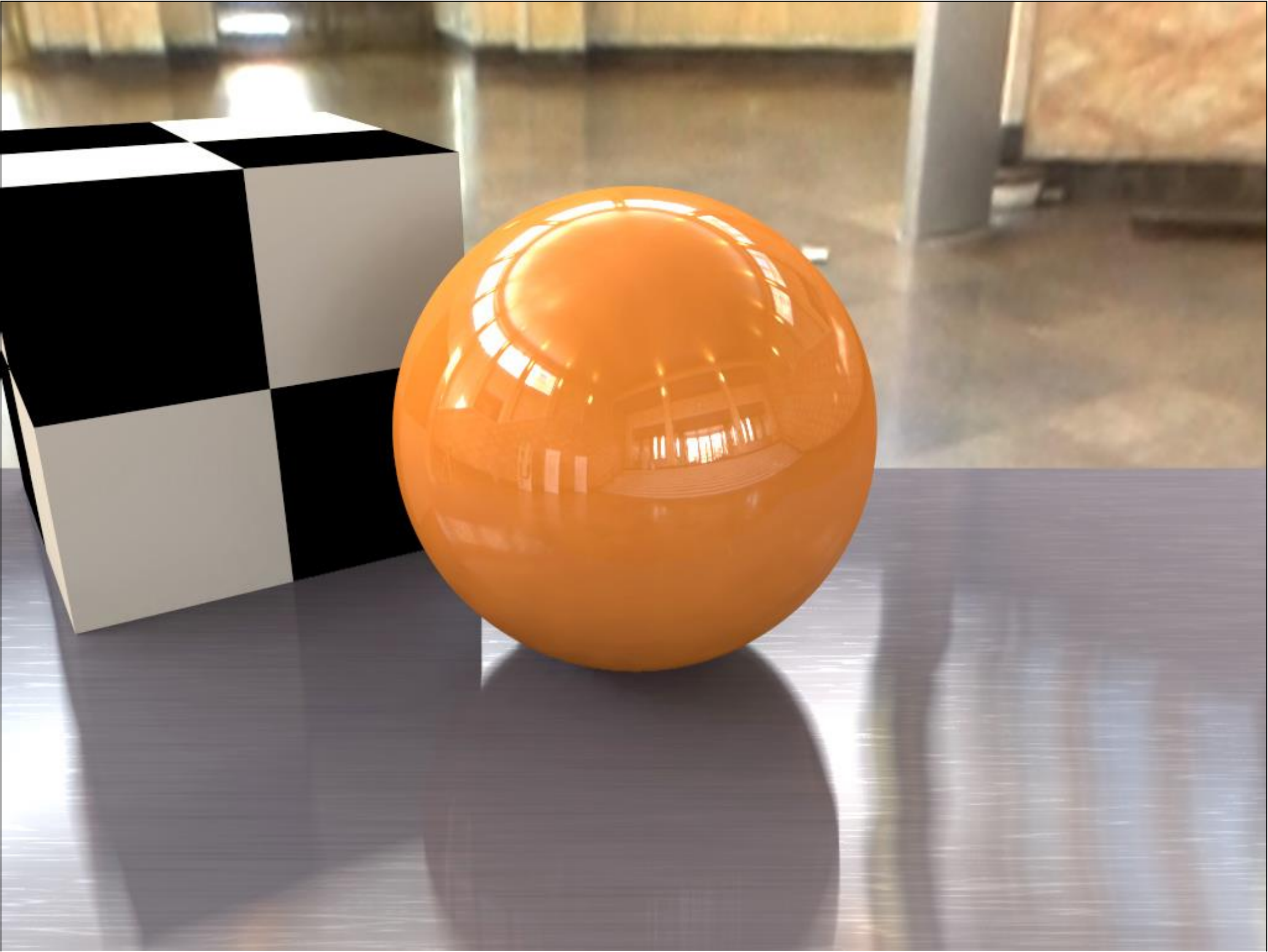
*Computer Science Dept.
Worcester Polytechnic Institute (WPI)*



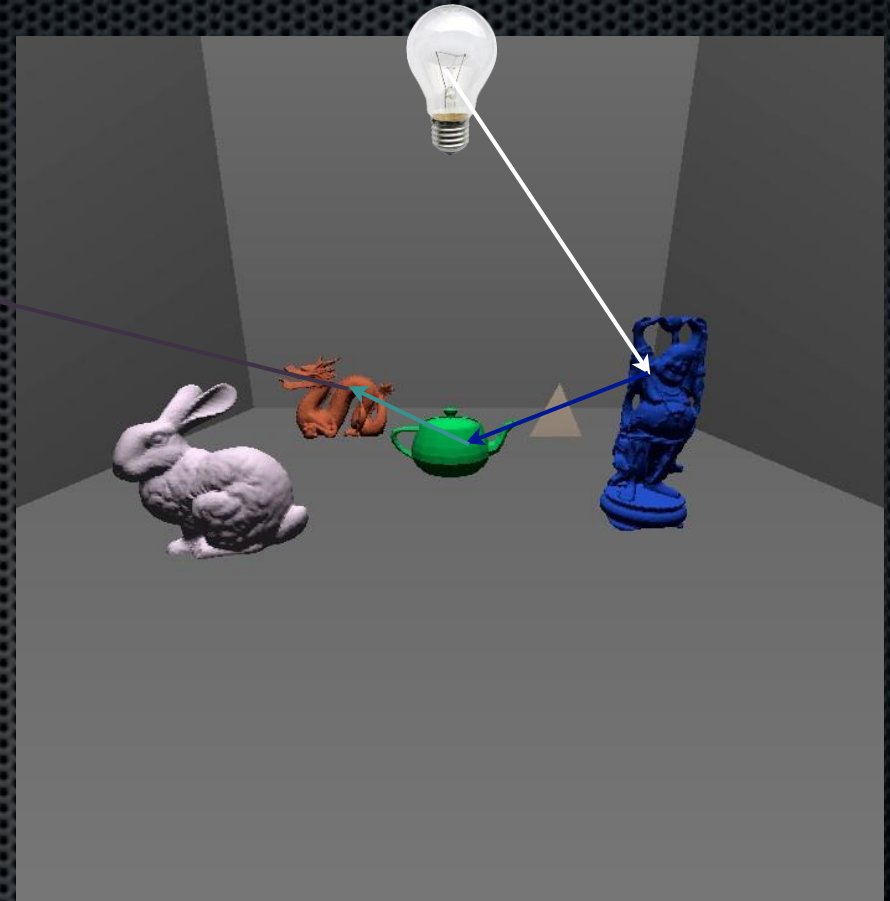
Some Slides Courtesy of Matt Stoker (Utah)





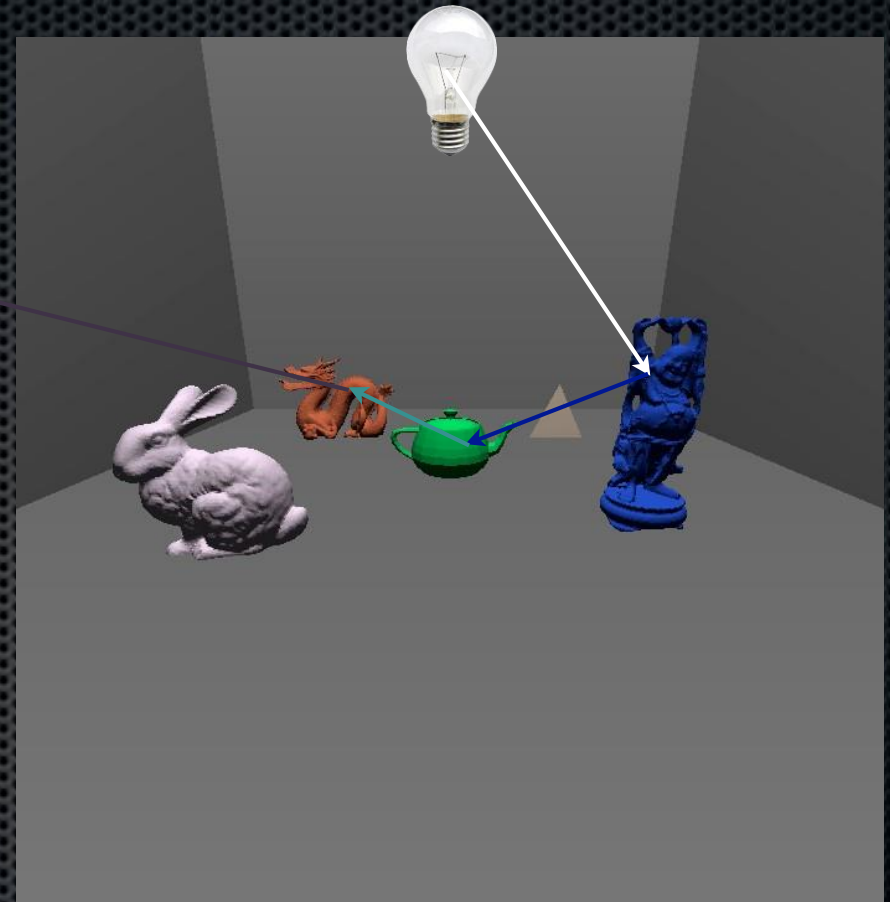


Ray Tracing



- For each light in scene
 - Emit 1,000,000,000 photons
 - For each photon
 - Find what geometry photon hits
 - Color photon
 - Scatter photon
 - Find what photon hits next
 - ...
 - Pray photon hits camera CCD
 - Light pixel that CCD micro-square represents

Ray Tracing

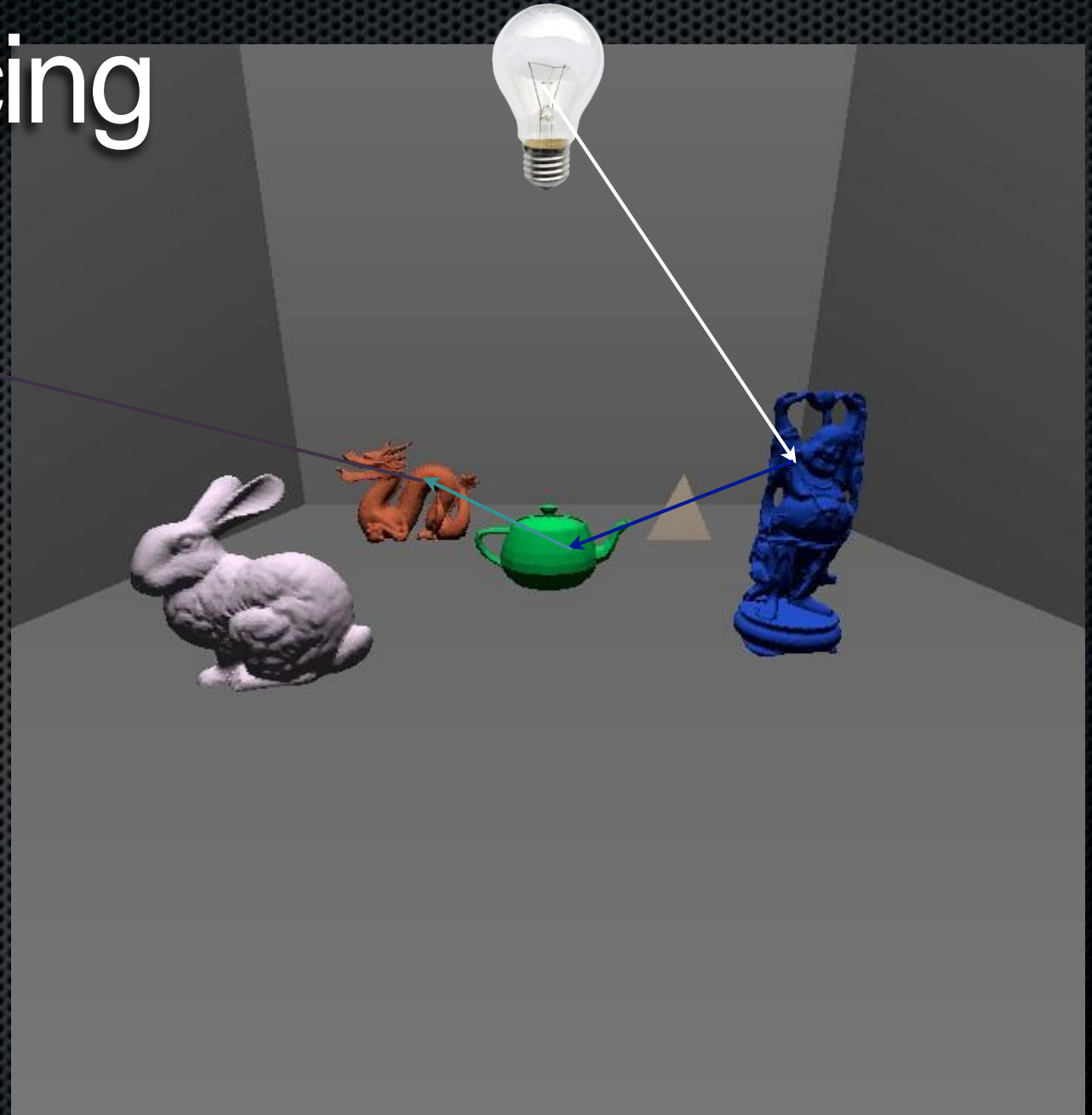


- For each light in scene
 - Emit 1,000,000,000 photons
 - For each photon
 - Photon hits

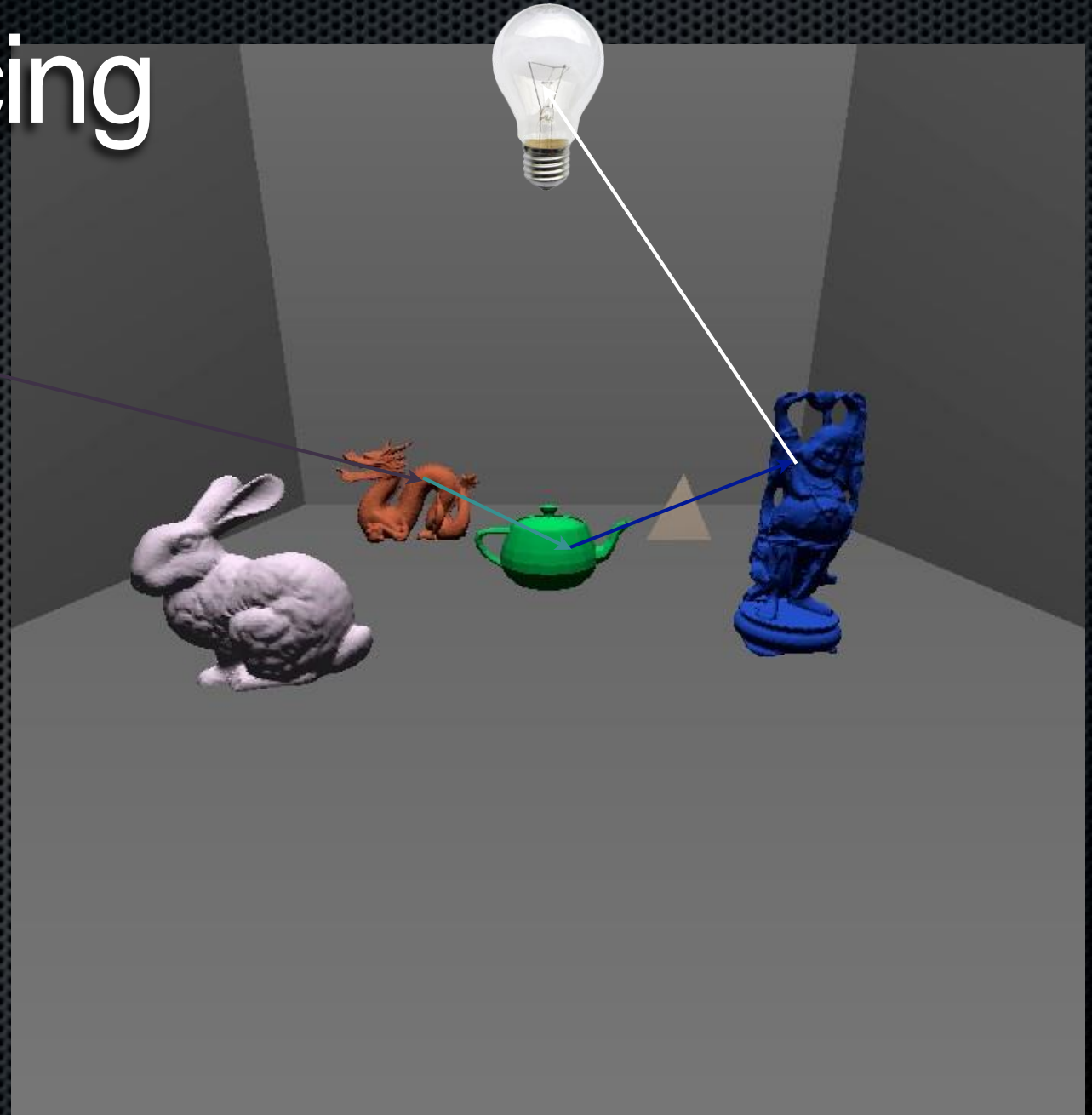


- ...
 - Pray photon hits camera CCD
 - Light pixel that CCD micro-square represents

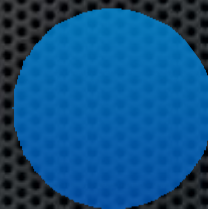
Ray Tracing



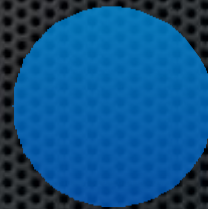
Ray Tracing



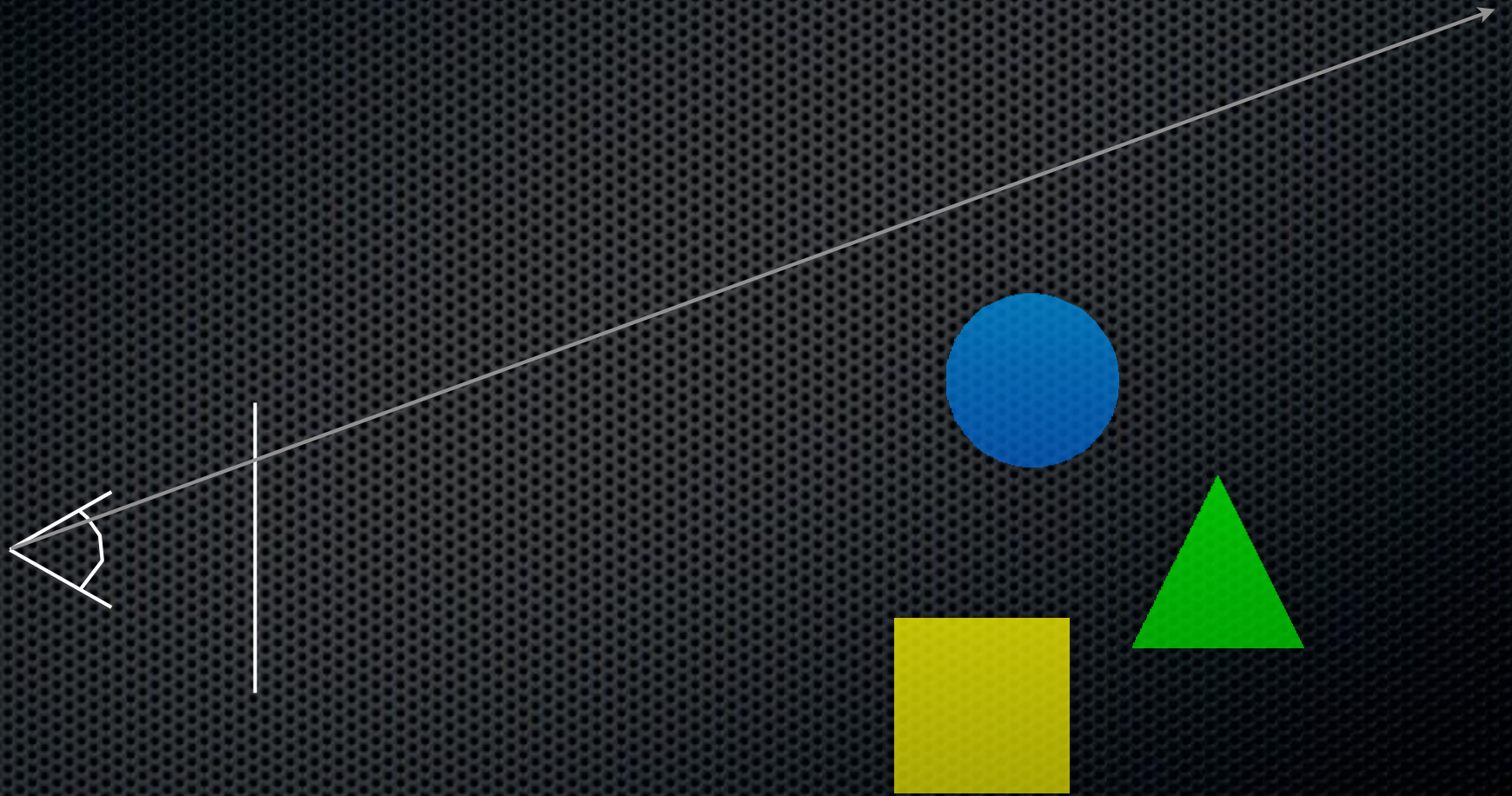
Ray Tracing



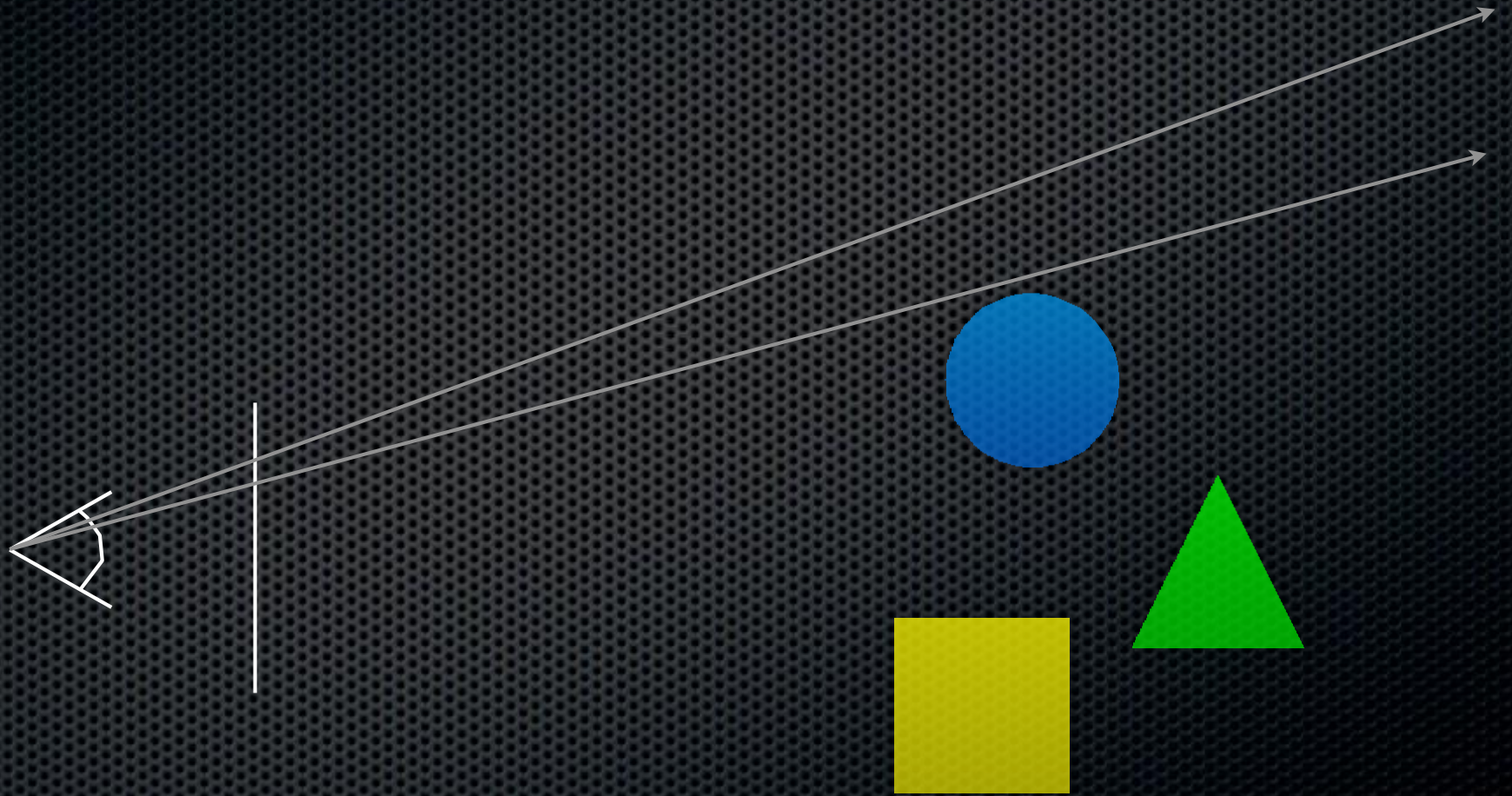
Ray Tracing



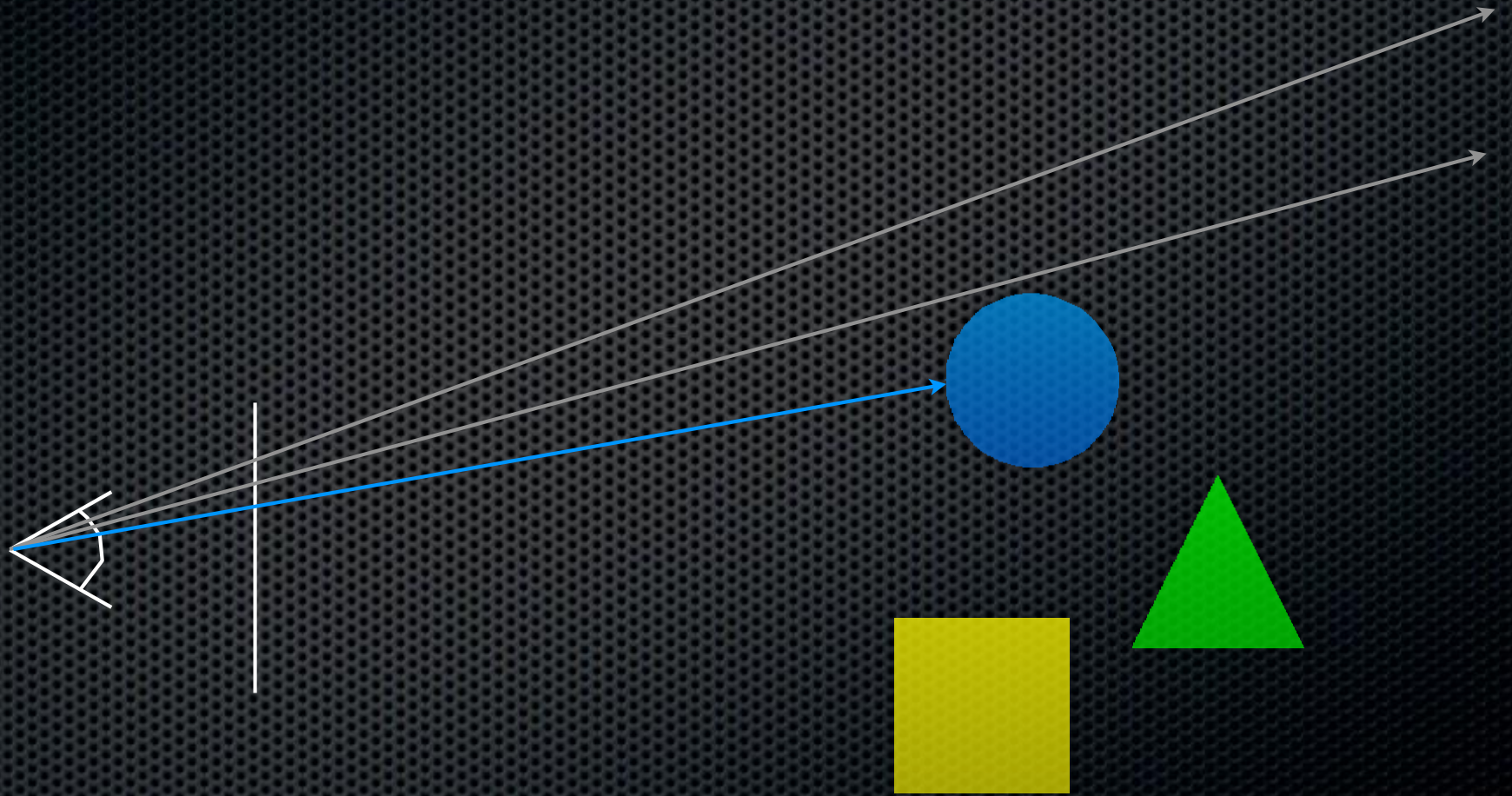
Ray Tracing



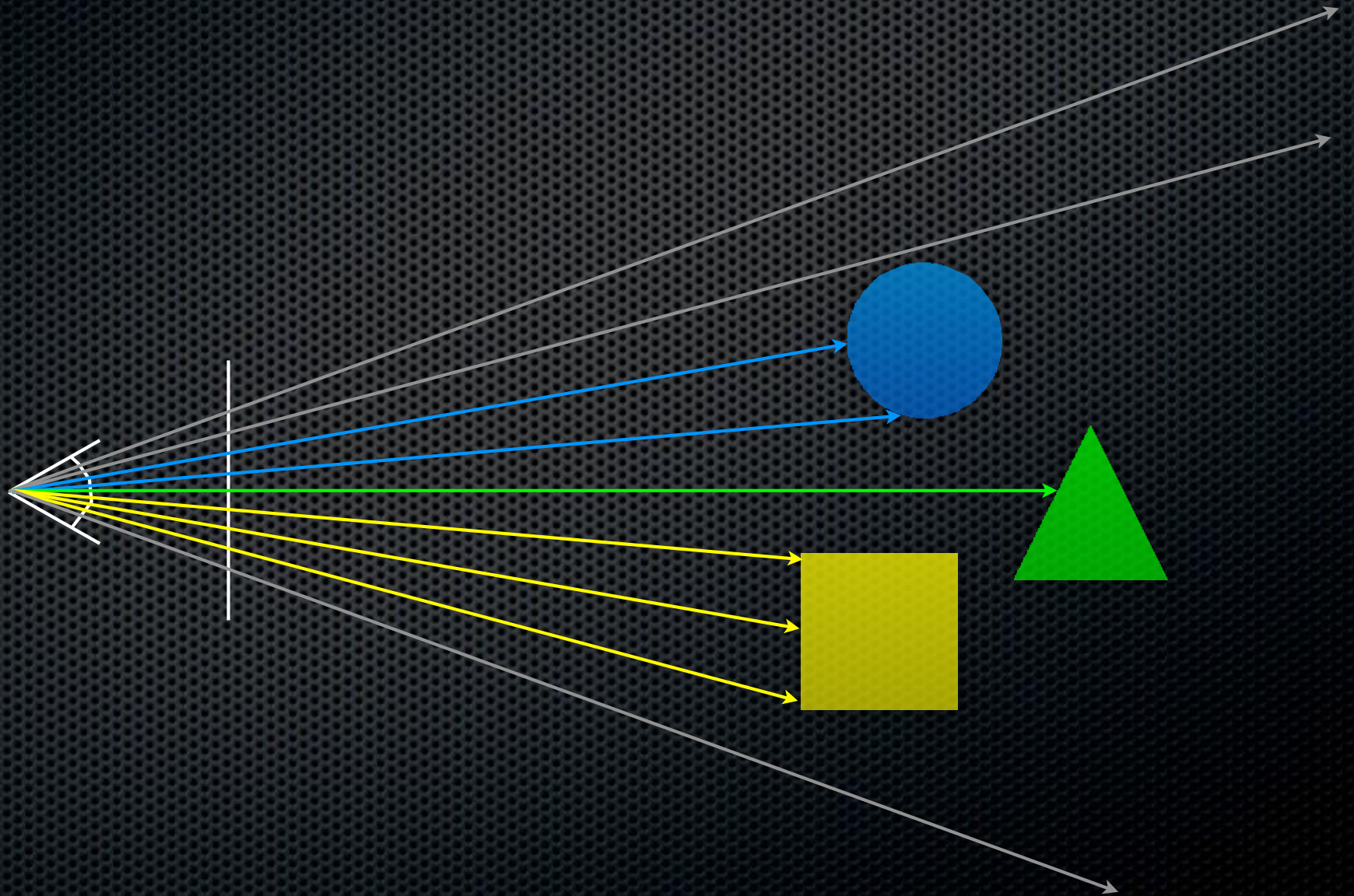
Ray Tracing



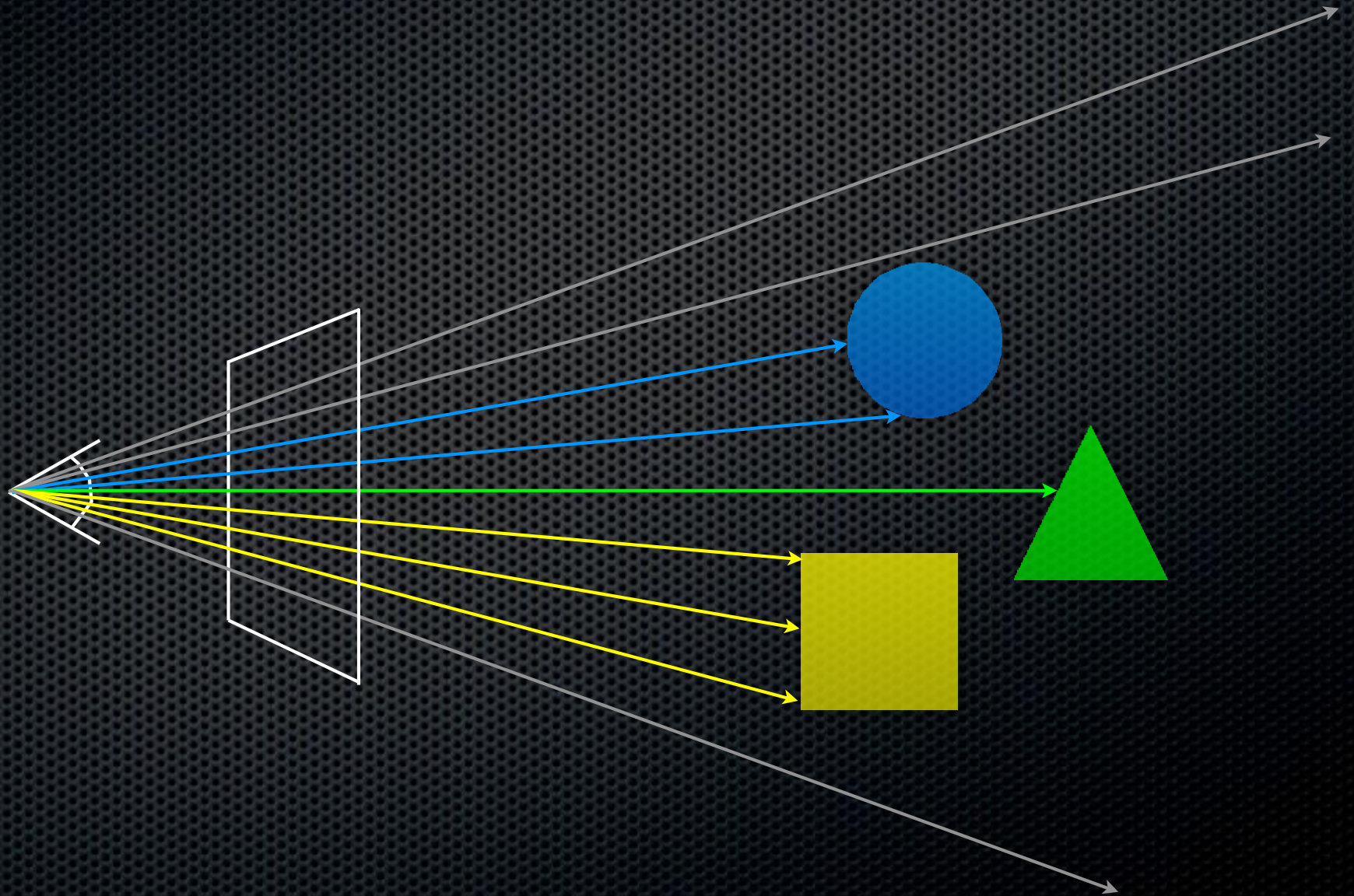
Ray Tracing



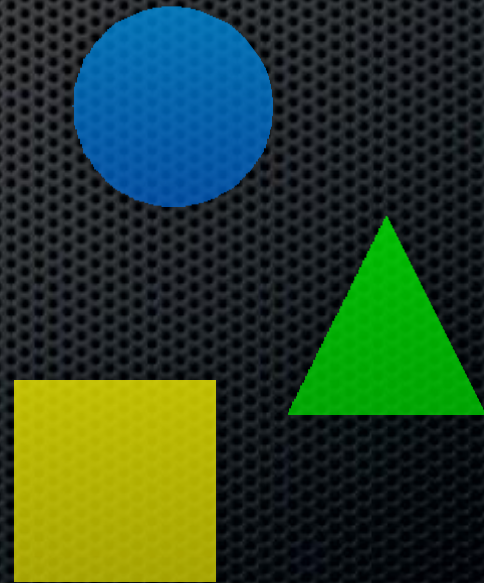
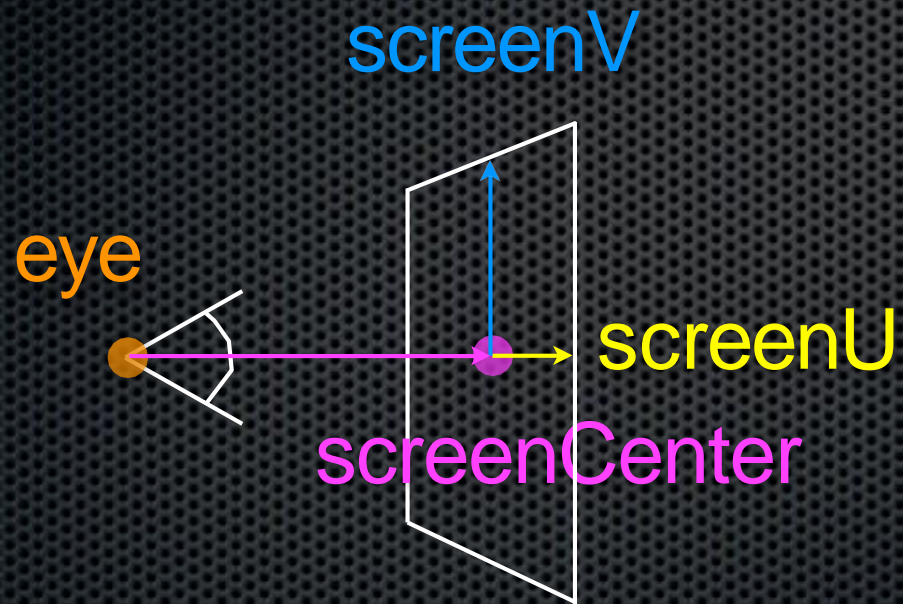
Ray Tracing



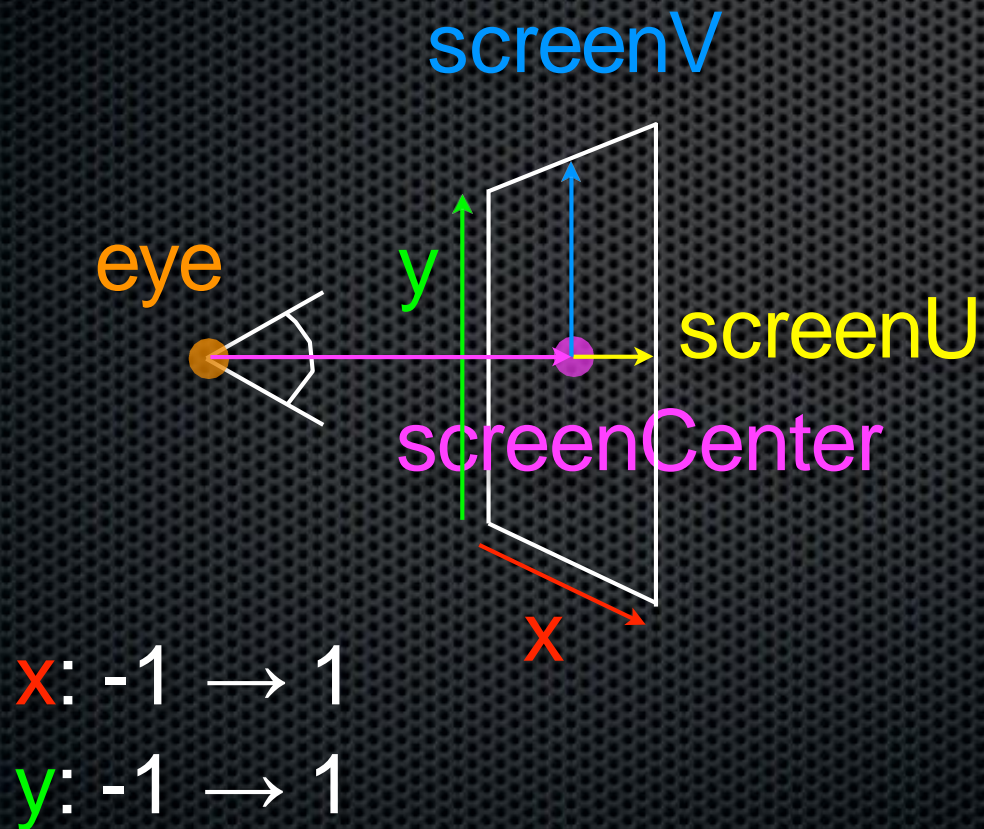
Ray Tracing



Ray Tracing



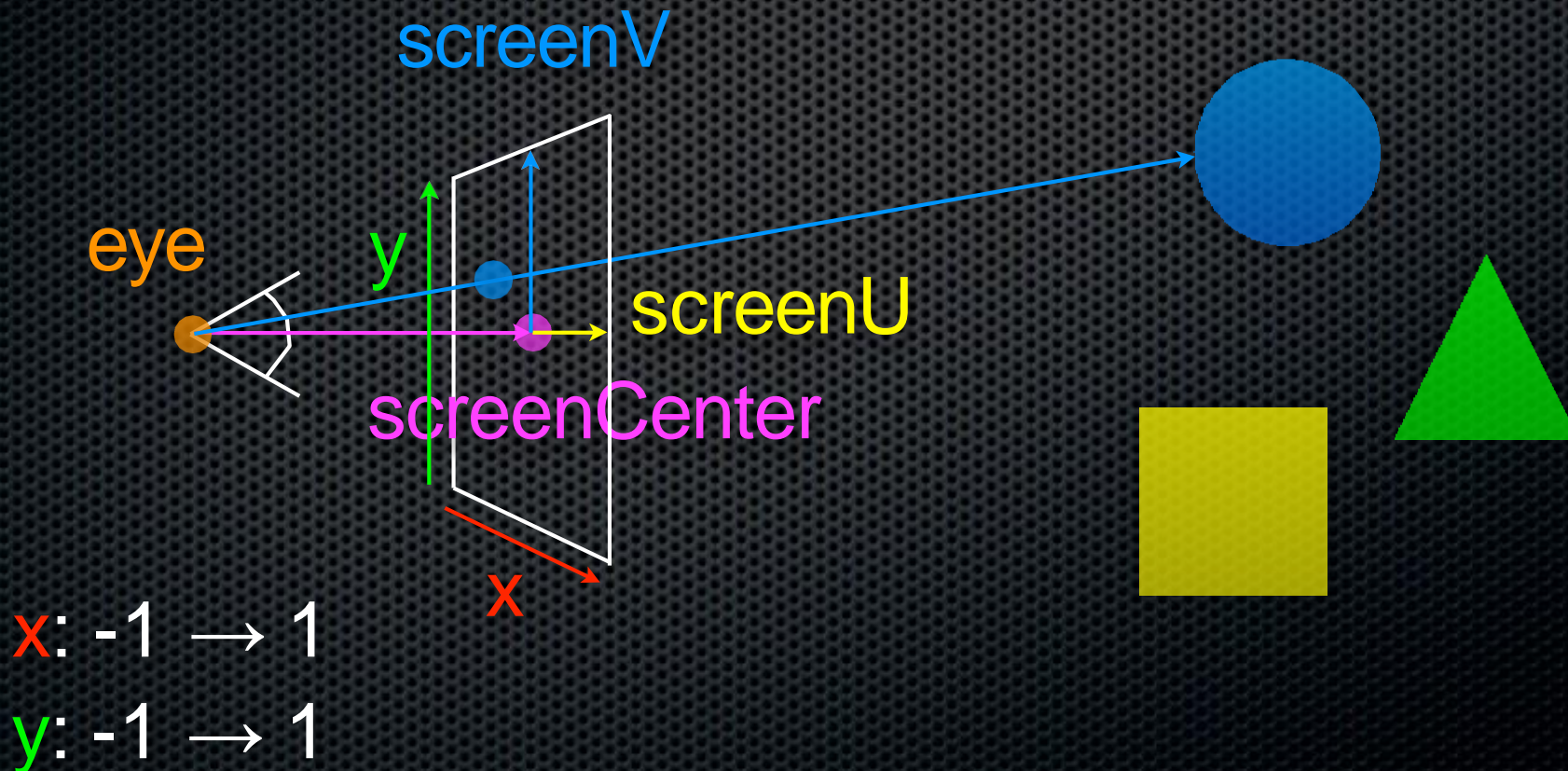
Ray Tracing



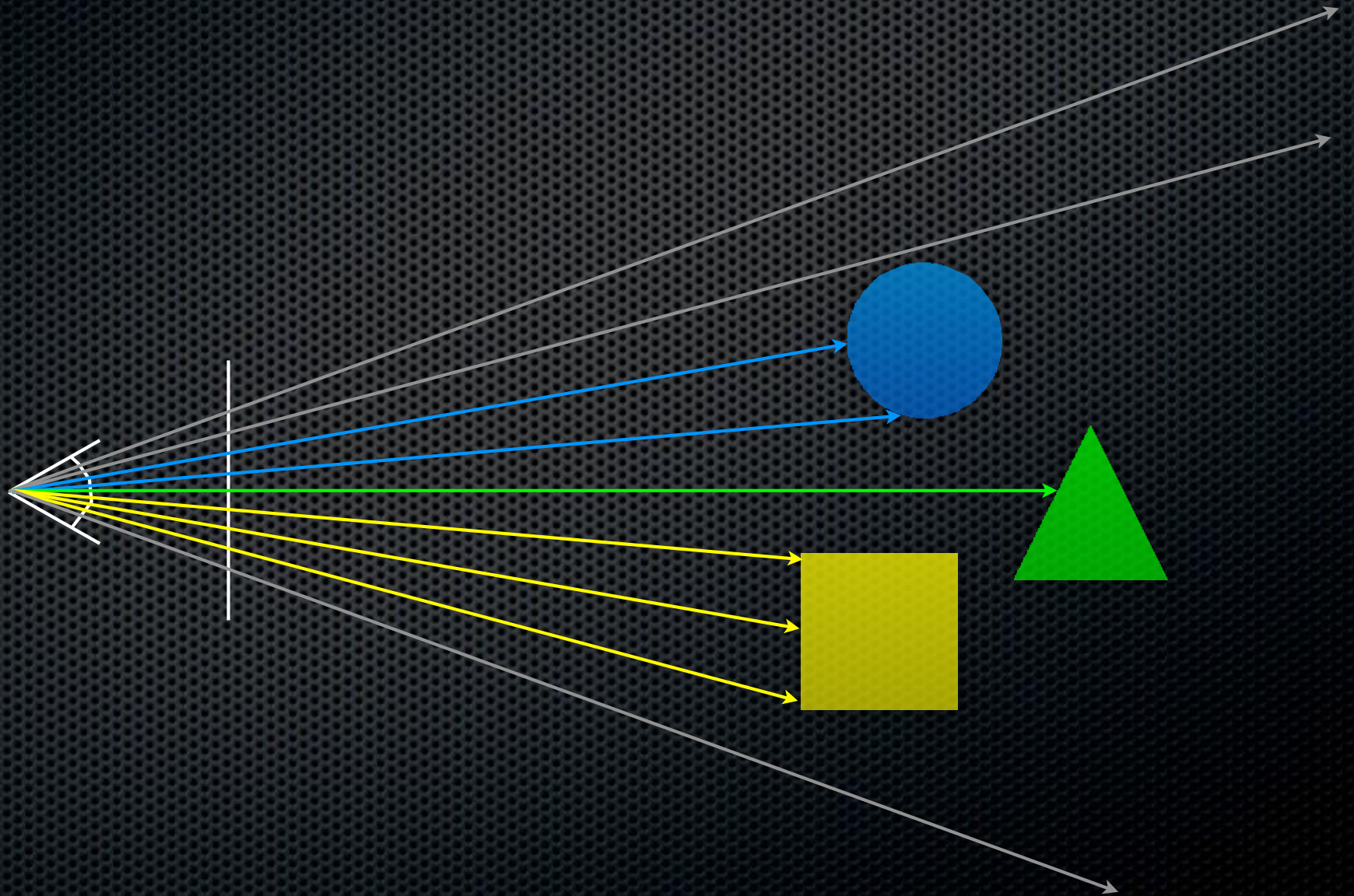
Ray Tracing

ray origin: **eye**

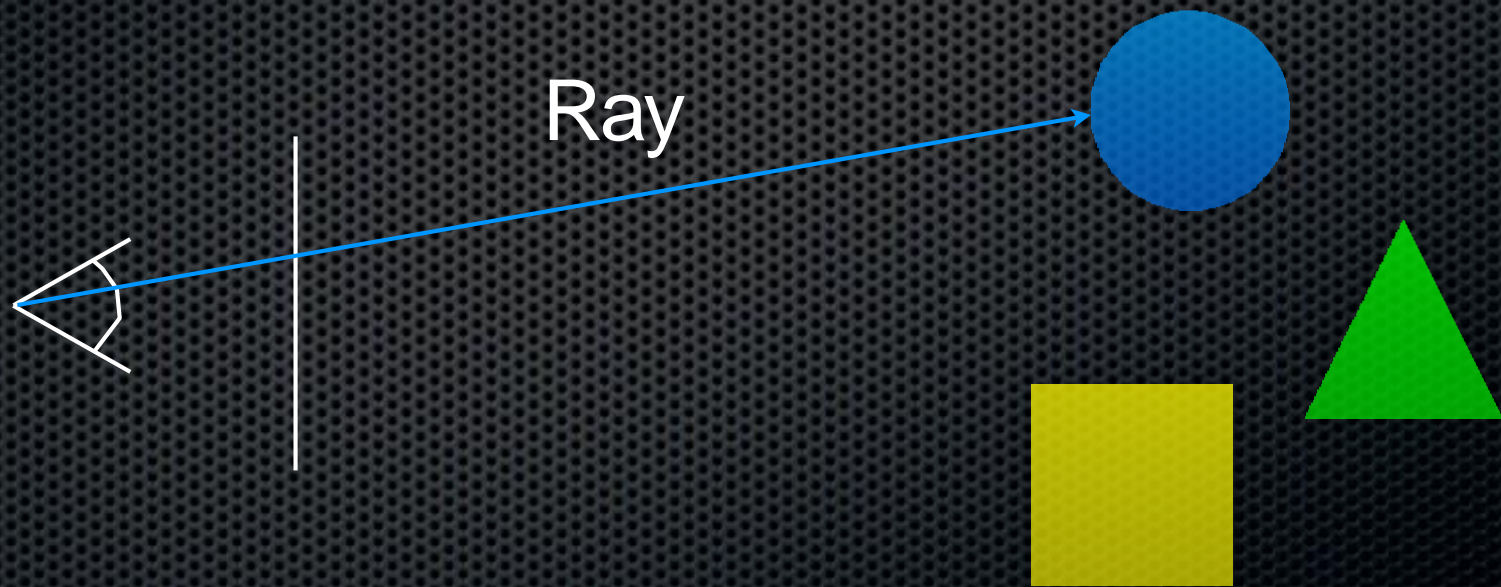
ray direction: **screenCenter** + **x**•**screenU** + **y**•**screenV**



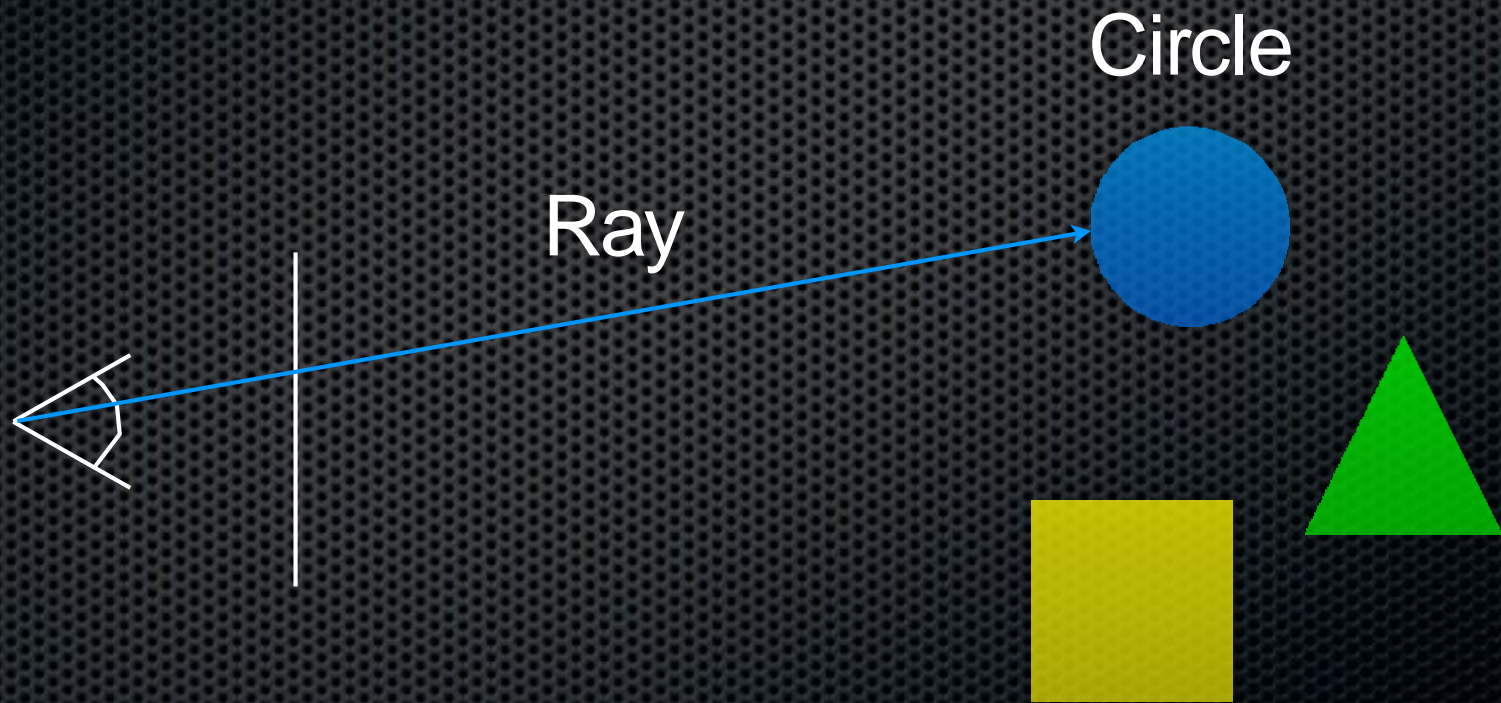
Ray Tracing



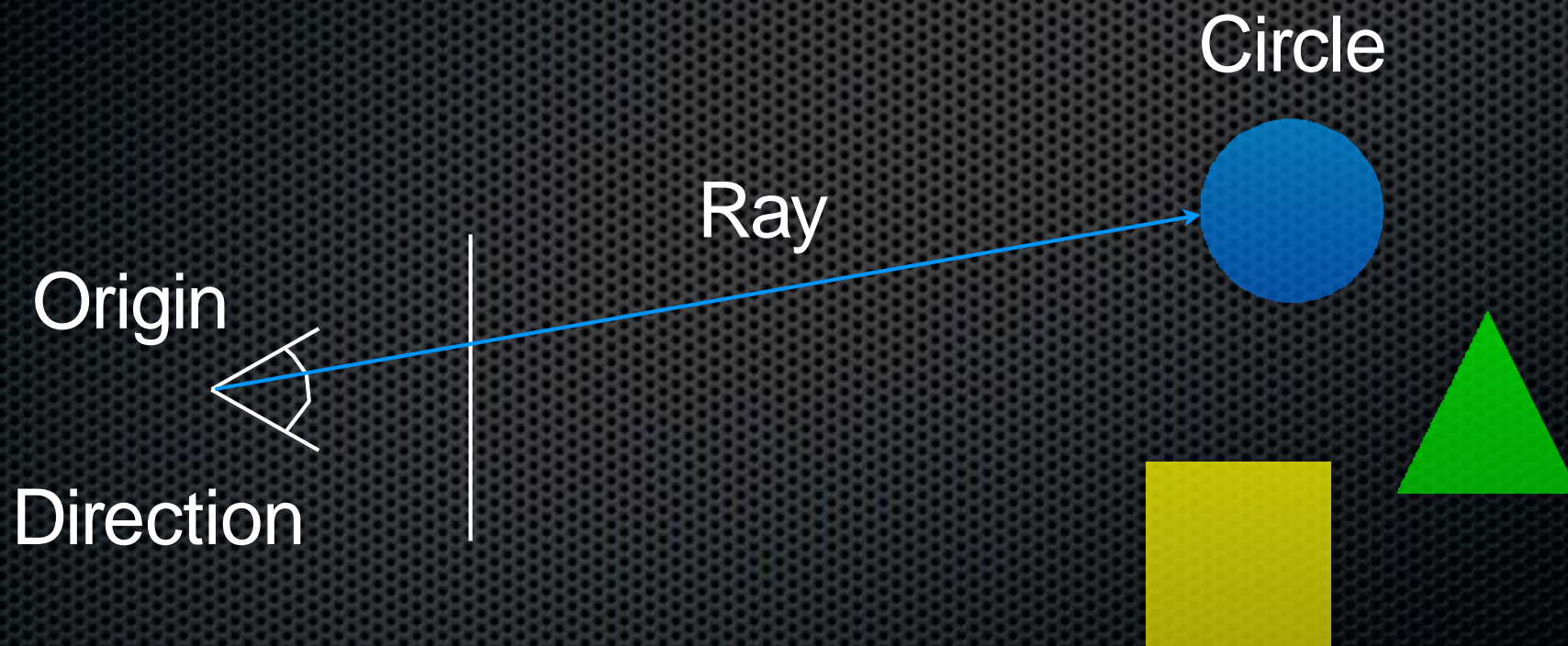
Ray Tracing



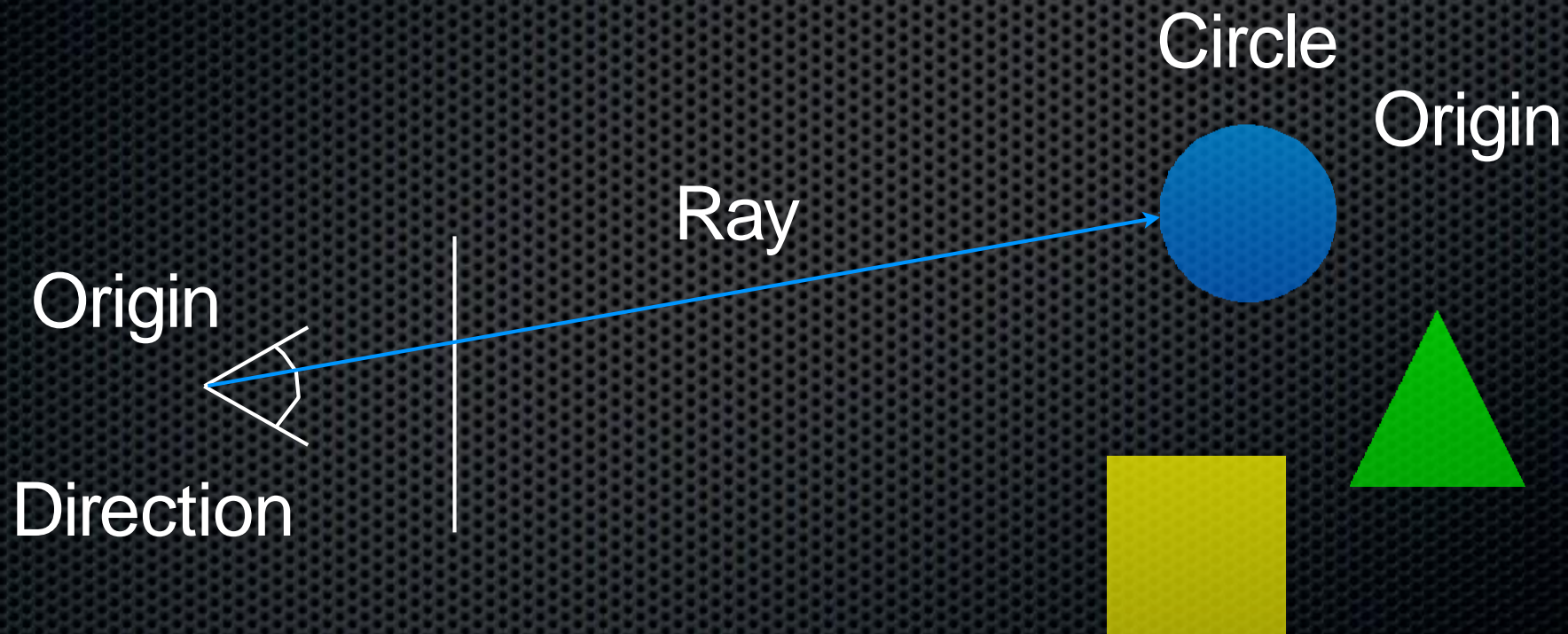
Ray Tracing



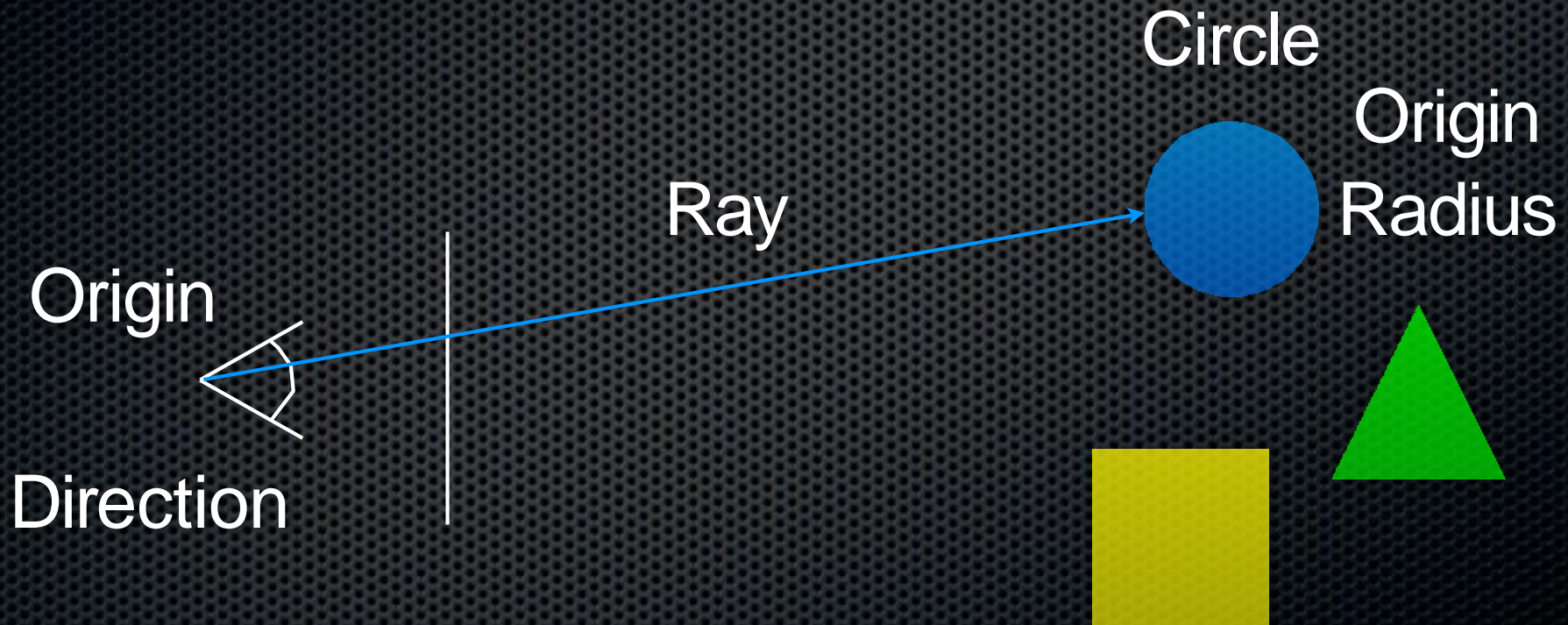
Ray Tracing



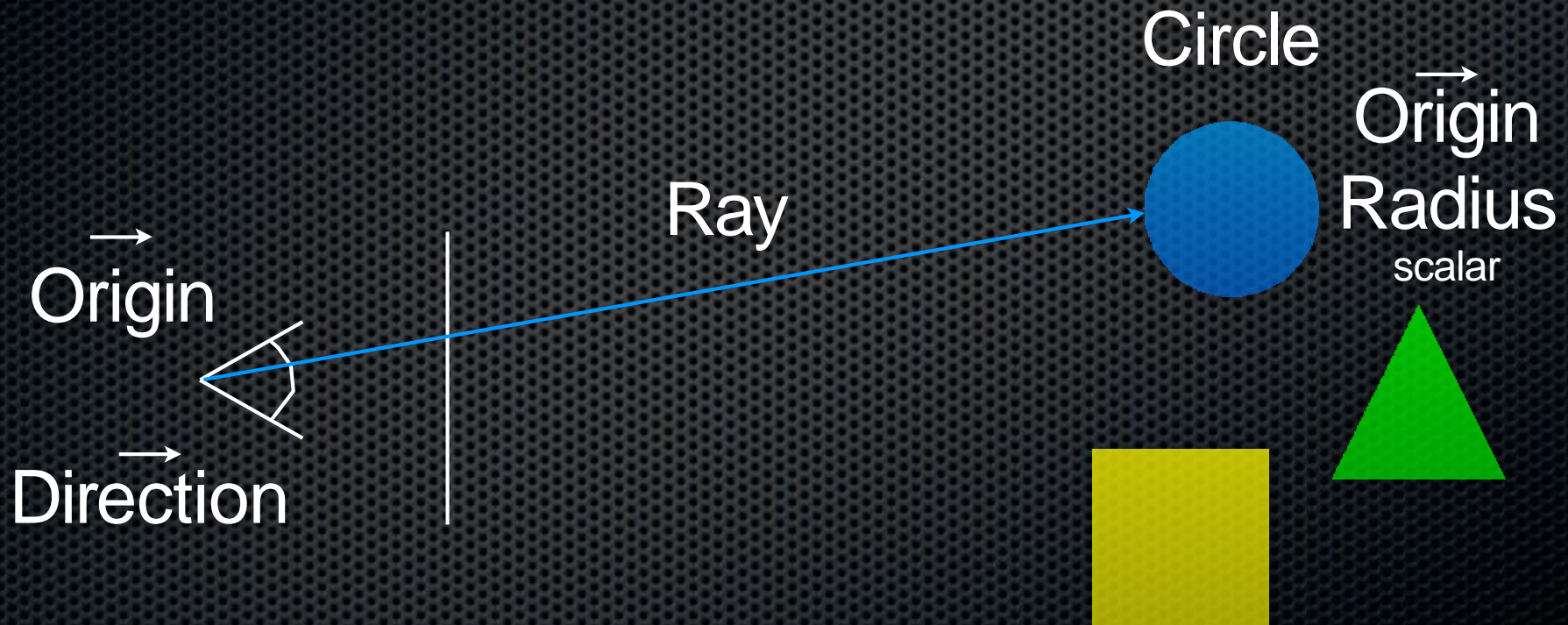
Ray Tracing



Ray Tracing

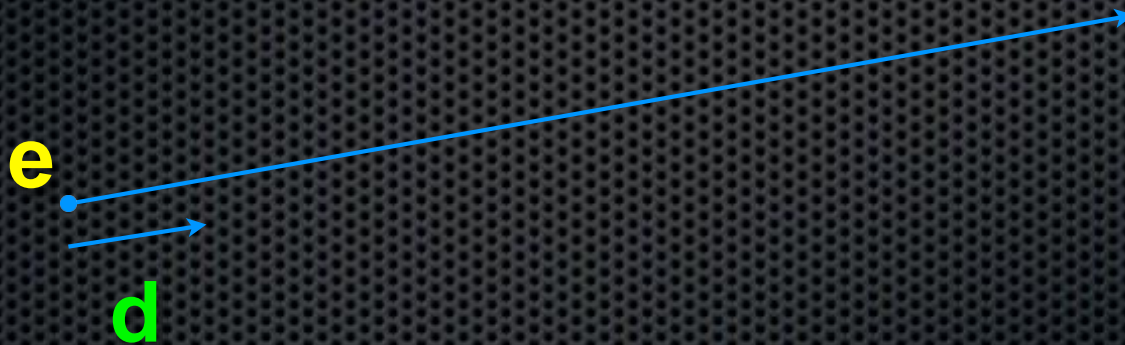


Ray Tracing



Ray Tracing

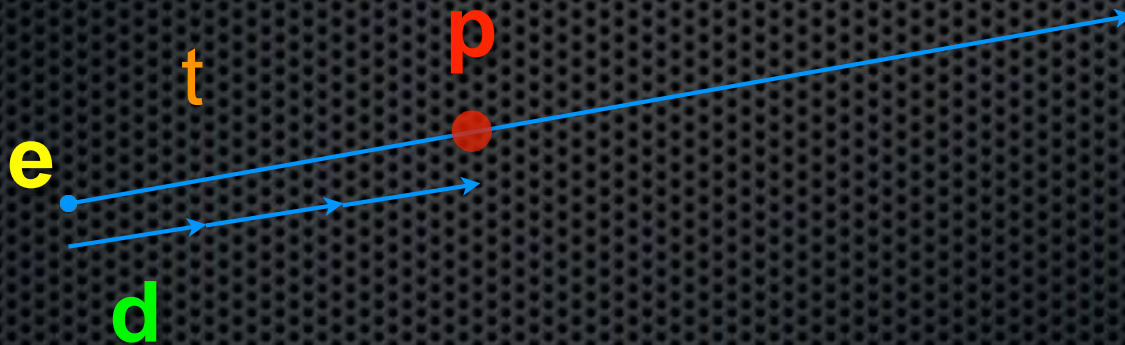
Ray: emission point (**e**) and direction (**d**)



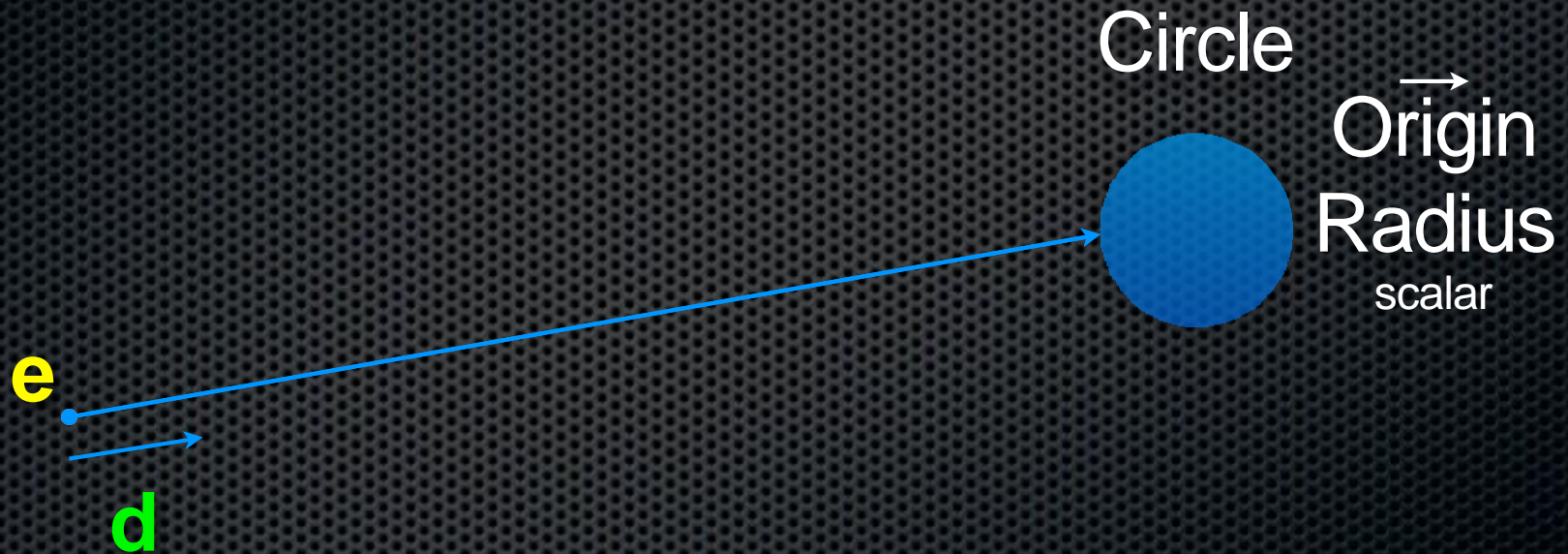
Ray Tracing

Ray: emission point (**e**) and direction (**d**)

Point on Ray: **p**(**t**) = **e** + **t****d**

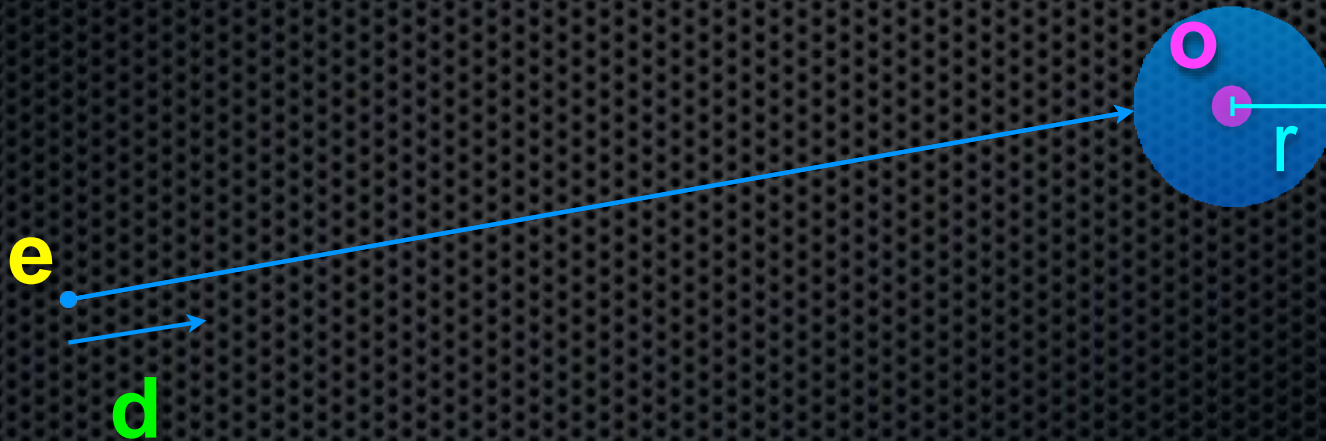


Ray Tracing



Ray Tracing

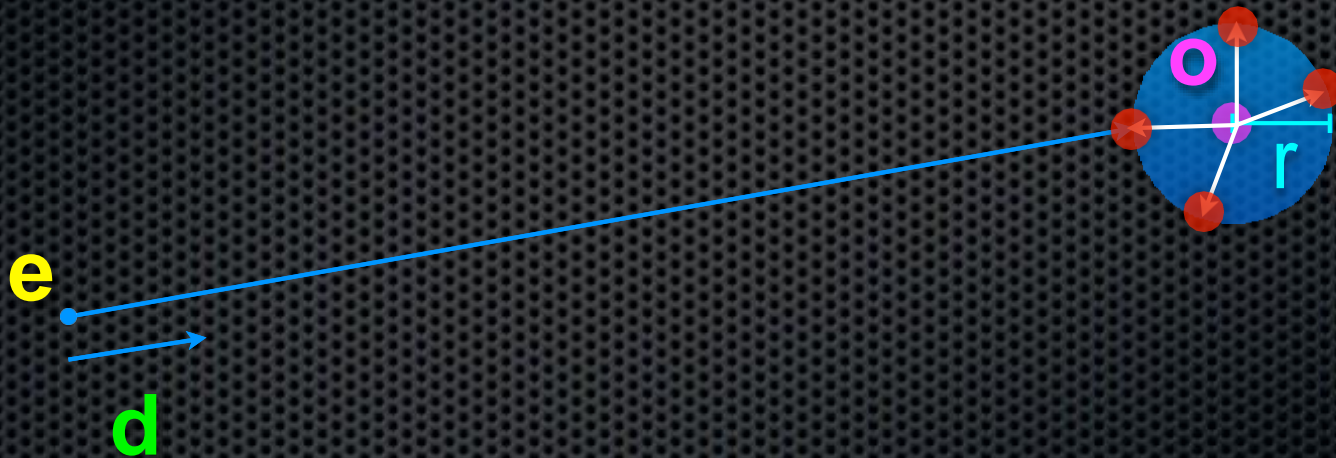
Circle: center point (o) and radius (r)



Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

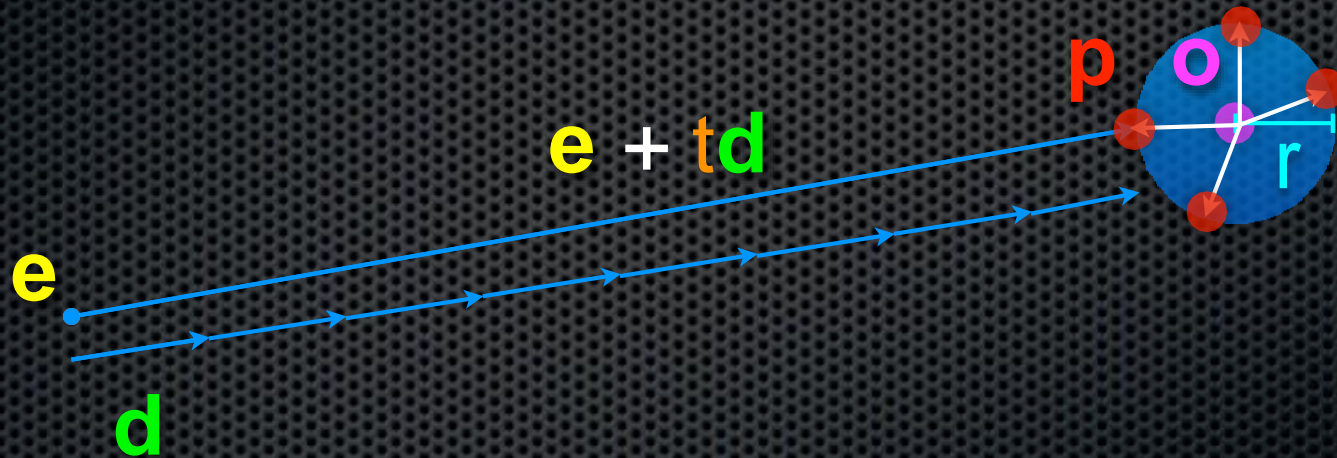


Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$

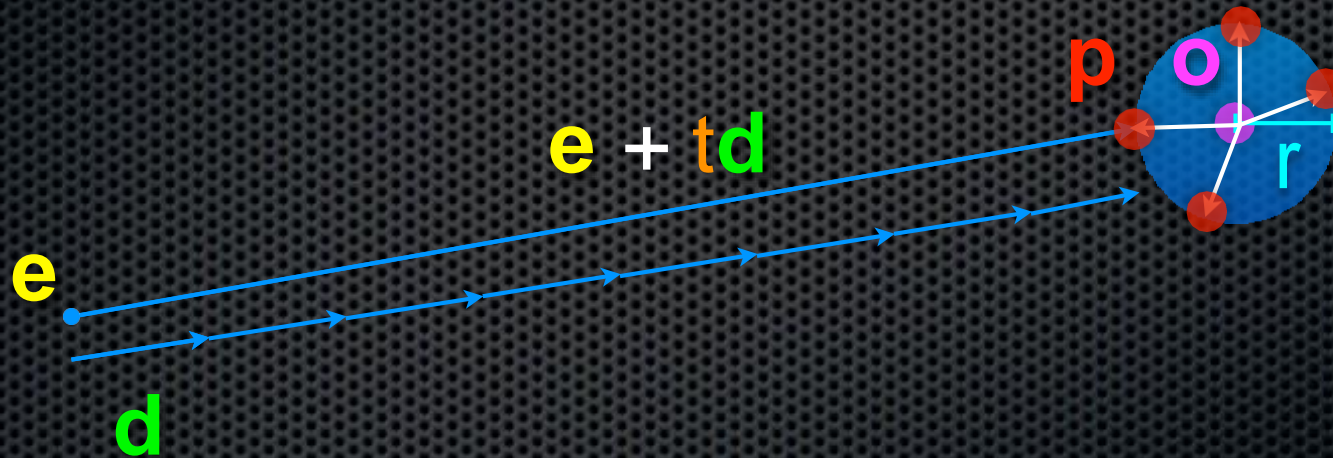


Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



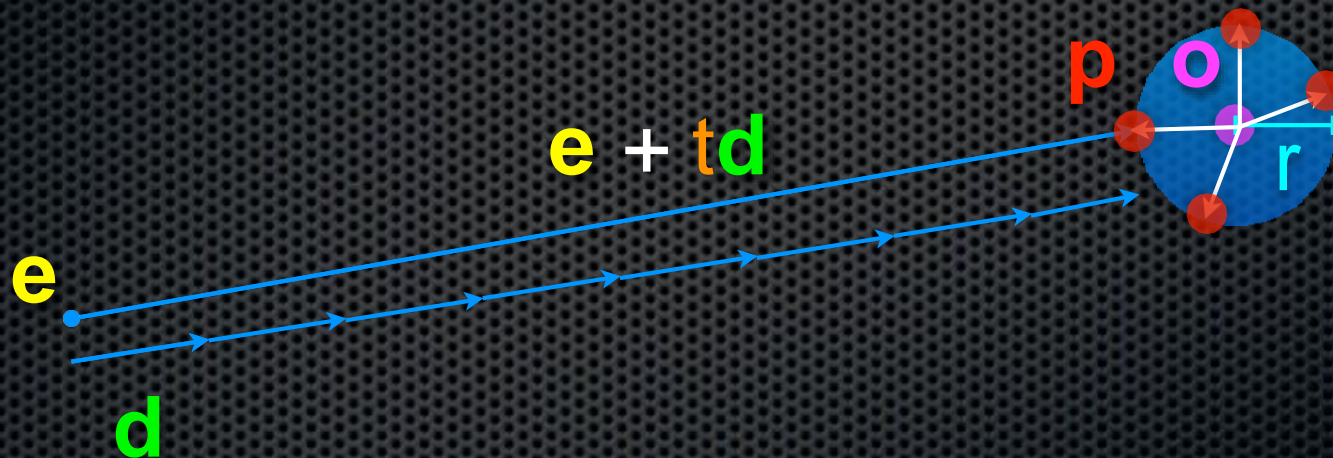
$$(\mathbf{e} + t\mathbf{d} - \mathbf{o}) \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{o}) - r^2 = 0$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



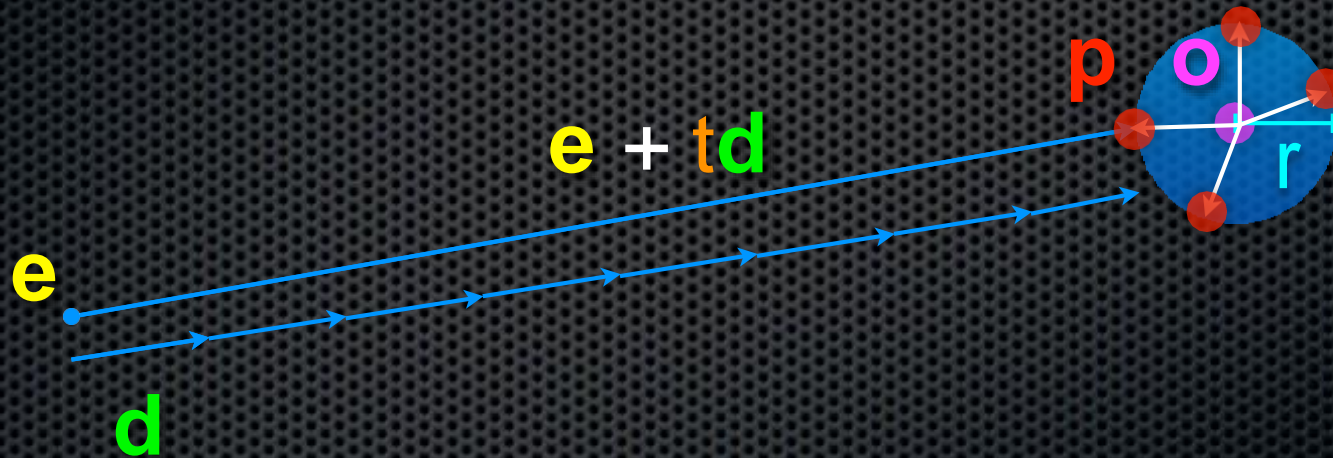
$$(t\mathbf{d} + \mathbf{e} - \mathbf{o}) \cdot (t\mathbf{d} + \mathbf{e} - \mathbf{o}) - r^2 = 0$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



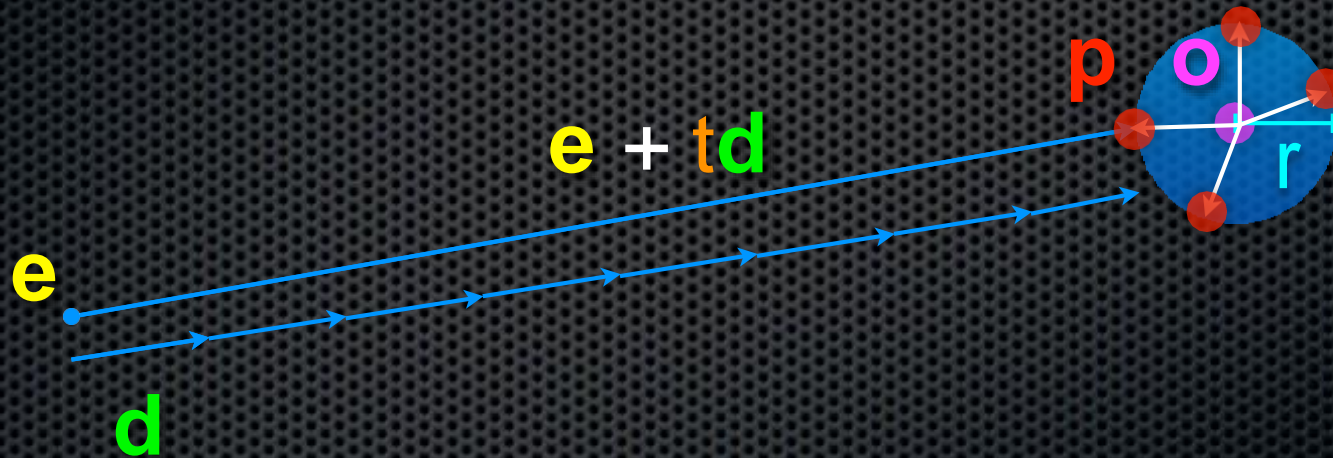
$$(t\mathbf{d} + (\mathbf{e} - \mathbf{o})) \cdot (t\mathbf{d} + (\mathbf{e} - \mathbf{o})) - r^2 = 0$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



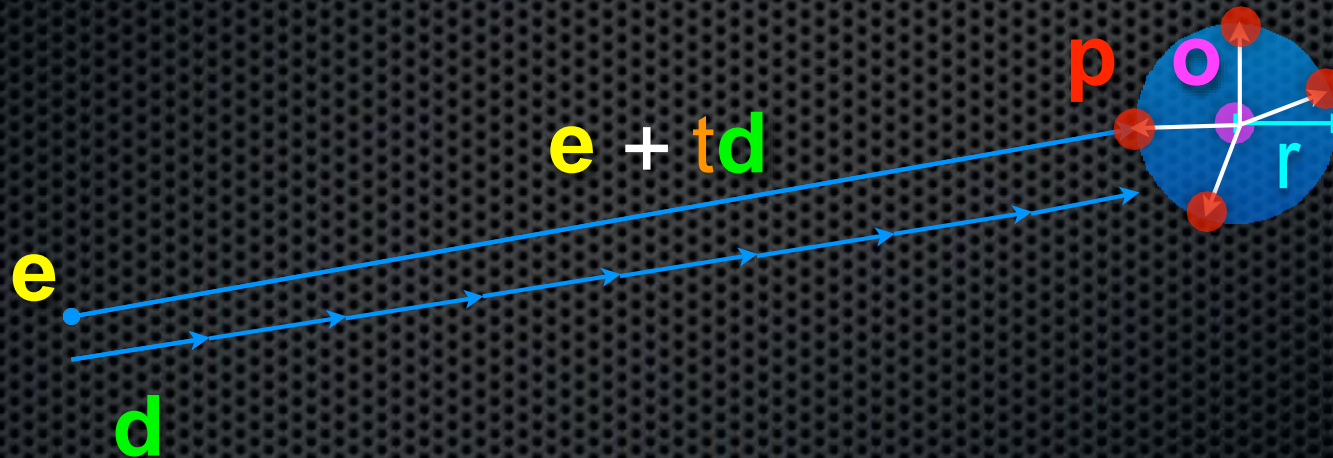
$$t^2(\mathbf{d} \cdot \mathbf{d}) + 2t\mathbf{d} \cdot (\mathbf{e} - \mathbf{o}) + (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2 = 0$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



$$at^2 + bt + c = 0$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{o})$$

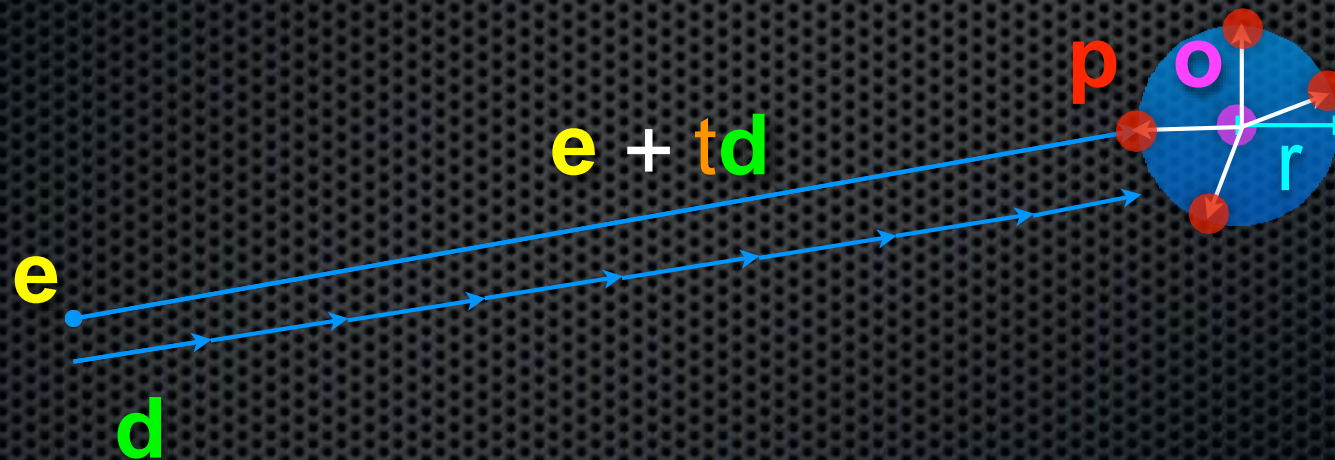
$$c = (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on circle: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{o})$$

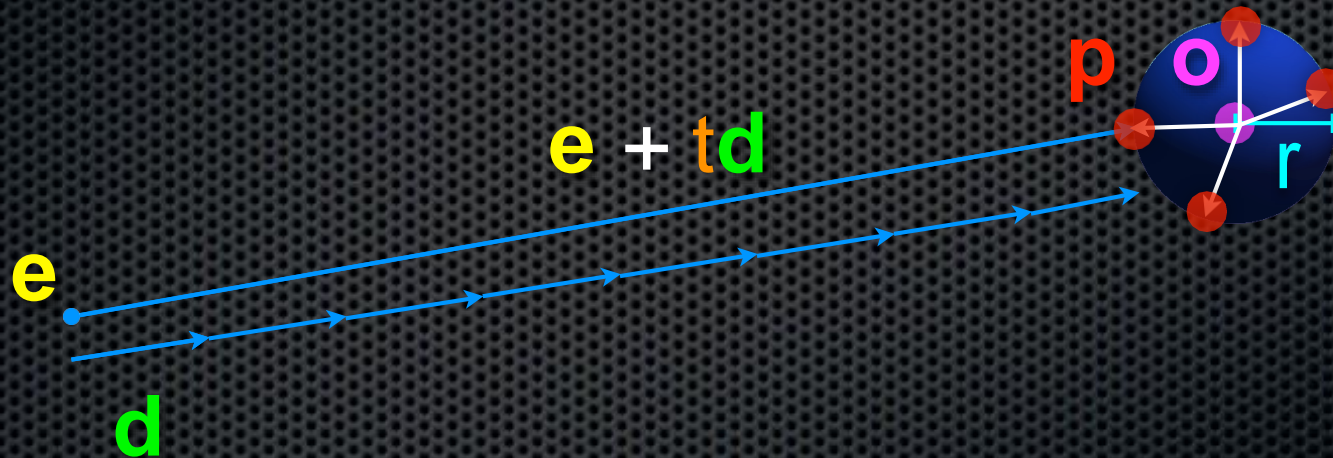
$$c = (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2$$

Ray Tracing

Circle: center point (\mathbf{o}) and radius (r)

Point on **sphere**: $(\mathbf{p} - \mathbf{o}) \cdot (\mathbf{p} - \mathbf{o}) - r^2 = 0$

Point on Ray: $\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}$



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{o})$$

$$c = (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2$$

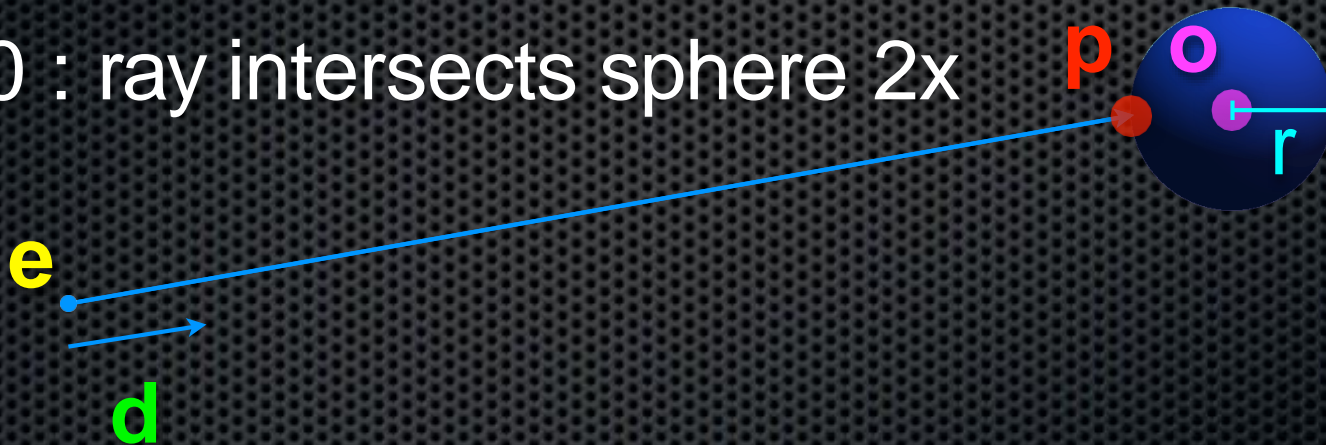
Ray Tracing

$$h = b^2 - 4ac$$

$h < 0$: ray misses sphere

$h = 0$: ray tangent to sphere

$h > 0$: ray intersects sphere 2x



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{o})$$

$$c = (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2$$

Ray Tracing

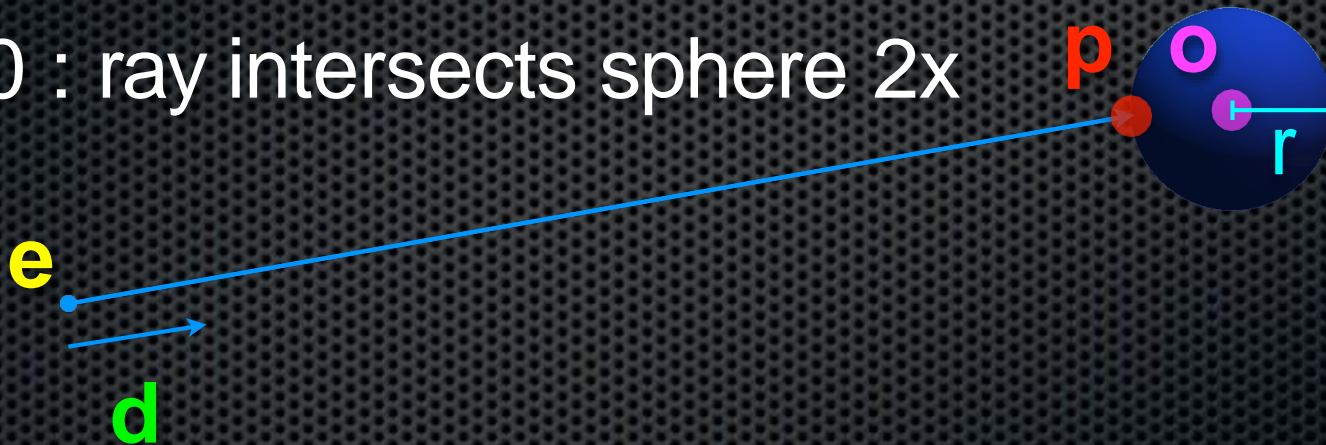
$$h = b^2 - 4ac$$

$h < 0$: ray misses sphere

$h = 0$: ray tangent to sphere

$h > 0$: ray intersects sphere 2x

**Which intersection
should be used?**



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \mathbf{d} \cdot \mathbf{d}$$

$$b = 2\mathbf{d} \cdot (\mathbf{e} - \mathbf{o})$$

$$c = (\mathbf{e} - \mathbf{o}) \cdot (\mathbf{e} - \mathbf{o}) - r^2$$