Computer Graphics (CS 4731)
Newell Method

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Slides Courtesy of Emmanuel Agu (WPI)

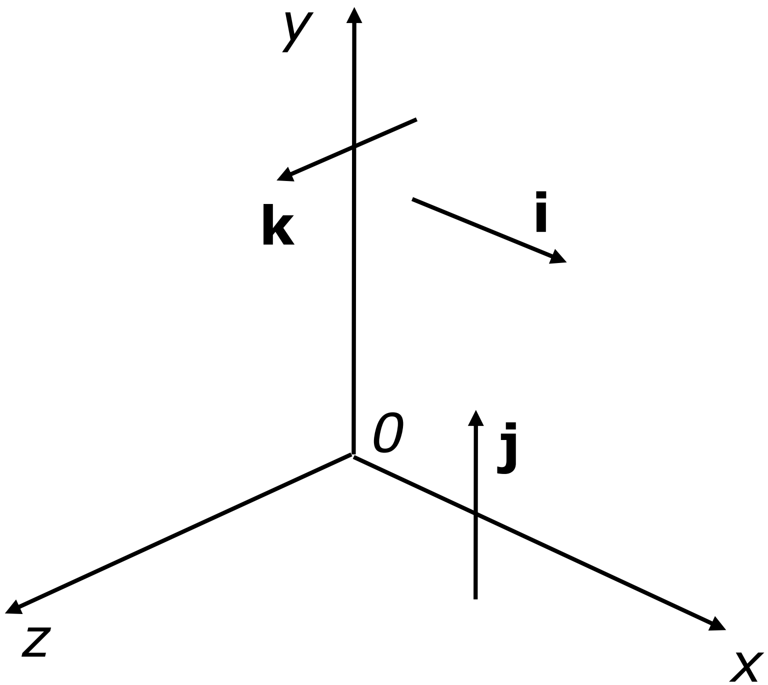
Standard Unit Vectors

Define

$$\mathbf{i} = (1,0,0)$$

$$\mathbf{j} = (0,1,0)$$

$$\mathbf{k} = (0,0,1)$$



So that any vector,

$$\mathbf{v} = (a,b,c) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

Cross Product (Vector product)

If

$$\mathbf{a} = \left(a_x, a_y, a_z\right) \qquad \mathbf{b} = \left(b_x, b_y, b_z\right)$$

Then

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} - (a_x b_z - a_z b_x)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}$$

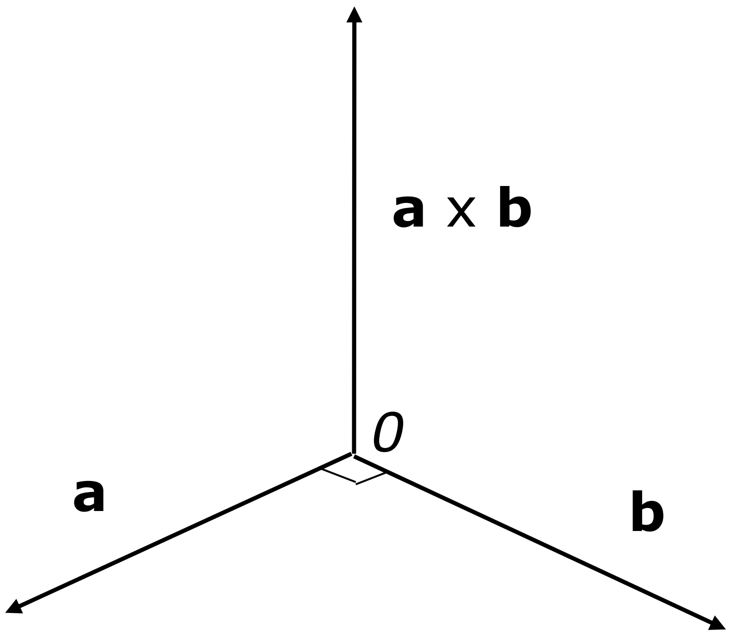
Remember using determinant

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Note: a x **b** is perpendicular to **a** and **b**

Cross Product

Note: a x **b** is perpendicular to both **a** and **b**



Cross Product (Vector product)

Calculate **a x b** if a = (3,0,2) and **b** = (4,1,8)

$$\mathbf{a} = (3,0,2) \qquad \mathbf{b} = (4,1,8)$$

Using determinant

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 4 & 1 & 8 \end{vmatrix}$$

Then

$$\mathbf{a} \times \mathbf{b} = (0-2)\mathbf{i} - (24-8)\mathbf{j} + (3-0)\mathbf{k}$$

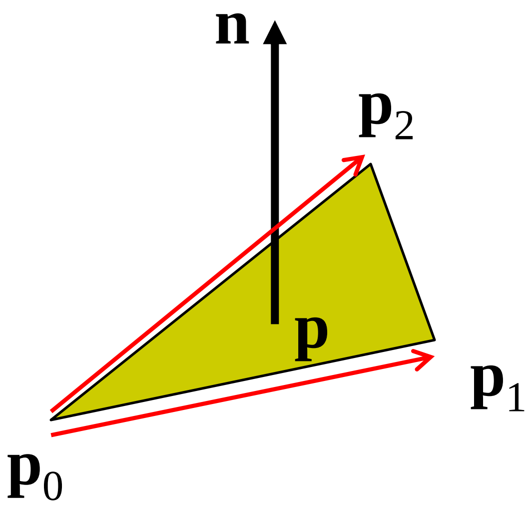
$$= -2\mathbf{i} - 16\mathbf{j} + 3\mathbf{k}$$

Normal for Triangle using Cross Product Method

plane $\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0$

$\mathbf{n} = (\mathbf{p}_2 - \mathbf{p}_0) \times (\mathbf{p}_1 - \mathbf{p}_0)$

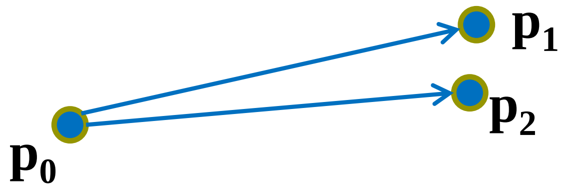
normalize $\mathbf{n} \leftarrow \mathbf{n} / |\mathbf{n}|$



Note that right-hand rule determines outward face

Newell Method for Normal Vectors

- Problems with cross product method:
  - calculation difficult by hand, tedious
  - If 2 vectors almost parallel, cross product is small
  - Numerical inaccuracy may result



- Proposed by Martin Newell at Utah (teapot guy)
  - Uses formulae, suitable for computer
  - Compute during mesh generation
  - Robust!

Newell Method Example

- Example: Find normal of polygon with vertices

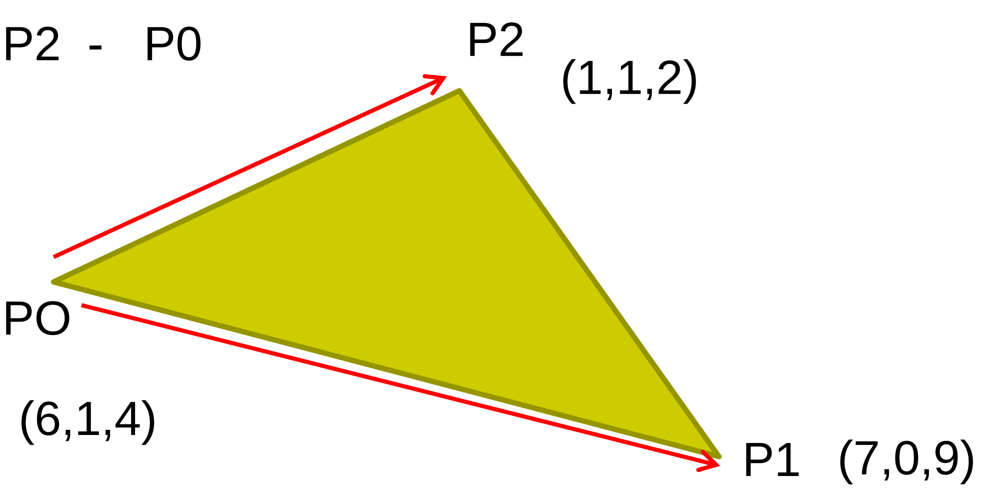
  P0 = (6,1,4), P1=(7,0,9) and P2 = (1,1,2)

- Using simple cross product:

  ((7,0,9)-(6,1,4)) X ((1,1,2)-(6,1,4)) = (2,-23,-5)

P1 - P0

P2 - P0



Newell Method for Normal Vectors

- Formulae: Normal N = (mx, my, mz)

$$m_x = \sum_{i=0}^{N-1} \left(y_i - y_{next(i)}\right)\left(z_i + z_{next(i)}\right)$$

$$m_y = \sum_{i=0}^{N-1} \left(z_i - z_{next(i)}\right)\left(x_i + x_{next(i)}\right)$$

$$m_z = \sum_{i=0}^{N-1} \left(x_i - x_{next(i)}\right)\left(y_i + y_{next(i)}\right)$$

Newell Method for Normal Vectors

- Calculate x component of normal

$$m_x = \sum_{i=0}^{N-1} \left(y_i - y_{next(i)}\right)\left(z_i + z_{next(i)}\right)$$

$$m_x = (1)(13) + (-1)(11) + (0)(6)$$

$$m_x = 13 - 11 + 0$$

$$m_x = 2$$

	x	y	z
P0	6	1	4
P1	7	0	9
P2	1	1	2
P0	6	1	4

Newell Method for Normal Vectors

- Calculate y component of normal

$$m_y = \sum_{i=0}^{N-1} \left(z_i - z_{next(i)}\right)\left(x_i + x_{next(i)}\right)$$

	x	y	z
P0	6	1	4
P1	7	0	9
P2	1	1	2
P0	6	1	4

$$m_y = (-5)(13) + (7)(8) + (-2)(7)$$

$$m_y = -65 + 56 - 14$$

$$m_y = -23$$

Newell Method for Normal Vectors

- Calculate z component of normal

$$m_z = \sum_{i=0}^{N-1} \left(x_i - x_{next(i)}\right)\left(y_i + y_{next(i)}\right)$$

$$m_z = (-1)(1) + (6)(1) + (-5)(2)$$

$$m_z = -1 + 6 - 10$$

$$m_z = -5$$

	x	y	z
P0	6	1	4
P1	7	0	9
P2	1	1	2
P0	6	1	4

Note: Using Newell method yields same result as Cross product method (2,-23,-5)