Fundamentals of exploration in GROOVE

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We globally assume a set of labels A.

A state space is a tuple $\langle S, \rightarrow, \uparrow, tdp \rangle$ with

- S a set of states;
- $\rightarrow \subseteq S \times A \times S$ a transition relation;
- $\uparrow \subseteq S$ a termination predicate;
- $tdp : S \rightarrow \mathbb{N}$ a *transient depth* (or *transience*) function.

State $s \in S$ is called *final* if $p\uparrow$, *transient* (denoted $trnt(s)$) if $tdp(s) > 0$ and *stable* (denoted $stable(s)$) if $tdp(s) = 0$.

State spaces are generated from a *pseudo-state spaces*.

A pseudo-state space is a tuple $\langle P, \mapsto, \rightarrowtail, \uparrow, tdp \rangle$ with

- P a finite set of pseudo-states;
- $\rightarrow \subseteq P \times A \times P$ a step relation;
- $\rightarrowtail : P \times P$ a partial, acyclic evolution relation;
- $\uparrow \subseteq P$ a termination predicate;
- $tdp : S \rightarrow \mathbb{N}$ a *transient depth* function.

Pseudo-state $p \in P$ is called *prime* (denoted $prime(p)$) if $p \not\leftarrowtail$, *closed* (denoted $closed(p)$) if $p \rightarrowtail$ and *open* (denoted $open(p)$) if it is not closed. A pseudo-state space is *well-formed* if it satisfies the following additional properties:

- Stepping is deterministic; i.e., $\rightarrow$ is a partial function from P to $A \times P$;
- Evolution is deterministic; i.e., $\rightarrowtail$ is a partial function from P to P;
- Evolution is injective; i.e., $\leftarrowtail$ is a partial function from P to P;
- All steps go from open to prime pseudo-states; i.e., $p \rightarrow q$ implies $open(p)$ and $prime(q)$;
- All final pseudo-states are stable and closed; i.e., $p\uparrow$ implies $stable(p)$ and $closed(p)$;
- Stepping cannot decrease transience; i.e., $p \rightarrow q$ implies $tdp(q) \geq tdp(p)$;

- Evolution cannot increase transience; i.e., $p \rightarrowtail q$ implies $tdp(q) \leq tdp(p)$.

From now on, we only deal with well-formed pseudo-states spaces. The *prime of* and *closure of* a pseudo-state p are defined as

$$\begin{aligned} \lhd p = q &\quad \text{where } prime(q) \text{ and } q \rightarrowtail^{*} p \\ p \rhd = q &\quad \text{where } closed(q) \text{ and } p \leftarrowtail^{*} q \ . \end{aligned}$$

Note that these are well-defined because P is finite and $\rightarrowtail$ is acyclic, deterministic and injective.

A pseudo-state space $\langle P, \mapsto, \rightarrowtail, \uparrow, tdp \rangle$ gives rise to a state space $\langle S, \rightarrow, \uparrow, tdp_S \rangle$ where

- $s \in S$ if s is a closed pseudo-state; i.e., $S = P \rhd$;
- $p \rhd \xrightarrow{a} p' \rhd$ for all $p \xrightarrow{a} p'$; i.e., $s \xrightarrow{a} s'$ if $s \leftarrowtail^{*} \xrightarrow{a} \rightarrowtail^{*} s'$;
- $\uparrow$ remains unchanged (noting that $p\uparrow$ implies $closed(p)$ and hence $p \in S$);
- tdp_S restricts tdp to S; i.e., $tdp_S = \{(p, tdp(p)) \mid p \in S\}$.

Given a pseudo-state space P, a *configuration* is a partial function $C : P \rhd \rightarrow P$. C gives rise to a state space $\langle S_C, \rightarrow_C, \uparrow_C, tdp_C \rangle$ where

- $S_C = \operatorname{dom} C$;
- For all $s, s' \in S_C$, $s \xrightarrow{a} s'$ if $C(s) \leftarrowtail^{*} \xrightarrow{a} \rightarrowtail^{*} C(s')$;
- For all $s \in S_C$, $s\uparrow_C$ if $C(s)\uparrow$;
- For all $s \in S_C$, $tdp_C(s) = tdp(C(s))$.