

$$T(n) = n^{1/2} + 10^{20}$$

Tengo que llegar a:

$$n^{1/2} + 10^{20} \leq cn^{1/2} \text{ para todo } n \geq n_0$$

$$\begin{array}{ll} n^{1/2} \leq c_1 n^{1/2} & 10^{20} \leq c_2 n^{1/2} \\ n^{1/2} \leq n^{1/2} & 1 \leq n^{1/2} \\ 1 * n^{1/2} \leq 1 * n^{1/2} & 10^{20} * 1 \leq 10^{20} * n^{1/2} \\ c_1 = 1 \quad n_0 = 0 & c_2 = 10^{20} \quad n_0 = 1 \end{array}$$

$$\begin{array}{l} 1 * n^{1/2} + 10^{20} * 1 \leq 1 * n^{1/2} + 10^{20} * n^{1/2} \\ n^{1/2} + 10^{20} \leq (1 + 10^{20}) n^{1/2} \end{array}$$

$$\begin{array}{l} c = c_1 + c_2 = 1 + 10^{20} \\ n_0 = 1 \end{array}$$

Claramente, se cumple que $T(n)$ es $O(n^{1/2})$, porque existen las constantes $c = 1 + 10^{20}$ y $n_0 = 1$, de modo que se cumple que $T(n) \leq cn^{1/2}$ para todo $n \geq 1$.

$$T(n) = n + \log_2 n$$

Hay que llegar a:

$$n + \log_2 n \leq cn \text{ para todo } n \geq n_0$$

$$\begin{array}{ll} n \leq c_1 n & \log_2 n \leq c_2 n \\ 1 * n \leq 1 * n & 1 * \log_2 n \leq 1 * n \\ c_1 = 1 \quad n_0 = 0 & c_2 = 1 \quad n_0 = 1 \end{array}$$

$$\begin{array}{l} n + \log_2 n \leq 1 * n + 1 * n \\ n + \log_2 n \leq (1 + 1) * n \\ n + \log_2 n \leq 2 * n \end{array}$$

$$\begin{array}{l} c = c_1 + c_2 = 1 + 1 = 2 \\ n_0 = 1 \end{array}$$

Claramente, se cumple que $T(n)$ es $O(n)$, porque existen las constantes $c = 2$ y $n_0 = 1$, de modo que se cumple que $T(n) \leq cn$ para todo $n \geq 1$.

$$T(n) = 6 + n * \log_2 n + \log_2 n - 1 \quad \text{es } O(n * \log_2 n)??$$

$$\begin{array}{llll} 6 \leq c_1 * (n * \log_2 n) & n * \log_2 n \leq c_2 * (n * \log_2 n) & \log_2 n \leq c_3 * (n * \log_2 n) & -1 \leq c_4 * (n * \log_2 n) \\ c_1 = 6 \quad n_0 = 2 & c_2 = 1 \quad n_0 = 1 & c_3 = 1 \quad n_0 = 1 & c_4 = 0 \quad n_0 = 1 \end{array}$$

$$\begin{array}{l} 6 + n * \log_2 n + \log_2 n - 1 \leq 6 * (n * \log_2 n) + 1 * (n * \log_2 n) + 1 * (n * \log_2 n) + 0 * (n * \log_2 n) \\ 6 + n * \log_2 n + \log_2 n - 1 \leq (6 + 1 + 1 + 0) (n * \log_2 n) \end{array}$$

$$6+n\log_2 n + \log_2 n - 1 \leq 8(n\log_2 n)$$

$$c=8$$

$$n_0=2$$

Claramente, se cumple que $T(n)$ es $O(n\log_2 n)$, porque existen las constantes $c=8$ y $n_0=2$, de modo que se cumple que $T(n) \leq c(n\log_2 n)$ para todo $n \geq 2$.

3^n es $O(2^n)$??

$$3^n \leq c2^n$$

$$3^n/2^n \leq c$$

$$(3/2)^n \leq c$$

Como $(3/2)^n$ es una función creciente, no se puede hallar un valor para c constante ni tampoco para un n_0 , de modo que se cumpla de definición de big-oh, entonces 3^n no es de orden 2^n .

$$2^{n+1} \leq c2^n$$

$$2 \cdot 2^n \leq c2^n$$

$$c=2 \quad n_0=1$$

$$2^{2n} \leq c2^n$$

$$2^{n+n} \leq c2^n$$

$$2^n \cdot 2^n \leq c2^n$$

$$(2^n \cdot 2^n)/2^n \leq c$$

$$2^n \leq c$$

Ejercicio 7.3)

`int x=1;`

`for (int i = 1; i < n; i = i+4)`

`for (int j = 1; j < n; j = j+|n/4|)`

`for (int k = 1; k < n; k = k*2)`

`x = x+1;`

c_1

sum de $i=1$ hasta $(n-1)/4$

sum de $j=1$ hasta 4

sum de $k=1$ hasta $\log_2 n$

c_2

Para i :

paso 0: $i=1=1+0 \cdot 4$

paso 1: $i=1+4=1+1 \cdot 4$

paso 2: $i=1+4+4=1+2 \cdot 4$

paso 3: $i=1+4+4+4=1+3 \cdot 4$

paso k : $i=1+4 \cdot k$

$$1+4k=n$$

$$4k=n-1$$

$$k=(n-1)/4$$

Para j:

paso 0: $j=1$

paso 1: $j=1+n/4$

paso 2: $j=1+2*n/4$

paso 3: $j=1+3*n/4$

paso 4: $j=1+4*n/4=1+n$

Para k:

paso 0: $k=1=2^0$

paso 1: $k=1*2=2^1$

paso 2: $k=1*2*2=2^2$

paso 3: $k=1*2*2*2=2^3$

paso p: $k=2^p$

$2^p=n$

$\log_2 2^p = \log_2 n$

$p = \log_2 n$

$$T(n) = c_1 + \sum_{i=1}^{(n-1)/4} \sum_{j=1}^4 \sum_{k=1}^{\log(n)} c_2$$

$$T(n) = c_1 + c_2 \sum_{i=1}^{(n-1)/4} \sum_{j=1}^4 \sum_{k=1}^{\log(n)} 1$$

$$T(n) = c_1 + c_2 \sum_{i=1}^{(n-1)/4} \sum_{j=1}^4 \log_2 n$$

$$T(n) = c_1 + c_2 * \log_2 n \sum_{i=1}^{(n-1)/4} \sum_{j=1}^4 1$$

$$T(n) = c_1 + c_2 * \log_2 n \sum_{i=1}^{(n-1)/4} 4$$

$$T(n) = c_1 + 4 * c_2 * \log_2 n \sum_{i=1}^{(n-1)/4} 1$$

$$T(n) = c_1 + 4 * c_2 * \log_2 n * (n-1)/4$$

$$T(n) = c_1 + (4 * c_2 * \log_2 n) * (n/4 - 1/4)$$

$$T(n) = c_1 + n/4 * (4 * c_2 * \log_2 n) - 1/4 * (4 * c_2 * \log_2 n)$$

$$T(n) = c_1 + c_2 * n * \log_2 n - c_2 * \log_2 n$$

$$c_1 \leq k_1 * (n * \log_2 n)$$

$$c_2 * (n * \log_2 n) \leq k_2 * (n * \log_2 n)$$

$$1 * c_1 \leq c_1 * (n * \log_2 n)$$

$$c_2 * (n * \log_2 n) \leq c_2 * (n * \log_2 n)$$

$$k_1 = c_1 \quad n_0 = 2$$

$$k_2 = c_2 \quad n_0 = 1$$

$$c_1 + c_2 * (n * \log_2 n) \leq c_1 * (n * \log_2 n) + c_2 * (n * \log_2 n)$$

$$c_1 + c_2 * (n * \log_2 n) \leq (c_1 + c_2) * (n * \log_2 n)$$

$$k = k_1 + k_2 = c_1 + c_2 \quad n_0 = 2$$

Ejercicio 7.4)

$$\sum_{i=1}^n (a+b) = \sum_{i=1}^n a + \sum_{i=1}^n b$$

```

j = 1;
while (j <= n) {
    for (i = n*n; i >= 1; i = i-3)
        x = x+1;
    j = j*2;
}

```

C_1
 sum desde $j=1$ hasta $\log_2(n+1)$
 sum desde $i=1$ hasta $n^2/3$
 C_2
 C_3

paso 0: $j=1=2^0$
 paso 1: $j=1*2=2^1$
 paso 2: $j=1*2*2=2^2$
 paso 3: $j=1*2*2*2=2^3$
 paso k: $j=2^k$
 $n+1=2^k$
 $\log_2(n+1)=k$

paso 0: $i=n^2=n^2-3*0$
 paso 1: $i=n^2-3=n^2-3*1$
 paso 2: $i=n^2-3-3=n^2-3*2$
 paso 3: $i=n^2-3-3-3=n^2-3*3$
 paso k: $i=n^2-3*k$
 $n^2-3*k=0$
 $-3k=-n^2$
 $k=n^2/3$

```

static public int rec3(int n){
    if ( n == 0 )
        return 0;
    else {
        if ( n == 1 )
            return 1;
        else
            return (rec3(n-2) * rec3(n-2));
    }
}

```

$$T(n) = c_1 \quad n \leq 1$$

$$c_2 + 2T(n-2) \quad n > 1$$

Paso 1: $T(n) = c_2 + 2T(n-2) \quad n > 1$

Paso 2: $T(n) = c_2 + 2[c_2 + 2T((n-2)-2)] \quad n-2 > 1$

$$T(n) = c_2 + 2[c_2 + 2T(n-4)]$$

$$T(n) = c_2 + 2c_2 + 2^2T(n-4)$$

Paso 3: $T(n) = 3c_2 + 2^2[c_2 + 2T((n-4)-2)] \quad n-4 > 1$

$$T(n) = c_2 + 2c_2 + 2^2c_2 + 2^3T(n-6)$$

Paso i: $T(n) = \sum_{k=0}^{i-1} 2^k c_2 + 2^i T(n-2i)$

$$T(n) = c_2 * \sum_{k=0}^{i-1} 2^k + 2^i T(n-2i)$$

$$T(n) = c_2 * (2^i - 1) + 2^i T(n-2i)$$

$$T(n) = c_2 * 2^i - c_2 + 2^i T(n-2i)$$

$n-2i=1$ CASO BASE

$$-2i=1-n$$

$$i=(-n+1)/(-2)$$

$$i=(n-1)/2$$

$$T(n) = c_2 * 2^{(n-1)/2} - c_2 + 2^{(n-1)/2} T(n-2((n-1)/2))$$

$$T(n) = c_2 * 2^{n/2-1/2} - c_2 + 2^{n/2-1/2} T(n-2(n/2-1/2))$$

$$T(n) = c_2 * (2^{n/2}/2^{1/2}) - c_2 + (2^{n/2}/2^{1/2}) T(n-n+1)$$

$$T(n) = c_2 * (2^{n/2}/2^{1/2}) - c_2 + (2^{n/2}/2^{1/2}) T(1)$$

$$T(n) = c_2 * (2^{n/2}/2^{1/2}) - c_2 + c_1 * (2^{n/2}/2^{1/2})$$

$$c_2 * (2^{n/2}/2^{1/2}) \leq k_1 2^{n/2}$$

$$(c_2/2^{1/2}) * 2^{n/2} \leq (c_2/2^{1/2}) * 2^{n/2}$$

$$k_1 = c_2/2^{1/2} \quad n_0 = 0$$

$$c_1 * (2^{n/2}/2^{1/2}) \leq k_2 * 2^{n/2}$$

$$(c_1/2^{1/2}) * 2^{n/2} \leq (c_1/2^{1/2}) * 2^{n/2}$$

$$k_2 = (c_1/2^{1/2}) \quad n_0 = 0$$

$$c_2 * (2^{n/2}/2^{1/2}) + c_1 * (2^{n/2}/2^{1/2}) \leq (c_2/2^{1/2}) * 2^{n/2} + (c_1/2^{1/2}) * 2^{n/2}$$

$$c_2 * (2^{n/2}/2^{1/2}) + c_1 * (2^{n/2}/2^{1/2}) \leq (c_2/2^{1/2} + c_1/2^{1/2}) * 2^{n/2}$$

$$k = c_2/2^{1/2} + c_1/2^{1/2} = k_1 + k_2 \quad n_0 = 0$$

```

preorden(){
    //procesar raiz
    LG<AG<T>> hijos = this.getHijos();
    hijos.comenzar();
    while(!hijos.fin())
        hijos.proximo().preorden();
}

```

Grafo<T> grafo;

Diferencias:

- 1- Puede haber ciclos.
- 2- Puede haber nodos inalcanzables desde algún otro.

```

dfs(){
    boolean[] marca = new boolean[grafo.listaDeVertices().tamanio()+1];
    for(i=1;i<=grafo.listaDeVertices().tamanio();i++){
        if(!marca[i])    dfs(i,marca);
    }
}

```

```

dfs(int i, boolean[] marca){
    marca[i]=true;
    Vertice<T> v = grafo.listaDeVertices().elemento(i);
    //procesar a v
    LG<Arista<T>> ady = grafo.listaDeAdyacentes(v);
    ady.comenzar();
    while(!ady.fin()){
        Arista<T> ar = ady.proximo();
        Vertice<T> destino = ar.verticeDestino();
        int pos = destino.posicion();
        if(!marca[pos])
            dfs(pos,marca);
    }
}

```

Ejercicio 6:

$$cte_1 + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \sum_{k=1}^j cte_2$$

$$cte_1 + cte_2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \sum_{k=1}^j 1$$

$$cte_1 + cte_2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n j$$

$$\sum_{j=i+1}^n j = \left(\sum_{j=1}^n j - \sum_{j=1}^i j \right)$$

$$cte_1 + cte_2 \sum_{i=1}^{n-1} \left(\frac{1}{2}(n^2 + n - i^2 - i) \right)$$

$$cte_1 + cte_2 * \frac{1}{2} * \sum_{i=1}^{n-1} (n^2 + n - i^2 - i)$$

$$n^2 \sum_{i=1}^{n-1} 1 + n \sum_{i=1}^{n-1} 1 - \sum_{i=1}^{n-1} i^2 - \sum_{i=1}^{n-1} i$$

$$n^2 * n - 1 + n * n - 1 -$$

12-3)

$$T(n) = \begin{cases} 1 & n=1 \\ 4T(n/2) + n^2, & n \geq 2 \end{cases}$$

$$\text{paso 1} = n^2 + 4T(n/2) \quad n \geq 2$$

$$\text{paso 2} = n^2 + 4[n^2 + 4T((n/2)/2)] \quad n/2 \geq 2$$

$$n^2 + 4n^2 + 4^2 T(n/4)$$

$$\text{paso 3} = n^2 + 4n^2 + 4^2[n^2 + 4T((n/4)/2)] \quad n/4 \geq 2$$

$$n^2 + 4n^2 + 4^2 n^2 + 4^3 T(n/8)$$

$$\text{paso } i = \sum_{k=0}^{i-1} 4^k n^2 + 4^i T(n/2^i)$$

$$n^2 * (4^{i-1+1} - 1) / (4 - 1) + 4^i T(n/2^i)$$

$$n^2 * [(4^i - 1) / 3] + 4^i T(n/2^i)$$

caso base: $n/2^i = 1$
 $n=2^i$
 $\log_2 n = \log_2 2^i$
 $\log_2 n = i$

$n^2 * [(4^{\log n} - 1) / 3] + 4^{\log n} * T(n/2^{\log n})$ Y AHORA ?!! xD