NCIA 202 - Bayesian Modeling of Brain and Behavior
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The self-calibrating Agent and its elusive prior

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1 Introduction

A Bayesian Agent, also known as an ideal observer is, by construction, optimal [1]. It is a decision-making system that (a) represents uncertainty about the environment through probabilistic beliefs and (b) chooses an action by combining these beliefs with sensory evidence using Bayes theorem [1]. Not all decision-making systems are Bayesian Agents: for an Agent to categorized as such, it is necessary that it meets (a) and (b).

A Bayesian Agent can have a fixed prior, or it can dynamically update its priors to adjust to the statistics of the environment. When dealing with the second case, the prior updating process can be formalized as Bayesian filtering, a probabilistic approach in which the Bayesian Agent updates its prior beliefs over latent states based on noisy observations. Notably, Kalman Filters (KF) are used for belief updating under the assumption of linear environmental dynamics and sensory mappings that are subject to a Gaussian noise. When state transitions or these sensory mappings are non-linear, bayesian inference becomes intractable. Under these scenarios, an approximate tractable alternative is provided by the Extended Kalman Filter (EKF) that linearizes these mappings locally around the current estimate [2].

To first analyze the prior of an Agent, it is necessary to assess whether it is Bayesian or not. Moreover, even when given the information that a given Agent is indeed a Bayesian Agent, attempting to model its training dynamics requires information about the training environment, which is sometimes unknown. In this report, we analyzed the behavior of an Agent to assess whether it was Bayesian. Also, we used KF and different versions of EKF as candidates of the learning model used by a Bayesian Agent, to characterize its training dynamics. In the following sections, we will explain the set up for the Agent analysis (Level 1), as well as the details for the planned assesment of its learning dynamics (Level 5), alongside the results of our analyses and our conclusions.

2 Research Question & Objectives

A Level 1 Agent observes a single noisy cue about a physical quantity S_1 and another single noisy cue about another physical quantity S_2. Then, it combines its prior belief about each of them with the sensory likelihood to form posterior estimates, compares these estimates $(\hat{S}_1, \hat{S}_2)$ and outputs 1 if $\hat{S}_1 > \hat{S}_2$ and 0 otherwise. From experimental responses only, we were instructed to identify the parametrization of the Agent's prior using the following information about the Agent: (1) it has a fixed Gaussian prior, (2) it performs Bayesian inference (3) it completes some binary decision process. Moreover, we knew we could make it complete experiments to understand it better.

In addition to Level 1, we chose to solve Level 5. In this level, we dealt with a self-calibrating Agent in two phases: training and experiment. During the training phase, our Agent learned the mean and uncertainty of its priors by itself (hence self-calibrating) to optimally match the environmental variability. Once this training phase was over, we could make it complete experiments. When running any experiment, this Agent would behave exactly like an Agent undergoing Level 1. We were instructed to show the dynamic evolution of the Agent's prior as a function of training length, and hypothesize a mechanism for the Agent's optimal updating of its prior mean and variance.

To summarize, we had the following objectives: (1) find a way to estimate the (static) prior distribution of our Agent when dealing with noisy inputs (Level 1), (2) show the dynamic evolution of the Agent's prior as a function of training length (Level 5), and (3) hypothesize a mechanism for the optimal updating of the Agent's prior mean and variance (Level 5). We used the following hypotheses to guide our analysis:

1. During experiments, the Agent's behavior is not deterministic (Objective 1)
2. During experiments, the Agent's behavior variability occurs due to internal noise (Objective 1)
3. During experiments, the Agent optimally incorporates both sensory noise and prior precision in its estimation process (i.e. it is a Bayesian Agent; Objective 1)
4. During the training phase, we hypothesize that the Agent updates its priors using a KF, corresponding to linear state-transition and observation dynamics (Objectives 2 and 3).
5. During the training phase, we hypothesize that the Agent updates its priors using an EKF, corresponding to a nonlinear state-transition and/or observation models that are locally linearized (Objectives 2 and 3).

3 Methods & Models

3.1 Hypothesis 1

If the Agent was making decisions deterministically, its output should be constant when presented with the same inputs. If the responses vary, then we cannot reject there is stochasticity in its decision process.

Experiment 1 : Identical repeated trials We created an experiment with constant $S_1 = S_2 = \sigma_1^2 = \sigma_2^2 = 0$. We generated 500 trials.

3.2 Hypothesis 2

If the Agent's behavior variability was due to external factors, then when presented with sensory inputs without noise, the resulting psychometric curve should resemble a step function. Conversely, if there is internal noise affecting the Agent's estimation, we should observe a smooth psychometric function.

Experiment 2: Psychometric curve with 0 sensory noise We created an experiment with variable S_1, S_2, $\Delta = S_1 - S2$, $\Delta \in [-2, 2]$, but constant $\sigma_1^2 = \sigma_2^2 = 0$. We generated 500 trials.

3.3 Hypothesis 3

If the Agent is behaving like a Bayesian Agent, then, when dealing with high external sensory noise, its posterior variance should increase. If the Agent was not incorporating sensory noise with an internal prior, adding extra noise to the stimulus should not change the slope of the psychometric function (see Appendix A for a proof).

Experiment 3: Loop over sensory noise levels We created an experiment with four different noise levels $\sigma_{S1} = \sigma_{S2} \in \{0.0,\ 0.2,\ 0.5,\ 1.0\}$. For each noise level, we also varied the stimulus difference $\Delta = S_1 - S_2 \in [-2,\ 2]$, $S_2 = 0$ and $S_1 = \Delta$. For each pair of (σ, Δ), we generated 500 trials.

3.4 Interim: Prior parameter estimation

Provided all of the above hypothesis held and our Agent is behaving like a Bayesian Agent, then we can estimate it's prior mean μ_{prior} under a high enough variance. This because the mean of the prior should exactly be the threshold of the psychometric function under infinite variance (see Appendix B for a proof).

Experiment 4: infinite σ to estimate μ_{prior} We created an experiment with $S_2 = 0$ and a very high variance $\sigma_2 = 8$. We generated experiments with S_1 going from -4 to 4 and $\sigma_1 = 2$. We generated 50 trials for each of these experiments. (Appendix B for explanation)

Experiment 5: MLE to estimate variance and internal noise We created an experiment with $S_2 = \mu_{\text{prior}}$, $\sigma_2 = 2$. We generated experiments with σ_1 going from 0 to 4 and S_1 varying from -4 to 4 with step 0.1. We generated 150 trials for each experiment. Then conducted Maximum Likelihood Estimation to estimate σ_{prior} and $\sigma_{internal}$ (see Appendix C for details).

3.5 Hypothesis 4 and 5

We used the fact that our Agent dynamically updates its priors in an optimal manner as a guiding assumption. This allowed us to expect that the Agent's prior evolution converged to the statistics of the learning environment after sufficiently long training. If the estimates converged for large enough training sessions, we could treat the final prior estimates as the environment statistics. Because the Agent updated its priors such that the posterior of one trial became the prior of the next, a static prior would indicate that the Agent had perfectly learned the environment statistics. In this case, the final prior estimates corresponded to the Agent's estimates of the environment statistics.

We used this information and the previously stablished method for estimating μ_{prior} and σ_{prior} (See 3.4) to calculate the μ_{prior} and σ_{prior} for different lengths of training. We conducts experiments to determine the initial prior of the Agent, the statistics of the environment and to track the learning dynamics of the Agent. The details of the experiments are in the Appendix F Table 1.

3.5.1 Parameter estimation and model comparison

After we retrieved the environment statistics, we modeled the Agent's prior evolution as a function of training length as a KF (see Appendix D for the derivation), an EKF (squared residual) and an EKF (absolute difference; see Appendix E for the derivation). This yielded a total of 5 learning models: prior mean and prior variance estimation with the same model, either KF or EKF (residuals or absolute; total of 3 models); and prior mean estimated with KF and then prior variance estimated with EKF (residuals or absolute; total of 2 models). We made the choice to mix the models for variance estimation because the variance estimation in EKF is mean-dependent. Thus, it would change if the mean is trained dynamically or if it is already trained a priori. We plotted the mean and variance as a function of training length for each of the models and compared goodness of fit using Mean Squared Error (MSE).

4 Results

Hypothesis 1 The Agent does not make deterministic decisions (Appendix F Figure-2), as its output is not constant under Experiment 1. Thus, we cannot reject there is stochasticity in its responses.

Hypothesis 2 The psychometric curve (Appendix F Figure-3) resulting from Experiment 2 does not resemble a step function. Thus, the Agent's behavior variability cannot be attributed to external factors. We conclude that the Agent's estimation must involve an internal noise parameter.

Hypothesis 3 The slope of the psychometric curve resulting from Experiment 3 changes as external sensory noise increases (Appendix F Figure-4). Thus, we conclude that the Agent incorporates both sensory noise and internal noise in its estimation process. Thus, we conclude the Agent is behaving like a Bayesian Agent.

Parameter Estimation Given that we could not reject Hypothesis 3, we conducted Experiment 4 and Experiment 5 to find $\mu_{\text{prior}} = -.841$, 95% CI [-.844, -.835] and $\sigma_{\text{prior}} = 1.121$ (95% CI [1.113, 1.122]), respectively.

Hypothesis 4 and 5 The results of our modeling for Hypotheses 4 and 5 are summarized in Appendix F Figure 1. We evaluated the ability of five models, the KF, the EKF with squared residuals, the EKF with absolute residuals, and both EKF models with a previous KF prior mean estimation, to recover the Agent's prior parameters (μ_{prior} and σ_{prior}) from Experiment 8 data.

The KF and the EKF with absolute residuals achieved similar performance in estimating the prior mean, with MSEs of 0.259 (95% CI: [0.017, 0.701]) and 0.259 (95% CI: [0.017, 0.689]), respectively. In contrast, the EKF with squared residuals showed substantially higher error (MSE = 0.547949, 95% CI: [0.144818, 1.101344]). These results indicate that the standard KF and the EKF absolute residual variant are both able to capture the central tendency of the Agent's prior accurately, while the EKF squared residuals variant overestimates the variability in the mean.

In terms of the prior variance, the KF alone yielded an MSE of 0.612 (95% CI: [0.506, 0.709]), indicating moderate error. The EKF with squared residuals with a pre-learnt mean (MSE = 0.053, 95% CI: [0.022, 0.0961]) and the EKF with absolute residuals (MSE = 0.048, 95% CI: [0.031, 0.068]) had similar performances. These results suggest that the EKF variants,especially the absolute residual version,improve estimation of prior variance by incorporating nonlinear state dynamics.

5 Discussion, Conclusion & Perspectives

We succesfully tested our 5 hypotheses. That is, during experiments, we could not reject that our Agent's behavior is stochastic, that its variability occurs due to internal noise, and that it incorporates sensory noise and prior prediction in its estimation process using Bayes Theorem. Thus, we concluded that our Agent, during experiments, was behaving as a Bayesian Agent. Moreover, we were able to show its prior evolution during training, and we found that its prior mean updating process was best represented by the dynamics of a KF Model or an EKF absolute residuals model, while its variance updating process was best represented by a EKF absolute residuals model. Thus, the Agent could be generating estimates of its prior mean using a KF and then proceeding to learn its prior variance using an EKF Absolute Residual, or be estimating both the prior mean and prior variance using an EKF with absolute residuals.

In terms of the present report's limitations, for the training dynamics analysis, we only compared five different models to represent the Agent's learning. We know that model comparison is a key element of behavioral modeling [3], and thus future studies should compare the prediction alternative models to better characterize the agent's learning evolution. Alternative models like Variational Bayes filter, which makes use of KL Divergence to align the priors to the environment variables, might could be good starting points for further comparison. Moreover, we were only able to get one run of trials from the observed Bayesian agent and were unable to remove the random noise this would create; by over multiple runs, one would be able to mitigate variations due to random stochasticity.

Code availability

The code to analyze the resulting model fits and to produce the figures in this paper is available at https://github.com/camila-maura/bayesian_Agents_project_s1.

References

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[3] Robert C Wilson and Anne GE Collins. Ten simple rules for the computational modeling of behavioral data. *eLife*, 8:e49547, nov 2019.

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Appendix A Psychometric function change due to sensory noise proof

Let the decision variable be $D = \mu_{post,1} - \mu_{post,2}$. It is clear, that the variance of the decision variable will be.

$$\sigma_D^2 = \sigma_{post,1}^2 + \sigma_{post,2}^2$$

Also, we know that for a posterior,

$$\sigma_{post,i}^2 = \frac{\sigma_0^2 \sigma_i^2}{\sigma_0^2 + \sigma_i^2} \qquad \forall i \in \{1, 2\}$$

Now, if we let σ_1 and σ_2 be 0, that is there is no variance in the input, the variance of D would be,

$$\sigma_D^2 = 0 + 0 = 0$$

Thus, the psychometric function of the decision variable must have an infinite slope, that is there must not be any stochasticity in the decisions and the psychometric curve must be a step function.
But the fact that there is a visible slope in the function tells us that,

$$\sigma_{i,effective}^2 = \sigma_i^2 + \sigma_{internal}^2 \qquad \forall i \in \{1, 2\}$$

Appendix B Estimating Prior Mean: Point of Subjective Equality (PSE) proof

We know that posterior mean can be calculated from the prior (μ_0, σ_0) on the basis of an observation x with variance σ_1^2 with the following

$$\mu_{post} = \frac{\sigma_1^2 \mu_0 + \sigma_0^2 x}{\sigma_1^2 + \sigma_0^2}$$

Now, The decision $S_1 > S_2$, can be modelled using a Decision variable $D = \mu_{post,1} - \mu_{post,2}$. This decision variable can be plotted using a psychometric function over multiple values of S_1 and a fixed reference S_2. Since the Agent performs Bayesian inference over its priors before comparing these variables, the psychometric function is written as,

$$\Phi\left(\frac{\mu_{post,1} - \mu_{post,2}}{\sigma_D}\right)$$

Where $\mu_{post,i}$ is the posterior for variable S_i and σ_D is the standard deviation for decision variable D. Now, we know that, for a psychometric function $\Phi(0) = 0.5$. With the reference being S_2, the value of S_1 when the psychometric function is equal to 0.5 is the $\mu_{threshold}$.
At this point (PSE),

$$\mu_{post,1} = \mu_{post,2} \tag{1}$$

$$\frac{\sigma_1^2\mu_0 + \sigma_0^2 s_1}{\sigma_1^2 + \sigma_0^2} = \frac{\sigma_1^2\mu_0 + \sigma_0^2 s_2}{\sigma_1^2 + \sigma_0^2} \tag{2}$$

$$\frac{\sigma_1^2\mu_0 + \sigma_0^2 \mu_{threshold}}{\sigma_1^2 + \sigma_0^2} = \frac{\sigma_1^2\mu_0 + \sigma_0^2 s_2}{\sigma_1^2 + \sigma_0^2} \tag{3}$$

Let the reference value of S_2 be 0. Thus, $s_2 = 0$. Now, if we assume that the variance of S_2 is large, we get,

$$\frac{\sigma_1^2\mu_0 + \sigma_0^2 \mu_{threshold}}{\sigma_1^2 + \sigma_0^2} = \mu_0 \tag{4}$$

$$\sigma_1^2\mu_0 + \sigma_0^2 \mu_{threshold} = \mu_0(\sigma_1^2 + \sigma_0^2) \tag{5}$$

$$\mu_{threshold} = \mu_0 \tag{6}$$

Thus, for a large variance, we can approximate the mean of the prior to be equal to the threshold of the psychometric function Φ at which the value of the function is 0.5

Appendix C Maximum Likelihood Estimation of prior mean

Once we estimate the mean of the bayesian Agent, and once we have estimated that there is an internal noise, we can find the variance using Maximum Likelihood Estimation.
Let

$$S_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$$

We can use the model of the decision variable as follows:

$$D = \mu_{post,1} - \mu_{post,2}$$

where,

$$\mu_{post,i} = \frac{\sigma_{i,effective}^2 \mu_0 + \sigma_0^2 s_i}{\sigma_{i,effective}^2 + \sigma_0^2}$$

where s_i is an instantiation of S_i. and $\sigma_{i,effective}^2 = \sigma_i^2 + \sigma_{internal}^2$. The variance of this Decision variable would thus be,

$$\sigma_D^2 = \frac{\sigma_0^2\sigma_{1,effective}^2}{\sigma_0^2 + \sigma_{1,effective}^2} + \frac{\sigma_0^2\sigma_{2,effective}^2}{\sigma_0^2 + \sigma_{2,effective}^2}$$

We thus, obtain a cumulative Gaussian function for the model,

$$\Phi_M\left(\frac{D}{\sigma_D}\right)$$

We attempt to maximise the likelihood of this function with the observed psychometric function as a target and $(\sigma_0, \sigma_{internal})$ as parameters.

$$\ell(\sigma_{internal}, \sigma_0) = \sum_i \left[k_i \log \Phi_M\left(\frac{D_i}{\sigma_D}\right) + (n_i - k_i)\log\left(1 - \Phi\left(\frac{D_i}{\sigma_D}\right)\right)\right]$$

$$(\hat{\sigma}_{internal}, \hat{\sigma}_0) = \arg\max_{(\sigma_{internal}, \sigma_0)} \ell(\sigma_{internal}, \sigma_0)$$

Appendix D Kalman Filter (KF)

Let x_t be the latent state at time t and y_t be the observation made at this time. We assume that there is a gaussian measurement noise σ_{meas}. Thus,

$$y_t = x_t + \epsilon \quad \text{where } \epsilon \sim \mathcal{N}(0, \sigma^2_{meas})$$

The Agent maintains its prior belief over the latent mean,

$$\mu_t \sim \mathcal{N}(\mu_t, P_t)$$

Thus, every observation can be modelled as

$$y_t = \mu_t + \eta_t \quad \text{where } \eta_t \sim \mathcal{N}(0, \sigma_{obs})$$

$$\sigma^2_{obs} = \sigma^2_{meas} + \sigma^2_{int}$$

This is clearly a linear-Gaussian, as required by the KF. At time step t, after making the observation y_t with prior beliefs (μ_{t-1}, P_{t-1}), the model will compute the posterior beliefs (μ_t, P_t) using Bayesian inference,

$$\mu_t = \frac{\sigma^2_{obs}\mu_{t-1} + P_{t-1}y_t}{\sigma^2_{obs} + P_{t-1}}$$

$$P_t = \frac{P_{t-1}\sigma^2_{obs}}{P_{t-1} + \sigma^2_{obs}}$$

Now, defining Kalman gain (K_t) at time t to be

$$K_t = \frac{P_{t-1}}{P_{t-1} + \sigma^2_{obs}}$$

We get the following belief update rules,

$$\mu_t = \mu_{t-1} + K_t(y_t - \mu_{t-1})$$

$$P_t = (1 - K_t)P_{t-1}$$

Appendix E Extended Kalman Filter (EKF, Absolute and Squared Residuals)

Let x_t be the latent state at time t and y_t be the observation made at this time. We assume that there is a gaussian measurement noise σ_{meas}. Thus,

$$y_t = x_t + \epsilon \quad \text{where } \epsilon \sim \mathcal{N}(0, \sigma^2_{meas})$$

The Agent maintains its prior belief over the latent mean with an uncertainty in the log space,

$$\mu_t \sim \mathcal{N}(\mu_t, \sigma_t)$$

and $s_t \in R$, such that

$$s_t = \log \sigma_t$$

with a Gaussian belief about s_t

$$s_t \sim \mathcal{N}(s_t, P_t)$$

E.1 Square - Residuals

Here, we use, as observations, the squared prediction errors,

$$z_t = (y_t - \mu_t)^2$$

where,

$$y_t = x_t + \eta_t \qquad \text{where } \eta_t \sim \mathcal{N}(0, \sigma_{obs})$$

The expected squared prediction error would thus be,

$$E[z_t] = E[(y_t - \mu_t)^2] = E[(x_t - \mu_t + \eta_t)^2] = E[(x_t - \mu_t)^2] + E[\eta_T^2] + 2E[(x_t - \mu_t)\eta_t]$$

Since η_t is independent of $x_t - \mu_t$ and since we know these expectations,

$$E[(x_t - \mu_t)^2] = Var(x_t) = \sigma_t^2$$

$$E[\eta_t^2] = Var(\eta_t) = \sigma_{obs}^2$$

$$E[z_t] = \sigma_t^2 + \sigma_{obs}^2$$

Now, talking about variance of z_t,

$$Var(z_t) = Var((y_t - \mu_t)^2) = 2Var((y_t - \mu_t)) = 2(\sigma_t^2 + \sigma_{obs}^2)$$

Now, we know that $s_t = log\sigma_t$, thus we can write this expectation as a function of s,

$$h(s_t) = (e^{s_t})^2 + \sigma_{obs}^2$$

This is a non-linear function, for EKF we must approximate it to a locally linear function using taylor expansion.

$$h(s_t) \approx h(s_0) + \frac{\partial h}{\partial s_t}(s_t - s_0)$$

We know that,

$$\frac{\partial h}{\partial s_t} = 2e^s$$

Let this partial be denoted by the Jacobian H_t Now, since $E(z_t) = h(s_t)$, $h(s_t)$ serves as the prediction at time time t where z_t is the actual observation. Looking at the variance of prediction error,

$$\begin{aligned} Var(z_t - h(s_t)) &= Var(z_t) + Var(h(s_t)) && (7)\\ &= 2(\sigma_t^2 + \sigma_{obs}^2) + Var(H_t(s_t - s_0)) && (8)\\ &= 2(\sigma_t^2 + \sigma_{obs}^2) + H_t Var(s_t) H_t && (9) \end{aligned}$$

We define Kalman gain similar to Kalman gain for KFs except with the introduction of H (Identity for KFs) to account for the non-linearity. This formulation of the Kalman gain comes from minimizing the variance of state estimate. [4]

$$K = PH^T(HPH^T + R)^{-1} \qquad \text{where } R = (\sigma_t^2 + \sigma_{obs^2})$$

Thus,

$$K_t = \frac{P_{t-1}H_{t-1}}{H_{t-1}P_{t-1}H_{t-1} + (\sigma_t^2 + \sigma_{obs}^2)}$$

With the update rule over s_t being,

$$s_t = s_{t-1} + K_t(z_t - h(s_{t-1})$$

And over mean being,

$$\mu_t = \mu_{t-1} + K_t(y_t - \mu_{t-1})$$

E.2 Absolute residuals

Here, we use, as observations, absolute prediction errors and highlight the changes this creates in further steps.

$$z_t = |y_t - \mu_t|$$

First, the expectation changes,

$$E(z_t) = \sqrt{\frac{2}{\pi}}\sqrt{\sigma_t^2 + \sigma_{obs}^2}$$

$$Var(z_t) = (\sigma_t^2 + \sigma_{obs}^2)(1 - \frac{2}{\pi}$$

We can derive this expectation similar to before, based on the knowledge that z_t is now a modded Gaussian. Now, we know that

$$h(s_t) = \sqrt{\frac{2}{\pi}}\sqrt{e^{2s_t} + \sigma_{obs}^2}$$

The Jacobian for this would now be,

$$H_t = \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{e^{2s_t} + \sigma_{obs}^2}}e^{2s}$$

We can use these values in the previously stated Kalman Gain to achieve the EKF updates using absolute residuals. Similar methods can be used for any non-linearity introduced at the observation.

In our study above, we estimate the measurement noise by minimising MSE between the observed behaviour and the behaviour of our model.

Appendix F Figures and Tables

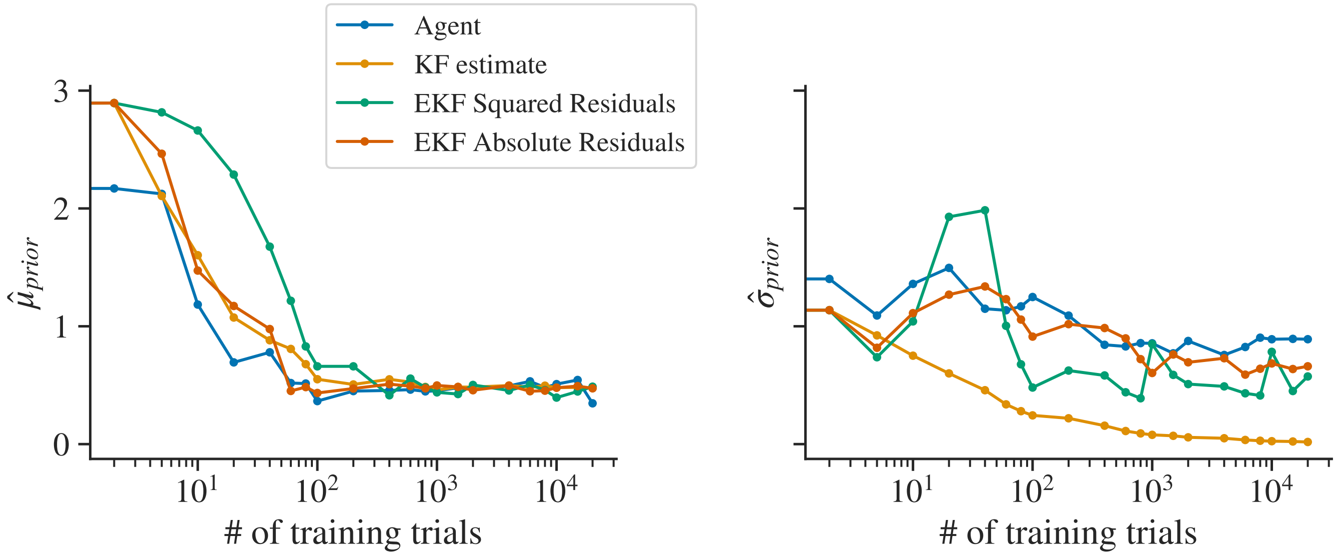

Figure 1: Estimates for μ_{prior} and σ_{prior} for the Agent's behavior and as the output of the 3 tested models: Kalman Filter (KF), Extended Kalman Filter (EKF) Squared Residuals and EKF Absolute Residuals. Importantly, both EKF Squared Residuals and EKF Absolute Residuals variance estimates orccur after a previous KF prior mean estimation.

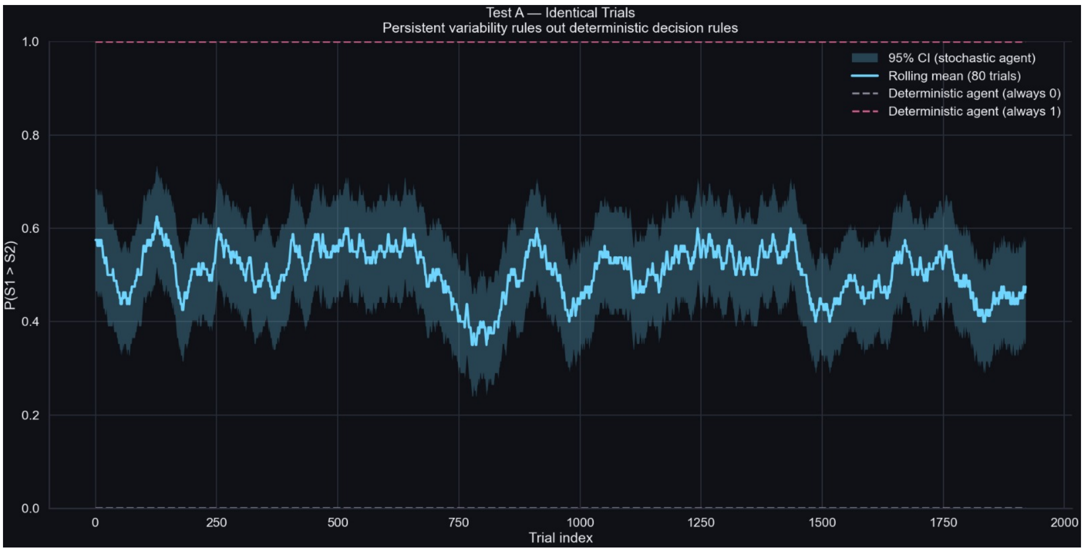

Figure 2: Experiment 1 - Same input over multiple trials gives different outputs

[1]training length $\in \{0, 2, 5, 10, 20, 40, 60, 80, 100, 200, 400, 600, 800, 1000, 1500, 2000, 4000, 6000, 8000, 10000, 15000, 20000\}$

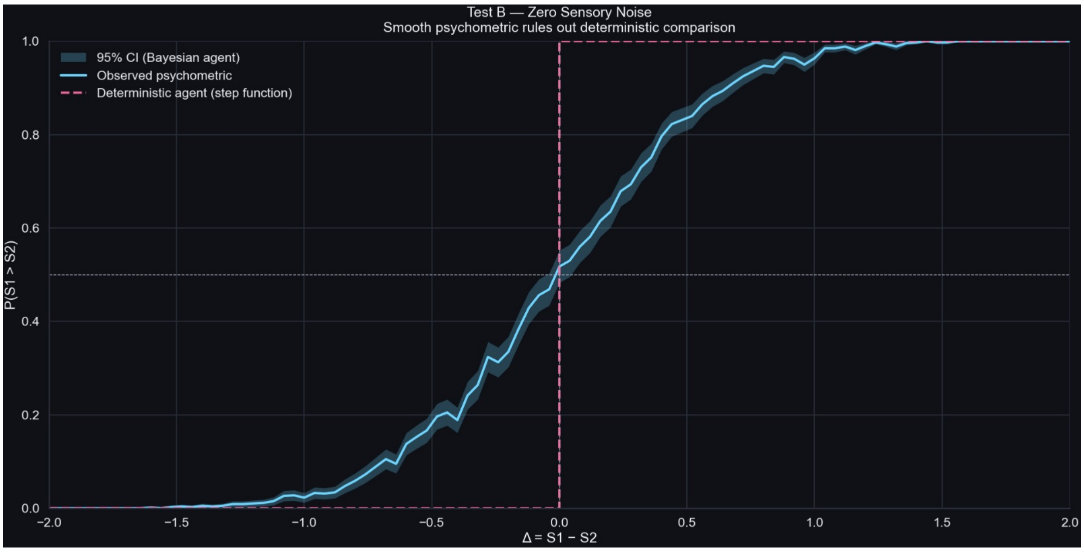

Figure 3: Experiment 2 - Psychometric curve at 0 input noise still not a step function

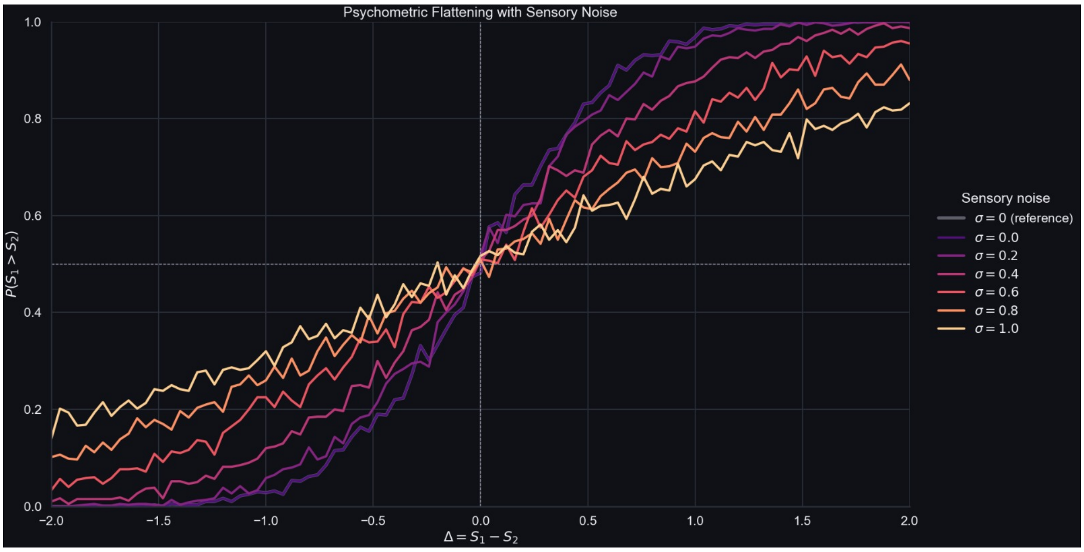

Figure 4: Experiment 3 - The slope decreases with increasing variance

Table 1: Summary of Experiments 6–8

Experiment	**Training length (in trials)**	**Purpose**	**Method / Model**
Experiment 6	0	Obtain initial prior parameters without any training	Parameter estimation at Level 1 (Experiment 5)
Experiment 7	50,000	Estimate environmental statistics and assess convergence of the Agent's prior to the environment	Parameter estimation at Level 1 (Experiment 5)
Experiment 8	custom[1]	Track the dynamics of the Agent's prior parameters over time	We initialised the model with the initial mean obtained from experiment 6 and P (See Appendix E) initialised to be 1. We set the measurement noise to be 1.96 (Appendix E. The samples received by the agent are from the environment estimated from experiment 7.