GRAPHS

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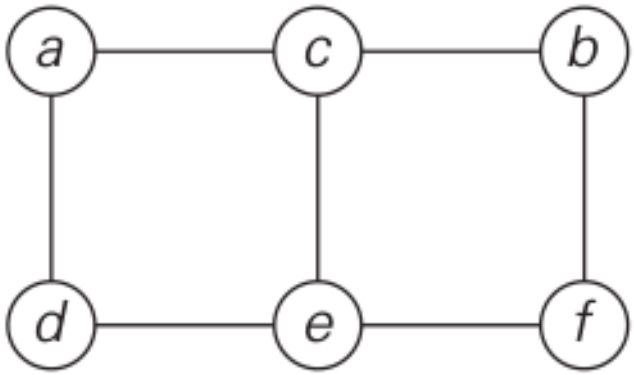
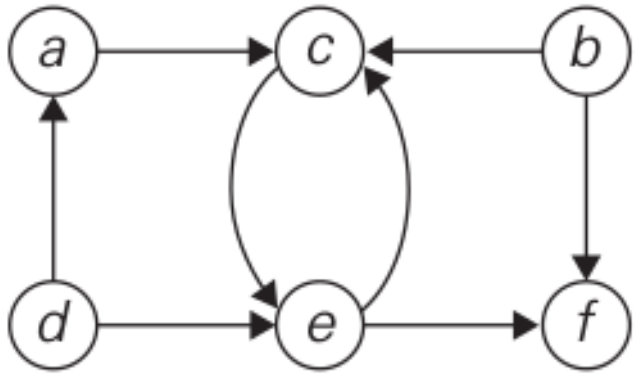
Agenda

Introduction[1]

Graph: a collection of nodes (vertices), some of them connected by edges (arcs) – $G = (V, E)$, where $V \neq \emptyset$, and $E \in V \leftrightarrow V$

- Undirected / Directed graphs (digraphs)





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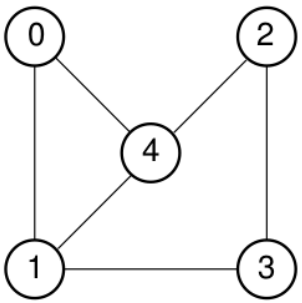
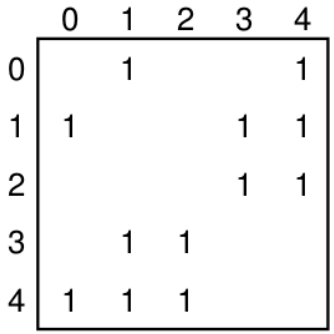
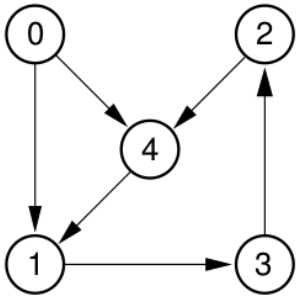
[1] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

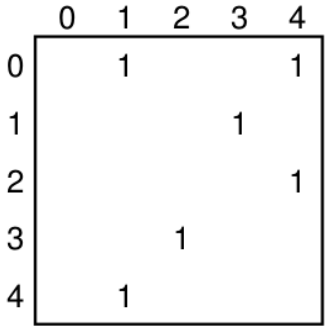
Possible implementations (unweighted graphs)[2]

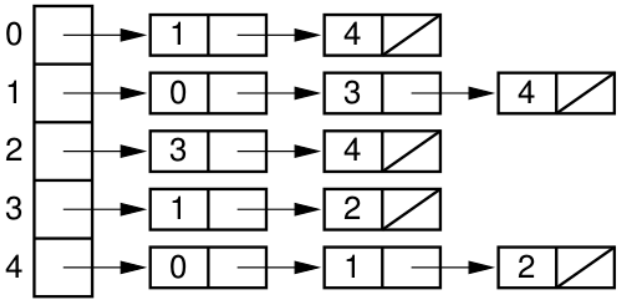
Adjacency matrix or adjacency list

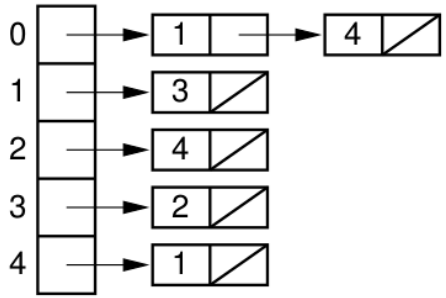








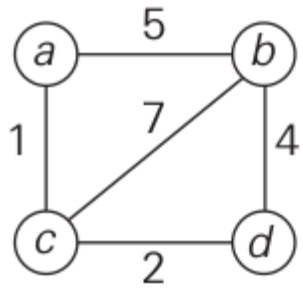




[2] Source: C. Shaffer. Data Structures and Algorithm Analysis. 2013.

Possible implementations (weighted graphs)[4]

Adjacency matrix or adjacency list



	a	b	c	d
a	∞	5	1	∞
b	5	∞	7	4
c	1	7	∞	2
d	∞	4	2	∞

a	→ b, 5 → c, 1
b	→ a, 5 → c, 7 → d, 4
c	→ a, 1 → b, 7 → d, 2
d	→ b, 4 → c, 2

[3]

Choosing the most appropriate implementation

- Space efficiency: dense graph *vs.* sparse graph
- Time efficiency (graph traversal): $\Theta(|V|^2)$ *vs.* $\Theta(|V| + |E|)$

[3] Fonte: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

[4] Source: C. Shaffer. Data Structures and Algorithm Analysis. 2013.

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Graph ADT

Operations

- int n(G g);
- int e(G g);
- int first(G g, int v);
- int next(G g, int v, int w);
- void setEdge(G g, int i, int j, int wt);
- void delEdge(G g, int i, int j);
- boolean isEdge(G g, int i, int j);
- int weight(G g, int i, int j);
- void setMark(G g, int v, int val);
- int getMark(G g, int v);

Graph ADT implemented as an adjacency matrix

Composite type:

```
1  int[][] matrix ;                          // adjacency matrix
2  int numEdge ;                              // number of edges
3  int[] Mark ;                          // auxiliary marking array
```

Algorithm: G create_graph(int n)

```
1  g.Mark ← new int[n];
2  g.matrix ← new int[n][n];
3  g.numEdge ← 0;
4  return g;
```

Graph ADT implemented as an adjacency matrix

Algorithm: int first(G g, int v)

```
1  for i ← 0 to n(g) − 1 do
2      if g.matrix[v][i] ≠ 0 then return i;
3  return n(g);
```

Algorithm: int next(G g, int v, int w)

```
1  for i ← w + 1 to n(g) − 1 do
2      if g.matrix[v][i] ≠ 0 then return i;
3  return n(g);
```

Implicit assumption: 0 denotes the absence of an edge between two nodes

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Graph ADT implemented as an adjacency matrix

Algorithm: void setEdge(G g, int i, int j, int wt)

```
1  if wt = 0 then error ;
2  if g.matrix[i][j] = 0 then g.numEdge++;
3  g.matrix[i][j] ← wt;
```

Algorithm: void delEdge(G g, int i, int j)

```
1  if g.matrix[i][j] ≠ 0 then g.numEdge--;
2  g.matrix[i][j] ← 0;
```

Implicit assumption: 0 denotes the absence of an edge between two nodes

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Graph traversals

In general terms (attention to unconnected graphs):

Algorithm: void graphTraverse(G g)

```
1  for v ← 0 to n(g) − 1 do
2      setMark(g, v, UNVISITED);
3  for v ← 0 to n(g) − 1 do
4      if getMark(g, v) = UNVISITED then
5          traverse(g, v);
```

Possible traversals

- DFS: depth-first search
- BFS: breadth-first search

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Graph traversals: DFS

Algorithm: void graphTraverse(G g)

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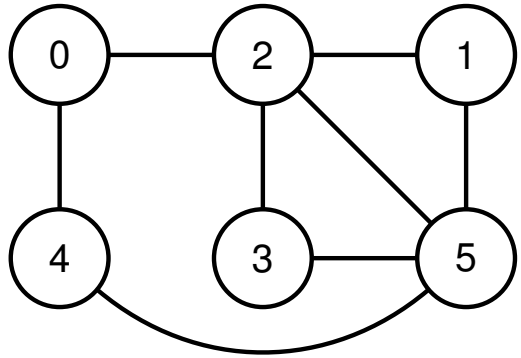
2      setMark(g, v, UNVISITED);
3  for v ← 0 to n(g) − 1 do
4      if getMark(g, v) = UNVISITED
         then DFS(g, v) ;
```

Algorithm: void DFS(G g, int v)

```
1  preVisit(g, v);
2  setMark(g, v, VISITED);
3  w ← first(g, v);
4  while w < n(g) do
5      if getMark(g, w) = UNVISITED
         then DFS(g, w) ;
6      w ← next(g, v, w);
7  posVisit(g, v);
```



	0	1	2	3	4	5
Mark	×	×	×	×	×	×

Graph traversals: DFS

Algorithm: void graphTraverse(G g)

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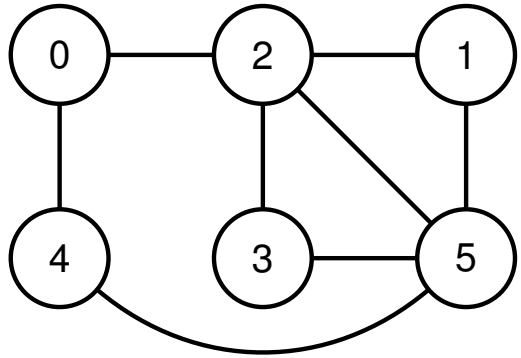

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- DFS(_, 0)

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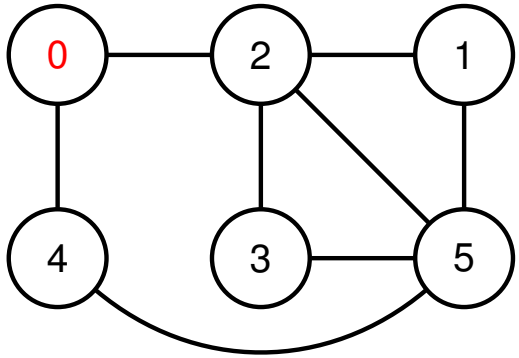

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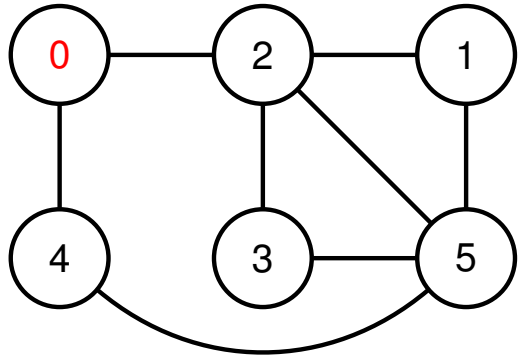

	0	1	2	3	4	5
Mark	✓	×	×	×	×	×

DFS calls:
- DFS(_, 0)
- DFS(_, 2)

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Graph traversals: DFS

Algorithm: void graphTraverse(G g)

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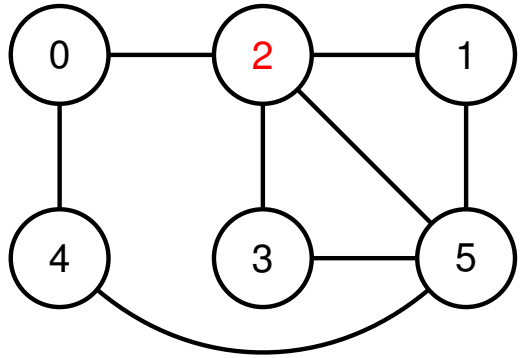

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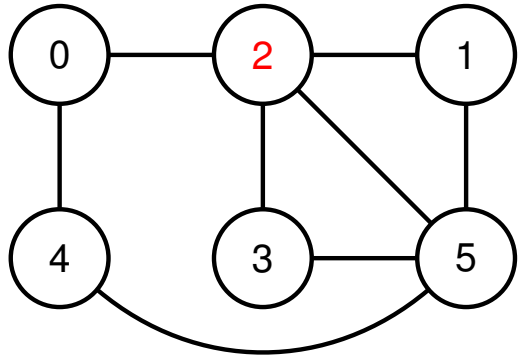

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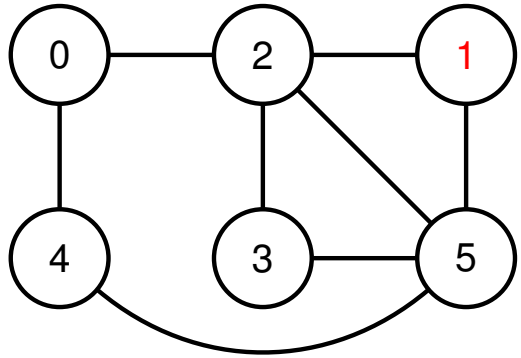

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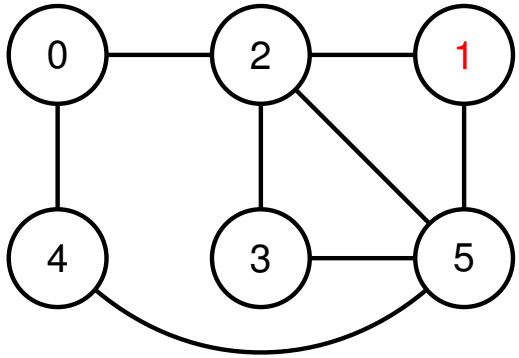

	0	1	2	3	4	5
Mark	✓	✓	✓	×	×	×

DFS calls:
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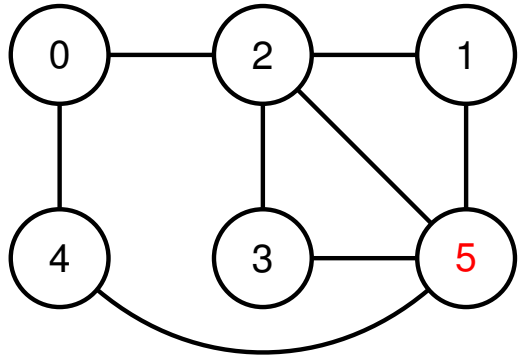

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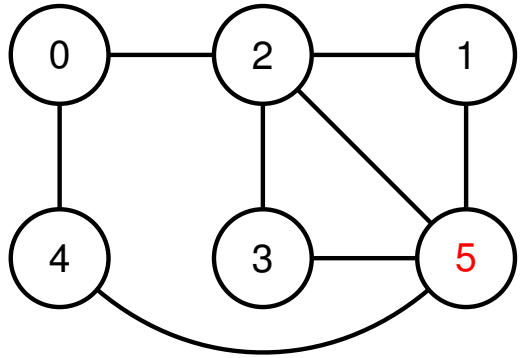

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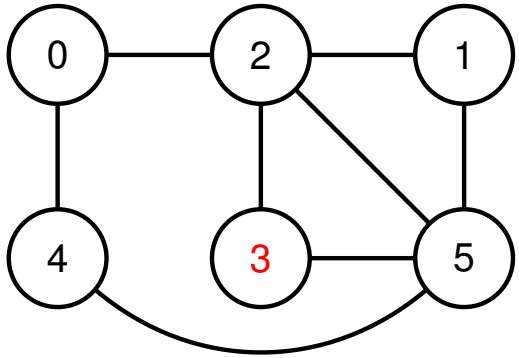

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	×	✓

DFS calls:
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Graph traversals: DFS

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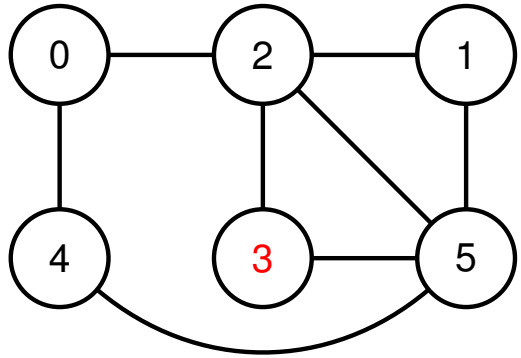

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	×	✓

DFS calls:
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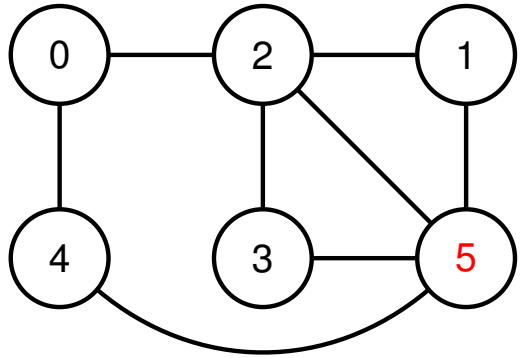

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- DFS(_, 5)
- DFS(_, 4)

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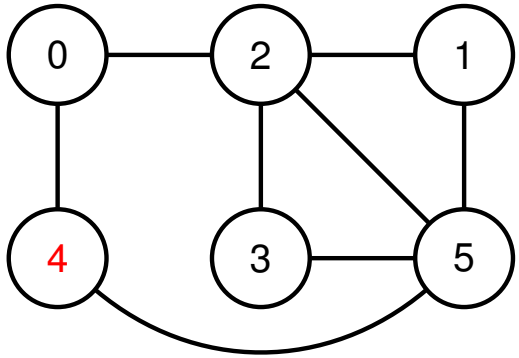
Graph traversals: DFS

Algorithm: void graphTraverse(G g)

```
1  for v ← 0 to n(g) − 1 do
2      setMark(g, v, UNVISITED);
3  for v ← 0 to n(g) − 1 do
4      if getMark(g, v) = UNVISITED
         then DFS(g, v) ;
```

Algorithm: void DFS(G g, int v)

```
1  preVisit(g, v);
2  setMark(g, v, VISITED);
3  w ← first(g, v);
4  while w < n(g) do
5      if getMark(g, w) = UNVISITED
         then DFS(g, w) ;
6      w ← next(g, v, w);
7  posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)
- DFS(_, 2)
- DFS(_, 1)
- DFS(_, 5)
- DFS(_, 4)

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Algorithm: void DFS(G g, int v)

```
1  preVisit(g, v);
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3  w ← first(g, v);
4  while w < n(g) do
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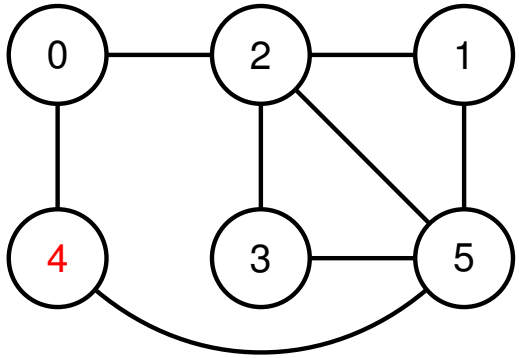

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)
- DFS(_, 2)
- DFS(_, 1)
- DFS(_, 5)
- DFS(_, 4)

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         then DFS(g, w) ;
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7  posVisit(g, v);
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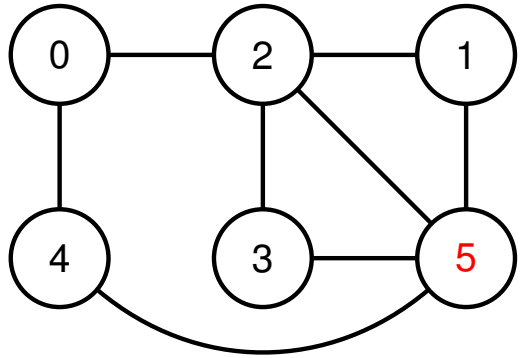

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)
- DFS(_, 2)
- DFS(_, 1)
- DFS(_, 5)

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Graph traversals: DFS

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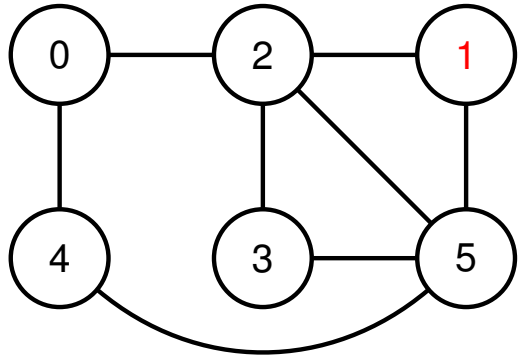

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)
- DFS(_, 2)
- DFS(_, 1)

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Graph traversals: DFS

Algorithm: void graphTraverse(G g)

```
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```
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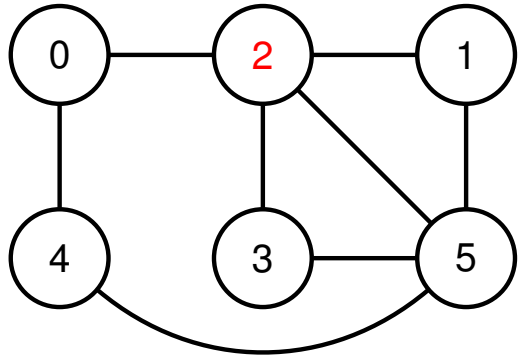
7  posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)
- DFS(_, 2)

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Graph traversals: DFS

Algorithm: void graphTraverse(G g)

```
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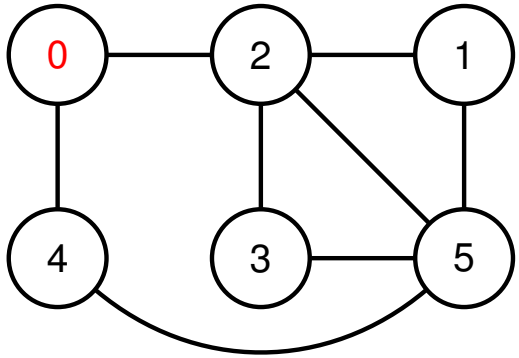

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

DFS calls:
- DFS(_, 0)

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Graph traversals: DFS

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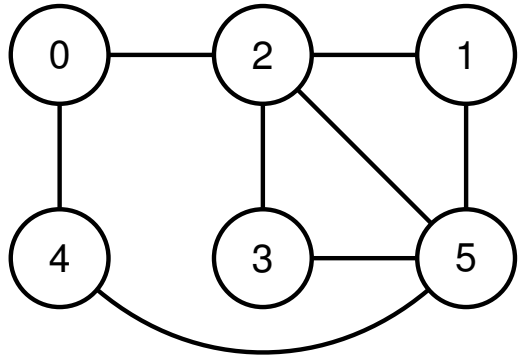

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

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Graph traversals: DFS

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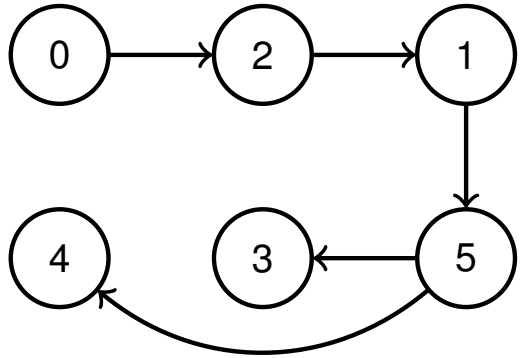
         then DFS(g, w) ;
6      w ← next(g, v, w);
7  posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

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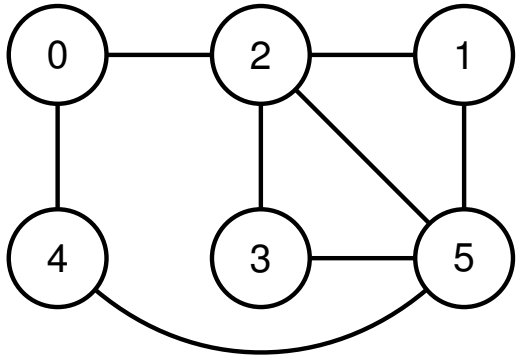
Graph traversals: BFS

Algorithm: void graphTraverse(G g)

```
1  for v ← 0 to n(g) − 1 do
2      setMark(g, v, UNVISITED);
3  for v ← 0 to n(g) − 1 do
4      if getMark(g, v) = UNVISITED
         then BFS(g, v) ;
```



	0	1	2	3	4	5
Mark	×	×	×	×	×	×

Graph traversals: BFS

Algorithm: void graphTraverse(G g)

```
1  for v ← 0 to n(g) − 1 do
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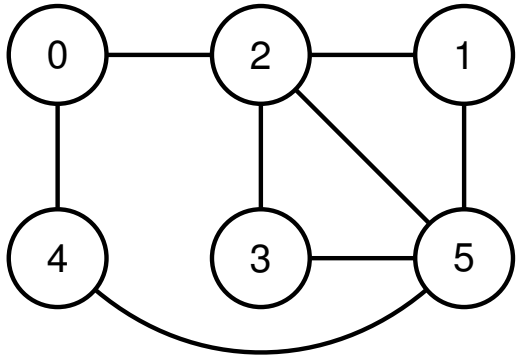


	0	1	2	3	4	5
Mark	×	×	×	×	×	×

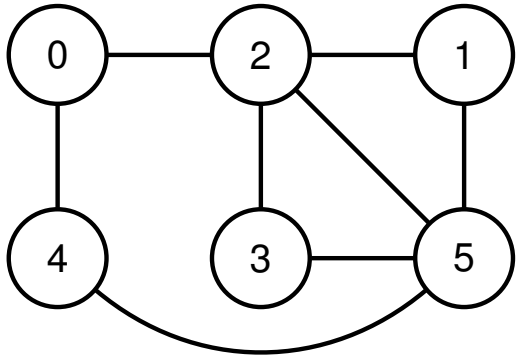
BFS calls:

- BFS(_, 0)

Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
4   while length(Q) > 0 do
5       v ← dequeue(Q);
6       preVisit(g, v);
7       w ← first(g, v);
8       while w < n(g) do
9           if getMark(g, w) =
              UNVISITED then
10              setMark(g, w, VISITED);
11              enqueue(Q, w);
12          w ← next(g, v, w);
13      posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	×	×	×	×	×

BFS calls:
- BFS(_, 0)

Queue: 0

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
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12          w ← next(g, v, w);
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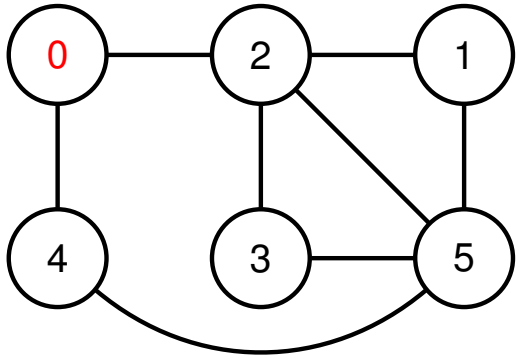
```



	0	1	2	3	4	5
Mark	✓	×	×	×	×	×

BFS calls:
- BFS(_, 0)

Queue: -

$v = 0$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
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12          w ← next(g, v, w);
13      posVisit(g, v);
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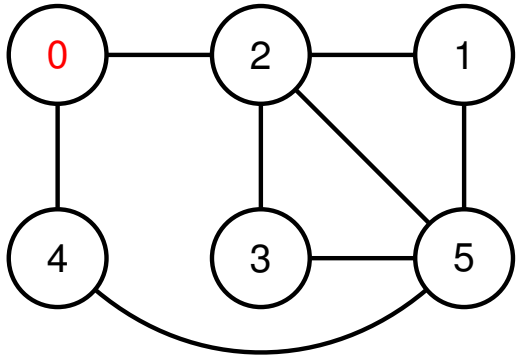

	0	1	2	3	4	5
Mark	✓	×	✓	×	✓	×

BFS calls:
- BFS(_, 0)

Queue: 2, 4

$v = 0$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
4   while length(Q) > 0 do
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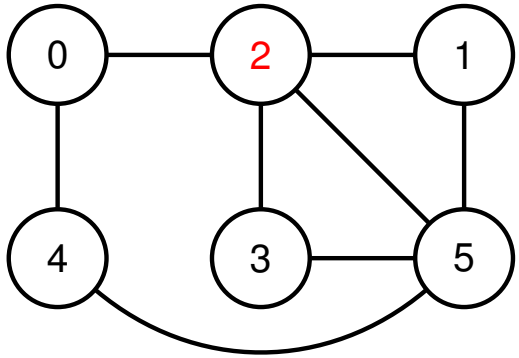

	0	1	2	3	4	5
Mark	✓	×	✓	×	✓	×

BFS calls:
- BFS(_, 0)

Queue: 4

$v = 2$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1  Q ← create_queue();
2  enqueue(Q, start);
3  setMark(g, start, VISITED);
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7      w ← first(g, v);
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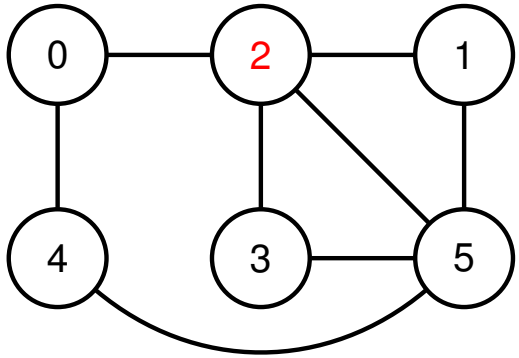
             UNVISITED then
10             setMark(g, w, VISITED);
11             enqueue(Q, w);
12         w ← next(g, v, w);
13     posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: 4, 1, 3, 5

$v = 2$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
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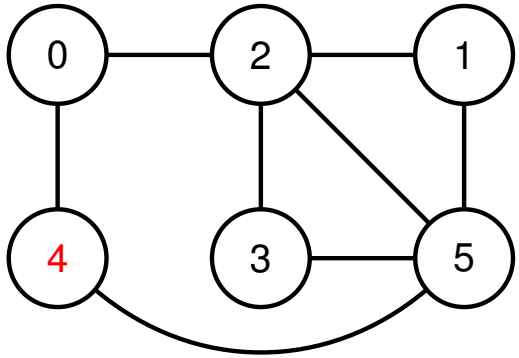

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: 1, 3, 5

$v = 4$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
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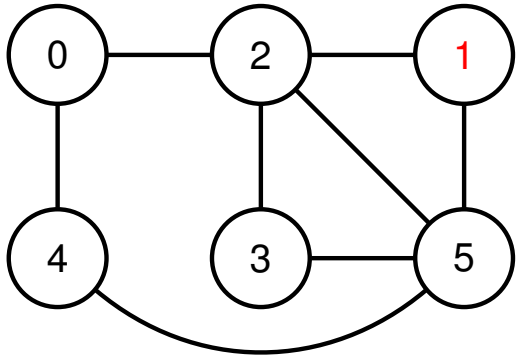
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: 3, 5

$v = 1$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
4   while length(Q) > 0 do
5       v ← dequeue(Q);
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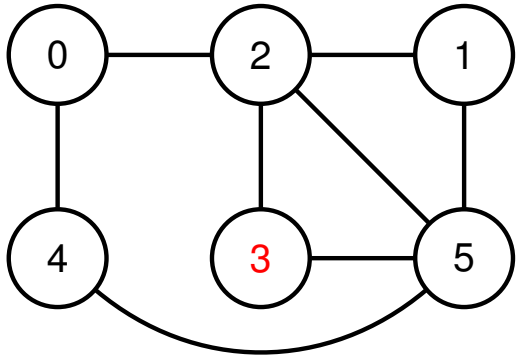
12          w ← next(g, v, w);
13      posVisit(g, v);
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: 5

$v = 3$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
3   setMark(g, start, VISITED);
4   while length(Q) > 0 do
5       v ← dequeue(Q);
6       preVisit(g, v);
7       w ← first(g, v);
8       while w < n(g) do
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              UNVISITED then
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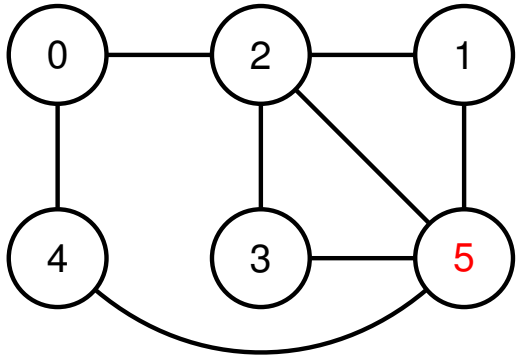
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: -

$v = 5$

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Graph traversals: BFS

Algorithm: void
BFS(G g, int start)

```
1   Q ← create_queue();
2   enqueue(Q, start);
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4   while length(Q) > 0 do
5       v ← dequeue(Q);
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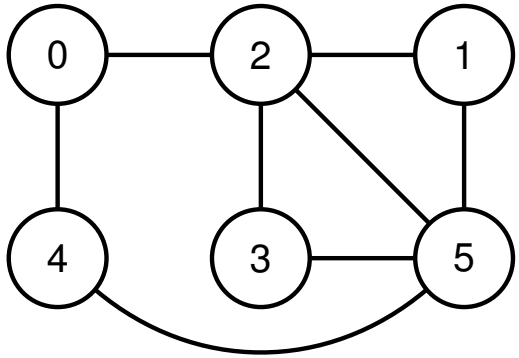

	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

BFS calls:
- BFS(_, 0)

Queue: -

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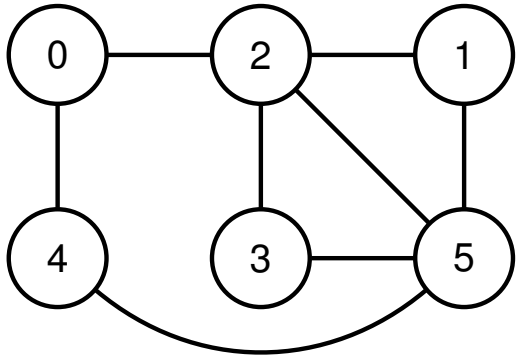
Graph traversals: BFS

Algorithm: void graphTraverse(G g)

```
1  for v ← 0 to n(g) − 1 do
2      setMark(g, v, UNVISITED);
3  for v ← 0 to n(g) − 1 do
4      if getMark(g, v) = UNVISITED
         then BFS(g, v) ;
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

Graph traversals: BFS

Algorithm: void graphTraverse(G g)

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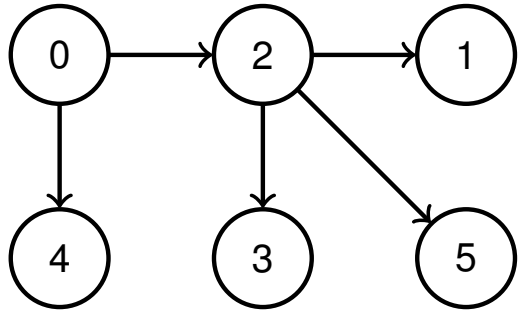
```



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓

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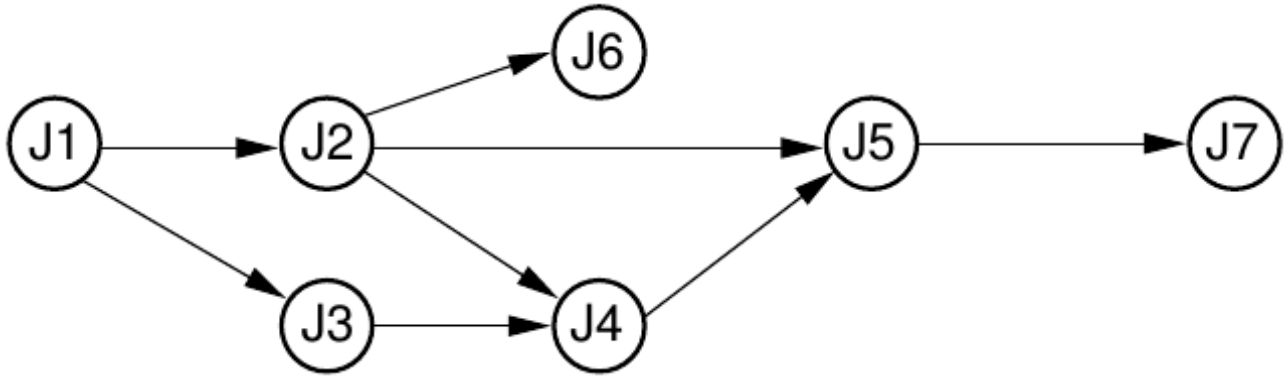
DFS: topological sorting[5]

Let G be a DAG (directed acyclic graph), find a solution considering the informed prerequisites

- Assumption: $u \rightarrow v$ means that u is a prerequisite of v



[5] Source: C. Shaffer. Data Structures and Algorithm Analysis. 2013.

DFS: topological sorting

Algorithm: void toposort(G g, int v, STACK s)

```
1  setMark(g, v, VISITED);
2  w ← first(g, v);
3  while w < n(g) do
4      if getMark(g, w) = UNVISITED then
5          toposort(g, w, s);
6      w ← next(g, v, w);
7  push(s, v);            // we will have in s a solution
```

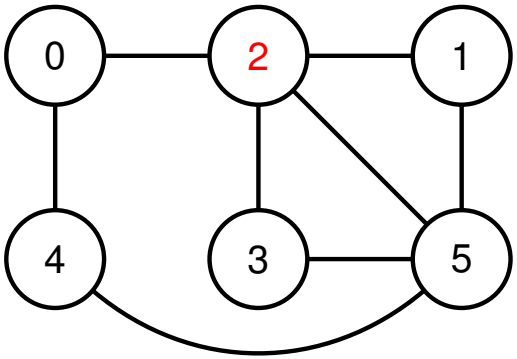
For the given example: J1, J3, J2, J6, J4, J5, J7

BFS: shortest path on unweighted graphs

Similar to the original BFS code

- Start the search on the origin
- When the destination is reached, stop the search
- To reconstruct the path: auxiliary array to store predecessors

Let the origin be 0 and the destination be 5, solution = $0 \rightarrow 2 \rightarrow 5$



	0	1	2	3	4	5
Mark	✓	✓	✓	✓	✓	✓
Pred	-	2	0	2	0	2

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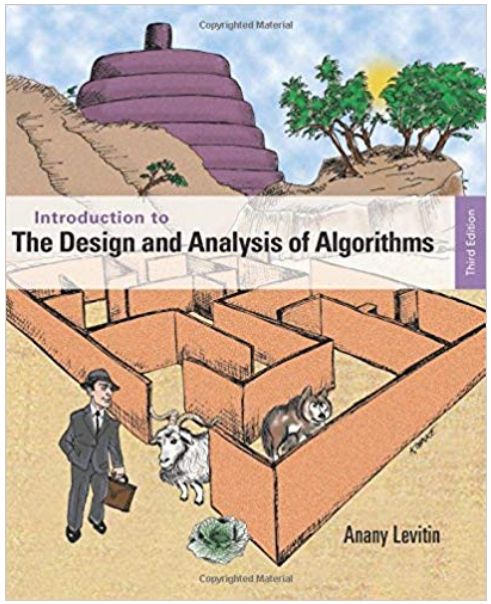
Bibliography



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GRAPHS

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