DIJKSTRA'S ALGORITHM

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Agenda

Greedy algorithms

Greedy algorithms: constructs a solution through a sequence of steps, each expanding a partially constructed solution

On each step, the choice made must be:

- Feasible: satisfies the problem's constraints
- Locally optimal: best feasible local choice
- Irrevocable: cannot be changed later

Dijkstra's algorithm

Let G be a weighted graph and $v \in V$ (*source*), finds the shortest path from v to all other nodes in V

- Single-source shortest paths

Applications

- Transport planning
- Communication networks
- Social networks
- Robotics
- Pathfinding
- Puzzles
- etc.

Dijkstra's algorithm: cannot be used on weighted graphs with negative weights

Agenda

Dijkstra's algorithm

First, find the closest node to v (itself)

On the i-th step:

- Knows the $(i-1)$-th closest nodes to v (they form a tree)
- Since there are no negative weights, the next closest one is adjacent to one of the $i-1$ closest nodes to v
- After chosing the $i-th$ closest node (w), updates the possible shortest paths to yet unchosen nodes (u) if $d_w + weight(w,u) < d_u$

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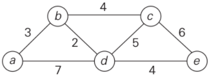
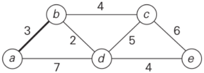
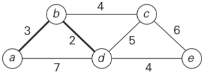
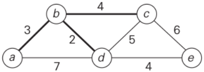
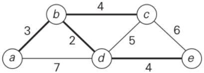
Dijkstra's algorithm[1]



Tree vertices	Remaining vertices	Illustration
a(−, 0)	**b(a, 3)** c(−, ∞) d(a, 7) e(−, ∞)	
b(a, 3)	c(b, 3 + 4) **d(b, 3 + 2)** e(−, ∞)	
d(b, 5)	**c(b, 7)** e(d, 5 + 4)	
c(b, 7)	**e(d, 9)**	
e(d, 9)		

[1] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

Dijkstra's algorithm

Algorithm: void Dijkstra(Graph G, int s, int[] D)

for $i \leftarrow 0$ **to** $n(G) - 1$ **do**
  $D[i] \leftarrow \infty$; $P[i] \leftarrow -$;
  $setMark(G, i, UNVISITED)$;
$H[1] \leftarrow (s, s, 0)$; $D[s] \leftarrow 0$;
for $i \leftarrow 0$ **to** $n(G) - 1$ **do**
  repeat
    $(p, v) \leftarrow removemin(H)$;
    if $v = NULL$ **then return**;
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  $w \leftarrow first(G, v)$;
  while $w < n(G)$ **do**
    if $getMark(G, w) \neq VISITED \wedge$
     $D[w] > D[v] + weight(G, v, w)$ **then**
      $D[w] \leftarrow D[v] + weight(G, v, w)$;
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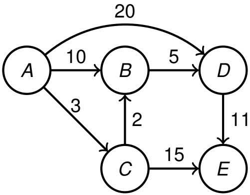
Let $s = A$



	A	B	C	D	E
Mark	×	×	×	×	×
Distance	0	∞	∞	∞	∞
Parent	–	–	–	–	–

(A,A,0)

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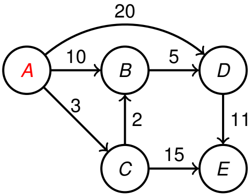

	A	B	C	D	E
Mark	✓	×	×	×	×
Distance	0	∞	∞	∞	∞
Parent	A	–	–	–	–

–

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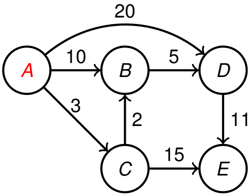
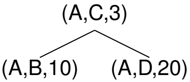


	A	B	C	D	E
Mark	✓	×	×	×	×
Distance	0	10	3	20	∞
Parent	A	–	–	–	–



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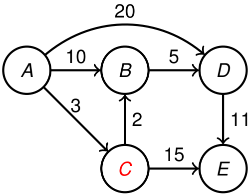
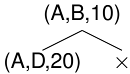


	A	B	C	D	E
Mark	✓	×	✓	×	×
Distance	0	10	3	20	∞
Parent	A	–	A	–	–



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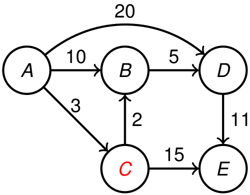
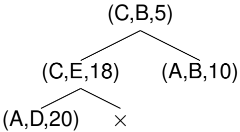


	A	B	C	D	E
Mark	✓	×	✓	×	×
Distance	0	5	3	20	18
Parent	A	–	A	–	–



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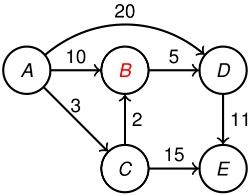
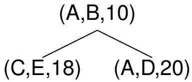


	A	B	C	D	E
Mark	✓	✓	✓	×	×
Distance	0	5	3	20	18
Parent	A	C	A	–	–



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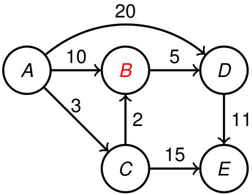
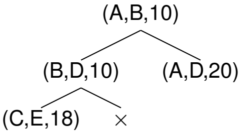


	A	B	C	D	E
Mark	✓	✓	✓	×	×
Distance	0	5	3	10	18
Parent	A	C	A	−	−



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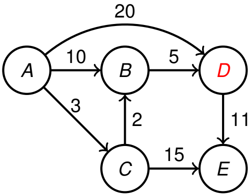
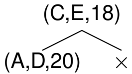


	A	B	C	D	E
Mark	✓	✓	✓	✓	×
Distance	0	5	3	10	18
Parent	A	C	A	B	–



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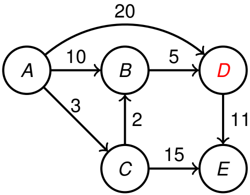

	A	B	C	D	E
Mark	✓	✓	✓	✓	×
Distance	0	5	3	10	18
Parent	A	C	A	B	–

(C,E,18)

(A,D,20) ×

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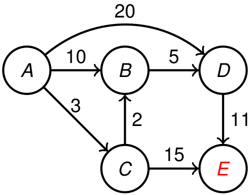

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
Distance	0	5	3	10	18
Parent	A	C	A	B	C

(A,D,20)

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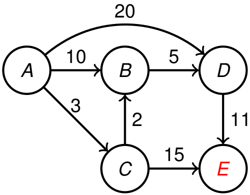

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
Distance	0	5	3	10	18
Parent	A	C	A	B	C

(A,D,20)

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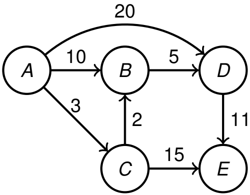

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
Distance	0	5	3	10	18
Parent	A	C	A	B	C

(A,D,20)

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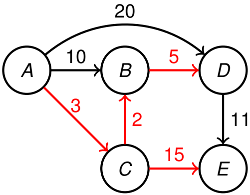

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
Distance	0	5	3	10	18
Parent	A	C	A	B	C

(A,D,20)

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```
for i ← 0 to n(G) − 1 do
    D[i] ← ∞ ; P[i] ← −;
    setMark(G, i, UNVISITED);
H[1] ← (s, s, 0) ; D[s] ← 0;
for i ← 0 to n(G) − 1 do
    repeat
        (p, v) ← removemin(H);
        if v = NULL then return;
    until getMark(G, v) = UNVISITED;
    setMark(G, v, VISITED) ; P[v] ← p;
    w ← first(G, v);
    while w < n(G) do
        if getMark(G, w) ≠ VISITED ∧
          D[w] > D[v] + weight(G, v, w) then
            D[w] ← D[v] + weight(G, v, w);
            insert(H, (v, w, D[w]));
        w ← next(G, v, w);
```

Time efficiency

- Matrix and no heap
  $\Theta(|V|^2 + E|) = \Theta(|V|^2)$, since $|E| \in O(|V|^2)$
  - Better for dense graphs
- List and heap
  $\Theta((|V| + |E|) \log |V|)$
  - Better for sparse graphs

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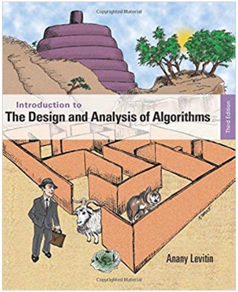

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DIJKSTRA'S ALGORITHM

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