SELECTION SORT | INSERTION SORT | MERGESORT

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Agenda

1. Selection sort (brute force)
2. Insertion sort (decrease-and-conquer)
3. Mergesort (divide-and-conquer)
4. Bibliography

Selection sort

Idea: find the smallest element and exchange it with the first one

Algorithm: SelectionSort(A[0..n - 1])

for $i \leftarrow 0$ **to** $n-2$ **do**
  $min \leftarrow i$;
  for $j \leftarrow i+1$ **to** $n-1$ **do**
    if $A[j] < A[min]$ **then** $min \leftarrow j$;
  swap $A[i]$ and $A[min]$;

index:	0	1	2	3	4	5	6
value:	89	45	68	90	29	34	17
	i						
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1. **for** $i \leftarrow 0$ **to** $n-2$ **do**
2. $min \leftarrow i$;
3.   **for** $j \leftarrow i+1$ **to** $n-1$ **do**
4.   **if** $A[j] < A[min]$ **then** $min \leftarrow j$;
5.   swap $A[i]$ and $A[min]$;

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Selection sort: complexity (basic op. = $A[j] < A[min]$)

$$
\begin{aligned}
C_{worst}(n) &= \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 \\
&= \sum_{i=0}^{n-2} ((n-1) - (i+1) + 1) \\
&= \sum_{i=0}^{n-2} (n-1-i) \\
&= \sum_{i=0}^{n-2} (n-1) - \sum_{i=0}^{n-2} i \\
&= (n-1) \sum_{i=0}^{n-2} 1 - \frac{(n-2)(n-1)}{2} \\
&= (n-1)^2 - \frac{(n-2)(n-1)}{2} \\
&= \frac{n^2-n}{2} \in \Theta(n^2)
\end{aligned}
$$

Remember that:

- $\sum_{i=l}^{u} c * a_i = c * \sum_{i=l}^{u} a_i$
- $\sum_{i=l}^{u} (a_i \pm b_i) = \sum_{i=l}^{u} a_i \pm \sum_{i=l}^{u} b_i$
- $S_n = \frac{(a_1 + a_n) * n}{2}$ (arithm. prog.)

Regarding the #swaps: $S_{worst}(n) = n - 1 \in \Theta(n)$

Bubble sort: $C(n) \in \Theta(n^2)$, $S_{worst}(n) \in \Theta(n^2)$

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Brute force *vs.* Decrease-and-conquer

Example: computing a^n

- Brute force ($\Theta(n)$): $\underbrace{a * \dots * a}_{n}$
- Decrease by a constant ($\Theta(n)$)

$$f(n) = \begin{cases} f(n-1) \cdot a & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

- Decrease by a constant factor ($\Theta(log\ n)$)

$$a^n = \begin{cases} (a^{n/2})^2 & \text{if } n \text{ is even and positive} \\ (a^{(n-1)/2})^2 \cdot a & \text{if } n \text{ is odd} \\ 1 & \text{if } n = 0 \end{cases}$$

Decrease-and-conquer

Design strategy: decrease-and-conquer

- Decrease by a constant
- Decrease by a constant factor
  - Other example: binary search
- Variable-size decrease
  - Example: $gcd(m, n) = gcd(n, m \bmod n)$

Insertion sort

Idea: provided that $A[0..n-i]$ is sorted, where $2 \leq i \leq n$, to sort $A[0..n-i+1]$, we "insert" $A[n-i+1]$ appropriately (dec. by a constant)

Algorithm: InsertionSort(A[0..n - 1])

```
for i ← 1 to n − 1 do
    v ← A[i]; j ← i − 1;
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        A[j + 1] ← A[j]; j ← j − 1;        // partial shift right
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$j = -1$

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						i	
	j						

$v = 34$

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Insertion sort

Idea: provided that $A[0..n-i]$ is sorted, where $2 \leq i \leq n$, to sort $A[0..n-i+1]$, we "insert" $A[n-i+1]$ appropriately (dec. by a constant)

Algorithm: InsertionSort(A[0..n - 1])

```
for i ← 1 to n − 1 do
    v ← A[i]; j ← i − 1;
    while j ≥ 0 ∧ A[j] > v do
        A[j + 1] ← A[j]; j ← j − 1;        // partial shift right
    A[j + 1] ← v;
```

index:	0	1	2	3	4	5	6
value:	29	34	45	68	89	90	17
							i
						j	

$v = 17$

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Insertion sort

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    A[j + 1] ← v;
```

index:	0	1	2	3	4	5	6
value:	29	29	34	45	68	89	90
							i

$v = 17$

$j = -1$

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Insertion sort

Idea: provided that $A[0..n-i]$ is sorted, where $2 \leq i \leq n$, to sort $A[0..n-i+1]$, we "insert" $A[n-i+1]$ appropriately (dec. by a constant)

Algorithm: InsertionSort(A[0..n - 1])

for $i \leftarrow 1$ **to** $n-1$ **do**
  $v \leftarrow A[i]; j \leftarrow i-1;$
  while $j \geq 0 \wedge A[j] > v$ **do**
    $A[j+1] \leftarrow A[j]; j \leftarrow j-1;$ `// partial shift right`
  $A[j+1] \leftarrow v;$

index:	0	1	2	3	4	5	6
value:	17	29	34	45	68	89	90
							i

$v = 17$

$j = -1$

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Idea: provided that $A[0..n-i]$ is sorted, where $2 \le i \le n$, to sort $A[0..n-i+1]$, we “insert” $A[n-i+1]$ appropriately (dec. by a constant)

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    A[j + 1] ← v;
```

index:	0	1	2	3	4	5	6
value:	17	29	34	45	68	89	90

$i = 7$

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Insertion sort: complexity (basic op.: $A[j] > v$)

$$
\begin{aligned}
C_{worst}(n) &= \sum_{i=1}^{n-1} \sum_{j=0}^{i-1} 1 \\
&= \sum_{i=1}^{n-1} ((i-1) - 0 + 1) \\
&= \sum_{i=1}^{n-1} i \\
&= \frac{(1+(n-1))((n-1)-1+1)}{2} \\
&= \frac{n^2-n}{2} \in \Theta(n^2)
\end{aligned}
$$

$$C_{best}(n) = \sum_{i=1}^{n-1} 1 = n - 1 - 1 + 1 = n - 1 \in \Theta(n)$$

$$C_{avg}(n) \approx \frac{n^2}{4} \in \Theta(n^2)$$

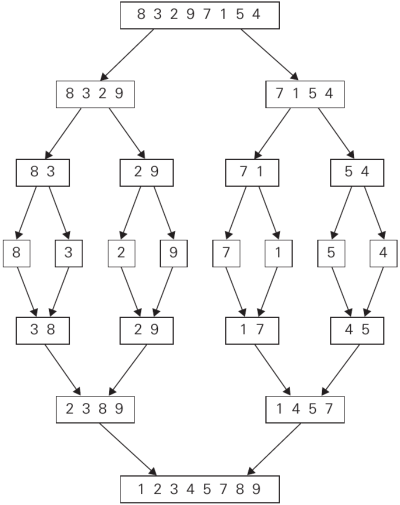
Agenda

1. Selection sort (brute force)
2. Insertion sort (decrease-and-conquer)
3. Mergesort (divide-and-conquer)
4. Bibliography

Divide-and-conquer

1. A problem is divided into several subproblems of the same type (ideally of about equal size)
2. The subproblems are solved
3. If necessary, the solutions to the subproblems are combined

Mergesort[1]



[1] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**
  $m \leftarrow \lfloor (l + r)/2 \rfloor$;
  $Mergesort(A, l, m)$; $Mergesort(A, m + 1, r)$; $Merge(A, l, r)$;

Algorithm: Merge(A[0..n-1], l, r)

for $i \leftarrow l$ **to** r **do** $temp[i] \leftarrow A[i]$;
$m \leftarrow \lfloor (l + r)/2 \rfloor$;
$i1 \leftarrow l$; $i2 \leftarrow m + 1$;
for $curr \leftarrow l$ **to** r **do**
  if $i1 = m + 1$ **then** $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then** $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

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Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

1. **if** $l < r$ **then**
2. $m \leftarrow \lfloor (l + r)/2 \rfloor$;
3. $Mergesort(A, l, m)$;
4. $Mergesort(A, m + 1, r)$;
5. $Merge(A, l, r)$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l				r

Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**
  $m \leftarrow \lfloor (l + r)/2 \rfloor$;
  $Mergesort(A, l, m)$;
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  $Merge(A, l, r)$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 1) \quad (\_, 2, 2)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l		r		

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  $Mergesort(A, m + 1, r)$;
  $Merge(A, l, r)$;

mergesort([5,2,1,7,0],0,4)

$$
\begin{array}{ll}
\downarrow & \\
(\_, 0, 4) & \\
\downarrow & \searrow \\
(\_, 0, 2) & (\_, 3, 4) \\
\downarrow & \searrow \\
(\_, 0, 1) & (\_, 2, 2) \\
\downarrow & \searrow \\
(\_, 0, 0) & (\_, 1, 1)
\end{array}
$$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l	r			

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```
mergesort(
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```

$$
\begin{array}{ll}
\downarrow & \\
(\_, 0, 4) & \\
\downarrow & \searrow \\
(\_, 0, 2) & (\_, 3, 4) \\
\downarrow & \searrow \\
(\_, 0, 1) & (\_, 2, 2) \\
\downarrow & \searrow \\
(\_, 0, 0) & (\_, 1, 1)
\end{array}
$$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l,r				

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if $l < r$ **then**
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  $Mergesort(A, l, m)$;
  $Mergesort(A, m + 1, r)$;
  $Merge(A, l, r)$;

`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 1) \quad (\_, 2, 2)$

$\downarrow \quad \searrow$

$(\_, 0, 0) \quad (\_, 1, 1)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l	r			

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```
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$$
\begin{array}{ll}
\downarrow & \\
(\_, 0, 4) & \\
\downarrow & \searrow \\
(\_, 0, 2) & (\_, 3, 4) \\
\downarrow & \searrow \\
(\_, 0, 1) & (\_, 2, 2) \\
\downarrow & \searrow \\
(\_, 0, 0) & (\_, 1, 1)
\end{array}
$$

index:	0	1	2	3	4
A:	5	2	1	7	0
		l,r			

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Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

```
if l < r then
    m ← ⌊(l + r)/2⌋;
    Mergesort(A, l, m);
    Mergesort(A, m + 1, r);
    Merge(A, l, r);
```

mergesort(
    [5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

↓ ↘

$(\_, 0, 1)$ $(\_, 2, 2)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l	r			

Mergesort

Algorithm: Merge(A[0..n-1], l, r)

```
for i ← l to r do temp[i] ← A[i];
m ← ⌊(l + r)/2⌋;
i1 ← l; i2 ← m + 1;
for curr ← l to r do
    if i1 = m + 1 then
        A[curr] ← temp[i2++];
    else if i2 > r then
        A[curr] ← temp[i1++];
    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

mergesort(
[5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

↓ ↘

$(\_, 0, 1)$ $(\_, 2, 2)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l	r			
temp:	5	2			
	i1	i2			

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$i1 \leftarrow l$; $i2 \leftarrow m+1$;
for $curr \leftarrow l$ **to** r **do**
  if $i1 = m+1$ **then**
    $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

`mergesort([5,2,1,7,0],0,4)`

$\downarrow$
$(\_,0,4)$
$\downarrow$ $\searrow$
$(\_,0,2)$ $(\_,3,4)$
$\downarrow$ $\searrow$
$(\_,0,1)$ $(\_,2,2)$

index:	0	1	2	3	4
A:	5	2	1	7	0
	l	r			
	c				
temp:	5	2			
	i1	i2			

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```
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    else if i2 > r then
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    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 2)$ $(\_, 3, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 1)$ $(\_, 2, 2)$

index:	0	1	2	3	4
A:	2	2	1	7	0
	l	r			
	c				
temp:	5	2			
	i1		i2		

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  else if $temp[i1] \leq temp[i2]$ **then**
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  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
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```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 1) \quad (\_, 2, 2)$

index:	0	1	2	3	4
A:	2	2	1	7	0
	l	r			
		c			
temp:	5	2			
	i1		i2		

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $(\_, 0, 2)$ $\searrow$ $(\_, 3, 4)$

$\downarrow$ $(\_, 0, 1)$ $\searrow$ $(\_, 2, 2)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l	r			
		c			
temp:	5	2			
		i1	i2		

Mergesort

Algorithm: Merge(A[0..n-1], l, r)

```
for i ← l to r do temp[i] ← A[i];
m ← ⌊(l + r)/2⌋;
i1 ← l; i2 ← m + 1;
for curr ← l to r do
    if i1 = m + 1 then
        A[curr] ← temp[i2++];
    else if i2 > r then
        A[curr] ← temp[i1++];
    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

mergesort([5,2,1,7,0],0,4)

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 2)$ $(\_, 3, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 1)$ $(\_, 2, 2)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l	r			
			c		
temp:	5	2			
		i1	i2		

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Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**
    $m \leftarrow \lfloor (l + r)/2 \rfloor$;
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    $Mergesort(A, m + 1, r)$;
    $Merge(A, l, r)$;

`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

$\downarrow \qquad \searrow$

$(\_, 0, 2) \qquad (\_, 3, 4)$

$\downarrow \qquad \searrow$

$(\_, 0, 1) \qquad (\_, 2, 2)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l		r		

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 1) \quad (\_, 2, 2)$

index:	0	1	2	3	4
A:	2	5	1	7	0
			l,r		

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`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l		r		

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    if $i1 = m+1$ **then** $A[curr] \leftarrow temp[i2{+}{+}]$;
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```
mergesort(
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```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \qquad \searrow$

$(\_, 0, 2) \qquad (\_, 3, 4)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l		r		
temp:	2	5	1		
	i1		i2		

Mergesort

Algorithm: Merge(A[0..n-1], l, r)

1. **for** $i \leftarrow l$ **to** r **do** $temp[i] \leftarrow A[i]$;
2. $m \leftarrow \lfloor (l+r)/2 \rfloor$;
3. $i1 \leftarrow l$; $i2 \leftarrow m+1$;
4. **for** $curr \leftarrow l$ **to** r **do**
5. **if** $i1 = m+1$ **then** $A[curr] \leftarrow temp[i2{+}{+}]$;
6. **else if** $i2 > r$ **then** $A[curr] \leftarrow temp[i1{+}{+}]$;
7. **else if** $temp[i1] \leq temp[i2]$ **then**
8. $A[curr] \leftarrow temp[i1{+}{+}]$;
9. **else** $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	2	5	1	7	0
	l		r		
	c				
temp:	2	5	1		
	i1		i2		

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow$ $\searrow$

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	5	1	7	0
	l		r		
	c				
temp:	2	5	1		
	i1			i2	

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        $A[curr] \leftarrow temp[i1{+}{+}]$;
    else if $temp[i1] \leq temp[i2]$ **then**
        $A[curr] \leftarrow temp[i1{+}{+}]$;
    else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \qquad \searrow$

$(\_, 0, 2) \qquad (\_, 3, 4)$

index:	0	1	2	3	4
A:	1	5	1	7	0
	l		r		
		c			
temp:	2	5	1		
	i1			i2	

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Mergesort

Algorithm: Merge(A[0..n-1], l, r)

for $i \leftarrow l$ **to** r **do** $temp[i] \leftarrow A[i]$;
$m \leftarrow \lfloor (l+r)/2 \rfloor$;
$i1 \leftarrow l$; $i2 \leftarrow m+1$;
for $curr \leftarrow l$ **to** r **do**
  if $i1 = m+1$ **then**
    $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_,0,4)$

$\downarrow \quad \searrow$

$(\_,0,2) \quad (\_,3,4)$

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A:	1	2	1	7	0
	l		r		
		c			
temp:	2	5	1		
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---

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```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	1	7	0
	l		r		
			c		
temp:	2	5	1		
		i1		i2	

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

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A:	1	2	5	7	0
	l		r		
			c		
temp:	2	5	1		
			i1	i2	

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
	l		r		
				c	
temp:	2	5	1		
			i1	i2	

Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

```
if l < r then
    m ← ⌊(l + r)/2⌋;
    Mergesort(A, l, m);
    Mergesort(A, m + 1, r);
    Merge(A, l, r);
```

`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
	l				r

Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**

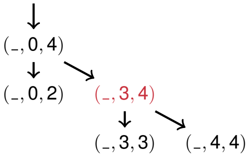
   $m \leftarrow \lfloor (l + r)/2 \rfloor$;

   $Mergesort(A, l, m)$;

   $Mergesort(A, m + 1, r)$;

   $Merge(A, l, r)$;

`mergesort([5,2,1,7,0],0,4)`



index:	0	1	2	3	4
A:	1	2	5	7	0
				l	r

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```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 3, 3) \quad (\_, 4, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
				l,r	

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Mergesort

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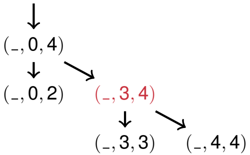
`mergesort([5,2,1,7,0],0,4)`



index:	0	1	2	3	4
A:	1	2	5	7	0
				l	r

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  $Mergesort(A, l, m)$;

  $Mergesort(A, m+1, r)$;

  $Merge(A, l, r)$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

$\downarrow \quad \searrow$

$(\_, 0, 2) \quad (\_, 3, 4)$

$\downarrow \quad \searrow$

$(\_, 3, 3) \quad (\_, 4, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
					l,r

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`mergesort([5,2,1,7,0],0,4)`

$\downarrow$

$(\_, 0, 4)$

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$(\_, 0, 2) \quad (\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
				l	r

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```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
				l	r
temp:				7	0
				i1	i2

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```
mergesort(
    [5,2,1,7,0],0,4)
```

↓
$(\_, 0, 4)$
↓ ↘
$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	7	0
				l	r
				c	
temp:				7	0
				i1	i2

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```
mergesort(
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```

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	0
				l	r
				c	
temp:				7	0
				i1	

$i2 = 5$

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Mergesort

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  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
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```

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	0
				l	r
					c
temp:				7	0
				i1	

$i2 = 5$

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```
mergesort(
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```

$\downarrow$
$(\_, 0, 4)$
$\downarrow \quad \searrow$
$(\_, 0, 2) \quad (\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	7
				l	r
					c
temp:				7	0
					i1

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`mergesort([5,2,1,7,0],0,4)`

↓

$(\_, 0, 4)$

↓ ↘

$(\_, 0, 2)$ $(\_, 3, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	7
				l	r
temp:				7	0
					i1

$c = 5$

$i2 = 5$

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Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**
- $m \leftarrow \lfloor (l + r)/2 \rfloor$;
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- $Merge(A, l, r)$;

```
mergesort(
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```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	7
	l				r

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```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	7
	l				r
temp:	1	2	5	0	7
	i1			i2	

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mergesort(
  [5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	1	2	5	0	7
	l				r
	c				
temp:	1	2	5	0	7
	i1			i2	

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  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

$\downarrow$

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	2	5	0	7
	l				r
	c				
temp:	1	2	5	0	7
	i1				i2

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Mergesort

Algorithm: Merge(A[0..n-1], l, r)

```
for i ← l to r do temp[i] ← A[i];
m ← ⌊(l + r)/2⌋;
i1 ← l; i2 ← m + 1;
for curr ← l to r do
    if i1 = m + 1 then
        A[curr] ← temp[i2++];
    else if i2 > r then
        A[curr] ← temp[i1++];
    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

mergesort(
    [5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	2	5	0	7
	l				r
		c			
temp:	1	2	5	0	7
	i1				i2

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mergesort([5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	5	0	7
	l				r
		c			
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		i1			i2

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```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	5	0	7
	l				r
			c		
temp:	1	2	5	0	7
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```
mergesort(
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```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	0	7
	l				r
			c		
temp:	1	2	5	0	7
			i1		i2

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for $curr \leftarrow l$ **to** r **do**
  if $i1 = m + 1$ **then**
   $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then**
   $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
   $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

`mergesort([5,2,1,7,0],0,4)`

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	0	7
	l				r
				c	
temp:	1	2	5	0	7
			i1		i2

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Mergesort

Algorithm: Merge(A[0..n-1], l, r)

for $i \leftarrow l$ **to** r **do** $temp[i] \leftarrow A[i]$;
$m \leftarrow \lfloor (l+r)/2 \rfloor$;
$i1 \leftarrow l$; $i2 \leftarrow m+1$;
for $curr \leftarrow l$ **to** r **do**
  if $i1 = m+1$ **then**
    $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	5	7
	l				r
				c	
temp:	1	2	5	0	7
				i1	i2

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Mergesort

Algorithm: Merge(A[0..n-1], l, r)

```
for i ← l to r do temp[i] ← A[i];
m ← ⌊(l + r)/2⌋;
i1 ← l; i2 ← m + 1;
for curr ← l to r do
    if i1 = m + 1 then
        A[curr] ← temp[i2++];
    else if i2 > r then
        A[curr] ← temp[i1++];
    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

mergesort([5,2,1,7,0],0,4)

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	5	7
	l				r
					c
temp:	1	2	5	0	7
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  if $i1 = m + 1$ **then**
    $A[curr] \leftarrow temp[i2{+}{+}]$;
  else if $i2 > r$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else if $temp[i1] \leq temp[i2]$ **then**
    $A[curr] \leftarrow temp[i1{+}{+}]$;
  else $A[curr] \leftarrow temp[i2{+}{+}]$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	5	7
	l				r
					c
temp:	1	2	5	0	7
				i1	

$i2 = 5$

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Mergesort

Algorithm: Merge(A[0..n-1], l, r)

```
for i ← l to r do temp[i] ← A[i];
m ← ⌊(l + r)/2⌋;
i1 ← l; i2 ← m + 1;
for curr ← l to r do
    if i1 = m + 1 then
        A[curr] ← temp[i2++];
    else if i2 > r then
        A[curr] ← temp[i1++];
    else if temp[i1] ≤ temp[i2] then
        A[curr] ← temp[i1++];
    else A[curr] ← temp[i2++];
```

`mergesort(`
`[5,2,1,7,0],0,4)`

↓

$(\_, 0, 4)$

index:	0	1	2	3	4
A:	0	1	2	5	7
	l				r
temp:	1	2	5	0	7
				i1	

$i2 = 5$

$c = 5$

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Mergesort

Algorithm: Mergesort(A[0..n-1], l, r)

if $l < r$ **then**
  $m \leftarrow \lfloor (l + r)/2 \rfloor$;
  $Mergesort(A, l, m)$;
  $Mergesort(A, m + 1, r)$;
  $Merge(A, l, r)$;

```
mergesort(
    [5,2,1,7,0],0,4)
```

index:	0	1	2	3	4
A:	0	1	2	5	7

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Divide-and-conquer

Complexity (in general terms):

- $C(n) = aC(n/b) + f(n)$, where $a \geq 1$, $b \geq 1$, and $a \leq b$
- n = size of the problem instance
- b = number of sub-instances
- a = number of sub-instances that need to be solved
- $f(n)$ = dividing and combining effort

Master theorem: if $f(n) \in \Theta(n^d)$ holds, where $d \geq 0$, then:

$$C(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

The same applies for O and Ω

Decrease by a constant factor: $a = 1$

Mergesort: complexity (basic op. = comp. of keys)

Assuming $n = 2^k$

- $C(n) = 2C(n/2) + C_{merge}(n)$ for $n > 1$
- $C(1) = 0$ for $n \leq 1$

Worst case: when it is necessary to interleave both portions of A

Dividing and combining effort: $C_{merge}(n) = (n - 1)$

- Basic operation: number of key comparisons

Therefore

- $C_{worst}(n) = 2C_{worst}(n/2) + n - 1$ for $n > 1$
- $C_{worst}(1) = 0$ for $n \leq 1$

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Mergesort: complexity (basic op. = comp. of keys)

Considering the master theorem

- $C_{worst}(n) = 2C_{worst}(n/2) + n - 1$ for $n > 1$, thus $a = b = 2$
- $f(n) = n - 1 \in \Theta(n) = \Theta(n^d)$, thus $d = 1$
- Since $a = b^d$ (i.e., $2 = 2^1$), $C_{worst}(n) \in \Theta(n^d log\ n) = \Theta(n\ log\ n)$

Pros and cons

- $\Uparrow$: stability
- $\Downarrow$: space efficiency

Complexity of sorting algorithms[2]

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	Θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	Θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	Θ(n+k)	O(n^2)	O(n)
Radix Sort	Ω(nk)	Θ(nk)	O(nk)	O(n+k)
Counting Sort	Ω(n+k)	Θ(n+k)	O(n+k)	O(k)
Cubesort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)

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[2] Source: http://bigocheatsheet.com/

Agenda

1 Selection sort (brute force)

2 Insertion sort (decrease-and-conquer)

3 Mergesort (divide-and-conquer)

4 Bibliography

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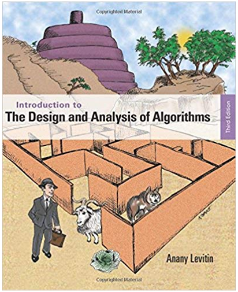

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SELECTION SORT | INSERTION SORT | MERGESORT

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