Technical Appendix

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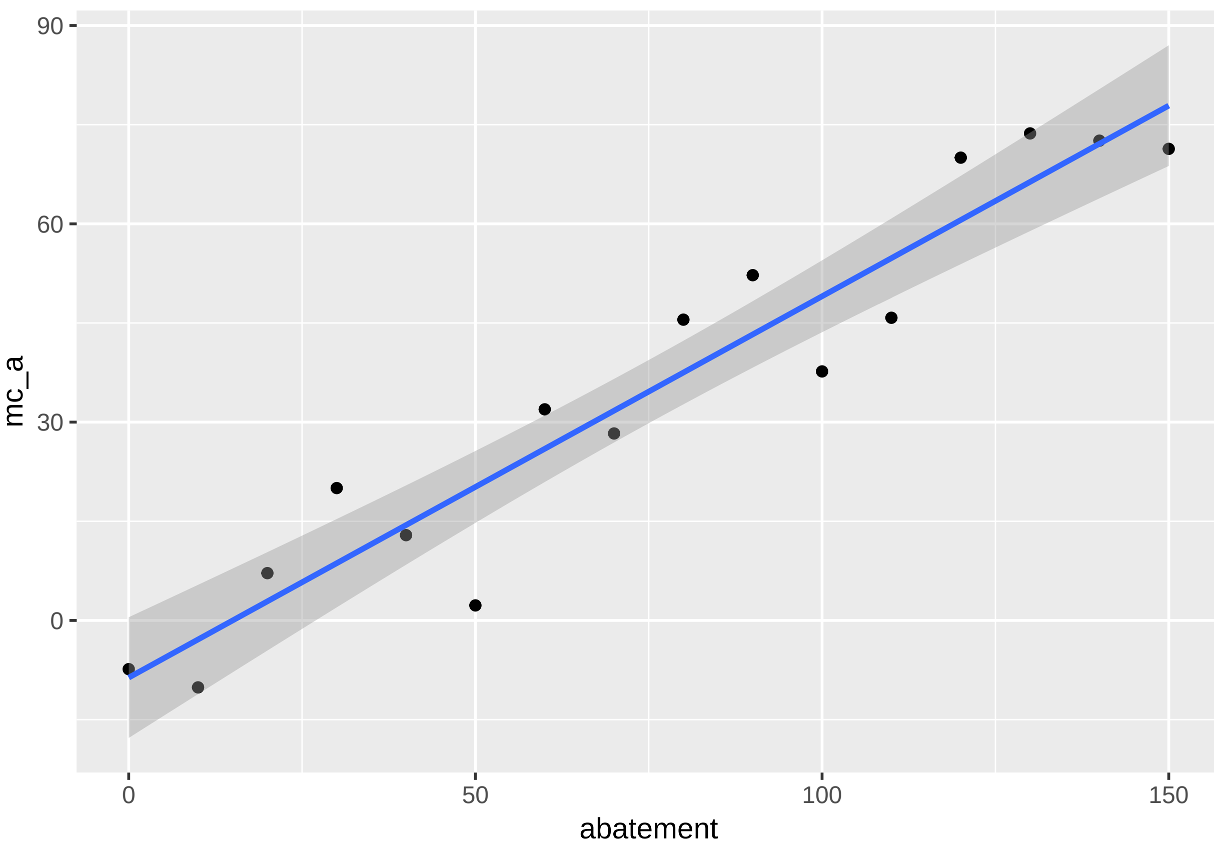
May 29, 2019

Part 1. Functional forms for the marginal cost of abatement by sector

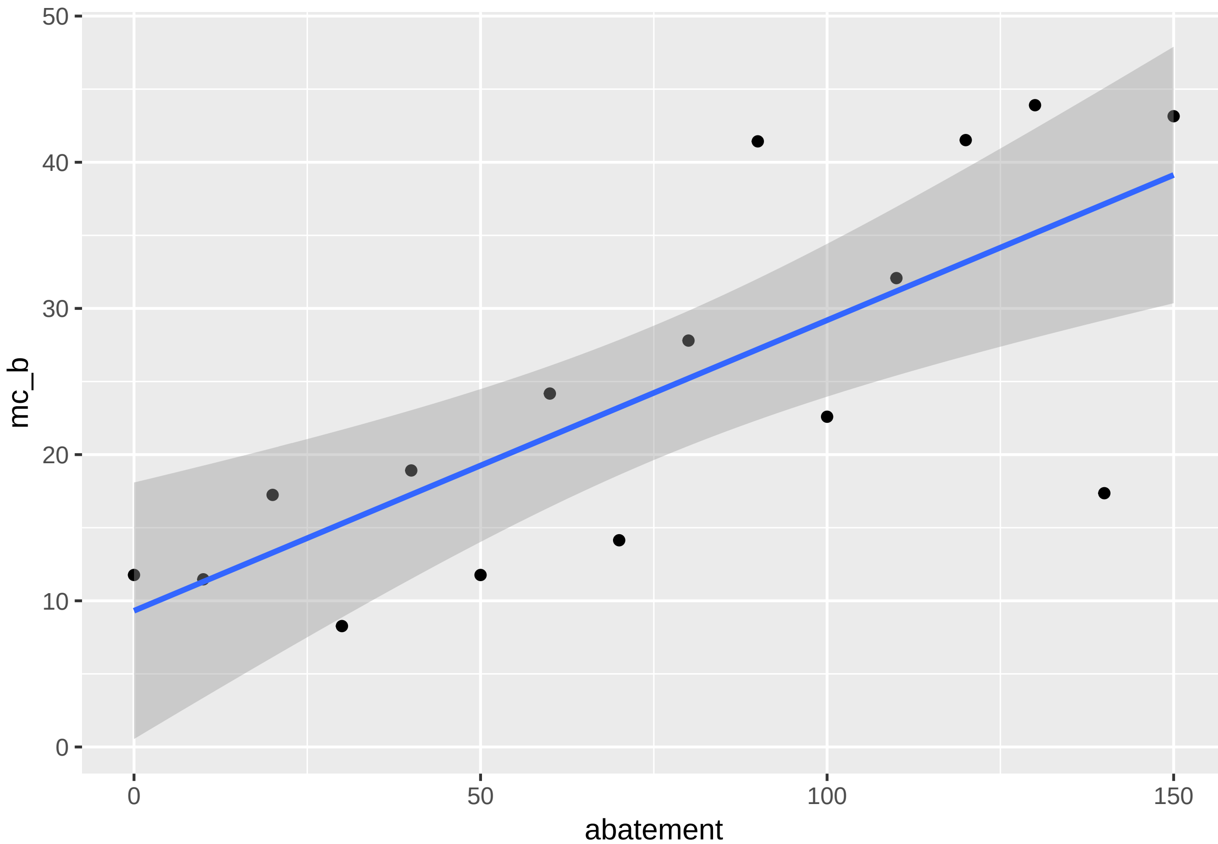
Write down a plausible functional form for the marginal cost of abatement for sector A. Use regression analysis to estimate the parameters of that function. Repeating this for sectors B, C, and D will give you a model of the marginal cost of abatement function for each sector. How well do your models fit the data for each sector? You may need to experiment with different functional forms. Produce a plot of the estimated marginal abatement cost functions in all four sectors (this plot should go in your memo).

Sector A



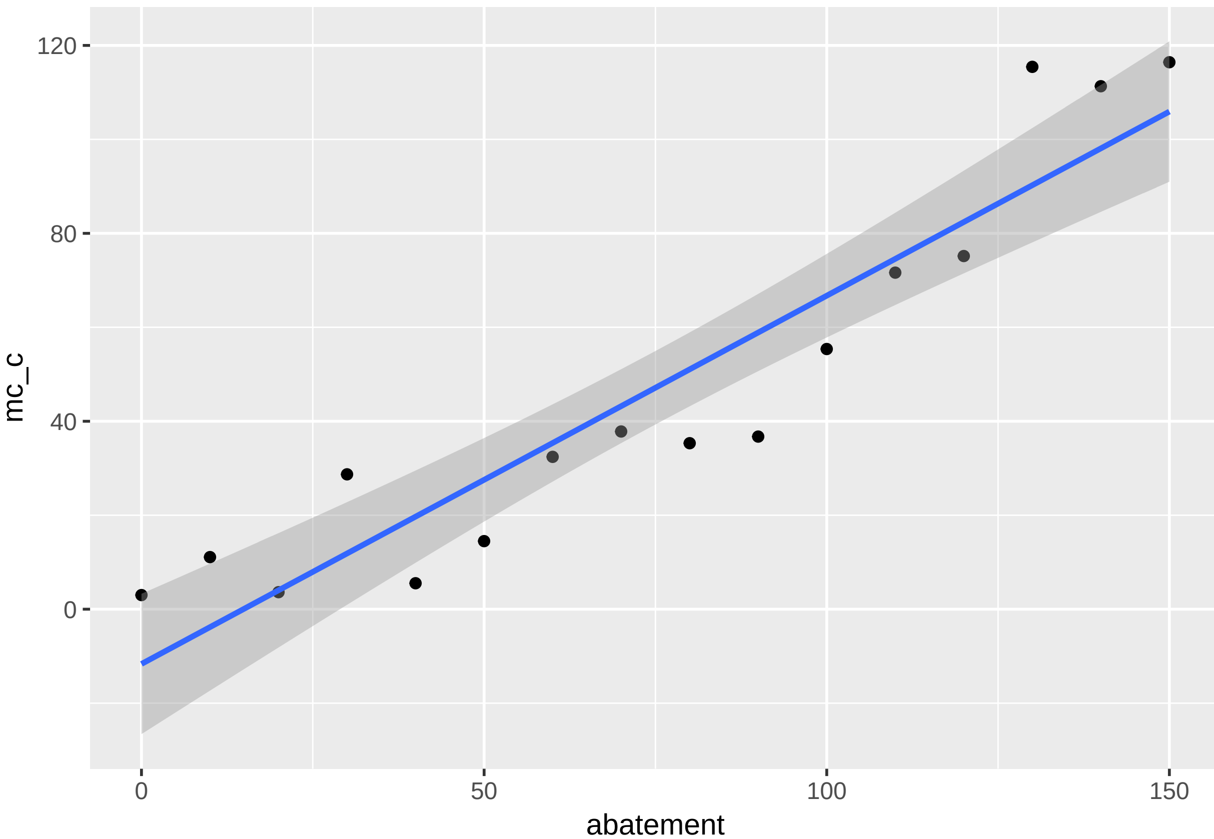
$Sector\ A\ Marinal\ Cost\ of\ Abatement = -8.6444767 + 0.5768419 * x$

Sector B



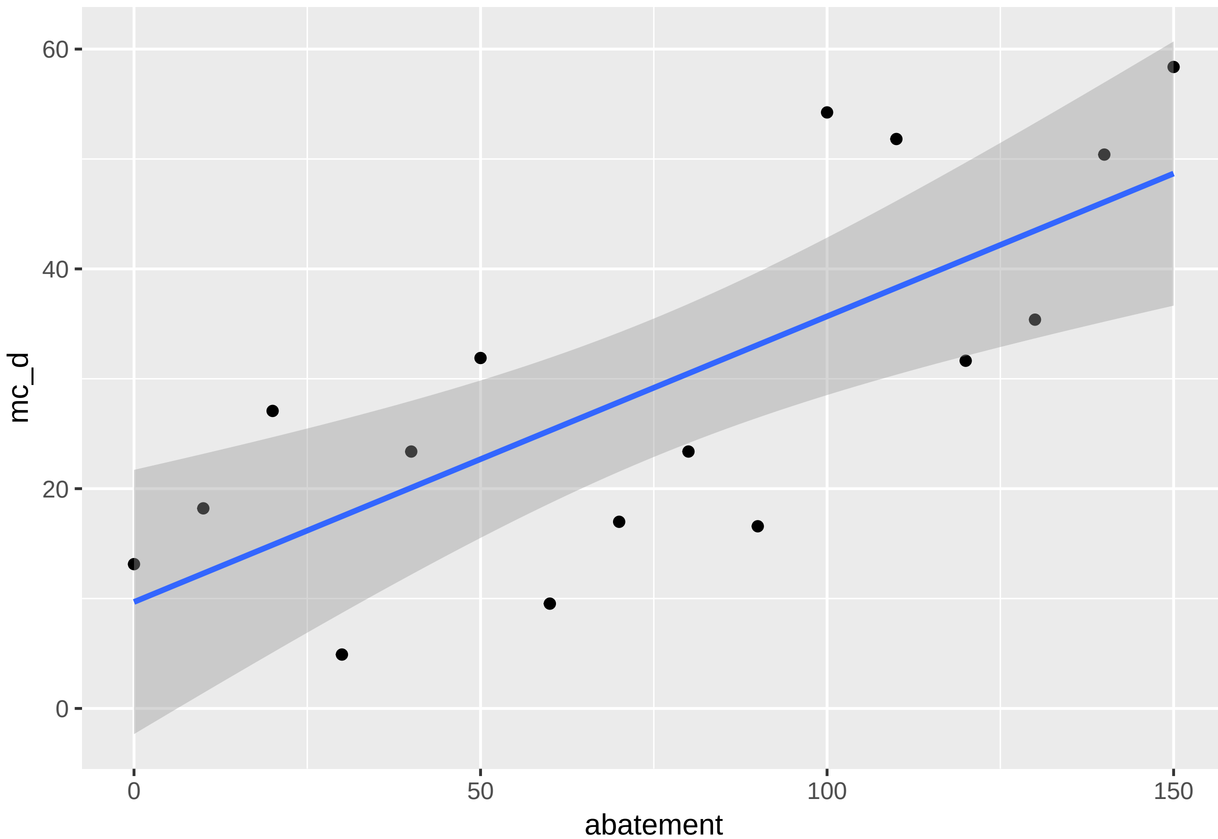
$Sector\ B\ Marinal\ Cost\ of\ Abatement = 9.3176977 + 0.1987443 * x$

Sector C



$Sector\ C\ Marinal\ Cost\ of\ Abatement = -11.6550307 + 0.7838266 * x^2$

Sector D



Add a table to answer: How well do your models fit the data for each sector?

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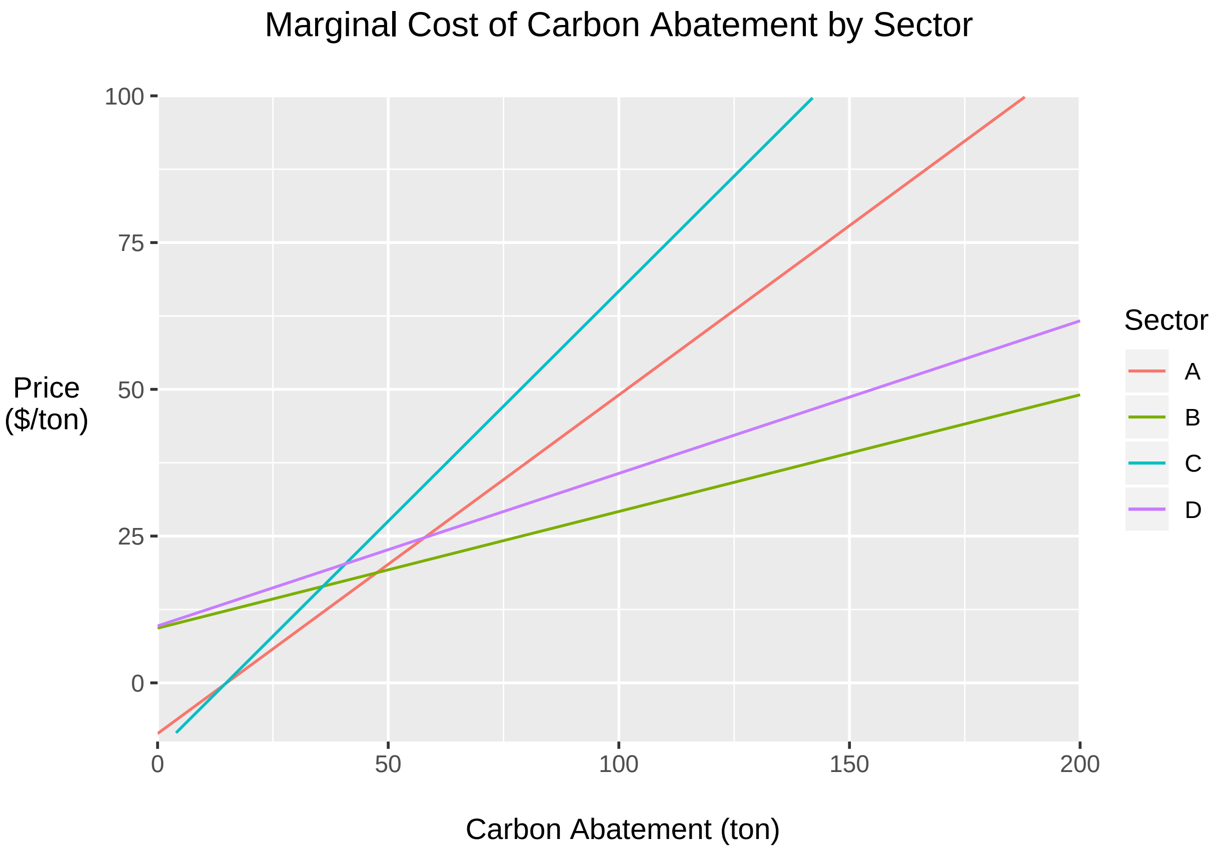
## Warning: Removed 6 rows containing missing values (geom_path).
```

```
## Warning: Removed 31 rows containing missing values (geom_path).
```



Part 2. Derive each sector's demand curve

Using these models and the current level of carbon emissions, derive each sector's demand curve for carbon emissions. In other words, how much would each sector be willing to pay for the right to pollute the first unit, second unit, etc? Draw these demand curves on a graph. Which sector is willing to pay the most for the first unit of carbon emissions?

```
# Demand curve for Sector A

a_demand_int <- a_curve(180)
a_demand_slope <- (0-a_demand_int)/(180-0)

#demand curve as a function
a_demand_curve <- function(x){
  a_demand_int + a_demand_slope*x
}

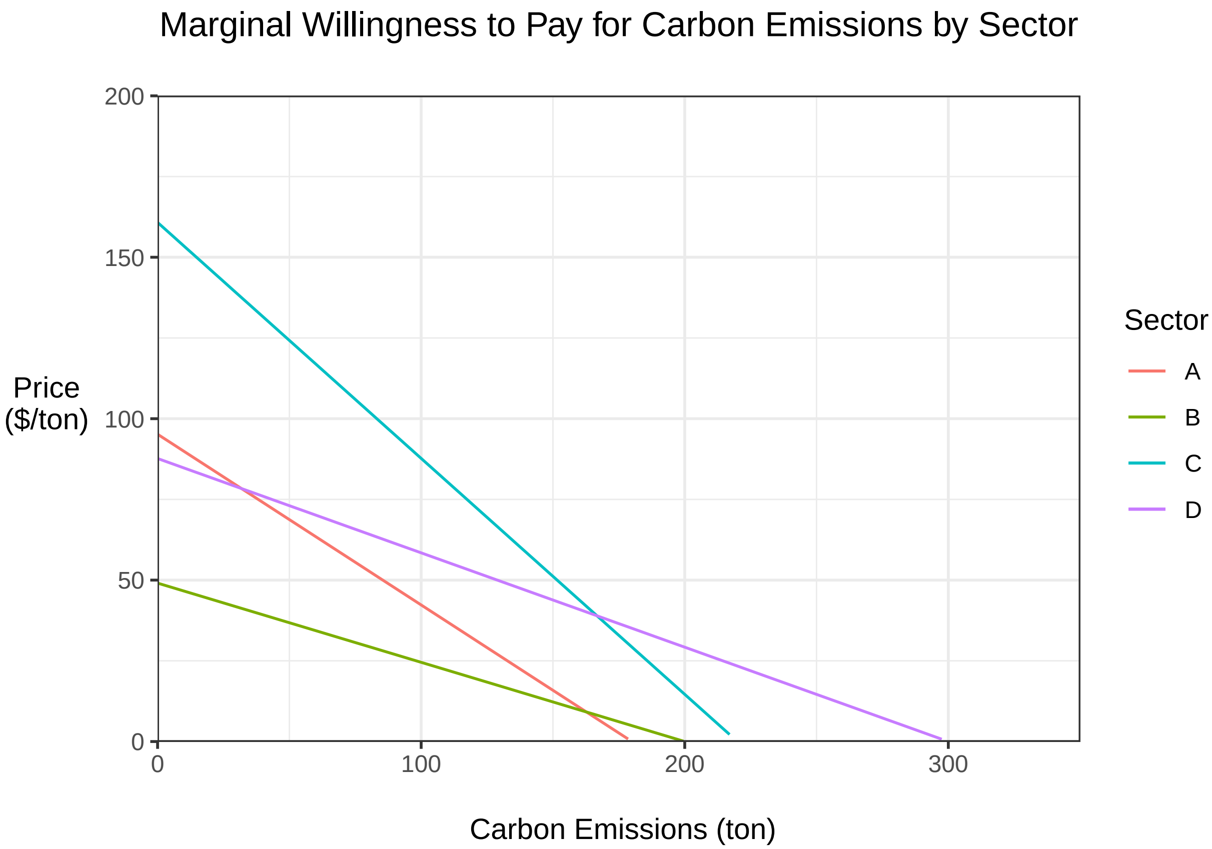
# Demand curve for Sector B

b_demand_int <- b_curve(200)
b_demand_slope <- (0-b_demand_int)/(200-0)

#demand curve as a function
b_demand_curve <- function(x){
  b_demand_int + b_demand_slope*x
}
```

```r
# Demand curve for Sector C

c_demand_int <- c_curve(220)
c_demand_slope <- (0-c_demand_int)/(220-0)

#demand curve as a function
c_demand_curve <- function(x){
  c_demand_int + c_demand_slope*x
}

# Demand curve for Sector D

d_demand_int <- d_curve(300)
d_demand_slope <- (0-d_demand_int)/(300-0)

#demand curve as a function
d_demand_curve <- function(x){
  d_demand_int + d_demand_slope*x
}

# Plot demand curves for each sector

ggplot(data.frame(x=c(0,200))) +
  stat_function(fun=a_demand_curve, geom ="line", aes(color = "A")) +
  stat_function(fun=b_demand_curve, geom ="line", aes(color = "B")) +
  stat_function(fun=c_demand_curve, geom ="line", aes(color = "C")) +
  stat_function(fun=d_demand_curve, geom ="line", aes(color = "D")) +
  labs(x = "Abatement", y = "Price")+
  scale_x_continuous(limits =c(0,350), expand = c(0,0))+
  scale_y_continuous(limits=c(0,200), expand=c(0,0))+
  theme_bw() +
  labs(fill = "", color = "Sector") +
  xlab("\n Carbon Emissions (ton)") +
  ylab("Price \n($/ton) \n") +
  ggtitle("Marginal Willingness to Pay for Carbon Emissions by Sector\n") +
  theme(plot.title = element_text(hjust = 0.5),
        axis.title.y = element_text(angle=0, hjust = 0.5, vjust = 0.5))
```

```
## Warning: Removed 49 rows containing missing values (geom_path).

## Warning: Removed 43 rows containing missing values (geom_path).

## Warning: Removed 38 rows containing missing values (geom_path).

## Warning: Removed 15 rows containing missing values (geom_path).
```



$Sector\ A\ Marinal\ Willingness\ to\ Pay = 95.187063 + -0.528817 * x$
$Sector\ B\ Marinal\ Willingness\ to\ Pay = 49.0665582 + -0.2453328 * x$
$Sector\ C\ Marinal\ Willingness\ to\ Pay = 160.7868179 + -0.7308492 * x$
$Sector\ D\ Marinal\ Willingness\ to\ Pay = 87.665766 + -0.2922192 * x$

Sector C is willing to pay the most for the first unit of carbon emissions.

Part 3. Country X

Now focus on country X (which contains sectors A, B, and C). Assume there are no "co-benefits" from carbon abatement (i.e. that there are no local pollutants that are affected by carbon abatement). Suppose to meet the Paris Accord commitments, country X needs to cut all carbon emissions in half. For each of the policy options listed below, derive: (1) the total cost of meeting the target in country X, (2) the cost (or benefit) to each sector, and (3) the tax revenue generated. a. Cap on carbon. Each sector (A, B, and C) must cut its carbon emissions by 100 tons (thus reducing total emissions from 600 down to 300). b. Tax on carbon. To emit a ton of carbon in country X, you must pay a tax of $t. You will need to find the tax that accomplishes the desired reduction. c. Cap and trade. Each sector (A, B, C) is allocated carbon permits equal to their current emissions minus 100 (same as in (a) above), thus achieving the total reduction of 300. Then, these three sectors are allowed to trade with each other. You will need to derive the outcome of that trading.

```
#Option A. Cap on Carbon - every sector reduces by 100 tons

#A now emits 80 tons
#B now emits 100 tons
#C now emits 120 tons
#All abate 100 tons
```

```r
#1. Total cost

#For each sector - how much does it cost to abate 100th unit?

a_100 <- a_curve(100)
#$49

b_100 <- b_curve(100)
#$29.2

c_100 <- c_curve(100)
#66.7

d_300 <- d_curve(300)
#87.7

#For each sector - how many units do they abate at $0? - for Sectors with abatement y-intercepts below 
#Re-write functions in terms of Abatement (not price)

a_abate <- function(x){(x - a_intercept)/a_slope}
c_abate <- function(x){(x - c_intercept)/c_slope}
b_abate <- function(x){(x - b_intercept)/b_slope}
d_abate <- function(x){(x - d_intercept)/d_slope}

a_0abate <- a_abate(0) #15 units
b_0abate <- b_abate(0) #-46.9 units
c_0abate <- c_abate(0) #14.9 units
d_0abate <- d_abate(0) #-37.3 units

#Find total cost of abatement for each sector

#A total cost
a_cost <- (a_100*(100-a_0abate)*1/2) + (a_0abate*a_intercept*1/2)
#$2020

#B total cost
b_cost <- ((b_intercept + b_100)/2)*100
#$1925

#C total cost
c_cost <- (c_100*(100-c_0abate)*1/2) + (c_0abate*a_intercept*1/2)
#$2776

d_cost <- (d_300*(300-d_0abate*1/2) + (d_0abate*d_intercept*1/2))
#27753

#Total cost for country X
total_cost_x <- c_cost + b_cost + a_cost
#$6721

#No tax revenue generated because no tax

#Total cost for country Y
```

```
total_cost_y <- d_cost
#$27753
```

```
#Option B. Tax on carbon. To emit a ton of carbon in country X, you must pay a tax of $t.
#Need to reduce 300 tons total across all sectors.

#Get aggregate abatement curve. Add curves for 3 sectors horizontally.

#A in terms of abatement
A1 <- 1/a_slope
A2 <- a_intercept/a_slope

#B in terms of abatement
B1 <- 1/b_slope
B2 <- b_intercept/b_slope

#C in terms of abatement
C1 <- 1/c_slope
C2 <- c_intercept/c_slope

#D in terms of abatement
D1 <- 1/d_slope
D2 <- d_intercept/d_slope

#Aggregate in terms of abatement
agg1 <- A1 + B1 + C1
agg2 <- A2 + B2 + C2

#Aggregate in terms of price (P)
P_agg1 <- 1/agg1
P_agg2 <- agg2/agg1

agg_abate<- function(q) {P_agg1*q + P_agg2}

#Optimal tax
tax <- agg_abate(300)
#$39.4


#1. Total Cost of Abatement for each Sector

#A Cost of Abatement#

#How many units?
a_tax_units <- a_abate(39.4)
#83.3 units

a_cost_tax <- (tax*(a_tax_units - a_0abate)*1/2) + (a_0abate*a_intercept*1/2) + ((180-a_tax_units)*tax)
#$5095

#B Cost of Abatement#

#How many units?
```

```r
b_tax_units <- b_abate(39.4)
#151 units

b_cost_tax <- (tax*(b_tax_units - b_0abate)*1/2) + (b_0abate*b_intercept*1/2) + ((200-b_tax_units)*tax)
#$5607

#C Cost of Abatement#

#How many units?
c_tax_units <- c_abate(39.4)
#65.1

d_tax_units <- d_abate(39.4)
#114

c_cost_tax <- (tax*(c_tax_units - c_0abate)*1/2) + (c_0abate*c_intercept*1/2) + ((220-c_tax_units)*tax)
#$7010

d_cost_tax <- (tax*(d_tax_units - d_0abate)*1/2) + (d_0abate*c_intercept*1/2) + ((220-d_tax_units)*tax)
#$7372

#2. Total Cost of abatement for country x
total_cost_tax <- a_cost_tax + b_cost_tax + c_cost_tax
#$17712

#3. Total tax revenue
total_tax_revenue <- ((180-a_tax_units)*tax) + ((200-b_tax_units)*tax) + ((220-c_tax_units)*tax)

#Net Social Benefit under Tax option
nsb_tax <- total_cost_tax - total_tax_revenue
```

```r
#Option C. Cap and trade. Each sector (A, B, C) is allocated carbon permits equal to their current emis

#Who buys, who sells and how much of each? Benefit and cost of trade for each?

#A sells credits
a_market_tons <- 100 - a_tax_units
#16.7 tons

#A benefit from sale
a_credit_benefit <- a_market_tons * tax
#$659

#B buys credits
b_market_tons <- 100 - b_tax_units
#-51.4 tons

#B cost from purchase
b_credit_cost <- b_market_tons * tax
#$2025

#C buys credits
c_market_tons <- 100 - c_tax_units
```

```r
#34.9

#C benefit from sale
c_credit_benefit <- c_market_tons * tax
#$1375

###Total Costs###

#Sector A
a_ct_cost <- (a_100*(100-a_0abate)*1/2) + (a_0abate*a_intercept*1/2) - a_credit_benefit
#$1360

#Sector B
b_ct_cost <- ((b_intercept + b_100)/2)*100 - b_credit_cost
#$3950

#Sector C
c_ct_cost <- (c_100*(100-c_0abate)*1/2) + (c_0abate*a_intercept*1/2) - c_credit_benefit
#$1401

#Total for country X
total_ct_cost <- a_ct_cost + b_ct_cost + c_ct_cost

total_ct_cost
```

```
## (Intercept)
##    6712.839
```

```r
#$6712
```

Part 4. Country Y

Again, without any co-benefits, suppose that country Y (which only has one carbon-emitting sector, D) has no obligation to reduce its emissions. Country X asks country Y to enter the country X carbon market. Doing so would require country Y to put a cap on carbon emissions at its current level of emissions (300 tons), but would then allow country Y to sell carbon offsets to sectors A, B, or C. Are there any incentives for country Y to enter country X's carbon market and to thus place a voluntary cap on its emissions? Are there any incentives for country X to try to attract country Y into its market?

```r
#For sector D - how much does it cost to abate 300th unit?
d_abate <- d_curve(300)
# 87.7
```

Country Y (which only has one carbon-emitting sector, D) has a marginal abatement cost of $87.7, which is less than country X (17,712 dollars). Until both Country X and Country Y both have the same marginal cost of abatement, both countries are better off being in the carbon market.

Incentives for Country Y to enter the the Country X carbon market: Benefit of selling carbon offsets to sectors A, B, and C of in Country X. Incentives for Country X to attract Country Y to the carbon market: Allowing sectors in Country X to emit more carbon by buying carbon offsets from Country Y and avoiding the increased costs of abatement.

Part 5. Air pollution externality

Now assume that every ton of carbon emissions creates 1 ton of local air pollution. Local air pollution causes economic damages (health, environmental, etc.), but only in the country in which it is emitted. Assume there are no local air pollution regulations in either country X or country Y.

a. In a carbon cap and trade market that only covers sectors in country X, how much local air pollution would you expect in country X? In country Y?

We expect 300 tons of pollution in country X given that the cap is set for 300 tons of carbon emissions.

b. If country Y enters the carbon market for country X (as in question 4 above), how much local pollution will there be in country X and country Y?

```
# Aggregate the slopes and intercepts
XY_agg_intercept <- A1 + B1 + C1 + D1
XY_agg_slope <- A2 + B2 + C2 + D2

# In terms of price
P_XYagg_demand_int <- (-XY_agg_intercept/XY_agg_slope)
P_XYagg_demand_slope <- (1/XY_agg_slope)

XY_agg_abate<- function(q) {P_XYagg_demand_int + P_XYagg_demand_slope*q}

XY_cost <- XY_agg_abate(600)
# 10.8
```

c. What advice can you give country X and country Y about the desirability of allowing international trade of carbon emissions credits?