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# FDTD Analysis of Horn Antennas

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## ABSTARCT

The field of antennas is vigorous and dynamic and over the past fifty years, antenna technology has been an indispensable partner of the communication revolution. There is a plethora of antennas and antenna elements, many of which exhibit intricate configurations. With the advent of super fast computers, numerical solution techniques are adapted extensively to analyze the problems with different boundary conditions and complex geometries. Over the past few years, there has been an increased reliance on Finite Difference Time Domain (FDTD) and Finite Element Method (FEM) for the characterization of electromagnetic problems.

## I. INTRODUCTION TO ANTENNAS

The IEEE Standard Definitions of Terms for Antennas (IEE Std 145-1983) defines an antenna as a means for radiating or receiving radio waves. In other words antenna is the transitional structure between free space and a guiding device. The guiding device may take the form of a co-axial line or a waveguide, and is used to transport electromagnetic energy from transmitting source to antenna, or from the antenna to the receiver.

## II. ELECTROMAGNETIC FIELD PROBLEMS

In the arena of Electromagnetics, nothing can move ahead without the involvement of Maxwell's equations. The four Maxwell's equations in are listed below:

Ñ.*D* = Ñ.*B* = 0

Ñ´E=-¶B/¶ *t*

Ñ ´ *H* = *J* + ¶ *B/*¶ *t*

where E and H are electric and magnetic fields respectively; D and B are electric displacement and magnetic flux density; ? A, ? V, ? l are small changes in area ,volume and length and is free charge density.

Field problems are broadly classified as:

· Static Field problems
· Quasi-static Field problems
· Transient Field problems

Work presented in the thesis belongs to "Transient field" problems.

There are two domains in electromagnetism. First, includes the study of electromagnetic waves and propagation, where the displacement current cannot be neglected. This is the *high frequency domain*. Second, includes the major part of electromagnetic devices like transformers, Motors etc. This domain refers to the *low frequency domain*. The work presented in the thesis belongs to "high frequency domain".

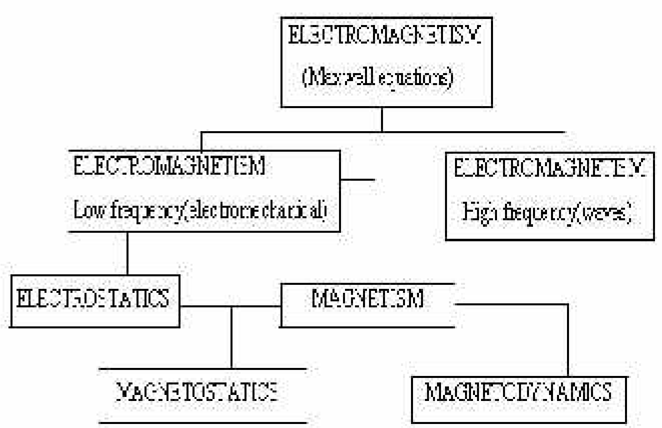


Figure 1.1 Types of field problems

Electromagnetic Field problems can be formulated using Maxwell's equations and need to satisfy certain constraints. These problems along with their constraints are expressed in the form of partial differential equations.
Method of Solution

1. Numerical Method
2. Analogue Methods
3. Graphical Methods
4. Analytical Methods

1. Numerical Methods

Today, computers and numerical methods provide an alternative for complicated calculations. To overcome the drawbacks faced by above listed methods, numerical methods are extensively been used. Different numerical techniques have been developed in recent past. Numerical methods are extremely powerful problem-solving tools. They are capable of handling large systems of equations, nonlinearities and complicated geometries that are not uncommon in engineering practice and they are impossible to solve analytically. As such they greatly enhance our problem solving skills.
Numerical methods can be broadly classified as:

· Finite Difference Method
· Boundary Element Method
· Finite Element Method

## III. FINITE DIFFERENCE TIME DOMAIN METHOD (FDTD)

Finite Difference Method is based on the replacement of a continuous domain with a grid of discrete points (nodes), the only ones at which the values of unknown quantities are to be computed. The idea is to replace derivatives and integrals with 'divided difference' approximations obtained as functions of the nodal values. This can be accomplished for instance, by using interpolating functions, which are not defined in specific sub-domains but simply in the neighborhood of a node. In its traditional versions, the grid is a regular one, i.e., a rectangular grid with nodes at intersection of orthogonal straight lines or a polar grid with nodes at the intersection of orthogonal straight lines or a polar grid with nodes at intersection of orthogonal circles and radii. However, the use of general curvilinear grids has not been successful. Severe difficulties are therefore, encountered in solving many problems by Finite Difference Method, so their efficiency is limited. This is essentially due, apart from other less important problems, to geometrical reasons related to fitting of grid to shapes of boundaries and interfaces involved. In fact, a regular grid is not suitable for problems with very steep variations of fields. Moreover, a regular grid is not suitable for curved boundaries or interfaces, because they intersect grid lines obliquely at points other than nodes. This may not be a problem under Dirichlet boundary conditions, but poses difficulties under Neumann Boundary conditions or under interface conditions involving normal derivatives. These situations require sophisticated interpolation schemes which are difficult to implement in an automatic form, and complicate the solution of algebraic equations resulting from discretization The finite difference method has thus been chosen as the tool for analysis of Pyramidal Horn which does not contain any curved boundary.

The method is arguably the most popular numerical method for the solution of problems in electromagnetics. Although the FDTD method has existed for nearly 30 years, its popularity continues to grow as computing costs continue to decline. The finite difference time domain method (FDTD) proposed by Yee in 1966, is a simple and elegant way to discretize the differential form of Maxwell's equations. Yee used an electric-field (E) grid, which was offset both spatially and temporally from a magnetic-field (H) grid, in terms of the past fields. The update equations are used in leapfrog scheme, to incrementally march the E and H fields forward in time. The original Yee FDTD algorithm is second-order accurate in both space and time. Numerical dispersion and grid-anisotropy errors can be kept small by having a sufficient number of grid spaces per wavelength. Taflove was also the first to present the correct stability criteria for the original orthogonal-grid Yee algorithm. The FDTD method can

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be used to calculate either scattered fields or total fields. When calculating only the scattered fields, the source of the fields is a function field, and the difference in material parameters from those of the background medium [4,5]. When using total fields, the total fields are often calculated only over an interior subsection of the computational domain [6-8], while scattered fields are calculated in the remaining (exterior) portion of the grid. By using scattered fields in this way, the field incident on the absorbing boundary condition is more readily absorbed of the known incident. To obtain this division of the computational domain, into scattered-field and total-field regions, the incident field must be specified over the boundary between these two regions. Holland and Williams presented a comparison of scattered field formulations (i.e., only the scattered fields were computed throughout the computational domain) and total-field formulations (i.e., the total fields were computed in a subdomain that contained the objected under study). They determined, due to numerical dispersion, that the total-field FDTD approach is superior to the scattered- field approach. Furthermore, the scattered-field approach has the disadvantage that it does not easily accommodate nonlinear media. The relative merits of the total-field and scattered-field formulations were also explored by Fang. Early techniques have included differential based absorbing boundary conditions (ABCs), such as those proposed by Merewether, Engquist and Madja , Lindman and Mur. These early techniques were vastly improved in the mid-1980s by formulations proposed by Higdon ,Liao et al and Keys .Many other extensions of these differential-based ABCs have since been proposed. One of the majoradvantages the FDTD method has over other numerical techniques is the ability to obtain wideband results using transient excitation. To obtain accurate results over a broad spectrum, it is often necessary to include the frequency-dependent properties of the material (i.e., it may not be possible to treat the permittivity, conductivity, or permeability as constants over the entire spectrum). Several techniques have been proposed to account for this frequency dependence. Furthermore, the FDTD algorithm has been extended to account for materials that are anisotropic and nonlinear. The analysis and design of antennas, using the FDTD method, have received considerable attention recently, and are areas of growing activity. In 1990, Maloney et al. presented accurate results for the radiation from rotationally symmetric simple antennas, such as cylindrical and conical monopoles. Boonzaaier and Pistorius presented results for the radiation from thin-wire dipoles and thin-wire Yagi antennas. More recently, Yagi antennas were also studied by Kashiwa et al.. In 1992, Tirkas and Balanis presented results for a monopole on a ground plane, while Luebbers and co-workers presented mutual-coupling and gain computations for a pair of wire dipoles. Subsequently, Maloney et al. presented a simple one-dimensional approximate TEM-feed model for the FDTD method, and used it to analyze a monopole backed by a plane reflector. In 1991, Katz et al. used the FDTD method to analyze both two-dimensional and three dimensional horn antennas, while a year later, Tirkas and Balanis also analyzed three- dimensional horn antennas In 1994, Tirkas and Balanis extended the contour-path FDTD method , and used it to analyze pyramidal horns withcomposite inner E-plane walls. With recent technological advancements, antenna elements have become smaller whereas the platforms they operate on become electrically larger. Traditional finite methods (FDTD and FEM) are second-order accurate thereby restricting the size of the domains that can be handled efficiently. In 2002,Georgakopoulos , Renaut ,Balanis, Birtcher and Panaretos proposed an approach which combined a subgridding technique with a higher-order scheme. FDTD subgridding techniques divided the simulation space into two separate grids; a fine one and a coarse one. The standard FDTD (2,2) was used to handle any of the fine features of the structure, whereas on the coarse grid FDTD(2,4), which is second-order accurate in time and fourth-order accurate in space, was used. Thus existing successfully-applied techniques in FDTD (2,2) were available for use on the fine grid. On the coarse mesh, away from phenomena associated with the complex structure, FDTD (2,4) was used mainly to simulate wave propagation in homogeneous media. With this approach, high accuracy was obtained both around fine geometric features, such as thin wires, thin slots, etc., as well as in the wave propagation. The numerical dispersion property of the two-dimensional alternating-direction implicit finite difference time-domain (2D ADI FDTD) method was studied by An Ping Zhao [30]in 2002. The original 2D ADI FDTD method was divided into two sub-ADI FDTD methods: the x-directional 2D ADI FDTD method or the y-directional 2D ADI FDTD method; methods: and secondly, the numerical dispersion relations were derived for both the ADI FDTD methods. Finally, the numerical dispersion errors caused by the two ADI FDTD methods were investigated. Numerical results indicated that the numerical dispersion error of the ADI FDTD methods depends highly on the selected time step and the shape and mesh resolution of the unit cell. It is also found that, to ensure the numerical dispersion error within certain accuracy, the maximum time steps allowed to be used in the two ADI FDTD methods are different and they can be numerically determined. The numerical performance of the envelope ADI–FDTD method and its applications were studied by C.T.M Choi and S.H Sun [31] in 2006. T. Kasuga and H. Inoue[32] proposed a novel FDTD simulation method using multiple-analysisspace for electromagnetic far field calculations in antennas in 2005. In 2006, S .Collardey, A. Sharaiha, and K.Mahdjoubi [33] calculated quality factor of small and complex antennas using FDTD Method by measuring the energy stored in them. FDTD was also used for analysis of electromagnetic crystal-based antenna for ultra - fast radiolocation applications by F. Ghanem, G.Y. Delisle and T.A. Denidni[34] in 2006. Jean – Pierre Berenger [46] suggested perfectly matched layer for the FDTD solution of wave structures interaction problems in 1999.

## IV. IMPLEMENTATION OF FIELD PATTERN OF PYRAMIDAL HORN IN FDTD

Electric and magnetic field intensities in a pyramidal horn**.**

In the present work, the pertinent equations for the far-zone radiated electromagnetic fields of a pyramidal horn and directivity expressions are presented.

These expressions are then implemented using a MATLAB simulation program. The horn-antenna geometry is shown in Figure 1.2. Figure 1.3 shows the E- and H-plane cross-sectional views. The radiation pattern for the pyramidal-horn antenna is well known. However, implementation of the radiation-pattern equations requires the programming of several integrals, which is done in MATLAB.

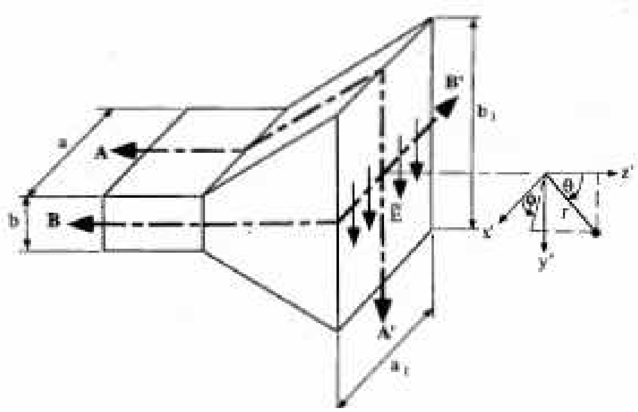

Figure 1.2 An Isotropic drawing of Pyramidal horn

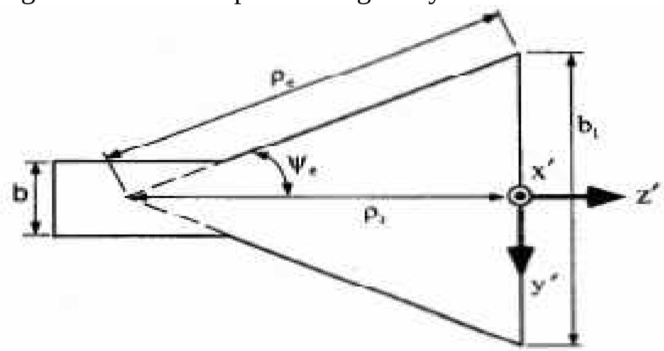


Figure 1.3 E Plane Cross-section of pyramidal horn

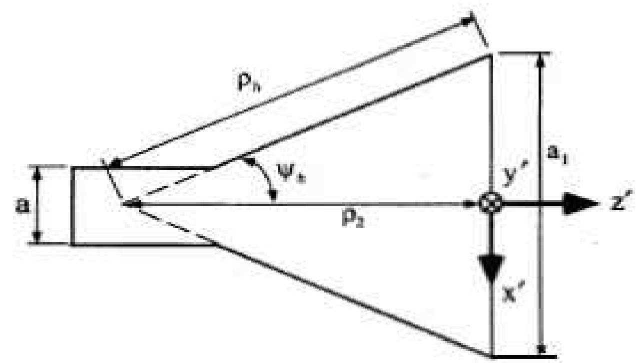


Figure 1.4 H Plane Cross-section of pyramidal horn

The far-field pattern is obtained using aperture integration. The tangential components of the E field over the aperture of the horn are approximated by

$$\bar{H} = \hat{z}\times\frac{\bar{E}}{\eta} = -\hat{x}\frac{E_0}{\eta}\cos\left(\frac{\pi}{a_1}x'\right)e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}+\frac{y'^2}{\rho_1}\right)} \quad (1.1)$$

where the plane wave propagates in the z direction. These fields can be used to construct equivalent electric and magnetic current densities as

$$\bar{M}_{x'} = -\hat{z}\times\bar{E} = \hat{x}E_0\cos\left(\frac{\pi}{a_1}x'\right)e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}+\frac{y'^2}{\rho_1}\right)} \quad (1.2)$$

$$\bar{J}_{y'} = \hat{z}\times\bar{H} = -\hat{y}\frac{E_0}{\eta}\cos\left(\frac{\pi}{a_1}x'\right)e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}+\frac{y'^2}{\rho_1}\right)} \quad (1.3)$$

over the aperture, which are used for computing the radiation pattern. As seen, the equivalent electric and magnetic currents have a sinusoidal amplitude distribution in the x′ direction, and quadratic phase variations in both the x' and y′ directions of the form

$$e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}+\frac{y'^2}{\rho_1}\right)} \quad (1.4)$$

## V. FAR FIELD COMPUTATIONS USING APERTURE INTEGRATION

The far-field expressions are determined by integrating the equivalent currents over the aperture Area. These are:

$$E_r \approx 0, \quad (1.5)$$

$$E_\theta \approx -\frac{jke^{-jkr}}{4\pi r}\left(L_\phi + \eta N_\theta\right) \quad (1.6)$$

$$E_\phi \approx \frac{jke^{-jkr}}{4\pi r}\left(L_\theta - \eta N_\phi\right) \quad (1.7)$$

where,

$$N_\theta = \int_{-\frac{b_1}{2}}^{\frac{b_1}{2}}\int_{-\frac{a_1}{2}}^{\frac{a_1}{2}} J_y(x',y')\cos\theta\sin\phi\, e^{+jk(x'\sin\theta\cos\phi+y'\sin\theta\sin\phi)}dx'dy' \quad (1.8)$$

$$N_\phi = \int_{-\frac{b_1}{2}}^{\frac{b_1}{2}}\int_{-\frac{a_1}{2}}^{\frac{a_1}{2}} J_y(x',y')\cos\phi\, e^{+jk(x'\sin\theta\cos\phi+y'\sin\theta\sin\phi)}dx'dy'$$

$$L_\theta = \int_{-\frac{b_1}{2}}^{\frac{b_1}{2}}\int_{-\frac{a_1}{2}}^{\frac{a_1}{2}} M_x(x',y')\cos\theta\cos\phi\, e^{+jk(x'\sin\theta\cos\phi+y'\sin\theta\sin\phi)}dx'dy'$$

(1.9, 1.10)

$$L_\phi = -\int_{-\frac{b_1}{2}}^{\frac{b_1}{2}}\int_{-\frac{a_1}{2}}^{\frac{a_1}{2}} M_x(x',y')\sin\phi\, e^{+jk(x'\sin\theta\cos\phi+y'\sin\theta\sin\phi)}dx'dy' \quad (1.11)$$

The PML technique is described here as a technique of free-space simulation to be used with the finite-difference time-domain method .The numerical experiments reported in have shown that this technique is very efficient for absorbing the electromagnetic waves and then for solving unbounded problems. For solving interaction problems with the finite-difference method, various techniques have been used in the past to absorb the outgoing waves, such as the matched layer or the one-way approximation of the wave equation .To obtain satisfactory solutions, the absorbing boundaries must be set at some distance from the scattering structure with the result that most of the computational domain is a surrounding vacuum. Correct solutions were computed with the PML layer set only two cells from the scatterer, showing that the computational domain may drastically reduce while using the PML technique. Unfortunately, due to the presence of some numerical reflection, such an achievement cannot be obtained using any PML layer. When the PML layer is set very close to the scatterer**,** the results are exact only if the thickness of the layer equals at least a certain value which appears to be directly related to the problem to be solved. The consequence is that the PML technique is not an efficient nor a reliable method of free-space simulation, as long as the relation between layer thickness and problem of concern is not clarified. If the layer is chosen at random, either the results may be erroneous or the computational requirements may be needlessly great. For this reason, the numerical reflection produced by PML layers was analyzed by Berenger. The parameters goveming this reflection were found, allowing an optimum PML layer to be specified in such a way that accurate solutions can be obtained with computational requirements as small as possible. This work was mainly based on the observation and analysis of numerical experiments, in both 2-D and 3-D cases. For simplicity and compactness, only the 2-D case is considered in the present work for the pyramidal horn analysis.

## VI. RESULT

Analysis of Pyramidal horn antenna in FDTD MATLAB M-file is written which produces the FDTD solution of a sectoral (2-D) Perfectly Electric Conducting (PEC) horn antenna excited by a sinusoidal voltage in a TEz computational domain. The computational domain is truncated with a Berenger Perfectly Matched Layer (PML) absorbing boundary conditions whose depth in layers is set by the variable number of PML's 'npmls'. The PML is introduced to eliminate reflections from the grid truncation and to simulate an outgoing traveling wave propagating in an unbounded medium. The M-file creates of the computational domain every 3rd time step. The horn is modeled by setting the necessary FDTD update equation coefficients to represent PEC material (sigma=infinity). Far-field pattern as amplitude level for E and H plane are obtained as a function of elevation angle.

The cell size of the space is: dx = 0.0025 meters

The time step is: dt = 4.23e-12 seconds

The frequency of excitation is: freq = 10 GHz
The wavelength is: lambda = 12*dx = 0.0305 meters.

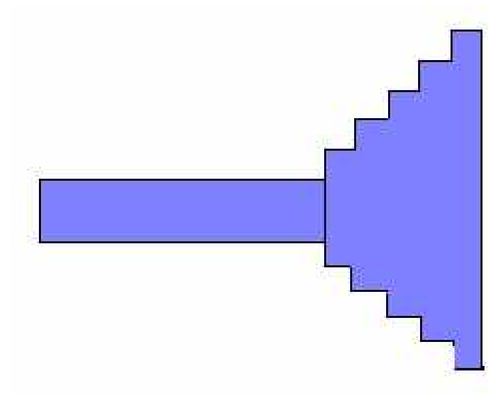

Figure 1.5 Modeled corrugated pyramidal horn geometry

The flare section of the horn is staircased. As modeled, the horn looks like in Figure 1.5. The following results were obtained after simulation.

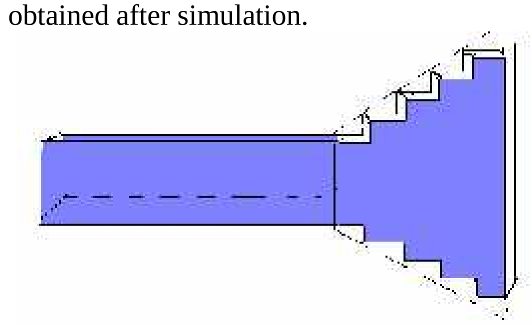

Figure 1.6 Actual geometry of pyramidal horn

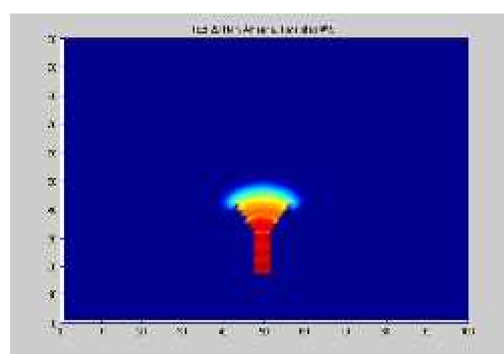

Figure 1.7 Radiation pattern at time step 45

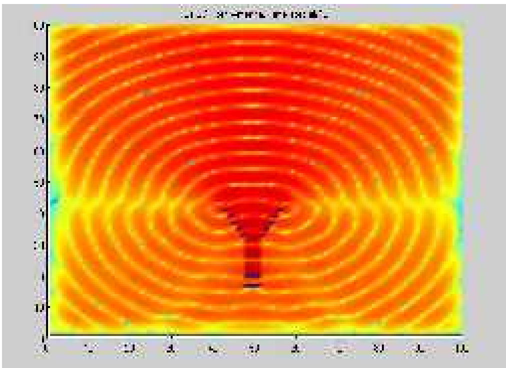

Figure 1.8 Radiation pattern at time step 210.

## REFERENCES

**[1]**. A. Taflove, “Review of the formulation and applications of the finite- difference time-domain method for numerical modeling of electromagnetic wave interactions with arbitrary structures,” *Wave Motion*, 10, 6, pp. 547-582, 1988.

**[2]**. K. S. Kunz and R. J. Luebbers, “The Finite Difference Time Domain Method for Electromagnetics”, *Boca Raton, FL, CRC Press*, 1993.

**[3]**. R. Luebbers and K. Kunz, “Finite difference time domain calculations of antenna mutual coupling,” *IEEE Transactions on Electromagnetic Compatibility*, EMC-34, 3, pp. 357-359, 1992.

**[4]**. R. J. Luebbers and J. Beggs, “FDTD calculation of wide-band antenna gain and efficiency,”*IEEE Transactions on Antennas and Propagation*, AP-40, 1 1 , pp.1403-1407, 1992.

**[5].** J. Fang, “Time Domain Finite Difference Computation for Maxwell’s Equations,” *PhD thesis University of California at Berkeley*, Berkeley, CA, 1989