Lecture V: Problems with Experiments: Non-compliance

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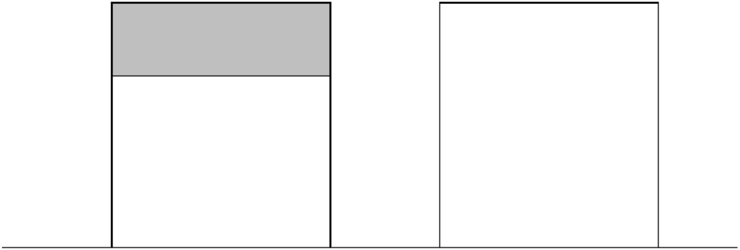
1. Introduction

- Experiments are subject to implementation issues that have profound consequences in the way they are analyzed and interpreted
- We focus on this lecture on non-compliance or failure to treat
  - Some of the units assigned to the treatment group do not receive the treatment (known as **one-side non-compliance**)
  - Some units assigned to treatment group go untreated and some assigned to the control group receives the treatment (**two-sided non-compliance**)
- Some examples?
- What's wrong with non-compliance?

What to compare?

- Example: Campaign to increase children vaccination
  - 2,000 households randomly divided in two groups (1,000 treated)
  - Only 300 households were actually treated by the campaign
  - 3 groups: 300 assigned to treatment and treated, 700 assigned to treatment but untreated, and 1,000 untreated. What would you compare?



Sources of non-compliance (Glennerster et al 2013)

- Some people in the treatment group are not treated
- People in the treatment group do not complete the treatment
- People in the control group receive the treatment
- Partner organization does not comply with the protocols for the delivery of the treatment
- People exhibit the opposite of compliance

2. Angrist-Imbens-Rubin Causal Model

- The model was proposed by Angrist, Imbens and Rubin (1996). The basic notation is the following:
  - i is an index for individuals in a population of size N
  - D_i is the treatment level
    - $D_i = 1$ if individual has been exposed to treatment
    - $D_i = 0$ if individual has not been exposed to treatment
  - $Y_i(D_i)$ is the outcome of interest
    - $Y_i(1)$ is the outcome in case of treatment
    - $Y_i(0)$ is the outcome in case of no treatment
  - Z is assignment to treatment
    - $Z_i(1)$ is the assignment to treatment status
    - $Z_i(0)$ is the assignment to no treatment status
- Participation into treatment: $D_i = D_i(Z)$

- Outcome: $Y_i = Y_i(Z, D)$
- Notice that 3 causal effects can be defined:
  - The effect of assignment Z on treatment D
  - The effect of assignment Z on outcome Y
  - The effect of treatment D on outcome Y
- The first two are called "intention to treat" (ITT) effects
- The AIR model defines the set of assumptions that ensures the identification of these effects

Assumptions of the AIR causal model

Assumption A.1: Stable treatment unit value assumption (SUTVA)

The potential outcomes and treatments for unit i are independent of the potential assignments, treatment and outcomes of unit $j \neq i$. Therefore:

$$D_i(Z) = D_i(Z_i) \tag{1}$$

$$Y_i(Z, D) = Y_i(Z_i, D_i) \tag{2}$$

- Then, we can define the ITT effects as follows:

Definition 1: Causal effect of Z on D

The causal effect of Z on D for unit i is $D_i(1) - D_i(0)$

Definition 2: Causal effect of Z on Y

The causal effect of Z on Y for unit i is $Y_i(1, D_i(1)) - Y_i(0, D_i(0))$

- Counterfactual logic requires to think for each individual!
  - Potential Y: $Y_i(1,1)$, $Y_i(1,0)$, $Y_i(0,1)$, $Y_i(0,0)$
  - Potential D: $D_i(0) = 0, D_i(0) = 1, D_i(1) = 0, D_i(1) = 1$
  - Potential Z: $Z_i = 0, Z_i = 1$
- Only one state is actually observed!
- If SUTVA holds, then we can classify individuals as follows:

Table 1: Classification of individuals according to assignment and treatment

		$Z_i = 0$	
		$D_i(0) = 0$	$D_i(0) = 1$
$Z_i = 1$	$D_i(1) = 0$	*Never-taker*	*Defier*
	$D_i(1) = 1$	*Complier*	*Always-taker*

Assumption A.2: Random assignment

All individuals have the same probability to be assigned to the treatment:

$$Pr(Z_i = 1) = Pr(Z_j = 1) \quad (3)$$

- Using A.1 and A.2, we can consistently estimate the two ITT effects:

$$ITT_{D,Z} = \mathbb{E}(D_i/Z_i = 1) - \mathbb{E}(D_i/Z_i = 0) = \frac{Cov(D_i Z_i)}{Var(Z_i)} \quad (4)$$

$$ITT_{Y,Z} = \mathbb{E}(Y_i/Z_i = 1) - \mathbb{E}(Y_i/Z_i = 0) = \frac{Cov(Y_i Z_i)}{Var(Z_i)} \quad (5)$$

- Note that the ratio between (4) and (5) gives the conventional IV estimator:

$$\beta_{IV} = \frac{\frac{Cov(Y_i Z_i)}{Var(Z_i)}}{\frac{Cov(D_i Z_i)}{Var(Z_i)}} = \frac{Cov(Y_i Z_i)}{Cov(D_i Z_i)} \tag{6}$$

- Questions:
  - Under which assumptions this IV estimator gives an estimate of the average causal effect of D on Y and for which group?
  - Does this estimate depends on the instrument we use?

Assumption A.3: Non-zero average causal effect of Z on D

The probability of treatment must be different in the two assignment groups:

$$Pr(D_i(1) = 1) \neq Pr(D_i(0) = 1) \tag{7}$$

- This is similar to the requirement of having the instrument correlated with the endogenous regressor

Assumption A.4: Exclusion restriction

The assignment affects the outcome only through the treatment,

$$Y_i(1, D_i) = Y_i(0, D_i) = Y_i(D_i) \tag{8}$$

- As in the standard IV case, A.3 can be tested but A.4 cannot

Definition 3: Causal effect of D on Y

The causal effect of D on Y for unit i is $Y_i(1) - Y_i(0)$

- Again: we cannot compute this because counterfactual is not observed!
- Solution: compare sample averages of the two components for individuals who are in the two treatment groups only because of different assignments (compliers and defiers)
- Are these assumptions enough?

$$\underbrace{Y_i(1, D_i(1)) - Y_i(0, D_i(0))}_{\text{Z} \rightarrow \text{Y}} = Y_i(D_i(1)) - Y_i(D_i(0))$$

$$\begin{aligned}
&= [Y_i(1)D_i(1) + Y_i(0)(1 - D_i(1))] \\
&- [Y_i(1)D_i(0) + Y_i(0)(1 - D_i(0))] \\
&= \underbrace{[D_i(1) - D_i(0)]}_{\text{Z} \rightarrow \text{D}} \underbrace{[Y_i(1) - Y_i(0)]}_{\text{D} \rightarrow \text{Y}}
\end{aligned}$$

- This holds at an individual level!
- Using sample averages:

$$\underbrace{\mathbb{E}[Y_i(1, D_i(1)) - Y_i(0, D_i(0))]}_{\mathrm{Z} \to \mathrm{Y}} = \mathbb{E}[\underbrace{[D_i(1) - D_i(0)]}_{\mathrm{Z} \to \mathrm{D}} \underbrace{[Y_i(1) - Y_i(0)]}_{\mathrm{D} \to \mathrm{Y}}]$$

$$= \mathbb{E}[Y_i(1) - Y_i(0) / D_i(1) - D_i(0) = 1] Pr(D_i(1) - D_i(0) = 1)$$

$$- \mathbb{E}[Y_i(1) - Y_i(0) / D_i(1) - D_i(0) = -1] Pr(D_i(1) - D_i(0) = -1)$$

- We still have an identification problem! (average effect for compliers may cancel with average effects for defiers)

Assumption A.5: Monotonicity

No one does the opposite of her assignment, no matter what the assignment is:
$D_i(1) \geq D_i(0)$, $\forall i$

- ATE for defiers in zero
- Notice that A.3 + A.5 implies strong monotonicity
  - There is no defiers
  - There exists at least one complier

Local average treatment effect (*LATE*)

- Given A.5, we can write equation 7 as:

$$\underbrace{\mathbb{E}[Y_i(1, D_i(1)) - Y_i(0, D_i(0))]}_{\text{Z} \to \text{Y}} = \mathbb{E}[\underbrace{[D_i(1) - D_i(0)]}_{\text{Z} \to \text{D}} \underbrace{[Y_i(1) - Y_i(0)]}_{\text{D} \to \text{Y}}]$$
$$= \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) - D_i(0) = 1]Pr(D_i(1) - D_i(0) = 1)$$

- Re-arranging this expression, we obtain an expression for *LATE*:

$$\mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) - D_i(0) = 1] = \frac{\mathbb{E}[Y_i(1, D_i(1)) - Y_i(0, D_i(0))]}{Pr(D_i(1) - D_i(0) = 1)}$$

Definition 4: *LATE*

LATE is the average effect of treatment for those who change treatment status because of a change of the instrument; i.e. the average effect of treatment for compliers

- It can be shown that:

$$\mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) - D_i(0) = 1] = \frac{Cov(Y_i Z_i)}{Cov(D_i Z_i)} = \beta_{IV} \quad (9)$$

- *IV* estimand is the *LATE*. *LATE* is the only treatment effect that can be estimated by *IV*

3. One-Side Non-Compliance

- When one-side non-compliance, the sample is composed only by **compliers** and **never-takers**
- Required assumptions are also simplified:
  - *SUTVA*
  - Random Assignment
  - Exclusion Restriction
  - Non-zero causal effect of Z on D
- What about monotonicity?
- *LATE* is also simplified:

$$\mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) = 1] = \frac{Cov(Y_i Z_i)}{Cov(D_i Z_i)} = \beta_{IV} \qquad (10)$$

Example: Potential Outcomes under One-Side Non-Compliance

Individual	Y(0) Bribes if no audit	Y(1) Bribes if audit	D(0) Assigned to no audit	D(1) Assigned to audit	Type
1	4	6	0	1	Complier
2	2	8	0	0	Never-Taker
3	1	5	0	1	Complier
4	5	7	0	1	Complier
5	6	10	0	1	Complier
6	2	10	0	0	Never-Taker
7	6	9	0	1	Complier
8	2	5	0	1	Complier
9	5	9	0	0	Never-Taker

- Computing ATE:

$$ATE = \mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y_i(1)] - \mathbb{E}[Y_i(0)] = 7.67 - 3.67 = 4$$

- Computing ITTs:

$$\begin{aligned} ITT_{Y,Z} &= \mathbb{E}[Y_i/Z_i = 1] - \mathbb{E}[Y_i/Z_i = 1] \\ &= \mathbb{E}[Y_i(1)/Z_i = 1] - \mathbb{E}[Y_i(0)/Z_i = 1] \\ &= (2+0+4+2+4+0+3+3+0)/9 = 2 \end{aligned}$$

$$\begin{aligned} ITT_{D,Z} &= \mathbb{E}[D_i/Z_i = 1] - \mathbb{E}[D_i/Z_i = 1] \\ &= \mathbb{E}[D_i(1)/Z_i = 1] - \mathbb{E}[D_i(0)/Z_i = 1] \\ &= (1+0+1+1+1+0+1+1+0)/9 = \frac{2}{3} \end{aligned}$$

- Computing LATE:

$$
\begin{aligned}
LATE &= \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) = 1] \\
&= (2 + 4 + 2 + 4 + 3 + 3)/6 = 3
\end{aligned}
$$

- Of course, only one potential outcome is observable for one assignment
- Recall:

$$
\begin{aligned}
LATE &= \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) - D_i(0) = 1] = \frac{ITT_{Y,Z}}{ITT_{D,Z}} \\
&= \frac{2}{2/3} = 3 = \beta_{IV}
\end{aligned}
$$

- Can we recover ATE?
  - *ATE* may be viewed as a weighted average of the treatment effect for compliers and never takers:

$$
\begin{aligned}
ATE = {} & \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) = 1] x ITT_{D,Z} \\
& + \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) = 0] x (1 - ITT_{D,Z})
\end{aligned}
$$

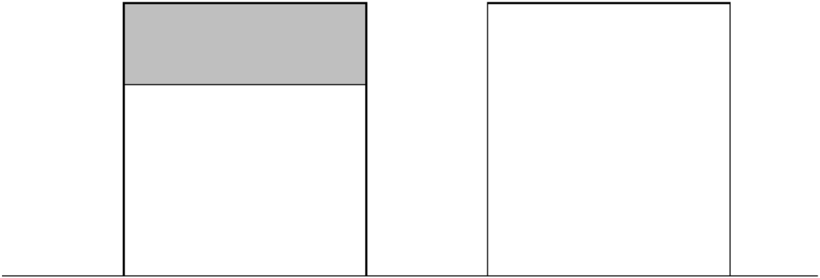
  - In our table:

$$ATE = 3.(2/3) + 6.(1/3) = 4$$

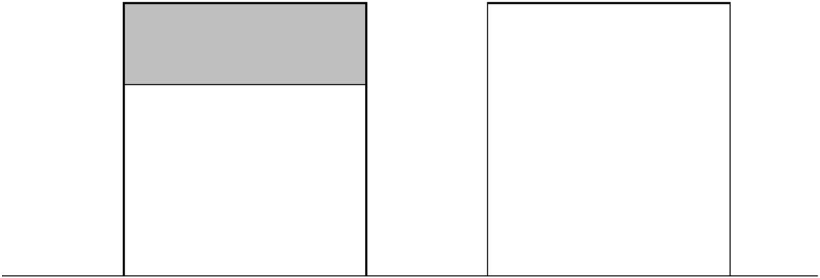
  - In practice?

- Few points to be emphasized:
  - An experiment with one-side non-compliance allows to estimate ITT and $LATE$ (ATE for compliers)
  - Whether a subject is a complier or never taker depends also on the experimental design
  - Exclusion restriction plays a critical role
  - When ITT for D is close to zero, small deviations from the exclusion restriction may severely bias the estimation of $LATE$

Recall 1: What if we ignore the failure to treat and compare assignment to treatment/control?



Recall 2: What if we compare those actually treated versus pure control?



- This comparison might be severely biased. Consider the following:

$$\begin{aligned}
MDO_{TvsC} &= \mathbb{E}[Y_i(1)/D_i(1) = 1] - \mathbb{E}[Y_i/Z_i = 0] \\
&= \mathbb{E}[Y_i(1)/D_i(1) = 1] - \mathbb{E}[Y_i(0)/D_i(1) = 1] \\
&+ \mathbb{E}[Y_i(0)/D_i(1) = 1] - \mathbb{E}[Y_i/Z_i = 0] \\
&= LATE + \mathbb{E}[Y_i(0)/D_i(1) = 1] - \mathbb{E}[Y_i/Z_i = 0]
\end{aligned}$$

4. The role of monotonicity

- Monotonicity plays a critical role in identifying LATE under two-side non-compliance

Example: Potential Outcomes under Two-Side Non-Compliance

Individual	Y(D=0)	Y(D=1)	D(0)	D(1)	Type	Y(Z=0)	Y(Z=1)
1	24	34	0	1	Complier	24	34
2	18	28	0	1	Complier	18	28
3	19	32	0	1	Complier	19	32
4	19	26	0	1	Complier	19	26
5	18	22	1	0	Defier	22	18
6	22	28	1	0	Defier	28	22
7	10	20	1	1	Always-Taker	20	20
8	11	12	0	0	Never-Taker	11	11
9	8	15	0	0	Never-Taker	8	8
10	11	18	0	0	Never-Taker	11	11

- Computing LATE:

$$
\begin{aligned}
LATE &= \mathbb{E}[Y_i(1) - Y_i(0)/D_i(1) = 1 - D_i(0) = 1] \\
&= (10 + 10 + 13 + 7)/4 = 10
\end{aligned}
$$

- We need to check whether we can recover LATE using the ratio of ITT:

$$
\begin{aligned}
ITT_{Y,Z} &= \mathbb{E}[Y_i/Z_i = 1] - \mathbb{E}[Y_i/Z_i = 1] \\
&= (10 + 10 + 13 + 7 - 4 - 6 + 0 + 0 + 0 + 0)/10 = 3
\end{aligned}
$$

$$
\begin{aligned}
ITT_{D,Z} &= \mathbb{E}[D_i/Z_i = 1] - \mathbb{E}[D_i/Z_i = 1] \\
&= (1 + 1 + 1 + 1 - 1 - 1 + 0 + 0 + 0 + 0)/10 = 2/10
\end{aligned}
$$

- Computing LATE using ITTs ratio:

$$\frac{ITT_{Y,Z}}{ITT_{D,Z}} = \frac{3}{2/10} = 15 \neq LATE$$

- Defiers in the sample violates the monotonicity assumption required for LATE to be identified. Dropping them from the sample:

$$\begin{aligned} ITT_{Y,Z} &= \mathbb{E}[Y_i/Z_i = 1] - \mathbb{E}[Y_i/Z_i = 1] \\ &= (10 + 10 + 13 + 7 + 0 + 0 + 0 + 0)/8 = 5 \end{aligned}$$

$$\begin{aligned} ITT_{D,Z} &= \mathbb{E}[D_i/Z_i = 1] - \mathbb{E}[D_i/Z_i = 1] \\ &= (1 + 1 + 1 + 1 + 0 + 0 + 0 + 0)/8 = 1/2 \end{aligned}$$

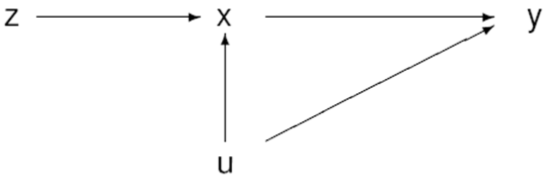
- Therefore, we can compute LATE using ITTs ratio:

$$\frac{ITT_{Y,Z}}{ITT_{D,Z}} = \frac{5}{1/2} = 10 = LATE$$

- We were able to recover LATE by imposing the monotonicity assumption

5. Evaluating the Assumptions in AIR Causal Model

- Identification of LATE depends on critical assumptions
- Violations of SUTVA
  - Potential outcomes of one individual can affect the outcomes of another one
  - Biased estimation of LATE (typically underestimation)
  - We will discuss this issue on detail later
- Violations of the exclusion restriction
  - Subject's treatment assignment does not matter once actual treatment has been accounted for
  - Biased estimation of LATE (both directions)
  - Play a central role on evaluating internal validity
  - Example:



- Violations of monotonicity
  - Rule out the existence of defiers
  - LATE cannot be identified if monotonicity is not imposed
  - Seems to be marginal in many settings