Lecture IX: Fixed Effects and Differences in Differences

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1. Motivation

- Our goal: to approximate an experimental design with observational data
- Panel data allow us to exploit temporal and cohort dimension to control for unobservable characteristics
- Two basic approaches:
  - Fixed Effects
  - Difference in Differences
- Both approaches are very popular in empirical economics and have a long tradition in the social sciences:
  - Snow (1854) on whether cholera was transmitted by air or water
  - Obenauer and von der Nienburg (1915) on the impact of minimum wage on wage rates

2. Fixed Effects: Motivation

- Motivation: what is the impact of prices on the demand of condoms for Colombian universities?
- Consider the following information:

Table 1: Single cross-section data

Universities	Year	Price	Per-capita Quantity
UNAL	2014	7.5	2.0
Uniandes	2014	5.0	1.0
Urosario	2014	6.0	1.5
U de la Sabana	2014	5.5	0.8

- What is the sign of the slope of the demand curve?

- Running a regression of quantities on prices will give us a positive coefficient! Why? Omitted bias...
- Assume you get extra data for the same units:

Table 2: Panel data

Universities	Year	Price	Per-capita Quantity
UNAL	2014	7.5	2.0
UNAL	2015	8.5	1.8
Uniandes	2014	5.0	1.0
Uniandes	2015	4.8	1.1
Urosario	2014	6.0	1.5
Urosario	2015	6.5	1.4
U de la Sabana	2014	5.5	0.8
U de la Sabana	2015	6.0	0.7

- Within variation in each unit is consistent with our demand theory
- Bottom line: using extra data for the same unit can help us to remove the omitted bias by exploiting the *within variation*
- Key assumption: unobservable factors that simultaneously affect the treatment and the outcome are *time-invariant*

2. Fixed Effects under the Potential Outcomes Framework

- Recall the basic (cross-sectional) model of potential outcomes previously discussed in class
- In this section we consider an extension for more periods:
  - Y_{it}: Observed outcome
  - D_{it}: Treatment status
- Recall that Y_{it} can be $Y_{it}(1)$ or $Y_{it}(0)$ depending on treatment status
- Also assume that: $\mathbb{E}(Y_{it}(0)|A_i, X_{it}, t, D_{it}) = \mathbb{E}(Y_{it}(0)|A_i, X_{it}, t)$
- Where:
  - X_{it}: Vector of observed time-varying covariates
  - A_i: Vector of unobserved bud fixed co-founders

- What does this assumption imply?
- We need to add some additional structure in order to obtain an estimable expression
- We impose some additional restrictions:
  - A_i does not depend on t in a linear model for: $\mathbb{E}(Y_{it}(0)|A_i, X_{it}, t) = \alpha + \lambda_t + A_i^{'}\gamma + X_{it}^{'}\beta$
  - Treatment effect is additive and constant: $\mathbb{E}(Y_{it}(1)|A_i, X_{it}, t) = \mathbb{E}(Y_{it}(0)|A_i, X_{it}, t) + \rho$
- Replacing: $\mathbb{E}(Y_{it}(1)|A_i, X_{it}, t) = \alpha + \lambda_t + \rho D_{it} + A_i^{'}\gamma + X_{it}^{'}\beta$

- This looks like the familiar fixed effects model you studied in intermediate econometrics:
  $Y_{it} = \alpha_i + \lambda_t + \rho D_{it} + X_{it}^{'}\beta + \epsilon_{it}$
- Where:

$$\epsilon_{it} \equiv Y_{it}(0) - \mathbb{E}(Y_{it}(0)|A_i, X_{it}, t)$$
$$\alpha_i \equiv \alpha + A_i^{'}\gamma$$

- Notice that α_i and λ_t are treated as parameters to be estimated in the fixed effects model

  Question: too many parameters to estimate?

Estimating Fixed Effects Models

- We need to get rid of the individual fixed effects to avoid the need of estimating too many parameters
- Two approaches:
  - Deviations from means
  - Diferencing
- Consider the basic fixed effects regression:
  $Y_{it} = \alpha_i + \lambda_t + \rho D_{it} + X_{it}'\beta + \epsilon_{it}$
- Deviations from means:
  - Compute individual averages:
    $\bar{Y}_i = \alpha_i + \bar{\lambda} + \rho \bar{D}_i + \bar{X}_i'\beta + \bar{\epsilon}_i$
  - Subtracting:
    $Y_{it} - \bar{Y}_i = (\lambda_t - \bar{\lambda}) + \rho(D_{it} - \bar{D}_i) + (X_{it}' - \bar{X}_i')\beta + (\epsilon_{it} - \bar{\epsilon}_i)$

- Differencing:
  - Computing difference with respect to previous period: $\Delta Y_{it} = \Delta\lambda_t + \rho\Delta D_{it} + \Delta X_{it}^{'}\beta + \Delta\epsilon_{it}$
  - Where: $\Delta Y_{it} = Y_{it} - Y_{it-1}$

3. Difference in Difference under Potential Outcomes

- DID is a very popular approach in applied economics. We lay out the identification assumptions for the causal interpretation of DID
- SUTVA is typically invoked in cross-sectional settings. In the case of DID, it has the following form:

Assumption 1: SUTVA

One, and only one, potential outcome is observed for every member of the population. Then,
$Y_t = DY_t(1) + (1 - D)Y_t(0), \forall t \in (0, 1)$

- Example of violation of SUTVA for **post-treatment state**: Treatment group do not look actively for employment during training making the likelihood of finding employment larger for non-participants

- Example of violation of SUTVA for **pre-treatment state**: Anticipation of future participation in training program reduces job search efforts making easier for future non-participants to find a job
- Now, consider the case of covariates X, so identification is conditional to covariates. This requires that the treatment does not affect X. Then,

Assumption 2: Exogeneity of X

X is exogenous, therefore
$X(1) = X(0) = X$, $\forall x \in \chi$

- Example of violation of Assumption 2: ?
- Assumptions 1 and 2 are standard in impact evaluation

- A specific assumption for DID is the following:

Assumption 3: No effect of the treatment on pre-treatment population

$\tau_0(x) = 0; \forall x \in \chi$

- Example of violation of Assumption 3: ?
- DID also requires the following condition:

Assumption 4: Common trends

$$
\begin{aligned}
&\mathbb{E}(Y_1(0)|X = x, D = 1) - \mathbb{E}(Y_0(0)|X = x, D = 1)\\
&= \mathbb{E}(Y_1(0)|X = x, D = 0) - \mathbb{E}(Y_0(0)|X = x, D = 0)\\
&= \mathbb{E}(Y_1(0)|X = x) - \mathbb{E}(Y_0(0)|X = x); \forall x \in \chi
\end{aligned}
$$

- Differences in the expected potential non-treatment outcomes over time (conditional on X) are unrelated to belonging to the treated or control group in the post-treatment period
- Example of violation of Assumption 4: ?
- Notice that Assumption 4 is based in a within-unit comparison. Can we think also in terms of a cross-sectional comparison?
- Consider the following expression:
  - $\mathbb{E}(Y_0(0)|X = x, D = 1) - \mathbb{E}(Y_0(0)|X = x, D = 0) =$? Is this expression biased?
  - Trivially, this is 0 when treatment is randomly assigned
  - If it is not random, then $\mathbb{E}(Y_0(0)|X = x, D = 1) - \mathbb{E}(Y_0(0)|X = x, D = 0) = Bias_0(x)$
  - Can a causal effect can be identified in this setting?

- If bias is the same in pre and post-treatment periods, then a causal effect can be identified

Assumption 4': Constant bias

$$\mathbb{E}(Y_0(0)|X = x, D = 1) - \mathbb{E}(Y_0(0)|X = x, D = 0)[= Bias_0(x)]$$
$$= \mathbb{E}(Y_1(0)|X = x, D = 1) - \mathbb{E}(Y_1(0)|X = x, D = 0)[= Bias_1(x)]; \forall x \in \chi$$

- It can be shown that Assumption 4 and Assumption 4' are equivalent
- Identification relies on the counterfactual difference $\mathbb{E}(Y_1(0)|X = x, D = 1) - \mathbb{E}(Y_0(0)|X = x, D = 1)$ being identifical to the observable difference $\mathbb{E}(Y_1|X = x, D = 0) - \mathbb{E}(Y_0|X = x, D = 0)$

- It is required that observations with characteristics x exist in all subsamples:

Assumption 5: Common support

$\mathbb{P}(TD = 1|X = x, [T, D] \in (t, d), (1, 1)) < 1;$
$\forall (t, d) \in (0, 1), (0, 0), (1, 0); \forall x \in \chi$

- Example of violation of Assumption 5: ?
- Notice that Assumption 5 is testable

Identification of ATET under DID Designs

- We want to identify $ATET_1$ (Notice that $ATET_0 = 0$ by definition)

Proof

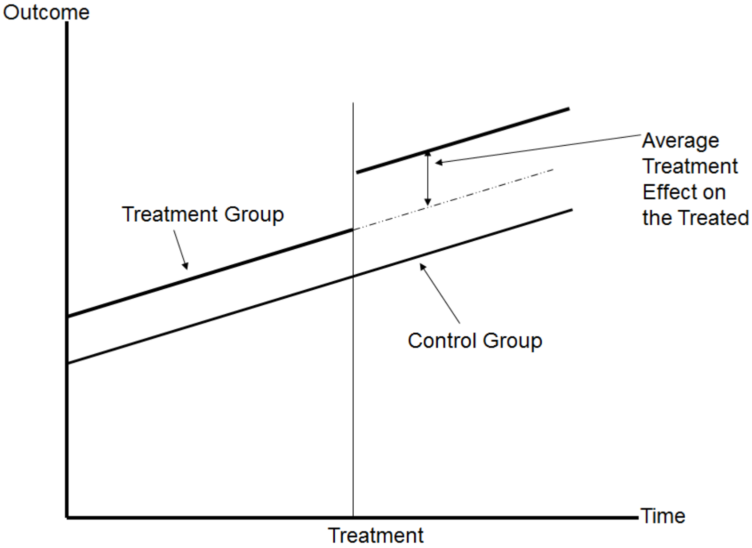
* $\tau_1^{ATET}(x) = \mathbb{E}(Y_1(1) - Y_1(0)|X = x, D = 1)$
$= \mathbb{E}(Y_1|X = x, D = 1) - \mathbb{E}(Y_1(0)|X = x, D = 1)$; due to *SUTVA*
* Consider the counterfactual $\mathbb{E}(Y_1(0)|X = x, D = 1)$
$\mathbb{E}(Y_1(0)|X = x, D = 1) = \mathbb{E}(Y_1(0)|X = x, D = 0) - \mathbb{E}(Y_0(0)|X = x, D = 0) + \mathbb{E}(Y_0(0)|X = x, D = 1)$; due to Common Trend Assumption
Then,
$\mathbb{E}(Y_1(0)|X = x, D = 1) = \mathbb{E}(Y_1|X = x, D = 0) - \mathbb{E}(Y_0|X = x, D = 0) + \mathbb{E}(Y_0(0)|X = x, D = 1)$; due to *SUTVA*

Proof...(Cont.)

*Consider now $\mathbb{E}(Y_0(0)|X = x, D = 1)$:
$\mathbb{E}(Y_0(0)|X = x, D = 1) = \mathbb{E}(Y_0(1)|X = x, D = 1)$; due to Assumption 3.
Then,
$\mathbb{E}(Y_0(1)|X = x, D = 1) = \mathbb{E}(Y_0|X = x, D = 1)$; due to *SUTVA*
*Considering all pieces together:
$\tau_1^{ATET}(x) = [\mathbb{E}(Y_1|X = x, D = 1) - \mathbb{E}(Y_0|X = x, D = 1)] -$
$= [\mathbb{E}(Y_1|X = x, D = 0) - \mathbb{E}(Y_0|X = x, D = 0)]$

- Can ATU or ATE be identified?
  - ATU would require a group that is treated in time 0 but become untreated in time 1. Therefore, it cannot be identified unless stronger assumptions are in place
  - Since ATE is a weighted average of ATT and ATU, it cannot be identified as well

Graphical Representation of DID designs



4. Difference in differences in differences designs (DDD)

- DID designs can be extended to account for variation within treatment units. Consider the case of minimum wage increase as Card and Krueger (1994):

$$\begin{aligned} Y_{ijt} = {} & \alpha + \beta_1 X_{ijt} + \beta_2 \tau_t + \beta_3 \delta_j + \beta_4 Treat_i \\ & + \beta_5(\delta_j x \tau_t) + \beta_6(\tau_t x Treat_i) + \beta_7(\delta_j x Treat_i) \\ & + \beta_8(\delta_j x \tau_t x Treat_i) + e_{ijt} \end{aligned}$$

- Where:
    - β_2, β_3 and β_4: time-series changes in Y, time-invariant characteristics of treatment states and time-invariant characteristics of treatment group respectively
    - β_5: changes over time in treatment states
    - β_6: changes over time for treatment group nationwide
    - β_7: time-invariant characteristics of treatment group in experimental states

5. Synthetic Controls

- When the sample of units are small and observations for many periods are available, it is possible to perform causal inference constructing synthetic control units
- Abadie and Gardeazabal (2003) propose this approach to analyze the impact of terrorism in Basque Country constructing a synthetic control based on the performance of other Spanish regions
- Suppose there is a sample of $J + 1$ units, being $j = 1$ the case of interests and $j = 2$ to $j = J + 1$ the potential comparisons observed at the same time periods $t = 1, ..., T$
- It is supposed enough pre and post-treatment periods are available

- A synthetic control is a weighted average of the units in the untreated group:
  - Consider a set of weights $W = (w_2, ..., w_{J+1})$ such that each of them $0 \leq w_j \leq 1$ and they sum to 1
  - How to choose W?
  - Choose W such that the characteristics of the treated unit are best resembled by the characteristics of the synthetic control
  - Let X_1 be a vector of the pre-intervention characteristics of the treatment unit and X_0 the characteristics of the donor pool
  - The difference of interests is $X_1 - X_0 W$
  - Abadie and Gardeazabal (2003) choose W^* as the value of W that minimizes:

$$\sum_{m=1}^{k} v_m (X_{1m} - X_{0m} W)^2$$

- Once pre-treatment balance is show with the pre-treatment data using the weights derived before, the synthetic control estimator of the effect is given by a comparison of post-treatment outcomes

$$Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt}$$

Example: Costs of Reunification of Germany

Table 1 Synthetic and Regression Weights for West Germany

Country	Synthetic Control Weight	Regression Weight	Country	Synthetic Control Weight	Regression Weight
Australia	0	0.12	Netherlands	0.09	0.14
Austria	0.42	0.26	New Zealand	0	0.12
Belgium	0	0	Norway	0	0.04
Denmark	0	0.08	Portugal	0	−0.08
France	0	0.04	Spain	0	−0.01
Greece	0	−0.09	Switzerland	0.11	0.05
Italy	0	−0.05	United Kingdom	0	0.06
Japan	0.16	0.19	United States	0.22	0.13

Notes: The synthetic weight is the country weight assigned by the synthetic control method. The regression weight is the weight assigned by linear regression. See text for details.

Example: Costs of Reunification of Germany

FIGURE 2 **Trends in per Capita GDP: West Germany versus Synthetic West Germany**