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Tiebout equilibria in local public good economies with spillovers

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Abstract

This paper analyzes the effects of spillovers on the equilibrium population distribution across jurisdictions in a local public good economy with free mobility. Spillovers are parametrized by a matrix $[\alpha_{ij}]$ where $\alpha_{ij} \in [0,1]$. When spillovers are symmetric and close to 0 or 1 (pure local public goods and pure public goods), all equilibrium jurisdiction structures are symmetric. However, any population distribution can be sustained in equilibrium for some value of the spillover parameter α. In the class of utility functions with additive externalities, we identify the unique family of utility functions for which equilibria are symmetric except for an isolated value of α. This is a class of utility functions which are linear in the public good and a power function of the private good, $u(c,\gamma) = -A(1-c)^{\beta} + \gamma$. With this specification of utility, we show that an increase in α results in a more fragmented equilibrium population distribution, and that when spillovers are asymmetric and large, a jurisdiction which is more centrally located (i.e. benefits more from the public goods provided in other jurisdictions) has a larger population in equilibrium.

JEL classification: H41; H49; H73
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1. Introduction

Since the seminal analysis of Tiebout (1956), an important literature has studied local public good economies where agents can freely move across jurisdictions.[2] For the most part, this literature supposes that local public goods are excludable and that agents do not benefit from the goods provided in other jurisdictions. In this paper, we argue that the presence of inter-jurisdictional benefits dramatically changes the equilibrium structure of jurisdictions. Consider a homogeneous population of individuals choosing between two possible locations. In the absence of spillovers, the only stable equilibrium is for all agents to live in a single location, as the equilibrium utility is increasing in the size of the jurisdiction. If on the other hand, agents benefit fully from the public goods offered in the other jurisdiction, the only equilibrium structure involves a uniform distribution of agents across the jurisdictions, as otherwise agents always have an incentive to move to the smaller jurisdiction.

The objective of this paper is to describe how the structure of inter-jurisdictional spillovers shapes the equilibrium distribution of individuals across locations. To this end, we consider a very simple model with a continuum of perfectly mobile identical agents and a fixed number of jurisdictions.[3] In this context, we ask the following questions: If spillovers are symmetric across jurisdictions, when is the equilibrium structure of jurisdictions symmetric? When is it unique? If multiple equilibria exist, which of the equilibrium population distributions is most efficient? How does an increase in spillovers (due for example to an improvement in communications technology) affect the equilibrium distribution of agents? If locations are asymmetric, how is this asymmetry reflected in the distribution of the population? Are centrally located jurisdictions (i.e. jurisdictions that benefit more from the public goods provided in other jurisdictions) more or less heavily populated?

Our analysis shows that, even in the simple model we consider, the answer to these questions is rather complex. We first note that in the two polar cases of local public goods ($\alpha=0$) and pure public goods ($\alpha=1$), all equilibrium population distributions are symmetric. For intermediate values of the symmetric spillover parameter, asymmetric equilibrium distributions emerge and the model admits multiple equilibrium distributions. In fact, for any possible distribution of the population across two jurisdictions, there exists a value of the spillover parameter for which this distribution is sustained in equilibrium. For a special class of utility functions (utility functions with additive externalities, which are linear in the local public good), we can rank the different equilibria, and show that agents obtain higher utility in an asymmetric equilibrium distribution than in a symmetric uniform distribution. We also characterize the unique family of utility functions with additive externalities for which equilibria are generically symmetric. For this family of power utility functions, an increase in the symmetric spillover parameter results in a greater fragmentation of the jurisdiction structure. (Better means of communications imply that agents will more likely be dispersed across locations). We also consider an asymmetric model with three jurisdictions, and show that, for high values of the spillover parameter, locations which are more central are also more populated.

The intuition underlying our main result on multiplicity of equilibria is easily grasped. When $\alpha=0$ (the case of local public goods), the equilibrium utility of an agent is monotonically

[2] We will not attempt to survey this large body of work here, and refer to Scotchmer (2002) for a recent discussion of the literature.

[3] Because the number of jurisdictions is given, we do not analyze the endogenous formation of jurisdictions, but the equilibrium population distribution across a fixed number of jurisdictions.

increasing in the size of the jurisdiction; when $\alpha = 1$ (the case of pure public goods), the equilibrium utility of an agent is monotonically decreasing in the size of the jurisdiction. Given that equilibrium utilities are continuous in α, for some intermediate values of the spillover parameter, the equilibrium utility will generically cease to be monotonic and asymmetric population distributions can be sustained in equilibrium. The only situation where this line of reasoning fails is the non-generic case where equilibrium utilities remain decreasing until they reach a specific value $\hat{\alpha}$, are constant at $\hat{\alpha}$, and then become increasing. This non-generic case allows us to characterize the family of utility functions for which equilibrium is unique except for a single value of the spillover parameter. For this family of power utility functions, an increase in the spillover parameter induces a change of regime from a local public good economy (where all agents reside in a single jurisdiction) to a pure public good economy (where agents are dispersed across all locations).

The literature on inter-jurisdictional spillovers has paid little attention to the effect of spillovers on household mobility. Starting with the seminal contribution of Oates (1972), most of the literature has centered around the comparison between centralization and decentralization of public good provision. Recent contributions to the literature have emphasized political economy arguments, modeling the choice of a centralized decision-making body made up of representatives of the different jurisdictions (Lockwood (2002), Besley and Coate (2003) and Redoano and Scharf (2004)). The design of matching grants and taxation schemes to remedy inefficiencies due to externalities has also recently been extensively studied Cremer et al. (1997), Lockwood (1999) and Figuières and Hindriks (2002)). In all these recent contributions, households are assumed to be immobile, and the jurisdiction structure is fixed.[4]

To the best of our knowledge, the only contributions combining household mobility with inter-jurisdictional spillovers are due to Wellisch (1993, 1994). The focus of these papers is not on the equilibrium population distribution across jurisdictions but on the efficiency of local public good provision. Wellisch (1994) finds that if households move across jurisdictions in response to changes in the level of local public good provision, local governments are led to produce the efficient amount of local public goods even in the presence of positive spillovers. The intuition underlying this result is that, if households are perfectly mobile, all households will have the same utility, and the local governments share the common objective of maximizing the utility of a representative household. The possibility of interregional transfers then guarantees that the optimal level of public good provision will be chosen by each government. By contrast, as we do not allow transfers across jurisdictions, the equilibrium level of public good provision in our model is always inefficient, so that agents in jurisdictions in different sizes do not necessarily achieve the same utility level, and the issue of equilibrium population distribution becomes meaningful.

Conley and Dix (1999) analyze the effect of spillovers on club formation, and show that an increase in the spillover parameter has ambiguous effects on the optimal club size. While the objective of Conley and Dix (1999)—understanding the relation between spillovers and the equilibrium club structure—is closely related to ours, differences between the Tiebout model of jurisdiction formation and their model of club formation preclude a direct comparison between the two papers.

The rest of the paper is organized as follows. We present the basic model in Section 2. Section 3 is devoted to the study of the model with symmetric spillovers and general utility functions.

[4] For example, Besley and Coate (2003, fn. 6 on p. 2614) note that they "ignore the issue of mobility in [their] analysis", and that "while such consideration are obviously important, incorporating them is sufficiently difficult that they are best left for a separate paper".

Section 4 focuses on the symmetric spillovers model for additive externalities. Section 5 contains our results on the effect of spillovers on the equilibrium population distribution. Finally, we conclude and give directions for further research in the last section.

2. The model

2.1. Agents and jurisdictions

We consider an economy with a continuum of identical agents on the interval [0,1] with Lebesgue measure λ. There are n distinct fixed jurisdictions in the economy, indexed by $i=1, 2, \ldots, n$. An assignment of agents to jurisdictions is a measurable mapping $\mathcal{A}:[0,1]\rightarrow\{1,2\ldots,n\}$. We define the distribution of the population across jurisdictions by a vector $\boldsymbol{m}$ in the $(n-1)$-dimensional simplex, $\boldsymbol{m}\in\Delta=\{(m_1,m_2,\ldots,m_n),\ m_i\geq 0,\ \sum m_i=1\}$where $m_i=\lambda(\{a,\mathcal{A}(a)=i\})$ denotes the fraction of the population living in jurisdiction i.

There are two goods in the economy: a private consumption good c and a local public good provided within each jurisdiction. The total amount of public goods provided in jurisdiction i is denoted G_i. We depart from traditional models of local public goods by assuming that there exist positive spillovers across jurisdictions.[5] Specifically, we suppose that spillovers are parametrized by a matrix $\mathbf{A}=[\alpha_{ij}]$ where $0\leq\alpha_{ij}\leq 1$ where denotes the spillovers from jurisdiction j to jurisdiction i. We interpret α_{ij} as the degree of spillovers that enable an agent in jurisdiction i to consume the local public good provided in jurisdiction j. All spillovers are assumed to be positive and $\alpha_{ii}=1$ for all i. For a large part of the analysis, we consider symmetric spillovers where $\alpha_{ij}=\alpha$ for all $i=j$. The two special cases where $\alpha_{ij}=0$ for all $i\neq 0$ and $\alpha_{ij}=1$ for all $i\neq j$ correspond to the two polar cases of pure local public goods and pure public goods, respectively.

Local public goods provided in different jurisdictions are taken to be perfect substitutes, and we define the composite public good consumed by an agent in jurisdiction i as

$$\gamma_i=\sum_j \alpha_{ij}G_j.$$

All agents share the same utility function over the private good and the composite public good given by

$$u(c,\gamma)$$

where u is twice continuously differentiable, strictly increasing in both arguments and strictly quasi-concave. We define the marginal rate of substitution of the consumption good for the public good by

$$f(c,\gamma)=\frac{u_c(c,\gamma)}{u_\gamma(c,\gamma)}.$$

All agents are endowed with one unit of the private good. We assume the following properties of the utility function.

[5] We thus rule out the case of negative spillovers in local public goods. This modeling choice is not motivated by economic considerations (we do believe that there are important instances where local public goods in one jurisdiction can produce negative effects on other jurisdictions), but because we want to keep pure local public goods and pure public goods as the two polar cases of our model.

Assumption 1. The marginal rate of substitution is nonincreasing in the consumption good $\left(\frac{\partial f}{\partial c} \leq 0\right)$ and nondecreasing in the public good $\left(\frac{\partial f}{\partial \gamma} \geq 0\right)$ with at least one strict inequality.

Assumption 2. The function u satisfies the following boundary conditions: (i) $\lim_{\gamma \to 0} f(c,\gamma) = 0$ for all c and (ii) $\lim_{c \to 0} f(c,m) \geq m$ for all $m \in [0,1]$.

Assumption 1 is a classical normality condition, requiring that both goods are normal (with at least one of them strongly normal). The boundary conditions of Assumption 2 guarantee that, in the case of pure local public goods, any jurisdiction of size m chooses an interior level of public good provision, $G \in (0,m)$.

In every jurisdiction i, the local public good is financed by a proportional tax on the consumption good. Given that all agents are identical, every agent in jurisdiction i thus provides the same amount of public good g_i and the total provision of public good in jurisdiction i is given by:

$$G_i = \int_{a,\mathcal{A}(a)=i} g_i d\lambda = m_i g_i.$$

Hence the utility of an agent in jurisdiction i can be rewritten as:

$$u\left(1 - \frac{G_i}{m_i}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right).$$

2.2. Equilibrium

We define a Tiebout equilibrium in our model by two simultaneous conditions: (i) every jurisdiction chooses its level of local public good provision in order to maximize the utility of its constituents and (ii) every agent optimally chooses the jurisdiction in which he resides. Formally,

Definition 1. An equilibrium is a vector $(\boldsymbol{G},\boldsymbol{m})$ in $\Re^n_+ \times \Delta$ satisfying: (i) $G_i \in \arg\max u\left(1 - \frac{G_i}{m_i}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right)$ for all i, and (ii) for all i such that $m_i > 0$, and all k, $u\left(1 - \frac{G_i}{m_i}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right) \geq u\left(1 - \frac{G_k}{m_k}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right)$.

Existence of equilibrium in our model derives from the powerful existence theorem of Konishi (1996). We simply adapt the proof of Konishi (1996) to our setting, where there is no fixed factor (land).[6]

Theorem 1. *(Konishi (1996)) The local public good economy with spillovers always admits an equilibrium.*

3. Symmetric spillovers

In this section, we consider the case of symmetric spillovers across jurisdictions. Spillovers are parametrized by a one-dimensional variable, $\alpha \in [0,1]$ which measures the degree of interaction between jurisdictions. The two extreme points of the segment, $\alpha = 0$ and $\alpha = 1$

[6] Because there is no land in our model, we cannot guarantee that all jurisdictions will be inhabited in equilibrium. As opposed to Konishi (1996), we thus allow for empty jurisdictions.

correspond to pure local public goods and pure public goods. We first map out the different steps of the analysis. The presence of spillovers creates an interdependency between the different jurisdictions. Hence, each jurisdiction's optimal choice of the level of public good depends on the public goods offered in the other jurisdictions. The first step of the analysis is thus to solve for the Nash equilibrium of the noncooperative game played among jurisdictions to determine the optimal level of public good provision of each jurisdiction. In a second step, we characterize the equilibrium population distributions resulting from the free-mobility condition. In our simple model, where all agents are identical, the free mobility condition amounts to two requirements:

1. *In any two nonempty jurisdictions, agents must receive the same payoff.* For any two jurisdictions i, k with $m_i>0$ and $m_k>0, u\left(1-\frac{G_i}{m_i}, G_i+\alpha\sum_{j\neq i}G_j\right)=u\left(1-\frac{G_k}{m_k}, G_k+\alpha\sum_{j\neq k}G_j\right)$.
2. *Agents must receive a higher payoff in a nonempty jurisdiction than in an empty jurisdiction.* For any two jurisdictions i, k with $m_i>0$ and $m_k=0$, $u\left(1-\frac{G_i}{m_i}, G_i+\alpha\sum_{j\neq i}G_j\right)\geq u\left(1, \alpha\sum_{j\neq k}G_j\right)$.

In our analysis of equilibrium population distributions, we will distinguish between *symmetric population distributions* (where all nonempty jurisdictions have the same size) and *asymmetric population distributions* (where two nonempty jurisdictions have different sizes).

3.1. Equilibrium of the game of public good provision

We first consider, for a fixed population distribution $\boldsymbol{m}$, the noncooperative game where every jurisdiction chooses its total level of public good provision. Our first result shows that when spillovers are symmetric, the equilibrium level of public good provision is unique.

Proposition 1. *Let the utility function satisfy Assumption 1 (with $\frac{\partial f}{\partial c}<0$ if $\alpha=1$). For a fixed population distribution $\boldsymbol{m}$ the equilibrium level of public good provision, $\mathbf{G}$ is unique for all $\alpha\in[0,1]$.*

Proposition 1 generalizes Bergstrom et al. (1992) who prove uniqueness of equilibrium in a game of private provision of pure public goods (for $\alpha=1$). In the particular case of pure public goods, the result only holds if the marginal rate of substitution is decreasing in the level of the private good. Otherwise (for example, when the utility function is quasi-linear), the total provision of public goods, $\gamma=\sum \boldsymbol{G}_j$ is unique but the distribution of public goods across jurisdictions is indeterminate. Interestingly, this result only holds for symmetric spillovers. In a companion paper, (Bloch and Zenginobuz (2004)), we provide an example with three jurisdictions to show that asymmetric spillovers can generate multiple equilibria.

As the equilibrium level of public good is unique, we define the *indirect utility function* of an agent in jurisdiction m_i when the population distribution is $\boldsymbol{m}$ and the spillover parameter α as

$$V_i(\boldsymbol{m},\alpha)=u\left(1-\frac{G_i}{m_i}, G_i+\alpha\sum_{j\neq i}G_j\right).$$

As a simple corollary to Proposition 1, we show that this indirect utility function is continuous in the parameters $\boldsymbol{m}$ and α.

Corollary 1. *Let the utility function satisfy Assumption 1 (with $\frac{\partial f}{\partial c}<0$ if $\alpha=1$). Then the equilibrium levels of public good and the indirect utility functions V_i ($\boldsymbol{m}$, α) are continuous in $\boldsymbol{m}$ and α.*

3.2. Symmetric equilibrium population distributions

We now turn our attention to the free-mobility conditions, determining the equilibrium population distribution. As the model is symmetric, *symmetric population distributions*, where all inhabited jurisdictions have the same size, are obvious candidates for equilibrium. For any integer p in $[1, n]$, let a p-symmetric population distribution be a symmetric distribution where p jurisdictions are inhabited, $\boldsymbol{m}^p = (1/p, 1/p, .., 0, 0)$. Two special cases are (i) the situation where a single jurisdiction is nonempty, $\boldsymbol{m}^1 = (1, 0, \ldots, 0)$ and (ii) the uniform distribution where all jurisdictions have equal size $\boldsymbol{m}^n$ $(1/n, \ldots, 1/n)$. We start with the following obvious observation.

Proposition 2. *Let the utility function satisfy Assumption 1. For all values of $\alpha \in [0, 1]$, the uniform population distribution is an equilibrium population distribution.*

We now consider the two extreme points of the interval of spillovers, $\alpha = 0$ and $\alpha = 1$ and fully characterize the equilibrium population distributions.

Proposition 3. *Let the utility function satisfy Assumptions 1 and 2. There exist $\underline{\alpha}$ and $\overline{\alpha}$ such that, for all $\alpha \leq \underline{\alpha}$ and $\alpha \geq \overline{\alpha}$, all equilibrium population distributions are symmetric. Furthermore, for $\alpha \leq \underline{\alpha}$, all p-symmetric population distributions are equilibrium population distributions, and for $\alpha \geq \overline{\alpha}$, the only equilibrium population distribution is the uniform distribution.*

Proposition 3 shows that in the two polar cases of pure local public goods and pure public goods, the only equilibria of the model are symmetric.[7] The intuition underlying this result is easily grasped. In a pure local public good economy, the indirect utility is increasing in the size of the jurisdiction. Hence, utility can only be equalized across jurisdictions if they have the same size. In a pure public good economy, individual contributions to the public good are increasing in the size of the jurisdiction, so that indirect utility is decreasing in the size of the jurisdiction. Again, utility can only be equalized among jurisdictions of the same size. Exploiting the continuity result of Corollary 1, we show that this property still holds for very low and very high values of the spillover parameter. Furthermore, in a pure local public good economy, agents obtain a very low utility by moving to an empty jurisdiction, and hence any p-symmetric population distribution can be sustained in equilibrium.[8] By contrast, in a pure public good economy, no jurisdiction can be left empty, as agents always benefit from moving to an empty jurisdiction, where they do not contribute but benefit fully from the public good.

3.3. Asymmetric equilibrium population distributions

For intermediate values of the spillover parameter, $\alpha \in (\underline{\alpha}, \overline{\alpha})$, the indirect utility is not necessarily monotonic in the size of the jurisdiction, leaving open the possibility of asymmetric

[7] This proposition only holds true if the boundary conditions are imposed. In the absence of boundary conditions, one could easily construct equilibrium levels of public goods where two jurisdictions of different sizes provide no public good— or, in the case of pure public goods, spend all their consumption good in the provision of the public good. In these cases, two agents living in jurisdictions of different sizes end up with the same indirect utility, and asymmetric population distributions could be sustained in equilibrium.

[8] We note however that the equilibrium where all agents reside in the same location is the only stable equilibrium. If we consider a dynamic adjustment process, and let agents make mistakes in their location choices, then any equilibrium with multiple jurisdictions will be unstable. As soon as the sizes of two jurisdictions become different, agents from the smaller jurisdiction have an incentive to move to the larger jurisdictions. This process will only end when all agents have moved to a single jurisdiction.

equilibrium population distributions. This is illustrated in Fig. 1, where we consider a model with two jurisdictions, of sizes m and $1-m$ and map the indirect utility as a function of m for different values of α. The indirect utility is increasing for $\alpha=0$ and decreasing for $\alpha=1$. As V is a continuous function of α, there exists an intermediate value of the spillover parameter, α^*, where the indirect utility is nonmonotonic. Fig. 1 then shows that this translates into an asymmetric equilibrium with $m^* \neq 1/2$.

We now provide our main result, showing that the case illustrated in Fig. 1 is not pathological. Instead, we show that any jurisdiction size can be sustained in a population equilibrium distribution for some spillover parameter α. This result will be more transparent in the case of two jurisdictions.

Proposition 4. *Let $n=2$ and the utility function satisfy Assumptions 1 and 2. For any population distribution $\mathbf{m}=(m, 1-m)$, there exists a value of α such that $\mathbf{m}$ can be sustained as an equilibrium population distribution.*

Proposition 4 is based on a simple mathematical argument. Consider two jurisdictions of unequal size, with the first jurisdiction smaller than the second. For $\alpha=0$, the utility is higher in the second jurisdiction than in the first. For $\alpha=1$, the comparison is reversed. As the indirect utility function is continuous in the spillover parameter, by the Intermediate Value Theorem, there must exist $\alpha \in (0,1)$ for which utilities are equalized in the two jurisdictions.

Proposition 4 also has a clear economic interpretation. For any asymmetric population distribution, members of the smaller jurisdiction provide less public good but benefit (through the spillover parameter α) from the larger amount of public good provided in the larger jurisdiction. Members of the larger jurisdiction provide a larger amount of public good and benefit from the smaller amount produced in the other jurisdiction. Proposition 4 shows that there exists a spillover parameter which exactly equates the utilities of the agents in the two jurisdictions.

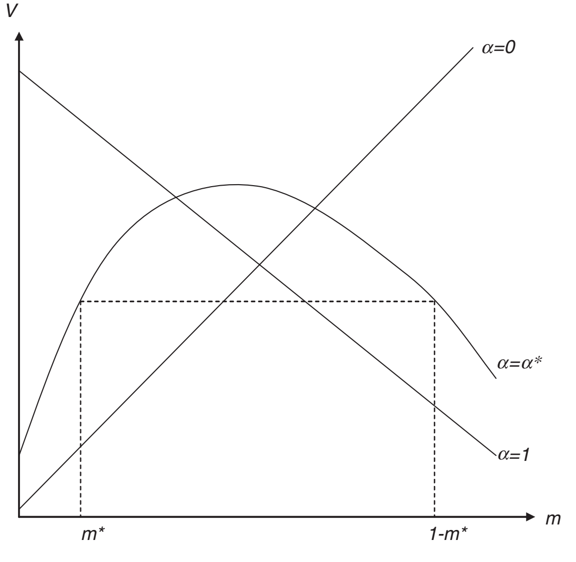

Fig. 1. Utility and jurisdiction sizes.

Proposition 4 can be extended to a larger number of jurisdictions in the following way. For any jurisdictions size $m \in [0,1]$, there exists a value of the spillover parameter α for which the asymmetric population distribution $\boldsymbol{m} = (m, (1-m)/(n-1), \ldots, (1-m)/(n-1))$ can be sustained in equilibrium. However, for $n > 2$, it is in general impossible to sustain in equilibrium n jurisdictions which all have different sizes. The reason is that the spillover parameter is a one-dimensional variable, which can only be used to equate the indirect utility between two jurisdictions.[9]

4. Additive externalities

In the preceding section, we have characterized equilibrium population distributions, and shown the existence of asymmetric equilibria. This analysis leaves open a number of questions, which cannot easily be answered in the general model. Can equilibria be ranked according to some efficiency criterion? What is the set of spillover parameters for which asymmetric equilibria exist?

In order to answer these questions, we turn to a simpler model, where utilities are linear in the public good.[10] We suppose that the utility function is of the form

$$u(c, \gamma) = u(c) + \gamma$$

where u is strictly increasing and continuously differentiable. The marginal rate of substitution is given by $f(c, \gamma) = u'(c)$ and is independent of γ. By Assumption 1, $u''(c) < 0$. The boundary conditions of Assumption 2 transpose as (i) $\lim_{c \to 1} u'(c) = 0$ and (ii) $\lim_{c \to 0} u'(c) \geq 1$.

This class of utility functions with additive externalities has been used by Ray and Vohra (2001) in their study of coalition formation with pure public goods. It allows for a tractable analysis because the *level of public good produced in any jurisdiction is independent of the public goods provided in other jurisdictions*. In other words, in the noncooperative game played across jurisdictions, every player has a dominant strategy. The characterization of equilibrium then boils down to the computation of the optimal public good level of each jurisdiction, which only depends on the size of the jurisdiction.

4.1. Efficiency of equilibria

We first consider the efficiency of the different equilibrium population distributions.

Proposition 5. *Consider a utility function with additive externalities satisfying Assumptions 1 and 2. For a fixed spillover parameter α, if there exist two symmetric equilibrium population distributions, the equilibrium utility is higher in the equilibrium with the smaller number of jurisdictions. If in addition $u'''(c) \leq 0$ and there exists an asymmetric equilibrium with two jurisdictions, the equilibrium utility is higher in the asymmetric equilibrium than in the uniform equilibrium.*

Proposition 5 compares, for a fixed spillover parameter α, the equilibrium utilities of the agents in different equilibrium configurations. It is easy to see that the most efficient population

[9] Formally, for n juridictions which all have different sizes, the equilibrium conditions give rise to a system of $(n-1)$ equations in a single variable α. This system will typically not admit any solution.

[10] We note that this class of utility functions is not the classical class of quasi-linear utility functions, where utility is linear in the consumption good.

distribution is to have all agents live in a single jurisdiction, as all spillovers are then internalized, and the jurisdiction chooses the optimal public good production level for all agents. When utilities are linear in the public good, we can further show that any symmetric equilibrium is dominated by a symmetric equilibrium with a smaller number of jurisdictions. If in addition we suppose that the third derivative of the subutility function $u(c)$ is negative, not only is the extreme population distribution $\boldsymbol{m}^1=(1,0)$ more efficient than the uniform distribution, $\boldsymbol{m}^2=(1/2,1/2)$, but any asymmetric equilibrium distribution dominates the uniform distribution. Hence, the uniform equilibrium appears to be the worst equilibrium configuration in the economy.

4.2. Spillover values for which all equilibria are symmetric

In the preceding section, we have shown that any population distribution can be sustained in equilibrium for some spillover value. However, it may be that the set of spillovers for which asymmetric equilibria exist is very small. In particular, there exist utility functions for which equilibrium population distributions are generically symmetric, namely symmetric except for a finite set of spillover parameters. The following Proposition characterizes the unique family of utility functions with additive externalities for which equilibria are generically symmetric.

Proposition 6. *Consider the family of utility functions:* $\mathcal{F}=\{u(c)=-A(1-c)^{\beta}+B$ *with is* $\beta>1, A\beta\geq 1$ *and* $B\in\Re\}$. *For any utility function u in* $\mathcal{F}$, *equilibrium population distributions are symmetric except for a single value of* α. *Furthermore, any utility function with additive externalities for which equilibria are generically symmetric must belong to the family* $\mathcal{F}$.

Proposition 6 is a technical result which requires a sophisticated mathematical argument that can be found in the Appendix. The logic of the argument can easily be illustrated in the case of two jurisdictions. The only case where equilibria are asymmetric except for a single spillover parameter $\hat{\alpha}$ is when the indirect utility function is increasing in m for all $\alpha<\hat{\alpha}$ decreasing for all $\alpha>\hat{\alpha}$ and constant at the value $\hat{\alpha}$. If the utility function is constant at $\hat{\alpha}$ then $\frac{\partial V(m,\hat{\alpha})}{\partial m}=0$ for all m. This differential equation can then be solved to characterize the family $\mathcal{F}$ of power utility functions.

5. The effect of spillovers on equilibrium population distributions

In the preceding section, we have characterized a family of utility functions for which asymmetric equilibria only exist for a single value of the spillover parameter. For this specific family of utility functions, we now study the effect of spillovers on the equilibrium population distribution.

5.1. Symmetric spillovers

With symmetric spillovers, for any utility function u in $\mathcal{F}$, the difference in utility between a nonempty jurisdiction i and an empty jurisdiction j in a k-symmetric equilibrium is given by:

$$V_i-V_j=(A\beta)^{-\frac{1}{\beta-1}}k^{-\frac{1}{\beta-1}}(1-\alpha-1/\beta).$$

Hence, for $\alpha<\hat{\alpha}=1-1/\beta$, as in the case of pure local public goods, all p-symmetric population distributions are sustained as equilibrium (and the equilibrium with a single jurisdiction is the only stable equilibrium), and for $\alpha>\hat{\alpha}$ as in the case of pure public goods, the only equilibrium is the uniform equilibrium where all jurisdictions are nonempty. This result shows that an increase in α (from $\alpha<\hat{\alpha}$ to $\alpha>\hat{\alpha}$) may induce a switch from an equilibrium with a single jurisdiction to an equilibrium where all jurisdictions are populated.

5.2. Asymmetric spillovers

We now turn to the study of two examples with asymmetric spillovers. We consider a quadratic utility function:

$$u(c) = -\frac{1}{2}(1-c)^2.$$

Example 1. $n=2$ and $\alpha_{12}=\alpha>\alpha'=\alpha_{21}$.

In this example, there are only two jurisdictions, but the spillovers are asymmetric: members of the first benefit more from the public good provided in the second jurisdiction. We can compute

$$V_1 = \frac{m^2}{2} + \alpha(1-m)^2 \text{ and } V_2 = \frac{(1-m)^2}{2} + \alpha' m^2,$$

where m is the size of jurisdiction 1, so that

$$\text{If } \alpha \le 1/2, m = \sqrt{\frac{1-2\alpha}{1-2\alpha'}} \Big/ \left(1+\sqrt{\frac{1-2\alpha}{1-2\alpha'}}\right) \le 1/2$$

$$\text{If } \alpha' \le 1/2 \text{ and } \alpha \ge 1/2, m = 1$$

$$\text{If } \alpha' \ge 1/2, m = \sqrt{\frac{2\alpha-1}{2\alpha'-1}} \Big/ \left(1+\sqrt{\frac{2\alpha-1}{2\alpha'-1}}\right) \ge 1/2.$$

This example shows that there is no general relation (for all values of the spillover parameter) between the asymmetry in spillovers and the asymmetry in jurisdiction sizes. If spillovers are low, then the jurisdiction which benefits most from spillovers is always smaller; if spillovers are high, then the jurisdiction which benefits most from spillovers is larger.

Example 2. $n=3$, $\alpha_{12}=\alpha>\alpha_{13}=\alpha'>\alpha_{23}=\alpha''$.

In this example, jurisdiction 1 is more centrally located and benefits most from the spillovers produced in other jurisdictions, followed by jurisdictions 2 and 3. We consider the case where spillovers are large in order to concentrate on a single equilibrium where all jurisdictions are inhabited and assume $\alpha''>1/2$. In that case, the equilibrium is given by the solution of the system of quadratic equations:

$$\frac{m_1^2}{2} + \alpha m_2^2 + \alpha' m_3^2 = \frac{m_2^2}{2} + \alpha m_1^2 + \alpha'' m_3^2$$

$$\frac{m_2^2}{2} + \alpha m_1^2 + \alpha'' m_3^2 = \frac{m_3^2}{2} + \alpha' m_1^2 + \alpha'' m_2^2.$$

We immediately obtain:

$$\left(m_1^2 - m_2^2\right)\left(\alpha - \frac{1}{2}\right) = m_3^2(\alpha' - \alpha'') > 0$$

$$\left(m_2^2 - m_3^2\right)\left(\alpha'' - \frac{1}{2}\right) = m_1^2(\alpha - \alpha') > 0$$

so that $m_1 > m_2 > m_3$. When spillovers are large, the size of the jurisdiction thus reflects the location advantages of the jurisdiction. Jurisdictions which benefit more from the public goods offered in other jurisdictions have a larger population.

6. Conclusion

In this paper, we analyze the effects of spillovers on the formation of jurisdictions in a local public good economy with free mobility. We suppose that the number of jurisdictions is fixed and model the spillover structure as a matrix $[\alpha_{ij}]$ where $\alpha_{ij} \in [0,1]$. When spillovers are symmetric and close to 0 or 1 (pure local public goods and pure public goods), all equilibrium jurisdiction structures are symmetric. However, any population distribution can be sustained in equilibrium for some value of the spillover parameter α. In the class of utility functions with additive externalities, we identify the unique family of utility functions for which equilibria are symmetric except for an isolated value of α. This is a class of utility functions which are linear in the public good and a power function of the private good, $u(c,\gamma) = -A(1-c)^{\beta} + \gamma$. With this specification of utility, we show that an increase in α results in a more fragmented equilibrium population distribution, and that when spillovers are asymmetric and large, a jurisdiction which is more centrally located (i.e. benefits more from the public goods provided in other jurisdictions) has a larger population in equilibrium.

The basic model we studied in this paper can be extended in a number of directions. For the most part of the analysis, we have considered symmetric spillovers only. In a companion paper (Bloch and Zenginobuz (2004)), we consider the effect of asymmetric spillovers on the provision of local public good for a fixed population distribution, without endogenizing the distribution of agents among jurisdictions. It will be interesting to see whether more general results can be obtained regarding the impact of asymmetric spillovers on equilibria with free mobility as well as for a fixed population distribution. We have also assumed that there is no congestion in the use of public good. In fact, congestion could be introduced in two different ways in our model, one of which derives from the existence of spillovers across jurisdictions. First, in the standard fashion, we could assume congestion at the level of the jurisdiction (due for example to the price of land), and model congestion as a negative factor ($-\upsilon(m_i)$) affecting every agent in jurisdiction i. Whenever equilibrium utility is decreasing in the size of the jurisdiction, the addition of a congestion effect will not change our results. However, when equilibrium utility is increasing in the size of the jurisdiction (as in the case of pure local public goods), the introduction of congestion may result in a nonmononotic utility, and by itself allow for asymmetric equilibrium population distributions. Secondly, we could also model congestion as occurring in the use of the local public good across jurisdictions, whenever there are spillover benefits, thereby introducing a new channel of interdependency among jurisdictions. In our view, this is a challenging problem to address, and, in addition to other possible extensions of our basic model, we plan to study this issue as well in future research.

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Appendix A.

Proof of Theorem 1. We first note that, given our assumptions, the optimal provision of public good is bounded above and hence, we can define a compact interval X such that $G \in X$ for all jurisdictions. As in Konishi (1996), define the population correspondence as a mapping μ: $X^n \twoheadrightarrow \Delta$ where $\mu = \{m, m_i > 0 \Rightarrow u(1 - \frac{G_i}{m_i}, \sum_j \mathbf{a}_{ij} G_j) \geq u(1 - \frac{G_k}{m_k}, \sum_j \mathbf{a}_{kj} G_j) \forall k\}$. Because it is always possible to find the maximum of a finite set, the mapping μ is nonempty valued. Suppose that m, $m' \in \mu$. For any $\lambda \in [0,1]$, the support of $\lambda m + (1-\lambda)\, m'$ is contained in the union of the supports of m and m'. Hence, if $\lambda m + (1-\lambda)\, m'$ assigns positive weight to some jurisdiction i, $u\left(1 - \frac{G_i}{m_i}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right) \geq u\left(1 - \frac{G_k}{m_k}, \sum_j \boldsymbol{\alpha}_{kj} G_j\right) \forall k$. So $\lambda m + (1-\lambda) m' \in \mu$ and μ is convex valued. Finally, consider two sequences $m^t \to m$ and $(g^t, G^t) \to (g, G)$ and suppose $m^t \in \mu(g^t, G^t) \forall n$. Suppose by contradiction that $m \notin \mu(g, G)$. Then there exists i, k such that $m_i > 0$ and $u\left(1 - \frac{G_k}{m_k}, \sum_j \boldsymbol{\alpha}_{kj} G_j\right) > u\left(1 - \frac{G_i}{m_i}, \sum_j \boldsymbol{\alpha}_{ij} G_j\right)$. Now because m^t converges to m, there exists t_1 such that $m_i^t > 0$ for all $t \geq t_1$. Furthermore, because utilities are continuous, there exists t_2 such that, for all $t \geq t_2$, $u\left(1 - \frac{G_k^t}{m_k} \sum_j \boldsymbol{\alpha}_{kj} G_j^t\right) > u\left(1 - \frac{G_i^t}{m_i} \sum_j \boldsymbol{\alpha}_{ij} G_j^t\right)$. For $t > \max\{t_1, t_2\}$ we thus obtain that $m^t \notin \mu(g^t, G^t)$, contradicting the original assumptions. We conclude that μ is upper hemi continuous. Next, define the correspondence: $\beta_1 : X^{n-1} \times \Delta X$ as $\beta_i = \operatorname{argmax} u\left(1 - \frac{G_i}{m_i}, \sum_j \mathbf{a}_{ij} G_j\right)$. Because utility functions are continuous β_1 is non-empty valued and upper hemi continuous. Given that μ is strongly quasi-concave, one can easily check that μ is a strictly concave function of G_i, so that β_1 is convex valued. Finally, let Φ: $X^n \times \Delta \twoheadrightarrow X^n \times \Delta$ be the correspondence defined by $\Phi = \mu \times \beta_1 \times \ldots . \times \beta_n$. This correspondence satisfies all the conditions of Kakutani's fixed point theorem, and admits a fixed point which is an equilibrium of the model. □

Proof of Proposition 1. Suppose that there exist two distinct equilibria $\boldsymbol{G}$ and $\boldsymbol{G}'$ and without loss of generality suppose that $G_1 > G_1'$. We first show that this implies $\gamma_1 > \gamma_1'$. In fact, suppose by contradiction that $\gamma_1 > \gamma_1'$. By Assumption A1,

$$f\left(1 - \frac{G_1}{m_1}, \gamma_1\right) > f\left(1 - \frac{G_1'}{m_1}, \gamma_1'\right),$$

contradicting the fact that G_1 and G_1' satisfy the first order condition, with $G_1 > G_1'$. As $\gamma_1 \leq \gamma_1'$ and $G_1 > G_1'$, there must exist another jurisdiction, say jurisdiction 2 for which $G_1' > G_2$. The same line of argument then shows that $\gamma_2' \leq \gamma_2$. The two inequalities in γ can be developed as:

$$G_1 + \alpha G_2 + \alpha \sum_{j>2} G_j \leq G_1' + \alpha G_2' + \alpha \sum_{j>2} G_j'$$

$$G_2' + \alpha G_1' + \alpha \sum_{j>2} G_j' \leq G_2 + \alpha G_1 + \alpha \sum_{j>2} G_j.$$

Adding the two inequalities and rearranging, we obtain:

$$(1-\alpha)(G_1 - G_1') \leq (1-\alpha)(G_2 - G_2').$$

If $\alpha < 1$, this implies that

$$0 < G_1 - G_1' \leq G_2 - G_2' < 0$$

a contradiction.

If $\alpha = 1$, the inequalities imply:

$$\gamma_1 = \gamma_1'.$$

If the public good is strongly normal, f is strictly decreasing in c and hence,

$$f\left(1 - \frac{G_1}{m_1}, \gamma_1\right) > f\left(1 - \frac{G_1'}{m_1}, \gamma_1'\right),$$

contradicting the fact that G_1 and G_1' satisfy the first order condition, with $G_1 > G_1'$. □

Proof of Corollary 1. Consider the game played across jurisdictions for a fixed population distribution and spillover parameter. The equilibrium correspondence $\boldsymbol{G}$ and equilibrium payoffs V_i are upper hemi continuous in those parameters. However, as equilibrium is unique by Proposition 1, $\boldsymbol{G}$ and V_i are both continuous functions of the parameters $\boldsymbol{m}$ and α. □

Proof of Proposition 3. Consider first the case $\alpha = 0$. Then $\gamma_i = G_i$. By Assumptions (A1) and (A2), the equilibrium level of public good provision is unique and characterized by the first order conditions:

$$f\left(1 - \frac{G_i}{m_i}, G_i\right) = m_i$$

We can easily compute

$$\frac{\partial V}{\partial m_i} = \frac{G_i}{m_i^2} u_c > 0.$$

Hence, $V_i(\boldsymbol{m}, 0) = V_j(\boldsymbol{m}, 0) \Rightarrow m_j$ and, if $m_i > m_j = 0$, then $V_i(\boldsymbol{m}, 0) > V_j(\boldsymbol{m}, 0)$. We conclude that for $\alpha = 0$, the set of equilibria is exactly equal to all p-symmetric population distributions.

Now consider any population distribution $\boldsymbol{m}$ which is not symmetric. There exist two jurisdictions i,j with $m_i \neq m_j$, and by the preceding argument $V_i(\boldsymbol{m}, 0) \neq V_j(\boldsymbol{m}, 0)$. Given that the indirect utility function is continuous in α by Corollary 1, there exists $\underline{\alpha}(\boldsymbol{m})$ such that for all $\alpha \leq \underline{\alpha}(\boldsymbol{m})$, $V_i(\boldsymbol{m}, \alpha) \neq V_j(\boldsymbol{m}, \alpha)$, and $\boldsymbol{m}$ cannot be an equilibrium population distribution. Pick then $\underline{\alpha} = \min_{\boldsymbol{m}} \{\underline{\alpha}(\boldsymbol{m})\}$. For any $\alpha \leq \underline{\alpha}$, all equilibrium population distributions must be symmetric, and the set of equilibria is equal to all p-symmetric population distributions.

Consider now the case $\alpha = 1$. As $\gamma_i = \gamma_j$ for all i, j, $u(1 - G_i/m_i, \gamma_i) = u(1 - G_j/m_j, \gamma_j) \Rightarrow G_i/m_i = G_j/m_j$. By Assumption (A2), $\lim_{\gamma \to 0} f(c, \gamma) = 0$ so there cannot exist an equilibrium where $G_i = 0$ for all i. Furthermore, by Assumption (A2), $\lim_{c \to 0} u_c(c, m)/u_\gamma(c, m) \geq m$. We claim that this boundary condition implies that there cannot be an equilibrium where $G_i = m_i$ and $\frac{i}{m_i} u_c(0, \gamma_i) + u_\gamma(0, \gamma_i) > 0$ for some jurisdiction i. Suppose that such an equilibrium existed. Then,

$$f(0, \gamma_i) < m_i$$

Because f is weakly increasing in the public good,

$$f(0, m_i) \leq f(m_i, \gamma_i) < m_i$$

in contradiction to the boundary condition: $f(0, m_i) \geq m_i$. We conclude that, in equilibrium, for any jurisdiction i,

$$f\left(1 - \frac{G_i}{m_i}, \gamma_i\right) = m_i$$

But then:

$$m_i = f\left(1 - \frac{G_i}{m_i}, \gamma_i\right) = f\left(1 - \frac{G_j}{m_j}, \gamma_j\right) = m_j$$

so that the only equilibrium is the uniform equilibrium.

Now consider any population distribution $\boldsymbol{m} \neq \boldsymbol{m}^n$. By the preceding argument, there exist two jurisdictions i, j such that $V_i(\boldsymbol{m}, 1) \neq V_j(\boldsymbol{m}, 1)$. If the public good is strongly normal, $\left(\frac{\partial f}{\partial c} < 0\right)$, by continuity of the indirect utility function, there exists $\overline{\alpha}(\boldsymbol{m})$ such that for all $\alpha \geq \underline{\alpha}(\boldsymbol{m})$, $V_i(\boldsymbol{m}, \alpha) \neq V_j(\boldsymbol{m}, \alpha)$ and $\boldsymbol{m}$ cannot be an equilibrium population distribution. Pick then $\overline{\alpha} = \max_{\boldsymbol{m}} \{\overline{\alpha}(\boldsymbol{m})\}$. For any $\alpha \geq \overline{\alpha}$, all equilibria involve a uniform population distribution. If the public good is not strongly normal $\left(\frac{\partial f}{\partial c} = 0\right)$, then consider the unique strategy profile defined by $\boldsymbol{G}(\boldsymbol{m}, 1) = \lim_{\alpha \to 1} \boldsymbol{G}(\boldsymbol{m}, \alpha)$. Because the equilibrium correspondence is upper hemi continuous, $\boldsymbol{G}(\boldsymbol{m}, 1)$ forms an equilibrium of the game played across jurisdictions, and hence for this equilibrium, there exist two jurisdictions i, j such that $V_i(\boldsymbol{m}, 1) \neq V_j(\boldsymbol{m}, 1)$. The same argument as above then shows that there exists $\overline{\alpha}$ such that, for any $\alpha \geq \overline{\alpha}$, all equilibria involve a uniform population distribution.

Proof of Proposition 4. Let $0 \leq m \leq 1/2$ denote the size of the first jurisdiction. Construct the difference in utilities between the two jurisdictions as:

$$\phi(m, \alpha) = V_i((m, 1 - m), \alpha) - V_2((m, 1 - m), \alpha).$$

We now show that, for all values $\boldsymbol{m} \in [0, 1/2)$, there exists a value of the spillover parameter for which $\phi(m, \alpha) = 0$.

For $\alpha = 0$, as $m < 1 - m$, $\phi(m, 0) < 0$. For $\alpha = 1$, we first show $G_2 > 0$. Suppose by contradiction that in equilibrium $G_2 = 0$ and $G_1 > 0$. (One of the two jurisdictions has to provide a positive amount of the public good by the boundary condition of Assumption 2.). Then:

$$1 - m \leq f(1, G_1) \leq f\left(1 - \frac{G_1}{m}, G_1\right) = m$$

where the first inequality and the last equality reflect the first order conditions, and the middle inequality is due to the fact that f is nonincreasing in the consumption good. But this clearly contradicts the hypothesis $m < 1 - m$. Consider then the two possible equilibrium configurations: $G_1 = 0$, $G_2 > 0$ and $G_1 > 0$, $G_2 > 0$. In the first case, clearly, $u(1, G_2) > u\left(1 - \frac{G_2}{1-m}, G_2\right)$ and hence $\phi(m, 1) > 0$. In the second case, the boundary condition of Assumption A2 guarantees that equilibria are interior and the first order conditions imply

$$1 - m = f\left(1 - \frac{G_2}{1 - m}, G_1 + G_2\right) > f\left(1 - \frac{G_1}{m}, G_1 + G_2\right) = m,$$

which can only be satisfied if $G_2/(1-m)>G_1/m$. Hence, $u\left(1-\frac{G_1}{m}, G_1+G_2\right)>u\left(1-\frac{G_2}{1-m}, G_1+G_2\right)$ and $\phi(m,1)>0$ for all possible equilibrium levels $\boldsymbol{G}$.

We have thus shown that for the continuous function[11] $\phi(m,\alpha)$ over the compact interval $[0,1]$, $\phi(m,0)>0$ and $\phi(m,1)<0$. By the Intermediate Value Theorem, there must exist a value $\alpha\in(0,1)$ such that $\phi(m,\alpha)=0$.

Proof of Proposition 5. We first compare equilibrium utilities in different symmetric equilibria. Given Assumption (A2), the equilibrium individual contribution of public good in every jurisdiction is interior, and given by:

$$u'(1-g)=\frac{1}{p}$$

Treating p as a continuous variable, we can compute

$$\frac{\partial g}{\partial p}=\frac{1/p^2}{u''(1-g)}<0$$

The equilibrium utility is given by $V=u(1-g)+\frac{1+\alpha(p-1)}{p}g$. Differentiating V with respect to p, and using the first order condition, we obtain:

$$\frac{\partial V}{\partial p}=\frac{(1-\alpha)}{p^2}+\frac{\alpha(p-1)}{p}\frac{\partial g}{\partial g}$$

establishing the result.

In order to compare equilibrium utilities in an asymmetric and symmetric equilibrium, consider any population distribution $\boldsymbol{m}=(m,1-m)$ with $m<1/2$. The equilibrium public good provision level in the two jurisdictions is given by the first order conditions:

$$u'\left(1-\frac{G_1}{m}\right)=m, u'\left(1-\frac{G_2}{1-m}\right)=1-m.$$

We consider the sum of utilities in the two jurisdictions:

$$V_1+V_2=u\left(1-\frac{G_1}{m}\right)+u\left(1-\frac{G_2}{1-m}\right)+(1+\alpha)(G_1+G_2).$$

Differentiating the sum of utilities with respect to m and using the first order conditions:

$$\frac{\partial(V_1+V_2)}{\partial m}=\frac{G_1}{m}-\frac{G_2}{(1-m)}+\alpha\frac{\partial G_1}{\partial m}+\alpha\frac{\partial G_2}{\partial m}$$

Next,

$$\frac{\partial G_1}{\partial m}=\frac{G_1}{m}-\frac{m}{u''\left(1-\frac{G_1}{m}\right)}, \frac{\partial G_2}{\partial m}=-\frac{G_2}{(1-m)}+\frac{1-m}{u''\left(1-\frac{G_2}{1-m}\right)}. \tag{1}$$

[11] If the public good is not strongly normal,we consider a continuous selection of the correspondence at $\alpha=1$ by letting $\boldsymbol{G}(\boldsymbol{m},1)=\lim_{\alpha\to 1}\boldsymbol{G}(\boldsymbol{m},\alpha)$.

Replacing, we get:

$$\frac{\partial(V_1+V_2)}{\partial m} = \left(\frac{G_1}{m} - \frac{G_2}{(1-m)}\right)(1+\alpha) + \alpha\left(\frac{1-m}{u''\left(1-\frac{G_2}{1-m}\right)} - \frac{1-m}{u''\left(1-\frac{G_1}{1-m}\right)}\right).$$

The first order conditions guarantee that, as long as $m<1-m$, the individual contributions are lower in the first jurisdiction $\frac{G_1}{m}<\frac{G_2}{(1-m)}$. Furthermore, as $1-\frac{G_2}{1-m}<1-\frac{G_1}{m}$ and $u'''<0$, $u''\left(1-\frac{G_2}{1-m}\right)\geq u''\left(1-\frac{G_1}{m}\right)$ so that $\frac{1-m}{u''\left(1-\frac{G_2}{1-m}\right)}-\frac{m}{u''\left(1-\frac{G_1}{m}\right)}<0$. This shows that the sum of utilities in the two jurisdictions is lower in the uniform population distribution ($m=1/2$) than in an asymmetric distribution ($m<1/2$). In equilibrium, $V_1=V_2$ so that the equilibrium utility of all the agents is higher in an asymmetric equilibrium than in the uniform equilibrium.

Proof of Proposition 6. Consider a utility function in the family $\mathcal{F}: u(c) = -A(1-c)^{\beta}+B$. Then,

$$G(m) = \left(\frac{m^{\beta}}{\beta A}\right)^{\frac{1}{\beta-1}}$$

For any distribution $\boldsymbol{m}$ and any m_i, m_j

$$V_i(\boldsymbol{m},\alpha) - V_j(\boldsymbol{m},\alpha) = (A\beta)^{-\frac{1}{\beta-1}}\left(m_i^{\frac{\beta}{\beta-1}} - m_j^{\frac{\beta}{\beta-1}}\right)(1-\alpha-1/\beta).$$

Hence, asymmetric equilibria, where $V_i(\boldsymbol{m},\alpha)-V_j(\boldsymbol{m},\alpha)=0$ and $m_i\neq m_j$, only exist for the spillover parameter value $\hat{\alpha}=1-1/\beta$.

Consider an arbitrary utility function u with additive externalities and denote, as above, the difference in utility between agents in two jurisdictions by

$$\phi(m,\alpha) = u\left(1-\frac{G_1}{m}\right) - u\left(1-\frac{G_2}{1-m}\right) - (G_2-G_1)(1-\alpha).$$

By the Implicit Function Theorem (Mas-Colell et al., 1995, p.941), as long as $\partial\phi/\partial\alpha\neq 0$ for some (α,m) satisfying $\phi(\alpha,m)=0$, we can compute

$$\frac{\partial\alpha}{\partial m} = -\frac{\partial\phi/\partial m}{\partial\phi/\partial\alpha}.$$

Hence, if there exists a couple (α,m) satisfying $\phi(\alpha,m)=0$ for which $\partial\phi/\partial m\neq 0$ and $\partial\phi/\partial\alpha\neq 0$, then there exists an open neighborhood around for which asymmetric equilibria exist. We conclude that a *necessary condition* for asymmetric equilibria to exist only for a finite set of spillover parameters is that:

For all (α,m) satisfying $\phi(m,\alpha)=0$, either $\partial\phi/\partial m=0$ or $\partial\phi/\partial\alpha=0$. Using the first order conditions, the equations $\partial\phi/\partial m=0$ or $\partial\phi/\partial\alpha=0$ can be rewritten as:

$$\frac{\partial\phi}{\partial m} = \frac{G_1}{m} - \alpha\frac{\partial G_1}{\partial m} + \frac{G_2}{1-m} + \alpha\frac{\partial G_2}{\partial m}, \tag{2}$$

$$\frac{\partial\phi}{\partial\alpha} = G_2 - G_1.$$

For all $m\in[0,1/2)$, $G_2-G_1>0$ and $\frac{\partial\phi}{\partial\alpha}\neq 0$. This implies that, if the set of spillovers for which asymmetric equilibria is finite, $\partial\alpha/\partial m=0$ for all $m\in[0,1/2)$. Hence, there exists a unique value

$\hat{\alpha}$ for which asymmetric equilibria exist. This value can easily be computed by setting $\phi(0,\hat{\alpha})=0$. Furthermore, using Eq. (1),

$$\frac{\partial \phi}{\partial m} = \frac{G_1}{m}(1-\hat{\alpha}) + \frac{\hat{\alpha} m}{u''\left(1-\frac{G_1}{m}\right)} + \frac{G_2}{1-m}(1-\hat{\alpha}) + \frac{\hat{\alpha}(1-m)}{u''\left(1-\frac{G_2}{1-m}\right)}.$$

Now for any jurisdiction size m and corresponding public good level G, define

$$h(m) \triangleq \frac{G}{m}(1-\hat{\alpha}) + \frac{\hat{\alpha} m}{u''\left(1-\frac{G_2}{m}\right)}$$

Eq. (2) thus amounts to:

$$h(m) = -h(1-m) \text{ for all } m \in [0,1/2). \tag{3}$$

Consider next an equilibrium with three jurisdictions, $\boldsymbol{m}=(m,m,1-2m)$ for $m\in[0,1/3)$. Let

$$\Psi(m,\alpha) = V_1(m,\alpha) - V_3(m,\alpha)$$

denote the difference in utility between members of the smaller jurisdiction of size m and of the larger jurisdiction of size $1-2m$. Given that the two first jurisdictions have equal size, an equilibrium is characterized by:

$$\Psi(m,\alpha) = 0$$

The previous argument can be replicated to show that, if equilibrium is symmetric for a generic set of spillover parameters, then there must exist a unique spillover parameter $\hat{\hat{\alpha}}$ such that $\frac{\partial \Psi}{\partial m}=0$ for all $m\in[0,1/3)$. This spillover parameter can be computed by setting $\Psi\left(0,\hat{\hat{\alpha}}\right)=0$. By construction, $\Psi(0,\alpha)=\phi(0,\alpha)$ and hence $\hat{\hat{\alpha}}=\hat{\alpha}$. The derivative $\frac{\partial \Psi}{\partial m}$ can thus be rewritten as:

$$\begin{aligned}\frac{\partial \Psi}{\partial m} &= \frac{G(m_1)}{m_1} - \hat{\alpha}\frac{\partial G(m_1)}{\partial m_1} + 2\left(\frac{G(m_3)}{m_3} - \hat{\alpha}\frac{\partial G(m_3)}{\partial m_3}\right) \\ &= h(m) + 2h(1-2m).\end{aligned}$$

Hence,

$$h(m) = -2h(1-2m) \text{ for all } m \in (1,1/3).$$

As $h(m)=-h(1-m)$ for all $m\in[0,1/3)$ by Eq. (3), we conclude that $h(1-2m)=-h(2m)$ and hence,

$$h(m) = 2h(2m) \text{ for all } m \in [0,1/3).$$

We now show that this implies that $h(m)=0$ for all $m<1/2$. Suppose by contradiction that $h(m)\neq 0$ for some $m<1/2$. Consider the sequence $x_n=m/2^n$. As $m<1/2$, $x_n<1/3$ for all $n\geq 1$ and hence

$$h(x_{n+1}) = 2h(x_n)$$

implying that

$$h(x_n) = 2^n h(m)$$

As the function h is continuous,

$$h(0)\lim_{n\to\infty} h(x_n) = \lim_{n\to\infty} 2^n h(m).$$

Now, if $h(m)>0$, $\lim_{n\to\infty} 2^n h(m)=\infty$ and if $h(m)<\lim_{n\to\infty} 2^n h(m)=\infty$. However, a direct computation shows that $h(0)=0$, yielding a contradiction.

$$\text{As } h(m) = h(1-m) = 0 \text{ for all } m \in [0,1/2), h(m) = 0 \text{ for all } m \in [0,1]. \tag{4}$$

Defining $c=1-G/m$, Eq. (4) can be rewritten

$$(1-c)(1-\hat{\alpha})u''(c) + \hat{\alpha}u'(c) = 0 \text{ for all } c \in (\underline{c}, 1]$$

where $\underline{c}$ is the value of c computed at $m=1$.

Solving this differential equation, we find

$$u'(c) = A(1-c)^{\hat{\alpha}/(1-\hat{\alpha})}.$$

The boundary conditions of Assumption 2 imply that $A \geq 1$, which also guarantees that $u''(c)>0$. Integrating, we finally obtain:

$$u(c) = -\frac{1}{1-\hat{\alpha}} A(1-c)^{1/(1-\hat{\alpha})} + B$$

for $A \geq 1$ *and* $B \in \Re$. This concludes the proof of the proposition. □

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