

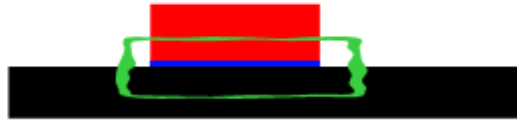
Lecture 12: Friction

- What is friction?
- Why is it important to know about friction for a control engineer?
- Static friction models
- Dynamic friction models

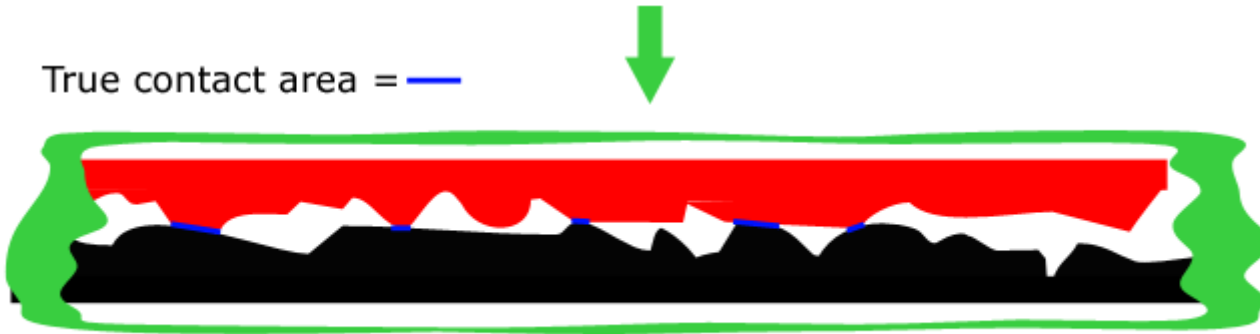
Book: Ch. 5

What is friction?

Apparent contact area = —



True contact area = —



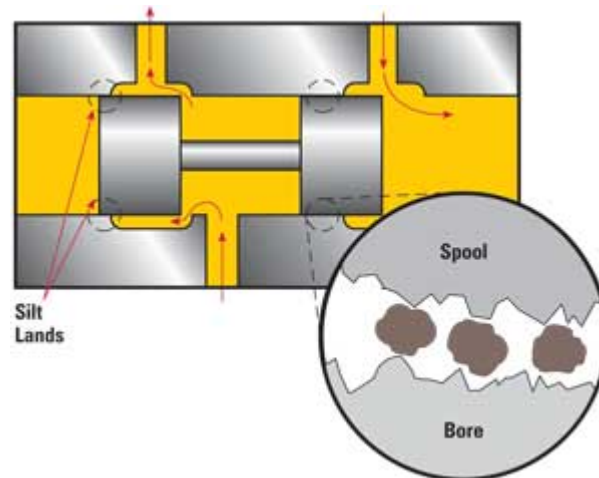
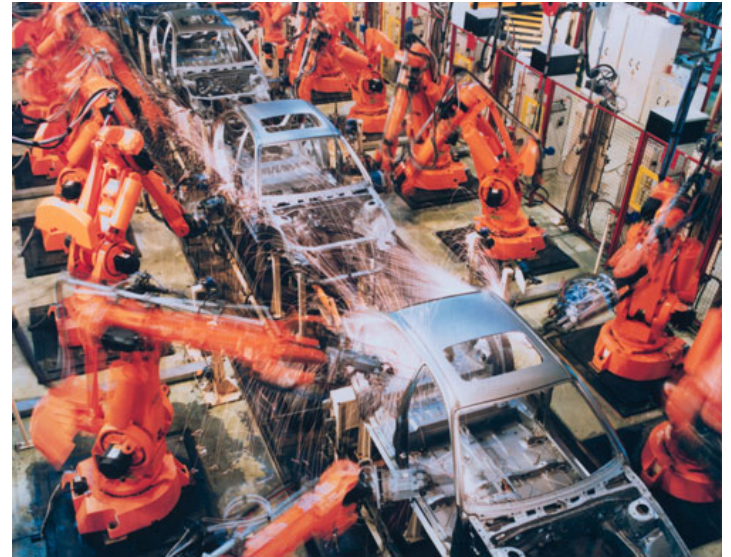
- “The evil of all motion”:
 - No matter which direction something is pushed, friction pulls it the other way
- But not all bad: Without friction we cannot move
 - Walking, cycling, driving, flying, ...



Control systems with friction, I

Friction is a problem for

- Control systems for positioning
 - Electrical and hydraulic actuators
 - Translational or rotational
- In process systems: Valves with friction
 - Often “stiction”



Control systems with friction, II

Friction can be used to control motion

- Electronic stability control (ESC), "anti-skidding"

Without ESC:



With ESC:



- Also ABS systems exploits friction characteristics

Static friction models

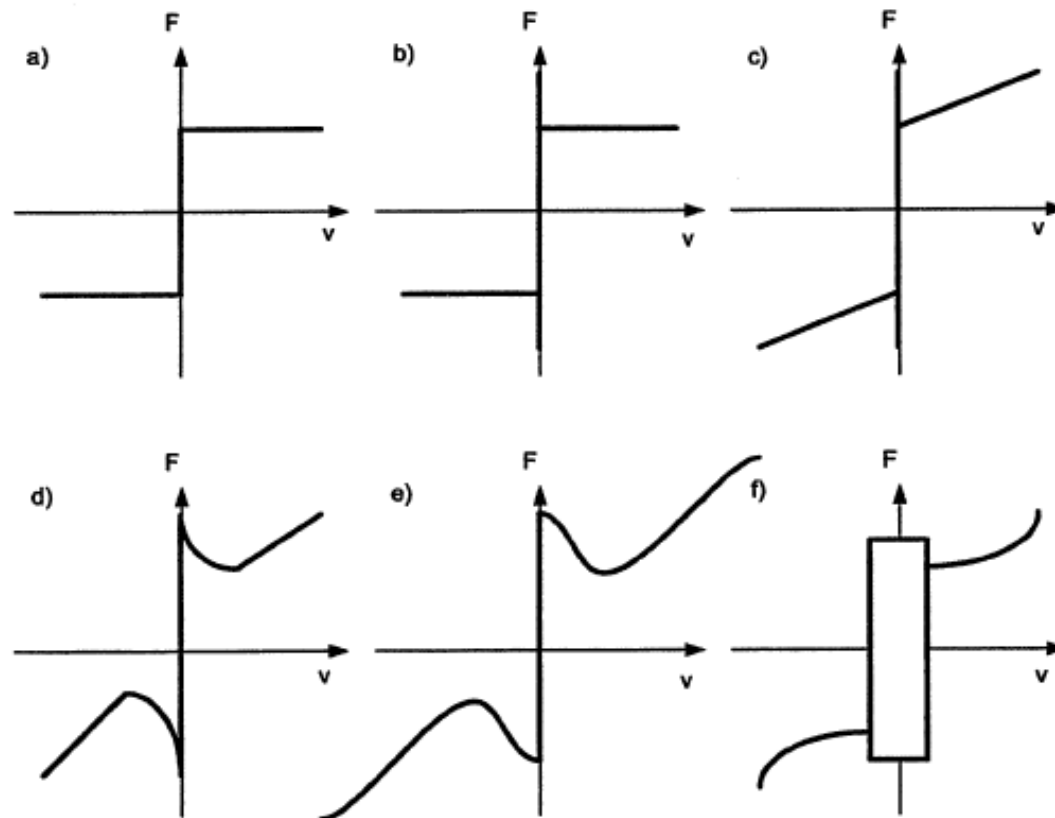
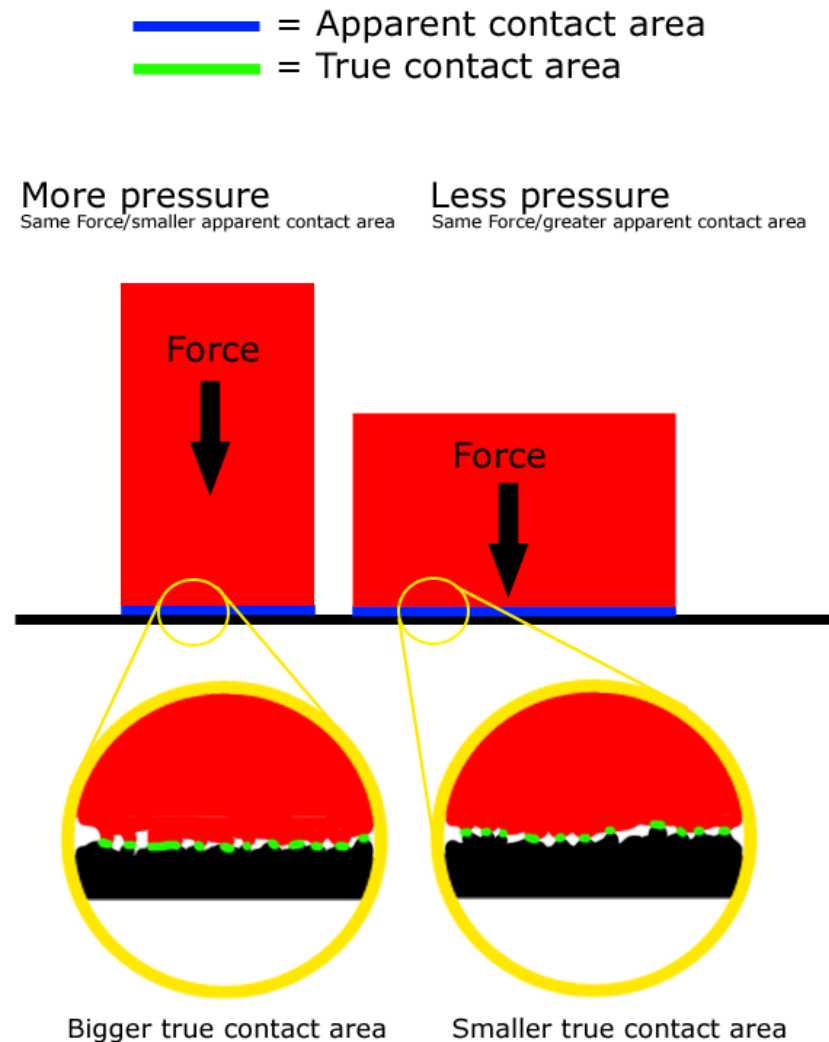


Figure 5.3: Static friction models: a) Coulomb friction b) Coulomb+stiction c) Coulomb+stiction+viscous d) Stribeck effect e) Hess and Soom; Armstrong f) Karnopp model

Dry friction is

- Independent of area
 - Da Vinci
- Proportional to normal force
 - Amonton, Euler
- Independent of velocity
 - Coloumb

$$F = \mu F_N$$



Generalized Stribeck curve

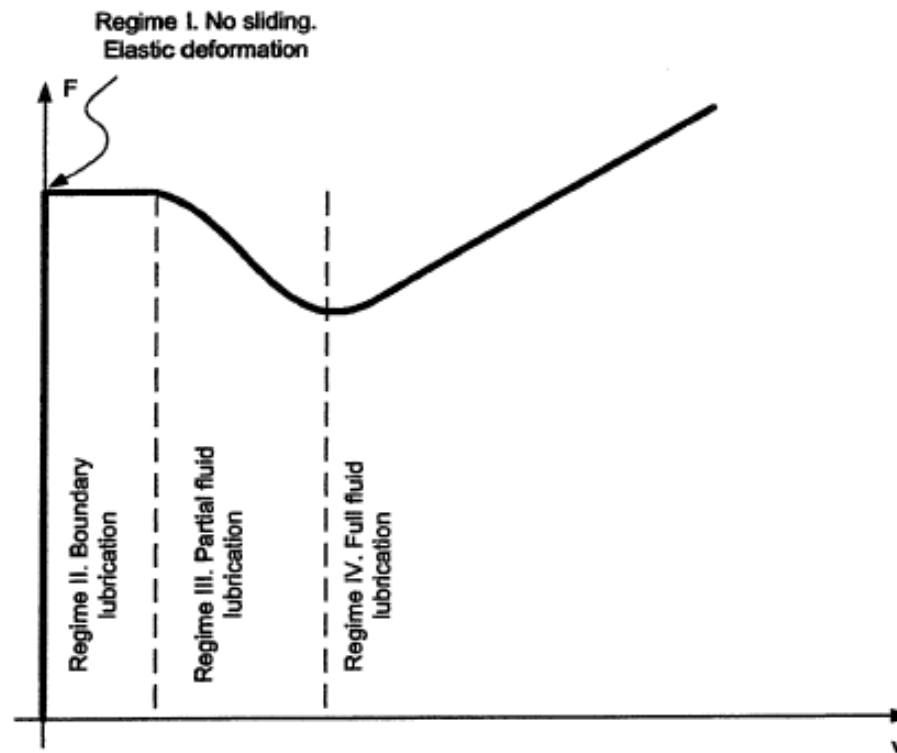


Figure 5.2: The generalized Stribeck curve, showing friction as a function of velocity for low velocities, (Armstrong-Hélouvry et al. 1994).

Dynamic friction models

The Dahl model

$$\frac{dF}{dt} = \sigma \left(v - |v| \frac{F}{F_c} \right)$$

The LuGre model

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v$$

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z$$

$$g(v) = F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2}$$

Why dynamic friction models?

- Easier to simulate
- Easier to analyze
- They reproduce (to some extent) dynamic friction phenomena
 - Presliding displacement
 - friction force act as a spring in sticking region
 - Frictional lag
 - Dynamic friction force depends on direction of velocity
 - Varying break-away force
 - Break-away force depends on rate-of-change of applied force

ABS-system – blokkeringsfrie bremseser

- Hva er det som gjør at **bremsing, gass, styring** får bilen til å endre hastighet?

Friksjon mellom hjul og vei

- Hva bestemmer friksjon?
 - Tyngde
 - Underlag og egenskaper ved dekk
 - tørr asfalt, våt asfalt, snø, is
 - **Relativ hastighetsforskjell** mellom bil og hjul
 - langsgående (longitudinal) slipp, side- (lateral) slipp



Slipp – relativ hastighetsforskjell

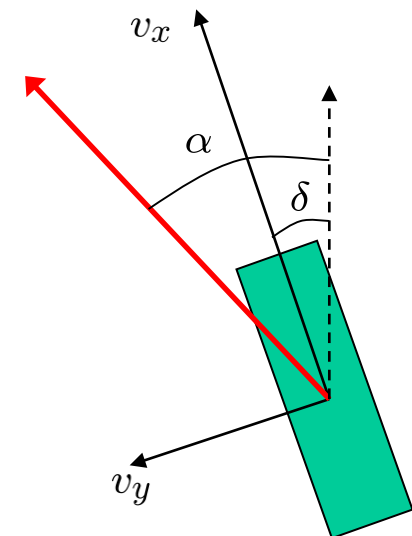
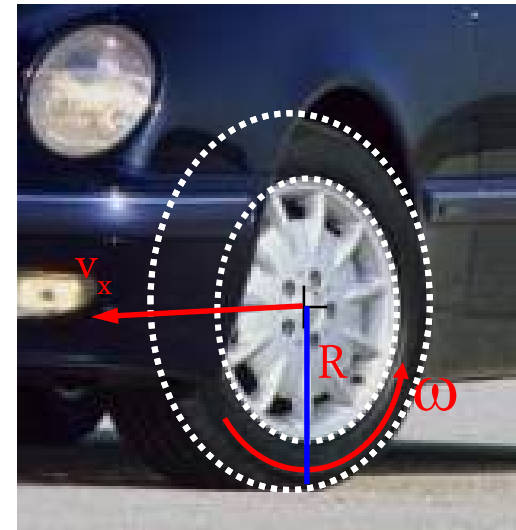
- I langsretning:

$$\lambda_x := \frac{v_x - R\omega}{v_x}$$

- I sideretning:

$$\lambda_y := \sin \alpha$$

$$\alpha := \delta + \arctan \frac{v_y}{v_x}$$



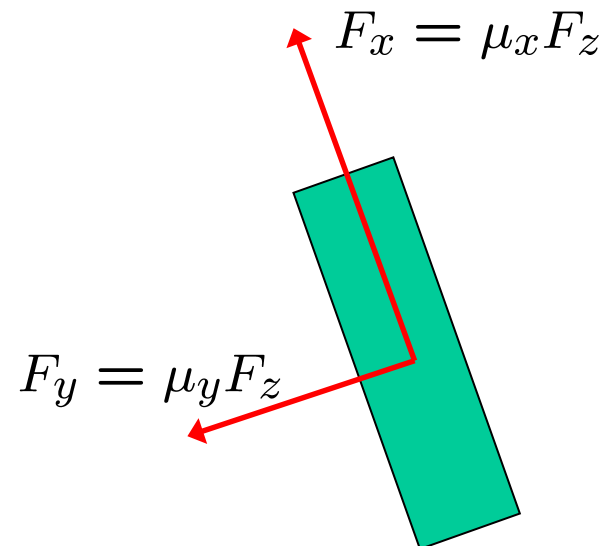
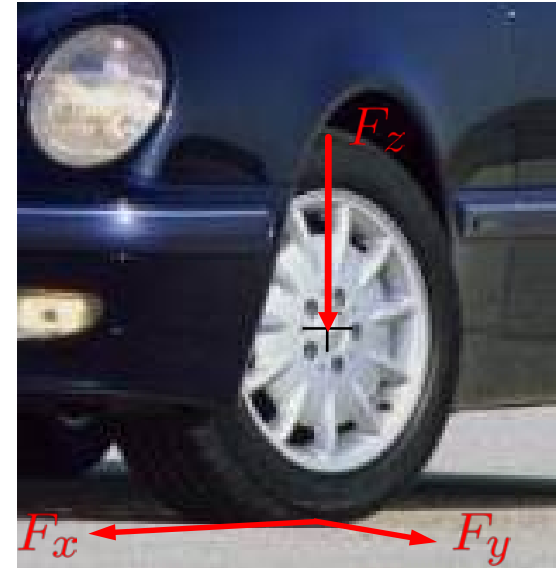
Friksjonskrefter

Coloumbs lov:

- Friksjonskrefter gitt av vertikale krefter og friksjonskoeffisient
- Friksjonskoeffisient gitt av slipp og underlag

$$\mu_x \approx \mu_x(\lambda_x, \lambda_y, \mu_H)$$

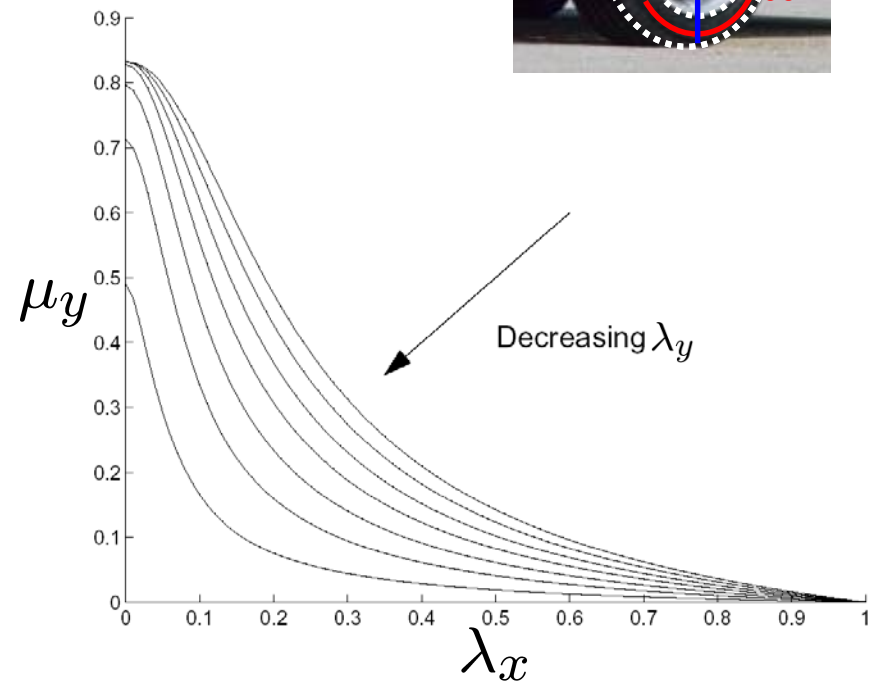
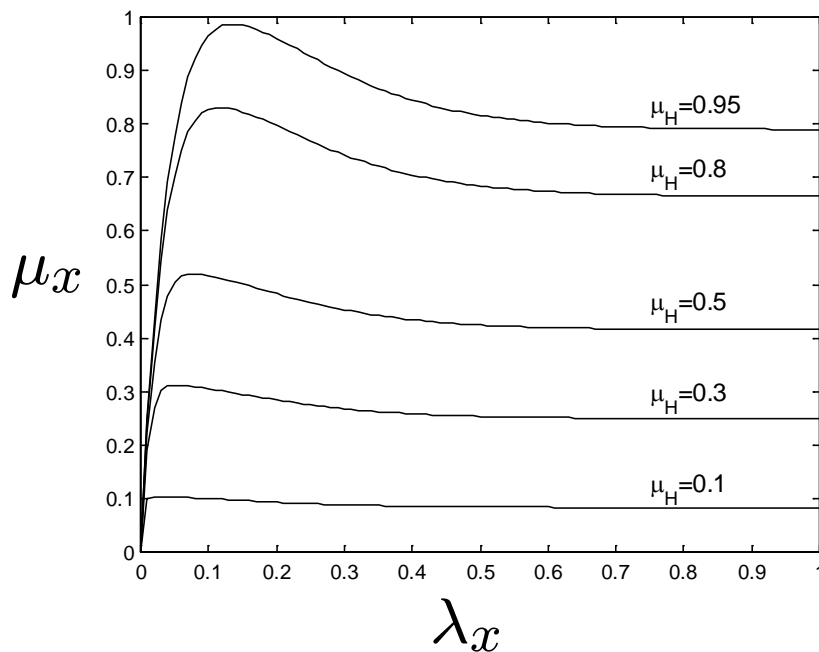
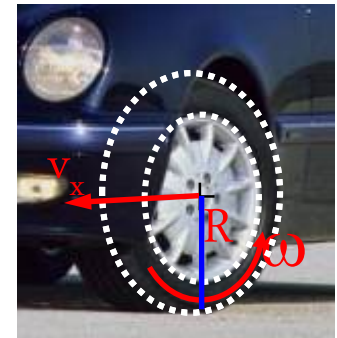
$$\mu_y \approx \mu_y(\lambda_y, \lambda_x, \mu_H)$$



Friksjonskoeffisienter under bremsing

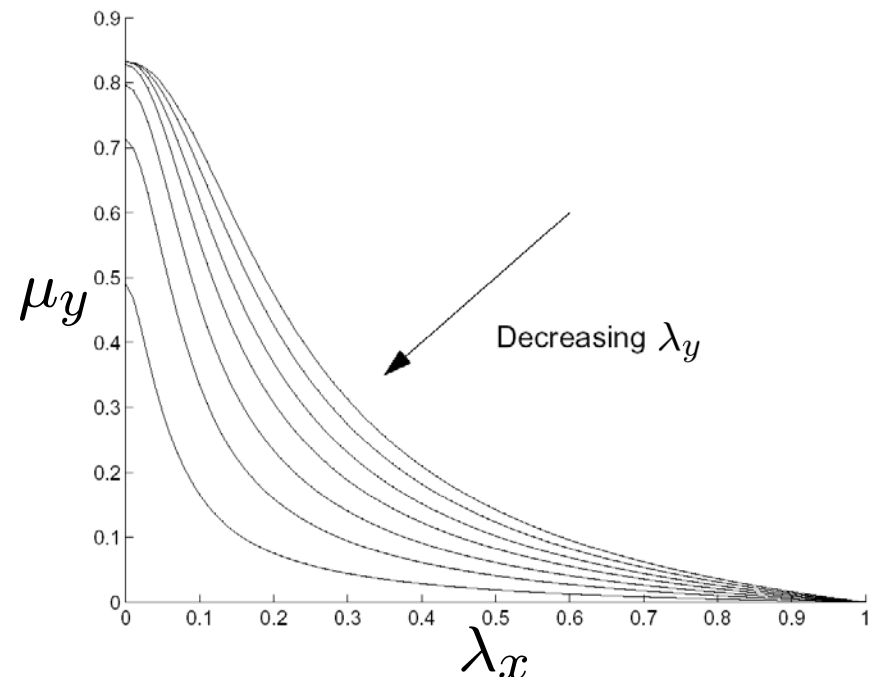
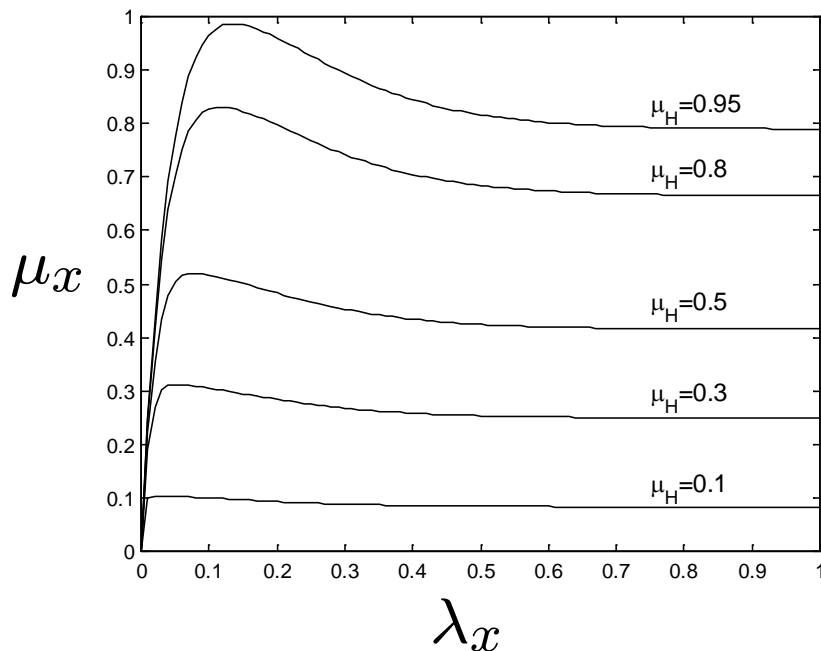
- Bremsing reduserer hjulhastighet i forhold til bilhastighet

$$\lambda_x := \frac{v_x - R\omega}{v_x}$$



Blokkeringsfrie bremsere – ABS

- Ønsker **konstant lav slipp** under bremsing fordi
 - Det gjør bremsing mest effektivt
 - Kan styre bilen under bremsing



ABS i praksis

Bremselengde:



Unnamanøver:

