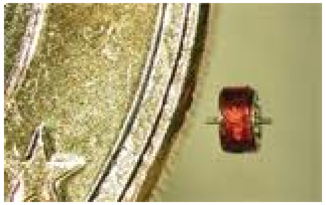
Lecture 7: Electromechanical systems

- Electrical motors
- DC motor with constant field
- Some network modeling, passivity, ...

Book: 3.2, 3.3

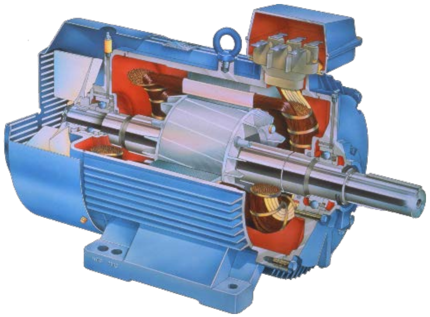
Why modeling of electrical motors?

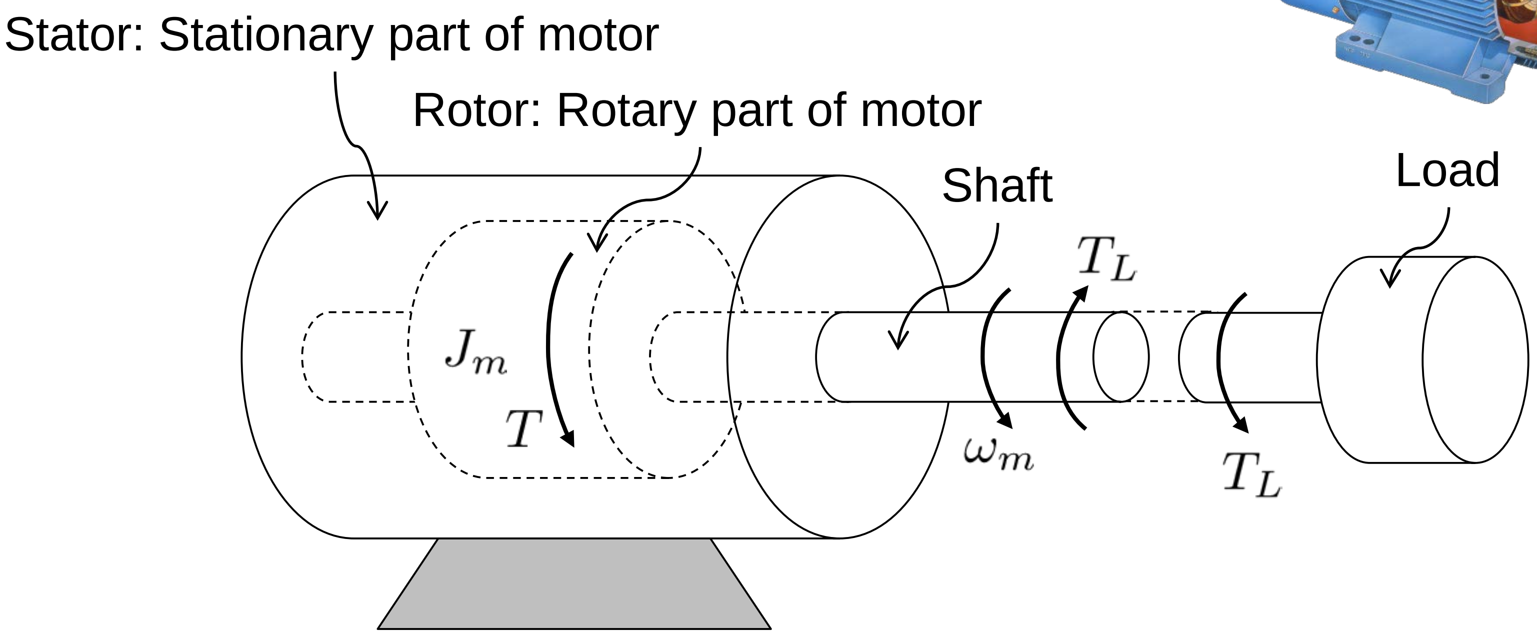
- Electrical (and hydraulic) motors are used when something should move
  - Used everywhere: Process industries, offshore oil&gas production, electromechanical systems, cars ...
  - Large and small



  - Often actuator (e.g. in a valve, in a compressor, ...)
- Example of modeling across domains (electrical + mechanical), and network modeling
  - Hydraulic motors another example (Ch. 4)
- Example of control-relevant modeling
  - Linear (transfer function) modeling

Motors





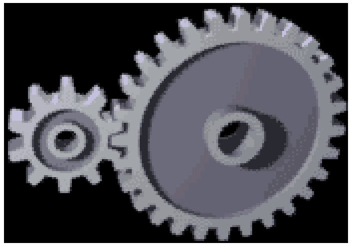
- Equation of motion for motor shaft:

$$J_m \dot{\omega}_m = T - T_L$$

  where
  - T : Motor torque (set up by some device, e.g. DC motor)
  - T_L : Load torque
  - J_m : Moment of inertia for rotor and shaft
  - ω_m: Angular velocity/motor speed [rad/s, or rev./min]

Gears

Rotational gear
(cogwheel)



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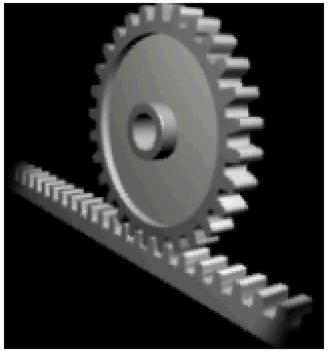
Translational gear
(rack and pinion)



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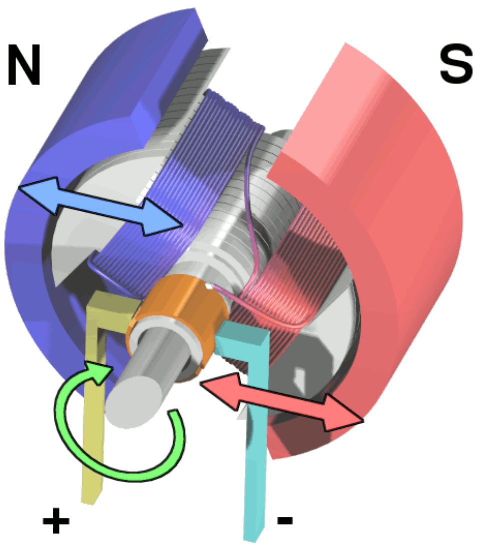
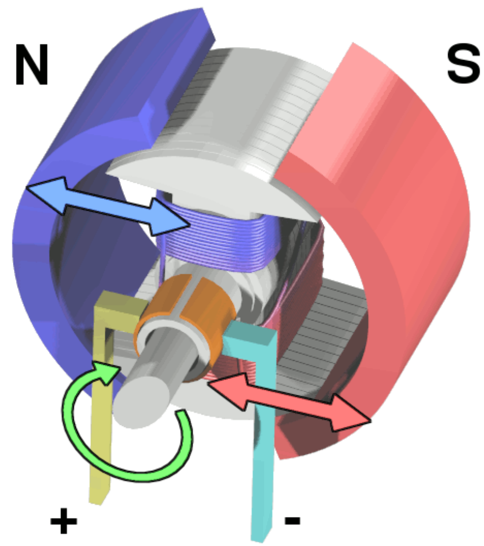
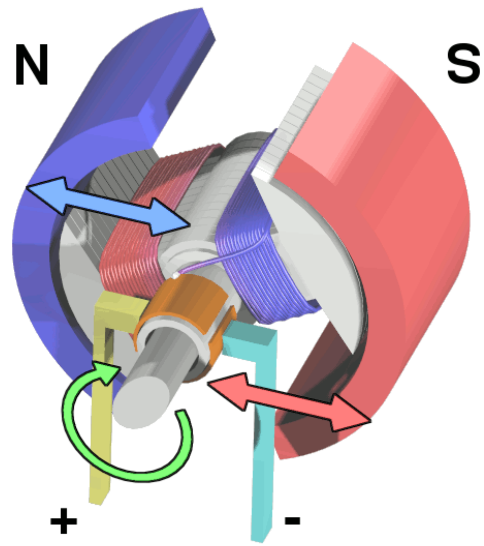
A simple DC electric motor



A simple DC electric motor. When the coil is powered, a magnetic field is generated around the armature. The left side of the armature is pushed away from the left magnet and drawn toward the right, causing rotation.



The armature continues to rotate.



When the armature becomes horizontally aligned, the commutator reverses the direction of current through the coil, reversing the magnetic field. The process then repeats.

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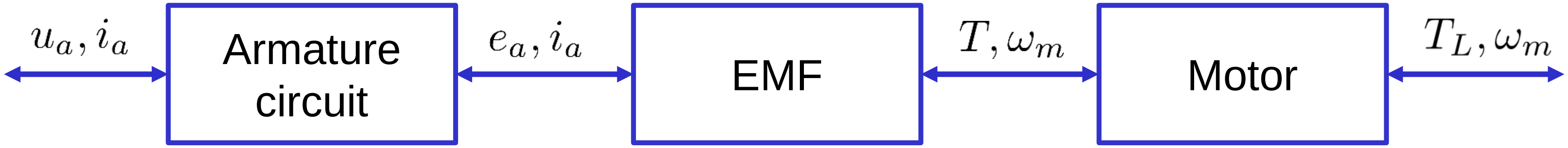
Network modeling of DC-motor:

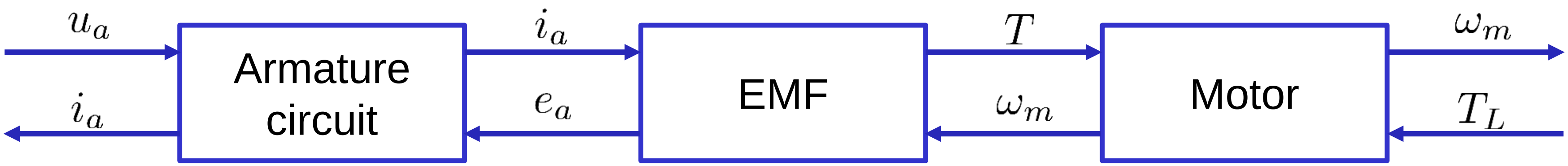


$$L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} = -R_a i_a - e_a + u_a$$

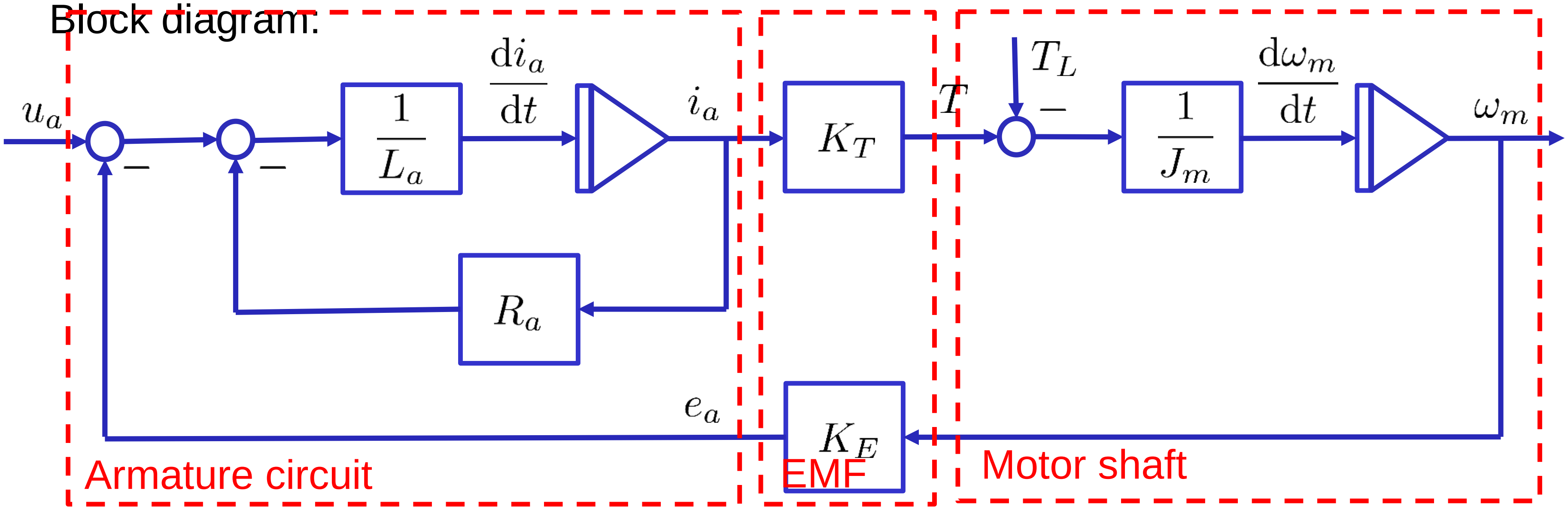
$$T = K_T i_a$$
$$e_a = K_E \omega_m$$

$$J_m \frac{\mathrm{d}\omega_m}{\mathrm{d}t} = T - T_L$$

Signal flow modeling of DC-motor:



Block diagram:



Dymola Demo: Motor Drive

- File -> Demos -> Motor Drive

- Modelica.Electrical.Machines

Passivity

- A system with input u and output y is passive if

$$\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$$

  for all $t \geq 0$, for all input trajectories.

- If the product yu has power as unit, then if
  - $\int_0^t y(\tau)u(\tau)d\tau \geq 0$: Energy is absorbed within the system, nothing delivered to the outside
  - $\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$: Some energy can be delivered to the outside, limited (typically) by the initial energy in the system.
  - $\int_0^t y(\tau)u(\tau)d\tau \to -\infty$: There is an inexhaustible energy source in the system. Not passive!

Recap: Explicit Runge-Kutta (ERK) methods

- IVP: $\dot{y} = f(y, t), \quad y(0) = y_0$
- One-step methods: $y_{n+1} = y_n + h\phi(y_n, t_n), \quad h = t_{n+1} - t_n$
- ERK:

$$
\begin{aligned}
k_1 &= f(y_n, t_n) \\
k_2 &= f(y_n + ha_{21}k_1, t_n + c_2 h) \\
k_3 &= f(y_n + h(a_{31}k_1 + a_{32}k_2), t_n + c_3 h) \\
&\vdots \\
k_\sigma &= f(y_n + h(a_{\sigma,1}k_1 + a_{\sigma,2}k_2 + \ldots + a_{\sigma,\sigma-1}k_{\sigma-1}), t_n + c_\sigma h) \\
y_{n+1} &= y_n + h(b_1k_1 + b_2k_2 + \ldots + b_\sigma k_\sigma)
\end{aligned}
$$

- Butcher array:

$$
\begin{array}{c|c}
\mathbf{c} & \mathbf{A} \\
\hline
 & \mathbf{b}^\mathsf{T}
\end{array}
\qquad
\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\vdots & \vdots & \vdots & \ddots & & \\
c_\sigma & a_{\sigma,1} & a_{\sigma,2} & \cdots & a_{\sigma,\sigma-1} & \\
\hline
 & b_1 & b_2 & \ldots & b_{\sigma-1} & b_\sigma
\end{array}
$$