Lecture 23: Process modeling & balance laws

- Balance laws
  - Differential balances
  - Material derivative

Book: 10.4, 11.1-11.4

Student Evaluation

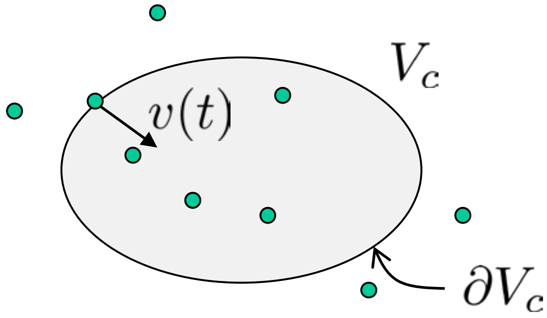
- Please fill out the student evaluation on itslearning

Q&A session / “Spørretime”

- There will be a Q&A session:
  - 10.05.2017 13.15 – 14.45 in EL3

- Please send me questions beforehand

The momentum balance



- In words

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{p} = \left\{ \begin{array}{c} \text{transfer of momentum into} \\ V_c \text{ by fluid flow} \\ \text{across surface } \partial V_c \end{array} \right\} + \left\{ \begin{array}{c} \text{generation of momentum} \\ \text{in } V_c \text{ due to forces} \\ \text{acting on } V_c \end{array} \right\}$$

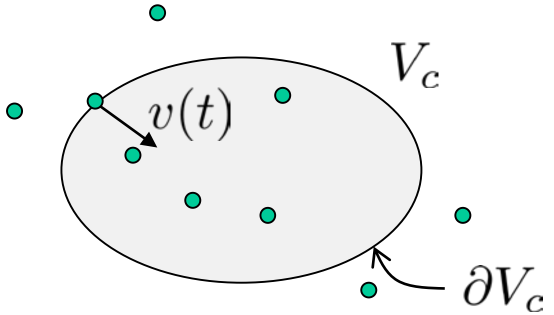
- Mathematically

$$\frac{{}^i\mathrm{d}}{\mathrm{d}t}\vec{p} = \frac{{}^i\mathrm{d}}{\mathrm{d}t}\iiint_{V_c} \rho\vec{v}\mathrm{d}V = -\iint_{\partial V_c} \rho\vec{v}\vec{v}\cdot\vec{n}\mathrm{d}A + \vec{F}^{(r)}$$

where $\vec{F}^{(r)}$ is resultant force on fluid in control volume

(often: gravity (hydrostatic) and/or friction (hydrodynamic))

The energy balance



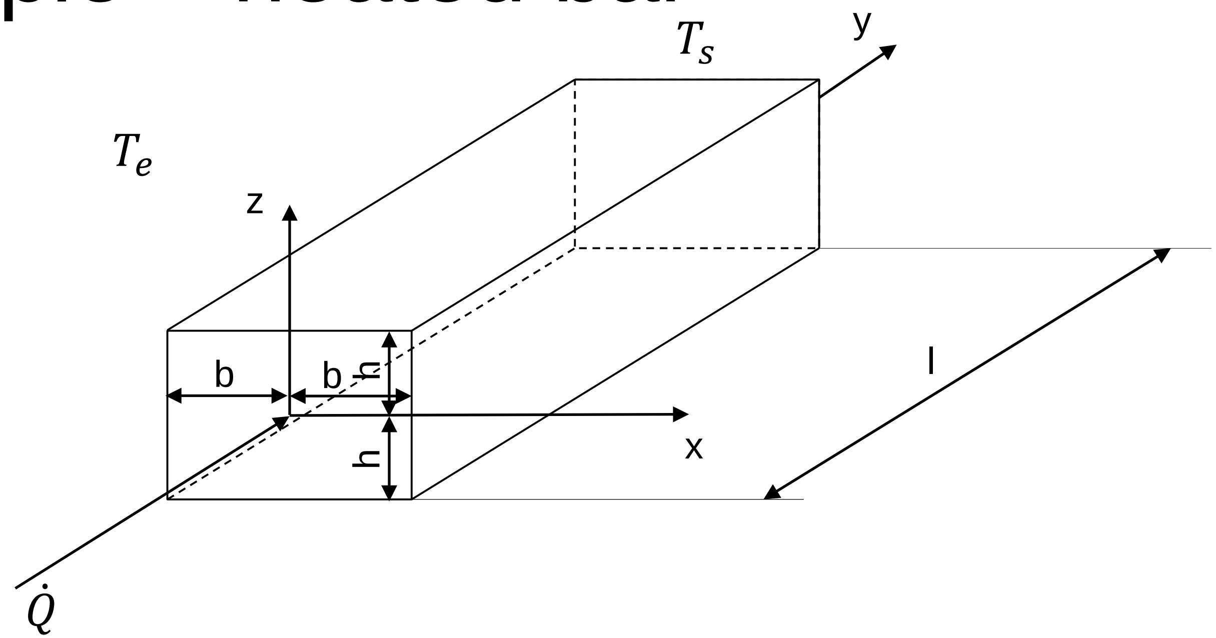
- In words

$$\frac{\mathrm{d}}{\mathrm{d}t}E = \left\{ \begin{array}{c} \text{transfer of energy into} \\ V_c \text{ by fluid flow} \\ \text{across surface } \partial V_c \end{array} \right\} + \left\{ \begin{array}{c} \text{transfer of energy into} \\ V_c \text{ by heat transfer} \\ \text{and by work} \end{array} \right\}$$

- Mathematically

$$\frac{\mathrm{d}}{\mathrm{d}t}E = \frac{\mathrm{d}}{\mathrm{d}t}\iiint_{V_c} \rho e \mathrm{d}V = \underbrace{-\iint_{\partial V_c} \rho e \vec{v}\cdot\vec{n}\mathrm{d}A}_{\text{Energy flow by convection}} + \dot{Q} - \dot{W}$$

Example – heated bar



- At all sidewalls there is heat exchange with the environment (T_e, heat exchange coefficient α)
- At the front side there is a heat flux $\dot{Q}$
- At the back side there is a constant temperature T_s