01.04.16

Kinematiske diff. ligninger

translasjon $\underline{v} \to \underline{r}$: $\dot{\underline{r}} = \underline{v}$

rotasjon $\underline{\omega}_{ab}^{a} \to R_b^a$: $\dot{R}_b^a = (\omega_{ab}^a)^\times R_b^a$

$\underline{\omega}_{ab}^{a} \to$ Euler vinkler $\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = ?$

$\underline{\omega}_{ab}^{a} \to$ Euler. par. $\dot{\eta} = ?$ $\dot{\epsilon} = ?$

KDL Eulervinkler

$$R_d^a = R_b^a R_c^b R_d^c = R_z(\psi) R_y(\theta) R_x(\phi)$$

$$\vec{\omega}_{ad} = \vec{\omega}_{ab} + \vec{\omega}_{bc} + \vec{\omega}_{cd}$$

$$= \dot{\psi}\vec{a}_3 + \dot{\theta}\vec{b}_2 + \dot{\phi}\vec{c}_1$$

Merk: Hvis $\theta = 90°$

- $\vec{c}_1$ blir parallell med $\vec{a}_3$

$\Rightarrow$ Vinkelhastigheter med komponenter langs $\vec{a}_3 \times \vec{b}_2$ kan kan ikke beskrives

$$\underline{\omega}_{ad}^{a} = \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} + R_z(\psi)\begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z(\psi)R_y(\theta)\begin{pmatrix} \dot{\varphi} \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\sin(\psi)\dot{\theta} + \cos(\psi)\cos(\theta)\dot{\varphi} \\ \cos(\psi)\dot{\theta} + \sin(\psi)\cos(\theta)\dot{\varphi} \\ \dot{\psi} - \sin(\theta)\dot{\varphi} \end{pmatrix}$$

$\cos = c$

$\sin = s$

$$= \begin{pmatrix} c(\psi)c(\theta) & -s(\psi) & 0 \\ s(\psi)c(\theta) & c(\psi) & 0 \\ -s(\theta) & 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = E_a(\underline{\Phi})\dot{\underline{\Phi}}$$

$$\Rightarrow \boxed{\dot{\underline{\Phi}} = E_a^{-1}(\underline{\Phi})\,\underline{\omega}_{ad}^{a}}$$

$$\det(E_a(\underline{\Phi})) = (\cos^2(\psi) + \sin^2(\psi))\cos(\theta) = \cos(\theta)$$

Singularitet når $\cos(\theta) = 0 \Rightarrow \theta = \frac{\pi}{2} + k\pi$, $k = 0, 1, 2 \ldots$

Kinetikk for satellitt:

$\dot{\omega}_{ad}^{a} = \ldots$ (Newton - Euler) / Momentbalanse

KDL Eulerparametre

Kan utledes $\quad R^a_b = R(\eta, \underline{\epsilon}), \quad \dot{R}^a_b = (\underline{\omega}^a_{ab})^\times R^a_b$

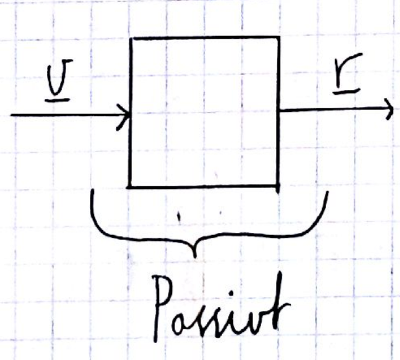
$$\dot{\eta} = -\frac{1}{2}\underline{\epsilon}^T \underline{\omega}^a_{ab}$$

$$\dot{\underline{\epsilon}} = \frac{1}{2}(\eta I - \underline{\epsilon}^\times)\underline{\omega}^a_{ab}$$

Passivitet av KDL

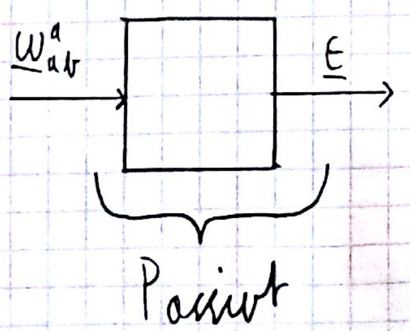
Translasjon: $\quad \dot{\underline{r}} = \underline{v}$

$$V = \frac{1}{2}\underline{r}^T\underline{r} > 0, \quad \dot{V} = \underline{r}^T\underline{v}$$



Rotasjon: $\quad V = 2(1-\eta) \geq 0$

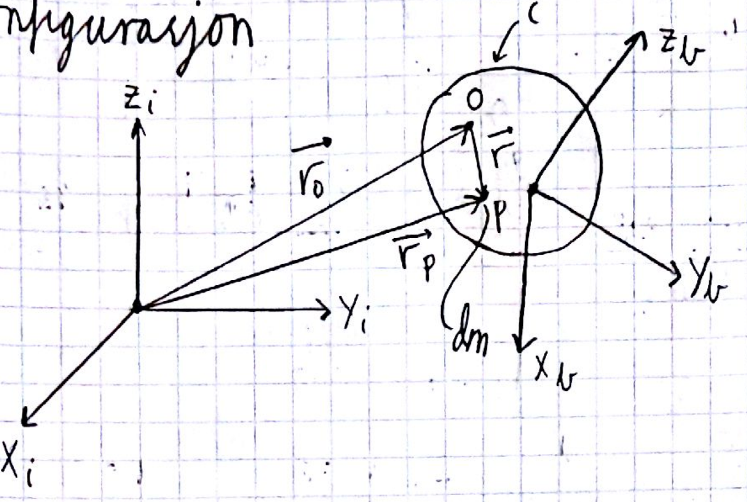
$$\dot{V} = -2\dot{\eta} = \underline{\epsilon}^T\underline{\omega}^a_{ab}$$



$(|\eta| = |\cos(\frac{\theta}{2})| \leq 1)$

Stive legemers kinematikk

Konfigurasjon



legeme-fast koordinatsys.

Inertielt koordinatsystem

0: fast punkt

p: vilkårlig punkt

$$\vec{r}_p = \vec{r}_0 + \vec{r}$$

$$\underline{r}^i = R^i_b \underline{r}^b$$

Hastighet

$$\vec{v}_0 = \frac{{}^i d}{dt}\vec{r}_0 \,, \quad \vec{v}_p = \frac{{}^i d}{dt}\vec{r}_p$$

$$\vec{v}_p = \frac{{}^i d}{dt}\vec{r}_p = \frac{{}^i d}{dt}\vec{r}_0 + \frac{{}^i d}{dt}\vec{r} = \vec{v}_0 + \frac{{}^i d}{dt}\vec{r}$$

$$\boxed{\vec{v}_p = \vec{v}_0 + \frac{{}^b d}{dt}\vec{r} + \vec{\omega}_{ib} \times \vec{r}}$$

Akselerasjon

$$\vec{a}_p = {}^i\frac{d^2}{dt^2}\vec{r}_p \ , \quad \vec{a}_0 = \frac{d^2}{dt^2}\vec{r}_0 \ , \quad \boxed{\begin{aligned} \vec{\alpha}_{ib} &= {}^i\frac{d}{dt}\vec{\omega}_{ib} \\ \vec{\alpha}_{ib} &= {}^b\frac{d}{dt}\vec{\omega}_{ib} \end{aligned}} + \underbrace{\cancel{\vec{\omega}_{ib} \times \vec{\omega}_{ib}}}_{0}$$

$$\vec{a}_p = {}^i\frac{d^2}{dt^2}\vec{r}_p = {}^i\frac{d^2}{dt^2}\vec{r}_0 + {}^i\frac{d^2}{dt^2}\vec{r}$$

$$= \vec{a}_0 + {}^i\frac{d}{dt}\left({}^i\frac{d}{dt}\vec{r}\right) = \vec{a}_0 + {}^i\frac{d}{dt}\left(\underbrace{{}^b\frac{d}{dt}\vec{r} + \vec{\omega}_{ib} \times \vec{r}}\right)$$

$$= \vec{a}_0 + {}^b\frac{d}{dt}\left(\quad\right) + \vec{\omega}_{ib} \times \left(\quad\right)$$

$$\vec{a}_p = \vec{a}_0 + \underbrace{2\vec{\omega}_{ib} \times {}^b\frac{d}{dt}\vec{r}}_{\text{coriolis}} + \underbrace{\vec{\alpha}_{ib} \times \vec{r}}_{\text{transversal}} + \underbrace{\vec{\omega}_{ib} \times (\vec{\omega}_{ib} \times \vec{r})}_{\text{sentripetal}}$$

Om $\vec{r}$ er fast i b:

$$\boxed{\begin{aligned} \vec{v}_p &= \vec{v}_0 + \vec{\omega}_{ib} \times \vec{r} \\ \vec{a}_p &= \vec{a}_0 + \vec{\alpha}_{ib} \times \vec{r} + \vec{\omega}_{ib} \times (\vec{\omega}_{ib} \times \vec{r}) \end{aligned}}$$

Massesenter

$$m = \int_b dm \qquad \vec{v}_c = \frac{\int_b \vec{v}_P \, dm}{m}$$

$$\vec{r}_c = \frac{\int_b \vec{r}_P \, dm}{m} \qquad \vec{a}_c = \frac{\int_b \vec{a}_P \, dm}{m}$$

$$\int_b \vec{r} \, dm = \int_b \vec{r}_P \, dm - \int \vec{r}_c \, dm = m\vec{r}_c - m\vec{r}_c = 0$$