21.01.16

2.3 Energibaserte metoder

2.3.2 Energifunksjon

Gitt et system beskrevet av

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u}, t)$$

Anta at vi har en funksjon $V(\underline{x}, t) \geq 0$ som er "energien" til systemet.

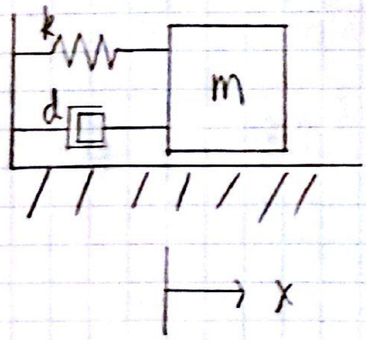
Deriver v.hj.a kjerneregelen

$$\dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \underline{x}} \cdot \frac{\partial x}{\partial t} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \underline{x}} \underline{f}(\underline{x}, \underline{u}, t)$$

Vi har

$$\boxed{\begin{array}{c} \dot{V} \leq 0 \ , \ \forall t \\ \Downarrow \\ \text{Energien avtar monotont} \end{array}} = \text{"stabilt"}$$

Eks. Masse-fjær-demper



$$m\ddot{x} + d\dot{x} + kx = 0$$

$$\left.\begin{array}{l} x_1 = x \\ x_2 = \dot{x} \end{array}\right\} \quad \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{d}{m} x_2 - \frac{k}{m} x_1 \end{array}$$

$\Longrightarrow$

Definerer energifunksjon

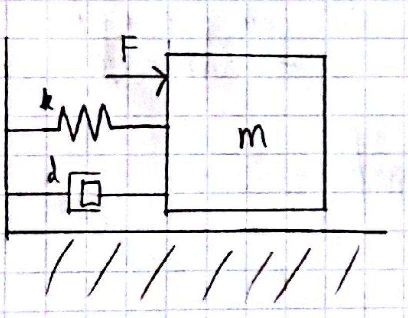
$$V = \underbrace{\frac{1}{2} k x_1^2}_{\text{potensiell energi}} + \underbrace{\frac{1}{2} m x_2^2}_{\text{kinetisk energi}}$$

$$\dot V = k x_1 \dot x_1 + m x_2 \dot x_2 = k x_1 x_2 + m x_2 \left(-\frac{d}{m} x_2 - \frac{k}{m} x_1\right) = -d x_2^2$$

$$V(t) \leq V(t_0) = V_0$$

Posisjon x_1: $\frac{1}{2} k x_1^2 \leq V_0 \Rightarrow x_1 \leq \sqrt{\frac{2V_0}{k}}$ $\qquad x_2 \leq \sqrt{\frac{V_0}{2m}}$

<u>Eks.</u> Masse-fjær-"demper"



$$m\ddot x + d\dot x + kx = F$$

$$\dot V = -d\dot x^2 + F\dot x$$

F pådrag: Velg $F = -k_d \dot x$

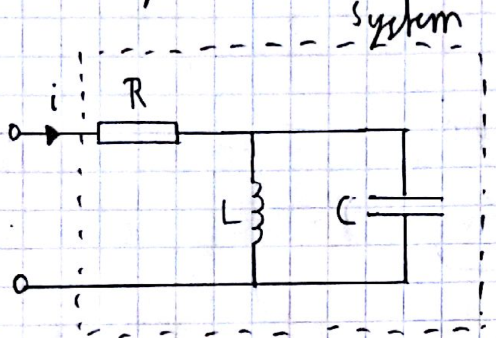
$$\Rightarrow \dot V = -(k_d + d)\dot x^2$$

Generelt: Energibasert regulator

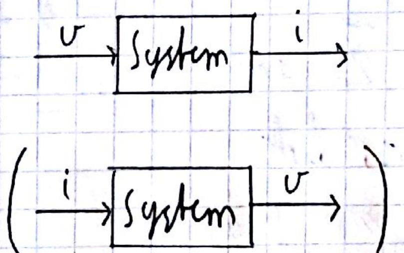
$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x},\underline{u},t) \\ \dot{V} &= \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \underline{x}}\,\underline{f}(\underline{x},\underline{u},t) \end{aligned}\right\} \text{ Velg } \underline{u} \text{ slik at } V \leq 0$$

2.4 Passivitet

Motivasjon:



Blokkdiagram



Effektflyt inn i systemet

$$P(t) = v(t)\cdot i(t)$$

$P(t) > 0$: Effekt levert til systemet

$P(t) < 0$: Effekt ut av systemet

Total energi levert til systemet

$$E(t) = E(t_0) + \int_{t_0}^{t} P(\tau)\,d\tau$$

$\Rightarrow$

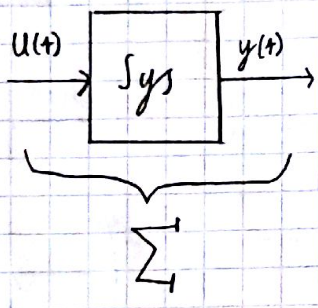
$E(t) > 0$: Energi lagret eller absorbert i systemet

$E(t) < 0$: Energi produsert til omgivelsene

Passivt system: $E(t) > 0$

$$\Rightarrow \int_{t_0}^{t} P(\tau)\,d\tau \geq -E(t_0) \Rightarrow \int_{t_0}^{t} i(\tau)\cdot v(\tau)\,d\tau \geq -E(t_0)$$

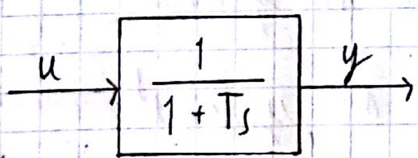
Def. Passivitet



Et system Σ er passivt hvis det finnes $E_0 \geq 0$ s.a. for alle inngangsforløp og alle $t \geq t_0$ gjelder at

$$\int_{t_0}^{t} y(t)\,u(t)\,d\tau \geq -E_0$$

Eks.



$$\dot{y} = \frac{1}{T}y + \frac{1}{T}u$$

$$(u = T\dot{y} + y)$$

$$\int_{t_0}^{t} y(\tau)u(\tau)\,d\tau = \int_{t_0}^{t} y(\tau)\left[T\dot{y}(\tau) + y(\tau)\right]d\tau$$

$$= T\int_{t_0}^{t} y(\tau)\dot{y}(\tau)\,d\tau + \int_{t_0}^{t} y^2(\tau)\,d\tau$$

$\Rightarrow$

$$= \frac{T}{2}\Big[\underbrace{y^2(t)}_{\geq 0} - y^2(t_0)\Big] + \int_{t_0}^{t} \underbrace{y^2(\tau)}_{\geq 0}\, d\tau \geq -\underbrace{\frac{T}{2} y^2(t_0)}_{E_0} = -E_0$$

$\Rightarrow$ Passivt

Eks. $m\ddot{x} + d\dot{x} + kx = F$

$$\xrightarrow{F} \boxed{\text{Sys}} \xrightarrow{\dot{x}}$$

$$\int_{t_0}^{t} F\dot{x}\, dx = \int_{t_0}^{t} (m\ddot{x} + d\dot{x} + kx)\dot{x}\, d\tau$$

$$= \int_{t_0}^{t} m\dot{x}\underbrace{\ddot{x}\, d\tau}_{d\dot{x}} + \int_{t_0}^{t} d\dot{x}^2\, d\tau + \int_{t_0}^{t} kx\underbrace{\dot{x}\, d\tau}_{dx}$$

$$= \frac{1}{2}m\Big[\underbrace{\dot{x}^2(t)}_{\geq 0} - \dot{x}^2(t_0)\Big] + \underbrace{d\int_{t_0}^{t} \dot{x}^2\, d\tau}_{\geq 0} + \frac{1}{2}k\Big[\underbrace{x^2(t)}_{\geq 0} - x^2(t_0)\Big]$$

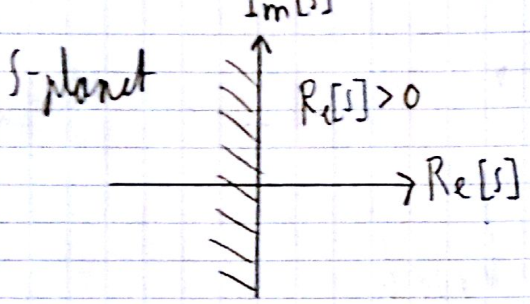
$$\geq -\frac{1}{2}m\dot{x}^2(t_0) - \frac{1}{2}kx^2(t_0) = -E_0 \quad \Rightarrow \text{Passivt!}$$

$E_0 = \frac{1}{2}m\dot{x}^2(t_0) + \frac{1}{2}kx^2(t_0)$ "initielt lagret energi"

2.4.5 Positivt relle transferfunksjoner

Def. $H(s)$ er positiv reell hvis

1. $H(s)$ er analytisk for alle $Re[s] > 0$
2. $H(s)$ er <u>reell</u> for alle positive og reelle s
3. $Re[H(s)] \geq 0$ for alle $Re[s] > 0$



Teorem. En proper, rasjonal $H(s)$ positiv reell hvis og bare hvis

1. Alle poler er i $Re[s] \leq 0$
2. $Re[H(j\omega)] \geq 0$ for alle ω som er s.a $j\omega$ ikke er en pol i $H(s)$
3. Hvis $j\omega_0$ <u>er</u> en enpol i $H(s)$, så er

$$\underset{s=j\omega_0}{Res}[H(s)] = \lim_{s \to j\omega_0} (s - j\omega_0) H(s)$$ relt positiv

Eks.

$$H(s) = \frac{1}{1+Ts} \quad \text{(rasjonal, proper)}$$

1. pol i $s = -\frac{1}{T}$ $\Rightarrow$ OK

2. $H(j\omega) = \frac{1}{1+j\omega T} = \frac{1-j\omega T}{1+(\omega T)^2}$

$$\mathrm{Re}\left[H(j\omega)\right] = \frac{1}{1+(\omega T)^2} \geq 0 \quad \forall \omega$$

3. OK $H(s)$ er positiv reell

Teorem

$$\underbrace{u \rightarrow \boxed{H(s)} \xrightarrow{y}}_{\text{Passiv}} \iff H(s) \text{ positiv reell}$$