14.7.1 Estimering av lokal feil 12.02.16

Gitt y_n

Beregn y_{n+1} med

$$\begin{array}{c|c} c & A \\ \hline & b^T \end{array}$$

- orden p
- Lokal feil $e_{n+1} = O(h^{p+1})$

Beregn $\hat{y}_{n+1}$ med

$$\begin{array}{c|c} \hat{c} & \hat{A} \\ \hline & \hat{b} \end{array}$$

- orden $p+1$
- Lokal feil $e_{n+1} = O(h^{p+2})$

Lokal løsning:

$$y_L(t_n; t_{n+1}) = y_{n+1} - e_{n+1}$$

$$= \hat{y}_{n+1} - \hat{e}_{n+1}$$

$$\Rightarrow \hat{y}_{n+1} - y_{n+1} = e_{n+1} - \hat{e}_{n+1} \approx e_{n+1}$$

$$\boxed{\text{Estimat av feil: } \quad e_{n+1} = \hat{y}_{n+1} - y_{n+1}}$$

Effektiv beregning

Velg $c = \hat{c}$, $A = \hat{A}$

→ Samme trinnberegninger

→ "Embedded Runge-Kutta"

$$\begin{array}{c|c} c & A \\ \hline & b^T \\ & \hat{b}^T \end{array}$$

14.7.3 Bruk av feilestimat til å justere steg lengde

Gitt estimat av feil:

$$l_{n+1} = \begin{pmatrix} l_{1,n+1} \\ l_{2,n+1} \\ \vdots \\ l_{d,n+1} \end{pmatrix}$$

Definer

$$\mathcal{E}_{n+1} \max_i |l_{i,n+1}| = O(h^{p+1})$$

Vi ønsker en toleranse l_{tol}, dvs vi krever

$$\mathcal{E}_{n+1} < l_{tol}$$

Vet $\mathcal{E}_{n+1} \approx C h^{p+1}$

Hvis $\mathcal{E}_{n+1} > l_{tol}$: Velg h_{new} slik at:

$$l_{new} \approx C h_{new}^{p+1} = l_{tol}$$

$$\Rightarrow \frac{l_{tol}}{\mathcal{E}_{n+1}} \approx \frac{C h_{new}^{p+1}}{h^{p+1}} = \left(\frac{h_{new}}{h}\right)^{p+1}$$

$$\boxed{h_{new} = h\left(\frac{l_{tol}}{\mathcal{E}_{n+1}}\right)^{\frac{1}{p+1}}}$$

Algoritme:

$\mathcal{E}_{n+1} \approx l_{tol}$: "gjør ingenting"

$\mathcal{E}_{n+1} \gg l_{tol}$: Sett $h = h_{new}$ (reduser h!)

$\mathcal{E}_{n+1} \ll l_{tol}$: Sett $h = h_{new}$ (øk h!)

I praksis: endrer mykere

(*) kan skrives

$$\underbrace{\ln(h_{new})}_{u_{n+1}} = \underbrace{\ln(h)}_{u_n} + \frac{1}{p+1}\left(\underbrace{\ln(e_{tol})}_{y_{ref}} - \underbrace{\ln(\mathcal{E}_{n+1})}_{y_n}\right)$$

Dette er en I-regulator

$$u_{n+1} = u_n + \frac{h}{T_i}\underbrace{(y_{ref} - y_n)}_{\tilde{y}_n}$$

Legg på P-virkning

$$u_{n+1} = u_n + K_p(\tilde{y}_n - \tilde{y}_{n-1}) + K_p\frac{h}{T_i}\tilde{y}_n$$

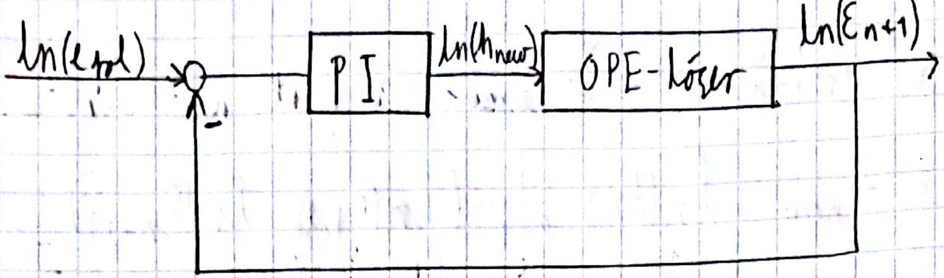
Dvs:

$$\ln(h_{new}) = \ln(h) + K_p(-\ln(\mathcal{E}_{n+1}) + \ln(\mathcal{E}_n)) + K_p\frac{h}{T_i}[\ln(e_{tol}) - \ln(\mathcal{E}_{n+1})]$$

$$\boxed{h_{new} = h\left(\frac{e_{tol}}{\mathcal{E}_{n+1}}\right)^{K_p\frac{h}{T_i}} \cdot \left(\frac{\mathcal{E}_n}{\mathcal{E}_{n+1}}\right)^{K_p}}$$

$K_p = \frac{0.4}{p+1}$ $\quad T_i = 1.3\,h$ $\quad$ Typiske verdier

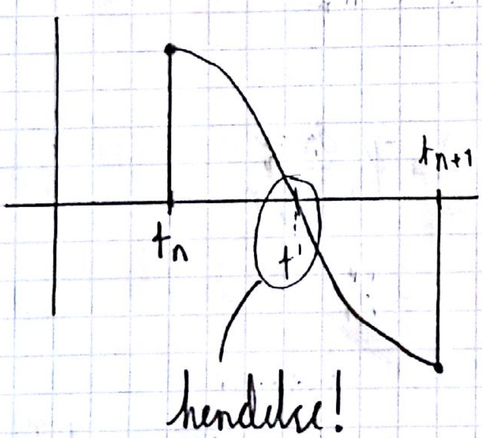
(*)



14.8.2/3 Interpolerte løsn. og deteksjon av hendelser

Hendelse ("event"-) funksjon

$$g(y, t) = 0$$



Vil finne t'

Interpolerer: $t' = t_n + \alpha h$

Lag polynom $y_n(\alpha)$

$y_n(0) = y_n$

$y_n(1) = y_{n+1}$

Finn α som løser $g(y_n(\alpha), t_n + \alpha h) = 0$