Lecture 11: Hydraulic motors, transmission lines

- Hydraulic motors
- Hydraulic transmission lines
- (Electrical transmission lines)

Book: 4.1-4.6, (1.6)

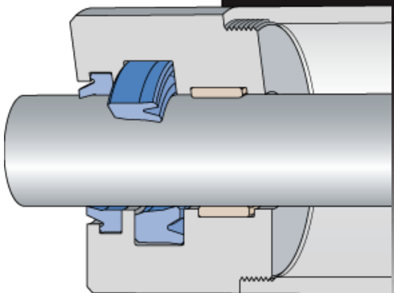
Systems using hydraulics to produce motion

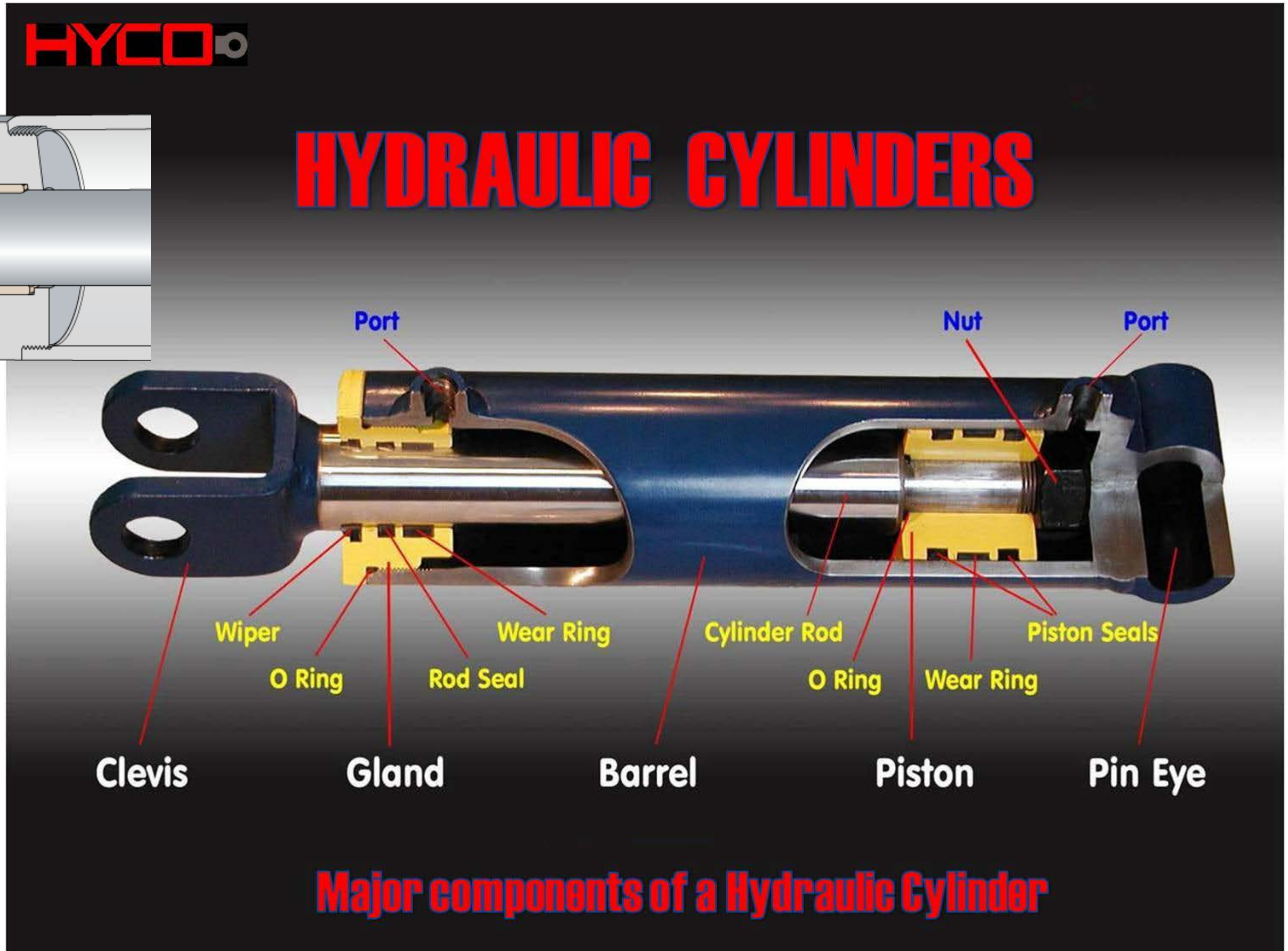
- Excavators

- Robots, cranes, etc.

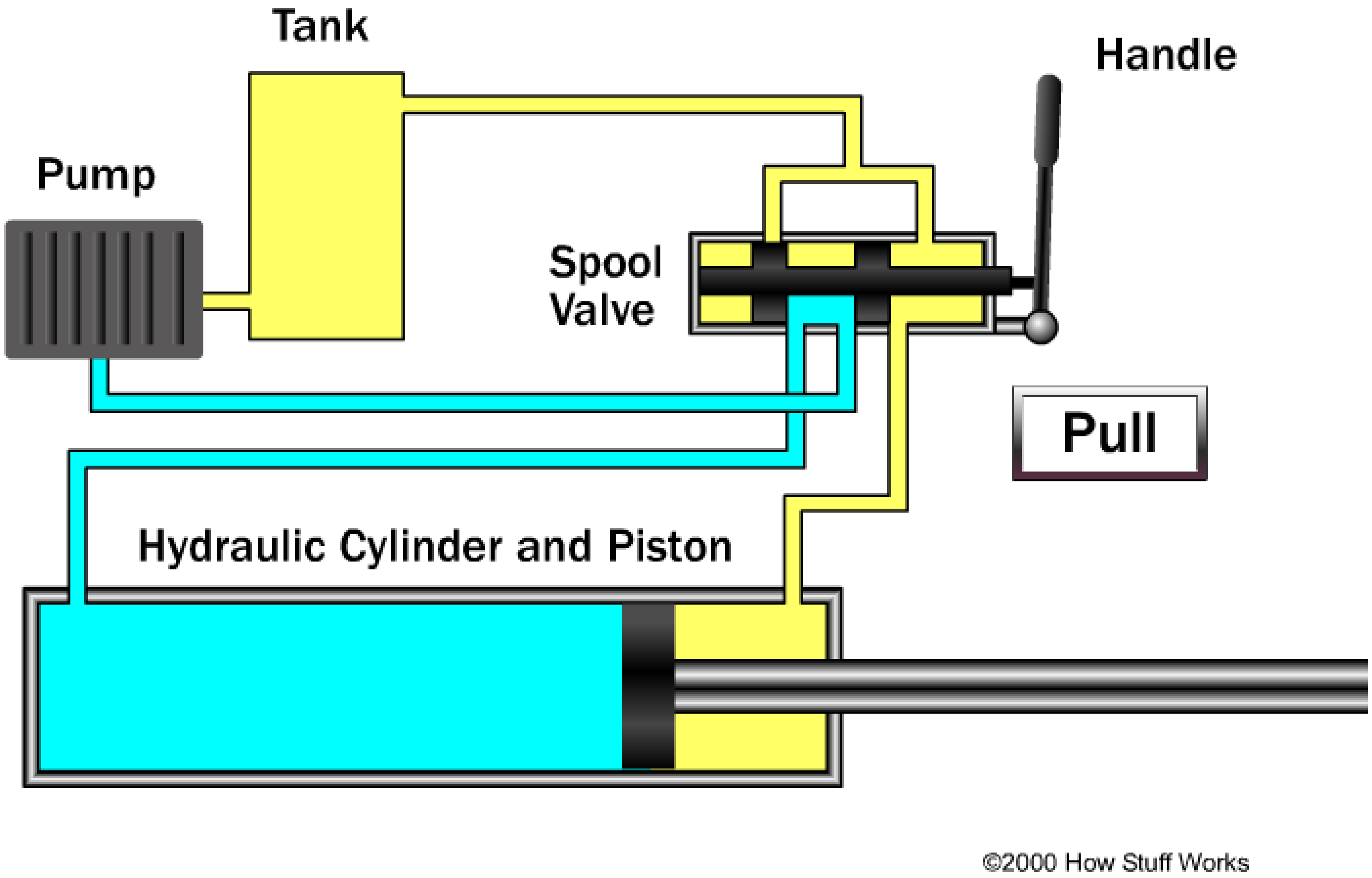
- To control motion of these systems, we need models of the hydraulic actuators

Hydraulic cylinder





Hydraulic system



Anna Konda – The fire fighting snake robot

Moody chart

- Circular pipe
- Darcy-Weisbach factor with Reynolds number and relative roughness

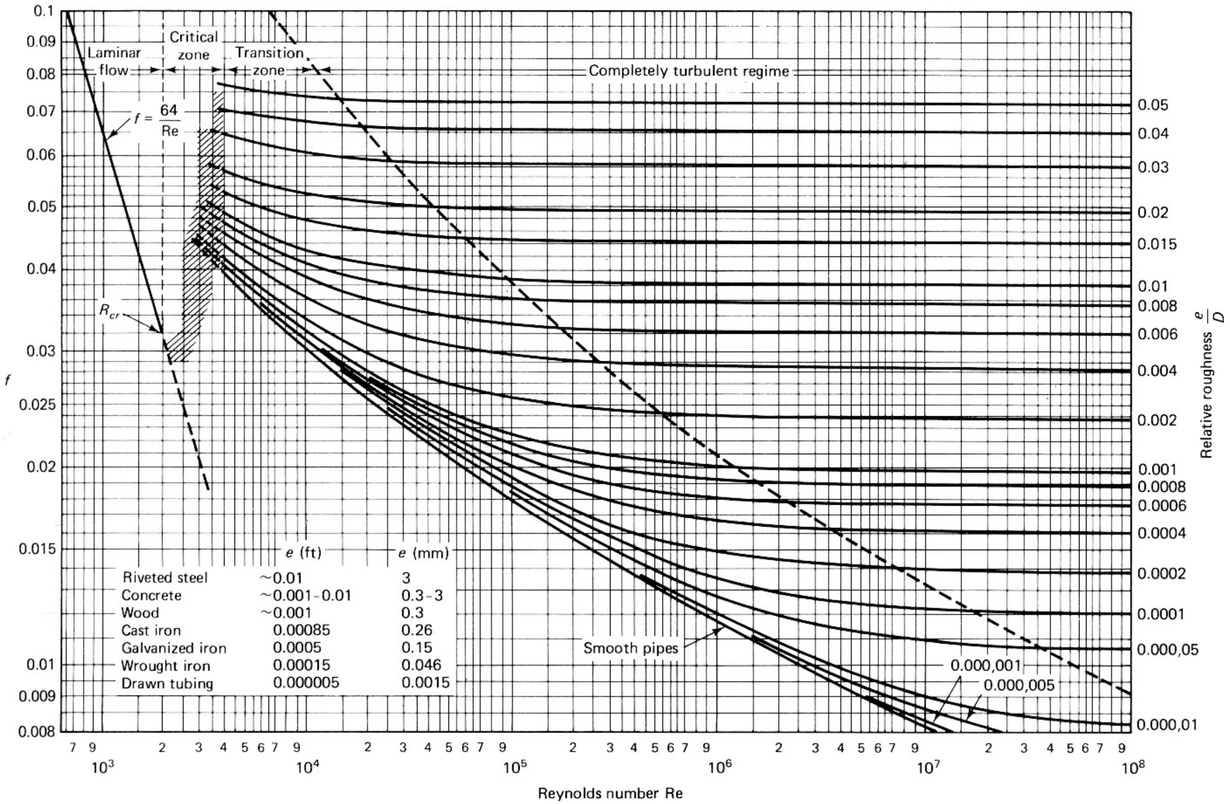


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Hydraulic cylinder

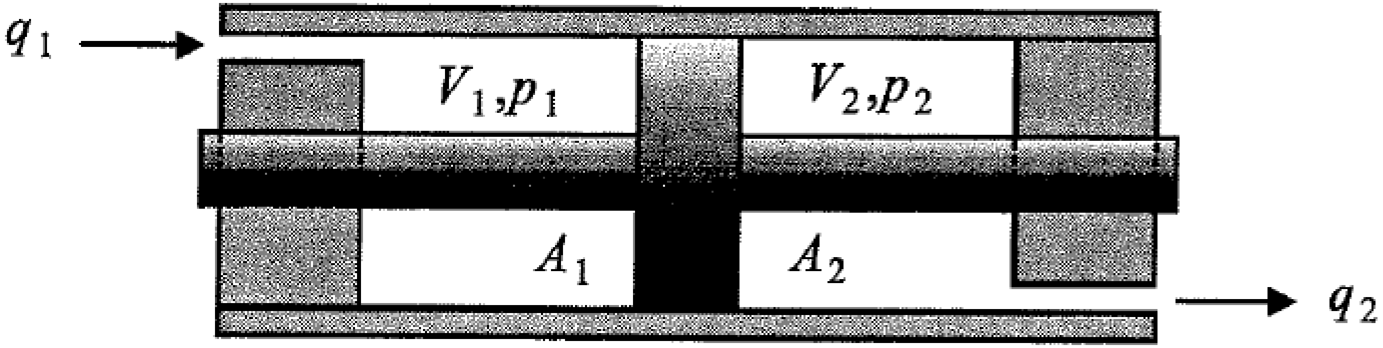


Figure 4.9: Symmetric hydraulic cylinder

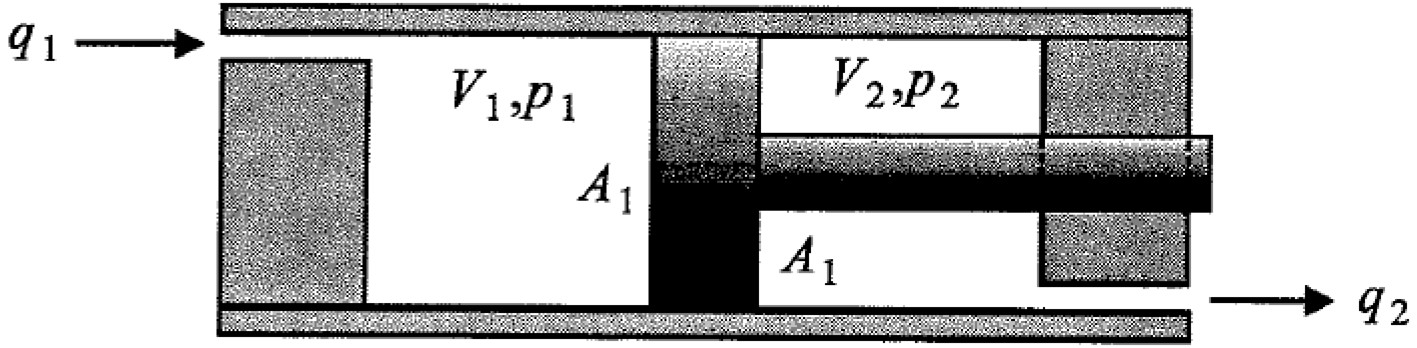


Figure 4.10: Single-rod hydraulic piston

Rotational hydraulic motor

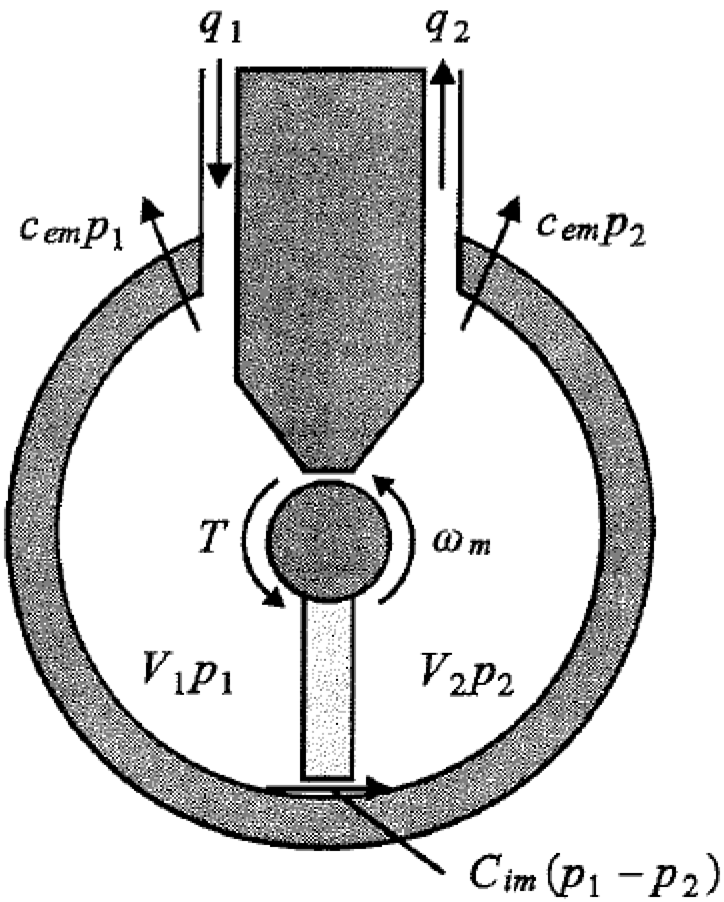
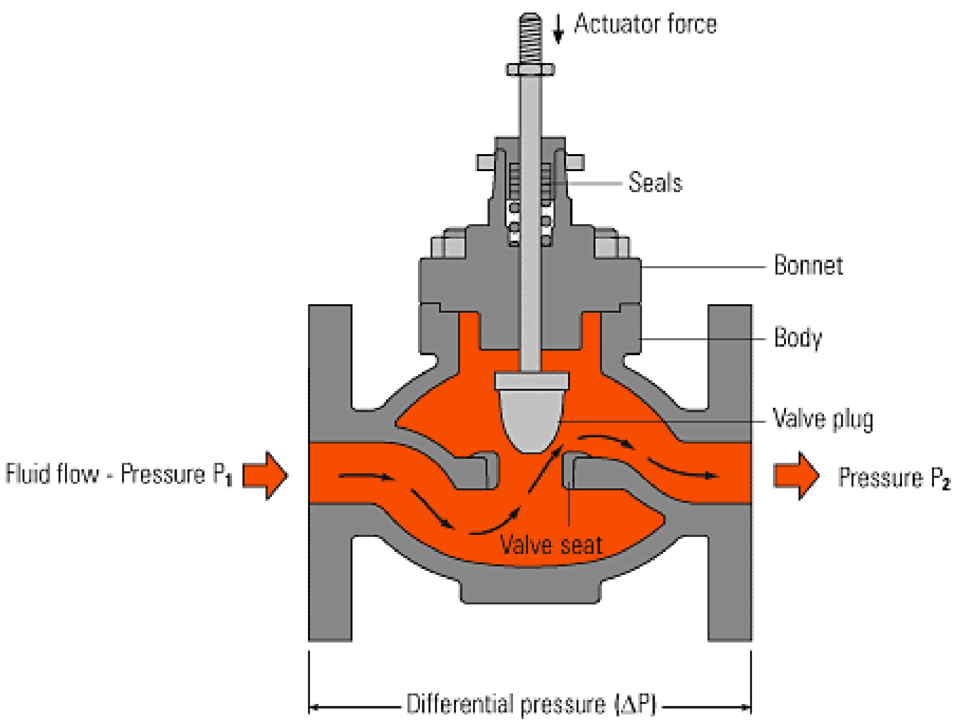
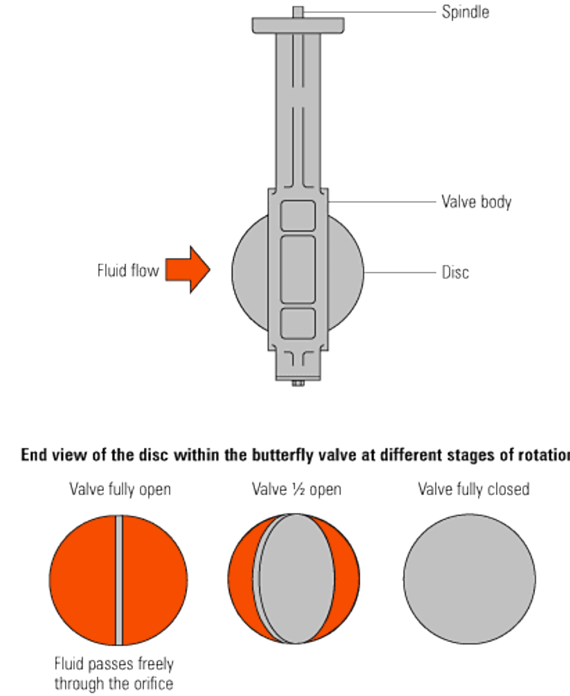


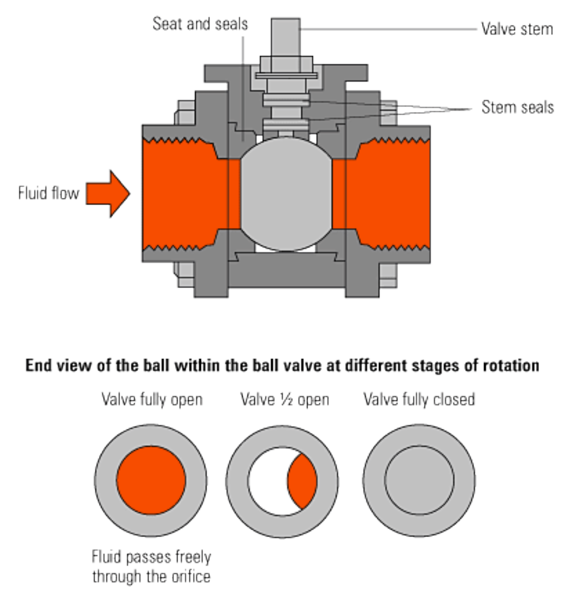
Figure 4.7: Rotational hydraulic motor of the single vane type with limited travel.

Valves

- Device that regulates flow
- Many different types of valves exist
  - Globe valve, ball valve, butterfly valve, ...





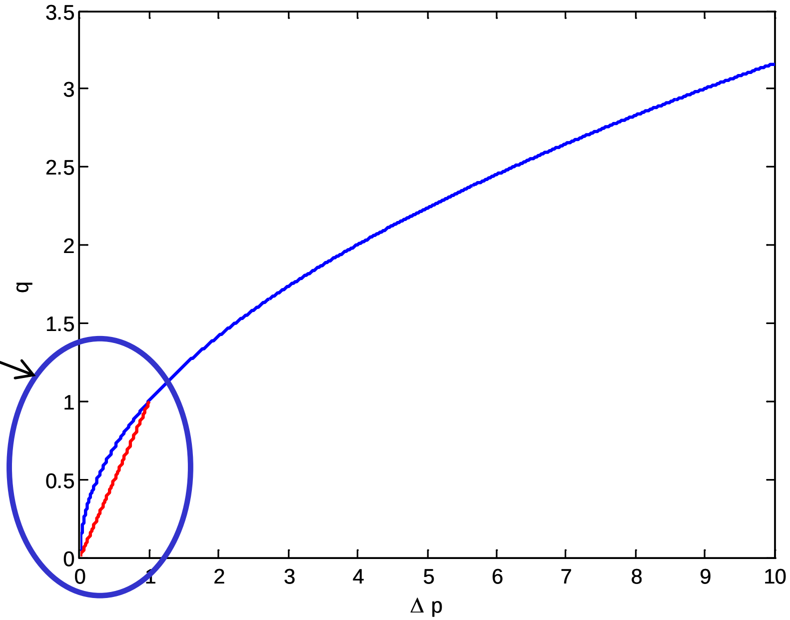


Valve models (book 4.2)

- Flow through a restriction is generally turbulent

$$q = C_d A \sqrt{\frac{2}{\rho}\Delta p}$$

Problematic!

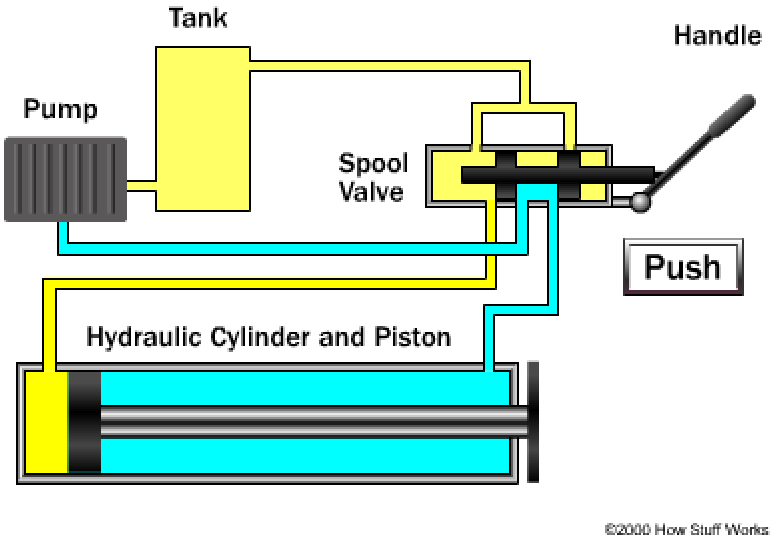


- Solution: Regularize by assuming laminar flow for small *Δp*

$$q = C_l \Delta p$$

- Book: Make transition smooth

Four-way valve



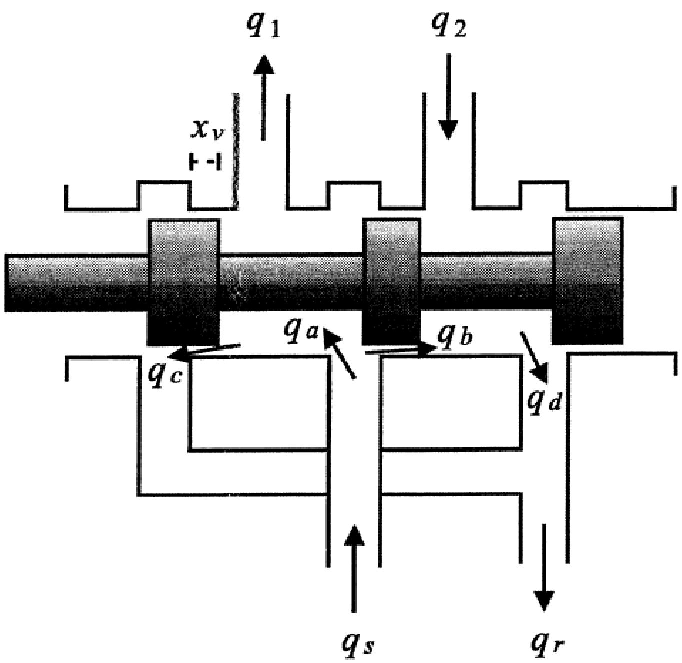
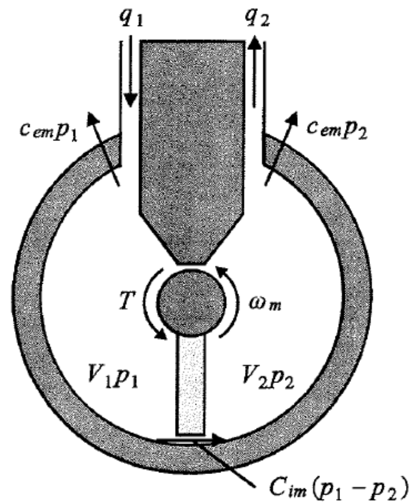


Figure 4.2: A matched and symmetric four-way valve.



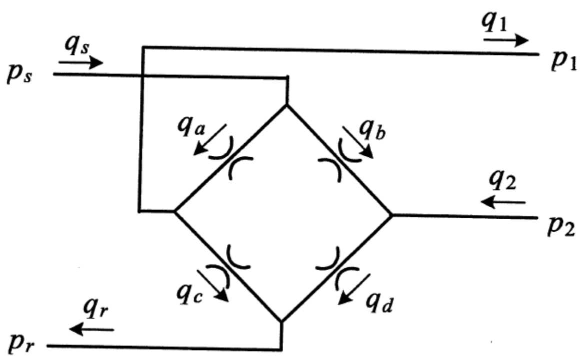


Figure 4.1: Four-way valve

Modeling of four-way valve

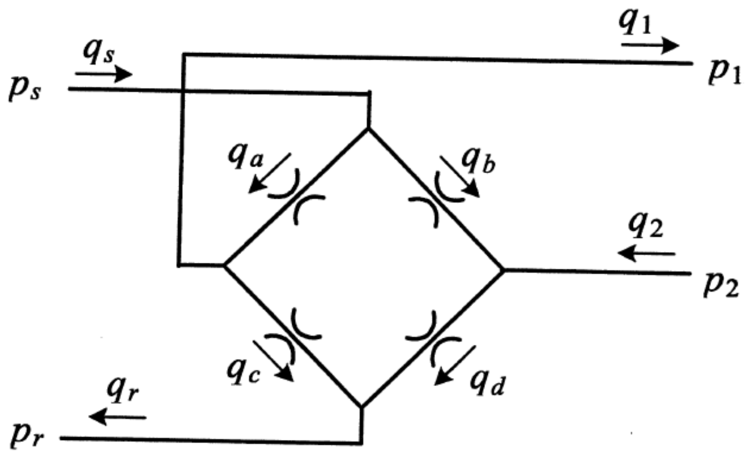


Figure 4.1: Four-way valve

- Define load pressure

$$p_L = p_1 - p_2$$

- Define load flow

$$q_L = \frac{q_1 + q_2}{2}$$

- Symmetric load assumption (motor)

$$q_1 = q_2$$

- Symmetric valve and symmetric load

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \operatorname{sign}(x_v) p_L\right)}$$

Characteristic of four-way valve

$$q_L = C_d b x_v \sqrt{\frac{1}{\rho}\left(p_s - \mathrm{sign}(x_v) p_L\right)}$$

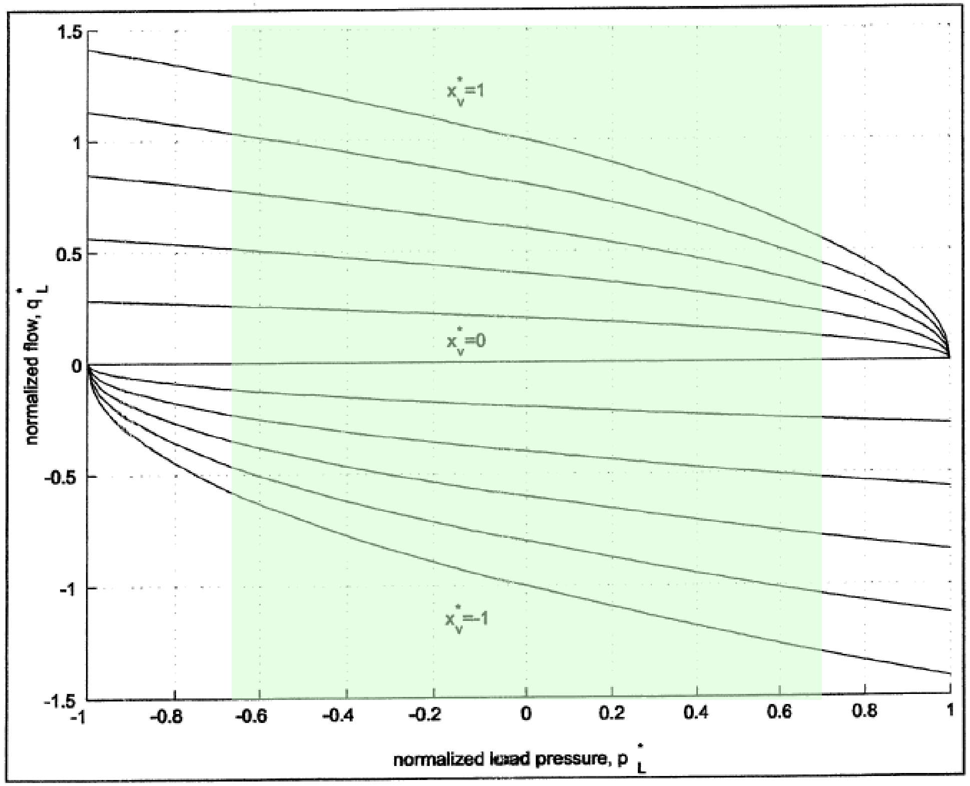


Figure 4.3: Valve characteristic

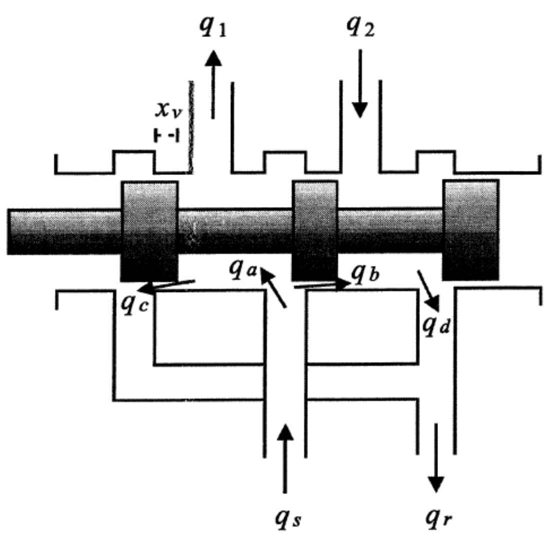
Linearized model:

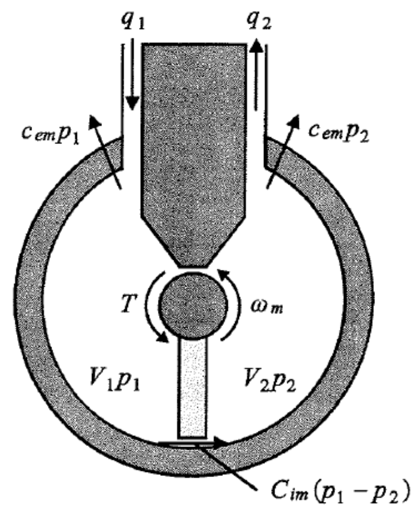
$$|p_L| \leq \frac{2}{3} p_s : \quad \boxed{q_L = K_q x_v - K_c p_L}$$

Gain uncertainty:

$$0.58 K_{q0} \leq K_q \leq 1.29 K_{q0}$$

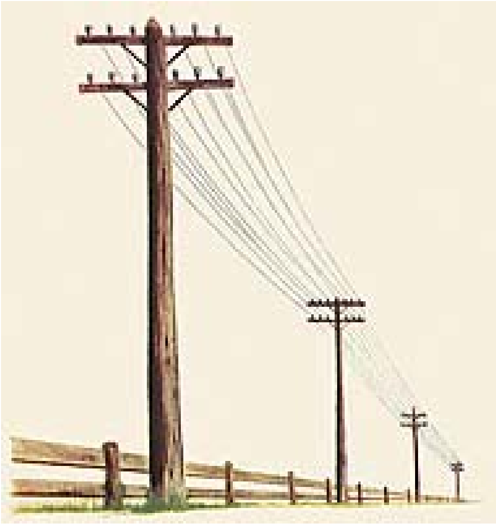
Transfer function valve+motor

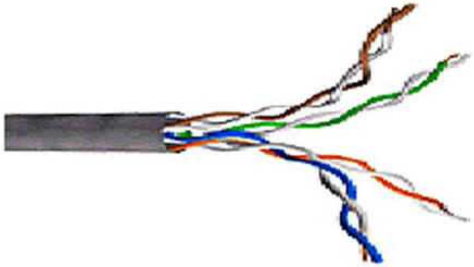


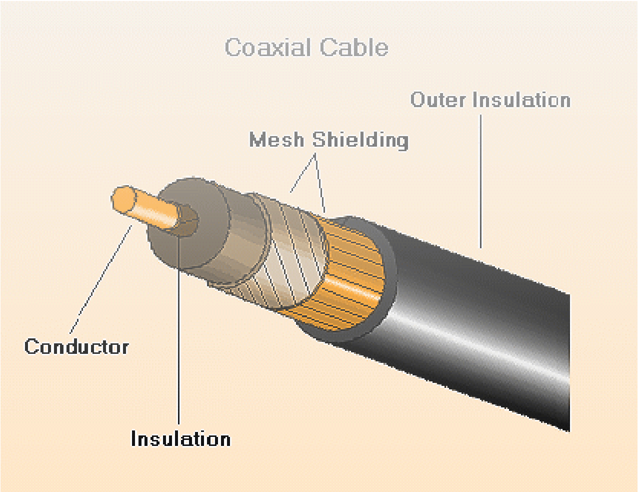


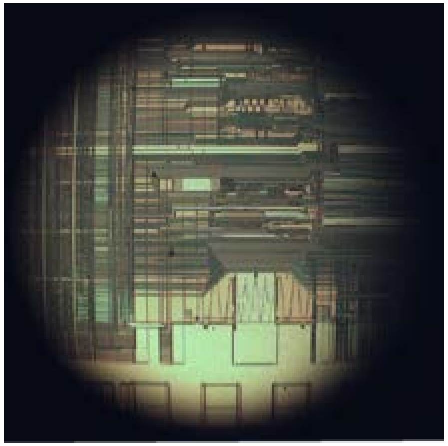
$$\theta_m(s) = \frac{\frac{K_q}{D_m}x_v(s) - \frac{K_{ce}}{D_m^2}\left(1+\frac{s}{\omega_t}\right)T_L(s)}{s\left(1+2\zeta_h\frac{s}{\omega_h}+\frac{s^2}{\omega_h^2}\right)}$$

Electrical transmission lines



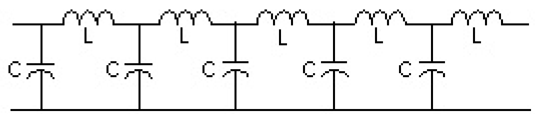




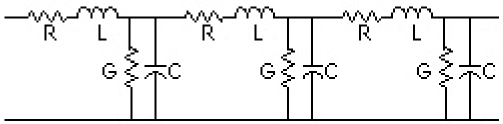


Telegrapher's equation (Wave equation)

- Lossless:



Lossy:



- Model (Ch. 1.6):

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

- Laplace:

$$\frac{\partial u(x,s)}{\partial x} = -X(s)i(x,s)$$
$$\frac{\partial i(x,s)}{\partial x} = -Y(s)u(x,s)$$

Series impedance:
$$X(s) = R + Ls$$
Parallel admittance:
$$Y(s) = G + Cs$$

Characteristic impedance:
$$Z_c(s) = \sqrt{\frac{X(s)}{Y(s)}}$$

Same equations for electrical and fluid/hydraulical transmission lines

Electrical transmission lines:

$$\frac{\partial u(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t}$$
$$\frac{\partial i(x,t)}{\partial x} = -Gu(x,t) - C\frac{\partial u(x,t)}{\partial t}$$

Fluid transmission lines:

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x}$$
$$\frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\frac{\partial p(x,t)}{\partial x} - \frac{F\left[q(x,t)\right]}{\rho}$$

- Current and flow “same” variables, as is voltage and pressure
- In both cases, we can define line impedance, characteristic impedance, propagation operator, etc.
- Solution to equations have same structure/form: waves propagating back and forth

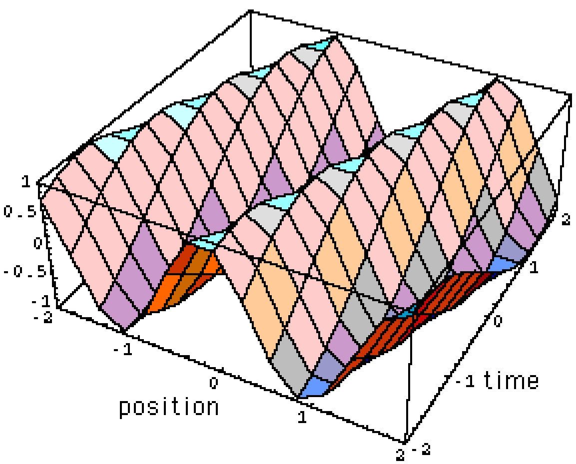
Solution: Waves

- Solution:

$$u_{out}(s) = e^{-\Gamma(s)} u_{in}(s)$$

- Propagation operator $\Gamma(s) = L\sqrt{X(s)Y(s)}$
  - Attenuation factor ® = Re[¡ (*j!*)]: How much is wave reduced
  - Phase factor: ¯ = Im[¡ (*j!*)]: How long does it take

- Lossless (*R* = *G* = 0): ¡ (*s*) = *Ts*
  - Attenuation factor: 0
  - Phase factor: Pure time-delay



When should we care?

- Solution lossless case: Time delay

$$e^{-Ts}$$

- Rule-of-thumb from control theory: We can ignore time-delay for frequencies much less than 1/*T*

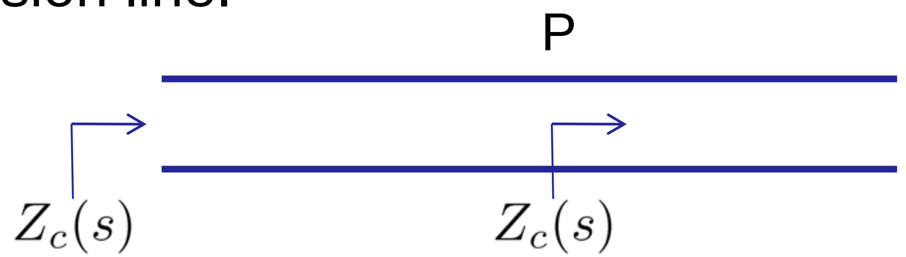
$$\omega \leq \frac{1}{T} \;\Rightarrow\; 2\pi\frac{c}{\lambda} \leq \frac{c}{L} \;\Rightarrow\; L \leq \frac{\lambda}{2\pi}$$

- Rule-of-thumb for transmission lines: When L is larger than one tenth of wavelength, treat as transmission line

- Power lines, *f* = 50Hz: ¸ = 6000km
- Personal computers, *f* = 10GHz: ¸ = 1.5cm

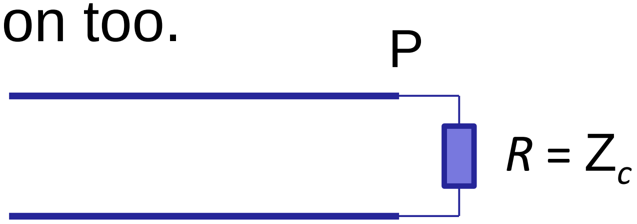
Impedance matching

- Suppose we have an imaginary joint at P in a very long transmission line.



The wave goes through the joint without reflection because there is actually no joint (just imagined).

- Now, let us terminate a resistance of value Z_c *at the same position of this* imaginary joint. The wave will go through without reflection too.



This is called a **matched load.**

http://cktse.eie.polyu.edu.hk/eie403/Transmissionline.pdf