22.01.16

Ex 32

$$H(s) = K\frac{(s+z_1)(s+z_2)\dots}{s(s+p_1)(s+p_2)\dots} \qquad (PR)$$

Anta $R_e[p_i] > 0$, $R_e[z_i] > 0$. Er $H(s)$ positiv reell?

1. Poler $s = -p_i$ OK

   $s = 0$ OK

3. Pol på imaginæraksen: $s = j0 = 0$

$$\underset{s=0}{\mathrm{Res}}[H(s)] = \lim_{s\to 0} sK\frac{(s+z_1)(s+z_2)\dots}{s(s+p_1)(s+p_2)\dots} = K\frac{z_1 z_2 \dots}{p_1 p_2 \dots} > 0 \quad \text{OK}$$

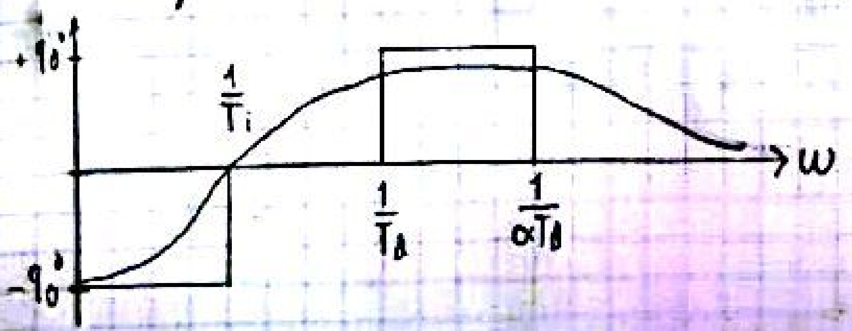
2. $R_e[H(j\omega)] \geq 0$

> Teorem: For (×)
>
> $H(s)$ PR $\iff R_e[H(j\omega)] \geq 0, \ \forall \omega \neq 0$

PID-regulator: $H_{PID}(s) = K_p \frac{1+T_i s}{T_i s} \cdot \frac{1+T_d s}{1+\alpha T_d s}$

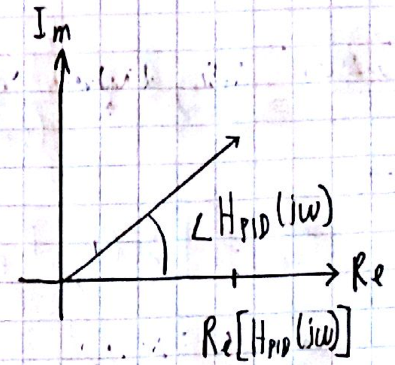
$K_p > 0$, $T_i > 0$, $T_d > 0$, $0 \leq \alpha \leq 1$

Fasediagram:



$\Rightarrow$

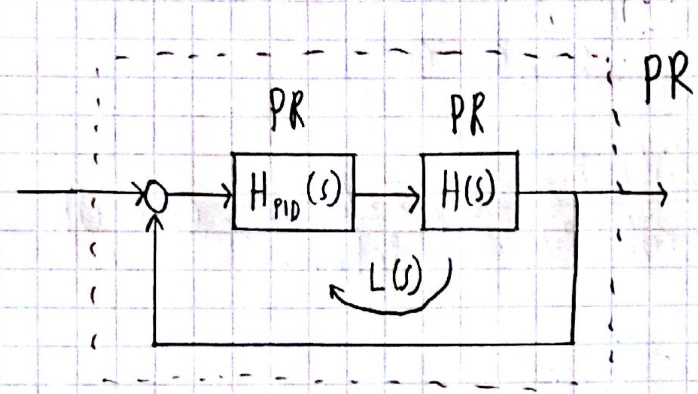
$$-90^\circ \leq \angle H_{PID}(j\omega) \leq 90^\circ$$



$$\Rightarrow Re\left[H_{PID}(j\omega)\right] \geq 0$$

$$\Rightarrow H_{PID}(s) \text{ PR}$$

$\Rightarrow H_{PID}(s)$ er passiv



$$\Rightarrow \frac{1}{1+L(s)}$$

$$L(s) = H_{PID}(s) \cdot H(s)$$

$$\angle L(j\omega) = \angle H_{PID}(j\omega) + \angle H(j\omega)$$

$\Rightarrow |\angle L(j\omega)| \leq 180^\circ \Rightarrow$ Stabilt!

Passivitet via lagringsfunksjoner

Gitt

$$\left.\begin{aligned}\dot{\underline{x}} &= \underline{f}(\underline{x}, \underline{u}) \\ \underline{y} &= \underline{h}(\underline{x})\end{aligned}\right\} \quad (*)$$

Anta at vi har

- Lagringsfunksjon $V(\underline{x}) \geq 0$
- Dissipasjonsfunksjon $g(\underline{x}) \geq 0$

Slik at

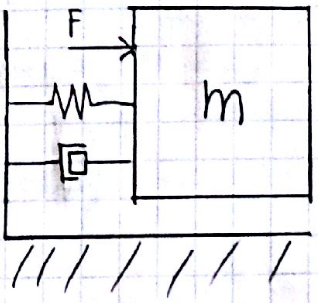
$$\dot{V} = \frac{\partial V}{\partial \underline{x}} \underline{f}(\underline{x}, \underline{u}) \leq \underline{u}^T \underline{y} - g(\underline{x})$$

Da er $(*)$ passiv, med inngang $\underline{u}$ og utgang $\underline{y}$

Bevis:

$$\int_{t_0}^{t} \underline{y}^T \underline{u} \, d\tau \geq \underbrace{V(\underline{x}(t))}_{\geq 0} - V(\underline{x}(t_0)) + \underbrace{\int_{t_0}^{t} g(\underline{x}(\tau)) \, d\tau}_{\geq 0} \geq \underbrace{-V(\underline{x}(t_0))}_{E_0}$$

Eks.



$$m\ddot{x} + d\dot{x} + kx = F$$

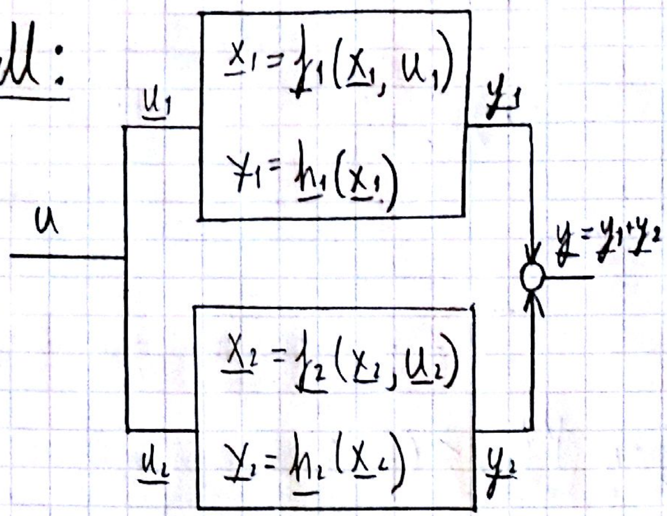
$$V = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$$

$$\dot{V} = \underbrace{F\dot{x}}_{u \cdot y} - \underbrace{d\dot{x}^2}_{g(x) \geq 0}$$

Passivt

2.4.15 Sammenkoblinger av passive system.

Parallell:



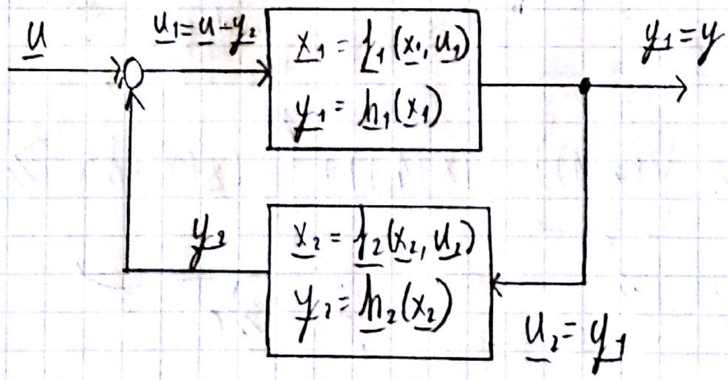
Anta begge sys passive, med lagringsfunksjoner $V_i \geq 0$, $g_i \geq 0$ s.a

$$\dot{V}_i = \frac{\partial V_i}{\partial x_i} f_i(x_i, u_i) \leq u_i^T y_i - g(x_i)$$

Definer $V = V_1 + V_2$, $g = g_1 + g_2$

$$\dot{V} \leq u_1^T y_1 + u_2^T y_2 - g_1 - g_2 = u^T(y_1 + y_2) - g = u^T y - g \quad \text{Passiv!}$$

Tilbakekobling:



$$\dot{V} \leq u_1^T y_1 + u_2^T y_2 \underbrace{- g_1 - g_2}$$

$$= (u - y_2)^T y_1 + y_1^T y_2 - g$$

$$= u^T y - g \qquad \text{Passiv!}$$