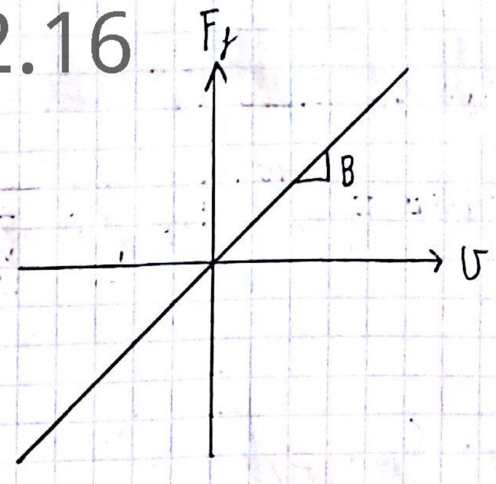
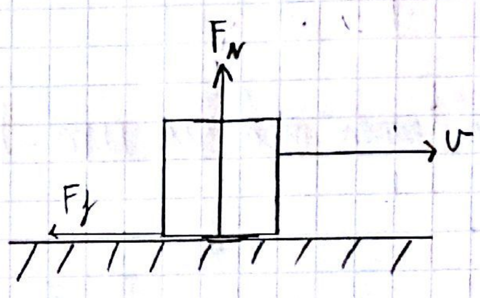
19.02.16

5. Friksjon

Viskøs friksjon: $F_f = Bv$

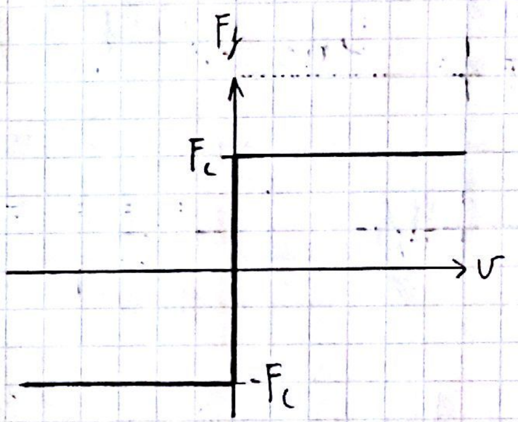


Coulumb-friksjon:

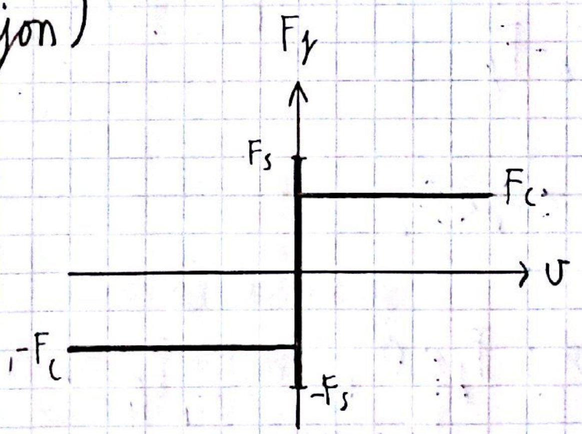


$F_c = \mu F_N$

$F_f = F_c \, \text{sign}(v), \quad v \neq 0$

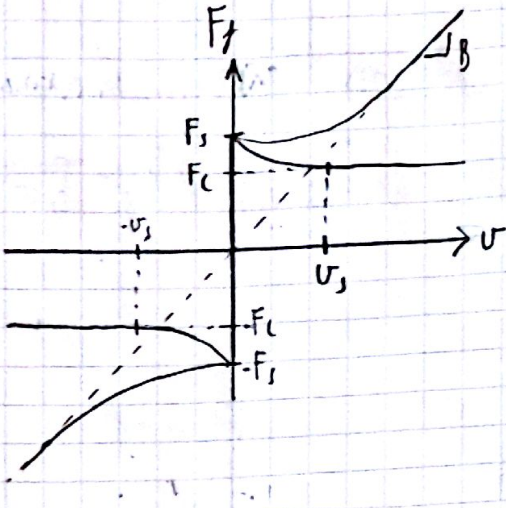


Statisk friksjon ("stiksjon")



Stribeck-effekten

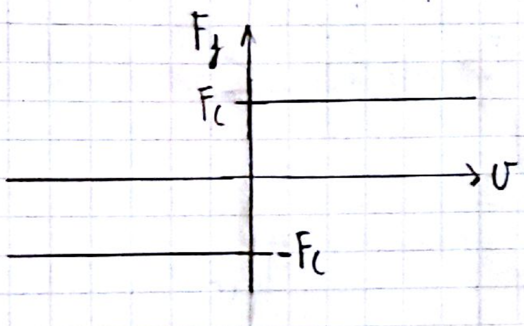
$$F_f = \left[F_c + (F_s - F_c)e^{-\left(\frac{v}{v_s}\right)^2}\right]\operatorname{sign}(v) + Bv$$



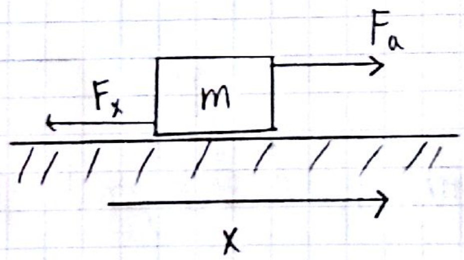
Merk: Empirisk!

5.2.4 Problemer med signum-ledd ved null hastighet

Coulomb



$$F_f = F_c \operatorname{sign}(v) = \begin{cases} -F_c, & v<0 \\ 0, & v=0 \\ F_c, & v>0 \end{cases}$$



$v = \dot{x}$

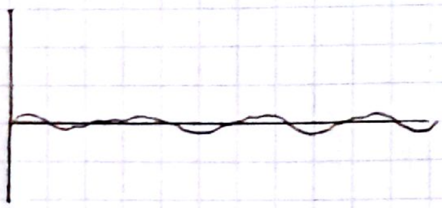
Newtons lov:

$m\dot{v} = F_a - F_f$

Anta $0 < F_a < F_c$

$v = 0$: $F_f = 0$, $m\dot{v} = F_a \Rightarrow v$ øker

$v > 0$: $m\dot{v} = F_a - F_f < 0 \Rightarrow v$ avtar



ufysisk

Enkel løsning: Karnoppsmodell (5.2.5)

$$m\dot{v} = F_a - F_f = \begin{cases} F_a + F_c &, v < 0 \\ F_a - F_c &, v > 0 \end{cases}$$

Hva med $v = 0$?

Vil ha $m\dot{v} = 0$ når $v = 0$ og $|F_a| < F_c$

Derfor: $F_f = F_a$ når $v = 0$ og $|F_a| = F_c$

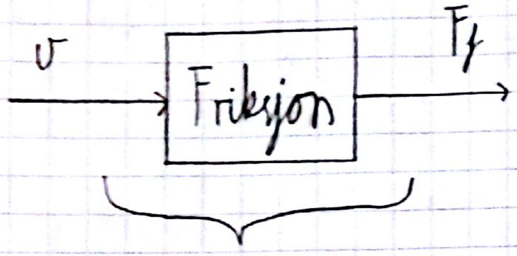
$$F_f = \begin{cases} \text{sat}(F_a, F_c), & v = 0 \\ F_c \,\text{sign}(v), & v \neq 0 \end{cases}$$

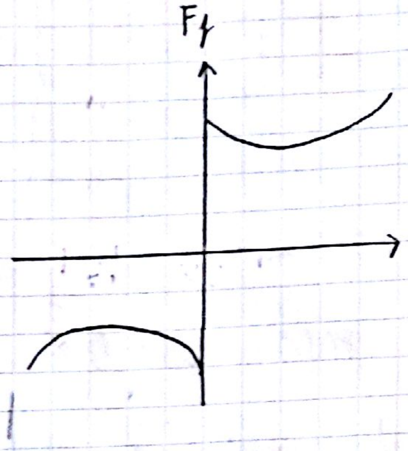
$$\text{sat}(x, \delta) = \begin{cases} x, & |x| \leq \delta \\ \delta \,\text{sign}(x), & |x| > \delta \end{cases}$$

- Trenger løser med "event-håndtering" (evt. dødbånd, men dårlig idé)

Kan utvides til stribeck.

5.2.7 Passivitet av friksjonsmodeller



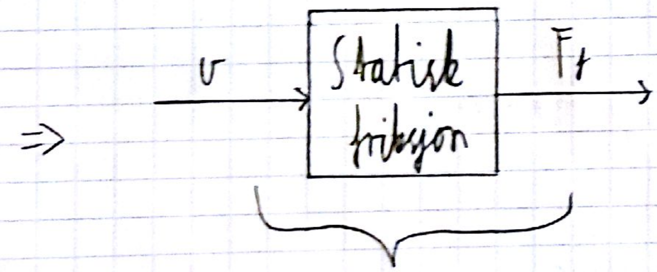


Passivt hvis

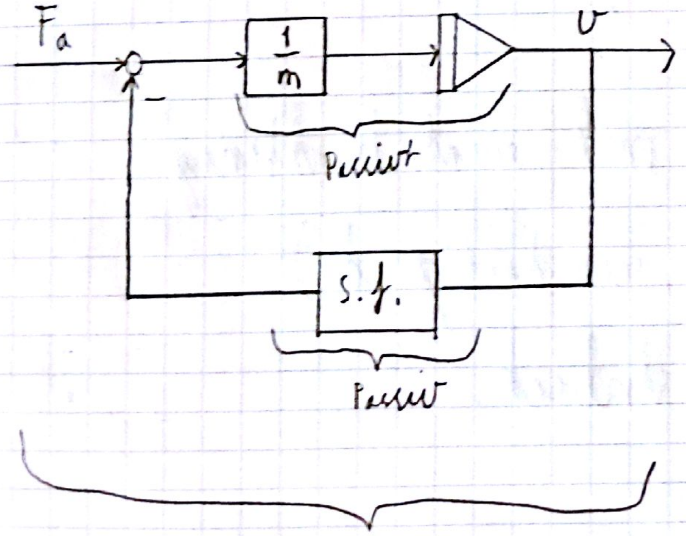
$$\int_{t_0}^{t} F_f(\tau)\, v(\tau)\, d\tau \geq -E_0$$

$$\Rightarrow \quad F_f \cdot v > 0 \quad \Rightarrow \quad \int_{t_0}^{t} F_f\, v\, d\tau \geq 0$$

$\Rightarrow$



Passivt



Passivt

5.3 Dynamisk friksjonsmodeller

5.3.1 Dahls modell

$$\frac{dF}{dt} = \sigma\left(v - |v|\frac{F}{F_c}\right)$$

Stasjonært (konstant hastighet):

$$\frac{dF}{dt} = 0 \Rightarrow v - |v|\frac{F_{ss}}{F_c} = 0$$

$$\Rightarrow F_{ss} = F_c\frac{v}{|v|} = F_c\,\text{sign}(v)$$

Coulomb!

Dynamisk:

$$\frac{dF}{dt} = -\sigma\frac{|v|}{F_c}F + \sigma \cdot v$$

$$\text{"}\dot{y} = -\frac{1}{T}y + u\text{"} \rightarrow \frac{1}{1+Ts}$$

"Tidskonstant" $T = \frac{F_c}{\sigma|v|}$

$$v \rightarrow 0 \Rightarrow T \rightarrow \infty$$

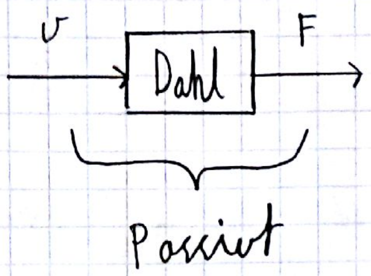
+ Bra for simulering

÷ Kan gi "drift" i stick-området ($v = 0$)

Passivitet av Dahls modell

Lagringsfunksjon $\quad V = \frac{1}{2\sigma} F^2$

$$\dot{V} = \frac{1}{\sigma} F \cdot \dot{F} = F v - \underbrace{|v| \frac{F^2}{F_c}}_{\geq 0} = y u - \underbrace{g(x)}_{\geq 0}$$



5.3.2 LuGre-modellen

$$F = \sigma_0 z + \underbrace{\sigma_1 \dot{z}}_{\substack{\text{ofte} \\ \sigma_1 = 0}} + \underbrace{\sigma_2 v}_{\substack{\text{viskøs} \\ \text{friksjon}}}$$

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z$$

$$g(v) = F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2}$$

Stasjonært (konstant hastighet)

$$\dot{z} = 0 \Rightarrow v - \sigma_0 \frac{v}{g(v)} z_{ss} = 0$$

$$\Rightarrow z_{ss} = \frac{g(v)}{\sigma_0} \cdot \frac{v}{|v|} = \frac{g(v)}{\sigma_0} \operatorname{sign}(v)$$

$$\Rightarrow F_{ss} = \left[F_c + (F_s - F_c) e^{-\left(\frac{v}{v_s}\right)^2} \right] \operatorname{sign}(v) + \sigma_2 v$$

Stribeck-effekt!

"Tidskonstant"

$$T = \frac{g(v)}{\sigma_0 |v|} \to \infty \quad \text{når} \quad |v| \to 0$$

- Samme fordel/ulempe som Dahls modell
- Mer realistisk dynamisk oppførsel

Passivitet

LuGre-modellen er passiv hvis σ_1 er liten nok.

$$P_1 + P_2 + \dots + P_n = P_T$$

$$(P_1 + P_2 + \dots + P_n)\,X_T = P_T X_T = P_M \Rightarrow X_T = \frac{P_M}{P_T}$$

$$P_1X_1 + P_2X_2 + \dots + P_nX_n = \cancel{P_T X_T} = P_M$$

P_1

$$P_1X_1 = P_T X_T - P_T X_T + P_1 X_1$$

$$P_1 \frac{X_1}{X_T} + P_2 \frac{X_2}{X_T} + \dots + P_n \frac{X_n}{X_T} = P_T$$