4.3 Motor-modeller

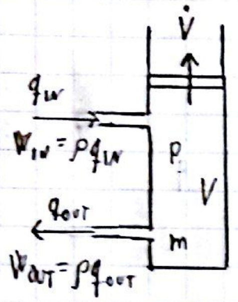
18.02.16

Kompressibilitet av væske er gitt av væskens **bulk modulus**

$$\beta = -V\frac{dp}{dV} = \rho\frac{dp}{d\rho} \quad \left(\begin{array}{c}\text{antatt} \\ \text{konstant}\end{array}\right)$$

$$\boxed{\begin{array}{c}\rho = \dfrac{m}{V} \\ \\ d\rho = -m\dfrac{1}{V^2}dV\end{array}}$$

Eks. olje: $\beta = 7000$ bar, vann: 22000 bar



Massebalanse

$$\frac{dm}{dt} = w_{IN} - w_{OUT}$$

$$\frac{d}{dt}(\rho V) = \rho q_{IN} - \rho q_{OUT}$$

$$\dot{\rho} V + \rho \dot{V} = \rho q_{IN} - \rho q_{OUT}$$

$$\frac{d\rho}{dt} = \frac{\rho}{\beta}\frac{dp}{dt}$$

$$\boxed{\frac{V}{\beta}\dot{p} + \dot{V} = q_{IN} - q_{OUT}}$$

Eks. Roterende hydraulisk motor

$$\frac{V_1}{\beta}\dot{p}_1 = -\dot{V}_1 - C_{im}(p_1 - p_2) - C_{em}p_1 + q_1$$

$$\frac{V_2}{\beta}\dot{p}_2 = -\dot{V}_2 - C_{im}(p_2 - p_1) - C_{em}p_2 - q_2$$

$$\dot{V}_1 = -\dot{V}_2 = D_m\omega_m$$

Moment balanse

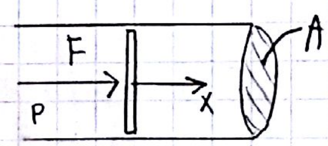
$$J\dot{\omega}_m = \underbrace{T_m}_{?} - \underbrace{B\omega_m}_{\text{Friksjon}} - \underbrace{T_L}_{\text{Last moment}}$$

Antar tapsfritt, effekt inn = effekt ut

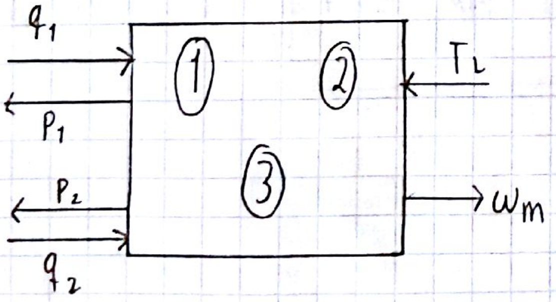
$$T_m\omega_m = p_1\dot{V}_1 + p_2\dot{V}_2$$

$$= (p_1 - p_2)D_m\omega_m$$

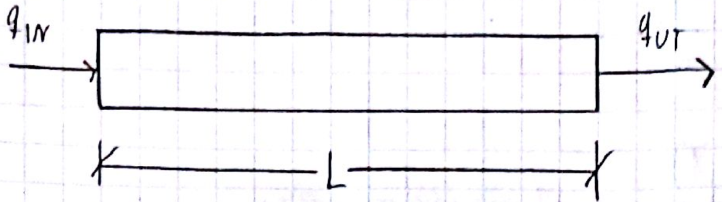
$$\Rightarrow \boxed{T_m = D_m(p_1 - p_2)}$$



$$F\dot{x} = p\underbrace{A\dot{x}}_{\dot{V}} = p\dot{V}$$



4.5/4.6 Transmisjonslinjer



L "liten": $\frac{V}{\beta}\dot{p} + \dot{V} = q_{IN} - q_{OUT}$

L stor: Må ta hensyn til trykkforplantning (= lydhastighet)

$$c = \sqrt{\frac{\beta}{\rho}}$$

Eks.

- Hydraulikkolje: $c = \sqrt{\frac{7000 \cdot 10^5 \, Pa}{870 \, \frac{kg}{m^3}}} \approx 900 \, m/s \approx 1000 \, m/s$

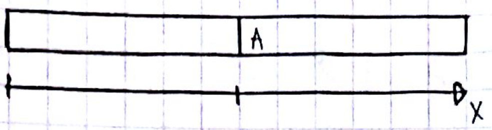
$c = \frac{L}{T}$

L	1m	10m	500 m
T	1ms	10ms	0.5s

- Borevæske $L = 10 \, km$, $c = \sqrt{\frac{15000 \cdot 10^5 \, Pa}{1600 \, kg/m^3}} = 970 \, m/s$

$T = 10 \, s$

PDE-modell av $p(x,t)$, $q(x,t)$



$$(11.93) \quad \frac{\partial p(x,t)}{\partial t} = -\frac{\beta}{A}\frac{\partial q(x,t)}{\partial x} \quad \text{(massebalanse)}$$

$$(11.94) \quad \frac{\partial q(x,t)}{\partial t} = -\frac{A}{\rho}\cdot\frac{\partial p(x,t)}{\partial x} - \underbrace{\frac{F(q(x,t))}{\rho}}_{\text{Friksjon}} \quad \text{(momentbalanse)}$$

$$\boxed{Z_0 = \frac{\rho c}{A} = \frac{\sqrt{\rho P}}{A}}$$

Laplace:

$$\frac{\partial q(x,s)}{\partial x} = -\frac{s}{c Z_0} p(x,s)$$

$$\frac{\partial q(x,s)}{\partial x} = -\frac{Z_0 s}{c} q(x,s) - \frac{Z_0 F[q(x,s)]}{c\rho}$$

$$= -\frac{Z_0 \Gamma(s)}{LT s} q(x,s)$$

Propagasjons-
operator

Spesialtilfeller:

- ingen friksjon ($F=0$):

$$\frac{Z_0 \Gamma^2(s)}{L T_s} = \frac{Z_0 s}{C} = \frac{Z_0 T_s}{L} \Rightarrow \Gamma(s) = T_s$$

- Lineær friksjon ($F = \rho B q$):

$$\Gamma(s) = T_s \sqrt{\frac{s+B}{s}} \qquad Z_c(s) = Z_0 \sqrt{\frac{s+B}{s}}$$

Bølgevariable:

$$\frac{\partial}{\partial x}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 0 & -\frac{T_s}{L Z_0} \\ -\frac{Z_0 \Gamma(s)}{L T_s} & 0 \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}$$

Diagonaliserer v/ koordinat-transformasjon

$$a(x,s) = p(x,s) + Z_c(s)\, q(x,s)$$

$$b(x,s) = p(x,s) - Z_c(s)\, q(x,s)$$

$$\Rightarrow \frac{\partial a}{\partial x} = -\frac{\Gamma(s)}{L} a$$

$$\frac{\partial b}{\partial x} = \frac{\Gamma(s)}{L} b$$

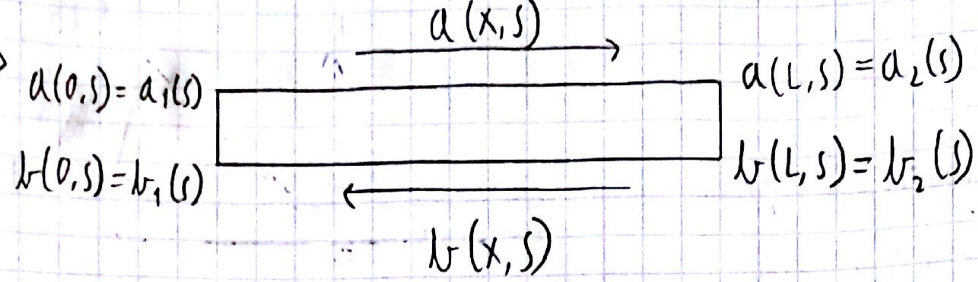
Kan enkelt løses:

$$a(x,s) = \exp\left(-\Gamma \frac{x}{L}\right) a(0,s)$$

$$b(x,s) = \exp\left(-\Gamma \frac{L-x}{L}\right) b(L,s)$$

$\Rightarrow$

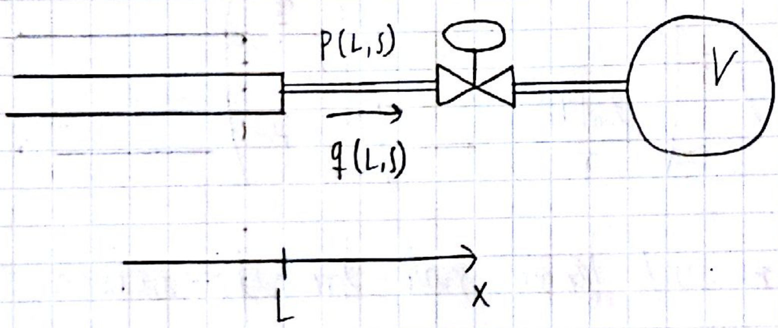
$\Rightarrow$



$$a_2(s) = e^{-\Gamma(s)}\, a_1(s) \qquad \text{Tapsfritt } \Gamma(s) = Ts$$

$$b_1(s) = \underbrace{e^{-\Gamma(s)}}_{e^{-Ts}}\, b_2(s) \qquad \rightarrow \text{ren tidsforsinkelse}$$

Anta linje "terminert" s.a $p(L,s) = Z_L(s)\, q(L,s)$



$$a_2(s) = a(L,s) = p(L,s) + Z_c(s)\, q(L,s) = (Z_L(s) + Z_c(s))\, q(L,s)$$

$$b_2(s) = b(L,s) = \quad \dots \quad = (Z_L(s) - Z_c(s))\, q(L,s)$$

$$\frac{b_2}{a_2}(s) = \frac{Z_L(s) - Z_c(s)}{Z_L(s) + Z_c(s)} = G_L(s) \qquad b_1 = b_2 e^{-\Gamma} = a_2 G_L e^{-\Gamma} = a_1 G_L e^{-2\Gamma}$$

$$\Rightarrow \frac{b_1}{a_1}(s) = G_L(s)\, e^{-2\Gamma(s)}$$

Impedanstilpassning:

$Z_L = Z_c \Rightarrow G_L = 0$

$Z_L = 0 \Rightarrow G_L = -1$ (åpen)

$Z_L = \infty \Rightarrow G_L = 1$ (lukket)