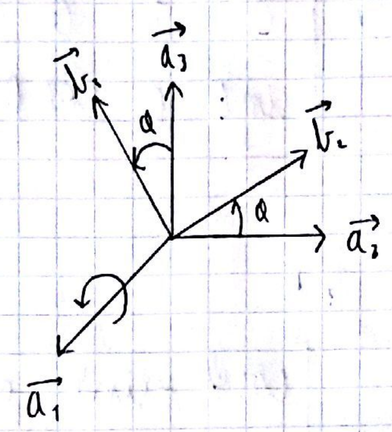
Enkel rotasjon = rotasjon om en fast akse

$R_x(\varphi)$ = rotasjon om x-akse

04.03.16



$$R_x(\varphi) = \begin{bmatrix} \vec{a}_1\cdot\vec{b}_1 & \vec{a}_1\vec{b}_2 & \vec{a}_1\vec{b}_3 \\ \vec{a}_2\vec{b}_1 & \vec{a}_2\vec{b}_2 & \vec{a}_2\vec{b}_3 \\ \vec{a}_3\vec{b}_1 & \vec{a}_3\vec{b}_2 & \vec{a}_3\vec{b}_3 \end{bmatrix}$$

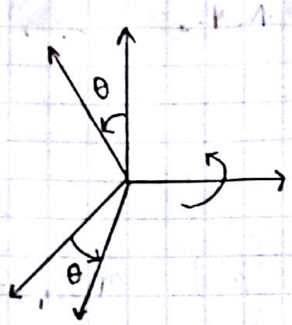
$$\vec{a}_1\cdot\vec{b}_1 = 1$$

$$\vec{a}_1\cdot\vec{b}_2 = 0 = \vec{a}_1\cdot\vec{b}_3 = \vec{a}_2\cdot\vec{b}_1 = \vec{a}_3\cdot\vec{b}_1$$

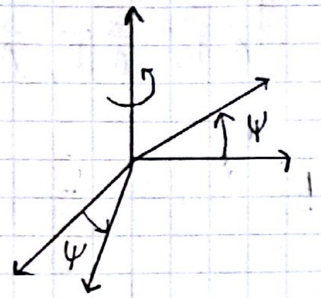
$$\vec{a}_2\vec{b}_2 = \cos\varphi \qquad \vec{a}_3\vec{b}_3 = \cos\varphi$$

$$\vec{a}_3\vec{b}_2 = \cos\left(\frac{\pi}{2}-\varphi\right) = \sin\varphi \qquad \vec{a}_2\vec{b}_3 = \cos\left(\frac{\pi}{2}+\varphi\right) = -\sin\varphi$$

$$R_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

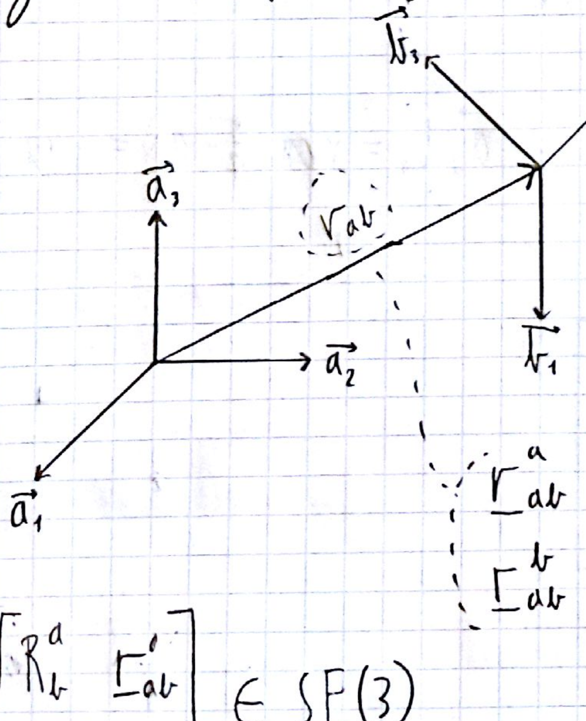


$$R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$$



$$R_z(\psi)=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}$$

Homogene transformasjonsmatriser



Orientering: R^a_b

Posisjon: $\vec{r}_{ab}$

av b i forhold til a

$$T^a_b=\begin{bmatrix}R^a_b & \underline{r}^a_{ab}\\ 0\,0\,0 & 1\end{bmatrix}\in SE(3)$$

$$SE(3): \left\{T \mid T=\begin{bmatrix}R & \underline{r}\\ 0 & 1\end{bmatrix},\ R\in SO(3),\ \underline{r}\in\mathbb{R}^3\right\}$$

$$\Rightarrow \begin{pmatrix} R^a_b & \underline{r}^a_{ab} \\ 0\,0\,0 & 1 \end{pmatrix} \begin{pmatrix} R^b_a & \underline{r}_{ba} \\ 0\,0\,0 & 1 \end{pmatrix} = \begin{pmatrix} I_{3\times3} & R^a_b \underline{r}^b_{ba} + \underline{r}^a_{ab} \\ 0\,0\,0 & 1 \end{pmatrix}$$

$$= \underline{r}^a_{ba} + \underline{r}^a_{ab} = 0$$

$$= I_{4\times4} \Rightarrow \left(T^a_b\right)^{-1} = T^b_a.$$

Sammensatte transformasjoner

$$= \underline{r}^a_{bc} + \underline{r}^a_{ab} = \underline{r}^a_{ac}$$

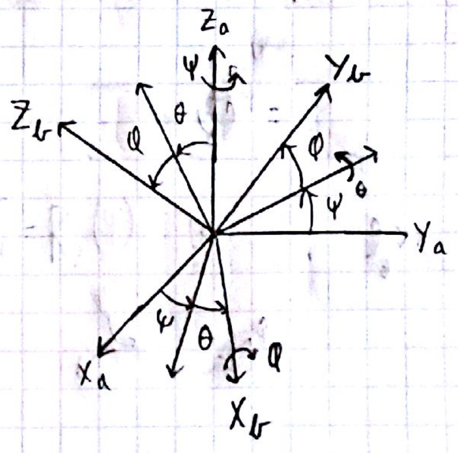
$$T^a_b T^b_c = \begin{pmatrix} R^a_b & \underline{r}^a_{ab} \\ 000 & 1 \end{pmatrix} \begin{pmatrix} R^b_c & \underline{r}_{bc} \\ 000 & 1 \end{pmatrix} = \begin{pmatrix} R^a_b R^b_c & R^a_b \underline{r}^b_{bc} + \underline{r}^a_{ab} \\ 000 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} R^a_c & \underline{r}^a_{ac} \\ 000 & 1 \end{pmatrix} = T^a_c$$

Euler-vinkler:

	(Roll)	(Pitch)	(Yaw)
	Rull	Stamp	Gir
Vinkel	ϕ	θ	ψ
Akse	x-akse	y-akse	z-akse

$$R^a_b = R_z(\psi) R_y(\theta) R_x(\phi)$$



Rekkefølge:

koord.trans $\longrightarrow$

rotasjon $\longleftarrow$

Vinkel-akse parameterisering

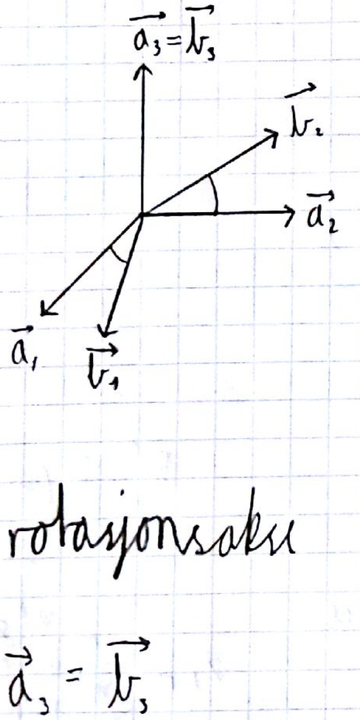
$$R_b^a \text{ ortogonal} \Rightarrow \lambda(R_b^a) = 1$$

Dvs. Det finnes en (egen-)vektor $\underline{k}$ s.a

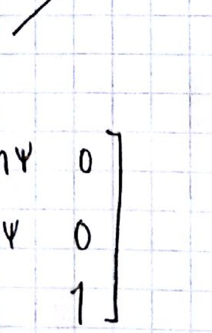
$$R_b^a \underline{k} = \underline{k} \qquad (\text{velg } \underline{k}^T \underline{k} = 1)$$

$$\underline{k} = \underline{k}^b : \underline{k}^a = R_b^a \underline{k}^b = \underline{k}^b$$

$\vec{k}$ har samme repr. i a og b



Alle rotasjoner kan beskrives av en vektor $\vec{k}$ (rot. akse) og en vinkel θ



$\vec{k}$ er en rotasjonsakse

$$\vec{k} = \vec{a}_3 = \vec{b}_3$$

$$R_b^a = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{k} = R_b^a \underline{k}$$

$$\vec{q} = \cos(\theta)\vec{p} + \sin(\theta)\vec{k} \times \vec{p} + (1-\cos(\theta))(\vec{k} \cdot \vec{p})\vec{k}$$

$$= \underbrace{\left(\cos(\theta)\vec{I} + \sin(\theta)\vec{k}^{\times} + (1-\cos(\theta)\vec{k}\vec{k}\right)}_{\vec{R}_{\vec{k},\theta}}\vec{p}$$

$$\vec{q} = \vec{R}_{\vec{k},\theta}\vec{p}$$

$$R_b^a = R_{\underline{k},\theta} = \cos(\theta)I + \sin(\theta)(\underline{k})^{\times} + (1-\cos(\theta))\underline{k}\,\underline{k}^T$$

$$\underline{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \theta = \Psi \Rightarrow R_z(\Psi)$$

$$s = \frac{1-z^{-1}}{h}$$

$$\frac{X(s)}{U(s)} = \frac{K}{(1+T_1 s)(1+T_2 s)} = \frac{K}{T_1T_2s^2+(T_1+T_2)s+1}$$

$$(1-z^{-1})^2 = 1-2z^{-1}+z^{-2}$$

$$\frac{X(z)}{U(z)} = \frac{K}{T_1T_2\left(\frac{1-z^{-1}}{h}\right)^2+(T_1+T_2)\frac{1-z^{-1}}{h}+1}$$

$$\frac{X(z)}{U(z)} = \frac{K}{\frac{T_1T_2}{h^2}(1-2z^{-1}+z^{-2})+\frac{(T_1+T_2)}{h}(1-z^{-1})+1}$$

$$\frac{X}{U} = \frac{K}{\left(\frac{T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}+1\right)-\left(\frac{2T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}\right)z^{-1}+\frac{T_1T_2}{h^2}z^{-2}}$$

$$\left[\left(\frac{T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}+1\right)-\left(\frac{2T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}\right)z^{-1}+\frac{T_1T_2}{h^2}z^{-2}\right]x = Ku$$

$$\left(\frac{T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}+1\right)x = \left(\frac{2T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}\right)x_{-1}-\frac{T_1T_2}{h^2}x_{-2}+Ku$$

$$x = \left(\frac{T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}+1\right)^{-1}\left[\left(\frac{2T_1T_2}{h^2}+\frac{(T_1+T_2)}{h}\right)x_{-1}-\frac{T_1T_2}{h^2}x_{-2}+Ku\right]$$