Lecture 19: Lagrangian mechanics (Lagrange's equation of motion)

Newton's law (for particles, or Newton-Euler EoM for rigid bodies) in combination with

- d'Alembert's principle
- Generalized coordinates

gives Lagrange's equations of motion

- Brief examples

Book: Ch. 7.7, 8.2

Newton-Euler equations of motion

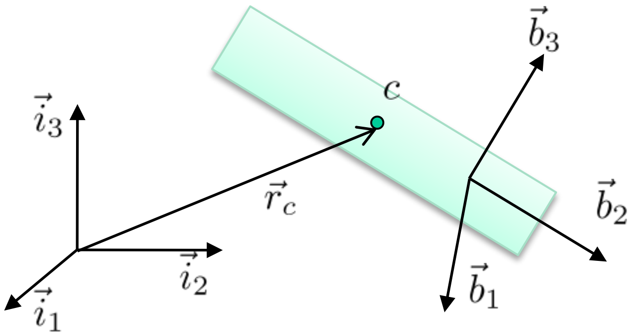
- Newton's law (for particle *k*)

$$m_k \vec{a}_k = \vec{F}^{(r)}$$

- Newton-Euler EoM for rigid bodies:
  - Integrate Newton's law over body, define center of mass
  - Define torque/moment and angular momentum to handle forces that give rotation about center of mass
  - Define inertia dyadic/matrix

$$\vec{F}_{bc} = m\vec{a}_c$$

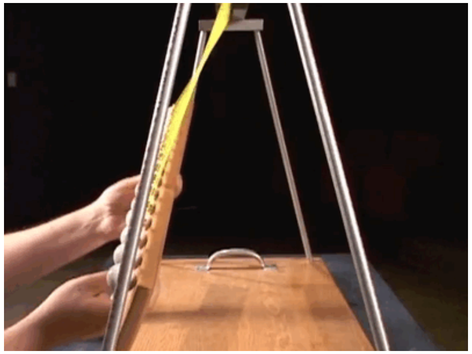
$$\vec{T}_{bc} = \vec{M}_{b/c} \cdot \vec{\alpha}_{ib} + \vec{\omega}_{ib} \times \left(\vec{M}_{b/c} \cdot \vec{\omega}_{ib}\right)$$



(Here: Referenced to center of mass)

- Implemented in e.g. Dymola (Modelica.Multibody library)

Is there a fundamental difference?



Pendulum

Lab helicopter

Quadrotor

Satellite

- Newton-Euler or Lagrange?
  - Newton-Euler can (of course) be used for everything, but if you are calculating by hand/symbolically, it is far easier to use Lagrange when you have constrained motion (forces of constraint)

Lagrange vs Newton-Euler

Newton-Euler

- Vectors
- Forces and moments
- Does not eliminate forces of constraints:
  - Obtains solutions for all forces and kinematic variables
  - ”Inefficient” (large DAE models)
- More general
  - Large systems can be handled, but for some configurations tricks are needed
  - Used in advanced modeling software

Lagrange

- Algebraic
- Energy
- Eliminates forces of constraints
  - Solutions only for generalized coordinates (and forces)
  - ”Efficient” (smaller ODE models)
- Less general
  - Need independent generalized coordinates
  - Difficult to automate for large/complex problems