Lecture 4: Passivity

Passivity (E2.4)

- Brief recap of passivity
- Positive Real (PR) transfer functions
- Passivity and storage functions

Recap: Energy functions and passivity

Using ”energy” as a concept for characterizing system behavior

- Energy functions (aka Lyapunov functions)
  - If the ”internal energy” of a system decreases, the system is stable
  - ”Introvert” (not concerned with surroundings)
- Passivity
  - Does a system produce ”energy” to its surroundings?
  - ”Extrovert” (mainly concerned with surroundings, via inputs and outputs)
- The above concepts are connected via storage functions

Passivity

- A system with input *u* and output *y* is passive if

$$\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$$

for all $t \geq 0$, for all input trajectories.

- If the product yu has power as unit, then if (for all *u*)
  - $\int_0^t y(\tau)u(\tau)d\tau \geq 0$: Energy is absorbed within the system, nothing delivered to the outside
  - $\int_0^t y(\tau)u(\tau)d\tau \geq -E_0$: Some energy can be delivered to the outside, limited (typically) by the initial energy in the system.
  - $\int_0^t y(\tau)u(\tau)d\tau \to -\infty$: There is an inexhaustible energy source in the system. Not passive!

Positive real transfer functions

$$\underbrace{\xrightarrow{\;u\;} \boxed{H(s)} \xrightarrow{\;y\;}}_{\text{Passive}} \quad \Leftrightarrow \quad H(s) \text{ is positive real}$$

Definition: The transfer function $H(s)$ (rational or irrational) is positive real if

1. $H(s)$ analytic in $\mathrm{Re}[s]>0$.
2. $H(s)$ is real for all positive and real s.
3. $\mathrm{Re}[H(s)] \geq 0$ for all $\mathrm{Re}[s]>0$.

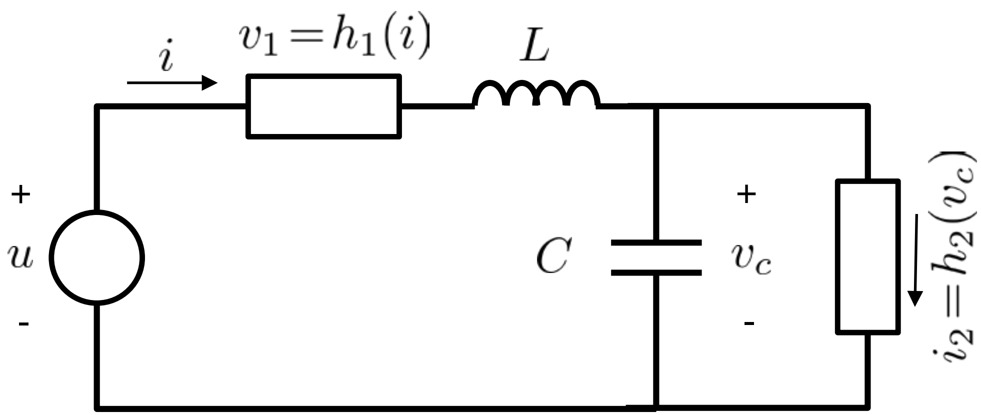
Check rational TFs for PRness

Theorem: A rational, proper transfer function $H(s)$ is positive real (and hence passive) if and only if

1. $H(s)$ has no poles in $\mathrm{Re}[s] > 0$.
2. $\mathrm{Re}[\mathrm{H}(j\omega)] \geq 0$ for all $\omega \in [-\infty, \infty]$ such that $j\omega$ is not a pole of $H(s)$.
3. If $j\omega_0$ is a pole of $H(s)$, then it is a simple pole, and the residual in $s = j\omega_0$ is real and greater than zero, that is,

$$\operatorname*{Res}_{s=j\omega_0} H(s) = \lim_{s \to j\omega_0} (s - j\omega_0) H(j\omega) > 0.$$

Example storage functions



- States: $x_1 = i, \quad x_2 = v_c$
- Model (Kirchoff's laws):

$$L\dot{x}_1 = u - h_1(x_1) - x_2$$
$$C\dot{x}_2 = x_1 - h_2(x_2)$$

- Output&input: $y = i, \quad u = u$
- Nonlinear resistors fulfilling

$$x_i h_i(x_i) \geq 0$$

- Storage (energy) function:

$$V(\mathbf{x}) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2$$

- Differentiate:

$$\begin{aligned}\dot{V} &= Lx_1\dot{x}_1 + Cx_2\dot{x}_2 \\ &= x_1(u - h_1(x_1) - x_2) + x_2(x_1 - h_2(x_2)) \\ &= yu - x_1h_1(x_1) - x_2h(x_2)\end{aligned}$$

- Passive!

Kahoot

- https://play.kahoot.it/#/k/c452fe59-cad5-4f8a-ba94-475d2a5569b6