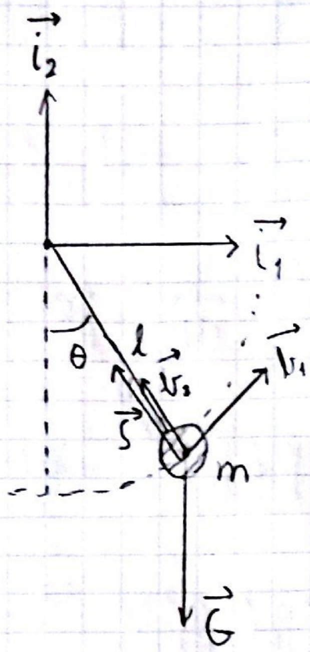
Pendel

15.04.16



$$\vec{r} = x\vec{i}_1 + y\vec{i}_2$$

$$\vec{v}_1 = \cos(\theta)\vec{i}_1 + \sin(\theta)\vec{i}_2$$

$$\vec{v}_2 = -\sin(\theta)\vec{i}_1 + \cos(\theta)\vec{i}_2$$

$$\vec{S} = S\vec{v}_2 \qquad \vec{G} = -mg\,\vec{i}_2$$

Newtonslov:

$$m\frac{d^2}{dt^2}\vec{r} = S\vec{v}_2 - mg\vec{i}_2$$

1) Utvikler i inertielt system:

$$m(\ddot{x}\vec{i}_1 + \ddot{y}\vec{i}_2) = S(-\sin(\theta)\vec{i}_1 + \cos(\theta)\vec{i}_2) - mg\vec{i}_2$$

$\vec{i}_1$: $\quad m\ddot{x} = -S\sin\theta \qquad , \sin(\theta) = \frac{x}{l}$

$\vec{i}_2$: $\quad m\ddot{y} = S\cos(\theta) - mg \quad , \cos(\theta) = \frac{-y}{l}$

$$x^2 + y^2 = l^2$$

2) Utvikle i body-system

$$\vec{r} = -l\vec{b}_2$$

$$\vec{v} = \frac{^i d}{dt}\vec{r} = \frac{^b d}{dt}\vec{r} + \vec{\omega}_{ib} \times \vec{r} \quad , \quad \vec{\omega}_{ib} = \dot{\theta}\vec{b}_3$$

$$= 0 + (\dot{\theta}\vec{b}_3) \times (-l\vec{b}_2) = l\dot{\theta}\vec{b}_1$$

$$\vec{a} = \frac{^i d}{dt}\vec{v} = \frac{^b d}{dt}(l\dot{\theta}\vec{b}_1) + (\dot{\theta}\vec{b}_3) \times (l\dot{\theta}\vec{b}_1)$$

$$= l\ddot{\theta}\vec{b}_1 + l\dot{\theta}^2\vec{b}_2$$

$$m(l\ddot{\theta}\vec{b}_1 + l\dot{\theta}^2\vec{b}_2) = S\vec{b}_2 - mg(\sin(\theta)\vec{b}_1 + \cos(\theta)\vec{b}_2)$$

$\vec{b}_1$: $\boxed{ml\ddot{\theta} = -mg\sin(\theta)} \longrightarrow \ddot{\theta} + \frac{g}{l}\sin(\theta) = 0$

$\vec{b}_2$: $ml\dot{\theta}^2 = S - mg\cos(\theta)$

Lagrange: generalisert koordinat θ,

$\vec{r} = -l\vec{b}_2$, $\vec{v} = l\dot{\theta}\vec{b}_1$

$T = \frac{1}{2} m \vec{v} \cdot \vec{v} = \frac{1}{2} m l^2 \dot{\theta}^2$

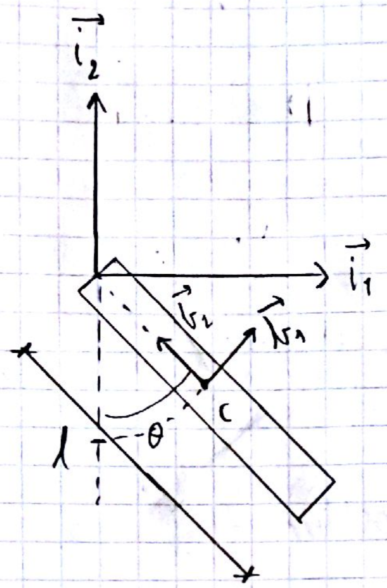
$U = mgl(1-\cos(\theta))$

$\mathcal{L} = T - U = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl(1-\cos(\theta))$

$\frac{d}{dt}(ml^2\dot{\theta}) + mgl\sin(\theta) = 0$

$\ddot{\theta} + \frac{g}{l}\sin(\theta) = 0$

"Rigid body"-pendel m/Lagrange



$$\vec{r}_c = -\frac{l}{2}\vec{b}_2 , \quad \vec{v}_c = \frac{l}{2}\dot{\theta}\vec{b}_1$$

$$\vec{\omega}_{ib} = \dot{\theta}\vec{b}_3 , \quad \underline{\omega}_{ib}^{b} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$$

$$M_{b/c}^{b} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_z \end{bmatrix}$$

$$I = \frac{ml^3}{12}$$

$$T = \frac{1}{2}m(\underline{v}_c^b)^T(\underline{v}_c^b) + \frac{1}{2}(\underline{\omega}_{ib}^b)^T M_{b/c}^b \underline{\omega}_{ib}^b$$

$$= \frac{1}{2}m\frac{l^2}{4}\dot{\theta}^2 + \frac{1}{2}I_z\dot{\theta}^2 = \frac{1}{6}ml^2\cdot\dot{\theta}^2$$

$$U = mg\frac{l}{2}(1-\cos(\theta)) , \quad \mathcal{L} = T - U$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \frac{1}{3}ml^2\dot{\theta} , \quad \frac{\partial \mathcal{L}}{\partial \theta} = -mg\frac{l}{2}\sin(\theta)$$

$$\frac{1}{3}\cancel{m}l^{\cancel{2}}\ddot{\theta} + \cancel{m}g\frac{\cancel{l}}{2}\sin(\theta) = 0$$

$$\boxed{\ddot{\theta} + \frac{3}{2}\frac{g}{l}\sin(\theta) = 0}$$

Eksamen 2017

$\omega_{ib}^b = (0, 0, \dot{q}_1)^T$

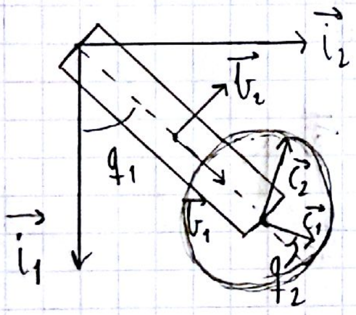
Problem 1

$$M_{b/c}^b = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_1 \end{bmatrix}$$

a) generalized coordinates $\quad q_1$ og q_2

generalized force $\quad 0$ og τ

b) Definerer koordinat system



$\vec{\omega}_{bc} = \dot{q}_2 \vec{b}_3 \qquad \vec{\omega}_{ic} = \vec{\omega}_{ib} + \vec{\omega}_{bc}$

$\vec{\omega}_{ib} = \dot{q}_1 \vec{i}_3 \qquad \vec{\omega}_{ic} = (\dot{q}_1 + \dot{q}_2)\vec{i}_3$

$$\underline{r}_{c1}^i = \begin{bmatrix} l_{c1}\cos(q_1) \\ l_{c1}\sin(q_1) \end{bmatrix}, \quad \underline{v}_{c1}^i = \begin{bmatrix} -l_{c1}\dot{q}_1\sin(q_1) \\ l_{c1}\dot{q}_1\cos(q_1) \end{bmatrix}$$

$$\underline{r}_{c2}^i = \begin{bmatrix} l\cos(q_1) \\ l\sin(q_1) \end{bmatrix}, \quad \underline{v}_{c2}^i = \begin{bmatrix} -l\dot{q}_1\sin(q_1) \\ l\dot{q}_1\cos(q_1) \end{bmatrix}$$

$$T_1 = \frac{1}{2} m_1 (\underline{v}_{c1}^i)^T \underline{v}_{c1}^i + \frac{1}{2}(\underline{\omega}_{ib}^b)^T M_{b/c}^b \underline{\omega}_{ib}^b$$

$$= \frac{1}{2} m_1 l_{c1}^2 \dot{q}_1^2 + \frac{1}{2} I_1 \dot{q}_1^2$$

$$T_2 = \frac{1}{2} m_2 l^2 \dot{q}_1^2 + \frac{1}{2} I_2 (\dot{q}_1 + \dot{q}_2)^2$$

$$U_1 = -m_1 l_{c1} g \cos(q_1)$$

$$U_2 = -m_2 l g \cos(q_1)$$

$$\mathcal{L} = T_1 + T_2 - U_1 - U_2$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m_1 l_{c1}^2 \dot{q}_1 + I_1 \dot{q}_1 + m_2 l^2 \dot{q}_1 + I_2 (\dot{q}_1 + \dot{q}_2)$$

$$\frac{\partial \mathcal{L}}{\partial q_1} = -m l_{c1} g \sin q_1 - m_2 l g \sin(q_1)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_1}\right) - \frac{\partial \mathcal{L}}{\partial q_1} = 0$$

$$\boxed{m_1 l_{c1}^2 \ddot{q}_1 + I_1 \ddot{q}_1 + m_2 l^2 \ddot{q}_1 + I_2 (\ddot{q}_1 + \ddot{q}_2) + m_1 l_{c1} g \sin(q_1) + m_2 l g \sin(q_1) = 0}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = I_2 (\dot{q}_1 + \dot{q}_2) \;, \quad \frac{\partial \mathcal{L}}{\partial q_2} = 0$$

$$\boxed{I_2 (\ddot{q}_1 + \ddot{q}_2) = \tau}$$