

Exercise 1 - TTK4130 Modeling and Simulation

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1 Problem 1

$$Ni = \phi(\mathcal{R}_a + \mathcal{R}_c + \mathcal{R}_b + \mathcal{R}_r). \quad (1)$$

$$\mathcal{R}_a = \frac{z}{A\mu_0}, \mathcal{R}_r = \text{const.} \quad (2)$$

$$\mathcal{R}_c, \mathcal{R}_b \ll \mathcal{R}_r, \mathcal{R}_a. \quad (3)$$

1.1 a

$$\mathcal{R}_r = \frac{z_0}{A\mu_0}, z_0 = \text{const.} \quad (4)$$

Using Equation 1 together with the relations from equations 2 and 4 we get

$$Ni = \phi\left(\frac{z}{A\mu_0} + \mathcal{R}_c + \mathcal{R}_b + \frac{z_0}{A\mu_0}\right).$$

Since \mathcal{R}_c and \mathcal{R}_b are negligible (Equation 3), the total magnetomotive force on the ball is

$$\underline{\underline{Ni = \frac{\phi}{A\mu_0}(z + z_0).}}$$

1.2 b

$$L(z) = \frac{N\phi}{i} = \frac{N^2 A\mu_0}{z + z_0}. \quad (5)$$

$$F = \frac{i^2}{2} \frac{\partial L(z)}{\partial z}. \quad (6)$$

Assume positive direction downwards and gravitational acceleration g .

$$\begin{aligned} ma &= \Sigma F \\ m\ddot{z} &= mg + F \\ m\ddot{z} &= mg + \frac{i^2}{2} N^2 A\mu_0 (z + z_0)^{-2}. \end{aligned}$$

$$\underline{\underline{\ddot{z} = g - \frac{1}{2m} i^2 N^2 A\mu_0 (z + z_0)^{-2}}} \quad (7)$$

1.3 c

Linearizing about z_d , $\dot{z}_d = 0$, $\ddot{z}_d = 0$.

$$0 = g - \frac{1}{2m} i_d^2 N^2 A \mu_0 (z_d + z_0)^{-2}$$

$$i_d = \frac{1}{N} \sqrt{\frac{2mg}{A\mu_0}} (z_d + z_0)$$

Equation 7 we now call f_1 . We define $z = z_d + \Delta z$, $i = i_d + \Delta i$ and thereby $\dot{z} = \Delta \dot{z}$. A Linearization of Equation 7 around the point z_d is then

$$\Delta \ddot{z} = \left. \frac{\partial f_1}{\partial \ddot{z}} \right|_{\substack{z=z_d \\ \dot{z}=\dot{z}_d \\ i=i_d}} \Delta \ddot{z} + \left. \frac{\partial f_1}{\partial \dot{z}} \right|_{\substack{z=z_d \\ \dot{z}=\dot{z}_d \\ i=i_d}} \Delta \dot{z} + \left. \frac{\partial f_1}{\partial i} \right|_{\substack{z=z_d \\ \dot{z}=\dot{z}_d \\ i=i_d}} \Delta i$$

$$\Delta \ddot{z} = \frac{1}{m} i_d^2 N^2 A \mu_0 (z_d + z_0)^{-3} \Delta z - \frac{1}{2m} i_d N^2 A \mu_0 (z_d + z_0)^{-2} \Delta i$$

$$\Delta \ddot{z} = \frac{2g}{z_d + z_0} \Delta z - \frac{N}{z_d + z_0} \sqrt{\frac{2gA\mu_0}{m}} \Delta i$$

2 Problem 2

2.1 b

Natural signal-flow inputs are T_i and ω_{i-1} . The outputs should be T_{i-1} and ω_i . For natural energy-flow, inputs should be T_{i-1} and $\omega_i - 1$, with T_i and ω_i as outputs.

2.2 c

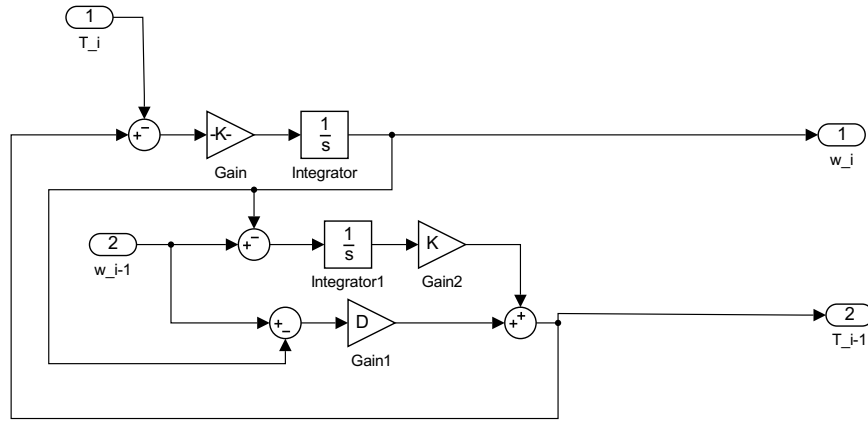


Figure 1: The elastic load implemented in Simulink

2.3 d

As seen in Figure 3, the angular velocity of the second load is delayed. This is due to the phase displacement between the motor and the first load, and the first and the second load.

2.4 e

In the bode plot in Figure 4 it is clear that the system has two resonance peaks at $\omega = 1.5 \frac{\text{rad}}{\text{s}}$ and $\omega = 0.7 \frac{\text{rad}}{\text{s}}$. For small frequencies the effects of integrators are present, which causes the phase to fall almost down to -450 . For large frequencies, the amplification of the input signal is small, and the phase stabilizes at about -270°

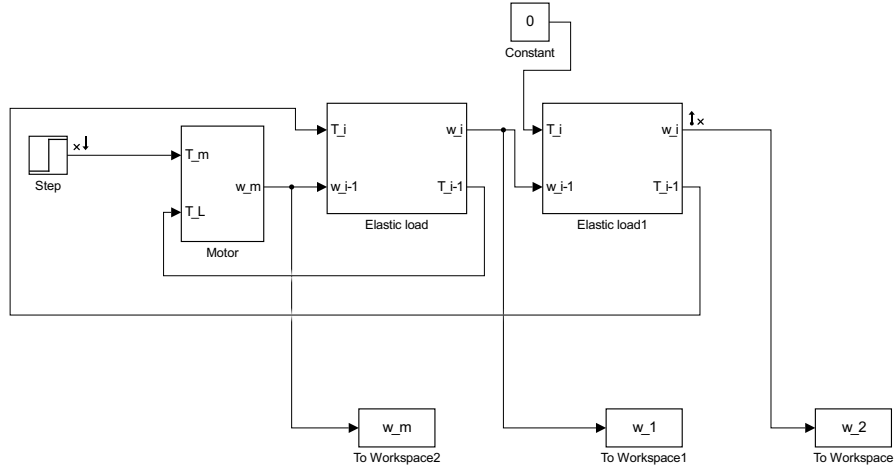


Figure 2: Two elastic loads in series with the motor.

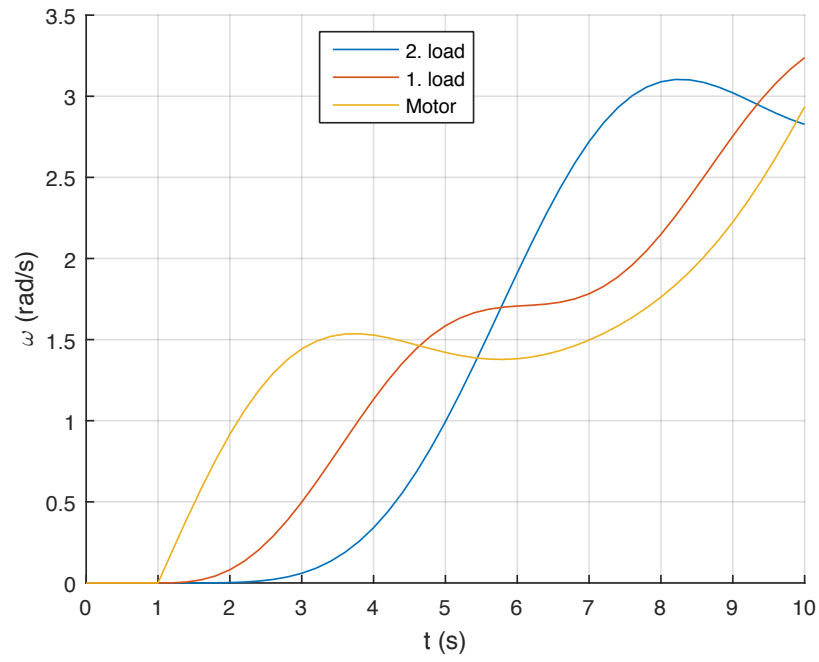
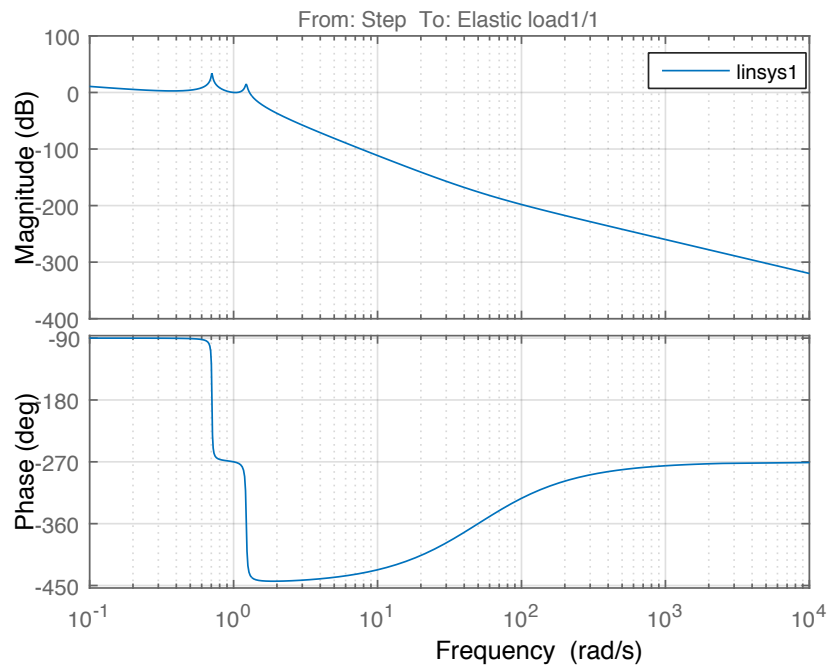


Figure 3: Plot of the rotational velocity of the motor and the two loads.



Figur 4: Bode plot of the transfer function from the step input to the rotational velocity of the second load. Made in with Simulink's linear analysis.

3 Problem 3

3.1 a

The rotational speed of the second load when simulated using Modelica can be seen in Figure 5. It looks just the same as when using Simulink (Figure 3).

The graphical view in Dymola gives a lot more visual information than the one in Simulink. By just looking at the model it is possible to understand what is to be simulated, while in Simulink you have to know and understand the mathematical model to get what is going on.

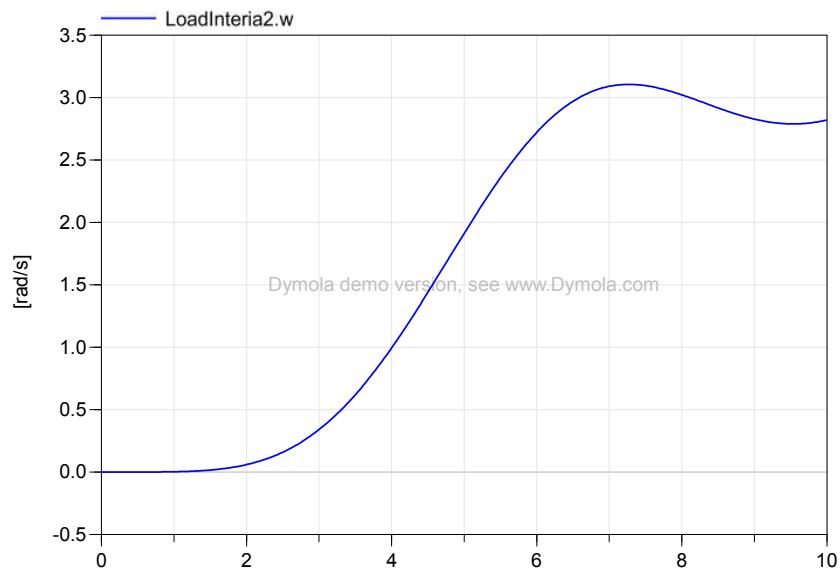


Figure 5: The rotational velocity of the second load when simulated using Dymola.

3.2 b

The Flange_a object in Dymola is a component that connects rotating mechanical systems. This can be compared to the input/output ports of Simulink subsystems. They both communicate with the outer system, but while the Simulink ports are purely objects to hold numbers, the Flange_a also holds physical attributes (rotation angle and torque). The Flange_a is a part of the simulation and emulates a physical connection, while Simulink ports merely are there to communicate with outer subsystems.

3.3 c

The bode plot created using Dymolas linear analysis tools can be seen in Figure 6. It looks very much the same as the one created with SIMulink (Figure 4).

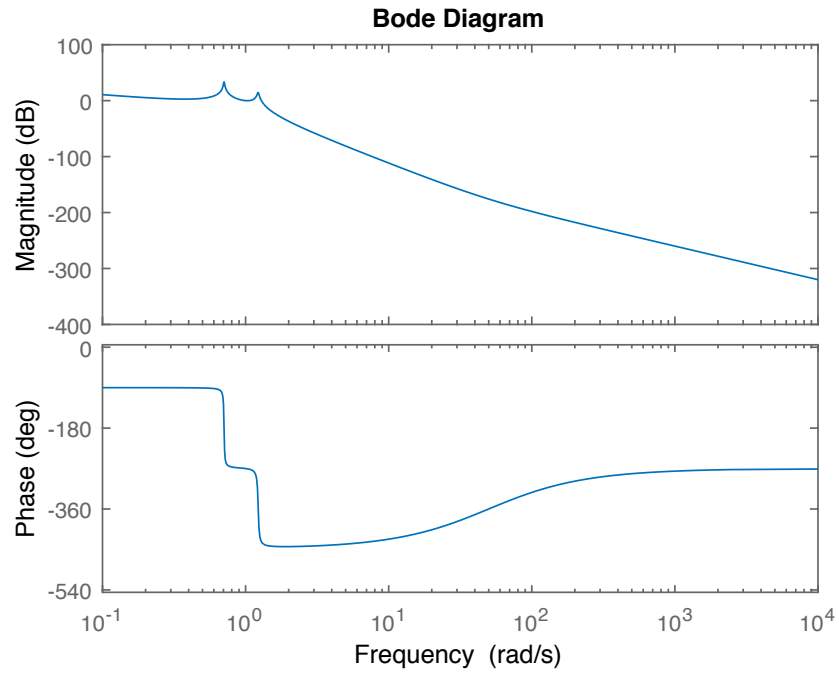


Figure 6: Bode plot of the system when linearized with Dymola.