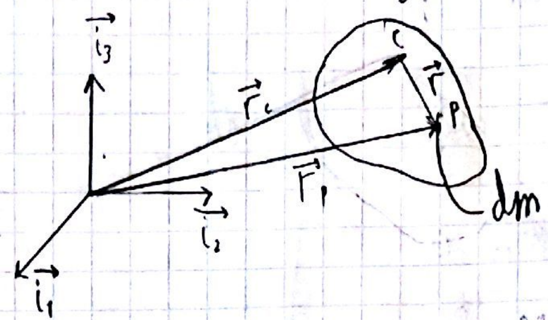
Kinetisk energi for stivt legeme 08.04.16

$$dK = \frac{1}{2} dm\, \vec{v}_p \cdot \vec{v}_p$$



$$K = \int_b dK = \frac{1}{2}\int_b \vec{v}_p \cdot \vec{v}_p\, dm, \quad \vec{v}_p = \vec{v}_c + \vec{\omega}_{ib} \times \vec{r}$$

$$= \frac{1}{2}\int_b \vec{v}_c \vec{v}_c\, dm + \cancel{\frac{1}{2}\int_b \vec{v}_c (\vec{\omega}_{ib} \times \vec{r})\, dm}$$

$$+ \underbrace{\cancel{\frac{1}{2}\int_b (\vec{\omega}_{ib} \times \vec{r}) \vec{v}_c\, dm}}_{\frac{1}{2}\vec{\omega}_{ib} \times \underbrace{\int_b \vec{r}\, dm}_{0} \vec{v}_c} + \frac{1}{2}\int_b (\vec{\omega}_{ib} \times \vec{r}) \cdot (\vec{\omega}_{ib} \times \vec{r})\, dm$$

$$K = \frac{1}{2}\vec{v}_c \cdot \vec{v}_c \int_b dm - \frac{1}{2}\vec{\omega}_{ib} \int_b \vec{r}^{\times} \cdot \vec{r}^{\times}\, dm\, \vec{\omega}_{ib}$$

$$= \frac{1}{2} m \vec{v}_c \cdot \vec{v}_c + \frac{1}{2}\vec{\omega}_{ib} \cdot \vec{M}_{b/c} \cdot \vec{\omega}_{ib}$$

$$\boxed{K = \frac{1}{2} m (\underline{v}_c^b)^T \underline{v}_c^b + \frac{1}{2} (\underline{\omega}_{ib}^b)^T M_{b/c}^b\, \underline{\omega}_{ib}^b}$$

Eks. satellitt

Anta body-system valgt slik at

$$M_{b/c}^{b} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$$

Rotasjonsdynamikk:

$$M_{b/c}^{b} \dot{\underline{\omega}}_{ib}^{b} + (\underline{\omega}_{ib}^{b})^{\times} M_{b/c}^{b} \underline{\omega}_{ib}^{b} = T_{bc}^{b}$$

$$\underline{\omega}_{ib}^{b} = (\omega_1, \omega_2, \omega_3)^T, \quad T_{bc}^{b} = (T_1, T_2, T_3)$$

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

$$m_{11} \dot{\omega}_1 + (m_{33} - m_{22}) \omega_2 \omega_3 = T_1$$

$$m_{22} \dot{\omega}_2 + (m_{11} - m_{33}) \omega_1 \omega_3 = T_2$$

$$m_{33} \dot{\omega}_3 + (m_{22} - m_{11}) \omega_1 \omega_2 = T_3$$

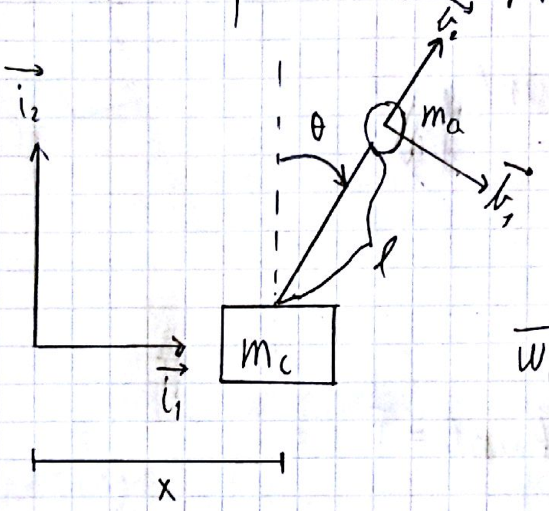
$$\dot{\underline{\Phi}} = E_{\alpha}^{-1}(\underline{\Phi}) \underline{\omega}_{ib}^{b}, \quad \underline{\Phi} = \begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix}$$

$$\left(\text{eller } \dot{\eta} = \frac{1}{2} \epsilon^T \underline{\omega}_{ib}^{b} \quad \dot{\underline{\epsilon}} = \frac{1}{2}(\eta I + \epsilon^{\times}) \underline{\omega}_{ib}^{b} \right)$$

$$\underline{v}_c^b = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \underline{\omega}_{ib}^b = \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \quad \underline{\varphi} = \begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix}$$

$$\underline{F}_{bc}^b = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \qquad \underline{T}_{bc}^b = \begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Eks. Invertert pendel (Newton Euler EoM for partikler)



masseløs stang

$$\vec{\omega}_{ib} = -\dot{\theta}\vec{b}_3 = -\dot{\theta}\vec{i}_3$$

$$\boxed{\begin{aligned} \vec{b}_1 \cdot \vec{i}_1 &= \cos\theta \\ \vec{b}_1 \cdot \vec{i}_2 &= -\sin\theta \\ \vec{b}_2 \cdot \vec{i}_1 &= \sin\theta \\ \vec{b}_2 \cdot \vec{i}_2 &= \cos\theta \end{aligned}}$$

Kinematikk

$$\boxed{\frac{{}^i d}{dt}\vec{u} = \frac{{}^b d}{dt}\vec{u} + \vec{\omega}_{ib} \times \vec{u}}$$

$$\vec{r}_a = x\vec{i}_1 + l\vec{b}_2$$

$$\vec{v}_a = \frac{{}^i d}{dt}\vec{r}_a = \frac{{}^i d}{dt}(x\vec{i}_1) + \frac{{}^i d}{dt}(l\vec{b}_2) = \dot{x}\vec{i}_1 + \frac{{}^b d}{dt}l\vec{b}_2 + (-\dot{\theta}\vec{b}_3) \times l\vec{b}_2$$

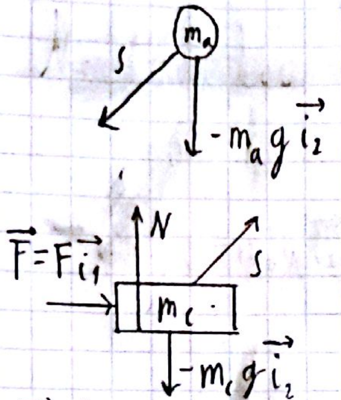
$$= \dot{x}\vec{i}_1 + l\dot{\theta}\vec{b}_1$$

$$\vec{a}_a = \ddot{x}\vec{i}_1 + l\ddot{\theta}\vec{b}_1 + (-\dot{\theta}\vec{b}_3) \times (l\dot{\theta}\vec{b}_1)$$

$$= \ddot{x}\vec{i}_1 + l\ddot{\theta}\vec{b}_1 - l\dot{\theta}^2\vec{b}_2$$

Kinetikk

Newtons lov



(1) $m_a \cdot \vec{a}_a = \vec{S} - m_a g \vec{i}_2$

(2) $m_c \vec{a}_c = \vec{F}$

$\vec{i}_1(1): \; m_a \ddot{x} + m_a l \ddot{\theta} \cos(\theta) - m_a l \dot{\theta}^2 \sin\theta = \vec{S} \cdot \vec{i}_1$

$\vec{b}_1(1): \; m_a \ddot{x} \cos(\theta) + m l \ddot{\theta} = m_a g \sin\theta$

$(2)\vec{i}_1: \; m_c \ddot{x} = F - \vec{S} \cdot \vec{i}_1$

$$(m_a + m_c)\ddot{x} + m_a l \ddot{\theta} \cos\theta - m_a l \dot{\theta}^2 \sin\theta = F$$