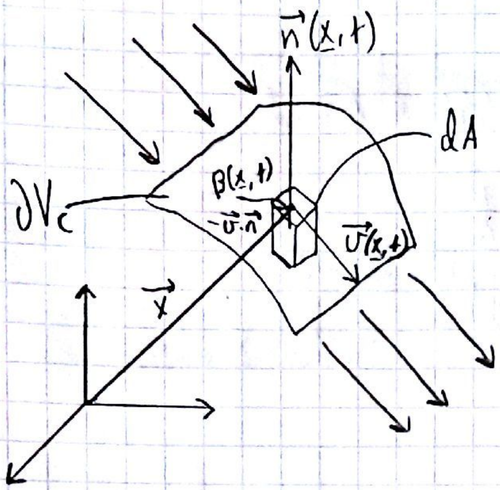
21.04.16



Innstrømming av β per tidsenhet

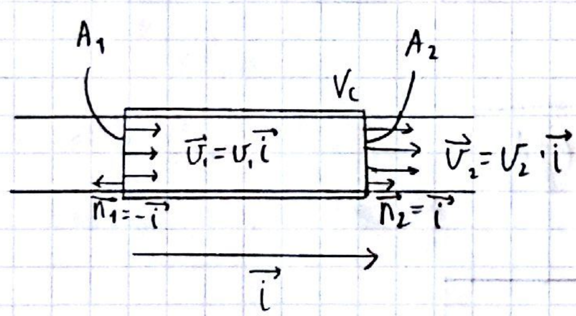
$$-\rho(\underline{x},t)\beta(\underline{x},t)\,\vec{v}(\underline{x},t)\cdot\vec{n}(\underline{x},t)\,dA$$

Gjennom hele ∂V_c

$$-\iint \rho(\underline{x},t)\beta(\underline{x},t)\vec{v}(\underline{x},t)\cdot\vec{n}(\underline{x},t)\,dA$$

$$= \iint_{\partial V_c} \rho\beta\,\vec{v}\cdot\vec{n}\,dA$$

Eks. Massestrøm i rør



$$-\iint_{\partial V_c} \rho\vec{v}\cdot\vec{n}\,dA = -\iint_{A_1} \rho_1 v_1\cdot\vec{n}_1\,dA - \iint_{A_2} \rho_2\vec{v}_2\cdot\vec{n}_2\,dA$$

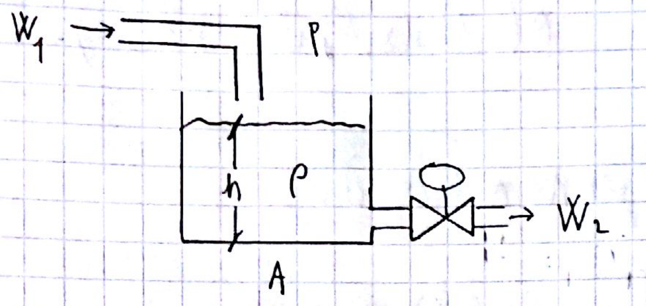
$$= -\rho_1(-v_1)A_1 - \rho_2 v_2 A_2 = \rho_1 v_1 A_1 - \rho_2 v_2 A_2 \qquad \Big|\; q_i = v_i A$$

$$= \rho_1 q_1 - \rho_2 q_2 \qquad \Big|\; w_i = \rho_i\cdot q_i$$

$$= w_1 - w_2$$

$$\boxed{\frac{d}{dt}m = w_1 - w_2}$$

<u>Eks.</u> Tank



massebalanse

$$\frac{d}{dt} m = w_1 - w_2$$

<u>kap. 4</u>

$$q_2 = C_d A_v \sqrt{\frac{2}{\rho} \Delta p}$$

$$\Delta p = p + \rho g h - p = \rho g h$$

$$m = \rho g h$$

$$\frac{d}{dt}(\rho g h) = w_1 - \rho C_d A_v \sqrt{\frac{2}{\rho} \rho g h}$$

$$\boxed{\frac{d}{dt} h = \frac{1}{\rho A} w_1 - \frac{1}{A} C_d A_v \sqrt{2 g h}}$$

Flerkomponent-system

$k = 1, \dots, n$ komponenter

Tetthet : ρ_k

dannelsesrate : r_k $\quad [r_k] = \frac{kg}{m^3 s}$

Balanselov, komponent k:

$$\frac{d}{dt} m_k = \frac{d}{dt} \iiint_{V_c} \rho_k \, dV = - \iint_{\partial V_c} \rho_k \vec{v} \cdot \vec{n} \, dA + \iiint_{V_c} r_k \, dV$$

Mest vanlig: kjemiske reaksjoner

$$\text{Eks.}: \underbrace{2H_2}_{2 \text{ molekyl}} + \overset{1 \text{ molekyl}}{O_2} \longrightarrow \underbrace{2H_2O}_{2 \text{ molekyl}}$$

mer praktisk med molekyl-balanse
heller enn masse-balanse.

\# molekyl måles i <u>mol</u>, symbol $\underline{n_k}$

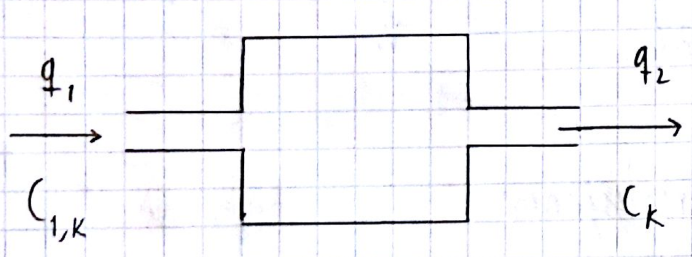
$$n_k = \frac{m_k}{M_k}, \quad M_k : \text{molvekt} \left[\frac{kg}{mol}\right]$$

Definer $c_k = \frac{n_k}{V} = \frac{m_k / M_k}{V} = \frac{\rho_k V / M_k}{V} = \frac{\rho_k}{M_k}$

$$\hat{r}_k = \frac{r_k}{M_k}$$

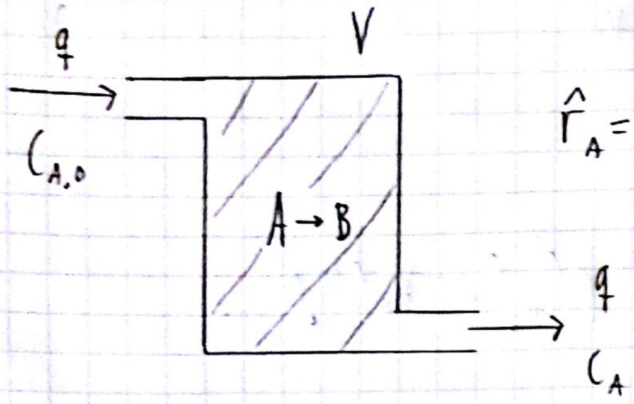
"Mol balance": $\boxed{\frac{d}{dt}\iiint_{\partial V_c} c_k \vec{v}_k \cdot \vec{n}\, dA + \iiint_{V_c} \hat{r}_k \, dV}$

Eks.



$$\frac{d}{dt} n_k = \frac{d}{dt}(c_k V) = q_1 c_{1,k} - q_2 c_k + \hat{r}_k V$$

Eks.



$\hat{r}_A = -\gamma C_A$

$$\frac{d}{dt}(C_A V) = q C_{A,0} - q C_A - \gamma C_A V$$

$$\boxed{\frac{d}{dt} C_A = -\left(\frac{q}{V} + \gamma\right) C_A + \frac{q}{V} C_{A,0}}$$

Generelt:

$$\hat{r}_n = f(C_1, C_2, \dots, C_n, T)$$

Eks.

$$\boxed{2H_2 + O_2 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} 2H_2O}$$

$k_1 = \gamma_1 C_{H_2}^2 C_{O_2}$

$k_2 = \gamma_2 C_{H_2O}^2$

$$\hat{r}_{O_2} = -\gamma_1 C_{H_2}^2 C_{O_2} + \gamma_2 C_{H_2O}^2$$

$$\hat{r}_{H_2} = 2\hat{r}_{O_2} \quad , \quad \hat{r}_{H_2O} = -2\hat{r}_{O_2}$$

→

$$\frac{d}{dt}\iiint_{V_c} c_k \, dV = -\iint_{\partial V_c} c_k \vec{v}\cdot\vec{n} \, dA + \iiint_{V_c} \hat{r}_k \, dV$$

$$\frac{d}{dt} c_k = \hat{r}_k \qquad \frac{d}{dt} c_{O_2} = -\gamma c_{H_2}^2 c_{O_2} + \gamma_2 c_{H_2O}$$

Støkiometriske koeffisienter

$$2H_2 + O_2 \rightleftharpoons 2H_2O$$

$$2 \qquad 1 \qquad 2$$

$$\Gamma = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} \qquad \underline{c} = \begin{bmatrix} c_{H_2} \\ c_{O_2} \\ c_{H_2O} \end{bmatrix}$$

$$\boxed{\frac{d}{dt}\underline{c} = \Gamma \hat{r}(\underline{c})} \longrightarrow \boxed{\hat{r}(\underline{c}) = \hat{r}_{O_2}}$$