1.2.1 Tilstandsrom-modeller

15.01.16

$$\dot{x}_1 = f_1(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_p, t)$$

$$\vdots$$

$$\dot{x}_n = f_n(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_p, t)$$

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u}, t)$$

$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, t)$$

LTI $\quad \underline{\dot{x}} = A\underline{x} + B\underline{u}$

$\quad \underline{y} = C\underline{x} + D\underline{u}$

1.2.2 Andre ordens modeller (av mekaniske systemer)

$$M(\underline{q})\underline{\ddot{q}} + f(\underline{q}, \underline{\dot{q}}) = \underline{u}$$

$$\left.\begin{aligned} \underline{x_1} &= \underline{q} \\ \underline{x_2} &= \underline{\dot{q}} \end{aligned}\right\} \quad \begin{aligned} \underline{\dot{x}_1} &= \underline{x_2} \\ \underline{\dot{x}_2} &= M^{-1}(\underline{x_1})(-f(\underline{x_1}, \underline{x_2}) + u) \end{aligned}$$

1.2.3 Lineariseringing

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u}, t)$$
$$\underline{y} = \underline{h}(\underline{x}, \underline{u}, t)$$

Vil linearisere om en løsning $(\underline{x}_0(t), \underline{u}_0(t))$ som oppfyller $\underline{\dot{x}}_0(t) = \underline{f}(\underline{x}_0(t), u_0(t), t)$

Definerer avvik Δx_1, Δu, Δy

$$\underline{x}(t) = \underline{x}_0(t) + \underline{\Delta x}(t)$$
$$\underline{u}(t) = \underline{u}_0(t) + \underline{\Delta u}(t)$$
$$\underline{y}(t) = \underline{h}(\underline{x}_0(t), u_0(t), t) + \underline{\Delta y}(t)$$

$$\underline{\dot{x}} = \underline{\dot{x}}_0 + \underline{\Delta \dot{x}}$$

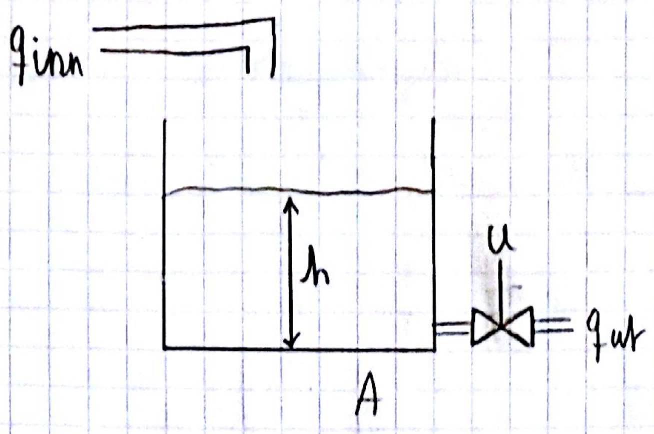
Taylor rekke utvikling om (x_0, u_0)

$$\underline{\dot{x}} = f(\underline{x}_0, \underline{u}_0, t) + \underbrace{\frac{\partial f}{\partial \underline{x}}\bigg|_{(x_0, u_0)}}_{A(t)} \cdot \underline{\Delta x} + \underbrace{\frac{\partial f}{\partial u}\bigg|_{(\underline{x_0}, \underline{u_0})}}_{B(t)} \cdot \underline{\Delta u} + h.o.t$$

$$\Rightarrow \boxed{\begin{aligned} \Delta \dot{x} &\approx A(t)\,\underline{\Delta x} + B(t)\,\underline{\Delta u} \\ \Delta y &\approx C(t)\,\underline{\Delta x} + D(t)\,\underline{\Delta y} \end{aligned}}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \ddots & \\ \vdots & & \ddots \end{bmatrix}$$

Eks. Buffertank



$q_{ut} = Cu\sqrt{h}$

Modell: $A\dot{h} = q_{inn} - q_{ut} = q_{inn} - Cu\sqrt{h}$

Lineariserer om $h = h_0$

stasjonært $q_{inn} = q_{ut} = Cu\sqrt{h} \Rightarrow u_0 = \frac{q_{inn}}{C\sqrt{h_0}}$

$$\Delta\dot{h} = -\frac{Cu_0}{2A\sqrt{h_0}} \cdot \Delta h - \frac{C\sqrt{h_0}}{A} \cdot \Delta u$$

1.3 Transferfunksjoner

For LTI-modeller

$$\underline{\dot{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t)$$

$$\underline{y}(t) = C\,\underline{x}(t) + D\,\underline{u}(t)$$

Laplace transform:

$\underline{x}(s) = \mathcal{L}\{\underline{x}(t)\}$, $\underline{u}(s) = \{\underline{u}(t)\}$, $\underline{y}(s) = \{\underline{y}(t)\}$

$$\mathcal{L}\{\underline{\dot{x}}(t)\} = s\,\mathcal{L}\{\underline{x}(t)\} - \underbrace{\underline{x}(t=0)}_{\text{antar}=0} = s\,\mathcal{L}\{\underline{x}(t)\}$$

$$s\underline{X}(s) = A\underline{x}(s) + B\underline{u}(s)$$

$$\underline{y}(s) = C\underline{x}(s) + D\underline{u}(s)$$

$$\underline{y}(s) = \underbrace{\left[(sI-A)^{-1}B + D\right]}_{H(s)}\underline{u}(s)$$

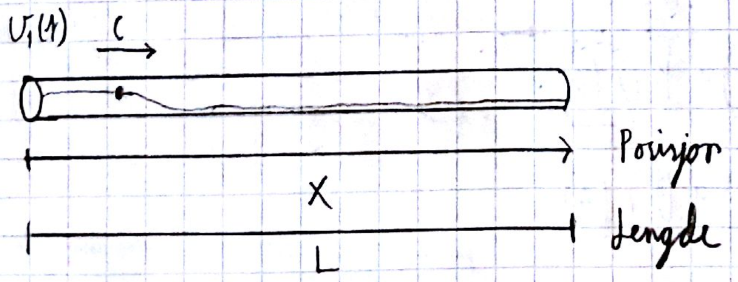
$$\frac{\underline{y}(s)}{\underline{u}(s)} = H(s)$$ $H(s)$: rasjonal funksjon

SISO: $$H(s) = K\frac{P(s)}{Q(s)} = K\frac{(s+z_1)\cdots(s+z_n)}{(s+p_1)\cdots(s+p_n)}$$

1.3.9 Partielle differensial ligninger

Eks. Første-ordens bølgeligning

$$\frac{\partial v(x,t)}{\partial t} = -c\frac{\partial v(x,t)}{\partial x}\ ; \quad v(0,t) = v_1(t)$$



$$\frac{v_2}{v_1}(s) = e^{-\frac{L}{c}s} = e^{-Ts}$$