Exercise 1 - TTK4130 Modeling and Simulation

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1 Problem 1

$$Ni = \phi(\mathcal{R}_a + \mathcal{R}_c + \mathcal{R}_b + \mathcal{R}_r). \tag{1}$$

$$\mathcal{R}_a = \frac{z}{A\mu_0}, \mathcal{R}_r = const. \tag{2}$$

$$\mathcal{R}_c, \mathcal{R}_b << \mathcal{R}_r, \mathcal{R}_a. \tag{3}$$

1.1 a

$$\mathcal{R}_r = \frac{z_0}{A\mu_0}, z_0 = const. \tag{4}$$

Using Equation 1 together with the relations from equations 2 and 4 we get

$$Ni = \phi(\frac{z}{A\mu_0} + \mathcal{R}_c + \mathcal{R}_b + \frac{z_0}{A\mu_0}).$$

Since $\mathcal{R}_c$ and $\mathcal{R}_b$ are neligible (Equation 3), the total magnetomotive force on the ball is

$$\underline{\underline{Ni = \frac{\phi}{A\mu_0}(z + z_0).}}$$

1.2 b

$$L(z) = \frac{N\phi}{i} = \frac{N^2 A\mu_0}{z + z_0}. \tag{5}$$

$$F = \frac{i^2}{2}\frac{\partial L(z)}{\partial z}. \tag{6}$$

Assume positive direction downwards and gravitational acceleration g.

$$\begin{aligned} ma &= \Sigma F \\ m\ddot{z} &= mg + F \\ m\ddot{z} &= mg + \frac{i^2}{2}N^2 A\mu_0(z + z_0)^{-2}. \end{aligned}$$

$$\underline{\underline{\ddot{z} = g - \frac{1}{2m}i^2 N^2 A\mu_0(z + z_0)^{-2}}} \tag{7}$$

1.3 c

Linearizing about z_d, $\dot{z}_d = 0$, $\ddot{z}_d = 0$.

$$0 = g - \frac{1}{2m} i_d^2 N^2 A\mu_0 (z_d + z_0)^{-2}$$

$$i_d = \frac{1}{N}\sqrt{\frac{2mg}{A\mu_0}}(z_d + z_0)$$

Equation 7 we now call f_1. We define $z = z_d + \Delta z, i = i_d + \Delta i$ and thereby $\dot{z} = \Delta\dot{z}$. A Linearization of Equation 7 around the point z_d is then

$$\Delta\ddot{z} = \frac{\partial f_1}{\partial \dot{z}}\Bigg|_{\substack{z = z_d \\ \dot{z} = \dot{z}_d \\ i = i_d}} \Delta\dot{z} + \frac{\partial f_1}{\partial z}\Bigg|_{\substack{z = z_d \\ \dot{z} = \dot{z}_d \\ i = i_d}} \Delta z + \frac{\partial f_1}{\partial i}\Bigg|_{\substack{z = z_d \\ \dot{z} = \dot{z}_d \\ i = i_d}} \Delta i$$

$$\Delta\ddot{z} = \frac{1}{m} i_d^2 N^2 A\mu_0 (z_d + z_0)^{-3}\Delta z - \frac{1}{2m} i_d N^2 A\mu_0 (z_d + z_0)^{-2}\Delta i$$

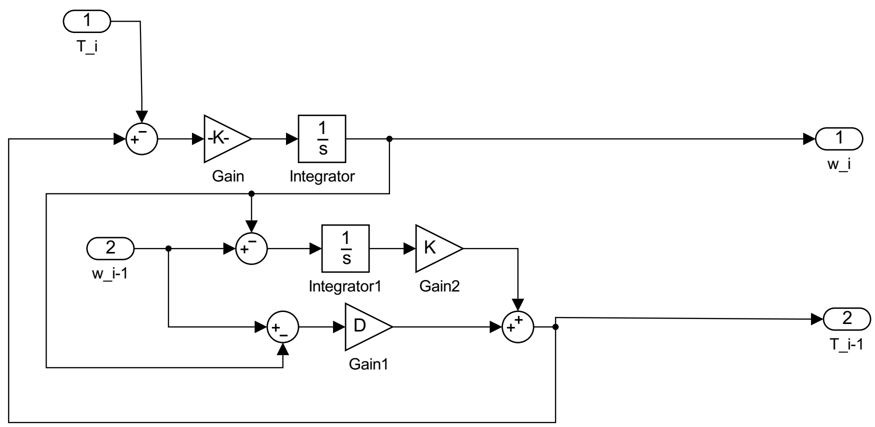
$$\underline{\underline{\Delta\ddot{z} = \frac{2g}{z_d + z_0}\Delta z - \frac{N}{z_d + z_0}\sqrt{\frac{2gA\mu_0}{m}}\Delta i}}$$

2 Problem 2

2.1 b

Natural signal-flow inputs are T_i and ω_{i-1}. The outputs should be T_{i-1} and ω_i. For natural energy-flow, inputs should be T_{i-1} and $\omega i - 1$, with T_i and ω_i as outputs.

2.2 c

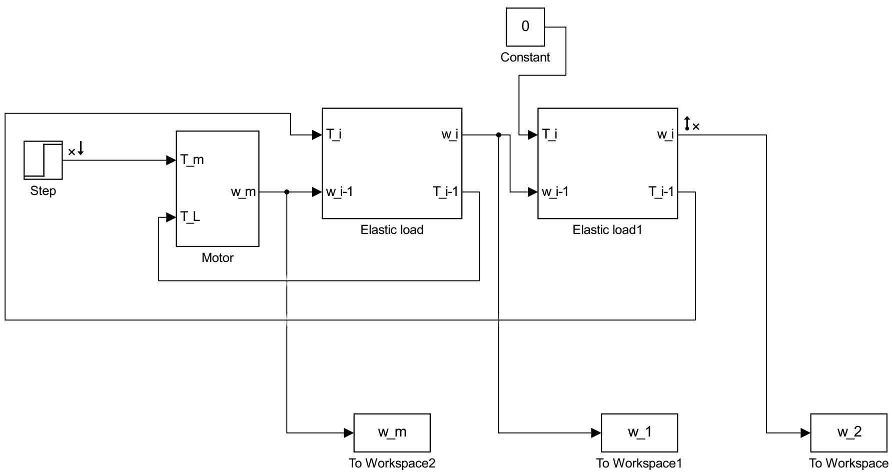


Figur 1: The elastic load implemented in Simulink

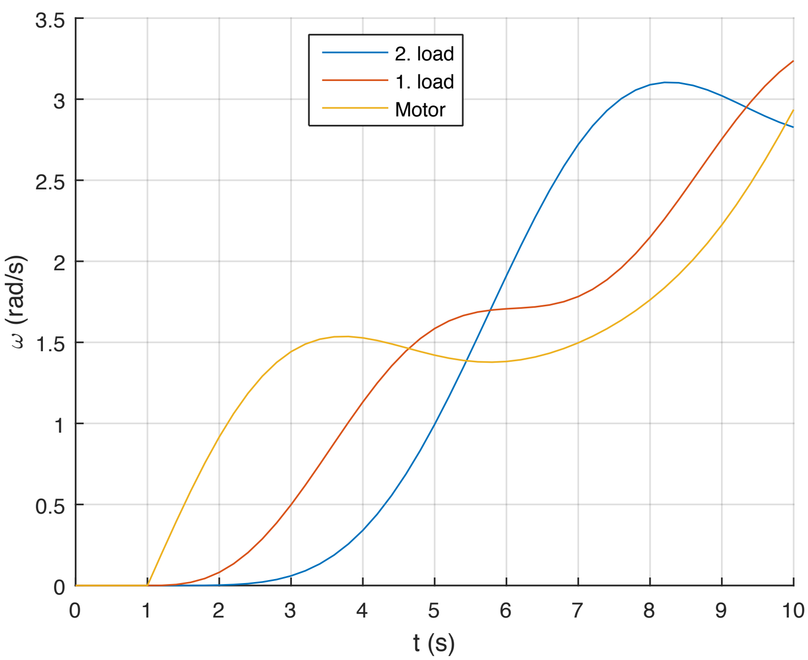
2.3 d

As seen in Figure 3, the angular velocity of the second load is greatly amplified. It is also delayed by a couple of seconds, which is natural to except of this system. Otherwise the rotational velocity of the last load behaves quite similary to that of the motor and the fist load.

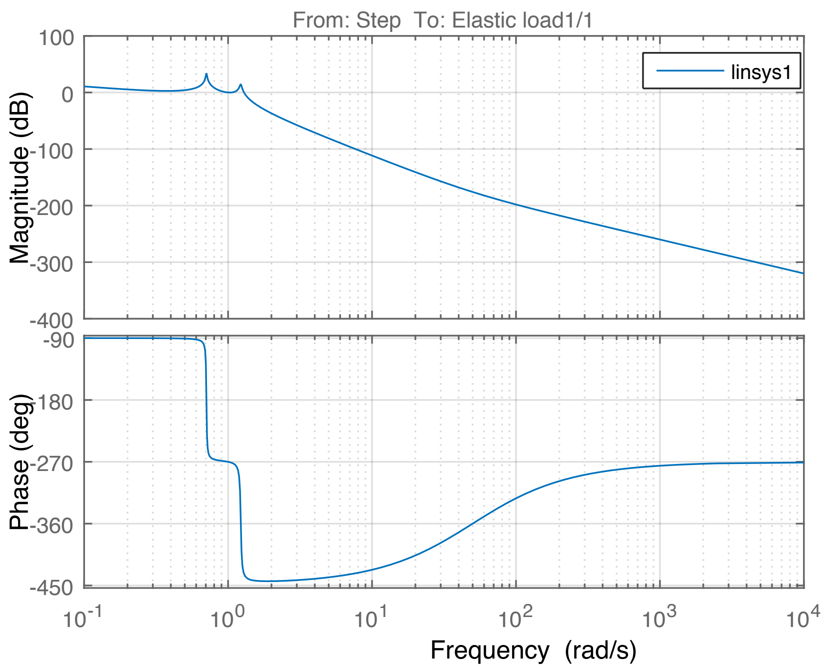
2.4 e



Figur 2: Two elastc loads in series with the motor.



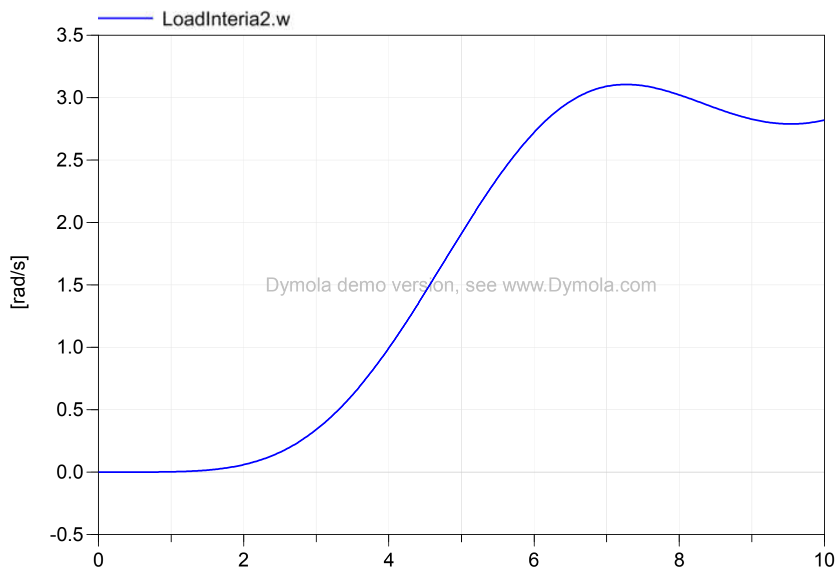
Figur 3: Plot of the rotational velocity of the motor and the two loads.



Figur 4: Bode plot of the transfer function from the step input to the rotational velocity of the second load.

3 Problem 3

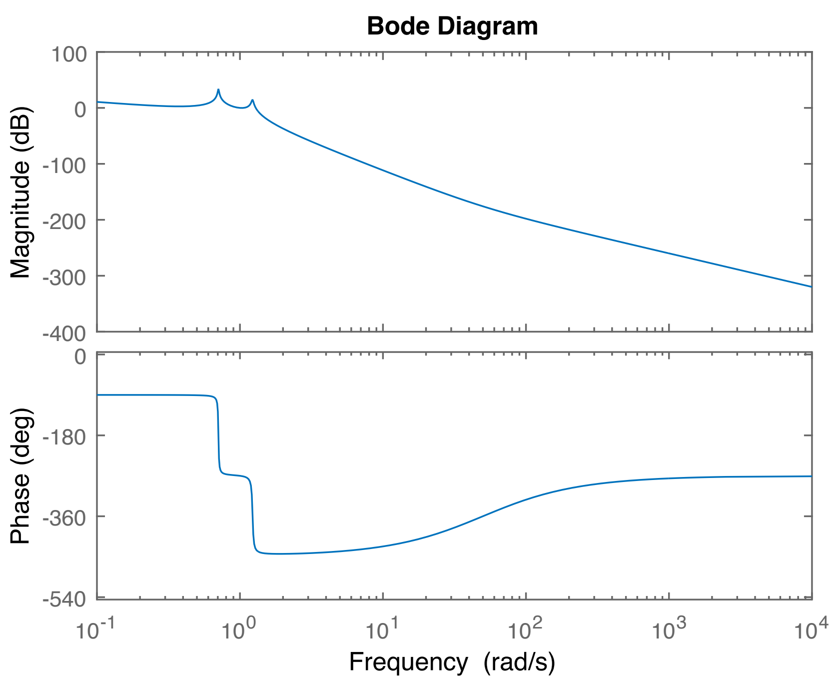
3.1 a



Figur 5: The rotational velocity of the second load when simulated using Dymola.

3.2 b

3.3 c



Figur 6: Bode plot of the system when linearized with Dymola.