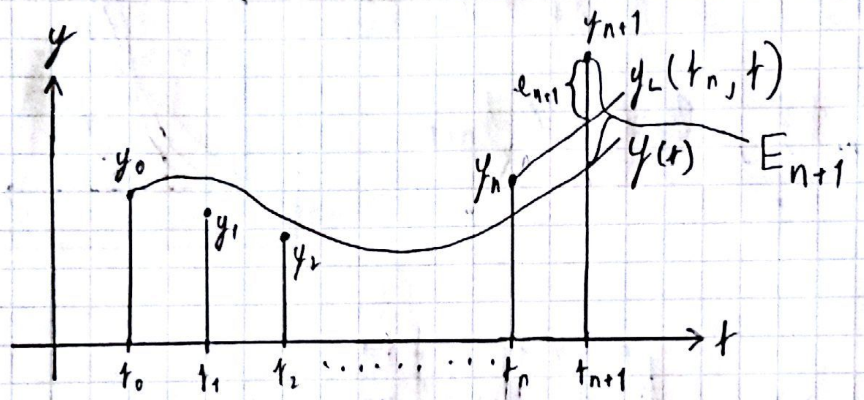
28.01.16

14 Simulering

$$\underbrace{\underbrace{\dot{y} = f(y,t)}_{\text{ODE}}, \quad y(t_0) = y_0}_{\text{IVP}}$$

Simulering:

Hvordan beregne $y(t)$?



Ettskrittsmetoder:

$$y_{n+1} = y_n + h\varphi(y_n, t_n)$$

Beregningsfeil (14.22)

Lokal løsning $y_L(t_n;t)$: $\dot{y}_L(t_n;t) = f(y_L(t_n;t))$, $y_L(t_n;t_n) = y_n$

Lokal feil: $e_{n+1} = y_{n+1} - y_L(t_n;t_{n+1})$

Global feil: $E_{n+1} = y_{n+1} - y(t_{n+1})$

Orden: En metode er orden p om p er minste heltall s.a.

$$\ell_{n+1} = O(h^{p+1})$$

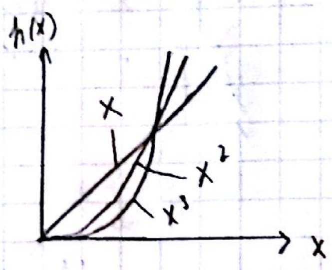
"Def"

$$h(x) = O(x^m)$$

$$\Updownarrow$$

$\exists C$ s.a

$$|h(x)| \leq C|x|^m$$



Taylor-rekkeutv. av $y_L(t_n; t_{n+1})$

$$y_L(t_n;t_{n+1}) = y_n + h\dot{y}_n + \frac{h^2}{2}\ddot{y}_n + \dots \frac{h^p}{p!} y_n^{(p)} + \frac{h^{p+1}}{(p+1)!} y_L^{(p+1)}(t_n, \tau), \quad t_n \leq \tau \leq t_{n+1}$$

$$= y_n + h f(y_n, t) + \frac{h^2}{2}\frac{d}{dt} f(y_n, t) + \dots + \frac{h^p}{p!}\frac{d^{p-1}}{dt^{p-1}} f(y_n, t_n) + \frac{h^{p+1}}{(p+1)!}\frac{d^p}{dt^p} f(y_L(t_n;\tau)), \quad t_n \leq \tau \leq t_{n+1}$$

Om vi kan vise

$$y_{n+1} = y_n + h f(y_n, t_n) + \dots \frac{h^p}{p!}\frac{d^{p-1}}{dt^{p-1}} f(y_n, t_n) + O(h^{p+1})$$

Så er

$$\ell_{n+1} = y_{n+1} - y_L(t_n, t_{n+1}) = O(h^{p+1}) - \frac{h^{p+1}}{(p+1)!}\frac{d^p f(y_L(t_n;\tau))}{dt^p}$$

$$= O(h^{p+1})$$

Dvs metoder er orden p (og: global feil $E_{n+1} = O(h^p)$)

Linear test-system

$$\dot{y} = \lambda y \quad \text{(skalart)}$$

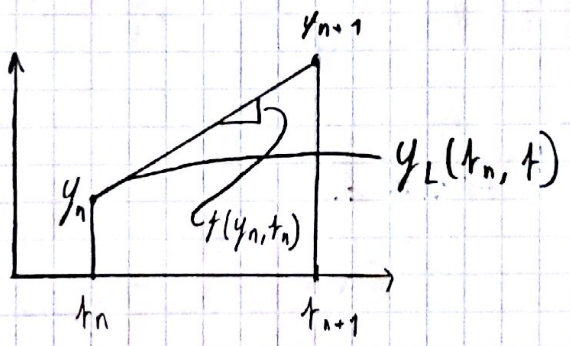
En ettskrittmetode gir numerisk løsning for (*)

$$y_{n+1} = R(h\lambda)\, y_n$$

Den numeriske løsningen er stabil hvis

$$|y_{n+1}| \leq |y_n| \iff |R(h\lambda)| \leq 1$$

14.3 Eulers metode



$$y_{n+1} = y_n + h f(y_n, t_n)$$

Orden: $e_{n+1} = O(h^2)$, $p = 1$

Stabilitet $\dot{y} = \lambda y$

$$y_{n+1} = y_n + h\lambda y_n = \underbrace{(1 + h\lambda)}_{R(h\lambda)} y_n$$

Stabilitet $|1 + h\lambda| \leq 1$