Euler parametre

31.03.16

Vinkel-akse: $\vec{k}, \theta$

Euler-parametre: $\eta = \cos\left(\frac{\theta}{2}\right)$, $\vec{\epsilon} = \vec{k}\sin\left(\frac{\theta}{2}\right)$

[kvaternion: $\underline{p} = \begin{pmatrix}\eta \\ \underline{\epsilon}\end{pmatrix}$]

Egenskap: $\eta^2 + \vec{\epsilon}\,\vec{\epsilon} = \cos^2\left(\frac{\theta}{2}\right) + \vec{k}\,\vec{k}\sin^2\left(\frac{\theta}{2}\right) = 1$

Rotasjonsmatrise $R_e(\eta, \underline{\epsilon})$:

$$\boxed{\begin{aligned} \sin\theta &= 2\sin\left(\tfrac{\theta}{2}\right)\cos\left(\tfrac{\theta}{2}\right) \\ \cos\theta &= \cos^2\left(\tfrac{\theta}{2}\right) - \sin^2\left(\tfrac{\theta}{2}\right) \\ &= 1 - 2\sin^2\left(\tfrac{\theta}{2}\right) \end{aligned}}$$

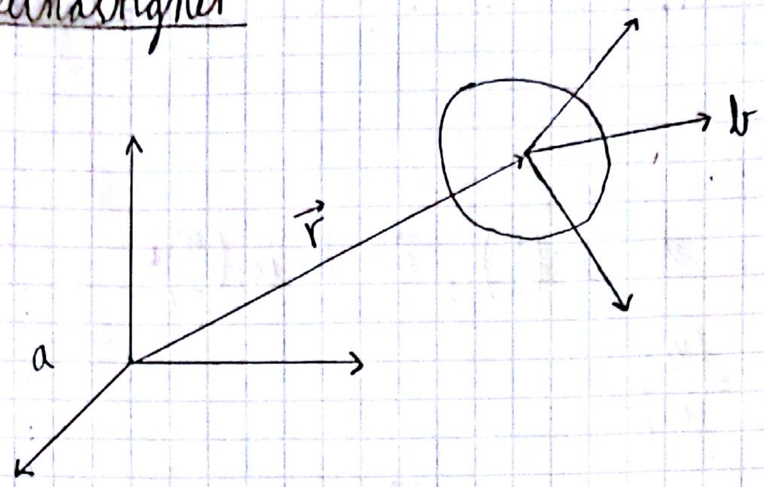
$$\underline{k}\sin\theta = 2\underline{k}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$\underline{k}^\times \sin(\theta) = 2\eta\,\underline{\epsilon}^\times$$

$$(1-\cos(\theta))\,\underline{k}^\times\underline{k}^\times = 2\sin^2\left(\frac{\theta}{2}\right)\underline{k}^\times\underline{k}^\times = 2\underline{\epsilon}^\times\underline{\epsilon}^\times$$

$$\boxed{R_e(\eta, \underline{\epsilon}) = I + 2\eta\,\underline{\epsilon}^\times + 2\underline{\epsilon}^\times\underline{\epsilon}^\times} \Rightarrow \boxed{\begin{aligned} R_e(-\eta, -\underline{\epsilon}) &= R_e(\eta, \underline{\epsilon}) \\ R_e(\eta, \underline{\epsilon})^T &= R_e(\eta, -\underline{\epsilon}) \end{aligned}}$$

Vinkelhastighet



Posisjon $\underline{r} \longrightarrow \underline{\dot{r}} = \underline{v}$

Orientering $R^a_b \longrightarrow \dot{R}^a_b = ?$

$$\begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix} \longrightarrow \begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = ?$$

$$\begin{pmatrix} \eta \\ \underline{\epsilon} \end{pmatrix} \longrightarrow \begin{pmatrix} \dot{\eta} \\ \underline{\dot{\epsilon}} \end{pmatrix} = ?$$

R^a_b orthogonal $\iff R^a_b (R^a_b)^T = I$

$$\frac{d}{dt}\left[R^a_b (R^a_b)^T\right] = \dot{R}^a_b (R^a_b)^T + R^a_b (\dot{R}^a_b)^T = 0$$

$S = \dot{R}^a_b (R^a_b)^T : \quad S + S^T = 0 \quad$ evt $\quad S = -S^T$

$$\underline{w}^a_{ab} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \qquad S = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} = (\underline{w}^a_{ab})^\times$$

$(\underline{w}^a_{ab})^\times = \dot{R}^a_b (R^a_b)^T$

$\boxed{\vec{w}_{ab} \text{ kalles vinkelhastigheten av } b \text{ relativt til } a}$

$$\boxed{\begin{aligned}
&\text{Før påske: koordinat transformasjon av matrise}\\
&(\underline{\omega}^b_{ab})^\times = R^b_a (\underline{\omega}^a_{ab})^\times R^a_b\\
&\dot{R}^a_b = R^a_b (\underline{\omega}^a_{ab})^\times R^b_a R^a_b\\
&\boxed{\dot{R}^a_b = R^a_b (\underline{\omega}^b_{ab})^\times}
\end{aligned}}$$

Vinkelhastighet for enkle rotasjoner

$$R_x(\phi),\ R_y(\theta),\ R_z(\psi)$$

$$\underline{[\omega_x(\dot{\phi})] = \dot{R}_x(\phi) R_x^T(\phi)}, \quad R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -s\phi & -c\phi \\ 0 & c\phi & -s\phi \end{bmatrix} \dot{\phi} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \dot{\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi} \\ 0 & \dot{\phi} & 0 \end{bmatrix},$$

$$\underline{\omega}_x(\dot{\phi}) = \begin{pmatrix} \phi \\ 0 \\ 0 \end{pmatrix} \quad \text{På samme vis: } \underline{\omega}_y(\dot{\theta}) = \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix}, \quad \underline{\omega}_z = \begin{pmatrix} 0 \\ 0 \\ \psi \end{pmatrix}$$

Merk: $\cancel{\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}}$ Nei!

For vinkel-akse:

$$R_b^a = R_{\underline{k},\theta} = I + \underline{k}^\times \sin\theta + \underline{k}^\times \underline{k}^\times (1-\cos\theta)$$

Anta $\underline{k}$ konstant

$$(\underline{\omega}_{ab}^a)^\times = \dot{R}_{ab}^a (R_b^a)^T = (\underline{k}^\times \cos(\theta) + \underline{k}^\times \underline{k}^\times \sin\theta)\,\dot{\theta}(I - \underline{k}^\times \sin(\theta) + \underline{k}^\times \underline{k}^\times (1-\cos(\theta)))$$

$$= \ldots \quad (\text{bruk } \underline{k}^\times \underline{k}^\times \underline{k}^\times = \underline{k}^\times(\underline{k}\,\underline{k}^T - \underline{k}^T \underline{k} I) = -\underline{k}^\times)$$

$$= \dot{\theta}\, \underline{k}^\times \quad \Rightarrow \quad \boxed{\underline{\omega}_{ab}^a = \dot{\theta}\,\underline{k}} \qquad \boxed{\omega_{ab} = \dot{\theta}\,\vec{k}}$$

Sammensatte rotasjoner

$$R_d^a = R_b^a R_c^b R_d^c$$

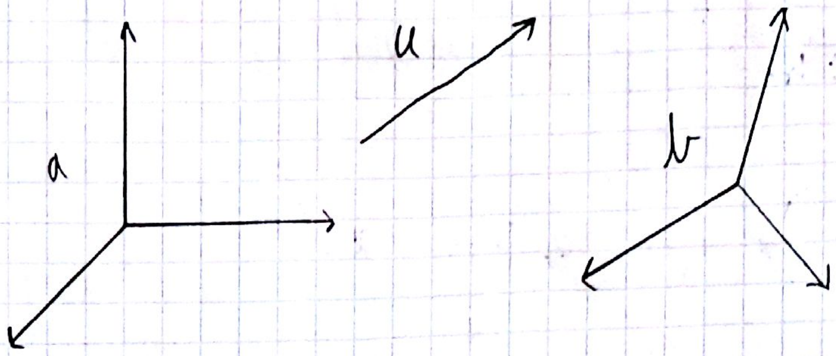
$$(\underline{\omega}_{ad}^a)^\times = \dot{R}_d^a (R_d^a)^T$$

$$= (\dot{R}_b^a R_c^b R_d^c + R_b^a \dot{R}_c^b R_d^c + R_b^a R_c^b \dot{R}_d^c)(R_d^c)^T (R_c^b)^T (R_b^a)^T$$

$$= \underbrace{\dot{R}_b^a (R_b^a)^T}_{(\underline{\omega}_{ab}^a)^\times} + R_b^a \underbrace{\dot{R}_c^b (R_c^b)^T}_{(\underline{\omega}_{bc}^b)^\times} (R_b^a)^T + R_c^a \underbrace{\dot{R}_d^c (R_d^c)^T}_{(\underline{\omega}_{cd}^c)^\times} (R_c^a)^T$$

$$= (\underline{\omega}_{ab}^a)^\times + (\underline{\omega}_{bc}^a)^\times + (\underline{\omega}_{cd}^a)^\times$$

dvs. $\boxed{\underline{\omega}_{ad}^a = \underline{\omega}_{ab}^a + \underline{\omega}_{bc}^a + \underline{\omega}_{cd}^a} \Rightarrow \boxed{\vec{\omega}_{ad} = \vec{\omega}_{ab} + \vec{\omega}_{bc} + \vec{\omega}_{cd}}$



$$\underline{u}^a = \begin{pmatrix} u_1^a \\ u_2^a \\ u_3^a \end{pmatrix} \qquad \underline{u}^b = \begin{pmatrix} u_1^b \\ u_2^b \\ u_3^b \end{pmatrix}$$

$$\underline{u}^a = R^a_b \underline{u}^b \qquad \cancel{\dot{u}^a = R^a_b \dot{u}^b}$$

$$\dot{u}^a = \dot{R}^a_b \underline{u}^b + R^a_b \dot{\underline{u}}^b = R^a_b \left[(\omega^b_{ab})^\times \underline{u}^b + \dot{\underline{u}}^b \right]$$

$$\boxed{\dot{u}^a = R^a_b \left[\underline{u}^b + (\underline{\omega}^b_{ab})^\times \underline{u}^b \right]}$$

Derivasjon av koordinatfri vektor

$$\frac{d}{dt}\vec{u} = ? \quad \text{udefinert}$$

$$\vec{u} - u_1^a \vec{a}_1 + u_2^a \vec{a}_2 + u_3^a \vec{a}_3$$

$$\boxed{{}^a\frac{d}{dt}\vec{u} = \dot{u}_1^a \vec{a}_1 + \dot{u}_2^a \vec{a}_2 + u_3^a \vec{a}_3}$$

$$\boxed{{}^b\frac{d}{dt}\vec{u} = \dot{u}_1^b \vec{b}_1 + \dot{u}_2^b \vec{b}_2 + u_3^b \vec{b}_3}$$

$$\boxed{{}^a\frac{d}{dt}\vec{u} = {}^b\frac{d}{dt}\vec{u} + \vec{\omega}_{ab} \times \vec{u}}$$