Multistep methods

02.06.16

Ex. Adams-Bashforth $\qquad \dot{y} = f(y,t)$

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} \underbrace{f(y(\tau),\tau)}_{\text{Adams: tilnerme med polynom } P_m(\tau)} d\tau$$

$P_m(\tau)$ er polynom som interpolerer

$f(y(\tau),\tau)$ på $t_{n+1-m}, \dots, t_n$

Ex. $m=2$: $\qquad \boxed{f_n = f(y_n, t_n)}$

$$P_2(\tau) = f_n + (\tau - t_n)\frac{f_n - f_{n-1}}{h}$$

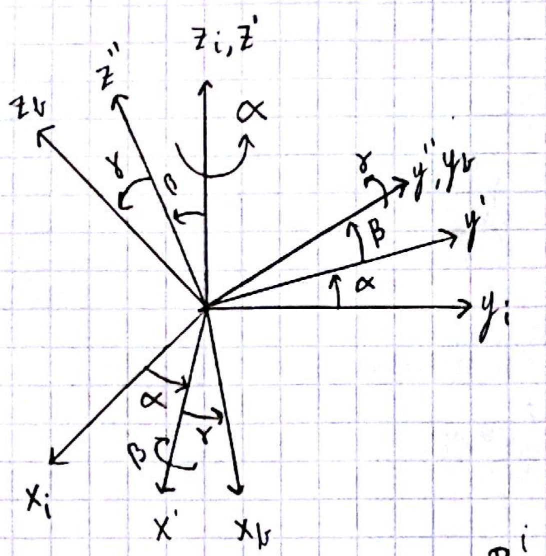
Interpolerer fordi $P_2(t_n) = f_n$

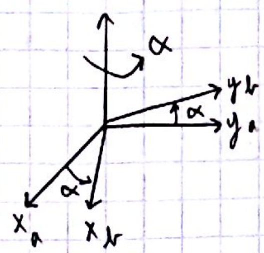
$P_2(t_{n-1}) = f_{n-1}$

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} P_2(\tau)d\tau = y_n + \left[f_n\tau + \frac{1}{2}(\tau - t_n)^2 \frac{f_n - f_{n-1}}{h}\right]_{t_n}^{t_{n+1}}$$

$$= y_n + f_n h + \frac{1}{2}h(f_n - f_{n-1}) = y_n + \frac{3}{2}hf_n - \frac{h}{2}f_{n-1}$$

Exercise 10 2b





$$R^a_b = R_z(\alpha)$$

$$\underline{v}^a = R^a_b \underline{v}^b$$

$$R^i_b = R_z(\alpha) R_x(\beta) R_y(\gamma)$$

$$R^b_i = R_y(-\gamma) R_x(-\beta) R_z(-\alpha)$$

$$\vec{w}_{ib} = \vec{w}_z(\dot\alpha) + \vec{w}_x(\dot\beta) + \vec{w}_y(\dot\gamma)$$

$$\underline{w}^i_{ib} = \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix} + R_z(\alpha) \begin{pmatrix} \dot\beta \\ 0 \\ 0 \end{pmatrix} + R_z(\alpha) R_x(\beta) \begin{pmatrix} 0 \\ \dot\gamma \\ 0 \end{pmatrix}$$

$$\underline{w}^b_{ib} = R^b_i \underline{w}^i_{ib} = R_y(-\gamma) R_x(-\beta) R_z(-\alpha) \underline{w}^i_{ib}$$

$$= R_y(-\gamma) \begin{pmatrix} 0 \\ \dot\gamma \\ 0 \end{pmatrix} + R_y(-\gamma) R_x(-\beta) \begin{pmatrix} \dot\beta \\ 0 \\ 0 \end{pmatrix} + R_y(-\gamma) R_x(-\beta) R_z(-\alpha) \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix}$$

Rotasjon rundt egen akse
(ingen påvirkning, kan fjernes)

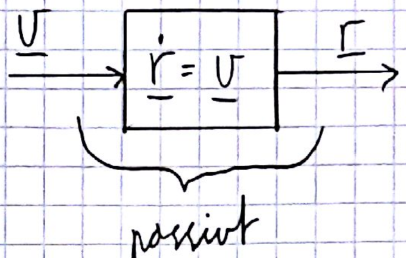
$$\begin{pmatrix} \cos(-\gamma) & 0 & \sin(-\gamma) \\ 0 & 1 & 0 \\ -\sin(-\gamma) & 0 & \cos(-\gamma) \end{pmatrix} \begin{pmatrix} 0 \\ \dot\gamma \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \dot\gamma \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \dot\gamma \\ 0 \end{pmatrix} + R_y(-\gamma) \begin{pmatrix} \dot\beta \\ 0 \\ 0 \end{pmatrix} + R_y(-\gamma) R_x(-\beta) \begin{pmatrix} 0 \\ 0 \\ \dot\alpha \end{pmatrix}$$

$\dot{\underline{r}} = \underline{v}$

$V(r) = \frac{1}{2}\underline{r}^T\underline{r}$

$\dot{V} = \underline{r}^T\underline{v}$

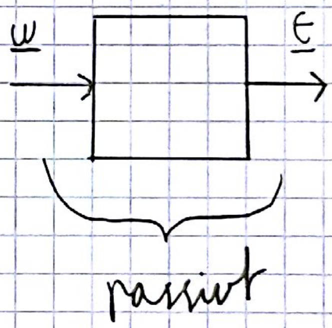


$\dot{\eta} = -\frac{1}{2}\epsilon^T\omega$

$\dot{\epsilon} = \frac{1}{2}(\eta\underline{I} - \epsilon^{\times})\underline{\omega}$

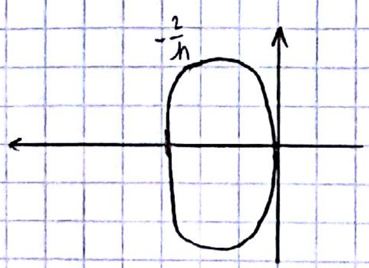
$V = 2(1-\eta) \geq 0$ $\qquad \eta = \cos\left(\frac{\theta}{2}\right)$

$\dot{V} = -2\dot{\eta} = \underline{\epsilon}^T\underline{\omega}$



Exam 2014 1c

$$A = \begin{pmatrix} 2y_1 - 2 & e^{y_2} \\ 0 & 100y_2 - 100 \end{pmatrix}\Bigg|_{(y_1,y_2)=(0,0)} = \begin{pmatrix} -2 & 1 \\ 0 & -100 \end{pmatrix}$$



$-\frac{2}{h} < -100$

$h < \frac{1}{50}$