Underdetermined linear systems a short story with some code

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Preface

This document is still in preparation. Please feel free to contact me with any suggestions, corrections or comments.

Keywords

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Backslash Pseudoinverse 500 150 400 300 -200 50 100 Least-squares Least-squares non-negative 500 150 400 -100 300 200 -0.05

Figure 1: Histograms of four solutions to an undetermined linear system.

3 Possible applications

References

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- [2] Gilbert Strang, Introduction to Linear Algebra, Fifth Edition, 2016

1 Introduction

For an undertermined linear system (where matrix \boldsymbol{A} is size $(n \times m)$ and n < m):

$$\mathbf{A}x = b \tag{1}$$

an infinite number of solutions exists. MATLAB implements various methods of computing possible solutions for x. In this paper we investigate the differences in the available solutions by analyzing their histograms. Four methods were selected: backslash \ operator, computing a pseudo-inverse pinv(), and two least-squares algorithms lsqnonneg() and lsqr().

2 Matlab example

We focus on an example where matrix \boldsymbol{A} is size (100×500) and a vector \boldsymbol{b} is size (100×1) ; both are populated by normally distributed random numbers.