

Underdetermined linear systems a short story with some code

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Preface

This document is still in preparation. Please feel free to contact me with any suggestions, corrections or comments.

Keywords

underdetermined linear systems, linear algebra, matrix decomposition, matrix approximation

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1 Introduction

For an underdetermined linear system (where matrix A is size $(n \times m)$ and $n < m$):

$$Ax = b \quad (1)$$

an infinite number of solutions exists. MATLAB implements various methods of computing possible solutions for x . In this paper we investigate the differences in the available solutions by analyzing their histograms. Four methods were selected: backslash `\` operator, computing a pseudo-inverse `pinv()`, and two least-squares algorithms `lsqnonneg()` and `lsqr()`.

2 Matlab example

We focus on an example where matrix A is size (100×500) and a vector b is size (100×1) ; both are populated by normally distributed random numbers.

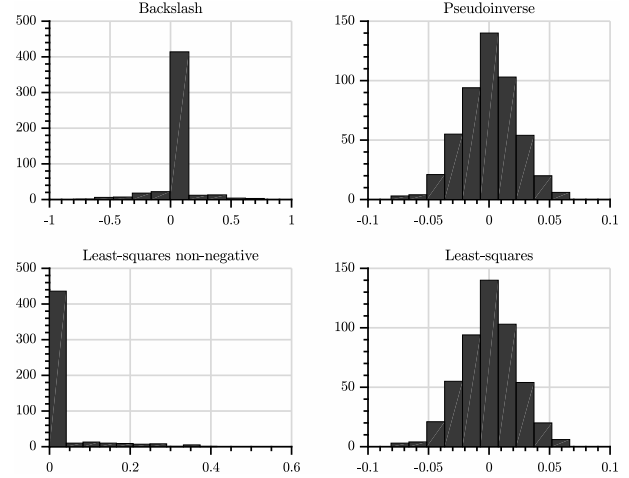


Figure 1: Histograms of four solutions to an undetermined linear system.

3 Possible applications

3.1 Principal Component Analysis

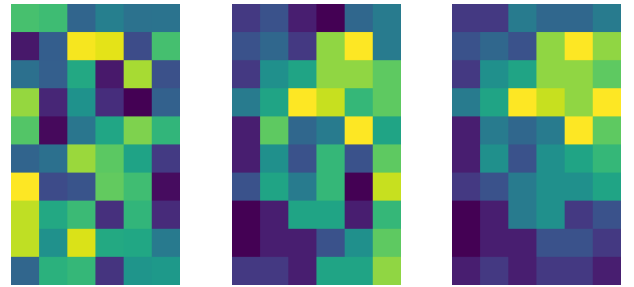


Figure 2: Random, semi-structured and structured raw data sets.

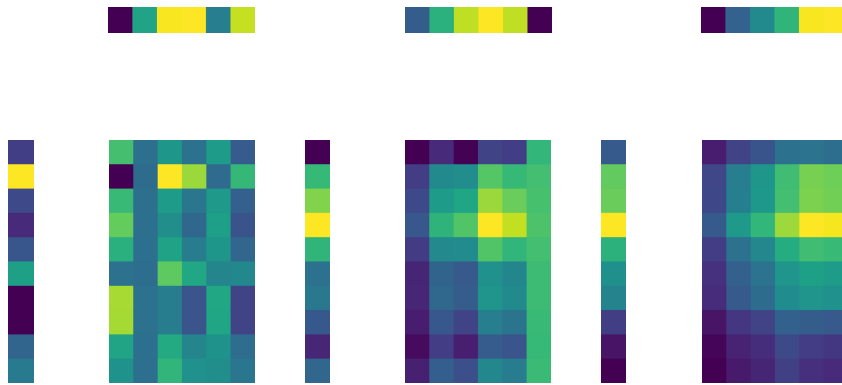


Figure 3: Random, semi-structured and structured raw data sets. PCA reconstruction with 1st Principal Component.

References

- [1] Nathan Kutz, *Data Driven Discovery of Dynamical Systems and PDEs*, an online lecture
- [2] Gilbert Strang, *Introduction to Linear Algebra*, Fifth Edition, 2016