



Figure 1: **A.** Schematic illustrating ENSD. Two datasets were generated using a latent factor model with 2 shared latent factors which are correlated. The shared subspace is 2 dimensional but the correlations reduce the *effective* shared dimensionality to ≈ 1.6 . **B.** ...

	shared eigenvectors		partially shared eigenvectors			
	flat λ	decay λ	flat λ		decay λ	
			largest 5	smallest 5	largest 5	smallest 5
ENSD	10	3.4	5	5	3.1	0.1
CCA	10	10	5	5	5	5

Table 1: *Strict vs effective dimensionality: comparing ENSD to CCA on synthetic data.* We report average estimated dimensionality over $n = 100$ synthetic datasets. We consider two scenarios involving paired datasets X and Y that each have non zero variance in 10 dimensions (X and Y are paired observations): i) shared eigenvectors, in which all eigenvectors in X and Y are shared and ii) partially shared, in which 5 eigenvectors are shared (X and Y also have 5 private, mutually orthogonal dimensions). For each of i) and ii), we consider the case where eigenvalue spectra are a) flat and b) decay from a maximum. Finally for the case of partially shared dimensions, ii), we consider: A) the first five eigenvectors overlap (ordered by variance explained) and B) the last 5 eigenvectors overlap. CCA correctly identifies the *strict* embedding dimensionality in each case. ENSD reports the *effective* dimensionality. When all eigenvectors are shared and eigenvalue spectra are flat (i and ii, a), ENSD and CCA agree on the dimensionality, however, when eigenspectra decay, the ENSD reflects the unequal distribution of variance along the shared dimensions, resulting in a lower effective dimensionality estimate.