MTE 203 - Advanced Calculus

Project 2

Multiple Integration

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Section 1: Summary of Analysis

**1a)** To plot the body and neck, the fsurf function was used. The constants were declared at the top of the program, then the equation of a sphere was plotted:

Next, the equation of an elliptic cone was rearranged to plot the neck on the same plot:

The cone needed to be offset in order for it to be visible above the plot. From the image below, the cone was plotted at . To cut off the top of the cone, the zlim function was used. The upper cut off point was at . The plots of the two cases can be seen below:

INSERT GRAPHS

Following is the Matlab code for this section:

syms x y

axis equal

z=sqrt(R^2-x^2-y^2);

fsurf(z)

hold on

fsurf(-z);

hold on

z=sqrt(x^2+y^2)\*tan(PHI\*pi/180);

fsurf(z+R-D-(sqrt(R^2-(R-D)^2)\*tan(PHI\*pi/180)));%-C1

UL=H\*tan(PHI\*pi/180);

zlim([-R H\*tan(PHI\*pi/180)+R-D-(sqrt(R^2-(R-D)^2)\*tan(PHI\*pi/180))]);

hold on;

**1b)** To find the volume of the body, the volume of the solid sphere was found, then the negative space was subtracted. This was calculated in just the positive octant, then multiplied by 8 to give the volume of the full sphere.

Where R is the radius of the body and T is the thickness of the hollow sphere. To find the volume of the head was calculated similarly to the body. Since it is a semisphere, it was multiplied by 4 instead of 8.

Where H is the height of the head. To find the volume of the neck, a truncated cylinder is used. We must first find the height of the upper bound. This is given as . Next, using similar triangles, . Thus, we get the following integral:

To calculate the volume of the socket, the volume was found of the upper cap of the body. Thus, the radius was isolated based on this location from the top of the sphere, giving. The equation of the integral is:

Finally, the total volume was found by adding the volumes of the body, head and neck and subtracting the volume of the socket. The Matlab code for this section is:

syms r theta z real

%solving for body volume

Pi=sym('pi');

I1=int(r,z,0,sqrt(R^2-r^2));%z integral outer

I2=int(I1,r,0,R);%r integral outer

I3=int(I2,theta,0,Pi/2);%theta integral outer

I4=int(r,z,0,sqrt((R-T)^2-r^2));%z integral inner

I5=int(I4,r,0,R-T);%r integral inner

I6=int(I5,theta,0,Pi/2);%theta integral iner

IBod=8\*(I3-I6)

volBody=double(IBod)

%solving for head volume

I7=int(r,z,0,sqrt(H^2-r^2));%z integral

I8=int(I7,r,0,H);%r integral ne

I9=int(I8,theta,0,Pi/2);%theta integral

IHead=4\*I9

volHead=double(IHead)

%solving for neck volume

I10=int(r,r,0,z/tan(PHI\*pi/180));

I11=int(I10,z,sqrt(R^2-(R-D)^2)\*tan(PHI\*pi/180),H\*tan(PHI\*pi/180));

I12=int(I11,theta,0,2\*Pi)

volNeck=double(I12)

%solving for socket volume

I13=int(r,z,R-D,sqrt(R^2-r^2));

I14=int(I13,r,0,sqrt(R^2-(R-D)^2));

I15=int(I14,theta,0,2\*Pi)

volSock=double(I15)

%solving total volume

Vtotal=volBody+volHead+volNeck-volSock

**2a)** Since this is a composite body, the center of mass of BB-8 can be calculated as follows:

Thus, the mass moment of inertia and mass of the head need to be calculated. The following equation is used to find the x coordinate of the center of mass:

For the x coordinate of the center of mass

Section 2: Summary of Results