Rosen-Roback Framework

Urban Economics

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Agenda

1. Recap Regressions: Non parametric estimation of conditional expectations
    - Local Constant Kernel Estimation
    - Local Linear Kernel Estimation

2. The Rosen Roback Framework

3. Further Readings

Non parametric estimation of conditional expectations

- In this part we are interested in the relationship between y and x where y is the dependent variable and x is the explanatory variable.

$$y = m(x) + u \tag{1}$$

- where we make no assumptions about $m(x)$
- The only assumption we add is $E(u|x) = 0$.

Non parametric estimation of conditional expectations

$$E(y|x) = m(x) \tag{2}$$

- This is true if $E(u|x) = 0$.
- The question is how we estimate this $m(x)$

Local Constant Kernel Estimation (Nadaraya–Watson)

- The question is how we estimate this $m(x)$

$$\hat{m}(x) = \frac{\hat{v}(x)}{\hat{g}(x)} = \frac{\frac{1}{nh^q}\sum \underline{K}\left(\frac{x_i - x}{h}\right) y_i}{\frac{1}{nh^q}\sum \underline{K}\left(\frac{x_i - x}{h}\right)} = \frac{\sum y_i \underline{K}\left(\frac{x_i - x}{h}\right)}{\sum \underline{K}\left(\frac{x_i - x}{h}\right)}$$

- Note that we can write $w_i = \frac{\underline{K}\left(\frac{x_i - x}{h}\right)}{\sum \underline{K}\left(\frac{x_i - x}{h}\right)}$

$$\hat{m}(x) = \sum y_i w_i$$

Local Constant Kernel Estimation (Nadaraya–Watson)

We can show that this is the same to solving the following problem

$$\underset{a}{Min} \sum_{i=1}^{n} w_i (y_i - a)^2$$

where $\hat{a}(x) = \hat{m}(x)$.

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Local Linear Kernel Estimation

We can think to fit a line instead of a constant

$$\underset{\{a,b\}}{Min} \sum_{i=1}^{n} w_i \left\{ y_i - \left[a + b \left(x_i - x \right) \right] \right\}^2$$

Intuition

- The question is how we can write $m(x_i)$ in terms of x.

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- The question is how we can write $m(x_i)$ in terms of x.
- With a linear approximation

$$m(x_i) \approxeq \underbrace{m(x)}_{a} + \underbrace{m'(x)}_{b}(x_i - x)$$

- We get a marginal effect

Local Linear Kernel Estimation

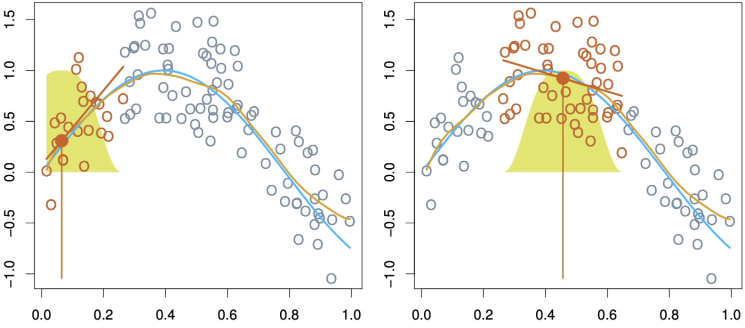
We can show that

$$\hat{a}(x) = y_w - \hat{b} \underbrace{(\bar{x}_w - x)}_{\text{weighted local mean}}$$

$$\hat{b}(x) = \frac{\sum (y_i - \bar{y}_w)(x_i - \bar{x}_w) w_i}{\sum (x_i - \bar{x}_w) w_i}$$

with $\bar{x}_w = \frac{\sum x_i w_i}{\sum w_i}$ y $\bar{y}_w = \frac{\sum y_i w_i}{\sum w_i}$ are the weighted means

Intuition



Example: Geographical Weighted Regressions

The determinant of educational achievement in Georgia (Fotheringham et al., 2002)

$$Bachelor\,or\,higher\,degrees_{county} = f(Rural, Eld, FB, Pov, Black) + u \tag{3}$$

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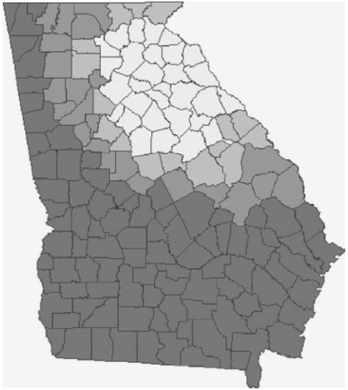
Parameter	*Minimum*	*First Quartile*	*Median*	*Third Quartile*	*Maximum*	*Range*	*Global (OLS)*
Intercept	14.170000	15.350000	17.050000	18.200000	18.860000	4.69	17.2437
Rural	−0.081350	−0.073480	−0.064850	−0.055110	−0.051080	0.03027	−0.0703
Eld	−0.191200	−0.094630	−0.065330	−0.032360	0.012500	0.2037	0.0114
FB	0.854300	1.282000	2.031000	2.796000	3.138000	2.2837	1.8525
Pov	−0.304800	−0.258100	−0.196100	−0.115100	−0.034210	0.27059	−0.2552
Black	−0.016900	0.006347	0.031610	0.060620	0.087210	0.07031	0.0491

Example: Geographical Weighted Regressions

The determinant of educational achievement in Georgia (Fotheringham et al., 2002)



The Rosen Roback Framework

- In the cross city context, spatial equilibrium requires consumers to be indifferent between living in a city and living somewhere else
- To simplify ignore intra-urban considerations like distance to the CBD
- We are going to assume that all housing is equivalent within a MSA and ignore commuting cost

The Rosen Roback Framework

- To illustrate let's begin assuming that utility depends only on net income and amenities:

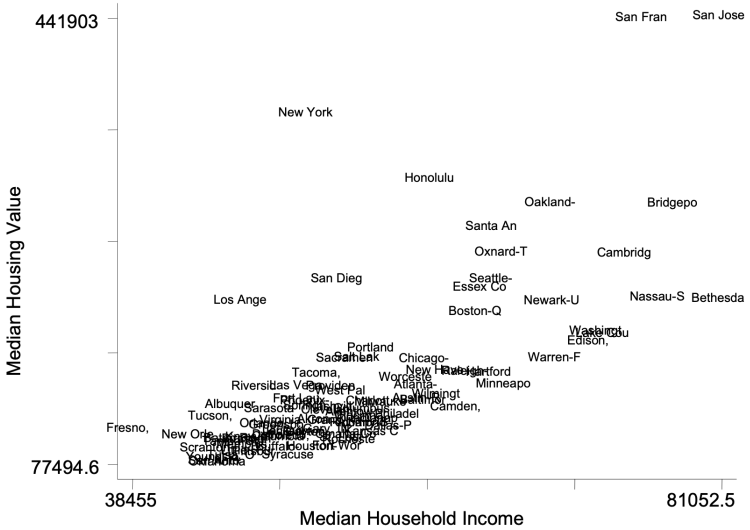
$$U(\text{Wage-Housing Costs}, \mathit{Amenities}) \tag{4}$$

- Spatial equilibrium requires that this is constant across space.
- Let's show how this extremely simple model has some empirical bite.

Housing Affordability

- Measuring affordability by Housing Costs / Income is a mistake
  - A common mistake in policy discussions
  - Bias understating the affordability of high-income areas

Income and Housing Prices



Housing Affordability

- Measuring affordability by Housing Costs / Income is a mistake
  - A common mistake in policy discussions
  - Bias understating the affordability of high-income areas
- House prices are strongly positively correlated with income levels

General Formulation of the Rosen Roback Framework

Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - If a city is growing, house prices equal construction costs
  - If a city is declining, house prices $<$ construction costs
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

Example Spatial Equilibrium

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value	Log real wage	Log wage	Log house value	Log real wage
Year:	2000	2000	2000	1990, 2000	1990, 2000	1990, 2000
Mean January temperature	−0.19	0.60	−0.33			
	[0.06]	[0.31]	[0.10]			
Mean January temperature × year 2000				−0.001	−0.43	0.19
				[0.05]	[0.11]	[0.03]
Year 2000 dummy				0.25	0.62	0.06
				[0.02]	[0.06]	[0.02]
Individual controls	Yes	—	Yes	Yes	—	Yes
Housing controls	—	Yes	—	—	Yes	—
MSA fixed effects	—	—	—	Yes	Yes	Yes
N	1,590,467	2,341,976	1,590,467	2,950,850	4,245,315	2,950,850
R^2	0.29	0.36	0.21	0.27	0.60	0.26

Notes: Individual-level data are from the Census Public Use Microdata Sample, as described in the Data Appendix. Metropolitan-area population is from the Census, as also described in the Data Appendix. Mean January temperature is from the City and County Data Book, 1994, and is measured in hundreds of degrees Fahrenheit. Real wage is controlled for with median house value, also from the Census as described in the Data Appendix. Individual controls include age and education. Location characteristics follow Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Standard errors are clustered by metropolitan area.

Further Readings

- Li, Q., & Racine, J. S. (2007). Nonparametric econometrics: theory and practice. Princeton University Press.
- Pagan, A. & Ullah (1999). Nonparametric Econometrics. Cambridge University Press.
- Silverman (1998). Density Estimation for Statistics and Data Analysis. Chapman & Hall/CRC.