Modelo Monocéntrico (cont.)

Urban Economics

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Agenda

Setup

- We assume a city has one unique center at $d = 0$, the central business district, CBD, where all firms are located.
- All workers have to commute to the CDB to work, and they face commuting costs.
- For simplicity we will assume the city is a line segment on $\mathbb{R}$
- This model allows us to study how house prices vary with distance from the CDB, along with housing consumption, land prices, construction density and population density.

Setup

- Consumers consume a numeraire composite good c and housing L, and

$$u(c, L)$$

is a utility function that's increasing in both arguments and strictly quasi-concave

- They all have identical preferences (in particular, nobody intrinsically values a certain location over another, given c, L).
- Housing is allocated competitively to the highest bidder at each location.

Setup

- Commuting costs are linear in distance $t(d) = td$
- If $r(d)$ is price of housing, and w is the wage, the budget constraint is

$$w - td = r(d)L(d) + c(d)$$

- There are N individuals living as workers in the city.

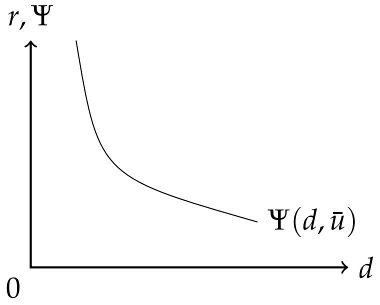
Two approaches to solve

- The Standard Approach or Marshallian approach
- The Bid-rent approach

Bid-rent approach

- The Alonso-Muth condition can be derived more directly using the so-called bid-rent approach approach .
- Define the bid-rent function for housing

$$\Psi(d,\bar{u}) = \max_{L(d)} \left\{ \frac{w - td - c(L(d),\bar{u})}{L(d)} \right\}$$



Results

- We got a bunch of gradients
  - The price of housing decreases with distance to the CBD (Alonso-Muth condition)
  - Consumption of housing increases with distance to the CDB.
  - The reduction in house price as one moves away from the CBD translates into a reduction in land prices. The construction industry then reacts to lower land prices by building with a lower capital to land ratio further away from the CBD.
  - The land price function as the upper envelope of consumers' bid rent and the agricultural land price is non-increasing in d.

Population Density

- The physical extent of the city must also be sufficient to hold its population N:

$$N = \int_0^{\bar{d}} n(d)dd$$

- where $n(d)$ denotes population density at a distance d from the CBD. We can express density as floor space at d relative to housing demand at d

$$n(d) = \frac{f(d)}{L(d)}$$

Population Density

$$n(d) = \frac{f(d)}{L(d)}$$

$$= \frac{\frac{dR(d)}{dd} / \frac{dr(d)}{dd}}{-t / \frac{dr(d)}{dd}}$$

$$= -t\frac{dR(d)}{dd}$$

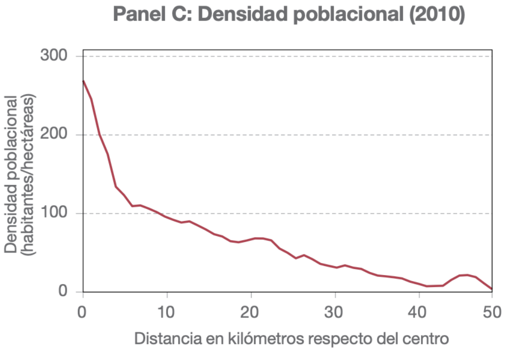
Population Density

- Given that $\frac{dR(d)}{dd} < 0$, density also declines with distance to the CBD,

$$\frac{dn(d)}{dd} < 0$$

- We get another gradient: Population Density is decreasing in distance from the CBD.
- This is a direct implication of two other gradients already discussed:
  - the increase in housing consumption and the decline in the capital
  - intensity of development as one moves farther from the CBD.

Population Density



Fuente: Daude (2017)

Supply of Housing

- Assume a neoclassical housing production function $H(K, L)$: capital and land.
- We assume that the parcel of land is given to the developer in intensive form:

$$S = \frac{K}{L},$$

$$h(S) = H(L, K)/L$$

- S is capital per unit of land, i.e. density of structure, or how much floor space per m^2 of lot size.

Supply of Housing

- Consumers: Now bid for price per unit of housing services h. Has the same properties as $r(d)$, but we will call it $P^h(d)$
- Developers
  - buy land at the land price $R(d)$ per unit of L
  - buy capital K at price r to build the house.
  - sell $h(s)$ units of housing space at price $P(d)$ to consumers.

Supply of Housing

- Developers maximize profit at location d

$$\Pi = (P(d)h(S) - rS - R(d))L$$

$$\pi = \frac{\Pi}{L} = P(d)h(S) - rS - R(d)$$

Supply of Housing

- FOC

$$P(d)h'(S) = r$$

- Zero profit condition

$$P(d)h(S) = rS + P(d)$$

Supply of Housing

- Total differential of FOC wrt to d

$$\frac{\partial P(d)}{\partial d} + \frac{\partial h'(S(d))}{\partial S(d)} \frac{dS(d)}{dd} = 0$$

$$S'(d) = -\frac{\partial P(d)}{\partial d} \frac{1}{h''(s)} < 0$$

- We get another gradient: capital intensity (building height) decreases with distance from the CDB.

Different Income Groups

- Suppose there are high and low income groups $w_2 > w_1$ with $\bar{u}_2 > \bar{u}_1$
- This means that higher w means higher bid (if housing is a normal good.)
- So, we have that

$$L_2(d) > L_1(d), \forall d$$

Different Income Groups

- But, by Alonso-Muth, this implies at a point of indifference $\tilde{d}$ that

$$\frac{d\Psi(d,\bar{u}_2)}{dd} = -\frac{t}{L_2(r_2(\tilde{d},\bar{u}_2))} > -\frac{t}{L_1(r_1(\tilde{d},\bar{u}_2))} = \frac{d\Psi(d,\bar{u}_1)}{dd}$$

- Note that $\frac{d\Psi(d,\bar{u})}{dd} < 0$ in general, so this means that $\Psi(d,\bar{u}_1)$ has a steeper gradient at $\tilde{d}$

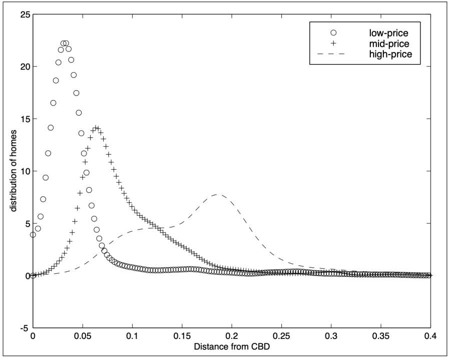
Different Income Groups

- If housing is a normal good (it's budget share increases with income) and commuting costs are the same across groups, poorer residents will locate closer to the CBD, richer ones further away.
- There is perfect separation between both groups.
- Rich people are more willing to pay greater commuting costs and live further away because their higher wage allows to consume more housing.

  (H.W. compare results with different commuting costs for rich and poor)

Different Income Groups

Figure 1: Distribution of low, medium and high priced homes versus distance in Toledo, OH



Fuente: LeSage (1999)

Intro to Spatial Econometrics

- Applied work in urban economics and regional science relies heavily on sample data that is collected with reference to locations
- What distinguishes spatial econometrics from traditional econometrics?
  - Spatial dependence between the observations and
  - Spatial heterogeneity in the relationships we are modeling.

Spatial Dependence
Cross-sectional iid non-spatial data

- Standard Cross-sectional models

$$y_i = X_i\beta + \epsilon_i \tag{1}$$
$$i = 1, \ldots, n \tag{2}$$

- Independent or statistically independent observations imply

$$E(\epsilon_i \epsilon_j) = E(\epsilon_i)E(\epsilon_j) = 0 \tag{3}$$

- Spatial dependence reflects a situation where values observed at one location or region, say observation i, depend on the values of neighboring observations at nearby locations.

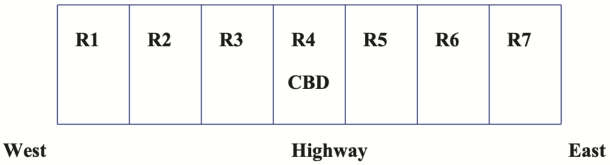
$$y_i = \alpha_i y_j + X_i\beta + \epsilon_i \tag{4}$$
$$y_j = \alpha_j y_i + X_j\beta + \epsilon_j \tag{5}$$

- This situation suggests a simultaneous data generating process, where the value taken by y_i depends on that of y_j and vice versa.

Spatial Dependence

Example

Figure 2: Regions east and west of the CBD



Fuente: LeSage & Pace (2009)

Spatial Dependence
Example

$$y = \begin{pmatrix} \mathit{Travel\ Times} \\ 42 \\ 37 \\ 30 \\ 26 \\ 30 \\ 37 \\ 42 \end{pmatrix} \tag{6}$$

$$X = \begin{pmatrix} \mathit{Density} & \mathit{Distance} \\ 10 & 30 \\ 20 & 20 \\ 30 & 10 \\ 50 & 0 \\ 30 & 10 \\ 20 & 20 \\ 10 & 30 \end{pmatrix} \tag{7}$$

Spatial heterogeneity

- The term spatial heterogeneity refers to variation in relationships over space.
- In the most general case we might expect a different relationship to hold for every point in space.

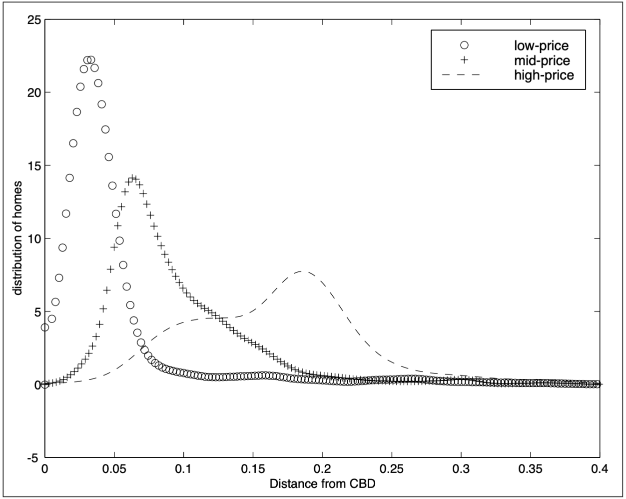
$$y_i = X_i \beta_i + \epsilon_i \quad (8)$$
$$i = 1, \ldots, n \quad (9)$$

Spatial heterogeneity

- We simply do not have enough sample data information with which to produce estimates for every point in space, (not enough degrees of freedom)
- To proceed with the analysis we need to provide a specification for variation over space.
- This specification must be parsimonious,that is,only a handful of parameters can be used in the specification.
- A large amount of spatial econometric research centers on alternative parsimonious specifications for modeling variation over space.

Spatial heterogeneity

Figure 3: Distribution of low, medium and high priced homes versus distance



The spatial autoregressive process

- The solution to the over-parameterization problem that arises when we allow each dependence relation to have relation-specific parameters is to impose structure on the spatial dependence relations.
- Ord (1975) proposed a parsimonious parameterization for the dependence relations (which built on early work by Whittle (1954)).
- The Spatial autoregressive process.

$$y_i = \rho \sum_{j=1}^{n} W_{ij} y_j + \epsilon_i \tag{10}$$

$$\epsilon_i \sim N(0, \sigma^2) \tag{11}$$

$$i = 1, \ldots, n \tag{12}$$

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