Rosen-Roback Framework

Urban Economics

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Recap: Rosen Roback Framework

Three Simultaneous Equilibria

- Individuals are optimally choosing which city to live in
  - There is a group of homogeneous individuals
  - Some of them are living in different cities
  - Their utility level is the same in all those cities
- Firms earn zero expected profits
  - Free entry of firms
  - Firm profits are equalized across cities
- The construction sector operates optimally
  - If a city is growing, house prices equal construction costs
  - If a city is declining, house prices $<$ construction costs
  - Free entry, zero profit for builders
  - Construction profits are equalized across cities

Recap: Rosen Roback Framework

Housing consumption

$$maxU(C,H) = \theta C^{1-\alpha} H^{\alpha} \tag{1}$$

$$st \tag{2}$$

$$W = C + r_H H \tag{3}$$

FOC

$$H^* = \alpha \frac{W}{r_H} \tag{4}$$

$$C^* = (1-\alpha)W \tag{5}$$

Indirect Utility

$$V = U^* = \theta \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{W}{r_H^{\alpha}} \tag{6}$$

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Production Sector

Cobb-Douglas production function with constant returns to scale:

$$y = AN^{\beta}K^{\gamma}\bar{Z}^{\zeta} \tag{7}$$

$$st \tag{8}$$

$$C = WN + p_k K + p_z Z \tag{9}$$

$\beta + \gamma + \zeta = 1$
The competitive wage in each city is

$$W = \beta\left(\left(\frac{\gamma}{p_k}\right)^{\gamma} A \left(\frac{\bar{Z}}{N}\right)^{\zeta}\right)^{\frac{1}{1-\gamma}} \tag{10}$$

small open economy $p_k = 1$

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Construction sector

- Exogenous amount of land $\bar{L}$ in each city
  - Natural and regulatory constraints
- Housing supply is the product of land L and building height h
- Height is built with tradable capital at a convex cost

$$\varphi p_K \left(\frac{h^\delta}{\delta} \right) \tag{11}$$

for $\phi > 0$ and $\delta > 1$

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Construction sector

Free entry to developers

$$max \left\{ p_H h - \varphi p_k \left(\frac{h}{\delta} \right)^{\delta} \right\}$$

$$h^* = \delta \left(\frac{p_H}{p_k} \right)^{\frac{1}{\delta - 1}}$$

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Eq in housing market

We assume

$$r_H = \mu p_H$$

the supply equal demand

$$\frac{\alpha W}{\mu p_H} N = \delta \left(\frac{p_H}{p_k} \right)^{\frac{1}{\delta - 1}} \bar{L}$$

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Eq in housing market

Market clearing price

$$p_H^* = \left[\varphi p_k \left(\frac{\alpha}{\delta\mu}\frac{WN}{\bar{L}}\right)^{\delta-1}\right]^{\frac{1}{\delta}}$$

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Spatial Equilibrium 3 conditions

1. Individual optimal Location choice

$$logW - \alpha log(p_H) + log\theta = \bar{u} + k_1$$

2. Firms labor demand

$$(1-\gamma)logW + \zeta(logN - log\bar{Z}) - logA = k_2 - \gamma logp_k$$

3. Housing Market equilibrium

$$\delta logp_H - (\delta - 1)(logw + logN - log\bar{L}) - log\varphi = logp_k + k_3$$

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Spatial Equilibrium 3 conditions

Our focus is in small ciites

Three endogenous variables

1. N
2. W
3. p_H

Three exogenous

1. θ
2. $\tilde{A} = A\bar{Z}^{\theta}$
3. $\tilde{L} = \bar{L}\varphi^{-1}$

Two economy wide characteristics

1. $\bar{u}$
2. p_k

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Equilibrium Solution

1. Equilibrium wages

$$logW = k_w + \frac{(\delta - 1)\alpha(log\tilde{A} - \zeta log\tilde{L}) - \delta\zeta log\theta}{\beta(\delta - 1)\alpha + \delta\zeta}$$

2. Equilibrium housing prices

$$logp_H = k_p + \frac{(\delta - 1)(log\tilde{A} + \beta log\theta - \zeta log\tilde{L})}{\beta(\delta - 1)\alpha + \delta\zeta}$$

3. Equilibrium population

$$logN = k_N + \frac{[\delta(1 - \alpha) + \alpha]log\tilde{A} + (\beta + \zeta)[\delta log\theta + (\delta - 1)\alpha log\tilde{L}]}{\beta(\delta - 1)\alpha + \delta\zeta}$$

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Researchers often try to estimate effect of some variable (X) on the amenities

$$log\theta = k_\theta + \xi_\theta X + \epsilon_\theta$$

$$log\tilde{A} = k_A + \xi_A X + \epsilon_A$$

$$log\tilde{L} = k_L + \xi_L X + \epsilon_L$$

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If we replace in th equilibrium conditions

$$logW = k_w + \xi_w X + \epsilon_w$$

$$logp_H = k_p + \xi_p X + \epsilon_p$$

$$logN = k_p + \xi_N X + \epsilon_N$$

where

$$\xi_W = \frac{(\delta - 1)\alpha(\xi_A - \zeta\xi_L) - \delta\zeta\xi_\theta}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_p = \frac{(\delta - 1)(\xi_A + \beta\xi_\theta - \zeta\xi_L)}{\beta(\delta - 1)\alpha + \delta\zeta}$$

$$\xi_N = \frac{[\delta(1 - \alpha) + \alpha]\xi_A + (\beta + \zeta)[\delta\xi_\theta + (\delta - 1)\alpha\xi_L]}{\beta(\delta - 1)\alpha + \delta\zeta}$$

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Solving and inverting we get

$$\xi_\theta = \alpha\xi_p - \xi_w$$

$$\xi_A = \zeta\xi_N + (1-\gamma)\xi_w$$

$$\xi_L = \xi_N + \xi_W - \frac{\delta}{\delta - 1}\xi_p$$

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

In the US, fastest growing areas have warm climates, something similar in Europe (what about Latam?) These areas, in the south and west of US, are known as the "sunbelt"

If we look across metropolitan areas, the relationship between January temperature and size is:

$$log(Population2000) = \underset{(0.2)}{12.2} + \underset{(0.005)}{0.017} JanuaryTemp$$

more if we look at changes

$$log\left(\frac{Population2000}{Population1990}\right) = \underset{(0.14)}{0.016} + \underset{(0.0004)}{0.003} \, JanuaryTemp$$

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

Why has population growth shifted to sunbelt?

1. Has productivity increased in South?
2. Have political institutions become more efficient (and less corrupt)?
3. Has advent of air conditioning made South more comfortable (amenities)?
4. Are people attracted to cheap housing, made possible by pro-building policies?

Recap: Rosen Roback Framework

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

- To differentiate between these hypothesis, we can use the spatial equilibrium model
- Glaeser and Gottileb run regressions of population, wages, and house values on temperature with controls
- Combine coefficients using model to look at effect of temperature on amenities, productivity, housing construction productivity

Example Spatial Equilibrium

An Example: Does the Rise of Sunbelt Cities Represent Amenities or Production?

TABLE 3
SPATIAL EQUILIBRIUM

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable	Log wage	Log house value	Log real wage	Log wage	Log house value	Log real wage
Year:	2000	2000	2000	1990, 2000	1990, 2000	1990, 2000
Mean January temperature	−0.19	0.60	−0.33			
	[0.06]	[0.31]	[0.10]			
Mean January temperature × year 2000				−0.001	−0.43	0.19
				[0.05]	[0.11]	[0.03]
Year 2000 dummy				0.25	0.62	0.06
				[0.02]	[0.06]	[0.02]
Individual controls	Yes	—	Yes	Yes	—	Yes
Housing controls	—	Yes	—	—	Yes	—
MSA fixed effects	—	—	—	Yes	Yes	Yes
N	1,590,467	2,341,976	1,590,467	2,950,850	4,245,315	2,950,850
R^2	0.29	0.36	0.21	0.27	0.60	0.26

Notes: Individual-level data are from the Census Public Use Microdata Sample, as described in the Data Appendix. Metropolitan-area population is from the Census, as also described in the Data Appendix. Mean January temperature is from the City and County Data Book, 1994, and is measured in hundreds of degrees Fahrenheit. Real wage is controlled for with median house value, also from the Census as described in the Data Appendix. Individual controls include age and education. Location characteristics follow Metropolitan Statistical Areas under the 1999 definitions, using Primary Metropolitan Statistical Areas rather than Consolidated Metropolitan Statistical Areas where applicable and New England County Metropolitan Areas where applicable. Standard errors are clustered by

Accounting for Unobservable Heterogeneity in Cross Section Using Spatial First Differences

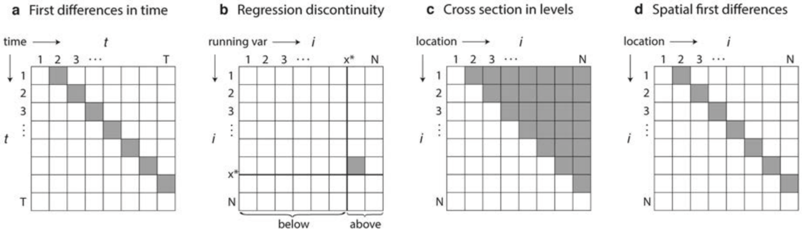
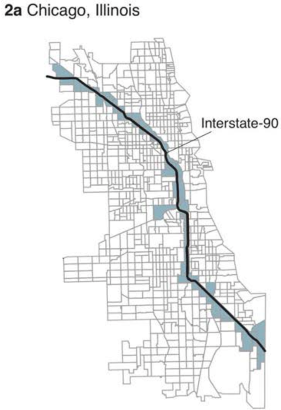


Figure 1: **Comparison of pair-wise assumptions regarding the comparability of observations needed for identification in different research designs.** Graphical depiction of the various comparisons exploited to identify causal effects in (a) FD time-series models, (b) regression discontinuity designs with discontinuity at x^*, (c) the cross-sectional approach in levels, and (d) SFD. Each observation in a data set appears on both a row and column for a grid. Squares are grey if the observations for that row and column are assumed to be comparable (i.e. expected potential outcomes are conditionally equal) when using the associated research design.

Accounting for Unobservable Heterogeneity in Cross Section Using Spatial First Differences

(a) New York



(b) Chicago

Accounting for Unobservable Heterogeneity in Cross Section Using Spatial First Differences

	Dependent variable: log average wage					
	10th Avenue, New York		I-90, Chicago		Staiger and Stock (1997)	
	Levels	SFD	Levels	SFD	OLS	IV
Average years of education	0.178***	0.089**	0.124***	0.072*	0.063***	0.098***
	(0.015)	(0.028)	(0.020)	(0.037)	(0.000)	(0.015)
Constant	4.682***	−0.007	5.355***	0.000	–	–
	(0.217)	(0.040)	(0.259)	(0.035)		
Observations	53	52	54	53	329, 509	329, 509
R squared	0.73	0.16	0.43	0.07	–	–

Table 1: **Cross-sectional estimates for returns to education using levels and SFD.** Data for the first four columns are for census tracts in Manhattan, New York along 10th Avenue and Chicago, Illinois along Interstate-90 for the year 2010. We report OLS standard errors, which, in this case, are more conservative than Newey-West standard errors. Asterisks indicate statistical significance at the 0.1%,***, 1%**, and 5%* levels.

Further Readings

- Druckenmiller, H. and Hsiang, S. (2018). Accounting for unobservable heterogeneity in cross section using spatial first differences. Technical report, National Bureau of Economic Research.
- Glaeser, E. L., & Gottlieb, J. D. (2009). The wealth of cities: Agglomeration economies and spatial equilibrium in the United States. Journal of economic literature, 47(4), 983-1028.
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- Ponzetto, G. (2012) Spatial Equilibrium Across Cities. Mimeo