Modelo Monocéntrico (cont.)

Urban Economics

Ignacio Sarmiento-Barbieri

Universidad de los Andes

February 3, 2022

Agenda

Setup

- We assume a city has one unique center at $d = 0$, the central business district, CBD, where all firms are located.
- All workers have to commute to the CDB to work, and they face commuting costs.
- For simplicity we will assume the city is a line segment on $\mathbb{R}$
- This model allows us to study how house prices vary with distance from the CDB, along with housing consumption, land prices, construction density and population density.

Setup

- Consumers consume a numeraire composite good c and housing L, and

$$u(c, L)$$

is a utility function that's increasing in both arguments and strictly quasi-concave

- They all have identical preferences (in particular, nobody intrinsically values a certain location over another, given c, L).
- Housing is allocated competitively to the highest bidder at each location.

Setup

- Commuting costs are linear in distance $t(d) = td$
- If $r(d)$ is price of housing, and w is the wage, the budget constraint is

$$w - td = r(d)L + c$$

- There are N individuals living as workers in the city.

First Simple Consumer Problem: von Thünen Consumers

- To start, assume there is no choice about housing

$$L = \bar{L}$$

- Then, given the price function, the consumer chooses where to locate

$$\max_{d>0} u(w - td - r(d)\bar{L}, \bar{L})$$

First Simple Consumer Problem: von Thünen Consumers

- Given perfect mobility (zero moving costs), utility is the same everywhere:

$$u(w - td - r(d)\bar{L}, \bar{L}) = \bar{u} \;\forall d \leq \bar{d}$$

- FOC

$$r(d)' = -\frac{t}{\bar{L}}$$

First Simple Consumer Problem: von Thünen Consumers

- To solve we add the assumption that the use of land beyond $\bar{d}$ yields rent $\bar{r} > 0$,

$$r(d) = \bar{r} + \frac{1}{\bar{L}} \int_{d}^{\bar{d}} t dt$$

Allow housing choice

- Now we allow for the choice of L
- Where to locate (d)?
- How much c?
- How are these choice going to influence the price function $r(d)$?

Allow housing choice

- 2 differences to standard model:
  - choose location d
  - choose between c and L, where $r(d)$ varies endogenously.

The Standard Approach or Marshallian approach

- This is a standard constrained utility maximization problem.
- How to bundle c, L in order to achieve max u under the budget constraint?

$$\max_{c(d)L(d)} u(c(d), L(d))$$

$$st$$

$$w - td = r(d)L(d) + c$$

The Standard Approach or Marshallian approach

- We can substitute for c in the utility function,

$$\max_{d} u(w - td - r(d)L(d), L(d))$$

- FOC yields a unique marshallian demand for housing at each location

$$r(d) = \frac{\frac{\partial u(.)}{\partial L}}{\frac{\partial u(.)}{\partial c}}$$

The Standard Approach or Marshallian approach

- Using the budget constraint we get the marshallian demand for $c(d)$

$$c(d) = w - td - r(d)L(d)$$

The Standard Approach or Marshallian approach

- In equilibrium, given that all individuals have the same income and are freely mobile, they must obtain the same level of utility $\bar{u}$
- **there are no gains from changing locations**

$$u(w - td - r(d)L(d), L(d)) = \bar{u}$$

The Standard Approach or Marshallian approach

- Totally differentiating with respect to d yields:

$$\frac{\partial u}{\partial L}\frac{\partial L}{\partial d} - \frac{\partial u}{\partial c} r(d)\frac{\partial L}{\partial d} - \frac{\partial u}{\partial c}\left(t + L(d)\frac{\partial r(d)}{\partial d}\right) = 0$$

- rearranging

$$\frac{\partial r(d)}{\partial d} = -\frac{t}{L(d)} < 0$$

- which is the Alonso-Muth condition.

The Standard Approach or Marshallian approach

$$\frac{\partial r(d)}{\partial d} = -\frac{t}{L(d)} < 0$$

- The Alonso-Muth condition states that,
  - at the residential equilibrium, if a resident move marginally away from the CBD, the cost of her current housing consumption falls just as much as her commuting costs increase.
  - Thus, the price of housing decreases with distance to the CBD.
  - This housing price gradient is the first of a series of gradients that occur in the monocentric model.

Bid-rent approach

- The main drawback of the Marshallian approach is that it gets to the solution in a roundabout way.

Bid-rent approach

- The main drawback of the Marshallian approach is that it gets to the solution in a roundabout way.
- It solves first for the consumer programme in a location before recovering the price of housing at this location through the residential equilibrium condition.
- Then, knowing the price of housing, it returns to the choice of consumption before solving for the optimal location.
- The main advantage of the Marshallian approach is to make clear that the price of housing at each location is endogenous and emerges within the model.

Bid-rent approach

- The Alonso-Muth condition can be derived more directly using the so-called bid-rent approach approach (also known as the direct approach).
- Define the bid-rent function for housing

$$\Psi(d,\bar{u}) = \max_{c(d),L(d)} \{r(d)|u(c,L) = \bar{u}, w - td = r(d)L(d) + c(d)\}$$

- as the maximum price a resident is willing to pay for housing at distance d from the CBD while enjoying utility u and satisfying the budget constraint
- we can write it as

$$\Psi(d,\bar{u}) = \max_{L(d)} \left\{ \frac{w - td - c(d)}{L(d)} |u(c,L) = \bar{u} \right\}$$

Bid-rent approach

- Recall the definition of the hicksian demand function in this case:

$$c(L(d), \bar{u}) = \underset{c}{argmin} \frac{w - td - c(d)}{L(d)}$$

$$st$$

$$u(L, c) = \bar{u}$$

Bid-rent approach

- Substitute the hicksian demand for c

$$\Psi(d,\bar{u}) = \max_{L(d)} \left\{ \frac{w - td - c(L(d),\bar{u})}{L(d)} \right\}$$

Bid-rent approach

- In equilibrium how do housing costs change as one moves a bit away from the CBD?

$$\left.\frac{d\Psi(d,\bar{u})}{dd}\right|_{L(d)=L\left(\underbrace{\Psi(d,\bar{u})}_{maximal\ r},u\right)} = -\frac{t}{L(d)} < 0$$

- we get the same condition, as we move away housing costs (the highest bid) decrease proportionally to transport costs.
- How much housing consumes?

Bid-rent approach

- How much housing consumes?
- It can be obtained from the FOC of

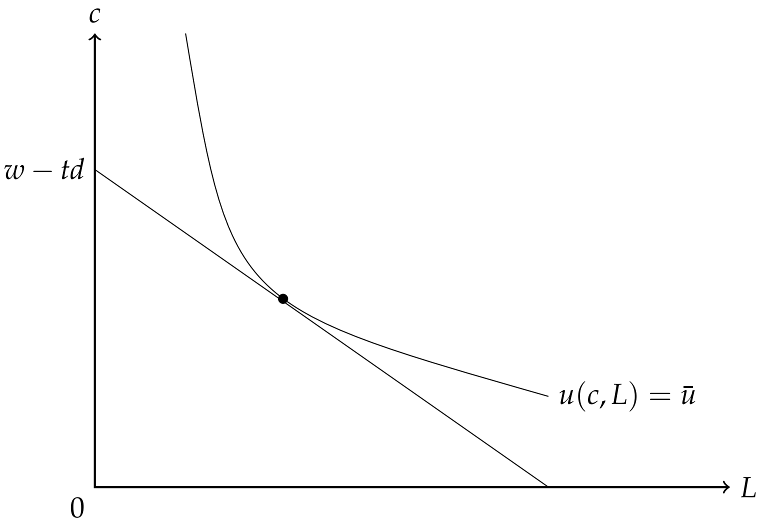
$$\Psi(d,\bar{u}) = \max_{L(d)} \left\{ \frac{w - td - c(L(d),\bar{u})}{L(d)} \right\}$$

$$\frac{\partial c(L(d),\bar{u})}{\partial L(d)} L(d) + w - td - c(L(d),\bar{u}) = 0$$

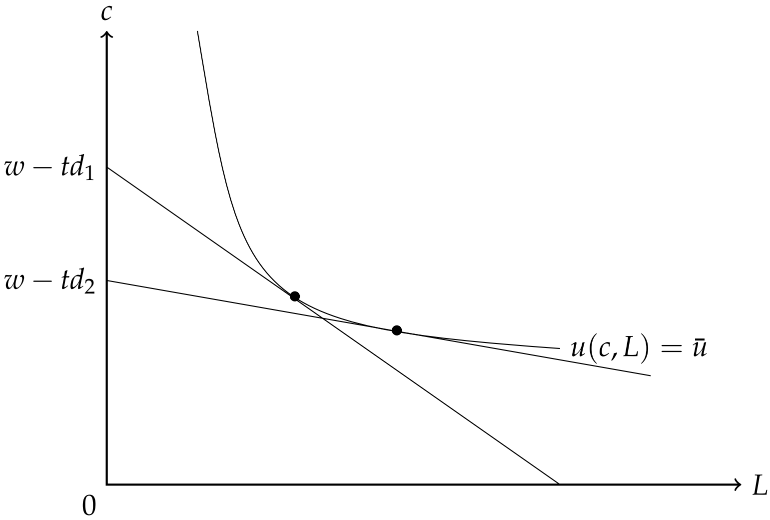
- that can be rewriten as

$$\underbrace{\frac{\partial c(L(d),\bar{u})}{\partial L(d)}}_{Slope\ Indif\ Curve} = \underbrace{\frac{w - td - c(L(d),\bar{u})}{L(d)}}_{Slope\ BC}$$

Bid-rent approach



Bid-rent approach



Bid-rent approach

- Corollary: lower price leads consumers to consume more housing the further they live from the CBD

Bid-rent approach

- Corollary: lower price leads consumers to consume more housing the further they live from the CBD
- Differentiate the hicksian demand for housing wrt d

$$\frac{\partial L(r(d),\bar{u})}{\partial d} = \underbrace{\frac{\partial L(r(d),\bar{u})}{\partial r(d)}}_{(-)} \underbrace{\frac{dr(d)}{dd}}_{(-)} \geq 0$$

- This is the second gradient: consumption of housing increases with distance to the CDB.
- Note: this is a pure substitution effect (away from c and towards more L) since $\bar{u}$ is fixed.

Supply

- Perfectly competitive house builders use CRS production function
- They supply $f(d)$ units of housing floor space per unit of land at distance d
- Ignore capital for now.
- The rental price of land is given by $R(d)$.
- There is zero profit:

$$\pi = r(d) - c(R(d)) = 0$$

Supply

- Totally differentiating this gives

$$\frac{dr(d)}{dd} = \frac{\partial c(R(d))}{\partial R(d)} \frac{dR(d)}{dd}$$

$$\frac{dR(d)}{dd} = \frac{dr(d)}{dd} \frac{1}{\frac{\partial c(R(d))}{\partial R(d)}} = \frac{dr(d)}{dd} f(d) < 0$$

- The reduction in house price r as one moves away from the CBD translates into a reduction in land prices.

- The construction industry then reacts to lower land prices by building with a lower capital to land ratio further away from the CBD.

- There are two other gradients: as we move away from the CBD we have declining land prices and declining capital intensity in housing (i.e. both larger gardens and properties with fewer stories).

Land Use Equilibrium

- What happens at the city edge $\bar{d}$?

Land Use Equilibrium

- What happens at the city edge $\bar{d}$?
- Land is built if the rent $R(d)$ it obtains from residential use is greater than the next best alternative
- Assume there is other use for land, here: agriculture.

Land Use Equilibrium

- Agricultural activity does not require commuting to CBD
- Therefore farmers' willingness to pay for land should be independent of d.
- The land market needs to be in equilibrium at any distance d.
- Land is built if the rent $R(d)$ it can fetch in residential use is at least as high as the rent R it can fetch from agriculture.
- Landlords lend land to the highest bidder at each location.

Land Use Equilibrium

- We know from equation that optimality of consumers required that

$$\frac{dr(d)}{dd} = \frac{d\Psi(d,\bar{u})}{dd},\ d < \bar{d}$$

- Landlords let land to the highest bidder at each location, i.e.

$$r(d) = max(\Psi(d,\bar{u}), farmer's\ \ bid)$$

- How much is the farmer going to bid for land?

Land Use Equilibrium

Farmer's Land Bid

- No commute $\rightarrow$ no importance of being close to CBD.
- Assume that produces $Q = aL$, where $a > 0$ and L is land.
- Profit:

$$\pi_A = p_q Q - R(d)L = (ap_q - R(d))L$$

- p_q is the price of agricultural good Q
- $R(d)$ is still the rental price of land
- Free entry:

$$\pi_A = 0$$

Land Use Equilibrium

Farmer's Land Bid

- This implies

$$R(d) = ap_q$$

$$R(d) = \bar{R}$$

- which is independent of the distance.

Land Use Equilibrium

- We can rewrite the price function as the upper envelope of those bids:

$$r(d) = max(\Psi(d, \bar{u}), \bar{R})$$

- Given the result on Ψ (ie. the Alonso-Muth condition), and the flatness of $\bar{R}$
- We get a new gradient: the land price function as the upper envelope of consumers' bid rent and the agricultural land price is non-increasing in d.

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