TABLAS Y FORMULARIOS

TABLA DE DERIVADAS

1. $D_x(u^n) = nu^{n-1} D_x u$
2. $D_x(u + v) = D_x u + D_x v$
3. $D_x(uv) = u D_x v + v D_x u$
4. $D_x\left(\frac{u}{v}\right) = \frac{v D_x u - u D_x v}{v^2}$
5. $D_x(e^u) = e^u D_x u$
6. $D_x(a^u) = a^u \ln a \, D_x u$
7. $D_x(\ln u) = \frac{1}{u} D_x u$
8. $D_x(\text{sen } u) = \cos u \, D_x u$
9. $D_x(\cos u) = -\text{sen } u \, D_x u$
10. $D_x(\tan u) = \sec^2 u \, D_x u$
11. $D_x(\cot u) = -\csc^2 u \, D_x u$
12. $D_x(\sec u) = \sec u \tan u \, D_x u$
13. $D_x(\csc u) = -\csc u \cot u \, D_x u$
14. $D_x(\text{sen}^{-1} u) = \frac{1}{\sqrt{1 - u^2}} D_x u$
15. $D_x(\cos^{-1} u) = \frac{-1}{\sqrt{1 - u^2}} D_x u$
16. $D_x(\tan^{-1} u) = \frac{1}{1 + u^2} D_x u$
17. $D_x(\cot^{-1} u) = \frac{-1}{1 + u^2} D_x u$
18. $D_x(\sec^{-1} u) = \frac{1}{u\sqrt{u^2 - 1}} D_x u$
19. $D_x(\csc^{-1} u) = \frac{-1}{u\sqrt{u^2 - 1}} D_x u$
20. $D_x(\text{senh } u) = \cosh u \, D_x u$
21. $D_x(\cosh u) = \text{senh } u \, D_x u$
22. $D_x(\tanh u) = \text{sech}^2 u \, D_x u$
23. $D_x(\coth u) = -\text{csch}^2 u \, D_x u$
24. $D_x(\text{sech } u) = -\text{sech } u \tanh u \, D_x u$
25. $D_x(\text{csch } u) = -\text{csch } u \coth u \, D_x u$

TABLA DE INTEGRALES

Algunas formas elementales

1. $\int du = u + C$
2. $\int a \, du = au + C$
3. $\int [f(u) + g(u)]du = \int f(u) \, du + \int g(u) \, du$
4. $\int u^n \, du = \frac{u^{n+1}}{n + 1} + C \qquad (n \neq -1)$
5. $\int \frac{du}{u} = \ln|u| + C$

Formas racionales que contienen $a + bu$

6. $\int \frac{u \, du}{a + bu} = \frac{1}{b^2}\left[a + bu - a \ln|a + bu|\right] + C$
7. $\int \frac{u^2 \, du}{a + bu} = \frac{1}{b^3}\left[\frac{1}{2}(a + bu)^2 - 2a(a + bu) + a^2 \ln|a + bu|\right] + C$
8. $\int \frac{u \, du}{(a + bu)^2} = \frac{1}{b^2}\left[\frac{a}{a + bu} + \ln|a + bu|\right] + C$
9. $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3}\left[a + bu - \frac{a^2}{a + bu} - 2a \ln|a + bu|\right] + C$
10. $\int \frac{u \, du}{(a + bu)^3} = \frac{1}{b^2}\left[\frac{a}{2(a + bu)^2} - \frac{1}{a + bu}\right] + C$

11. $$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

12. $$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

13. $$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$$

Formas que contienen $\sqrt{a+bu}$

14. $$\int u\sqrt{a+bu}\,du = \frac{2}{15b^3}(3bu-2a)(a+bu)^{3/2} + C$$

15. $$\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(15b^2u^2-12abu+8a^2)(a+bu)^{3/2} + C$$

16. $$\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$$

17. $$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

18. $$\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu} + C$$

19. $$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$$

20. $$\int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C & \text{si } a > 0 \\ \dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bu}{-a}} + C & \text{si } a < 0 \end{cases}$$

21. $$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

22. $$\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

23. $$\int \frac{\sqrt{a+bu}\,du}{u^n} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bu}\,du}{u^{n-1}}$$

Formas que contienen $a^2 \pm u^2$

24. $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

25. $$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C = \begin{cases} \dfrac{1}{a}\tanh^{-1}\dfrac{u}{a} + C & \text{si } |u| < a \\ \dfrac{1}{a}\coth^{-1}\dfrac{u}{a} + C & \text{si } |u| > a \end{cases}$$

26. $$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C = \begin{cases} -\dfrac{1}{a}\tanh^{-1}\dfrac{u}{a} + C & \text{si } |u| < a \\ -\dfrac{1}{a}\coth^{-1}\dfrac{u}{a} + C & \text{si } |u| > a \end{cases}$$

Formas que contienen $\sqrt{u^2 \pm a^2}$

En las formulas 27 a 38, se puede sustituir

$\ln\left(u + \sqrt{u^2 + a^2}\right)$ por $\operatorname{senh}^{-1}\frac{u}{a}$

$\ln\left|u + \sqrt{u^2 - a^2}\right|$ por $\cosh^{-1}\frac{u}{a}$

$\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right|$ por $\operatorname{senh}^{-1}\frac{a}{u}$

27. $\displaystyle\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

28. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

29. $\displaystyle\int u^2\sqrt{u^2 \pm a^2}\,du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

30. $\displaystyle\int \frac{\sqrt{u^2 + a^2}\,du}{u} = \sqrt{u^2 + a^2} - a\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

31. $\displaystyle\int \frac{\sqrt{u^2 - a^2}\,du}{u} = \sqrt{u^2 - a^2} - a\sec^{-1}\frac{u}{a} + C$

32. $\displaystyle\int \frac{\sqrt{u^2 \pm a^2}\,du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

33. $\displaystyle\int \frac{u^2\,du}{\sqrt{u^2 \pm a^2}} = \frac{u}{2}\sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

34. $\displaystyle\int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{u^2 + a^2}}{u}\right| + C$

35. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

36. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$

37. $\displaystyle\int (u^2 \pm a^2)^{3/2}\,du = \frac{u}{8}(2u^2 \pm 5a^2)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8}\ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$

38. $\displaystyle\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{u}{\pm a^2\sqrt{u^2 \pm a^2}} + C$

Formas que contienen $\sqrt{a^2 - u^2}$

39. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1}\frac{u}{a} + C$

40. $\displaystyle\int \sqrt{a^2 - u^2}\,du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

41. $\displaystyle\int u^2\sqrt{a^2 - u^2}\,du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

42. $\displaystyle\int \frac{\sqrt{a^2 - u^2}\,du}{u} = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

$\displaystyle= \sqrt{a^2 - u^2} - a\cosh^{-1}\frac{a}{u} + C$

43. $\int \frac{\sqrt{a^2 - u^2}\, du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \operatorname{sen}^{-1}\frac{u}{a} + C$

44. $\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

45. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

$= -\frac{1}{a}\cosh^{-1}\frac{a}{u} + C$

46. $\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$

47. $\int (a^2 - u^2)^{3/2}\, du = -\frac{u}{8}(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

48. $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Formas que contienen $2au - u^2$

49. $\int \sqrt{2au - u^2}\, du = \frac{u - a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(1 - \frac{u}{a}\right) + C$

50. $\int u\sqrt{2au - u^2}\, du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(1 - \frac{u}{a}\right) + C$

51. $\int \frac{\sqrt{2au - u^2}\, du}{u} = \sqrt{2au - u^2} + a\cos^{-1}\left(1 - \frac{u}{a}\right) + C$

52. $\int \frac{\sqrt{2au - u^2}\, du}{u^2} = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(1 - \frac{u}{a}\right) + C$

53. $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(1 - \frac{u}{a}\right) + C$

54. $\int \frac{u\, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a\cos^{-1}\left(1 - \frac{u}{a}\right) + C$

55. $\int \frac{u^2\, du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2}\sqrt{2au - u^2} + \frac{3a^2}{2}\cos^{-1}\left(1 - \frac{u}{a}\right) + C$

56. $\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$

57. $\int \frac{du}{(2au - u^2)^{3/2}} = \frac{u - a}{a^2\sqrt{2au - u^2}} + C$

58. $\int \frac{u\, du}{(2au - u^2)^{3/2}} = \frac{u}{a\sqrt{2au - u^2}} + C$

Formas que contienen funciones trigonométricas

59. $\int \operatorname{sen} u\, du = -\cos u + C$

60. $\int \cos u\, du = \operatorname{sen} u + C$

61. $\int \tan u\, du = \ln|\sec u| + C$

62. $\int \cot u\, du = \ln|\operatorname{sen} u| + C$

63. $\int \sec u\, du = \ln|\sec u + \tan u| + C = \ln\left|\tan\left(\frac{1}{4}\pi + \frac{1}{2}u\right)\right| + C$

64. $\int \csc u\, du = \ln|\csc u - \cot u| + C = \ln\left|\tan\frac{1}{2}u\right| + C$

65. $\int \sec^2 u\,du = \tan u + C$

66. $\int \csc^2 u\,du = -\cot u + C$

67. $\int \sec u \tan u\,du = \sec u + C$

68. $\int \csc u \cot u\,du = -\csc u + C$

69. $\int \operatorname{sen}^2 u\,du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$

70. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$

71. $\int \tan^2 u\,du = \tan u - u + C$

72. $\int \cot^2 u\,du = -\cot u - u + C$

73. $\int \operatorname{sen}^n u\,du = -\frac{1}{n}\operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2} u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

75. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

76. $\int \cot^n u\,du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

78. $\int \csc^n u\,du = -\frac{1}{n-1}\csc^{n-2} u \cot u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

79. $\int \operatorname{sen} mu \operatorname{sen} nu\,du = -\frac{\operatorname{sen}(m+n)u}{2(m+n)} + \frac{\operatorname{sen}(m-n)u}{2(m-n)} + C$

80. $\int \cos mu \cos nu\,du = \frac{\operatorname{sen}(m+n)u}{2(m+n)} + \frac{\operatorname{sen}(m-n)u}{2(m-n)} + C$

81. $\int \operatorname{sen} mu \cos nu\,du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

82. $\int u \operatorname{sen} u\,du = \operatorname{sen} u - u \cos u + C$

83. $\int u \cos u\,du = \cos u + u \operatorname{sen} u + C$

84. $\int u^2 \operatorname{sen} u\,du = 2u \operatorname{sen} u + (2 - u^2)\cos u + C$

85. $\int u^2 \cos u\,du = 2u \cos u + (u^2 - 2)\operatorname{sen} u + C$

86. $\int u^n \operatorname{sen} u\,du = -u^n \cos u + n\int u^{n-1} \cos u\,du$

87. $\int u^n \cos u\,du = u^n \operatorname{sen} u - n\int u^{n-1} \operatorname{sen} u\,du$

88. $$\int \operatorname{sen}^m u \cos^n u\,du = \frac{\operatorname{sen}^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n}\int \operatorname{sen}^{m-2} u \cos^n u\,du$$
$$= \frac{\operatorname{sen}^{m+1} u \cos^{n-1} u}{m+n} + \frac{n-1}{m+n}\int \operatorname{sen}^m u \cos^{n-2} u\,du$$

Formas que contienen funciones trigonométricas inversas

89. $\int \operatorname{sen}^{-1} u\,du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$

90. $\int \cos^{-1} u\,du = u \cos^{-1} u - \sqrt{1-u^2} + C$

91. $\int \tan^{-1} u\,du = u \tan^{-1} u - \ln\sqrt{1+u^2} + C$

92. $\int \cot^{-1} u\,du = u \cot^{-1} u + \ln\sqrt{1+u^2} + C$

93. $$\int \sec^{-1} u\,du = u \sec^{-1} u - \ln\left|u + \sqrt{u^2-1}\right| + C$$
$$= u \sec^{-1} u - \cosh^{-1} u + C$$

94. $$\int \csc^{-1} u\,du = u \csc^{-1} u + \ln\left|u + \sqrt{u^2-1}\right| + C$$
$$= u \csc^{-1} u + \cosh^{-1} u + C$$

Formas que contienen funciones exponenciales y logarítmicas

95. $\int e^u\,du = e^u + C$

96. $\int a^u\,du = \frac{a^u}{\ln a} + C$

97. $\int ue^u\,du = e^u(u-1) + C$

98. $\int u^n e^u\,du = u^n e^u - n\int u^{n-1} e^u\,du$

99. $\int u^n a^u\,du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a}\int u^{n-1} a^u\,du + C$

100. $\int \frac{e^u\,du}{u^n} = -\frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^u\,du}{u^{n-1}}$

101. $\int \frac{a^u\,du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1}\int \frac{a^u\,du}{u^{n-1}}$

102. $\int \ln u\,du = u\ln u - u + C$

103. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

104. $\int \frac{du}{u\ln u} = \ln|\ln u| + C$

105. $\int e^{au}\operatorname{sen} nu\,du = \frac{e^{au}}{a^2+n^2}(a\operatorname{sen} nu - n\cos nu) + C$

106. $\int e^{au}\cos nu\,du = \frac{e^{au}}{a^2+n^2}(a\cos nu + n\operatorname{sen} nu) + C$

Formas que contienen funciones hiperbólicas

107. $\int \operatorname{senh} u\,du = \cosh u + C$

108. $\int \cosh u\,du = \operatorname{senh} u + C$

109. $\int \tanh u\,du = \ln|\cosh u| + C$

110. $\int \coth u\,du = \ln|\operatorname{senh} u| + C$

111. $\int \operatorname{sech} u\,du = \tan^{-1}(\operatorname{senh} u) + C$

112. $\int \operatorname{csch} u\,du = \ln|\tanh \tfrac{1}{2}u| + C$

113. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

114. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

115. $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

116. $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

117. $\int \operatorname{senh}^2 u\,du = \tfrac{1}{4}\operatorname{senh} 2u - \tfrac{1}{2}u + C$

118. $\int \cosh^2 u\,du = \tfrac{1}{4}\operatorname{senh} 2u + \tfrac{1}{2}u + C$

119. $\int \tanh^2 u\,du = u - \tanh u + C$

120. $\int \coth^2 u\,du = u - \coth u + C$

121. $\int u\operatorname{senh} u\,du = u\cosh u - \operatorname{senh} u + C$

122. $\int u\cosh u\,du = u\operatorname{senh} u - \cosh u + C$

123. $\int e^{au}\operatorname{senh} nu\,du = \frac{e^{au}}{a^2-n^2}(a\operatorname{senh} nu - n\cosh nu) + C$

124. $\int e^{au}\cosh nu\,du = \frac{e^{au}}{a^2-n^2}(a\cosh nu - n\operatorname{senh} nu) + C$