Mock Midterm Prep

Week 2

Definitions

- **Global Values**

  This is the definition for global values, blah blah blah

- **Local Values**

  A local value is like a global value but only around its neighbourhood

- **Saddle Point**

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- **Critical Point**
  This definition is critical, which is why it's red. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.

- **Quadratic Form**

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- **Critical Points**

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- **Principal Minor**

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Theorems

- **Second Derivative Test**

  Theorem description. Thing A if and only if Thing B, but don't confuse this with a definition, or Jaworski will dock you points!

- **Third Derivative Test**

  The same as the second derivative test, but again. Exercise: Induction.

- **Theorem**

  This one is unnamed so I didn't put a title.

- **Proposition**

  Like the last one but a proposition. Lorem ipsum dolor sit amet

- **Lemma**

  Like the last one but a lemma bladjfslkfjlskjflasd

- **A really important theorem**
  Theorem description. Thing A if and only if Thing B, but don't confuse this with a definition, or Jaworski will dock you points!

- **Corollary**

  And one of its corollaries.

Computations

1. **Lecture 2**. Given $f(x, y) = xye^{-x^2-y^2}$, $f : \mathbb{R}^2 \to \mathbb{R}$, with critical point $(0, 0)$, classify its nature.

2. **Lecture 2**. Given $f(x, y, z) = x^3 - y^2 + 3xy + z^2 - 2z$, $f : \mathbb{R}^3 \to \mathbb{R}$, find and classify its critical points.

3. **Lecture 2**. Let $f(x, y) = x^4 + y^4$. Find and classify its critical points.

4. **Homework 1**. Find the critical points of the following functions and determine their nature

   1. $f(x, y) = x^3 + y^3 - 9xy + 1$.
   2. $f(x, y) = (x - 1)(x^2 - y^2)$.
   3. $f(x, y, z) = 2x^2 + 3y^2 + 4z^2 - 3xy + 8z$.

5. **Homework 1**. Let $f(x, y) = x^3y^3$. Show that f has a saddle point at $(0, 0)$.

6. **Homework 1**. Find and classify the critical points $f(x, y) = sinx + siny + sin(x + y)$ inside the square $0 \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq \frac{\pi}{2}$.

Proofs

7. **Lecture 1**. Let Q be the quadratic form associated with an $n \times n$ matrix A. Prove that

    1. Q is positive definite $\iff \lambda_i > 0 \ \forall i \in \mathbb{N}$.
    2. Q is negative definite $\iff \lambda_i < 0 \ \forall i \in \mathbb{N}$.
    3. Q is indefinite $\iff$ there are positive and negative eigenvalues.

8. **Lecture 1**. Let A be an $n \times n$ matrix. If $detA \neq 0$, then,

    1. A is positive definite $\iff det(A_k) > 0, \forall k \in \mathbb{N}$.
    2. A is negative definite $\iff (-1)^k \cdot det(A_k) > 0, \forall k \in \mathbb{N}$.
    3. A is indefinite $\iff$ A is neither positive nor negative definite.

9. **Homework 1**. Prove or disprove:

1.