Optics Winter Notes

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Note: Theorem numbers come from the order they are presented in lecture, and do not correspond to any textbook or written course material.

Week 1

Radiant Units

Example of Laser Radiance:
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1. Solid angle:
2. Irradiance: $E_e = \frac{\text{power}}{\text{area}} =$
3. Irradiance of Lightbulb:
4. Divergence: $\tan \alpha_{\frac{1}{2}} = \frac{2.5 \times 10^{-3}\,\text{m}}{15\,\text{m}} \implies$

Definition of geometrical optics:
Under **geometrical optics**, we make the following assumptions:

1. **Space** is 3D and is isomorphic to $\mathbb{R}^3$ together with the Euclidean metric
2. **Time** is isomorphic to $\mathbb{R}$ together with the Euclidean metric,
3. Mediums are closed, connected subsets of $\mathbb{R}^3$. Every point in space belongs to a medium, and any two mediums are interior-disjoint.
4. Light is treated as a particle. That is, it has a well-defined position in space, and its wave properties are ignored.

Remark: The content of the next few lectures will be valid under geometrical optics.

Definition of a ray:
For a particular wave, a **wavefront** of that wave is a set of all connected points that show the same amplitude over time. A ray describes the direction of propagation of that wave, and is normal to the wavefront.

Proposition 1.1 Huygens' Principle
For light under geometrical optics, the following is true:

1. Light can be envisioned as a continuous array of discrete sources of light
2. The initial arrangement of these sources is called the **primary wavefront**
3. Each point source emits spherical waves, which we call **wavelets**
4. Wavelets form a **secondary wavefront**

Proposition 1.2 Fermat's Principle
When light propagates between two points, it takes the path of least time.
Remark: This was an evolution upon Hero's Principle, which states it takes the path of least distance. Fermat's model is superior in that it accounts for the difference in the speed of light in different mediums, and so shows refraction.

Definition of index of refraction:
The **index of refraction** of a medium is

Definition of the incident plane:
The **incident plane** of a ray incident on a surface is the plane defined by the direction vector of the ray and the vector normal to the surface. The angle between these vectors is called the **angle of incidence**.

Theorem 1.3 Law of Reflection
When a ray reflects at an interface between two media with different optical properties, the reflected ray:

1. remains within the incident plane
2. the angle of refraction, θ_r, is equal to the angle of incidence, θ_i

Question
When a ray enters a new medium, does it always reflect *and* refract?

Theorem 1.4 Snell's Law
We can find the index of refraction of a medium by

$$n = \frac{c}{v}$$

where n is the index of refraction, c is the speed of light in a vacuum, and v is the speed of light in that medium.

Theorem 1.5 Law of Refraction
When light is refracted at two media, the transmitted ray:

1. remains in the incident plane
2. $n_i \sin\theta_i = n_t \sin\theta_t$ (Snell's Law)

Example of:

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1. The distance the light travelled to the surface is shorter than it would be if refraction did not occur.
2. by Snell's Law:

$$\begin{aligned}
&n_1 \sin\theta_1 = n_2 \sin\theta_2 \\
&\sin\theta \approx \tan\theta \approx \theta \\
&\tan\theta_1 = \frac{x}{h} \\
&\tan\theta_1' = \frac{x}{h'} = \tan\theta_2 \\
&n_1 \frac{x}{h} = n_2 \frac{x}{h'} \\
&h' = \frac{n_2}{n_1} h = \frac{1}{1.33} 3\,\mathrm{m} = 2.25\,\mathrm{m}
\end{aligned}$$

Definition of total internal reflection:
Consider a ray entering a medium with lower index of refraction than the one it is entering from. In this case, the angle at which the ray exits will always be larger than the angle at which it enters. At some **critical angle of incidence**, the refracted light will propagate tangent to the surface. Above this critical angle of incidence, the ray will not exit the medium, and we have **total internal reflection**.

Theorem 1.6

Since $n_1 > n_2$, we have $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > \sin\theta_1$.
Since $\theta_2 \in [0, \frac{\pi}{2}]$, and sin is increasing in this range, $\theta_2 > \theta_1$.
At some critical angle θ_c, no light is transmitted, i.e. $\theta_2 = \frac{\pi}{2}$.
We compute $\sin\theta_c = \frac{n_2}{n_1}\sin\frac{\pi}{2} = \frac{n_2}{n_1}$.
For $\theta_1 > \theta_c$, the light stays within the media, we call this **total internal reflection**

Week 2

Imaging by Reflection and Refraction

2.1 Imaging by an Optical System

Definition of an optical system:
An **optical system** is a combination of reflecting and refracting surfaces (such as lenses and mirrors) that may alter the propagation direction of light.
Rays enter the optical system from an **object point**, then exit, and converge to an **image point**.

Proposition 2.1
Every ray from an object point to an image point has the same transit time.

Theorem 2.2 Principle of Reversibility
Consider an arbitrary object at O with its image at I. If we placed an object at I, the path of the rays would be exactly reversed, and that object would have an image at I.

2.1.1 Optical Axis and Paraxial Rays

Definition of an optical axis:
The **optical axis** defines the path along which light propagates, on average.
A system of simple lenses and mirrors is one in which the optical axis passes through all of their centres of curvature.
A **paraxial ray** is a ray at a small enough angle near the optical axis that the small angle approximation is appropriate.

2.2 Reflections at Spherical Surfaces

Definition of the curvature of a mirror:
For spherical mirrors in a system of simple mirrors, we have:

1. the **centre of curvature** is the point equidistant to every point on the surface of the mirror. Positive for mirror facing light, negative for facing away.

2. the **radius of curvature** is that distance between the center and the surface of the mirror. Positive for convex mirrors, negative for concave mirrors.

3. the **vertex** of the mirror is where the mirror intersects with the optical axis.

Theorem 2.3
For convex or concave mirrors under the paraxial approximation, we have

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$$

Proof. This proof is valid for convex mirrors only, but the rest is similar. Note from the geometry that $\theta = \alpha + \phi$, and $2\theta = \alpha + \alpha' \implies \alpha - \alpha' = -2\phi$.
Applying the small angle approximation (for paraxial rays), we find $\frac{h}{s} - \frac{h}{s'} = -2\frac{h}{R} \implies \frac{1}{s} - \frac{1}{s'} = -2\frac{2}{R}$. $\square$

2.2.1 Objects at Infinity

Definition of the focal length:
Consider an object on the optical axis, infinitely far from the mirror. All the rays of light in this case would be parallel to the optical axis, and the reflected rays' projections would intersect exactly at $s' = -\frac{R}{2}$.
We call this point the **focal point**,

$$f = -\frac{R}{2}$$
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Note: The focal point of a convex mirror will always be negative, and for a concave mirror will always be positive.

2.2.2 Lateral Magnification

Definition of the magnification factor:
The ratio of the image height to the object height, multiplied by its inversion along the y-axis, is

$$m = \frac{h_i}{h_o} = -\frac{s'}{s}$$

where h_i is negative for inverted images.

Remark: Optics using the paraxial approximation are referred to as **Gaussian** or **first-order** optics.

2.3 Refraction by Spherical Surfaces

Theorem 2.4
When light is refracted through a spherical surface, going from an n_1-medium to an n_2-medium, we have

$$\frac{n_1}{s} + \frac{n_2}{s} = \frac{n_2 - n_1}{R}$$

where s, s' are distances between the vertex and object, and vertex and image, respectively. The magnification ratio is

$$m = \frac{n_1 s'}{n_2 s}$$

Proof. By Snell's Law, we see $\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$. If $n_2 > n_1$: $\theta_2 < \theta_1$, then applying the paraxial approximating, we find $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$, with magnification factor $m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$. □