Geometrical Optics

PHYS 2202: Wave Motion and Optics

Lab Section A2

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0. Contents

1. Introduction

1.1 Objectives

This experiment was designed to explore a variety of different techniques that can be used to measure the focal length of a spherical lens. In the following pages we describe how theoretical equations were manipulated to make focal length depend on different measurable quantities, the different methods created for measuring those quantities, and how well those methods compare with each other.

In exploring the following techniques, we also gained a deeper intuitive understanding of the geometrical optics principles underlying the techniques. Namely, the laws of reflection, refraction, and magnification.

1.2 Thin Lens Equation

First, we use the "thin lens" equation:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

which tells us the relation between the focal length of a lens f, the distance of the object from the lens along the optical axis s_o, and the distance of the image along the optical axis s_i. Note that this equation assumes all rays are paraxial, and that the lens is spherical and of negligible thickness. Importantly, for an object on the optical axis, s_i is where the image will be in focus.

Then perhaps the most straightforward approach to measure the focal length is to place an object along the optical axis and measure where the image is in focus. In order to make use of these quantities, we rearrange the thin lens equation to find:

$$f = \frac{s_o s_i}{s_o + s_i} \tag{1.1}$$

Alternatively, using many trials, we may plot $\frac{1}{s_o}$ against $\frac{1}{s_i}$ and find the focal length at the y-intercept, that is, as $\frac{1}{s_i} \to 0$. This concept is exploited further in the next section.

1.2.1 Limits at Infinity

There is yet more we can do. Note that in the thin lens equation, if $s_o \to \infty$, then that respective term goes to zero, and we obtain an extremely simple relation for f. Namely,

$$\begin{aligned} \frac{1}{f} &= \lim_{s_o \to \infty} \left(\frac{1}{s_o} + \frac{1}{s_i} \right) \\ &= \frac{1}{s_i}, \end{aligned}$$

or equivalently,

$$f = s_i \tag{1.2}$$

A similar process with $s_i \to \infty$ gives us

$$f = s_o \tag{1.3}$$

Finally, we exploit the concept of an "object at infinity" by noting that it is equivalently an object emanating parallel rays of light. Thus we can approximate an object at infinity by a distant object, and simulate an image at infinity by a mirror.

1.2.2 Using Magnification

For this method we use what we know about magnification under the previously stated approximations, namely,

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}, \tag{1.4}$$

which we combine with the thin lens equation to find

$$\frac{1}{M} = -\frac{s_o}{f} + 1 \tag{1.5}$$

So may plot $\frac{1}{M}$ against s_o to find f as the slope.

1.3 Lensmaker Equation

The lensmaker equation gives us a way to find the focal point of a double lens based on its geomtry. In particular,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Downarrow$$

$$f = \frac{R_1 R_2}{(R_2 - R_1)(n-1)} \tag{1.6}$$

where R_1, R_2 are the radii of curvature of each side of the lens, and n is the refractive index of the lens' material. Using a spherometer, these radii are found by

$$R = \frac{r^2 + d^2}{2r} \tag{1.7}$$

where r is the spherometer reading, and d is the distance from the leg of the spherometer to the screw. So by simply taking some readings of the lens using the spherometer, we are able to find its focal length.

2. Experimental setup

2.1 Apparatus

The instruments used were as follows:

- A lamp
- A mirror
- A screen
- A biconvex BK7 glass lens with refractive index 1.51502
- A metre stick (± 0.5 mm)
- A ruler (0 to 30cm) (± 0.5 mm)
- Two vertex pointers of length (152.47 ± 0.02)mm
- A G&G Spherometer (-8 to 8mm) ($\pm 25\,\mu$m) with a leg-to-screw distance of 22.5 mm (measured by ruler)

2.2 Procedure

Distant Object For this part of the procedure, we used the ceiling lights as our object, estimating them to be about 2.5m above the desk, which we used as our screen. A lens was held above the desk until the image of the ceiling lights was in sharp focus, at which point the height of the lens was measured. This was repeated over 5 trials.

Mirror Method Next, the lens was secured onto a flat surface to ensure the precision of the optical axis. A lamp, our object, was placed to the left, and a mirror to the right (as in fig. 2.1). The distance between the object and the lens was then adjusted until the image was in sharp focus, at which point all distances were measured. This was repeated over 5 trials.

Thin Lens (Imprecise) At this point, the mirror was replaced by a screen (as in fig. 2.2), and the lens and screen position were adjusted until the image was in sharp focus, at which point all distances were measured. This was repeated over 10 trials.

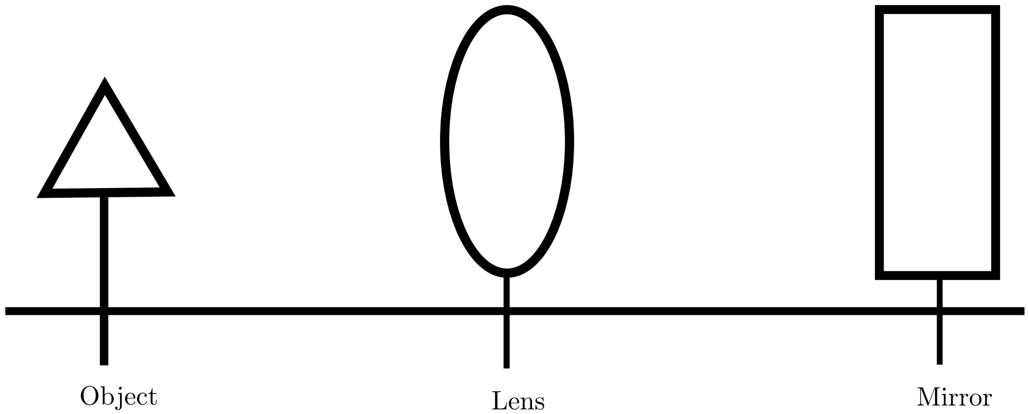

Figure 2.1: Experimental setup for the mirror method portion of the experiment

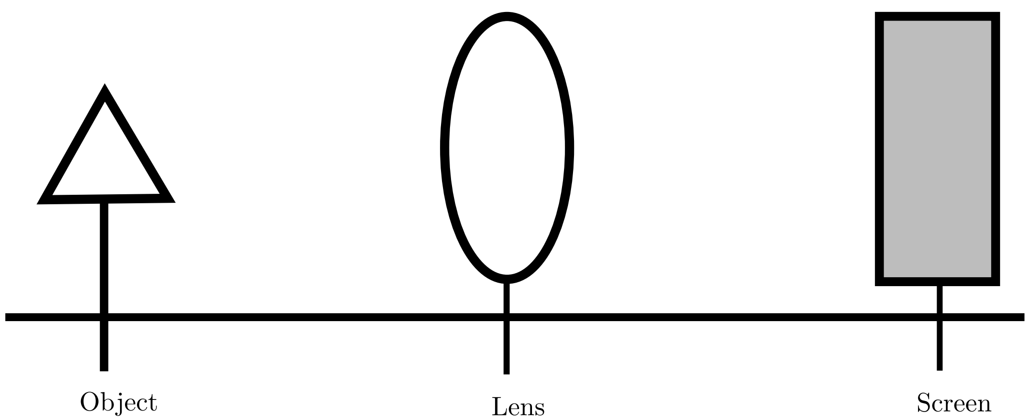

Figure 2.2: Experimental setup for the imprecise thin lens portion of the experiment

Thin Lens and Magnification First, the height of the object was measured with a ruler. To add a final layer of precision to our previous setup, two vertex pointers were used to measure distance. One was placed between the lamp and the lens, the other between the lens and the screen. Once again, the lens and the screen's positions were adjusted until the image was in sharp focus. At this point, four lateral measurements were taken: the position of the left vertex pointer touching the lamp; the position of the left vertex pointer touching the lens; the position of the right vertex pointer touching the lens; and the position of the right vertex pointer touching the screen. This setup is illustrated in fig. 2.3. In addition to these, the height of the image was also measured using a ruler.

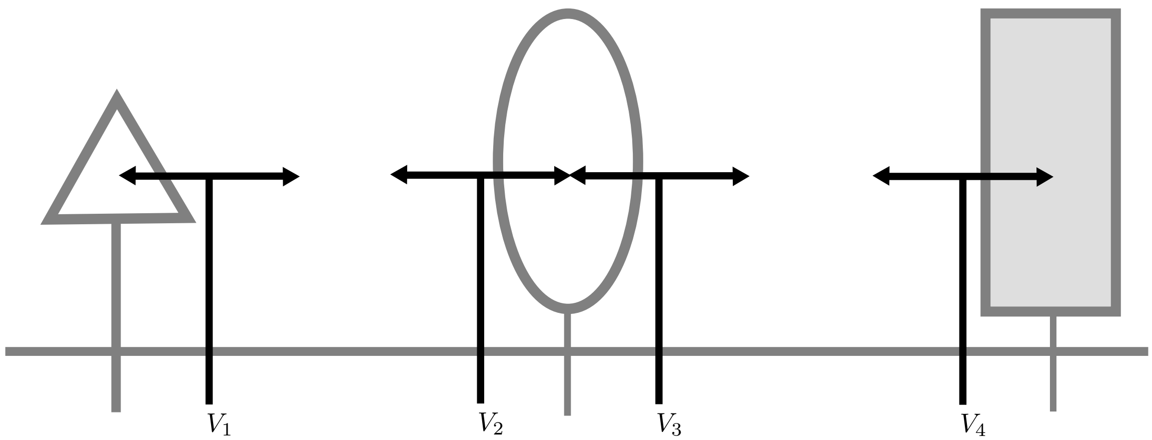

Figure 2.3: Experimental setup for the thin lens and magnification portions of the experiment

Lensmaker First, the distance between the central screw of the spherometer and the leg of the spherometer was measured using a ruler. Then the spherometer was placed atop one face of the lens, and the screw's vertical position adjusted until it touched the face of the lens. This spherometer reading was recorded, and the procedure was repeated on the lens' second side. The entire procedure was then repeated over 3 trials.

3. Observations

All measurements taken with ruler, metre stick, or spherometer. Any uncertainties not listed are equal to the reading uncertainty of the metre stick, that is, $\pm 0.5\,[\mathrm{mm}]$.

3.1 Distant Object

Trial	Image Distance [mm] $\left(\pm\frac{\sqrt{2}}{2}\right)$
1	190
2	190
3	195
4	190
5	190

Table 3.1: Measurements taken of image distance for the distant object method

3.2 Mirror Method

Trial	Object Position [mm]	Lens Position [mm]
1	150	343
2	170	366
3	200	395
4	250	445
5	300	494

Table 3.2: Measurements taken of object and lens when image in focus

3.3 Thin Lens Equation

Trial	Object Position [mm]	Lens Position [mm]	Screen Position [mm]
1	150	550	940
2	150	500	952
3	150	600	957.5
4	150	650	983
5	150	520	944
6	150	540	942
7	150	560	948
8	150	580	950
9	150	480	966.5
10	150	460	998

Table 3.3: Measurements taken of object and lens when image in focus

3.4 Magnification and Thin Lens: Vertex Pointers

Object Height [mm] $\left(\pm\frac{\sqrt{2}}{2}\right)$	Distance (no Lens) [mm] $\left(\pm\frac{\sqrt{2}}{2}\right)$	Distance (Lens) [mm] $\left(\pm\frac{\sqrt{2}}{2}\right)$
19.5	101	107

Table 3.4: Measurements taken once, at the start of this portion

Trial	V_1 [mm]	V_2 [mm]	V_3 [mm]	V_4 [mm]	Img. Height [mm] $\left(\pm\frac{\sqrt{2}}{2}\right)$
1	221	516	685.5	877.5	-15
2	221	496.5	665	869	-16
3	221	477	645	868.5	-17.5
4	221	456.5	624.5	861	-19
5	221	437	645	867	-21
6	221	417	584.5	866	-25
7	221	396.5	565	887.5	-27.5
8	221	375.5	444.5	925.5	-33
9	221	545	712	894	-14

Table 3.5: Measurements taken of object, lens, and image height when image in focus

3.5 Lensmaker Equation

Trial	Front Reading [mm] (± 0.025)	Back Reading [mm] (± 0.025)
1	1.34	1.41
2	1.275	1.3
3	1.28	1.285

Table 3.6: Spherometer readings of front and back lenses

4. Data analysis

4.1 Distant Object

Since the focal length for this method is the same as the image distance, which was directly measured, we simply take the focal length to be the average of the measurements recorded in table 3.1:

$$
\begin{aligned}
f_{\mathrm{DO}} &= \frac{\sum s_i}{N} \\
&= \frac{200\,\mathrm{mm} + 190\,\mathrm{mm} + 195\,\mathrm{mm} + 190\,\mathrm{mm} + 200\,\mathrm{mm}}{5} \\
&= 195\,\mathrm{mm}
\end{aligned}
$$

Now the reading error of the ruler was 0.5 mm, and each of the 5 measurements was the result of making 2 readings. As such,

$$
\begin{aligned}
\sigma_{\mathrm{reading}} &= \frac{1}{N}\sqrt{\sum(2\sigma^2)} \\
&= \frac{1}{5}\sqrt{10(0.5\,\mathrm{mm})^2} \\
&= 0.316\,\mathrm{mm};
\end{aligned}
$$

the standard deviation was

$$
\begin{aligned}
\sigma_{\mathrm{st.\ dev.}} &= \sqrt{\frac{\sum(s_i - f_{\mathrm{DO}})^2}{N-1}} \\
&= \sqrt{\frac{(5\,\mathrm{mm})^2 + (5\,\mathrm{mm})^2 + +(0\,\mathrm{mm})^2 + (5\,\mathrm{mm})^2 + (5\,\mathrm{mm})^2}{4}} \\
&= 5\,\mathrm{mm};
\end{aligned}
$$

and the systematic error was

$$\begin{aligned}\sigma_{\text{sys}} &= \frac{f_{\text{DO}}^2}{2500\,\text{mm} - f_{\text{DO}}} \\ &= \frac{(195\,\text{mm})^2}{2305\,\text{mm}} \\ &= 16.5\,\text{mm}.\end{aligned}$$

Since the systematic error is the highest of these, we take $\sigma_{\text{DO}} = \sigma_{\text{sys}}$ and

$$f_{\text{DO}} = (195 \pm 17)\text{mm}.$$

4.2 Mirror Method

The object distance was found by subtracting the object positions from the image positions from table 3.2, and the results summarized in table 4.1.

Trial	s_o[mm]
1	193
2	196
3	195
4	195
5	193

Table 4.1: Results of object distance calculations for mirror method

By the same method as in **Distant Object**, we find $f_{\text{MM}} = 194.6\,\text{mm}$, $\sigma_{\text{reading}} = 0.316\,\text{mm}$, and $\sigma_{\text{st. dev.}} = 1.14\,\text{mm}$. Since $\sigma_{\text{st. dev.}} > 2\sigma_{\text{reading}}$, we take $\sigma_{\text{MM}} = \sigma_{\text{st. dev.}}$ and obtain

$$f_{\text{MM}} = (194.6 \pm 1.1)\text{mm}.$$

4.3 Thin Lens (Individual)

As in the mirror method, the object distance was found by taking the difference between the lens and object positions; similarly, image distance was found by taking the difference between the image and lens. The values were taken from table 3.3, and the results obtained are summarized in table 4.2.

As in the previous two methods, the reading error on each of these is $\sigma_s = \sqrt{2\left(\frac{1}{2}\right)^2} = 0.707\,\text{mm}$.

From these values, the focal length f is found by eq. (1.1):

Trial	s_o[mm]	s_i[mm]
1	400	390
2	350	452
3	450	357.5
4	500	333
5	370	424
6	390	402
7	410	388
8	430	370
9	330	486.5
10	310	538

Table 4.2: Results of distance calculations for thin lens method (individual)

$$\begin{aligned} f &= \frac{s_o s_i}{s_o + s_i} \\ &= \frac{(400\,\mathrm{mm})\,(390\,\mathrm{mm})}{(400\,\mathrm{mm}) + (390\,\mathrm{mm})} \\ &= 197.47\,\mathrm{mm} \end{aligned}$$

and the error by

$$\begin{aligned} \sigma_f &= \frac{\sigma_s}{s_o + s_i}\sqrt{s_i^2 + s_o^2} \\ &= \frac{0.707\,\mathrm{mm}}{400\,\mathrm{mm} + 390\,\mathrm{mm}}\sqrt{(400\,\mathrm{mm})^2 + (390\,\mathrm{mm})^2} \\ &= 0.5\,\mathrm{mm}. \end{aligned}$$

The results of these calculations are summarized in table 4.3. Fun fact: as $s_i \to s_o$, $\sigma_f \to 0.5\,\mathrm{mm}$. The proof is left as an exercise to the reader :)

From here we take the average, reading uncertainty, and standard deviation as in **Distant Object** to find $f_{\mathrm{TLI}} = 198.09\,\mathrm{mm}$, $\sigma_{\mathrm{reading}} = 0.16\,\mathrm{mm}$, and $\sigma_{\mathrm{st.\ dev.}} = 1.16\,\mathrm{mm}$. Since $\sigma_{\mathrm{st.\ dev.}} > 2\sigma_{\mathrm{reading}}$, we take $\sigma_{\mathrm{TLI}} = \sigma_{\mathrm{st.\ dev.}}$ and obtain

$$f_{\mathrm{TLI}} = (198.1 \pm 1.2)\mathrm{mm}.$$

4.4 Thin Lens and Magnification (Plotted)

First, the width of the lens was found by subtracting the distance without it from the distance with it (from table 3.4), and its error as in **Distant Object**. It was found to be $d_{\mathrm{lens}} = 107\,\mathrm{mm} - 101\,\mathrm{mm} \pm \sqrt{4(0.5\,\mathrm{mm})} = (6.0 \pm 1.0)\mathrm{mm}$.

Because of how they were measured, each object distance was calculated by

Trial	f[mm]	σ_f[mm]
1	197.47	0.50
2	197.26	0.50
3	199.23	0.50
4	199.88	0.51
5	197.58	0.50
6	197.95	0.50
7	199.35	0.50
8	198.88	0.50
9	196.63	0.51
10	196.67	0.52

Table 4.3: Results of focal length calculations for thin lens method (individual)

$$\begin{aligned} s_o &= V_2 - V_1 + d_{\text{vp}} + \frac{d_{\text{lens}}}{2} \\ &= 516\,\text{mm} - 221\,\text{mm} + 152.47\,\text{mm} + \frac{6\,\text{mm}}{2} \\ &= 450.47\,\text{mm} \end{aligned}$$

and similarly for image distance, each with reading error

$$\begin{aligned} \sigma_s &= \sqrt{\sigma_{V_1}^2 + \sigma_{V_2}^2 + \sigma_{\text{vp}}^2 + \frac{\sigma_{\text{lens}}^2}{4}} \\ &= \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2500} + \frac{1}{4}} \\ &= 1.118\,\text{mm}. \end{aligned}$$

From these, $\frac{1}{s_o}, \frac{1}{s_i}$ were found simply by inversion.
Finally, magnification was found by eq. (1.4):

$$\begin{aligned} \frac{1}{M} &= \frac{h_o}{h_i} \\ &= \frac{19.5\,\text{mm}}{-15\,\text{mm}} \\ &= -1.3. \end{aligned}$$

with error

$$
\begin{aligned}
\sigma_{\frac{1}{M}} &= \frac{\sigma_h}{h_i}\sqrt{1+\left(\frac{1}{M}\right)^2} \\
&= \frac{0.707\,\text{mm}}{-15\,\text{mm}}\sqrt{1+1.3^2} \\
&= -0.0773.
\end{aligned}
$$

The results of all these calculations are summarized in table 4.4 and table 4.5.

Trial	$\frac{1}{s_i}$[mm^{-1}]	$\frac{1}{s_o}$[mm^{-1}]	$\sigma_{\frac{1}{s_i}}$[mm^{-1}]	$\sigma_{\frac{1}{s_o}}$[mm^{-1}]
1	2.22E-03	2.88E-03	6.97E-06	1.17E-05
2	2.32E-03	2.78E-03	7.61E-06	1.09E-05
3	2.43E-03	2.64E-03	8.35E-06	9.85E-06
4	2.56E-03	2.55E-03	9.25E-06	9.21E-06
5	2.69E-03	2.65E-03	1.02E-05	9.93E-06
6	2.85E-03	2.29E-03	1.14E-05	7.41E-06
7	3.02E-03	2.09E-03	1.29E-05	6.19E-06
8	3.23E-03	1.57E-03	1.47E-05	3.49E-06
9	2.09E-03	2.96E-03	6.15E-06	1.24E-05

Table 4.4: Values to plot in thin lens (plotted) method

Trial	s_o[mm]	$\frac{1}{M}$	σ_{s_o}[mm]	$\sigma_{\frac{1}{M}}$
1	450.470	-1.300	1.118	-0.077
2	430.970	-1.219	1.118	-0.070
3	411.470	-1.114	1.118	-0.060
4	390.970	-1.026	1.118	-0.053
5	371.470	-0.929	1.118	-0.046
6	351.470	-0.780	1.118	-0.036
7	330.970	-0.709	1.118	-0.032
8	309.970	-0.591	1.118	-0.025
9	479.470	-1.393	1.118	-0.087

Table 4.5: Values to plot in magnification method

Following the Least Squares Method, the delta value was calculated for the magnification method by

$$
\begin{aligned}
\Delta &= N\sum^{N} s_o^2 - \left(\sum^{N} s_o\right)^2 \\
&= 9\left((450.47\,\text{mm})^2 + (430.97\,\text{mm})^2 + \cdots\right) - (450.47\,\text{mm} + 430.97\,\text{mm} + \cdots)^2 \\
&= 229\,376\,\text{mm}^2.
\end{aligned}
$$

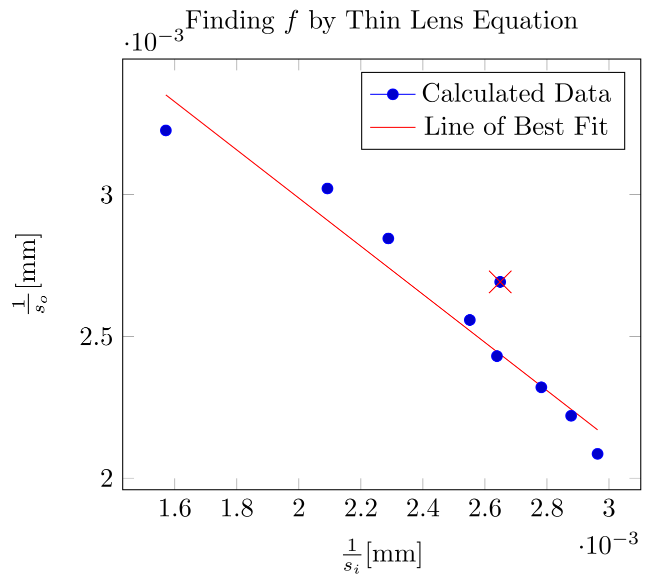

Figure 4.1: Plot of the values from table 4.4

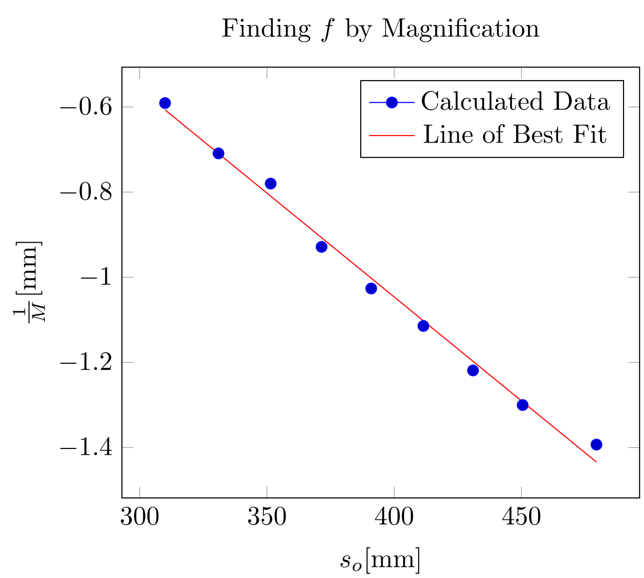

Figure 4.2: Plot of the values from table 4.5

With the delta value in hand, the slope was calculated by

$$
\begin{aligned}
m &= \frac{1}{\Delta}\left(N\sum^{N} s_o \frac{1}{M} - \sum^{N} s_o \sum^{N} \frac{1}{M}\right) \\
&= \frac{1}{229\,376\,\mathrm{mm}^2}\left(9\left((450.47\,\mathrm{mm})(-1.3) + \cdots\right) - (450.47\,\mathrm{mm} + \cdots)(-1.3 + \cdots)\right) \\
&= -0.004\,88\,\mathrm{mm}^{-1},
\end{aligned}
$$

and the intercept by

$$
\begin{aligned}
b &= \frac{1}{\Delta}\left(\sum^{N} s_o^2 \sum^{N} \frac{1}{M} - \sum^{N} s_o \sum^{N} s_o \frac{1}{M}\right) \\
&= \frac{1}{229\,376\,\mathrm{mm}^2}\left(\left((450.47\,\mathrm{mm})^2 + \cdots\right)(-1.3 + \cdots) - (450.47\,\mathrm{mm} + \cdots)\left((450.47\,\mathrm{mm})(-1.3) + \cdots\right)\right) \\
&= 0.905.
\end{aligned}
$$

These values were also determined for the thin lens plot by the same procedure, and the results summarized in table 4.6. Note that one point was excluded (shown in the figure) since it is an outlier.

Method	m	b	σ_m	σ_b
Thin Lens	-0.848	$0.004\,68\,\mathrm{mm}^{-1}$	1.55	$0.003\,89\,\mathrm{mm}^{-1}$
Magnification	-$0.004\,88\,\mathrm{mm}^{-1}$	0.905	$0.000\,159\,\mathrm{mm}^{-1}$	0.06298

Table 4.6: Slope and intercept values for lines of best fit

Finally, we invert the Thin Lens intercept and the Magnification slope to obtain:

$$
\begin{aligned}
f_{\mathrm{TLP}} &= (210 \pm 180)\mathrm{mm} \\
f_{\mathrm{MAG}} &= (205 \pm 7)\mathrm{mm}
\end{aligned}
$$

(for $f = \frac{1}{x}$, $\sigma_f = \frac{\sigma_x}{x^2}$)

4.5 Lensmaker

We calculate the radius of curvature from the spherometer readings by eq. (1.7)

$$
\begin{aligned}
R &= \frac{r^2 + d^2}{2r} \\
&= \frac{(1.34\,\mathrm{mm})^2 + (22.5\,\mathrm{mm})^2}{2\,(1.34\,\mathrm{mm})} \\
&= 189.6\,\mathrm{mm}
\end{aligned}
$$

with error

$$\begin{aligned}
\sigma_R &= \sqrt{\frac{\left(r^2-d^2\right)^2}{4r^4}\sigma_r^2 + \frac{d^2}{r^2}\sigma_d^2} \\
&= \sqrt{\frac{\left(\left(1.34\,\mathrm{mm}\right)^2 - \left(22.5\,\mathrm{mm}\right)^2\right)^2}{4\left(1.34\,\mathrm{mm}\right)^4}\left(0.025\,\mathrm{mm}\right)^2 + \frac{\left(22.5\,\mathrm{mm}\right)^2}{\left(1.34\,\mathrm{mm}\right)^2}\left(0.707\,\mathrm{mm}\right)^2} \\
&= 12.4\,\mathrm{mm}
\end{aligned}$$

making sure to respect the sign convention. With the radii in hand, then the focal length for each trial is calculated by eq. (1.6)

$$\begin{aligned}
f &= \frac{R_1 R_2}{\left(R_2 - R_1\right)\left(n-1\right)} \\
&= \frac{\left(189.6\,\mathrm{mm}\right)\left(-180.2\,\mathrm{mm}\right)}{\left(\left(-180.2\,\mathrm{mm}\right) - \left(189.6\,\mathrm{mm}\right)\right)\left(1.51502 - 1\right)} \\
&= 179.39\,\mathrm{mm}
\end{aligned}$$

with error

$$\begin{aligned}
\sigma_f &= \frac{\sqrt{R_2^2\sigma_{R_1}^2 + R_1^2\sigma_{R_2}^2}}{(n-1)|R_2 - R_1|} \\
&= \frac{\sqrt{\left(-180.2\,\mathrm{mm}\right)^2\left(12.4\,\mathrm{mm}\right)^2 + \left(189.6\,\mathrm{mm}\right)^2\left(11.7\,\mathrm{mm}\right)^2}}{(n-1)|-180.2\,\mathrm{mm} - 189.6\,\mathrm{mm}|} \\
&= 16.5\,\mathrm{mm}
\end{aligned}$$

The full results of these calculations are summarized in table 4.7

Trial	R_1[mm]	R_2[mm]	σ_{R_1}[mm]	σ_{R_2}[mm]	f[mm]	σ_f[mm]
1	189.5693	-180.226	12.38151	11.72059	179.3908	16.5343
2	199.1669	-195.362	13.06773	12.79475	191.4934	17.7524
3	198.3939	-197.627	13.01218	12.95711	192.2349	17.82745

Table 4.7: Results of focal length calculations for lensmaker method

For our final focal length, we take the average of these values, and as in **Distant Object**, obtain $f_{\mathrm{LNS}} = 187.7\,\mathrm{mm}$, $\sigma_{\mathrm{reading}} = 10\,\mathrm{mm}$, and $\sigma_{\mathrm{st.\ dev.}} = 7.2\,\mathrm{mm}$. Since $\sigma_{\mathrm{st.\ dev.}} \leq 2\sigma_{\mathrm{reading}}$, we take $\sigma_{\mathrm{LNS}} = \sigma_{\mathrm{reading}}$, and obtain

$$f_{\mathrm{LNS}} = (188 \pm 10)\mathrm{mm}.$$

4.6 Combination of Measurements

Happily, all of our results are in agreement! In particular, note $f = 196\,\text{mm}$ is within two uncertainties of every value. As such, we include them all in our weighted average, as follows:

$$\begin{aligned}
f_{\text{weighted average}} &= \frac{\sum \frac{f}{\sigma_f^2}}{\sum \frac{1}{\sigma_f^2}} \\
&= \frac{\frac{195\,\text{mm}}{(17\,\text{mm})^2} + \frac{194.6\,\text{mm}}{(1.1\,\text{mm})^2} + \cdots}{\frac{1}{(17\,\text{mm})^2} + \frac{1}{(1.1\,\text{mm})^2} + \cdots} \\
&= 196.2\,\text{mm}
\end{aligned}$$

with error

$$\begin{aligned}
\sigma_{\text{weighted}} &= \frac{1}{\sqrt{\sum \frac{1}{\sigma_f^2}}} \\
&= \frac{1}{\sqrt{\frac{1}{(17\,\text{mm})^2} + \frac{1}{(1.1\,\text{mm})^2} + \cdots}} \\
&= 0.8\,\text{mm}
\end{aligned}$$

for a final value of

$$f = (196.2 \pm 0.8)\text{mm}.$$

5. Results

Happily, every single value was in agreement within two uncertainties. In order from most to least precise, the **Mirror Method** yielded $f_{\text{MM}} = (194.6 \pm 1.1)$mm; the **Thin Lens (Individual)** method yielded $f_{\text{TLI}} = (198.1 \pm 1.2)$mm; the **Magnification** method yielded $f_{\text{MAG}} = (205 \pm 7)$mm; the **Lensmaker** technique yielded $f_{\text{LNS}} = (188 \pm 10)$mm; the **Distant Object** method yielded $f_{\text{DO}} = (195 \pm 17)$mm; and finally, the **Thin Lens (Plotted)** method yielded $f_{\text{TLP}} = (210 \pm 180)$mm.

The weighted average of all of these was $f = (196.2 \pm 0.8)$mm.

6. Discussion

Technically, the most accurate technique was the **Distant Object** technique, yielding $f = (195 \pm 17)$mm, though it was extremely imprecise. Both the **Mirror** and **Thin Lens (Individual)** methods were extremely precise, and only slightly less accurate. The mirror method in particular is notable for how simple and quick it was to set up.

The **Magnification** technique was reasonably effective, as was the **Lensmaker** technique, but the graphical **Thin Lens** method was uniquely ineffective at $f = (210 \pm 180)$mm being both extremely imprecise, and the least accurate. It also was the part of the experiment that took us the most time to perform, *and* had a more complicated analysis. As this was based on the graph's y-intercept, that is, when $\frac{1}{s_o} \to 0$, it is even worse considering how much more effective the distant object technique was with the same theoretical justification and less precise instruments.

Aside from this, one notable flaw in the distant object technique that was not properly accounted for, was how imprecise the readings taken of the image distance were. In real-time, hands were wobbling, and the smallest division of the ruler was certainly not the smallest division we were able to differentiate; as such, the reading error should have been much higher. The experiment also required us to simply estimate the height of the ceiling, which is totally arbitrary and drastically affects the systematic error. While we could have also estimated a higher reading error, there is no reason to introduce such arbitrary "measurements" into our labs unless it's strictly necessary. The first problem would have been easily solved with a retort stand; the second, with a measuring tape and a ladder.