Intro Lab - Review Exercise Solutions

By Camila Restrepo 101230443, Lab Section A2

(Code for clearing workspace every time the file is run)

```
clear
clc
```

First, for ease of use, the data provided for each question was inputted into an external intro.xlsx file, attached with the solution. The following code was used to identify and separate the data as needed:

```
file_path = 'intro.xlsx';
Q1_data = table2array(readtable(file_path,'Sheet',1));
```

```
Warning: Column headers from the file were modified to make them valid MATLAB identifiers before creating
variable names for the table. The original column headers are saved in the VariableDescriptions property.
Set 'VariableNamingRule' to 'preserve' to use the original column headers as table variable names.
```

```
Q2_data = table2array(readtable(file_path,'Sheet',2));
```

```
Warning: Column headers from the file were modified to make them valid MATLAB identifiers before creating
variable names for the table. The original column headers are saved in the VariableDescriptions property.
Set 'VariableNamingRule' to 'preserve' to use the original column headers as table variable names.
```

```
Q3_data = table2array(readtable(file_path,'Sheet',3));
```

```
Warning: Column headers from the file were modified to make them valid MATLAB identifiers before creating
variable names for the table. The original column headers are saved in the VariableDescriptions property.
Set 'VariableNamingRule' to 'preserve' to use the original column headers as table variable names.
```

S1. Reading Error vs. Statistical Error

Since the order of the trials should not matter for any of these calculations, we isolate the focal length data:

```
focal_lengths = Q1_data(:,2);
```

The mean, $\overline{x}$, of n values is calculated by $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ where x_i stands for each value. In this case,

$$\frac{12.35 + 11.41 + 10.97 + 13.33 + 11.55 + 12.12}{6} = 11.955$$

So we have:

```
n = 6; mean = 11.955;
```

Next, we find the standard deviation, σ, of our set by $\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2}$.

We automate this computation by

```
standard_deviation = sqrt((1 / (n - 1)) * sum((focal_lengths - mean).^2))
```

```
standard_deviation = 0.8373
```

We note now that twice the reading error of an analog instrument is the same as that instrument's smallest division. In this case, $0.1\,\mathrm{cm}$. Since the standard deviation is $\approx 0.8\,\mathrm{cm} > 0.1\,\mathrm{cm}$, we will need to calculate the standard deviation of the mean, σ_{mean}, to use as our error.

For this purpose, we use $\sigma_{\mathrm{mean}} = \dfrac{\sigma}{n} = \dfrac{0.8373\,\mathrm{cm}}{6} = 0.13955\,\mathrm{cm}$.

So we report our final measurement as $(11.96 \pm 0.14)\,\mathrm{cm}$.

S2. Error Propagation

First, we load our data using

```
diffraction_order = Q2_data(1,1);
wavelength = Q2_data(1,2); sigma_wavelength = Q2_data(1,3);
angle_degrees = Q2_data(1,4); sigma_angle_degrees = Q2_data(1,5);
```

Then, we convert the angle measurement to radians by $x^\circ = \dfrac{x\pi}{180}\,\mathrm{rad}$ to find

```
angle = angle_degrees * pi / 180
```

```
angle = 0.0032
```

```
sigma_angle = sigma_angle_degrees * pi / 180
```

```
sigma_angle = 1.7453e-04
```

Now we can calculate the width of the slit by $a = \dfrac{m\lambda}{\sin\theta} = \dfrac{1(632.832\,\mathrm{nm})}{\sin(0.0032)}$, or

```
slit_width = diffraction_order * wavelength / sin(angle)
```

```
slit_width = 2.0032e+05
```

To find the uncertainty, we apply error propagation, in this case

$$\sigma_a = \sqrt{\left(\frac{\partial}{\partial\lambda}a\right)^2\sigma_\lambda^2 + \left(\frac{\partial}{\partial\theta}a\right)^2\sigma_\theta^2} = \frac{m}{\sin\theta}\sqrt{\sigma_\lambda^2 + \frac{\lambda^2\sigma_\theta^2}{\tan^2\theta}}, \text{ or}$$

```
sigma_a = (1 / sin(angle)) * sqrt(sigma_wavelength^2 + (wavelength * sigma_angle /
tan(angle))^2)
```

```
sigma_a = 1.1068e+04
```

So we state our final result as $(200 \pm 11)\,\mu\mathrm{m}$.

S3. Least Squares Method and χ^2 Goodness of Fit

First, we load our data using

```
displacements = Q3_data(:,1);
phases = Q3_data(:,2); sigma_phases = Q3_data(:,3);
```

We plot our data with

```
errorbar(displacements, phases, sigma_phases);
hold on
xlabel('Displacement [m]'); ylabel('Phase [rad]');
```

Using the weighted least squares method, that is, using the equations:

$$\sigma_y \approx \sqrt{\frac{1}{n-2}\sum_i (y_i - y(x_i))^2}$$

$$\Delta = \sum_i \frac{1}{\sigma_{y_i}^2}\sum_i \frac{x_i^2}{\sigma_{y_i}^2} - \left(\sum_i \frac{x_k}{\sigma_{y_i}^2}\right)^2$$

$$m = \frac{1}{\Delta}\left(\sum_i \frac{1}{\sigma_{y_i}^2}\sum_i \frac{x_i y_i}{\sigma_{y_i}^2} - \left(\sum_i \frac{x_i}{\sigma_{y_i}^2}\right)\left(\sum_i \frac{y_i}{\sigma_{y_i}^2}\right)\right), \text{ and}$$

$$b = \frac{1}{\Delta}\left(\sum_i \frac{x_i^2}{\sigma_{y_i}^2}\sum_i \frac{y_i}{\sigma_{y_i}^2} - \left(\sum_i \frac{x_i}{\sigma_{y_i}^2}\right)\left(\sum_i \frac{x_i y_i}{\sigma_{y_i}^2}\right)\right)$$

we find a line of best fit.

```
n = length(phases);
delta = n * sum(displacements.^2) - sum(displacements)^2;
m = (1 / delta) * (n * sum(displacements .* phases) - sum(displacements) *
sum(phases));
b = (1 / delta) * (sum(displacements.^2) * sum(phases) - sum(displacements) *
sum(displacements .* phases));

sigma_y = sqrt((1/(n-2)) * sum((phases - (m * displacements + b)).^2));
delta_weighted = sum(1 ./ sigma_phases.^2) * sum(displacements.^2 ./
sigma_phases.^2) - (sum(displacements ./ sigma_phases.^2))^2;
m_weighted = (1 / delta_weighted) * (sum(1 ./ sigma_phases.^2) *
sum(displacements .* phases ./ sigma_phases.^2) - sum(displacements ./
sigma_phases.^2) * sum(phases ./ sigma_phases.^2))
```

```
m_weighted = 1.2400
```

```
b_weighted = (1 / delta_weighted) * (sum(displacements.^2 ./ sigma_phases.^2)
* sum(phases ./ sigma_phases.^2) - sum(displacements ./ sigma_phases.^2) *
sum(displacements .* phases ./ sigma_phases.^2))
```

```
b_weighted = -0.1049
```

So the slope and intercept of the best fit line are $1.214\ \frac{\mathrm{rad}}{\mathrm{m}}$ and $-0.1181\ \mathrm{rad}$, respectively.

```
yfit = m_weighted * displacements + b_weighted;
plot(displacements, yfit, 'r-');
```

We calculate $\chi^2_{\min}$ by $\sum_{i=1}^{n} \frac{(x_i - x_{\mathrm{expected}})^2}{x_{\mathrm{expected}}}$ and find

```
min_chi_squared = sum((phases - yfit).^2 ./ abs(yfit))
```

```
min_chi_squared = 0.0964
```

Since we have 2 parameters, there are $n - 2 = 8$ degrees of freedom.

```
s = 8;
pearson_function = @(x) (1 / (gamma(s/2) * 2^(s/2))) .* x.^(s/2 - 1) .* exp(-x./2);
p = integral(pearson_function, min_chi_squared, Inf)
```

```
p = 1.0000
```

Since the p-value is greater than 0.95, this is data is well described by our model.

We rearrange to conclude $\varphi = 2\pi f \frac{\Delta d}{c} \Rightarrow c = 2\pi f \frac{\Delta d}{\varphi} = \frac{2\pi f}{m} = \frac{2\pi(60\ \mathrm{MHz})}{1.24\ \frac{\mathrm{m}}{\mathrm{rad}}}$, or

```
c = 2 * pi * 60 * 10^6 / 1.24
```

```
c = 3.0403e+08
```

Which is very close to the actual value! Finally, let's find the error. We use error propagation to find

$\sigma_c = -\frac{2\pi f}{m^2}\sigma_m$, so using $\sigma_m = \sqrt{\frac{1}{\Delta}\sum_i \frac{1}{\sigma_{y_i}^2}}$ we write

```
sigma_c = - (2 * pi * 60 * 10^6 / m^2) * sqrt((1 / delta_weighted) * sum(1 ./
sigma_phases.^2))
```

```
sigma_c = -1.0041e+07
```

So our final result is $(3.04 \pm 0.10) \times 10^8\ \frac{\mathrm{m}}{\mathrm{s}}$, which is in agreement with the accepted value! Don't worry Maxwell, we got you.