Visvesvaraya National Institute of Technology
Department of Mathematics
II sessional Examination-2018
Linear Algebra and Applications (MAL-206)

Time: 1 hour Marks: 15

Answer any five questions.

Q.1 Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (x - 3y - 2z, y - 4z, z)$.
(a) Show that T is invertible.
(b) Find T^{-1} and T^{-2}.
[1 + 2]

Q.2 Let T be a linear transformation on the finite dimesional space V and let P be the change-of-basis matrix from a basis $\mathcal{B}$ to a basis $\mathcal{B}'$. Then show that $[T]_{\mathcal{B}'} = P^{-1}[T]_{\mathcal{B}}P$. [3]

Q.3 Suppose the x and y axes in the $\mathbb{R}^2$ are rotated counterclockwise by the angle $\frac{\pi}{4}$ so that the new x' and y' axes are along the lines $y = x$ and $y = -x$ respectively. Then
(a) Find the change of basis matrix.
(b) Find the co-ordinate of the point (a, b) under the given rotation.
[1.5 + 1.5]

Q.4 Let $A = \begin{pmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{pmatrix}$. Recall that A determines a mapping $F : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $F(v) = Av$, where vectors are written as columns. Find the matrix $[F]_{S,S'}$ that represents the mapping relative to the following bases of $\mathbb{R}^3$ and $\mathbb{R}^2$: $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $S' = \{(1, 3), (2, 5)\}$. [3]

Q.5 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (7x + 3y, 3x - y)$. Find invertible orthogonal matrix P such that $P^{-1}[T]_{\mathcal{B}}P = D$, where D is a diagonal matrix and $\mathcal{B}$ is standard basis of $\mathbb{R}^2$. [3]

Q.6 Let T be a linear operator on an n-dimensional vector space V. Then show that the characteristic and minimal polynomial for T have the same roots except for multiplicities. [3]

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