COMPENSATOR DESIGN FOR ACTIVE ANALOG CIRCUIT

Presented by
Shalini Shrivastava
Under the guidance of
Prof. V.R.Sule and Prof. M.S.Baghini

Department of Electrical Engineering
Indian Institute of Technology, Bombay

M.Tech. First stage

Compensation Design for Active Analog Circuit

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Motivation & Introduction

- Active network, under certain circumstances, may become unstable and therefore break into oscillation.
- As technology is scaling down,and require high speed and low power consumption, the main issue is the stability of complete circuit.
- Existing graphical approach does not provide the compensation network, While with coprime factorization approach using state space gives set of compensators.
- State space is the modern approach, which enables to shortcomings of transfer function approach [9].
- State-space description of a network or system emphasizes the internal structure of that system, as well as its input-output performance [2], used in analysis and synthesis of the system.

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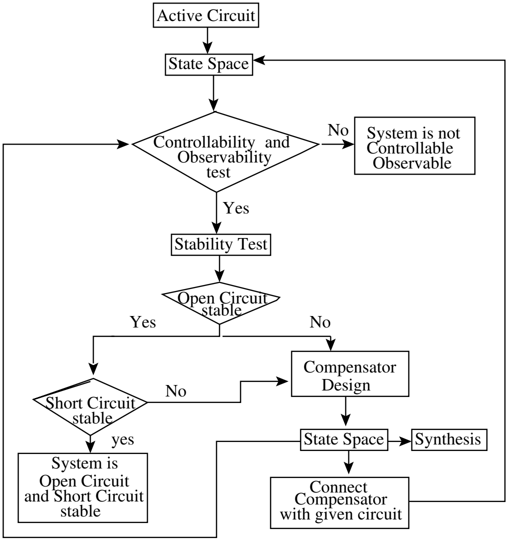
[9]Sanjit Kumar Mitra, Analysis and Synthesis of Linear Active Networks, John Wiley and Sons, Inc,1969.
[2]B.D.O. Anderson and S. Vongpanitlerd, Network analysis and synthesis: a modern systems theory approach, Courier Dover Publications, 2006.

Motivation & Introduction



Compensation Design For Active Analog Circuit

Stability

- Network is said to be stable if as a result of subjecting it to an excitation which dies out with time, the response remains bounded in amplitude as time grows indefinitely [1].
- Open circuit stability test: Network is said to be open circuit stable, when poles of driving point impedance lies in left half of s-plane. For open circuit unstable network, compensator is to be connected in parallel with given circuit
- Short circuit stability test: Network is said to be short circuit stable, when poles of driving point admittance lies in left half of s-plane. For short circuit unstable network, compensator is to be connected in series with given circuit.

[1]S.S.Haykin, Active Network Theory, Addison-Wesley Publishing Company, 1970.

Compensator Design

- Let one port network is specified by its minimal driving point impedance z(s)

$$z(s) = \frac{n(s)}{d(s)} \tag{1}$$

  where n(s) and d(s) are coprime.
- By Euclidean algorithm, it is possible to obtain two polynomials x(s) & y(s) satisfying Bezout identity:

$$n(s)x(s) + d(s)y(s) = 1 \tag{2}$$

  where controller impedance is $z_c(s) = \frac{y(s)}{x(s)}$
- It may be possible that x(s) may be zero, for which controller is undefined.
- The solution to overcome this problem is to arrange these four polynomials n(s), d(s), x(s) and y(s) as rational expressions as elements of $\wp$ which represents the family of all stable, proper and real rational functions [2].

[2] John C. Doyle, Bruce A. Francis, Allen R. Tannenbaum, Feedback Control Theory, Dover Publications, Inc. Mineola, New York, 2009.

Coprime Factorization

- N(s), D(s), X(s) and Y(s) as elements of $\wp$ instead of polynomials.
- Let Z(s) be impedance function of one port network given as,

$$Z(s) = \frac{N(s)}{D(s)} \tag{3}$$

- By Euclidean algorithm, it is possible to obtain two polynomials X(s) & Y(s) satisfying Bezout identity:

$$N(s)X(s) + D(s)Y(s) = 1 \tag{4}$$

  where, N(s), D(s) $\in \wp$ and are coprime. This is called coprime factorization of Z(s) over $\wp$.
- It is required to get N(s),D(s), X(s) and Y(s), all in $\wp$ using state space method.
- Let the input and output of Z(s) be denoted as u and y respectively. Then the state model of Z(s) is given as, $\dot{x} = Ax + Bu$ and y=Cx+du.

Coprime Factorization

- Now choose a real matrix F, $1 \times n$, such that A + BF is stable i.e. all the eigenvalues in Re $s < 0$.
- Define a signal v:= u Fx u = Fx + v. Substituting this in above state model we get,

$$\dot{x} = (A + BF)x + Bv; \qquad y = (C + DF)x + Dv \tag{5}$$

- The transfer function from v to u is given by,

$$D(s) = 1 + F[sI - (A + BF)]^{-1}B \tag{6}$$

- The transfer function from v to y is given by,

$$N(s) = D + (C + DF)[sI - (A + BF)]^{-1}B \tag{7}$$

- Similarly, choose a real matrix H, $n \times 1$, such that A + HC is stable i.e. all the eigenvalues in Re $s < 0$.

$$X(s) = F[sI - (A + HC)]^{-1}H \tag{8}$$

$$Y(s) = 1 + F[sI - (A + BF)]^{-1}(-B - HD) \tag{9}$$

Compensator

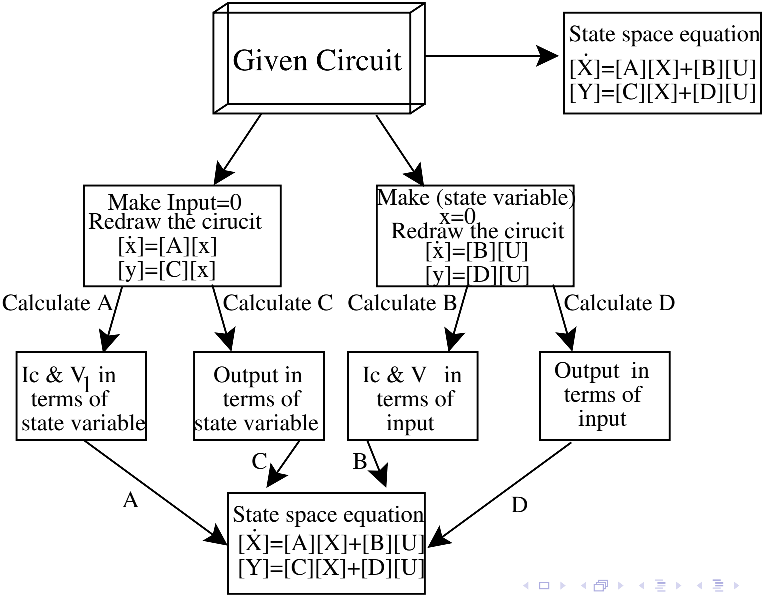
- Theorem: If Z(s)=$\frac{N(s)}{D(s)}$ be a coprime factorization over $\wp$ with X(s) and Y(s) be two functions in $\wp$ satisfying the equation N(s)X(s)+D(s)Y(s)=1, then the set of all compensators that can stabilize Z is given by,

$$Z_c(s) = \frac{Y(s) - N(s)Q}{X(s) + D(s)Q} \tag{10}$$

where Q $\in \wp$, known as free parameter.

Compensation Design For Active Analog Circuit

Algorithm


Given Circuit
State space equation
[Ẋ]=[A][X]+[B][U]
[Y]=[C][X]+[D][U]
Make Input=0
Redraw the cirucit
[ẋ]=[A][x]
[y]=[C][x]
Make (state variable)
x=0
Redraw the cirucit
[ẋ]=[B][U]
[y]=[D][U]
Calculate A
Calculate C
Calculate B
Calculate D
Ic & V1 in terms of state variable
Output in terms of state variable
Ic & V in terms of input
Output in terms of input
A
C
B
D
State space equation
[Ẋ]=[A][X]+[B][U]
[Y]=[C][X]+[D][U]

Example

- Figure 1 (a) shows a circuit model of a transistor tuned amplifier based on the y-parameter model of the transistor.Develop state-space equation for circuit. Assume first that Y_{12} and Y_{21} and real constant. Example is unsolved example of [2].

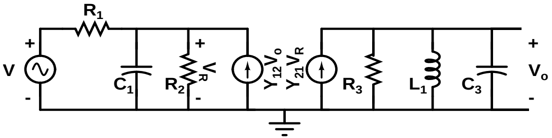


Figure: Example 2

[2]B.D.O. Anderson and S. Vongpanitlerd, Network analysis and synthesis: a modern systems theory approach, Courier Dover Publications, 2006.

Example

- Step 1: Write the state space equation of the given network in the form of A (3 × 3), B(3 × 1), C(1 × 3), D (1 × 1) matrices with input $[V]$, output voltage $[V_o]$, and state variables as $x = [V_{c1} \quad V_{c2} \quad I_l]^T$.
  State Space equations are:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_l \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_l \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{bmatrix} \begin{bmatrix} V \end{bmatrix}$$

  Output equation:

$$\begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_l \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} V \end{bmatrix}$$

Example

- Step 2: Make input V as zero, i.e. voltage source (V) should be short, as shown in Fig. 2. And Substitute Inductor by current source I_l and Capacitors by voltage source V_{c1} & V_{c2}

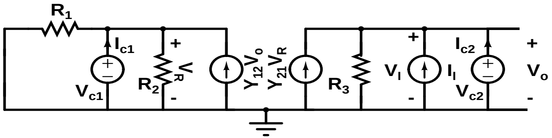


Figure: Step 2

- Step 3: As here we have two sources we will use the superposition theorem to find the coefficient of matrix [A] and [C]
  - Let V_{c2} and $I_l = 0$, State space equation become as:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_l \end{bmatrix} = \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} \begin{bmatrix} V_{c1} \end{bmatrix}; \begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} C_{11} \end{bmatrix} \begin{bmatrix} V_{c1} \end{bmatrix}$$

Example

- Calculate I_{c1} in terms of V_{c1}

$$I_{c1} = \frac{1}{(R_1||R_2)} V_{c1}; \qquad \dot{V}_{c1} = \frac{(R_1 + R_2)}{C_1 R_1 R_2} V_{c1}; \qquad A_{11} = \frac{(R_1 + R_2)}{C_1 R_1 R_2} \tag{11}$$

- Calculate I_{c2} in terms of V_{c1}

$$I_{c2} = -Y_{21} V_R; \qquad \dot{V}_{c1} = -\frac{Y_{21}}{C_2} V_{c1}; \qquad A_{12} = -\frac{Y_{21}}{C_2} \tag{12}$$

- Calculate V_l in terms of V_{c1}

$$V_l = 0; \qquad \dot{I}_l = 0 V_{c1} \qquad A_{13} = 0 \tag{13}$$

- Calculate V_o in terms of V_{c1}

$$V_o = 0; \qquad C_{11} = 0 \tag{14}$$

Example

- Let $V_{c1} = 0$ and $I_l = 0$, State space equation become as:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_l \end{bmatrix} = \begin{bmatrix} A_{21} \\ A_{22} \\ A_{32} \end{bmatrix} \begin{bmatrix} V_{c2} \end{bmatrix} ; \begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} C_{12} \end{bmatrix} \begin{bmatrix} V_{c2} \end{bmatrix}$$

- Calculate I_{c1} in terms of V_{c2}

$$I_{c1} = -Y_{12} V_o; \qquad \dot{V}_{c1} = -\frac{Y_{12}}{C_1} V_{c2}; \qquad A_{21} = -\frac{Y_{12}}{C_1} \tag{15}$$

- Calculate I_{c2} in terms of V_{c2}

$$I_{c2} = \frac{1}{R_3} V_{c2}; \qquad \dot{V}_{c2} = \frac{1}{R_3 C_2} V_{c2}; \qquad A_{22} = \frac{1}{R_3 C_2} \tag{16}$$

- Calculate V_l in terms of V_{c2}

$$V_l = V_{c2}; \qquad \dot{I}_l = \frac{1}{L} V_{c2} \qquad A_{32} = \frac{1}{L} \tag{17}$$

Example

- Calculate V_o in terms of V_{c2}

$$V_o = V_{c2}; \qquad C_{12} = 1 \tag{18}$$

- Let $V_{c1} = 0$ and $V_{c2} = 0$, State space equation become as:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{i}_l \end{bmatrix} = \begin{bmatrix} A_{31} \\ A_{32} \\ A_{33} \end{bmatrix} \begin{bmatrix} I_l \end{bmatrix}; \begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} C_{13} \end{bmatrix} \begin{bmatrix} I_l \end{bmatrix}$$

- Calculate I_{c1} in terms of I_l

$$I_{c1} = -Y_{12} V_o; \qquad I_{c1} = 0; \qquad \dot{V}_{c1} = 0 I_l; \qquad A_{31} = 0 \tag{19}$$

- Calculate I_{c2} in terms of I_l

$$I_{c2} = -I_l; \qquad \dot{V}_{c2} = -\frac{1}{C_2} I_l; \qquad A_{32} = \frac{1}{C_2} \tag{20}$$

Example

- Calculate V_l in terms of I_l

$$V_l = 0; \qquad \dot{I}_l = 0 I_l; \qquad A_{33} = 0 \tag{21}$$

- Calculate V_o in terms of I_l

$$V_o = 0; \qquad C_{13} = 0 \tag{22}$$

Example

- Step 5: Make state variable x zero, i.e. capacitor should be short, and inductor open circuit as shown in Fig. 3.

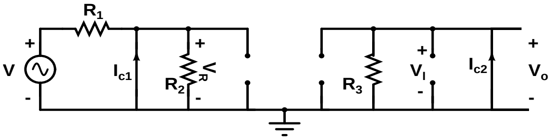


Figure: Step 2

State Space equations are:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{i}_l \end{bmatrix} = \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{bmatrix} \begin{bmatrix} V \end{bmatrix}; \qquad \begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} V \end{bmatrix}$$

Example

- Calculate I_{c1} in terms of V

$$I_{c1} = -\frac{1}{R_1}V; \qquad \dot{V}_{c1} = -\frac{1}{C_1 R_1}V; \qquad B_{11} = -\frac{1}{C_1 R_1} \tag{23}$$

- Calculate I_{c2} in terms of V

$$I_{c2} = -Y_{12}V_R; \qquad \dot{V}_{c1} = 0V; \qquad B_{12} = 0 \tag{24}$$

- Calculate V_l in terms of V

$$V_l = 0; \qquad \dot{I}_l = 0V \qquad B_{13} = 0 \tag{25}$$

- Calculate V_o in terms of V

$$V_o = 0; \qquad D = 0 \tag{26}$$

Example

- State Space equations are:

$$\begin{bmatrix} \dot{V}_{c1} \\ \dot{V}_{c2} \\ \dot{I}_l \end{bmatrix} = \begin{bmatrix} \frac{R_1+R_2}{C_1 R_1 R_2} & -\frac{Y_{12}}{C_2} & 0 \\ -\frac{Y_2 1}{C1} & \frac{1}{R_3 C_2} & \frac{1}{C_2} \\ 0 & \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_l \end{bmatrix} + \begin{bmatrix} -\frac{1}{R_1 C_1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} V \end{bmatrix}$$

Output equation:

$$\begin{bmatrix} V_o \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{c1} \\ V_{c2} \\ I_l \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} V \end{bmatrix}$$

Limitations

- Circuit should have finite components & finite number of controlled sources.
- The network does not contain a node with the only elements incident at node comprising inductor and/or current generator. i.e the network does not contain a cut set.
- The network does not contain a loop with the only branches in the loop comprising capacitor and/or voltage generator. i.e. the network does not contain a tie set.

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Example

- In this section we summarized the whole research work with an example. The circuit of Fig.5 is a first-order circuit with voltage controlled current source, taken from unsolved example of [7].

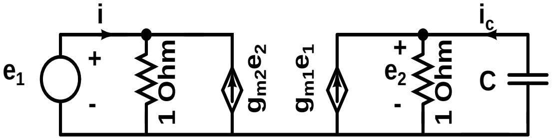

Figure: Example to find the Compensator

[7]L.O.Chua, C.A.Deoser and E.H.Kuh, Linear and Nonlinear Circuits. New York: Mcgraw-Hill,1987

Example

- Step 1: Write the state space equation of the given network in the form of A, B, C, D matrices with x as V_c, u input as e_1 voltage and y as output current i.

$$[\dot{V}_c] = [A][V_c] + [B][e_1] \tag{27}$$

$$[i] = [C][V_c] + [D][e_1] \tag{28}$$

- Step 2: Make input e_1 as zero, i.e. voltage source should be short, and substitute capacitor with voltage source of V_c as shown in Fig. 5.

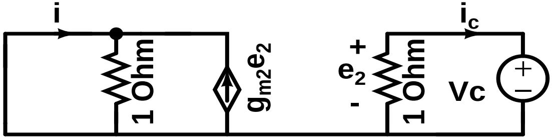

Figure: Input=0

- State space equation modified as:

$$[\dot{V}_c] = [A][V_c]; \qquad [i] = [C][V_c] \tag{29}$$

Example

- Step 3: Calculate I_c in terms of V_c.

$$I_c = V_c; \quad C\dot{V}_c = V_c; \quad \dot{V}_c = \frac{1}{C}V_c; \quad [A] = 1 \tag{30}$$

- Step 4:Determine the output i in terms of state variable V_c.

$$i = -g_{m2}e_2; \quad e_2 = V_c; \quad i = -g_{m2}V_c \quad i = -2V_c; \quad [C] = -2 \tag{31}$$

- Step 5: Short circuits the capacitor to make state variable x=0 in original circuit and Redraw the circuit shown in Fig.6.

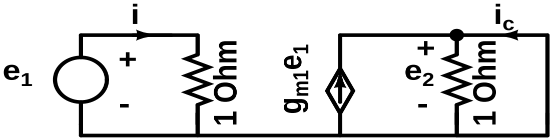

Figure: State variable=0

- Step 6: Calculate I_c in terms of input voltage e_1.

$$I_c = -g_{m1}e_1 = -e_1; \quad C\dot{V}_c = -e_1; \quad \dot{V}_c = -\frac{1}{C}e_1; \quad [B] = -\frac{1}{C} = -1 \tag{32}$$

Example

- Step 7: Determine the output i, in terms of input e_1.

$$i = e_1; \qquad [D] = 1 \tag{33}$$

- Hence State Space Representation of the circuit shown in Fig.4.

$$\dot{V}_c = [1][V_c] + [-1][e_1]; \qquad [i] = [-2][V_c] + [1][e_1] \tag{34}$$

- Controllablity and Observablity matrix are as:
C_o=[B]=[-1] $\neq$ 0 and Q_o = [C^T]=[-2]$\neq$ 0, respectively
therefore system is controllable, as well as Observable.
- As the Eigen values of matrix A is $+1$, i.e. pole of Z_{in} lie is RHS of s plane therefore, system is Open circuit unstable.

Example

- Now we will make the compensator by state space approach[7]
- **Step 1:** A, B, C, D for the given network are:
  A=1, B=-1, C=-2, D=1
- **Step 2:** Coprime factorization of Z(s) can be done so as to get four rational functions N(s),D(s), X(s) and Y(s) [7], all in $\wp$, such that Z(s)=$\frac{N(s)}{D(s)}$
  N(s)X(s)+D(s)Y(s)=1
  - Calculation of N(s), D(s): Choose a real matrix F $= [f_1]$, 1×1, such that $A + BF$ is stable i.e. all the eigenvalues in Re s<0. Let the eigenvalue of A + BF be located at -1. The desired characteristics equation is (s+1).
    $|sI - (A + BF)| = |s(1) - (1 + (-1)f_1| = s + (-1 + f_1)$
    Comparing coefficients of above equation with those of desired equation we get, $f_1 = 2$ thus F=[2].

[7]L.O.Chua, C.A.Deoser and E.H.Kuh, Linear and Nonlinear Circuits. New York: Mcgraw-Hill,1987

Example

- The transfer function from v to u is given by, $D(s) = 1 + F[sI - (A + BF)]^{-1}B)$ which can be represent as

$$\left[\begin{array}{c|c} A+BF & B \\ \hline F & 1 \end{array}\right].$$

$$D(s) = \frac{s-1}{s+1} \tag{35}$$

- The transfer from v to y is given by, $N(s) = D + (C + DF)[sI - (A + BF)]^{-1}B)$ which can be represent as

$$\left[\begin{array}{c|c} A+BF & B \\ \hline C+DF & D \end{array}\right]$$

$$N(s) = 1 \tag{36}$$

Example

- Calculation of X(s), Y(s).Choose a real matrix H $= [h_1]$, 1×1, such that $A + HC$ is stable i.e. all the eigenvalues in Re s<0. Let the eigenvalue of A + BF be located at -1. The desired characteristics equation is (s+1). $|sI - (A + HC)| = |s(1) - (1 + (-2)h_1| = s + (-1 + h_1(-2))$ Comparing coefficients of above equation with those of desired equation we get, $h_1 = 1$ thus H=[1].
- The transfer function from v to u is given by, $X(s) = F[sI - (A + HC)]^{-1}H)$ which can be represent as

$$\left[\begin{array}{c|c} A + HC & H \\ \hline F & 0 \end{array}\right]$$

$$X(s) = \frac{2}{s+1} \tag{37}$$

Example

- The transfer from v to y is given by,
  $Y(s) = 1 + F[sI - (A + HC)]^{-1}(-B - HD)$ which can be represent as

$$\left[\begin{array}{c|c} A + HC & -B - HD \\ \hline F & 1 \end{array}\right]$$

$$Y(s) = 1 \tag{38}$$

- Thus we have following four rational function satisfying Bezout's identity

$$N(s) = 1; \qquad D(s) = \frac{s-1}{s+1}; \qquad X(s) = \frac{2}{s+1}; \qquad Y(s) = 1 \tag{39}$$

Example

- The compensator transfer function is given as:

$$Z_c = \frac{Y(s) - N(s)Q(s)}{X(s) + D(s)Q(s)} \tag{40}$$

where Q(S) is free parameter $\in \wp$
Let us assune, Q=0.5, then Z_c is given as,

$$Z_c = \frac{(s+1)}{(s+3)} \tag{41}$$

- Total impedance after connecting the compensator is
Z_T=N(s)[Y(s)-N(s)Q]=1[1-(0.5)1]=0.5
Hence Overall system after connecting the compensator become stable.

Example

- Same example is simulated in the Scilab code, Bode Plot obtain is shown in Fig.7:

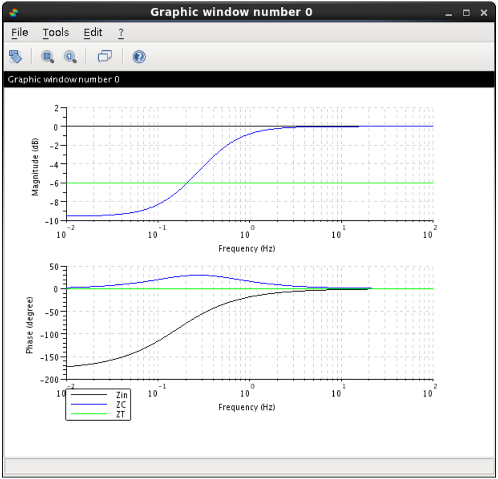

Figure: Bode Plot from Scilab Code

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Conclusion

- Stabilization problem for one port networks has been studied and solved.
- The set of compensating networks depending on the Q free parameter which will stabilize the given one port network can be obtained by the application of coprime factorization theory via state space realization of the networks.
- Finding the state space representation of given network by method discussed has certain limitation
- The network should not have node with the only elements incident at node comprising inductor and/or current generator i.e the network should contain a cut set.The network does not contain a loop with the only branches in the loop comprising capacitor and/or voltage generator. i.e. the network does not have a tie set.

Future Work

- Solving the stabilization problem for two port networks.
- Applying coprime factorization approach to multivariate system and to obtain set of compensating networks.
- Synthesis the compensator network.
- Applying the algorithm to find state space with simulator, and try to find state space for more practical case without any limitation.
- Application of these concepts to practical circuits such as multistage amplifiers to design a suitable compensator and to evaluate its performance with various process technology.

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Thank You