Samuel Mazumowskij | ASEN 3128 AS 4 | due 10/3/19

1. $\Delta\dot{p} = \frac{1}{I_x}\Delta L_c = \frac{1}{I_x}(-k_1 \cdot \Delta p - k_2 \Delta\phi)$ Lateral

$\Delta\dot{\phi} = \Delta p$

$\dot{y} = F(y,t)$

$$\dot{y} = \begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{\phi} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{k_1}{I_x} & -\frac{k_2}{I_x} \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta p \\ \Delta\phi \end{bmatrix}}_{y}$$

$\dot{y} = A \cdot y$

$(A - \lambda I)y^* = 0$

$\det(A - \lambda I) = 0$

$$\det(A-\lambda I) = \begin{vmatrix} -\frac{k_1}{I_x} - \lambda & -\frac{k_2}{I_x} \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + \frac{k_1 \lambda}{I_x} + \frac{k_2}{I_x} = 0$$

$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{k_2}{I_x}} \qquad \zeta = \frac{k_1}{I_x} \cdot \frac{1}{2\sqrt{\frac{k_2}{I_x}}}$$

$$\zeta = \frac{k_1}{2\sqrt{I_x k_2}}$$

$$\lambda = -\zeta\omega_n \pm \sqrt{(\zeta\omega_n)^2 - \omega_n^2}$$

$$\tau = \frac{1}{-\zeta\omega_n + \sqrt{(\zeta\omega_n)^2 - \omega_n^2}} = \frac{1}{2}$$

$$\lambda_1 = -\zeta\omega_n + \sqrt{\omega_n^2(\zeta^2-1)}$$

$$\omega_n = \frac{2}{\zeta + \sqrt{\zeta^2 - 1}}$$

$$\lambda_2 = -\zeta\omega_n - \sqrt{\omega_n^2(\zeta^2-1)}$$

$\lambda_1 = 10 \cdot \lambda_2$

$$-\zeta\omega_n + \sqrt{\omega_n^2(\zeta^2-1)} = 10(-\zeta\omega_n - \sqrt{\omega_n^2(\zeta^2-1)})$$

$$9\zeta\omega_n + 11\sqrt{\omega_n^2(\zeta^2-1)} = 0$$

$$9\zeta\omega_n = -11\sqrt{\omega_n^2(\zeta^2-1)}$$

$$\frac{81}{121}\zeta^2\omega_n^2 = \omega_n^2\zeta^2 + \omega_n^2$$

$$\frac{81}{121}\zeta^2 = \zeta^2 + 1$$

$$-\zeta^2 \cdot \frac{40}{121} = 1$$

$$\zeta = \sqrt{\frac{121}{40}}$$

$$\zeta = 1.739$$

$$\omega_n = \frac{2}{\zeta - \sqrt{\zeta^2 - 1}}$$

$\omega_n = 6.324$

$K_2 = w_n^2 I_x = (6.324)^2 \cdot 5.8\times10^{-5}$ $\quad I_x = 5.8\cdot10^{-5}\ kgm^2$

$\boxed{K_2 = 0.00232}$

$K_1 = \zeta \cdot 2\sqrt{I_x \cdot K_2} = 1.739 \cdot 2\sqrt{5.8\times10^{-5}\cdot 0.00232}$

$\boxed{K_1 = 0.001276}$

longitudinal

since $\lambda^2 + \frac{K_3}{I_y}\lambda + \frac{K_4}{I_y}$

is the same equation as the lateral
we can use the same final derivations

$\zeta = 1.734$

$w_n = 6.324$

$K_2 = (6.324)^2 \cdot 7.2\times10^{-5} = 0.00288$

$K_1 = \zeta \cdot 2 \cdot \sqrt{I_y \cdot K_2} = 1.739 \cdot 2\sqrt{7.2\times10^{-5}\cdot 0.00288} = 0.001584$

$$\boxed{\begin{aligned} K_3 &= 0.001584 \\ K_4 &= 0.00288 \end{aligned}}$$

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2. The results make sense since $-K_1*P_1$ will be countering $-K_2*\phi$ so the settling time is longer than just the proportional. Steady hover is not a steady flight condition since it doesn't recover to steady hover conditions.

3. The two models have nearly identical results.

4. like in the simulations the copter continues to in the x and y directions while maintaining altitude. At about the 6 second mark you can see the command was given to change theta to 5°, and after that point the graph looks similar to problems 2b and 3b.