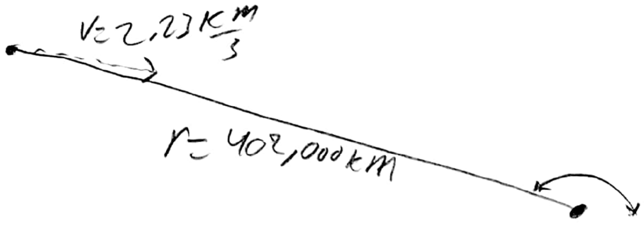
(2.38) given: $r = 402{,}000\ km$ $\quad \theta^* = -150^\circ$ $\quad V = 2.23\ \frac{km}{s}$

find; a) e , b) alt_p , c) V_p

Solution;



$$r = a\,\frac{e^2-1}{1+e\cos\theta^*}$$

$$r + re\cos\theta^* = ae^2 - a$$

$$0 = ae^2 - re\cos\theta^* - a - r$$

$$0 = 133319e^2 - 402000\cos(-150^\circ)e - 535319$$

$$e = (-3.647,\ 1.086)$$

since e is positive it must be

$$\boxed{e = 1.086}$$

$$\frac{V^2}{2} - \frac{V_{esc}^2}{2} = \frac{V_\infty^2}{2}$$

$$V_\infty = 2\sqrt{\frac{V^2}{2} - \frac{V_{esc}^2}{2}}$$

$$V_{esc} = \sqrt{\frac{2\mu}{r}}$$

$$V_{esc} = 1.408\ \frac{km}{s}$$

$$V_\infty = 2\sqrt{\frac{2.23^2}{2} - \frac{1.408^2}{2}}$$

$$V_\infty = 1.729\ \frac{km}{s}$$

$$V_\infty = \sqrt{\frac{\mu}{a}}$$

$$|a| = \frac{\mu}{V_\infty^2} = \frac{398600}{1.729^2} = 133319\ km$$

$$r_p = a(1-e) = -133319\cdot(1-1.086)$$

$$r_p = 11465.6\ km$$

$$alt_p = r_p - 6378 = 5088\ km$$

$$\boxed{alt_p = 5088\ km}$$

$$V_{esc\,p} = \sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2\cdot 398600}{11463.6\ km}} = 8.34\ \frac{km}{s}$$

$$V_p = \sqrt{2\left(\frac{V_\infty^2}{2} + \frac{V_{esc\,p}^2}{2}\right)} = \sqrt{2\left(\frac{1.729^2}{2} + \frac{8.34^2}{2}\right)} = 8.516\ \frac{km}{s}$$

$$\boxed{V_p = 8.516\ \frac{km}{s}}$$

(3.8) given: $alt_p = 200km$ $alt_a = 600km$

find: time $t_B - t_A$ alt > 400km

solution:

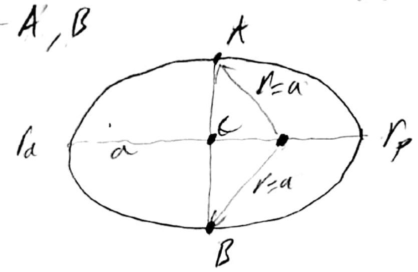
$$r = a(1 - e\cos E)$$

$$r_p = alt_p + 6378 = 6578 km$$

$$r_a = alt_a + 6378 = 6978 km$$

$$a = \frac{r_a + r_p}{2} = \frac{6578 + 6978}{2} = 6778 km$$

if $r_{400} = 6778$ then $r_{400} = a$ which means the location is at A, B



$$e = 1 - \frac{r_p}{a} = 1 - \frac{6578}{6778} = 0.0295$$

$$t_A - t_p = \frac{\frac{\pi}{2} - e\sin(\frac{\pi}{2})}{n} \qquad n = \sqrt{\frac{\mu}{a^3}}$$

$$t_B - t_p = \frac{\frac{3\pi}{2} - e\sin(\frac{3\pi}{2})}{n}$$

$$t_B - t_A = \frac{\frac{\pi}{2} - e\sin(\frac{\pi}{2})}{n} - \frac{\frac{3\pi}{2} - e\sin(\frac{3\pi}{2})}{n}$$

$$= \frac{\frac{\pi}{2} - 0.0295\sin(\frac{\pi}{2}) - \frac{3\pi}{2} - 0.0295\sin(\frac{3\pi}{2})}{\sqrt{\frac{398600}{6778^3}}}$$

$$t_B - t_A = 2828.95 \cdot \frac{1 min}{60s} = 47.1 min$$

$$\boxed{t_B - t_A = 47.1 min}$$

3.10 given; $T = 14h$ $r_p = 10{,}000km$ $t = 10h$

find; a) r, b) V, c) V_r

Solution;

a) $T = 2\pi\sqrt{\frac{\mu}{a^3}}$ $a^3 = \frac{\mu}{\frac{T^2}{4\pi^2}} = \sqrt[3]{\frac{4\pi^2\mu}{T^2}} = \sqrt[3]{\frac{4\pi^2 \cdot 398600}{50400^2}}$

$\left(\frac{T}{2\pi}\right)^2 = \frac{\mu}{a^3}$ $a = 29{,}490\ KM$

$r_p = a(1-e)$

$e = 1 - \frac{r_p}{a} = 1 - \frac{10{,}000}{29{,}490} \approx 0.6609$

$n(t_a - t_p) = E - e\sin(E) = M$

$M = \frac{2\pi}{T}(t_a - t_p) = \frac{2\pi}{50400} \cdot 36000 = 4.4879$

Newton Raphson; $E_{i+1} = E_i - \frac{E_i - e\sin E_i - M}{1 - e\cos E_i}$ initial guess; $M e - \frac{e}{2} = 3.1288$

Matlab $E = 3.9415$ $r = a(1 - e\cos E)$

$r = 29490(1 - 0.6609\cos(3.9415)) = 42354$

$\boxed{r = 42354.86\,KM}$

b) $V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \sqrt{\frac{2 \cdot 398600}{42354} - \frac{398600}{29490}} = 2.303\ \frac{km}{s}$

c) $V_r = \frac{\mu}{h} \cdot e\sin\theta^* = \frac{\mu}{\sqrt{r_p\mu(1+e)}}\, e\, \frac{a\sqrt{1-e^2}}{r}\sin E$

$= \frac{398600}{\sqrt{10000 \cdot 398600(1+0.609)}} \cdot (0.609)\ \frac{29490\sqrt{1-0.609^2}}{42354}$

$\boxed{V_r = -1.271\ \frac{KM}{s}}$

(3.11) given: $r_p = 7500\,km$ $r_a = 16000\,km$

find: θ_B^* at $t = 40\,min$ after $\theta_A^* = 80°$

Solution: $a = \frac{r_p + r_a}{2} = \frac{7500 + 16000}{2} = 11750\,km$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{16000 - 7500}{16000 + 7500} = 0.3617$$

$$E_A = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\ \tan\frac{\theta^*}{2}\right)$$

$$E_A = 2\tan^{-1}\left(\sqrt{\frac{1+0.3617}{1-0.3617}}\ \tan\left(\frac{80°}{2}\right)\right) = 1.7728$$

$$M_A = E_A - e\sin(E_A) = 1.7728 - (0.3617)\sin(1.7728)) = 1.4185$$

$$M_A = \frac{2\pi}{T}(t - t_p)$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{11750^3}{398600}} = 12675\,s$$

$$(t_A - t_p) = \frac{M_A\,T}{2\pi} = \frac{1.4185 \cdot 12675}{2\pi} = 2861.6\ s$$

$$t_B = t_A + 40\,min \cdot \frac{60\,sec}{1\,min} = 5261.6\,s$$

$$M_B = \frac{2\pi}{T}(t_B - t_p) = \frac{2\pi}{12675}(5261.6\,s) = 2.608\ rad$$

find E_B: Newton Raphson: initial guess

$$M_B + \frac{e}{2} = 2.608 + \frac{0.3617}{2} = 2.789\ rad$$

Matlab

$$E_B = 2.747$$

$$\theta_B^* = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\ \tan\left(\frac{E_B}{2}\right)\right) = 2\tan^{-1}\left(\sqrt{\frac{1+0.3617}{1-0.3617}}\ \tan\left(\frac{2.747}{2}\right)\right)$$

$$\theta_B^* = 4.011\ rad$$

$$\theta_B^* = 4.011 \cdot \frac{180}{\pi}$$

$$\boxed{\theta_B^* = 229.8°}$$

(Q5) given; $T = 3.5627\,hr$ $p = 4403\,km$

find; a) t_1 @ alt 600 km, r, v b) diagram

Solution;

$r_1 = 600 + 3346 = 3946\,km$

$T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$a = \sqrt[3]{\frac{\mu T^2}{4\pi^2}} = \sqrt[3]{\frac{42828 \cdot (3.5627 \cdot 3600)^2}{4 \cdot \pi^2}} = 5630\ km$

$p = a(1 - e^2)$

$e = \sqrt{1 - \frac{p}{a}} = \sqrt{1 - \frac{4403}{5630}} = 0.359$

$r = a(1 - e\cos E)$

$e\cos E = 1 - \frac{r}{a}$

$E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right) = \cos^{-1}\left(\frac{1 - \frac{3946}{5630}}{0.359}\right) = 0.6306\ rad$

$E_1 = \frac{\pi}{2} - E = 5.65$ rad → since t_1 is past apoapsis we needed to subtract from 2π to bring it to the correct quadrant

t_0 is half the period so $t_0 = \frac{T}{2} = \frac{3.5627}{2}$

$t_0 = 1.78135\ hr$

$n(t_1 - t_p) = E - e\sin E$

$t_1 - t_p = \frac{(\ldots - 0.359 \cdot \sin(\ldots))}{\frac{2\pi}{3.5627 \cdot 3600}} = 11466\ s$

$t_1 - t_0 = 11466 - 1.78135 \cdot 3600 = 5053.3\,s$

$\boxed{t_1 - t_0 = 5053.3\,s}$

Q5 continued

$$\theta^* = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\right) = 2\tan^{-1}\left(\sqrt{\frac{1+0.359}{1-0.359}}\tan\left(\frac{\ }{2}\right)\right)$$

$\theta^* = -0.887 rad$

$\theta^* = -2.7 \cdot \frac{180}{\pi} = -50.83°$

$h = \sqrt{\mu a(1-e^2)} = \sqrt{42828(5930)(1-.359^2)}$

$h = 14491 \frac{km^2}{s}$

$$\boxed{(\hat{r}, \hat{\theta}, \hat{h}) = (3996, -50.83, 14491) km}$$

Velocity

$v = v_r \hat{r} + v_\theta \hat{\theta}$

$v_r = \frac{\mu}{h} e \sin(\theta^*) = \frac{42828}{14491} \cdot 0.359 \cdot \sin(-154.75)$

$v_r = -0.453 \frac{km}{s}$

$v_\theta = \frac{\mu}{h}(1 + e\cos\theta^*) = \frac{42828}{14491}(1 + 0.359\cos(-154.75))$

$v_\theta = 1.495 \frac{km}{s}$

$$\boxed{v = -0.453\hat{r} + 1.495\hat{\theta} \frac{km}{s}}$$

Perifocal

$r = \frac{h^2}{\mu} \cdot \frac{1}{1+e\cos\theta^*}(\cos\theta^* \hat{p} + \sin\theta^* \hat{q})$

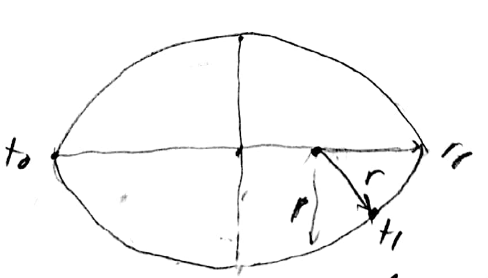
$= \frac{14491^2}{42828} \cdot \frac{1}{1+0.359\cos(-50.83)} \cdot (\cos(-50.83)\hat{p} + \sin(-50.83)\hat{q})$

$$\boxed{r = -2524\hat{p} - 3098\hat{q}\ [km]}$$

$v = \frac{\mu}{h}(-\sin\theta^* \hat{p} + (e + \cos\theta^*)\hat{q}) = \frac{42828}{14491}(-\sin(-50.83)\hat{p} + (0.359 + \cos(-50.83))\hat{q})$

$$\boxed{v = 2.29\hat{p} + 2.93\hat{q}\ [\frac{km}{s}]}$$

b)



since $r < r$ and not equal to r_p it must be in the bottom right quadrant.

```
% Samuel Razumovskiy
% 1/29/2020

clear,clc,close all
```

Problem 3

```
M = 4.4879;
E0 = 3.1288;
e = 0.6609;

En = E0 - (E0-e*cos(E0)-M)/(1-e*cos(E0));
i = 0;

while abs(En - E0)>0.0001
    E0 = En;
    En = E0 - (E0-e*sin(E0)-M)/(1-e*cos(E0));
    i = i+1;
end
fprintf('E = %.3f\n', En)
```

```
E = 3.991
```

Problem 4

```
E0 = 2.789;
e = 0.3617;
M = 2.60814;

En = E0 - (E0-e*cos(E0)-M)/(1-e*cos(E0));
i = 0;

while abs(En - E0)>0.0001
    E0 = En;
    En = E0 - (E0-e*sin(E0)-M)/(1-e*cos(E0));
    i = i+1;
end
fprintf('E = %.3f\n', En)
```

```
E = 2.747
```

Published with MATLAB® R2019b