1. $\dot{u} = \dfrac{X - mg\sin\theta - mqw + mrv}{m}$

$u_0 = v_0 = w_0 = 0 \quad L_0 = M_0 = N_0 = 0$

$p_0 = q_0 = r_0 = 0$

$\psi_0 = \theta_0 = \phi_0 = 0$

$X_0 = Y_0 = Z_0 = 0$

$\dot{u} = \underbrace{\left(\dfrac{X}{m} - g\sin\theta - qw + rv\right.}_{h_1}$

$\Delta\dot{u} = \dfrac{\partial h_1}{\partial X}\Big|_0 \cdot \Delta X + \dfrac{\partial h_1}{\partial q}\Big|_0 \cdot \Delta q + \dfrac{\partial h_1}{\partial w}\Big|_0 \cdot \Delta w + \dfrac{\partial h_1}{\partial r}\Big|_0 \cdot \Delta r + \dfrac{\partial h_1}{\partial v}\Big|_0 \cdot \Delta v + \dfrac{\partial h_1}{\partial \theta}\Big|_0 \cdot \Delta\theta$

$\Delta\dot{u} = \dfrac{\Delta X}{m} + g\cancel{\cos\theta_0}\Delta\theta - \cancel{w_0}\Delta q - \cancel{q_0}\Delta w + \cancel{v_0}\Delta r + \cancel{r_0}\Delta v$

$\Delta\dot{u} = \dfrac{\Delta X}{m} + g\Delta\theta \quad (1)$

$\dot{v} = \dfrac{Y + mg\cos\theta\sin\phi - mru + mpw}{m}$

$\dot{v} = \dfrac{Y}{m} + g\cos\theta\sin\phi - ru + pw$

$\dot{v} = \dot{v}_0^{\,0} + \Delta\dot{v}$

$= h_2(\qquad\qquad)$

$\Delta\dot{v} = \dfrac{\partial h_2}{\partial Y}\Big|_0 \Delta Y + \dfrac{\partial h_2}{\partial \theta}\Big|_0 \Delta\theta + \dfrac{\partial h_2}{\partial \phi}\Big|_0 \Delta\phi + \dfrac{\partial h_2}{\partial r}\Big|_0 \Delta r + \dfrac{\partial h_2}{\partial u}\Big|_0 \Delta u + \dfrac{\partial h_2}{\partial p}\Big|_0 \Delta p + \dfrac{\partial h_2}{\partial w}\Big|_0 \Delta w$

$\Delta\dot{v} = \dfrac{\Delta Y}{m} - \cancel{g\sin\theta_0\sin\phi_0\Delta\theta} + g\cancel{\cos\theta_0}\cos\phi_0\Delta\phi - \cancel{u_0}\Delta r - \cancel{r_0}\Delta u + \cancel{w_0}\Delta p + \cancel{p_0}\Delta w$

$\Delta\dot{v} = \dfrac{\Delta Y}{m} + g\Delta\phi \quad (2)$

$\dot{w} = \dfrac{Z + mg\cos\theta\cos\phi - mpv + mqu}{m}$

$\dot{w} = \dfrac{Z}{m} + g\cos\theta\cos\phi - pv + qu$

$= h_3(\qquad\qquad)$

$\Delta\dot{w} = \dfrac{\partial h_3}{\partial Z}\Big|_0 \Delta Z + \dfrac{\partial h_3}{\partial \theta}\Big|_0 \Delta\theta + \dfrac{\partial h_3}{\partial \phi}\Big|_0 \Delta\phi + \dfrac{\partial h_3}{\partial p}\Big|_0 \Delta p + \dfrac{\partial h_3}{\partial v}\Big|_0 \Delta v + \dfrac{\partial h_3}{\partial q}\Big|_0 \Delta q + \dfrac{\partial h_3}{\partial u}\Big|_0 \Delta u$

$\Delta\dot{w} = \dfrac{\Delta Z}{m} + g\cancel{\sin\theta_0}\cos\phi_0\Delta\theta - g\cos\theta_0\cancel{\sin\phi_0}\Delta\phi - \cancel{v_0}\Delta p - \cancel{p_0}\Delta v + \Delta q\cancel{u_0} + \cancel{q_0}\Delta u$

$\Delta\dot{w} = \dfrac{\Delta Z}{m} \quad (3)$

$$X = -\eta\sqrt{u^2+v^2+w^2}\cdot u$$

$$= g_1(\quad)$$

$$\Delta X = \frac{\partial g_1}{\partial u}\Big|_0 \Delta u + \frac{\partial g_1}{\partial v}\Big|_0 \Delta v + \frac{\partial g_1}{\partial w}\Big|_0 \Delta w$$

$$\Delta X = -\eta\cancel{\frac{2u_0^2+v_0^2+w_0^2}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} + \cancel{\frac{-\eta u_0 v_0}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} + \cancel{\frac{-\eta u_0 w_0}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} = 0$$

(1) $\boxed{\Delta \dot{u} = g\Delta\theta}$

$$Y = -\eta\sqrt{u^2+v^2+w^2}\cdot v$$

$$\Delta Y = 0$$

(2) $\boxed{\Delta \dot{v} = g\Delta\phi}$

$$Z = \eta\sqrt{u^2+v^2+w^2}\cdot w + Z_c$$

$$= g_3(\quad$$

$$\Delta Z = \frac{\partial g_3}{\partial u}\Big|_0 \Delta u + \frac{\partial g_3}{\partial v}\Big|_0 \Delta v + \frac{\partial g_3}{\partial w}\Big|_0 \Delta w + \frac{\partial g_3}{\partial f_1}\Big|_0 \Delta f_1 \ldots \frac{\partial g_3}{\partial f_4}\Big|_0 \Delta f_4$$

$$\Delta Z = -\eta\cancel{\frac{2u_0^2+v_0^2+w_0^2}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} - \eta\cancel{\frac{u_0 v_0}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} - \eta\cancel{\frac{u_0 w_0}{\sqrt{u_0^2+v_0^2+w_0^2}}}^{0} + \Delta f_1 + \Delta f_2 + \Delta f_3 + \Delta f_4$$

$$\Delta Z = \Delta Z_c$$

(3) $\boxed{\Delta \dot{w} = \frac{\Delta Z_c}{m}}$

$$\dot{p} = \frac{L + I_{zx}\dot{r} + qr(I_z - I_y) - I_{zx}pq}{I_x} \qquad I_{zx} = 0$$

$$\dot{p} = \frac{L}{I_x} + \frac{qr(I_z - I_y)}{I_x} = h_4(\quad) \qquad \Delta\dot{p} = \frac{\partial h_4}{\partial r}\Big|_0 \Delta r + \frac{\partial h_4}{\partial L}\Big|_0 \Delta L + \frac{\partial h_4}{\partial q}\Big|_0 \Delta q$$

$$\Delta\dot{p} = \frac{q_0 \Delta r \cancel{(I_z - I_y)}}{\cancel{I_x}} + \frac{\Delta L}{I_x} + \frac{\Delta q \cancel{r_0}(I_z - I_y)}{\cancel{I_x}} = \frac{\Delta L}{I_x} \qquad \Delta\dot{p} = \frac{\Delta L}{I_x} \ (4)$$

$$L = -\alpha\sqrt{p^2 + q^2 + r^2} \cdot p + L_c = g_4(\quad) \qquad \Delta L = \frac{\partial g_4}{\partial p}\Big|_0 \Delta p + \frac{\partial g_4}{\partial q}\Big|_0 \Delta q + \frac{\partial g_4}{\partial r}\Big|_0 \Delta r + \frac{\partial g_4}{\partial L_c}\Big|_0 \Delta L_c$$

$$\Delta L = -\alpha \frac{2p_0^2 + \cancel{q_0^2 + r_0^2}}{\cancel{\sqrt{p_0^2 + q_0^2 + r_0^2}}} \Delta p - \alpha \frac{p_0 q_0}{\cancel{\sqrt{p_0^2 + q_0^2 + r_0^2}}} - \alpha \frac{p_0 r_0}{\cancel{\sqrt{p_0^2 + q_0^2 + r_0^2}}} + \Delta L_c$$

$$\Delta L_c = \frac{r}{\sqrt{2}}(\Delta f_1 + \Delta f_2 - \Delta f_3 - \Delta f_4)$$

$$\Delta L = \Delta L_c$$

$$\boxed{\Delta\dot{p} = \frac{\Delta L_c}{I_x}} \ (4)$$

$$\dot{q} = \frac{M - rp(I_x - I_z) - I_{zx}(p^2 - r^2)}{I_y} = \frac{M}{I_y} - \frac{rp(I_x - I_z)}{I_y} = h_5(\quad)$$

$$\Delta\dot{q} = \frac{\partial h_5}{\partial r}\Big|_0 \Delta r + \frac{\partial h_5}{\partial p}\Big|_0 \Delta p + \frac{\partial h_5}{\partial M}\Big|_0 \Delta M = -\Delta r \cancel{p_0} \frac{(I_x - I_z)}{I_y} - \cancel{r_0} \Delta p \frac{(I_x - I_z)}{I_y} + \frac{\Delta M}{I_y}$$

$$\Delta\dot{q} = \frac{\Delta M}{I_y} \ (5)$$

$$M = -\alpha\sqrt{p^2 + q^2 + r^2} \cdot q + M_c = g_5(\quad)$$

$$\Delta M = \frac{\partial g_5}{\partial p}\Big|_0 \Delta p + \frac{\partial g_5}{\partial q}\Big|_0 \Delta q + \frac{\partial g_5}{\partial r}\Big|_0 \Delta r + \frac{\partial g_5}{\partial M_c}\Big|_0 \Delta M_c$$

$$\Delta M = -\alpha \frac{p_0 q_0}{\cancel{\sqrt{p_0^2 + q_0^2 + r_0^2}}} \Delta p - \alpha \frac{p_0^2 + 2q_0^2 + r_0^2}{\sqrt{p_0^2 + q_0^2 + r_0^2}} \Delta q - \alpha \frac{\cancel{q_0} r_0}{\cancel{\sqrt{p_0^2 + q_0^2 + r_0^2}}} \Delta r + \Delta M_c$$

$$\Delta M = \Delta M_c$$

$$\boxed{\Delta\dot{q} = \frac{\Delta M_c}{I_y}} \ (5)$$

Scanned by CamScanner

$$\dot r = \underbrace{\frac{N + \cancel{I_{zx}\dot p} - pq(I_y - I_x) + \cancel{I_{zx} q r}}{I_z}} = \frac{N}{I_z} - \frac{pq(I_y - I_x)}{I_z} = h_6(\quad)$$

$$\Delta\dot r = \frac{\partial h_6}{\partial p}\Big|_0 \Delta p + \frac{\partial h_6}{\partial q}\Big|_0 \Delta q + \frac{\partial h_6}{\partial N}\Big|_0 \Delta N = -\cancel{\frac{\Delta p\, q_0 (I_y - I_x)}{I_z}} - \cancel{\frac{p_0 \Delta q (I_y - I_x)}{I_z}} + \frac{\Delta N}{I_z}$$

$$\Delta\dot r = \frac{\Delta N}{I_z} \quad (6)$$

$$N = -\alpha\sqrt{p^2+q^2+r^2}\, r + N_c = g_6(\quad)$$

$$\Delta N = \frac{\partial h_6}{p}\Big|_0 \Delta p + \frac{\partial h_6}{q}\Big|_0 \Delta q + \frac{\partial h_6}{\partial r}\Big|_0 \Delta r + \frac{\partial h_6}{\partial N_c}\Big|_0 \Delta N_c$$

$$\Delta N = -\alpha \cancel{\frac{p_0 r_0}{\sqrt{p_0^2+q_0^2+r_0^2}}} \Delta p - \alpha \cancel{\frac{q_0 r_0}{\sqrt{p_0^2+q_0^2+r_0^2}}} \Delta q - \alpha \cancel{\frac{p_0^2+q_0^2+2r_0^2}{\sqrt{p_0^2+q_0^2+r_0^2}}} \Delta r + \Delta N_c$$

$$\Delta N = \Delta N_c$$

$$\boxed{\Delta\dot r = \frac{\Delta N_c}{I_z}} \quad (6)$$

$$\dot\theta = q\cos\phi - r\sin\phi = h_7(\quad)$$

$$\Delta\dot\theta = \frac{\partial h_7}{\partial q}\Big|_0 \Delta q + \frac{\partial h_7}{\partial \phi}\Big|_0 \Delta\phi + \frac{\partial h_7}{\partial r}\Big|_0 \Delta r = \cos\phi_0 \Delta q + \cancel{(q_0 \sin\phi_0 - r_0\cos\phi_0)\Delta\phi} - \cancel{\sin\phi_0 \Delta r}$$

$$\boxed{\Delta\dot\theta = \Delta q} \quad (7)$$

$$\dot\phi = p + (q\sin\phi + r\cos\phi)\tan\theta = h_8(\quad)$$

$$\Delta\dot\phi = \frac{\partial h_8}{\partial p}\Big|_0 \Delta p + \frac{\partial h_8}{\partial q}\Big|_0 \Delta q + \frac{\partial h_8}{\partial r}\Big|_0 \Delta r + \frac{\partial h_8}{\partial \phi}\Big|_0 \Delta\phi + \frac{\partial h_8}{\partial \theta}\Big|_0 \Delta\theta$$

$$\Delta\dot\phi = \Delta r + \cancel{\sin\phi_0 \tan\theta_0 \Delta q} + \cancel{\cos\phi_0 \tan\theta_0 \Delta r} + \cancel{(q_0\cos\phi_0 - r_0\sin\phi_0)\tan\theta\,\Delta\phi}$$
$$+ \cancel{(q_0\sin\phi_0 + r_0\cos\phi_0)\sec^2\theta\,\Delta\theta}$$

$$\boxed{\Delta\dot\phi = \Delta p} \quad (8)$$

2. The results make some sense since hover is not a stable flight condition. Any small deviation in the flight conditions cause the aircraft to leave hover.

3. The linearized version seems very similar to the non-linearized results. There is a noticeable difference in the rate changes since the rates in the linearized model don't have drag.

4. The control law makes the quad copter semi stable since it still deviats a little over time.

5. The control law doesn't do much in terms of stablizing the quadcopter. The copter starts oscillating for a few seconds before crashing.