1. $Alt = 20{,}000\,ft$ $\quad M = 0.5 \quad V = 518\,\frac{ft}{s} \quad W = 6.366\times10^{5}\,lb$

$I_x = 1.82\times10^{7}\,slug\,ft^2 \quad I_y = 3.31\times10^{7}\,slug\,ft^2 \quad I_z = 4.97\times10^{7}\,slug\,ft^2 \quad I_{zx} = 9.7\times10^{5}\,slug\,ft^2$

$\xi = -6.8^\circ \quad C_D = 0.04$

$\frac{dX}{du} = -4.883\times10^{1}\,\frac{lbs}{ft} \quad \frac{dZ}{du} = -1.342\times10^{3}\,\frac{lbs}{ft} \quad \frac{dM}{du} = 8.176\times10^{3}\,lbs$

$\frac{dX}{dw} = 1.546\times10^{3}\,\frac{lbs}{ft} \quad \frac{dZ}{dw} = -8.561\times10^{3}\,\frac{lbs}{ft} \quad \frac{dM}{dw} = -5.627\times10^{4}\,lbs$

$\frac{dX}{dq} = 0 \quad \frac{dZ}{dq} = -1.263\times10^{5}\,\frac{lbs}{rad} \quad \frac{dM}{dq} = -1.394\times10^{7}\,\frac{ft\,lbs}{rad}$

$\frac{dX}{d\dot{w}} = 0 \quad \frac{dZ}{d\dot{w}} = 3.104\times10^{2}\,\frac{lbs^2}{ft} \quad \frac{dM}{d\dot{w}} = -4.138\times10^{3}\,lbs^2$

$\frac{dX}{\delta_e} = 3.994\times10^{4}\,\frac{lb}{rad} \quad \frac{dZ}{\delta_e} = -3.341\times10^{5}\,\frac{lb}{rad} \quad \frac{dM}{\delta_e} = -3.608\times10^{7}\,\frac{ft\,lb}{rad}$

$4.448\,N = 1\,lb$ $\quad 1\,slug = 14.5939\,kg$

$1\,ft = 0.3048\,m$

$\Downarrow$

	X	Z	M
u	$-712.6\,\frac{Ns}{m}$	$-19584\,\frac{Ns}{m}$	$36367\,Ns$
w	$22561\,\frac{Ns}{m}$	$-124932\,\frac{Ns}{m}$	$-250289\,Ns$
q	$0\,\frac{Ns}{rad}$	$-561780\,\frac{Ns}{rad}$	$-1.84\times10^{7}\,\frac{mNs}{rad}$
$\dot{w}$	$0\,\frac{Ns^2}{m}$	$4529.7\,\frac{Ns^2}{m}$	$-18406\,Ns^2$
δ_e	$177653\,\frac{N}{rad}$	$-1.49\times10^{6}\,\frac{N}{rad}$	$-4.84\times10^{7}\,\frac{mN}{rad}$

$Alt = 6096\,m \quad V = 157.9\,\frac{m}{s}$

$W = 2.832\times10^{6}\,N$

$I_x = 2.468\times10^{7}\,kg\,m^2$

$I_y = 4.488\times10^{7}\,kg\,m^2$

$I_z = 6.738\times10^{7}\,kg\,m^2$

$I_{zx} = 1.315\times10^{6}\,kg\,m^2$

4) using MATLAB

$$\zeta = \sqrt{\frac{1}{1+(\frac{\omega}{\epsilon})^2}}$$

$$\omega_n = -\frac{\epsilon}{\zeta}$$

$\lambda_1 = -0.462 + 0.9725i$

$\lambda_2 = -0.462 - 0.9725i$

$\lambda_3 = -0.002 + 0.0825i$

$\lambda_4 = -0.002 - 0.0825i$

Phugoid: $\zeta = 0.0243$ $\quad \omega_n = 0.0805$

Short period: $\zeta = 0.9979$ $\quad \omega_n = 0.4630$

5) $\begin{bmatrix} M_q/I_y & M_\alpha u_0/I_y \\ 1 & 0 \end{bmatrix}$ MATLAB $\Downarrow$

short period: eigenvalues

$$\boxed{\begin{aligned}\lambda_1 &= -0.2124 + 0.9206i \\ \lambda_2 &= -0.2124 - 0.9206\end{aligned}}$$

the real parts are more than double in the actual and imaginary is similar

Phugoid: $T_{ph} = \dfrac{2\pi}{\omega_{n_{ph}}\sqrt{1-\zeta_{ph}^2}} = \dfrac{2\pi}{0.4610\sqrt{1-0.0243^2}} = 78.1\ s$

$\boxed{T_{ph} = 78.1 s}$

$u_0 = 156.784\ \frac{m}{s}$

$T_{approx} = \dfrac{\pi\sqrt{2}\, u_0}{g}$

$$\boxed{T_{approx} = \frac{\pi\sqrt{2}}{9.81\frac{m}{s^2}} \cdot 156.8\frac{m}{s} = 71.01\ S}$$

The two periods are similar but still have a big difference.

6. b. The osscilations in the phugoid mode have a period very similar to the ones calculated and are quite consistant throughout the graphs.

The short period osscilations can be seen in the plots but it goes away quickly as the dampening ratio is much higher.

The short period is excited by w and q deviations

The phugoid mode is excited by any deviation

Table of Contents


```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Author: Samuel Razumovskiy
% Date written: 10/17/19
% Date modified: 10/24/19
%
% Purpose: Finding the eigenvalues of a 737 and observing its flight
% characteristics
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

clear,clc,close all
```

Question 2

```
% Defining all the variables
Xu = -712.6;
Xw = 22561;
Xq = 0;
Xwd = 0;
Xdel = 177653;
Zu = -19584;
Zw = -124932;
Zq = -561782;
Zwd = 4529.7;
Zedl = -1.48e6;
Mu = 36367;
Mw = -250289;
Mq = -1.89e7;
Mwd = -18406;
Mdel = -4.89e7;
exz = -6.8;
Ix = 2.468e7;
Iy = 4.488e7;
Iz = 6.738e7;
Izx = 1.315e6;
m = 6.366e5*4.448/9.81;
V = 157.9;
g = 9.81;
```

```
u0 = V*cosd(exz);

% Defining equations using variables
Xup = Xu*cosd(exz)^2-(Xw+Zu)*sind(exz)*cosd(exz)+Zw*sind(exz)^2;
Xwp = Xw*cosd(exz)^2+(Xu-Zw)*sind(exz)*cosd(exz)-Zu*sind(exz)^2;
Xqp = Xq*cosd(exz)-Zq*sind(exz);
Xudp = Zwd*sin(exz)^2;
Xwdp = -Zwd*sind(exz)*cosd(exz);
Zup = Zu*cosd(exz)^2-(Zw-Xu)*sind(exz)*cosd(exz)-Xw*sind(exz)^2;
Zwp = Zw*cosd(exz)^2+(Zu+Xw)*sind(exz)*cosd(exz)+Xu*sind(exz)^2;
Zqp = Zq*cosd(exz)+Xq*sind(exz);
Zudp = -Zwd*sind(exz)*cosd(exz);
Zwdp = Zwd*cosd(exz)^2;
Mup = Mu*cosd(exz)-Mw*sind(exz);
Mwp = Mw*cosd(exz)+Mu*sind(exz);
Mqp = Mq;
Mudp = -Mwd*sind(exz);
Mwdp = Mwd*cosd(exz);
```

Question 3

Calculating the A matrix

```
A = [Xup/m,Xwp/m,0,-g;Zup/(m-Zwdp),Zwp/(m-Zwdp),(Zqp+m*u0)/(m-
Zwdp),0;...
    1/Iy*(Mup+(Mwdp*Zup/(m-Zwdp))),1/Iy*(Mwp+(Mwdp*Zwp/(m-Zwdp))),1/
Iy*(Mqp+(Mwdp*(Zup+m*u0)/(m-Zwdp))),0;
    0,0,1,0];

fprintf('A = \n')
disp(A)
```

```
A =
   -0.0073    0.0274         0   -9.8100
   -0.1205   -0.4347  157.2904         0
    0.0002   -0.0055   -0.4859         0
         0         0    1.0000         0
```

Question 4

```
% Calculating Eigen Values
[V,D] = eig(A);

lam1 = D(1);
lam2 = D(2,2);
lam3 = D(3,3);
lam4 = D(4,4);
figure
hold on
grid on
plot(D(1),'*b')
plot(D(2,2),'*b')
```

```
plot(D(3,3),'*r')
plot(D(4,4),'*r')
title('Eigen values')
legend('Short Period','~','Short Period')

% Finding the Damping ratio and the natural frequency
eta_sp = real(lam1);
nu_sp = imag(lam1);
eta_ph = real(lam3);
nu_ph = imag(lam3);

z_sp = sqrt(1/(1+(nu_sp/eta_sp)^2));
wn_sp = -eta_sp/z_sp;

z_ph = sqrt(1/(1+(nu_ph/eta_ph)^2));
wn_ph = -eta_ph/z_ph;
```

Question 5

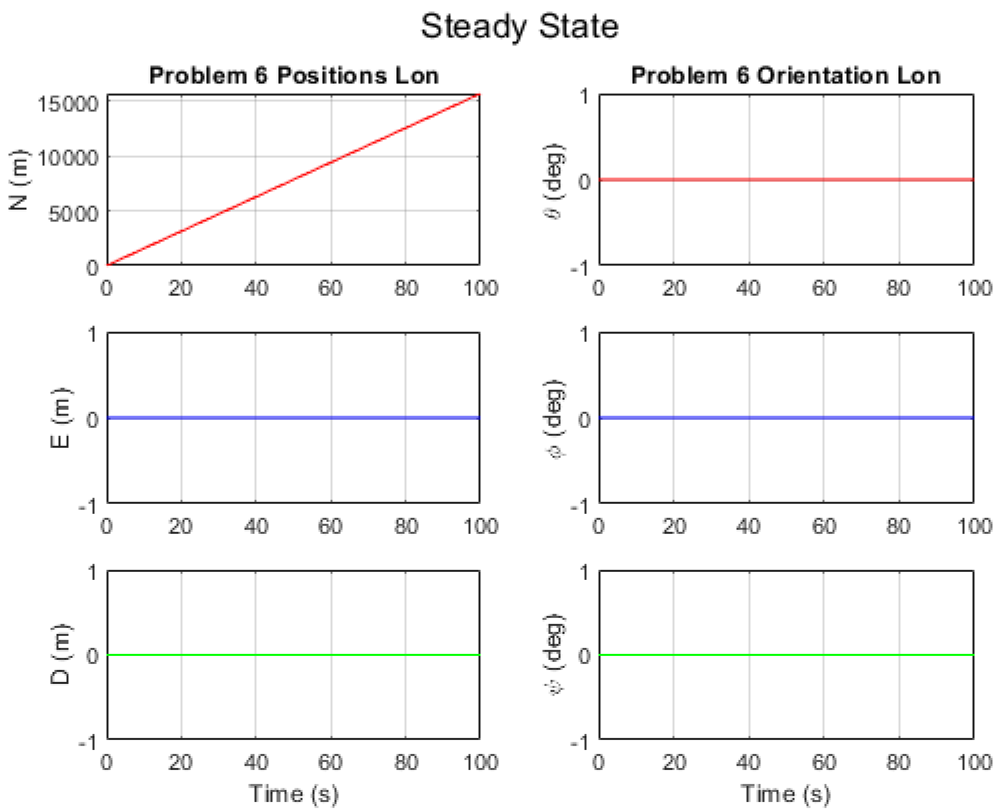
```
m = [Mqp/Iy,Mup*u0/Iy;1,0];
[V2,D2] = eig(m);
```

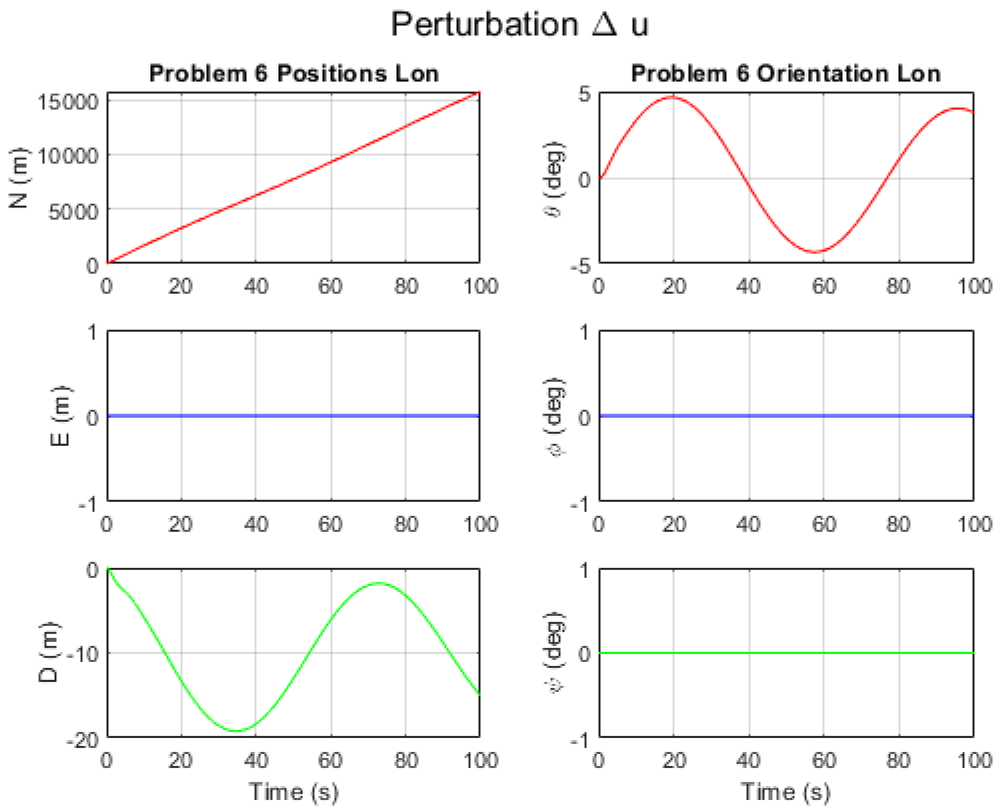
Question 6

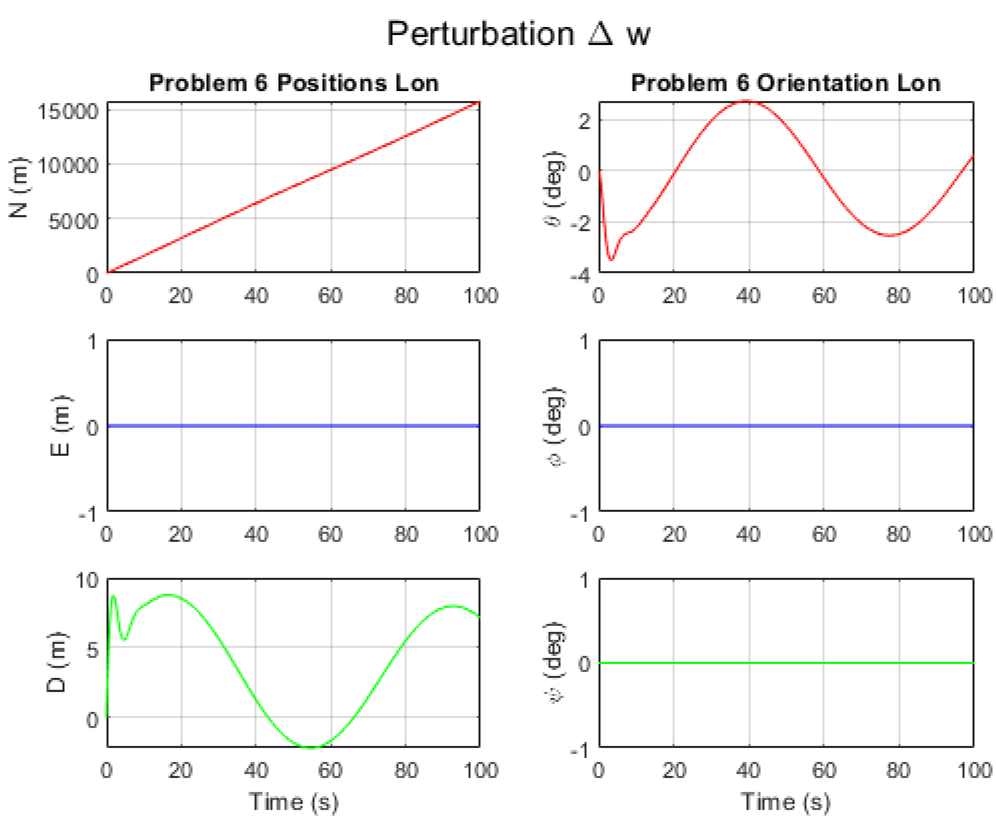
```
dvelU = 10;
```

```
dvelW = 10;
dq = .1;
dtheta = .1;

changes = [0,0,0,0;dvelU,0,0,0;0,dvelW,0,0;0,0,dq,0;0,0,0,dtheta];
names = ["Steady State","Perturbation \Delta u","Perturbation \Delta
 w","Perturbation \Delta q","Perturbation \Delta \theta"];


for i = 1:length(changes)
    CH = changes(i,:);
    tspan = [0 100];

    pos_N = 0; pos_E = 0; pos_D = 0; % m E frame
    u = CH(1); v = 0; w = CH(2); % m/s B frame
    y_trans = [pos_N; pos_E; pos_D; u; v; w];

    psi = 0; theta = CH(4); phi = 0; % rad
    p = 0; q = CH(3); r = 0; % rad/s B frame
    y_rot = [psi; theta; phi; p; q; r];

    y = [y_trans;y_rot];
    opt = odeset('maxstep',0.001);

    [t,pos] =
 ode45(@(t,y)linearized_Aircraft_ODE(t,y,u0,A),tspan,y,opt);

    figure
    sgtitle(names(i))
    subplot(3,2,1)
    plot(t,pos(:,1),'r')
    title("Problem 6 Positions Lon")
    ylabel('N (m)')
    grid on

    subplot(3,2,3)
    plot(t,pos(:,2),'b')
    ylabel('E (m)')
    grid on

    subplot(3,2,5)
    plot(t,pos(:,3),'g')
    ylabel('D (m)')
    xlabel('Time (s)')
    grid on

    subplot(3,2,2)
    plot(t,pos(:,8).*180./pi,'r')
    title("Problem 6 Orientation Lon")
    ylabel("\theta (deg)")
    grid on

    subplot(3,2,4)
    plot(t,pos(:,9).*180./pi,'b')
```

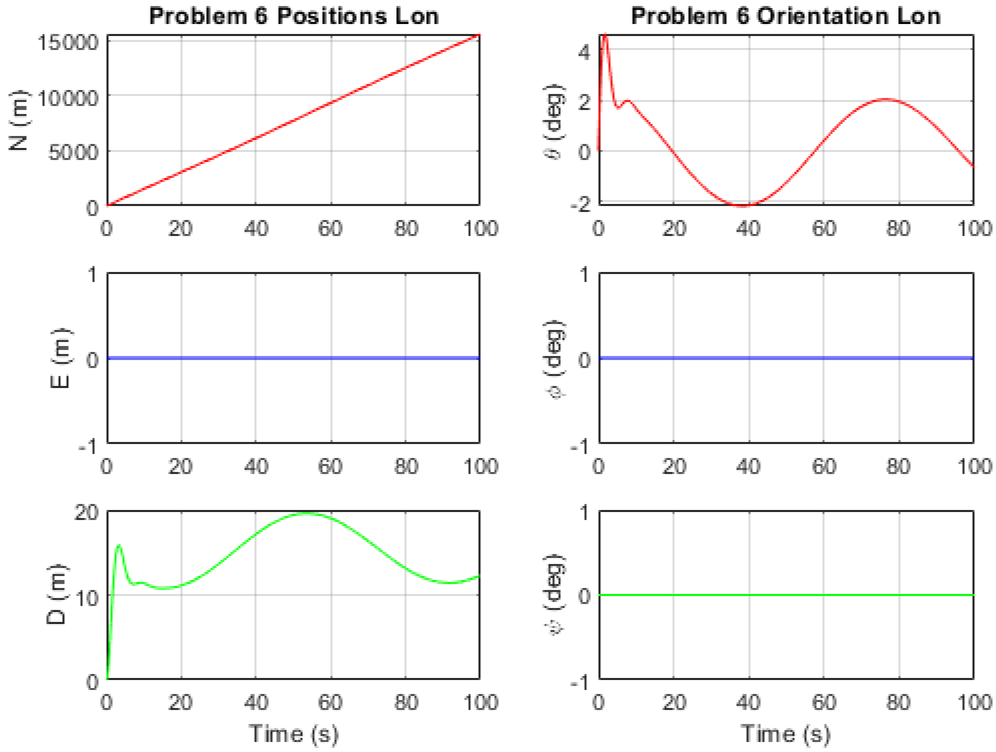
```
    ylabel("\phi (deg)")
    grid on

    subplot(3,2,6)
    plot(t,pos(:,7).*180./pi,'g')
    ylabel("\psi (deg)")
    xlabel('Time (s)')
    grid on

end
```



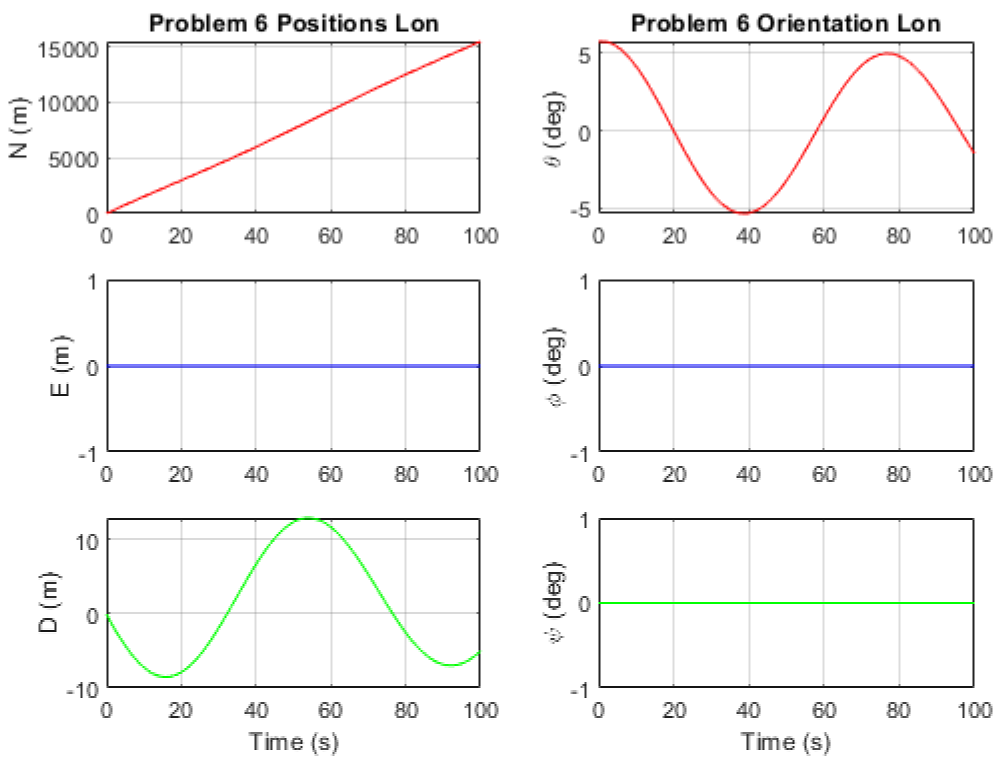

Perturbation Δ u
Problem 6 Positions Lon
Problem 6 Orientation Lon
N (m)
E (m)
D (m)
θ (deg)
φ (deg)
ψ (deg)
Time (s)
Time (s)


Perturbation Δ w
Problem 6 Positions Lon
Problem 6 Orientation Lon
N (m)
E (m)
D (m)
θ (deg)
φ (deg)
ψ (deg)
Time (s)
Time (s)

Perturbation Δ q



Perturbation Δ θ



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