Bottle Rocket Simulation

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11/20/18

ASEN 2012
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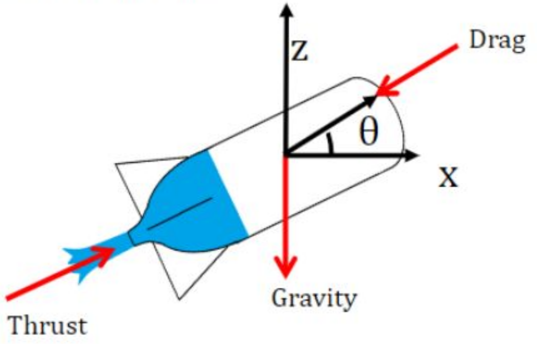
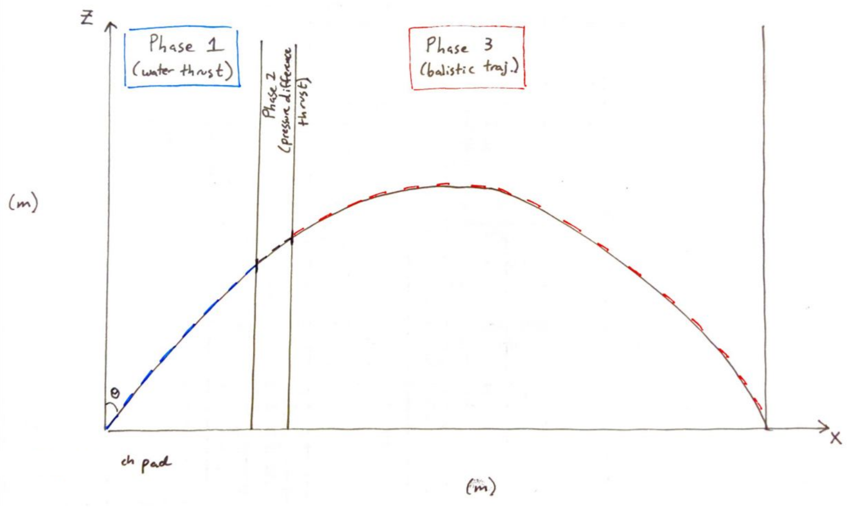
Background

- 10 step model
  - Purpose
  - Info
  - Required to find
  - Assumptions
    - incompressible ambient flow , steady flow, stable rocket, stable thrust, no air/water thrust mixture, no effects of wind, standard atmosphere.
- How a bottle rocket works
- Bottle rocket parameters
  - Pressure
  - Drag coefficient

Free body diagram rocket



Phase trajectory visualization



Equations Phase 1

$$\frac{p}{p^{i}_{air}} = \left(\frac{v^{i}_{air}}{v}\right)^{\gamma}$$

$$\dot{m} = c_d \rho_w A_t V_e$$

$$\frac{dv}{dt} = c_d A_t V_e = c_d A_t \sqrt{\frac{2(p-p_a)}{\rho_w}} = c_d A_t \sqrt{\frac{2}{\rho_w}\left[p_0\left(\frac{v_0}{v}\right)^{\gamma} - p_a\right]}$$

$$(p - p_a) = \frac{\rho_w}{2} V_e^2$$

$$F = \dot{m} V_e + (p_e - p_a) A_t$$

$$\dot{m}_R = -\dot{m} = -\rho_w c_d A_t V_e = -c_d A_t \sqrt{2\rho_w (p - p_a)}$$

$$V_e = \sqrt{\frac{2(p-p_a)}{\rho_w}}$$

$$F = \dot{m} V_e = 2 c_d A_t (p - p_a)$$

$$m^{i}_{R} = m_B + \rho_w \left(v_B - v^{i}_{air}\right) + \left(\frac{p^{i}_{air}}{R T^{i}_{air}}\right) v^{i}_{air}$$

Equations Phase 2

$$p_{end} = p^{i}_{air}\left(\frac{v^{i}_{air}}{v_B}\right)^{\gamma} ; T_{end} = T^{i}_{air}\left(\frac{v^{i}_{air}}{v_B}\right)^{\gamma-1}$$

$$\frac{p}{p_{end}} = \left(\frac{m_{air}}{m^{i}_{air}}\right)^{\gamma} \qquad \rho = \frac{m_{air}}{v_B} ; \ T = \frac{p}{\rho R}$$

$$V_e = M_e\sqrt{\gamma R T_e}$$

$$\dot{m}_{air} = c_d \rho_e A_t V_e$$

$$F = \dot{m}_{air} V_e + \left(p_e - p_a\right) A_t$$

$$p_* = p\left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}$$

Choked flow $p_* > p_a$: $T_e = \left(\frac{2}{\gamma + 1}\right) T \quad \rho_e = \frac{p_e}{R T_e}$

Non Choked flow $p_* \le p_a$: $\frac{p}{p_a} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)}$

$$\frac{T}{T_e} = \left(1 + \frac{\gamma - 1}{2} M_e{}^2\right) \ , \rho_e = \frac{p_a}{R T_e}$$

Methodology

Problem formulation

- Main Function:
  - Initial conditions and constants
  - Call to ODE45
  - Plotting
- ODE45 Function:
  - Calculations of all equations and derivatives
  - Outputs volume, mass, velocity, and position

10 step method

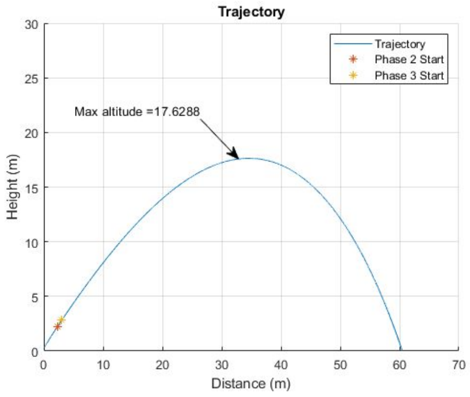
- Fundamental principles
- Alternate approaches
  - C++, Python, Mathematica
  - Runge-Kutta
- Step by step process
  - Phase 1
  - Phase 2
  - Phase 3

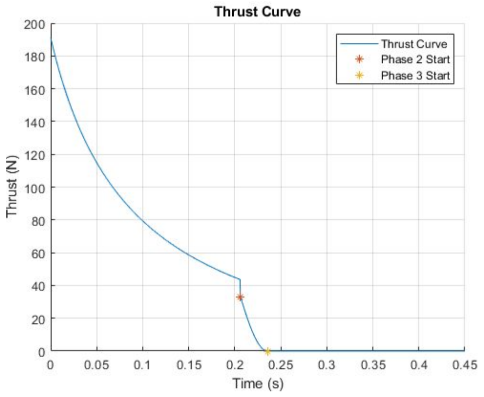
Results

- 10 step method
  - Code verification (test case)
  - Does answer make sense?

Launch angle: 45 deg.
Initial Air pressure: 50 psi
Max distance: 60.47 m

Vol. Air: 0.0015 m^3
Drag coefficient: 0.5
Max height:17.62 m



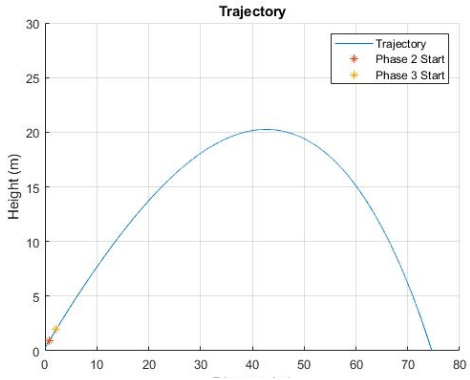


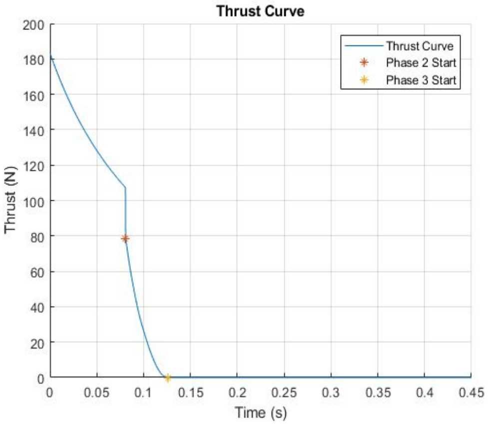
Final Varied Parameters

Launch angle: 40 deg
Initial Air pressure: 48 psi

Vol. Air: 0.0015 m^3
Drag coefficient: 0.4

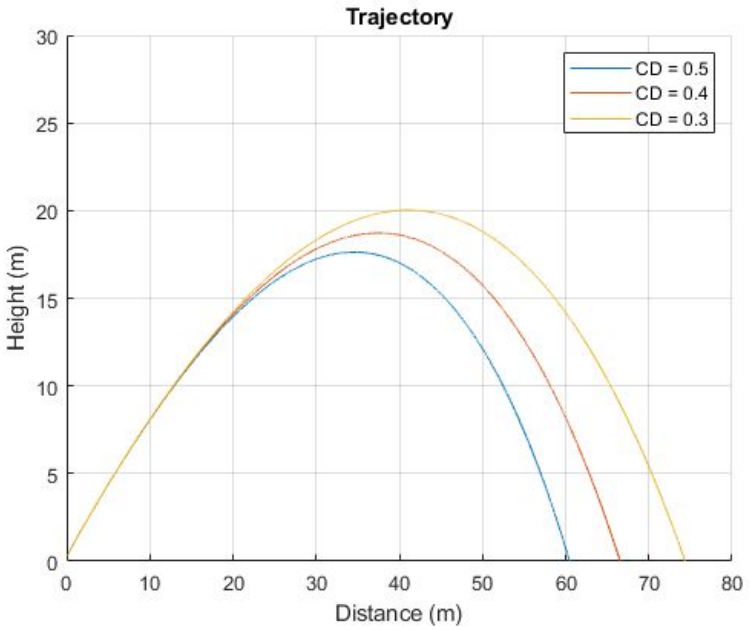
Max distance: 74.64 m



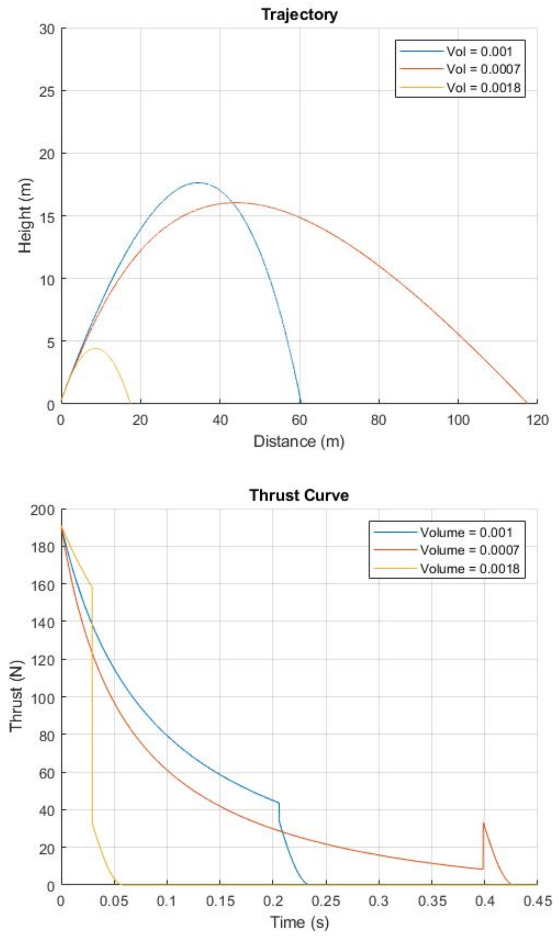


Results (cont.)

Drag Coefficient variation

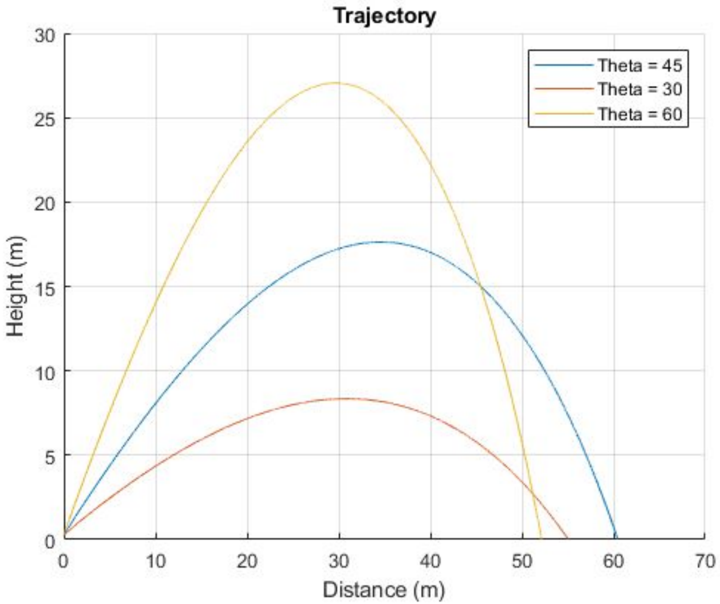


Water/air ratio variation

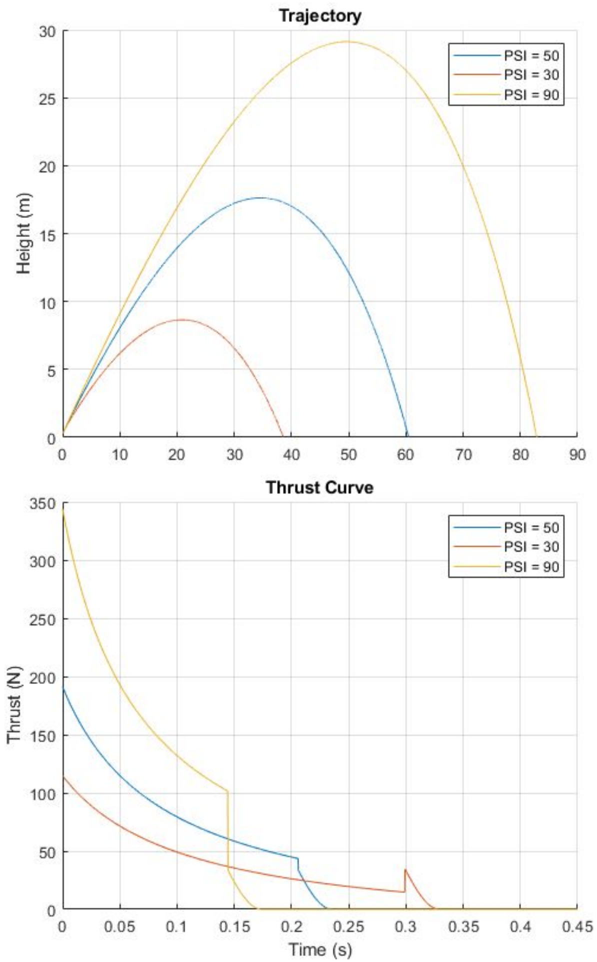


Results (cont.)

Launch Angle variation

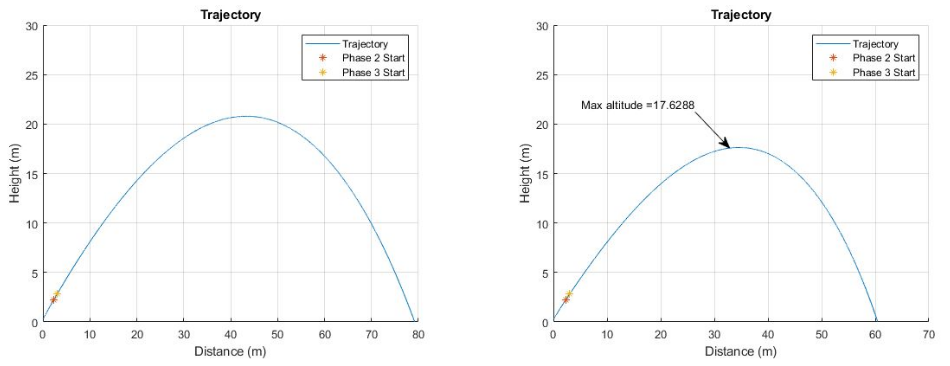


Pressure variation



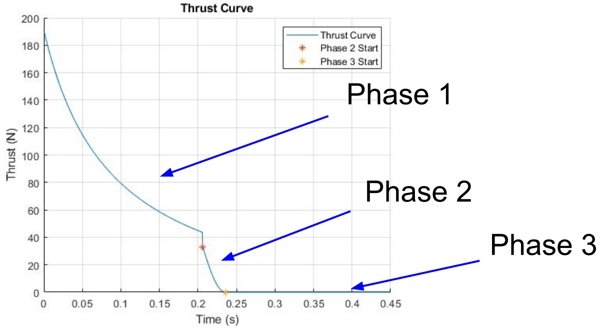
Discussion

- Shape of the trajectory
- Thrust profile
- Velocity profile
- Effects of different parameters
  - Initial Pressure
  - Initial volume ratio
  - Coefficient of drag
  - Angle of launch
- Trade offs
  - Less water means less thrust
  - Goldilocks principle with respect to air-water ratio
  - Higher pressure means higher initial thrust (thrust duration)



Drag coefficient of 0.25

Drag coefficient of 0.5



Conclusion

- Found the correct trajectory of the bottle rocket
- Used ODE45 and simplified the code as much as possible
- Found the effects each parameter has on the trajectory

References

Torin Clark, “Project 2 Bottle Rocket Modeling” Project document, (2018).

“Water-Bottle Rockets. - Ppt Download.” SlidePlayer, SlidePlayer, slideplayer.com/slide/4878182/.

“WATER ROCKET SAFETY.” NASA, NASA, www.grc.nasa.gov/www/k-12/rocket/BottleRocket/safety.htm.

W., Mike, “How much to fill a bottle rocket?” Ask the Van, Department of Illinois Physics [online], https://van.physics.illinois.edu/qa/listing.php?id=17080 [retrieved 20 November 2018].