Samuel Mazumdysky | HW # 11 | 4/12/19

21.3 given: $m = 4kg$ $x_0 = 0$ $P = 0.5s$ $x_1 = 0.1m$ $t_1 = 0s$

find: x_2 if $t_2 = 0.4s$

solution: $T = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$ $V = \frac{1}{2} k x^2$

$$\frac{d}{dt}(T+V) = \frac{m\left(\cancel{\frac{dx}{dt}}\right)\left(\frac{d^2x}{dt^2}\right) + kx\cancel{\frac{dx}{dt}} = 0}{m\frac{dx}{dt}}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\ddot{x}(t) + \cancel{2\zeta\omega_n\dot{x}(t)} + \omega_n^2 x(t) = \cancel{\omega^2 x(t)}$$

$$\omega_n^2 = \frac{k}{m}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$ $P = \frac{2\pi}{\omega_n}$ $\omega_n = \frac{2\pi}{P}$

$\sqrt{\frac{k}{m}} = \frac{2\pi}{P}$

$$\ddot{x}(t) + \omega_n^2 x(t) = 0$$

$k = \left(\frac{2\pi}{P}\right)^2 m$

since $\zeta = 0$

$$x(t) = x_0 \cos(\omega_n t) + \cancel{\frac{\dot{x}_0}{\omega_n}\sin(\omega_n t)}$$

$k = 631.7 \frac{N}{m}$

$$x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$x(0.4) = 0.1\cos\left(\sqrt{\frac{631.7}{4}}\, 0.4\right)$$

$$\boxed{x(0.4) = 0.031m}$$

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Samuel Morcmoskey | HW #11 | 4/15/19

(21.4) given: $m = 4kg$ $f = 6Hz$ $x_1 = 0.1m$

$\frac{dx}{dt} = 5\frac{m}{s}$ $t=0$

Find: E (amplitude)

Solution:

$$x(t) = A\cos(\omega_n t) + B\cos(\omega_n t)$$

$$E = \sqrt{A^2 + B^2} \qquad A = x_0 \qquad B = \frac{\dot{x}_0}{\omega_n}$$

$$E = \sqrt{0.1^2 + \left(\frac{5}{37.7}\right)^2} \qquad \omega_n = 2\pi f = 2\pi(6) = 37.7\frac{rad}{s}$$

$$\boxed{E = 0.166\,m}$$

Samuel Muramovsky | HW # 11 | 4/15/14

(21.44) given: $m = 4kg$ $\quad l = 2m$ $\quad M_f = 1.4\frac{d\theta}{dt}\ Nm$

Find a) P, f $\quad$ b) t @ $\frac{A}{2}$

Solution:

$$\Sigma M_O = M_f - mg\frac{l}{2}\sin(\theta) = I\ddot{\theta}$$

$$I\ddot{\theta} - M_f + mg\frac{l}{2}\sin\theta = 0$$

$$\ddot{\theta} - \frac{1.4\dot{\theta}}{\frac{1}{3}ml^2} + \frac{mg\sin\theta}{\frac{1}{3}ml^2} = 0$$

$$\ddot{\theta} - \frac{4.2\dot{\theta}}{ml^2} + \frac{3g}{2l}\theta = 0$$

$$\zeta = \frac{\frac{4.2}{ml^2}}{\omega_n^2} = 0.048$$

$$\omega_n^2 = \frac{3g}{2l} = \sqrt{\frac{3g}{2l}} = 2.71\ \frac{rad}{s}$$

$$P = \frac{2\pi}{\sqrt{\frac{3g}{2l}}} = 2.32S$$

$$f = \frac{\sqrt{\frac{3g}{2l}}}{2\pi} = 0.432\ Hz$$

$$\boxed{\begin{array}{l} P = 2.32S \\ f = 0.432Hz \end{array}}$$

$$\theta(t) = \theta_0 e^{-\zeta\omega_n t}\left\{\cos(\omega_d t) + \frac{\zeta}{\sqrt{1+\zeta^2}}\sin(\omega_d t)\right\} + \frac{\dot{\theta}_0}{\omega_d}e^{-\zeta\omega_n t}\sin(\omega_d t)$$

$$\frac{\cancel{\theta_0}}{2} = \cancel{\theta_0}e^{-\zeta\omega_n t}$$

$$\ln\left(\tfrac{1}{2}\right) = -\zeta\omega_n t$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-\zeta\omega_n} = 5.28S$$

$$\boxed{t = 5.28S}$$

Samuel Marcinovsky | HW #11 | 4/15/14

21.45 given: $\theta_0 = 2°$

find: $\theta(2)$

Solution:

$$\theta(t) = \theta_0 e^{-\zeta\omega_n t}\left\{\cos(\omega_d t) + \frac{\zeta}{\sqrt{1+\zeta^2}}\sin(\omega_d t)\right\} + \cancel{\frac{\dot{\theta}_0}{\omega_d} e^{-\zeta\omega_n t}} \sin(\omega_d t)$$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.709$

$\omega_n = 2.71 \frac{rad}{s}$

$\zeta = 0.048$

$$\theta(2) = 2e^{-0.048 \cdot 2.71 \cdot 2}\left\{\cos(2.709 \cdot 2) + \frac{0.048}{\sqrt{1+0.048^2}}\sin(2.709 \cdot 2)\right\}$$

$$\boxed{\theta(2) = 0.94^{\circ}}$$

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Samuel Arazumovskiy HW#11 4/15/19

(3) given: $\dot{\beta}+0.5\beta=u$

Find:

Solution: $L[\dot{\beta}+0.5\beta]=L[u]$

$$s\beta(s)-\beta(0)+0.5\beta(s)=U(s) \qquad \beta(0)=0$$

$$\beta(s)(s+0.5)=U(s)$$

a) $\boxed{\frac{\beta(s)}{U(s)}=\frac{1}{s+0.5}}$

$\boxed{\text{b) } u=-K_p(\beta-\beta_R)}$

c) $\dot{\beta}+0.5\beta=-K_p(\beta-\beta_R)$

$$s\beta(s)+0.5\beta(s)=-K_p(\beta(s)-\beta_R(s))$$

$$\beta(s)(s+0.5+K_p)=K_p\beta_R(s)$$

$\boxed{\text{c) } \frac{\beta(s)}{\beta_R(s)}=\frac{K_p}{s+0.5+K_p}}$

d) $K_p=10, 30$

e) $\boxed{t_{10}=0.28s \qquad t_{30}=0.09s}$

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