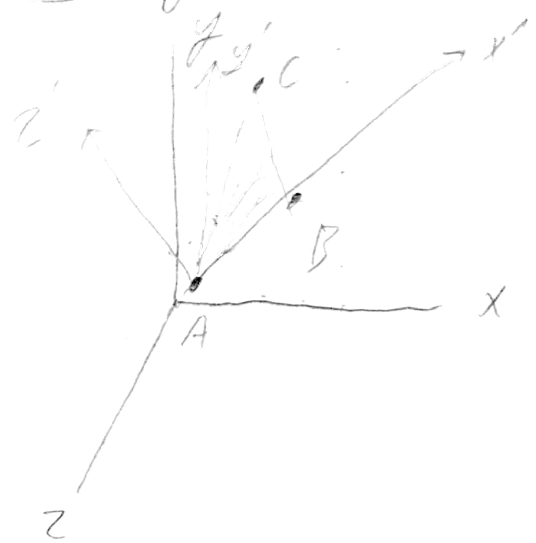
4.8 given:



find; a) Q b) find $[2,-1,3]^T$ in unprimed

solution;

$$x' = (4-1)\hat{i} + (6-2)\hat{j} + (5-3)\hat{k} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$

$$z' = \vec{AB} \times \vec{AC} = [3,4,2]^T \times [2,7,-5]^T = -34\hat{i} + 19\hat{j} + 13\hat{k}$$

$$y' = x' \times z' = [3,4,2] \times [-34,19,13] = [14,-107,193]$$

$$\hat{i} = \frac{x'}{|x'|} = 0.557\hat{i} + 0.743\hat{j} + 0.3714\hat{k}$$

$$\hat{j} = \frac{y'}{|y'|} = 0.0633\hat{i} + 0.4839\hat{j} - 0.8728\hat{k}$$

$$\hat{k} = \frac{z'}{|z'|} = -0.8281\hat{i} + 0.4627\hat{j} + 0.3166\hat{k}$$

a) $$\boxed{Q = \begin{bmatrix} 0.557 & 0.743 & 0.3714 \\ -0.0663 & 0.484 & -0.873 \\ -0.828 & 0.463 & 0.3166 \end{bmatrix}}$$

b) $$V = Q^T \cdot [2,-1,3]^T = \boxed{-1.3066\hat{i} + 2.3848\hat{j} + 2.5654\hat{k}}$$

(4.10) given: $\alpha = 40°$ $\beta = 25°$

find: Q

Solution:

$$Q' = R_1(\alpha) \cdot R_2(\beta)$$

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(40) & \sin(40) \\ 0 & -\sin(40) & \cos(40) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.766 & 0.643 \\ 0 & -0.643 & 0.766 \end{bmatrix}$$

$$R_2(\beta) = \begin{bmatrix} 0.906 & 0 & -0.4226 \\ 0 & 1 & 0 \\ 0.4226 & 0 & 0.906 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.766 & 0.643 \\ 0 & -0.643 & 0.766 \end{bmatrix} \cdot \begin{bmatrix} 0.906 & 0 & -0.4226 \\ 0 & 1 & 0 \\ 0.4226 & 0 & 0.906 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0.9063 & 0 & -0.4226 \\ 0.2716 & 0.766 & 0.5826 \\ 0.3237 & -0.643 & 0.6943 \end{bmatrix}$$

$$\boxed{Q = Q'^T = \begin{bmatrix} 0.9063 & 0.2716 & 0.3237 \\ 0 & 0.766 & -0.6428 \\ -0.4226 & 0.5825 & 0.6943 \end{bmatrix}}$$

Table of Contents


```
%%%%% Problem 3
```

Knock Knock, Housekeeping

```
clear, clc, close all
```

Givens

```
alpha = 45;
beta = 45;
gamma = 45;

r1 = [1, 0, 0; 0, cosd(alpha), sind(alpha); 0, -sind(alpha),
 cosd(alpha)];
r2 = [cosd(beta), 0, -sind(beta);0, 1, 0; sind(beta), 0, cosd(beta)];
r3 = [cosd(gamma), sind(gamma), 0; -sind(gamma), cosd(gamma), 0; 0, 0,
 1];
```

a) Sequences

```
Q1 = r3*r2*r1;
Q2 = r1*r2*r3;
Q3 = r3*r1*r3;

disp('Sequence 1')
disp(Q1)
disp('Sequence 2')
disp(Q2)
disp('Sequence 3')
disp(Q3)
```

```
Sequence 1
    0.5000    0.8536    0.1464
   -0.5000    0.1464    0.8536
    0.7071   -0.5000    0.5000

Sequence 2
    0.5000    0.5000   -0.7071
   -0.1464    0.8536    0.5000
    0.8536   -0.1464    0.5000
```

```
Sequence 3
    0.1464    0.8536    0.5000
   -0.8536   -0.1464    0.5000
    0.5000   -0.5000    0.7071
```

b) Axis and angle of rotation

```
[evec1,ev1] = eig(Q1);
[evec2,ev2] = eig(Q2);
[evec3,ev3] = eig(Q3);

[phi1,u1] = get_phi_u(evec1,ev1);
[phi2,u2] = get_phi_u(evec2,ev2);
[phi3,u3] = get_phi_u(evec3,ev3);

fprintf('Angle of rotation for sequence 1 = %.3f\n', phi1)
disp('Axis of rotation for sequence 1')
disp(u1)
fprintf('Angle of rotation for sequence 2 = %.3f\n', phi2)
disp('Axis of rotation for sequence 2')
disp(u2)
fprintf('Angle of rotation for sequence 3 = %.3f\n', phi3)
disp('Axis of rotation for sequence 3')
disp(u3)

disp('The DCMs all have the same values just in different locations.
 One')
disp('one thing of note is that the eigenvectors of sequences 2 and 3
 have')
disp('an equal value. The axes of rotation are all different, and have
 a wide range')
function [phi,u] = get_phi_u(evec,ev)
    [r,~] = size(ev);
    for i = 1:r
        if(floor(ev(i,i)) == 1)
            u = evec(:,i);
        else
            phi = acosd(real(ev(i,i)));
        end
    end
end
```

```
Angle of rotation for sequence 1 = 85.801
Axis of rotation for sequence 1
    0.6786
    0.2811
    0.6786

Angle of rotation for sequence 2 = 64.737
Axis of rotation for sequence 2
    0.3574
    0.8629
```

```
    0.3574

Angle of rotation for sequence 3 = 98.421
Axis of rotation for sequence 3
    0.5054
   -0.0000
    0.8629
```

The DCMs all have the same values just in different locations. One one thing of note is that the eigenvectors of sequences 2 and 3 have an equal value. The axes of rotation are all different, and have a wide range

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(9.3) given:

find a) ω_{plate}, b) α_{plate}, c) a_c

solution: a)

$$\omega_{plate} = \dot{\theta}\hat{K} + \dot{\phi}\hat{J} + \dot{\nu}\hat{m} + \dot{\psi}\hat{n}$$

$$\hat{m} = \sin(\phi)\hat{I} + \cos(\phi)\hat{K}$$

$$\hat{n} = -\cos(\phi)\sin(\nu)\hat{I} + \cos(\nu)\hat{J} + \sin(\phi)\sin(\nu)\hat{K}$$

$$\omega_{plate} = \dot{\theta}\hat{K} + \dot{\phi}\hat{J} + \dot{\nu}\sin(\phi)\hat{I} + \dot{\nu}\cos(\phi)\hat{K} - \dot{\psi}\cos(\phi)\sin(\nu)\hat{I} + \dot{\psi}\cos(\nu)\hat{J} + \dot{\psi}\sin(\phi)\sin(\nu)\hat{K}$$

$$\boxed{\omega_{plate} = (\dot{\nu}\sin(\phi) - \dot{\psi}\cos(\phi)\sin(\nu))\hat{I} + (\dot{\phi} + \dot{\psi}\cos(\nu))\hat{J} + (\dot{\theta} + \dot{\nu}\cos(\phi) + \dot{\psi}\sin(\phi)\sin(\nu))\hat{K}}$$

b) $\alpha_{plate} = \left.\frac{d\omega_{plate}}{dt}\right)_{rel} + \Omega \times \omega_{plate}$ where $\Omega = \dot{\theta}\hat{K}$

$$= \dot{\nu}\dot{\phi}\cos(\phi) - \dot{\psi}\dot{\phi}\sin(\phi)\sin(\nu) + \dot{\psi}\dot{\nu}\cos(\phi)\cos(\nu)\,\hat{I} + \dot{\psi}\dot{\nu}\cos(\nu)\hat{J} + (\dot{\nu}\dot{\phi}\cos(\phi) + \dot{\psi}\dot{\phi}\cos(\phi)\sin(\nu) + \dot{\psi}\dot{\nu}\sin(\phi)\cos(\nu))\hat{K}$$

$$+ \dot{\theta}(\dot{\phi} + \dot{\psi}\cos(\nu))\hat{I} + \dot{\theta}(\dot{\nu}\sin(\phi) - \dot{\psi}\cos(\phi)\sin(\nu))\hat{J}$$

$$\boxed{\begin{aligned}\alpha_{plate} = &(\dot{\nu}(\dot{\phi}\cos(\phi) + \dot{\psi}\cos(\phi)\cos(\nu)) + \dot{\psi}\dot{\phi}\sin(\phi)\sin(\nu) + \dot{\psi}\dot{\theta}\cos(\nu) + \dot{\phi}\dot{\theta})\hat{I} \\ &+ \dot{\nu}(\dot{\theta}\sin(\phi) - \dot{\psi}\sin(\nu)) - \dot{\psi}\dot{\theta}\cos(\phi)\sin(\nu)\hat{J} \\ &+ \dot{\psi}\dot{\nu}\cos(\nu)\sin(\phi) + \dot{\psi}\dot{\phi}\cos(\phi)\sin(\nu) - \dot{\phi}\dot{\nu}\sin(\phi)\hat{K}\end{aligned}}$$

(9.5) given: $\vec{r}_A = 2\hat{i} + 2\hat{j} - 2\hat{k}\,m$ $\vec{v}_A = 1\hat{i} + 2\hat{j} + 3\hat{k}\ \frac{m}{s}$

$\vec{r}_B = 1\hat{i} + 1\hat{j} - 1\hat{k}\,m$

find: v_B

solution: $|r_A| = \sqrt{2^2+2^2+2^2} = 3.464\,m$ $|\vec{r}_B| = \sqrt{1^2+1^2+1^2} = 1.732\,m$

$|v_A| = \sqrt{1^2+2^2+3^2} = 3.74165\ \frac{m}{s}$

$w = \frac{v_A}{r_A} = \frac{v_B}{r_B}$

$v_B = \frac{v_A}{r_A} \cdot r_B = \frac{3.74165}{3.464} \cdot 1.732 = 1.8708\ \frac{m}{s}$

$\boxed{v_B = 1.8708\ \frac{m}{s}}$

9.6 given: $\vec{w} = w_x \hat{i} + w_y \hat{j} + w_z \hat{k}$

$\vec{\Omega} = w_x \hat{i} + w_y \hat{j}$

find: α

solution: $\alpha = \left(\frac{d\vec{w}}{dt}\right)_{rel} + \vec{\Omega} \times \vec{w}$

$$= 0 + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ w_x & w_y & 0 \\ w_x & w_y & w_z \end{vmatrix}$$

$$= w_y w_z \hat{i} - w_x w_z \hat{j} + (\cancel{w_x w_y - w_x w_y}) \hat{k}$$

$$\boxed{\alpha = w_y w_z \hat{i} - w_x w_z \hat{j}}$$