Samuel Rauzmovksiy
ASEN 3128 Assignment 11
12/5/2019

1. Spiral Approximation = -0.0051, Spiral Actual = -0.0051
   Roll Approximation = -0.6831, Roll Actual = -0.569
   The approximations in this case are fairly close to the actual eigenvalues, and using them seems useful.

2. a)
   B matrix

0	1.6694
-0.1386	0.1455
0.0074	-0.4784
0	0

b)
A_{aug}

-0.0809	0	-157.9000	9.8100	0	0
-0.0172	-0.5659	0.6219	0	0	0
0.0064	0.0065	-0.2521	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
1	0	0	0	157	0

B_{aug}

0	1.6694
-0.1386	0.1455
0.0074	-0.4784
0	0
0	0
0	0

3. Case a: None of the stability derivatives are effected. We see an increase in stability of the spiral mode, decrease in stability of the roll mode, and an increase in stability of the dutch roll mode with a fairly constant damping ratio. The spiral and roll modes both merge into a second mode with enough of an increase in gain value.

   Case b: $\mathcal{Y}_p$, $\mathcal{L}_p$, and $\mathcal{N}_p$ are affected by the change in K_{ap}. There is a slight decrease in the dutch roll, spiral, and roll stability. Where at one point the roll and spiral modes go positive.

   Case c: $\mathcal{Y}_r$, $\mathcal{L}_r$, and $\mathcal{N}_r$ are affected by the change in K_{ar}. The spiral mode gets unstable while the spiral and dutch roll mode increase in stability. Where the dutch roll mode has relatively the same damping.

   Case d: none of the stability derivatives are effected. Here we see the roll and dutch roll mode getting more stable and more damped. While the spiral mode splits into a new mode by combining with one of the augmented eigenvalues and goes unstable.

   Case e: $\mathcal{Y}_v$, $\mathcal{L}_v$, and $\mathcal{N}_v$ are affected by a change in K_{rv}. Here the dutch roll mode gets more stable but with much higher oscillations. The roll mode gets less stable as well as the spiral, which goes unstable.

   Case f: $\mathcal{Y}_p$, $\mathcal{L}_p$, and $\mathcal{N}_p$ are affected by a change in K_{rp}. The dutch roll mode gets very stable and increases the damping for a bit. But, the roll mode gets less stable and meets up with the spiral mode which briefly gets more stable. Once the two modes meet, they split and move over to the right and meet again at the x axis splitting again along the positive real axis.

   Case g: $\mathcal{Y}_r$, $\mathcal{L}_r$, and $\mathcal{N}_r$ are affected by a change in K_{rr}. For this case the dutch roll mode gets critically or overly damped with a high enough gain value. The spiral mode also gets more stable as it and the roll mode(which is getting less stable) meet up and split vertically. Moving over to the right creating oscillations but not going unstable.

Case h: There is no effect on the stability derivatives by $K_{r\phi}$. As $K_{r\phi}$ increases the spiral mode goes unstable, the roll mode gets more stable, and the dutch roll has an increase in stability but as well as a decrease in damping and an increase in oscillations.

Case i: There is no effect on the stability derivatives by $K_{r\psi}$. As $K_{r\psi}$ increase the spiral mode splits and goes unstable, the roll mode gets more stable, and the dutch roll has an increase in stability but as well as a decrease in damping and an increase in oscillations.

Overall the most effective at damping the dutch roll mode would be K_{rr} since it can completely damp out the dutch roll. But increasing K_{rr} too much causes the roll and spiral modes to go unstable. So a K_{rr} value of around 1 can damp the dutch roll mode significantly and increase stability of the spiral mode while keeping the roll mode stable.

$K_{r\phi}$ can increase stability in the roll mode, but it can cause the spiral mode to become unstable. So a combination must be used to keep the spiral mode stable. For instance the K_{rr} gain value can increase the stability of the spiral mode, although the increase in stability of the spiral mode from K_{rr} is less than the decrease from $K_{r\phi}$.

Table of Contents

Header

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%
% Author: Samuel Razumovskiy
% Date written: 11/22/19
% Date modified: 12/5/19
%
% Purpose: Finding the eigenvalues of a 737 and observing its lateral
% flight characteristics
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%

clear,clc,close all
```

Question 1

Defining all the given parameters

```
exz = -6.8*pi/180;
Ix = 2.46767e7;
Iy = 4.488e7;
Iz = 6.7384e7;
Izx = 1.315e6;
m = 6.366e5*4.448/9.81;
u0 = 157.9;
S = 5500*0.3048^2;
b = 195.68*0.3048;
c = 27.31*0.3048;
g = 9.81;
rho = 0.66011;

Ixs = Ix*cos(exz)^2+Iz*sin(exz)^2+Izx*sin(2*exz);
Izs = Ix*sin(exz)^2+Iz*cos(exz)^2-Izx*sin(2*exz);
Izxs = -.5*(Ix-Iz)*sin(2*exz)-Izx*sin(sin(exz)^2-cos(exz)^2);

% Making a matrix of the coefficients
Coeff = [-0.8771, -0.2797,   0.1946;...
               0, -0.3295, -0.04073;...
               0,   0.304,  -0.2737];
```

```
% Making a matrix of the conversion factors
Conv = [1/2*rho*u0*S,1/2*rho*u0*b*S,1/2*rho*u0*b*S;...
    1/4*rho*u0*b*S,1/4*rho*u0*b^2*S,1/4*rho*u0*b^2*S;...
    1/4*rho*u0*b*S,1/4*rho*u0*b^2*S,1/4*rho*u0*b^2*S];

Dim= Conv.*Coeff;

% Dimensionalized stability derivatives
Yv = Dim(1,1); Lv = Dim(1,2); Nv = Dim(1,3);
Yp = Dim(2,1); Lp = Dim(2,2); Np = Dim(2,3);
Yr = Dim(3,1); Lr = Dim(3,2); Nr = Dim(3,3);

% Converting the Stability derivatives to Stability frame
Yvp = Yv;
Ypp = Yp*cos(exz)-Yr*sin(exz);
Yrp = Yr*cos(exz)+Yp*sin(exz);
Lvp = Lv*cos(exz)-Nv*sin(exz);
Lpp = Lp*cos(exz)^2-(Lr+Np)*sin(exz)*cos(exz)+Nr*sin(exz)^2;
Lrp = Lr*cos(exz)^2-(Nr-Lp)*sin(exz)*cos(exz)-Np*sin(exz)^2;
Nvp = Nv*cos(exz)+Lv*sin(exz);
Npp = Np*cos(exz)^2-(Nr-Lp)*sin(exz)*cos(exz)-Lr*sin(exz)^2;
Nrp = Nr*cos(exz)^2+(Lr+Np)*sin(exz)*cos(exz)+Lp*sin(exz)^2;

Ixp = (Ixs*Izs-Izxs^2)/Izs;
Izp = (Ixs*Izs-Izxs^2)/Ixs;
Izxp = Izxs/(Ixs*Izs-Izxs^2);

% A matrix
A = [        Yvp/m,            Ypp/m,         Yr/m-u0, g;...
    Lvp/Ixp+Izxp*Nvp, Lpp/Ixp+Izxp*Npp, Lrp/Ixp+Izxp*Nrp, 0;...
    Izxp*Lvp+Nvp/Izp, Izxp*Lpp+Npp/Izp, Izxp*Lrp+Nrp/Izp, 0;...
                   0,                1,                0, 0];

[V,D] = eig(A);

% Specifying the fancy letter values
Yv = A(1,1); Yp = A(1,2); Yr = A(1,3);
Lv = A(2,1); Lp = A(2,2); Lr = A(2,3);
Nv = A(3,1); Np = A(3,2); Nr = A(3,3);

Re = real(D);
Im = imag(D);

% Roll approximation
RollAp = A(2,2);
e = g*(Lv*Nr-Lr*Nv);
d = -g*Lv+Yv*(Lr*Np-Lv*Np)+Yr*(Lp*Nv-Lv*Np);
SpiralAp = -e/d;
```

Question 2 a

```
% Coefficients from the book are multiplied against a conversion
 matrix
```

```
Coeff = [    0, -1.368e-2, -1.973e-4;...
        0.1146,  6.976e-3,   -0.1257];
conv = [1/2*rho*u0^2*S, 1/2*rho*u0^2*S*b, 1/2*rho*u0^2*S*b;...
        1/2*rho*u0^2*S, 1/2*rho*u0^2*S*b, 1/2*rho*u0^2*S*b];
dim = Coeff.*conv;

Yda = dim(1,1);
Ydr = dim(2,1);
Lda = dim(1,2);
Ldr = dim(2,2);
Nda = dim(1,3);
Ndr = dim(2,3);

% B matrix
B = [          Yda/m,              Ydr/m;
    Lda/Ixp+Izxp*Nda, Ldr/Ixp+Izxp*Ndr;
    Izxp*Lda+Nda/Izp, Izxp*Ldr+Ndr/Izp;
                   0,                0];
```

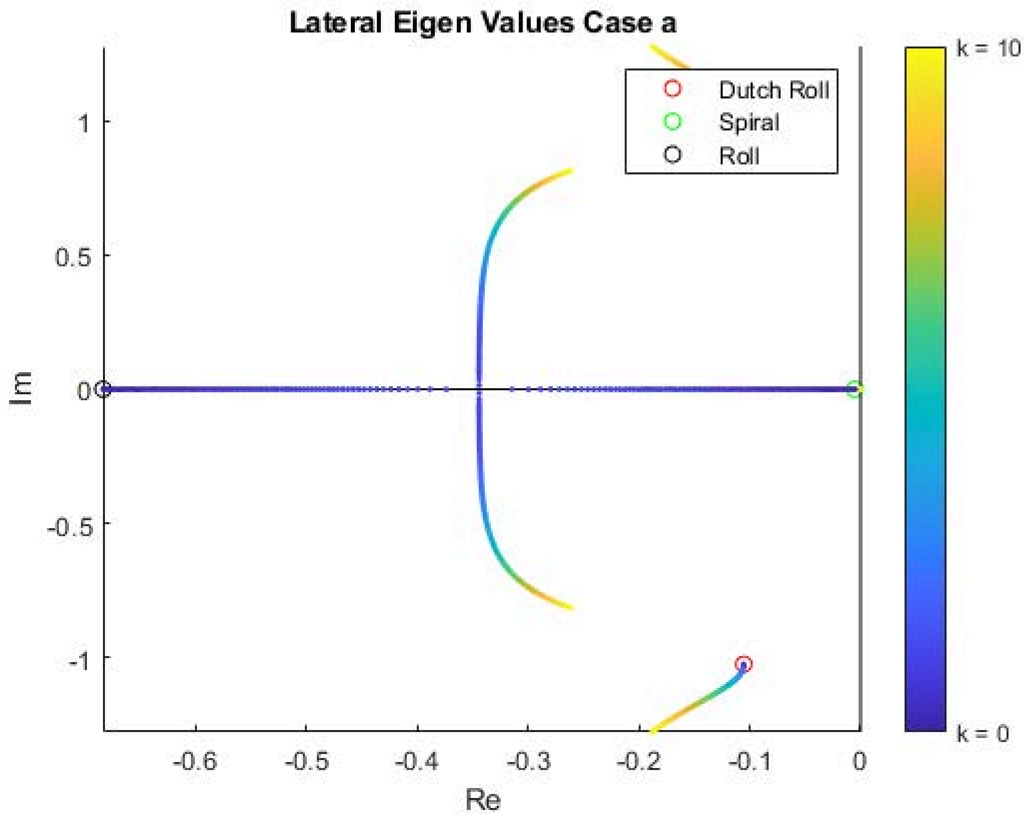
Question 2 b

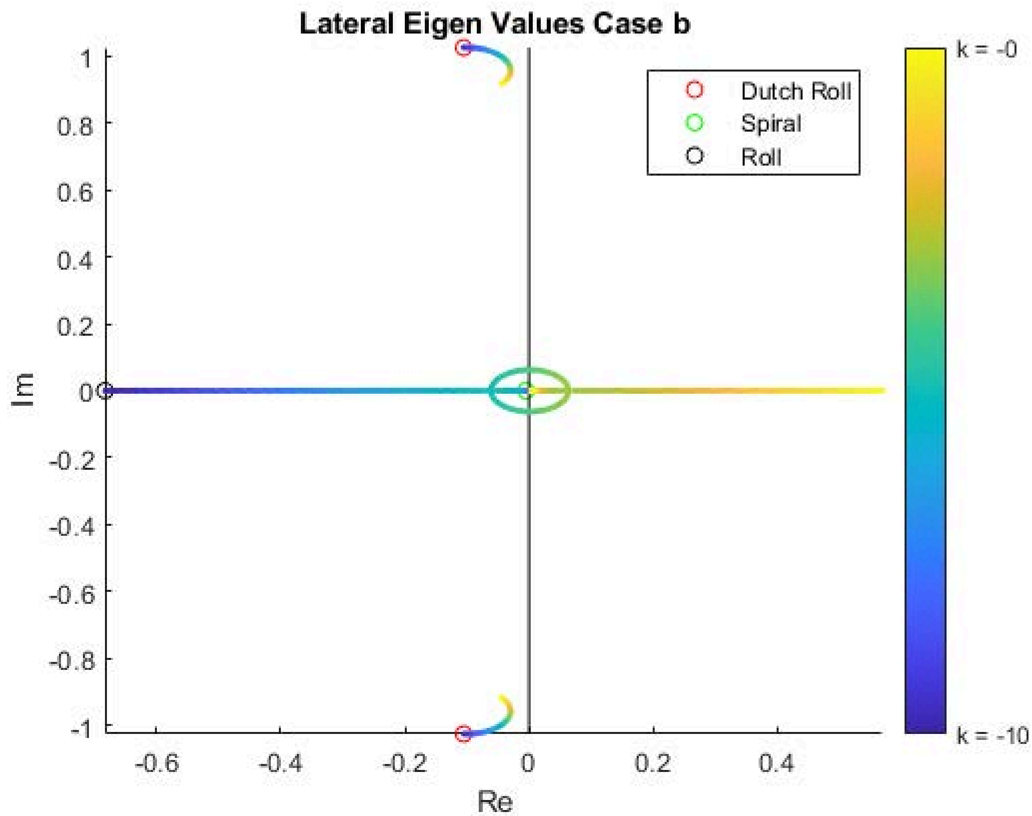
```
% Augmenting the A and B matricies
Aaug =
 [A(1,:),0,0;A(2,:),0,0;A(3,:),0,0;A(4,:),0,0;0,0,1,0,0,0;1,0,0,0,u0,0];

Baug = [B(1,:);B(2,:);B(3,:);B(4,:);0,0;0,0];
```

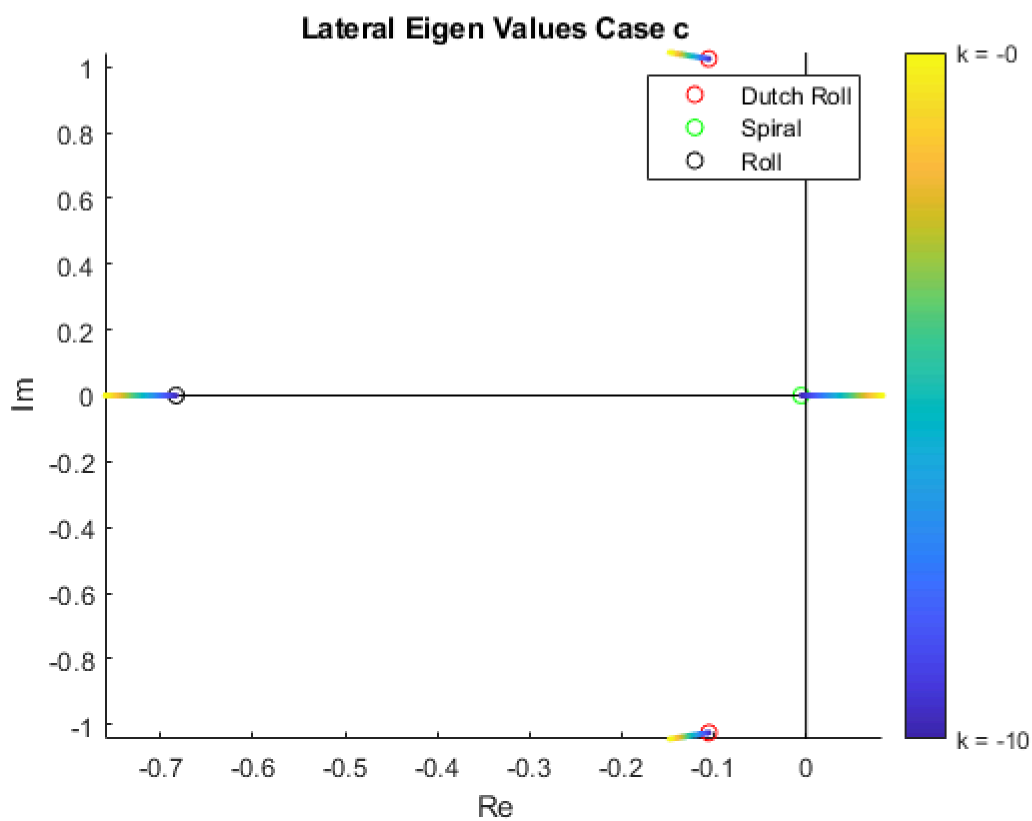
Question 3

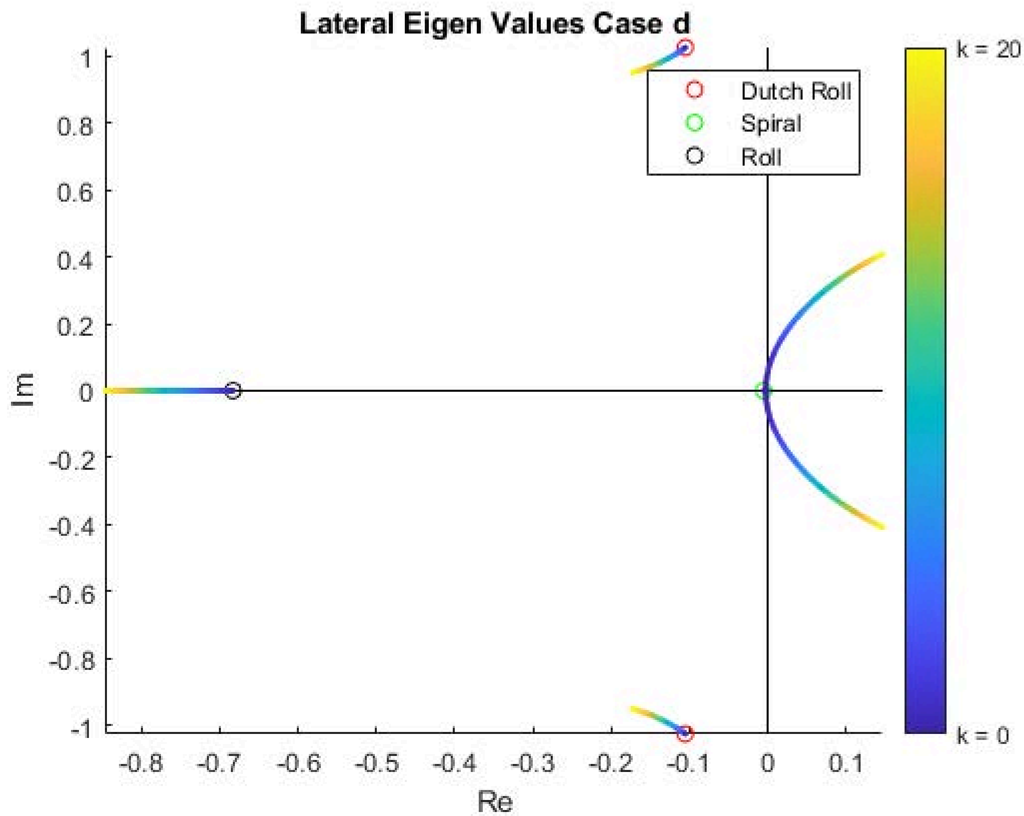
```
% Setting all the variations
Ddaphi = 0:0.01:10;
Ddap = -(0:0.01:10);
Ddar = -(0:0.01:10);
Ddapsi = 0:0.01:20;
Ddrv = -(0:0.0001:0.1);
Ddrp = -(0:0.01:2);
Ddrr = 0:0.01:5;
Ddrphi = -(0:0.01:5);
Ddrpsi = 0:0.01:5;
name = {'Case a';'Case b';'Case c';'Case d';'Case e';'Case f';'Case
 g';...
    'Case h';'Case i'};

% Creating all the K matrices for each case
K = {{   0,     0,     0, Ddaphi,       0,0;...
         0,     0,     0,      0,       0,0},...
     {   0, Ddap,     0,      0,       0,0;...
         0,     0,     0,      0,       0,0},...
     {   0,     0, Ddar,      0,       0,0;...
         0,     0,     0,      0,       0,0},...
     {   0,     0,     0,      0, Ddapsi,0;...
         0,     0,     0,      0,       0,0},...
```

```
        {  0,     0,     0,       0,       0,0;...
        Ddrv,     0,     0,       0,       0,0},...
        {  0,     0,     0,       0,       0,0;...
           0,  Ddrp,     0,       0,       0,0},...
        {  0,     0,     0,       0,       0,0;...
           0,     0,  Ddrr,       0,       0,0},...
        {  0,     0,     0,       0,       0,0;...
           0,     0,     0,  Ddrphi,       0,0},...
        {  0,     0,     0,       0,       0,0;...
           0,     0,     0,       0,  Ddrpsi,0}};

% Loops through all the cases and plots the eigenvalues
for i = 1: numel(K)
    PlotEig(Aaug,Baug,K{i},name{i})
end
```

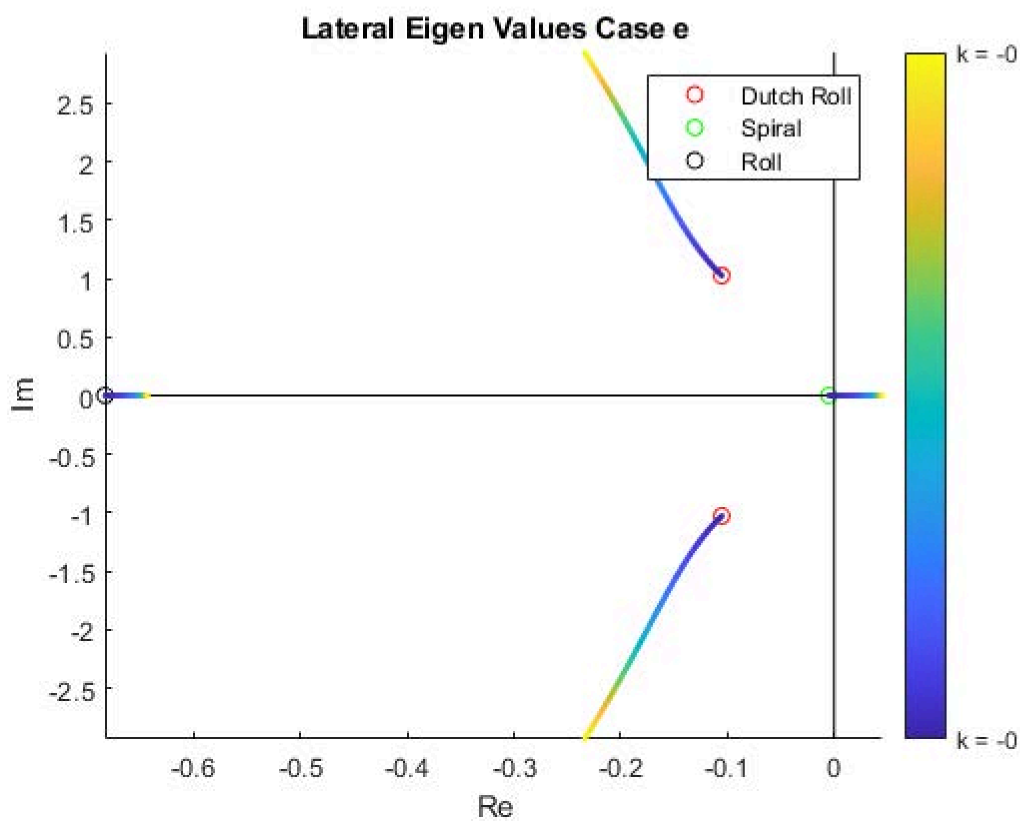



Lateral Eigen Values Case b
Dutch Roll
Spiral
Roll
k = -0
k = -10
Im
Re
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.6
-0.4
-0.2
0
0.2
0.4

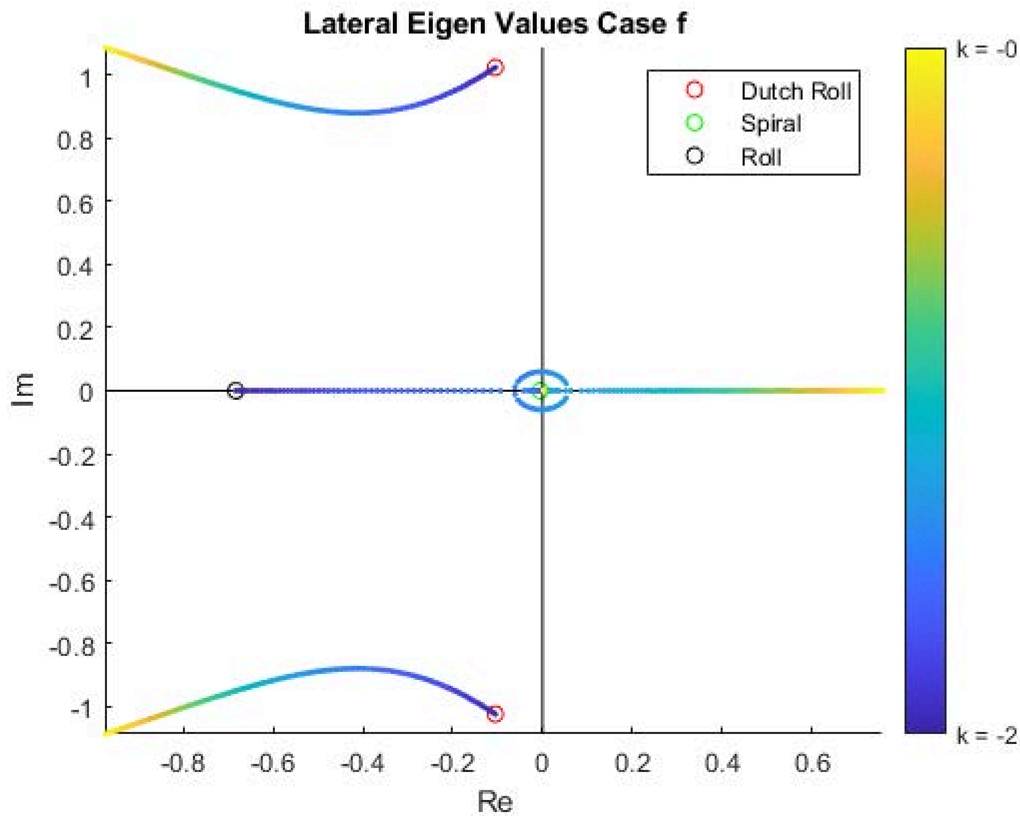

Lateral Eigen Values Case c
Dutch Roll
Spiral
Roll
k = -0
k = -10
Im
Re
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

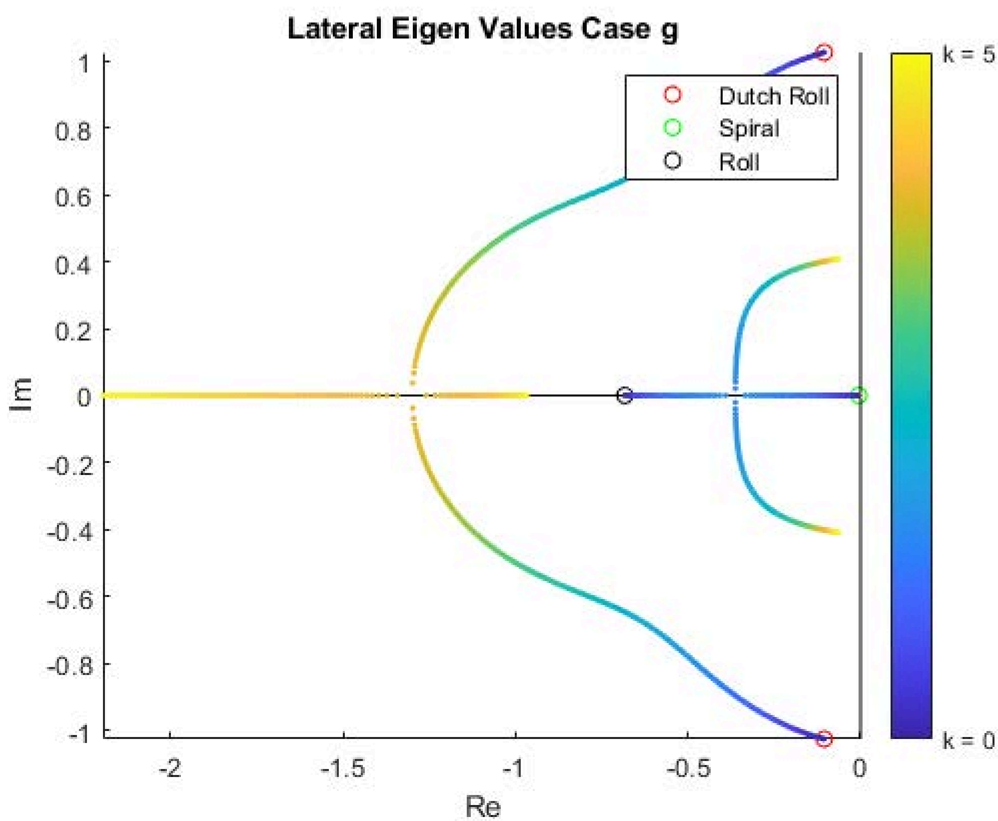


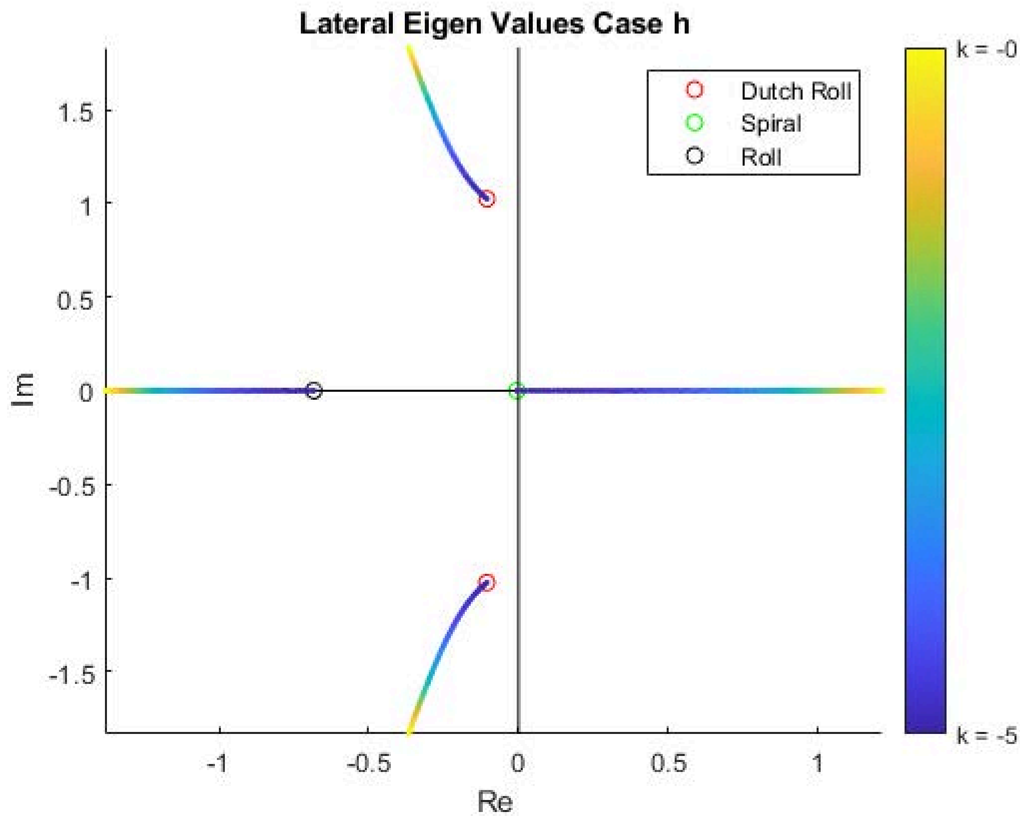
Lateral Eigen Values Case d
Dutch Roll
Spiral
Roll
k = 20
k = 0
Im
Re
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1



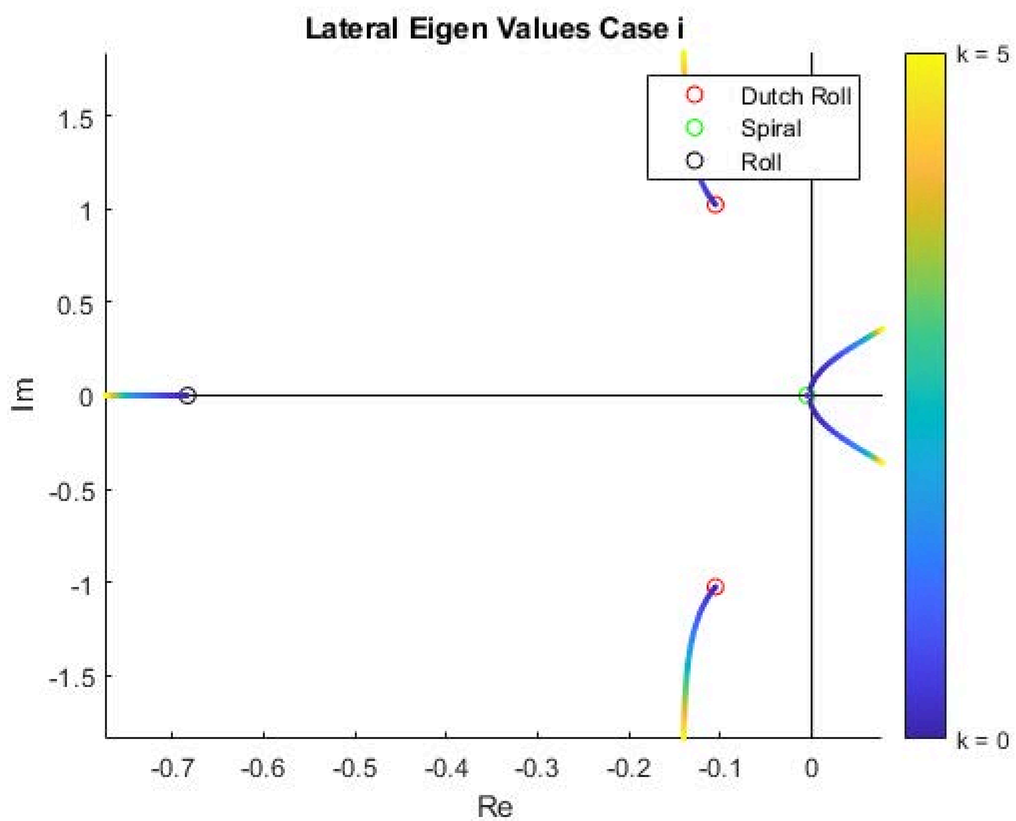
Lateral Eigen Values Case e
Dutch Roll
Spiral
Roll
k = -0
k = -0
Im
Re
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0


Lateral Eigen Values Case f
Dutch Roll
Spiral
Roll
k = -0
k = -2
Im
Re
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


Lateral Eigen Values Case g
Dutch Roll
Spiral
Roll
k = 5
k = 0
Im
Re
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0



Lateral Eigen Values Case h
Dutch Roll
Spiral
Roll
k = -0
k = -5
Im
Re
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1



Lateral Eigen Values Case i
Dutch Roll
Spiral
Roll
k = 5
k = 0
Im
Re
1.5
1
0.5
0
-0.5
-1
-1.5
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

Functions Called

The following functions were built and called as part of this assignment.

```
function PlotEig(A,B,Kin,name)
% PlotEig is used to generate the eigen values of a bunch of K values

% Creates the K matrix that will be used for math
K = zeros(size(Kin));
[r,c] = size(Kin);

% This loop finds the index of the changing K value
for i = 1:r
    for j = 1:c
        if numel(Kin{i,j}) > 1
            k = Kin{i,j};
            x = j;
            y = i;
            break
        end
    end
end
lam1 = zeros(size(k));
lam2 = zeros(size(k));
lam3 = zeros(size(k));
lam4 = zeros(size(k));
lam5 = zeros(size(k));
lam6 = zeros(size(k));

% This loop calculates the eigen values by iterating through each k
 value
% and creates a new K everytime to calculate the closed loop A matrix
for i = 1:numel(k)
    K(y,x) = k(i);
    Acl = A+B*K;
    [~,D]   = eig(Acl);
    lam1(i) = D(1,1);
    lam2(i) = D(2,2);
    lam3(i) = D(3,3);
    lam4(i) = D(4,4);
    lam5(i) = D(5,5);
    lam6(i) = D(6,6);
end

%% Plotting
maxX = max(real([lam1,lam2,lam3,lam4,lam5,lam6]));
minX = min(real([lam1,lam2,lam3,lam4,lam5,lam6]));
maxY = max(imag([lam1,lam2,lam3,lam4,lam5,lam6]));
minY = min(imag([lam1,lam2,lam3,lam4,lam5,lam6]));
figure
hold on
c = linspace(1,10,length(k));
```

```
s(1) = plot(real([lam3(1),lam4(1)]),imag([lam3(1),lam4(1)]),'ro');
s(2) = plot(real(lam5(1)),imag(lam5(1)),'go');
s(3) = plot(real(lam6(1)),imag(lam6(1)),'ko');
s(4) = plot([20,-20],[0,0],'k');
s(4) = plot([0,0],[20,-20],'k');
s(4) = scatter(real(lam1),imag(lam1),5,c,'filled');
s(5) = scatter(real(lam2),imag(lam2),5,c,'filled');
s(6) = scatter(real(lam3),imag(lam3),5,c,'filled');
s(7) = scatter(real(lam4),imag(lam4),5,c,'filled');
s(8) = scatter(real(lam5),imag(lam5),5,c,'filled');
s(9) = scatter(real(lam6),imag(lam6),5,c,'filled');
colorbar('Ticks',[1,10],'TickLabels',{sprintf('k =
 %.0f',min(k)),sprintf('k = %.0f',max(k))});
title(name)
xlabel('Re')
ylabel('Im')
title(['Lateral Eigen Values ',name])
legend(s([1, 2, 3]),'Dutch Roll','Spiral','Roll')
xlim([minX,maxX])
ylim([minY,maxY])
end
```

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