Solution série 6
partie 1

1 Exercise 1

1.1 Question A

Matrice M :

$$\begin{bmatrix} 0 & 50 & 80 & 86 \\ & 0 & 150 & 160 \\ & & 0 & 60 \\ & & & 0 \end{bmatrix}$$

Matrice frontiere :

$$\begin{bmatrix} & 1 & 2 & 3 \\ & & 2 & 1 \\ & & & 3 \\ & & & \end{bmatrix}$$

Calculs :

$$\begin{aligned} M[1,2] &= min\Big\{M[1,1] + M[2,2] + 1 \times 5 \times 10 = 50 \\ &= 50 \end{aligned}$$

$$\begin{aligned} M[2,3] &= min\Big\{M[2,2] + M[3,3] + 5 \times 10 \times 3 = 150 \\ &= 150 \end{aligned}$$

$$\begin{aligned} M[3,4] &= min\Big\{M[3,3] + M[4,4] + 10 \times 3 \times 2 = 60 \\ &= 60 \end{aligned}$$

$$M[1,3] = min \begin{cases} M[1,1] + M[2,3] + 1 \times 5 \times 3 = 165 \\ M[1,2] + M[3,3] + 1 \times 10 \times 3 = 80 \end{cases}$$
$$= 80$$

$$M[2,4] = min \begin{cases} M[2,2] + M[3,4] + 5 \times 10 \times 2 = 160 \\ M[2,3] + M[4,4] + 5 \times 3 \times 2 = 180 \end{cases}$$
$$= 160$$

$$M[1,4] = min \begin{cases} M[1,1] + M[2,4] + 1 \times 5 \times 3 = 170 \\ M[1,2] + M[3,4] + 1 \times 10 \times 3 = 130 \\ M[1,3] + M[4,4] + 1 \times 5 \times 3 = 86 \end{cases}$$
$$= 86$$

Le paranthèsage optimale est le suivant :

$$((A_1 A_2) A_3) A_4$$

1.2 Question B

Matrice M :

$$\begin{bmatrix} 0 & 8 & 12 & 28 & 22 & 28 \\ & 0 & 8 & 18 & 20 & 23 \\ & & 0 & 40 & 18 & 30 \\ & & & 0 & 10 & 16 \\ & & & & 0 & 15 \\ & & & & & 0 \end{bmatrix}$$

Matrice frontière :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 5 \\ & 2 & 3 & 3 & 5 \\ & & 3 & 3 & 5 \\ & & & 4 & 5 \\ & & & & 5 \\ & & & & \end{bmatrix}$$

Calculs :

$$M[1,2] = min \begin{cases} M[1,1] + M[2,2] + 2 \times 1 \times 4 = 8 \end{cases}$$
$$= 8$$

$$M[2,3] = min\Big\{M[2,2] + M[3,3] + 1 \times 4 \times 2 = 8$$
$$= 8$$

$$M[3,4] = min\Big\{M[3,3] + M[4,4] + 4 \times 2 \times 5 = 40$$
$$= 40$$

$$M[4,5] = min\Big\{M[3,3] + M[4,4] + 2 \times 5 \times 1 = 10$$
$$= 10$$

$$M[5,6] = min\Big\{M[3,3] + M[4,4] + 5 \times 1 \times 3 = 15$$
$$= 15$$

$$M[1,3] = min\begin{cases} M[1,1] + M[2,3] + 2 \times 1 \times 2 = 12 \\ M[1,2] + M[3,3] + 2 \times 4 \times 2 = 24 \end{cases}$$
$$= 12$$

$$M[2,4] = min\begin{cases} M[2,2] + M[3,4] + 1 \times 4 \times 5 = 60 \\ M[2,3] + M[4,4] + 1 \times 2 \times 5 = 18 \end{cases}$$
$$= 18$$

$$M[3,5] = min\begin{cases} M[3,3] + M[4,5] + 4 \times 2 \times 1 = 18 \\ M[3,4] + M[5,5] + 4 \times 5 \times 1 = 60 \end{cases}$$
$$= 18$$

$$M[4,6] = min\begin{cases} M[4,4] + M[5,6] + 2 \times 5 \times 3 = 45 \\ M[4,5] + M[6,6] + 2 \times 1 \times 3 = 16 \end{cases}$$
$$= 16$$

$$M[1,4] = min \begin{cases} M[1,1] + M[2,4] + 2 \times 1 \times 5 = 28 \\ M[1,2] + M[3,4] + 2 \times 4 \times 5 = 88 \\ M[1,3] + M[4,4] + 2 \times 2 \times 5 = 32 \end{cases}$$
$$= 28$$

$$M[2,5] = min \begin{cases} M[1,1] + M[2,4] + 1 \times 4 \times 1 = 22 \\ M[1,2] + M[3,4] + 1 \times 2 \times 1 = 20 \\ M[1,3] + M[4,4] + 1 \times 5 \times 1 = 23 \end{cases}$$
$$= 22$$

$$M[3,6] = min \begin{cases} M[1,1] + M[2,4] + 4 \times 2 \times 3 = 18 \\ M[1,2] + M[3,4] + 4 \times 5 \times 3 = 60 \\ M[1,3] + M[4,4] + 4 \times 1 \times 3 = 86 \end{cases}$$
$$= 18$$

$$M[1,5] = min \begin{cases} M[1,1] + M[2,5] + 2 \times 1 \times 1 = 18 \\ M[1,2] + M[3,5] + 2 \times 4 \times 1 = 60 \\ M[1,3] + M[4,5] + 2 \times 2 \times 1 = 86 \\ M[1,4] + M[5,5] + 2 \times 5 \times 1 = 86 \end{cases}$$
$$= 18$$

$$M[2,6] = min \begin{cases} M[2,2] + M[3,6] + 1 \times 4 \times 3 = 18 \\ M[2,3] + M[4,6] + 1 \times 2 \times 3 = 60 \\ M[2,4] + M[5,6] + 1 \times 5 \times 3 = 86 \\ M[2,5] + M[6,6] + 1 \times 1 \times 3 = 86 \end{cases}$$
$$= 18$$

$$M[1,6] = min \begin{cases} M[1,1] + M[2,5] + 2 \times 1 \times 3 = 29 \\ M[1,2] + M[3,5] + 2 \times 4 \times 3 = 62 \\ M[1,3] + M[4,5] + 2 \times 2 \times 3 = 40 \\ M[1,4] + M[5,5] + 2 \times 5 \times 3 = 73 \\ M[1,5] + M[6,6] + 2 \times 1 \times 3 = 28 \end{cases}$$
$$= 28$$

Le paranthèsage optimal est le suivant :

$$(A_1((A_2A_3)(A_4A_5)))A_6$$

Note : les réponses suivantes sont valides, mais préférez la première solution :

1. $(A_1(A_2A_3(A_4A_5)))A_6$
2. $(A_1((A_2)A_3(A_4A_5)))A_6$

2 Question 2

$$C = \begin{bmatrix} 0 & 0,5 & 1,0 & 1,30 & 1,69 & 1,92 \\ & 0 & 0,25 & 0,45 & 0,69 & 0,92 \\ & & 0 & 0,1 & 0,26 & 0,42 \\ & & & 0 & 0,08 & 0,22 \\ & & & & 0 & 0,07 \\ & & & & & 0 \end{bmatrix}$$

$$racine = \begin{bmatrix} 1 & 1 & 2 & 1 & 1,2 \\ & 2 & 2 & 2 & 2 \\ & & 3 & 3 & 4 \\ & & & 4 & 4 \\ & & & & 5 \end{bmatrix}$$

Calculs :

$$C[1,1] = min\Big\{C[1,0] + C[2,1] + \textstyle\sum_{s=i}^{j} P_s = 0 + 0 + 0,5 = 0,5$$
$$= 0,5$$

$$C[2,2] = min\Big\{C[1,1] + C[3,2] + \textstyle\sum_{s=i}^{j} P_s = 0 + 0 + 0,25 = 0,25$$
$$= 0,25$$

$$C[3,3] = min\Big\{C[3,2] + C[4,3] + \textstyle\sum_{s=i}^{j} P_s = 0 + 0 + 0,1 = 0,1$$
$$= 0,1$$

$$C[4,4] = min\Big\{C[1,3] + C[5,2] + \textstyle\sum_{s=i}^{j} P_s = 0 + 0 + 0,08 = 0,08$$
$$= 0,08$$

$$C[5,5] = min\left\{C[5,4] + C[6,5] + \sum_{s=i}^{j} P_s = 0 + 0 + 0,07 = 0,07\right.$$
$$= 0,07$$

$$C[1,2] = min\begin{cases} C[1,0] + C[2,2] = 0 + 0,25 = 0,25 \\ C[1,1] + C[3,2] = 0,5 + 0 = 0,5 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,25 + 0,5 + 0,25$$
$$= 1,0$$

$$C[2,3] = min\begin{cases} C[2,1] + C[3,3] = 0 + 0,1 = 0,1 \\ C[2,2] + C[4,3] = 0,25 + 0 = 0,25 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,1 + 0,25 + 0,1$$
$$= 0,45$$

$$C[3,4] = min\begin{cases} C[3,2] + C[4,4] = 0 + 0,08 = 0,08 \\ C[3,3] + C[5,4] = 0,1 + 0 = 0,1 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,08 + 0,1 + 0,08$$
$$= 0,26$$

$$C[4,5] = min\begin{cases} C[4,3] + C[5,5] = 0 + 0,07 = 0,07 \\ C[4,4] + C[6,5] = 0,08 + 0 = 0,08 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,07 + 0,08 + 0,07$$
$$= 0,22$$

$$C[1,3] = min\begin{cases} C[1,0] + C[2,3] = 0 + 0,45 = 0,45 \\ C[1,1] + C[3,3] = 0,5 + 0,1 = 0,6 \\ C[1,2] + C[4,3] = 1,0 + 0 = 1,0 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,45 + 0,5 + 0,25 + 0,1$$
$$= 1,30$$

$$C[2,4] = min \begin{cases} C[2,1] + C[3,4] = 0 + 0,26 \\ C[2,2] + C[4,4] = 0,1 + 0,07 \\ C[2,3] + C[5,4] = 0,26 + 0 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,17 + 0,25 + 0,1 + 0,08$$
$$= 0,69$$

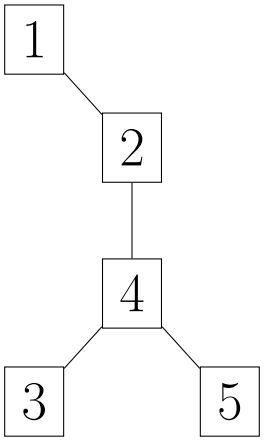
$$C[3,5] = min \begin{cases} C[3,2] + C[4,5] = 0 + 0,22 = 0,22 \\ C[3,3] + C[5,5] = 0,1 + 0,07 = 0,17 \\ C[3,4] + C[6,5] = 0,26 + 0 = 0,26 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,17 + 0,1 + 0,08 + 0,07$$
$$= 0,42$$

$$C[1,4] = min \begin{cases} C[1,0] + C[2,4] = 0 + 0,69 = 0,69 \\ C[1,1] + C[3,4] = 0,5 + 0,26 = 0,76 \\ C[1,2] + C[4,4] = 1,0 + 0,08 = 1,08 \\ C[1,3] + C[5,4] = 1,3 + 0 = 1,3 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,76 + 0,5 + 0,25 + 0,1 + 0,08$$
$$= 1,69$$

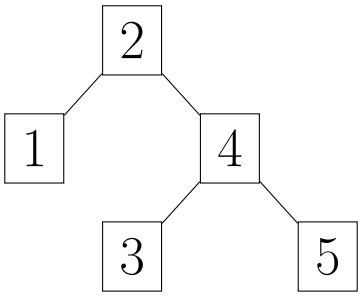
$$C[2,5] = min \begin{cases} C[2,1] + C[3,5] = 0 + 0,42 = 0,42 \\ C[2,2] + C[4,5] = 0,25 + 0,22 = 0,47 \\ C[2,3] + C[5,5] = 0,45 + 0,07 = 0,52 \\ C[2,4] + C[6,5] = 0,69 + 0 = 0,69 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,42 + 0,25 + 0,1 + 0,08 + 0,07$$
$$= 0,92$$

$$C[1,5] = min \begin{cases} C[1,0] + C[2,5] = 0 + 0,92 = 0,92 \\ C[1,1] + C[3,5] = 0,5 + 0,42 = 0,92 \\ C[1,2] + C[4,5] = 1,0 + 0,22 = 1,22 \\ C[1,3] + C[5,5] = 1,3 + 0,07 = 1,37 \\ C[1,3] + C[6,5] = 1,69 + 0 = 1,69 \end{cases} + \sum_{s=i}^{j} P_s$$
$$= 0,92 + 0,5 + 0,25 + 0,1 + 0,08 + 0,07$$
$$= 1,92$$

Arbre optimal #1 :



Arbre optimal #2 :



Espérance du temps de recherche : $1,92$.