

Solution série 6 partie 2

1 Exercice 1

$$C = \begin{bmatrix} 0 & 0,5 & 1,0 & 1,30 & 1,69 & 1,92 \\ & 0 & 0,25 & 0,45 & 0,69 & 0,92 \\ & & 0 & 0,1 & 0,26 & 0,42 \\ & & & 0 & 0,08 & 0,22 \\ & & & & 0 & 0,07 \\ & & & & & 0 \end{bmatrix}$$

$$\text{racine} = \begin{bmatrix} 1 & 1 & 2 & 1 & 1,2 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 4 \\ 4 & 4 \\ 5 \end{bmatrix}$$

Calculs :

$$\begin{aligned} C[1,1] &= \min \left\{ C[1,0] + C[2,1] + \sum_{s=i}^j P_s = 0 + 0 + 0,5 = 0,5 \right. \\ &\quad \left. = 0,5 \right. \end{aligned}$$

$$\begin{aligned} C[2,2] &= \min \left\{ C[1,1] + C[3,2] + \sum_{s=i}^j P_s = 0 + 0 + 0,25 = 0,25 \right. \\ &\quad \left. = 0,25 \right. \end{aligned}$$

$$\begin{aligned} C[3,3] &= \min \left\{ C[3,2] + C[4,3] + \sum_{s=i}^j P_s = 0 + 0 + 0,1 = 0,1 \right. \\ &\quad \left. = 0,1 \right. \end{aligned}$$

$$\begin{aligned} C[4,4] &= \min \left\{ C[1,3] + C[5,2] + \sum_{s=i}^j P_s = 0 + 0 + 0,08 = 0,08 \right. \\ &\quad \left. = 0,08 \right. \end{aligned}$$

$$C[5,5] = \min \left\{ C[5,4] + C[6,5] + \sum_{s=i}^j P_s = 0 + 0 + 0,07 = 0,07 \right. \\ \left. = 0,07 \right.$$

$$C[1,2] = \min \left\{ C[1,0] + C[2,2] = 0 + 0,25 = 0,25 \right. \\ \left. + \sum_{s=i}^j P_s \right. \\ \left. = 0,25 + 0,5 + 0,25 \right. \\ \left. = 1,0 \right.$$

$$C[2,3] = \min \left\{ C[2,1] + C[3,3] = 0 + 0,1 = 0,1 \right. \\ \left. + \sum_{s=i}^j P_s \right. \\ \left. = 0,1 + 0,25 + 0,1 \right. \\ \left. = 0,45 \right.$$

$$C[3,4] = \min \left\{ C[3,2] + C[4,4] = 0 + 0,08 = 0,08 \right. \\ \left. + \sum_{s=i}^j P_s \right. \\ \left. = 0,08 + 0,1 + 0,08 \right. \\ \left. = 0,26 \right.$$

$$C[4,5] = \min \left\{ C[4,3] + C[5,5] = 0 + 0,07 = 0,07 \right. \\ \left. + \sum_{s=i}^j P_s \right. \\ \left. = 0,07 + 0,08 + 0,07 \right. \\ \left. = 0,22 \right.$$

$$C[1,3] = \min \left\{ C[1,0] + C[2,3] = 0 + 0,45 = 0,45 \right. \\ \left. + \sum_{s=i}^j P_s \right. \\ \left. = 0,45 + 0,5 + 0,25 + 0,1 \right. \\ \left. = 1,30 \right.$$

$$\begin{aligned}
C[2, 4] &= \min \left\{ \begin{array}{l} C[2, 1] + C[3, 4] = 0 + 0, 26 \\ C[2, 2] + C[4, 4] = 0, 1 + 0, 07 \\ C[2, 3] + C[5, 4] = 0, 26 + 0 \end{array} \right. + \sum_{s=i}^j P_s \\
&= 0, 17 + 0, 25 + 0, 1 + 0, 08 \\
&= 0, 69
\end{aligned}$$

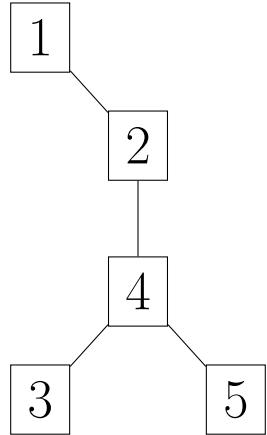
$$\begin{aligned}
C[3, 5] &= \min \left\{ \begin{array}{l} C[3, 2] + C[4, 5] = 0 + 0, 22 = 0, 22 \\ C[3, 3] + C[5, 5] = 0, 1 + 0, 07 = 0, 17 \\ C[3, 4] + C[6, 5] = 0, 26 + 0 = 0, 26 \end{array} \right. + \sum_{s=i}^j P_s \\
&= 0, 17 + 0, 1 + 0, 08 + 0, 07 \\
&= 0, 42
\end{aligned}$$

$$\begin{aligned}
C[1, 4] &= \min \left\{ \begin{array}{l} C[1, 0] + C[2, 4] = 0 + 0, 69 = 0, 69 \\ C[1, 1] + C[3, 4] = 0, 5 + 0, 26 = 0, 76 \\ C[1, 2] + C[4, 4] = 1, 0 + 0, 08 = 1, 08 \\ C[1, 3] + C[5, 4] = 1, 3 + 0 = 1, 3 \end{array} \right. + \sum_{s=i}^j P_s \\
&= 0, 76 + 0, 5 + 0, 25 + 0, 1 + 0, 08 \\
&= 1, 69
\end{aligned}$$

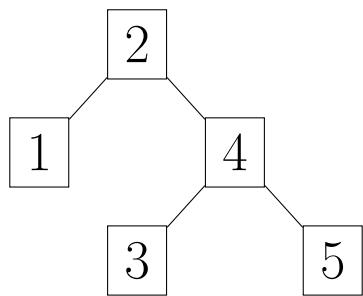
$$\begin{aligned}
C[2, 5] &= \min \left\{ \begin{array}{l} C[2, 1] + C[3, 5] = 0 + 0, 42 = 0, 42 \\ C[2, 2] + C[4, 5] = 0, 25 + 0, 22 = 0, 47 \\ C[2, 3] + C[5, 5] = 0, 45 + 0, 07 = 0, 52 \\ C[2, 4] + C[6, 5] = 0, 69 + 0 = 0, 69 \end{array} \right. + \sum_{s=i}^j P_s \\
&= 0, 42 + 0, 25 + 0, 1 + 0, 08 + 0, 07 \\
&= 0, 92
\end{aligned}$$

$$\begin{aligned}
C[1, 5] &= \min \left\{ \begin{array}{l} C[1, 0] + C[2, 5] = 0 + 0, 92 = 0, 92 \\ C[1, 1] + C[3, 5] = 0, 5 + 0, 42 = 0, 92 \\ C[1, 2] + C[4, 5] = 1, 0 + 0, 22 = 1, 22 \\ C[1, 3] + C[5, 5] = 1, 3 + 0, 07 = 1, 37 \\ C[1, 3] + C[6, 5] = 1, 69 + 0 = 1, 69 \end{array} \right. + \sum_{s=i}^j P_s \\
&= 0, 92 + 0, 5 + 0, 25 + 0, 1 + 0, 08 + 0, 07 \\
&= 1, 92
\end{aligned}$$

Arbre optimal #1 :



Arbre optimal #2 :



Espérance du temps de recherche : 1,92.

2 Exercice 2

Légende :

Abrévation	opération
I	Insertion
D	Suppression
S	Substitution

2.1 Question A

$\frac{Y}{X}$		G	A	C	T	C	A	G	T
	0	1	2	3	4	5	6	7	8
A	1	1	1	2	3	4	5	6	7
C	2	2	2	1	2	3	4	5	6
G	3	2	3	2	2	3	4	4	5
T	4	3	3	3	2	3	4	5	4
C	5	4	4	3	3	2	3	4	5

Tableau des distances :

↖	↖	↖	↖, ←	↖	↖	↖
↑, ←	↖, ↑	↖	↖, ←	↖	↖	↖
↖	↖, ↑, ←	↖	↖	↖, ←	↖, ←	↖
↑	↖	↑	↖	↖, ←	↖, ←	↖, ↑, ←
↑	↖, ↑	↖	↑	↖	↖	↑, ←

Un alignement optimal possible :

```
X      -ACGTC---
| | | | | | |
ISSDSSIII
| | | | | | |
Y      GAC-TCAGT
```

Coût : 5

2.2 Question B

$\frac{Y}{X}$		G	A	C	T	C	A	G	T
	0	4	8	12	16	20	24	28	32
A	3	2	4	8	12	16	20	24	28
C	6	5	4	4	8	12	16	20	24
G	9	6	7	7	7	11	14	16	20
T	12	9	7	9	7	9	12	16	16
C	15	12	10	7	10	7	11	15	18

Tableau des distances :

↖	↖	↖	↖	↖	↖	↖	↖
↑	↖	↖	↖	↖, ←	↖	↖	↖
↖	↖, ↑	↖, ↑	↖	↖, ←	↖	↖	↖
↑	↖	↑	↖	↖	↖	↖	↖
↑	↑	↖	↑	↖	↖, ←	↖, ←	↖

Un alignement optimal possible :

X -ACG-T-C
 |||||||||
 ISSSISIS
 |||||||||
 y GACTCAGT

Coût : 18

3 Exercise 3

3.1 Question A

$$C[1, j] = \begin{cases} \infty & \text{si } P_1 \mod i \neq 0 \\ \frac{j}{P_1} & \text{sinon} \end{cases}$$

3.2 Question B

0

3.3 Question C

$$P[1, 1] = \begin{cases} 1 & \text{si } P_1 = 1 \\ \infty & \text{sinon} \end{cases}$$

3.4 Question D

j_0	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8	j_9	j_{10}	j_{11}	j_{12}	j_{13}	j_{14}	j_{15}	j_{16}	j_{17}	j_{18}	j_{19}	j_{20}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	2	3	4	1	2	3	4	5	2	3	4	5	6	3	4	5	6	7	8
0	1	2	3	4	1	2	3	4	5	2	1	2	3	4	5	2	3	4	5	6
0	1	2	3	4	1	2	3	4	5	2	1	2	3	4	5	2	3	4	5	6

3.5 Question E

Une formulation possible :

$$C[i, j] = \begin{cases} 0 & \text{si } j = 0 \\ \frac{j}{P_i} & \text{si } i = 1 \text{ et } j \mod P_i = 0 \\ \infty & \text{si } i = 1 \text{ et } j \mod P_i \neq 0 \\ C[i - 1, j] & \text{si } j < P_i \\ \min(C[i - 1, j], C[i, j - P_i] + 1) & \text{sinon} \end{cases}$$

3.6 Question F

1. Fonction MONNAIE(P , montant) :
2. $C \leftarrow \text{initialiserTab(montant} + 1)$
3. Pour $i \leftarrow 1$ haut montant + 1 faire
4. Si $(i - 1) \bmod P[1] = 0$ alors
5. $C[i] \leftarrow \frac{i-1}{P[1]}$
6. Sinon
7. $C[i] \leftarrow \infty$
8. Fin Si
9. Fin pour
10. $\text{nbPieces} \leftarrow \text{Taille}(P)$
11. Pour $i \leftarrow 2$ haut nbPieces faire
12. Pour $j \leftarrow 3$ haut montant + 1 faire
13. Si $j \geq P[i]$ alors
14. $C[j] = \min(C[j - 1], C[j - P[i]] + 1)$
15. Fin Si
16. Fin pour
17. Fin Pour
18. Renvoyer C
19. Fin Fonction

3.7 Question G

Complexité de l'algorithme : $\Theta(Ln)$

4 Exercice 4

Regarder le solutionnaire de l'exercice 4 du devoir d'automne 2022.