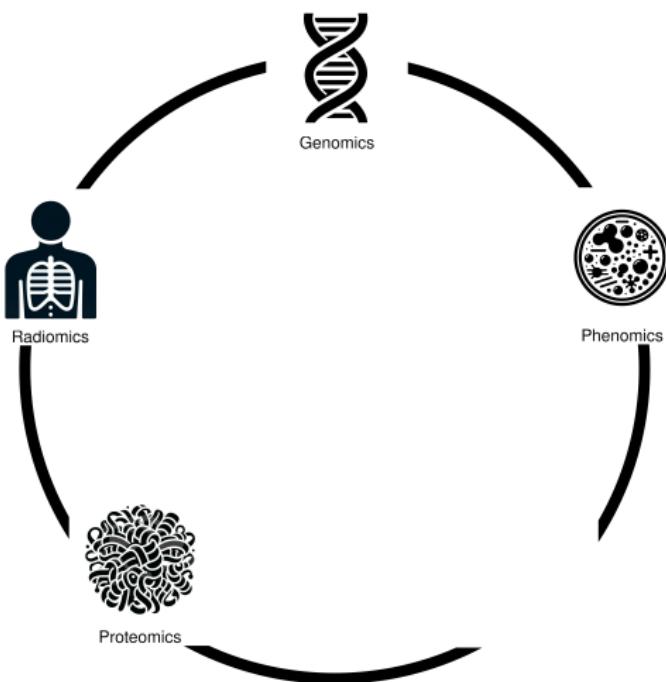


# jaxFlowSim

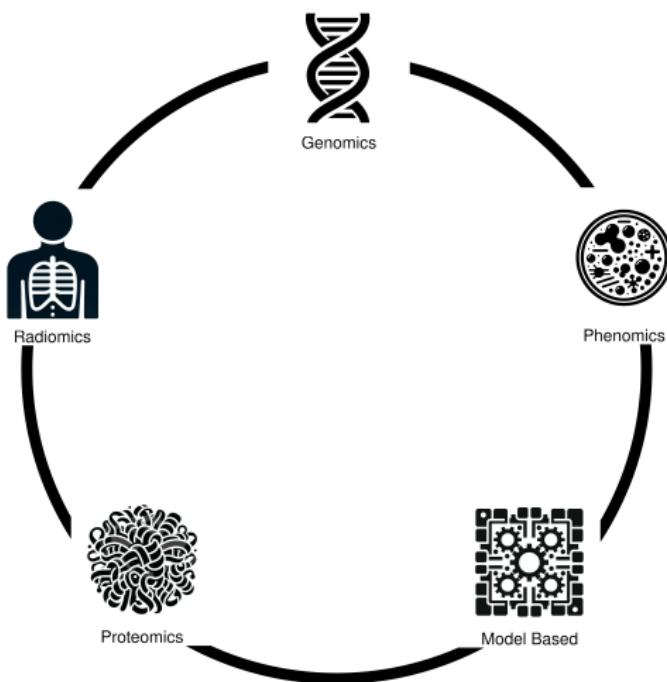
Diego Renner

January 21, 2024

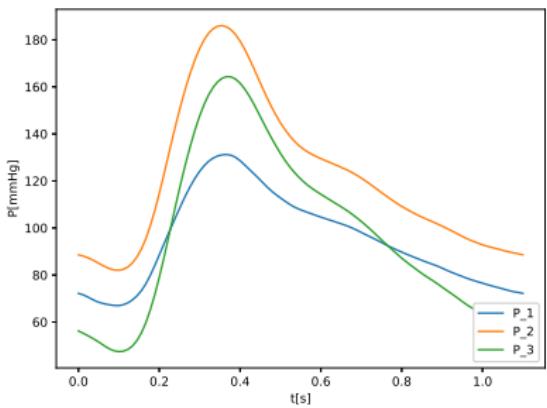
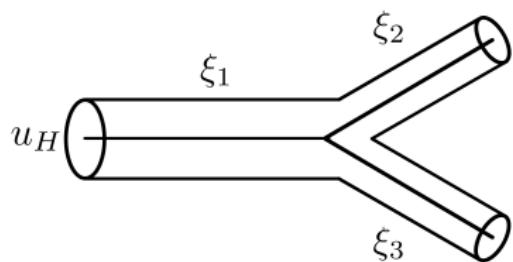
# Towards Personalised Medicine: Current Approaches



# Towards Personalised Medicine: Future Approaches



# Example: Bifurcation



# Motivation

## Use and Novelty

- towards personalised medicine
- parameter inference
- sensitivity analysis

# 1D-Navier Stokes Equations

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial z} = \mathbf{S}(\mathbf{U}), \quad t > 0, \quad z \in [0, l], \\ \mathbf{U}(z; 0) = \mathbf{U}_0(z), \quad z \in [0, l], \quad (1)$$

$$\mathbf{U}(0; t) = \mathbf{U}_L(t), \quad t > 0,$$

$$\mathbf{U}(l; t) = \mathbf{U}_R(t), \quad t > 0,$$

$$\mathbf{U} := \begin{bmatrix} A \\ Q \end{bmatrix}, \quad \mathbf{F}(\mathbf{U}) := \begin{bmatrix} Q \\ \frac{Q^2}{A} + \frac{\beta A^{\frac{3}{2}}}{3\rho\sqrt{A_0}} \end{bmatrix}, \quad \mathbf{S}(\mathbf{U}) := \begin{bmatrix} 0 \\ -22\frac{\mu}{\rho} \frac{Q}{A} \end{bmatrix}. \quad (2)$$

$\mathbf{U}_0 \hat{=} \text{initial condition}$ ,  $\mathbf{U}_L \hat{=} \text{left boundary values}$ ,  $\mathbf{U}_R \hat{=} \text{right boundary values}$ ,  
 $A \hat{=} \text{cross-section}$ ,  $A_0 \hat{=} \text{reference cross-section}$ ,  $Q \hat{=} \text{volumetric flow-rate}$ ,  
 $\beta \hat{=} \text{elasticity coefficient}$ ,  $\rho \hat{=} \text{blood density}$ ,  $\mu \hat{=} \text{blood dynamic viscosity}$ .

# Tube Law

$$P(z; t) := P_{\text{ext}}(z; t) + \beta \left( \sqrt{\frac{A(z; t)}{A_0(z)}} - 1 \right), \quad (3)$$

$$\beta(z) := \frac{\sqrt{\pi} E h_0(z)}{(1 - \nu^2) \sqrt{A_0(z)}}. \quad (4)$$

$P \hat{=} \text{pressure}$ ,  $P_{\text{ext}} \hat{=} \text{external pressure}$ ,  
 $E \hat{=} \text{Young's modulus}$ ,  $h_0 \hat{=} \text{reference vessel wall thickness}$ ,  $\nu \hat{=} \text{Poisson's ratio (elasticity parameter)}$ .

# Initial Conditions

$$u(z; 0) \equiv 0, \quad z \in [0, l], \quad (5)$$

$$A(z; 0) = A_0(z), \quad z \in [0, l], \quad (6)$$

$$Q(z; 0) = u(z; 0)A(z; 0) \equiv 0, \quad z \in [0, l]. \quad (7)$$

# Inlets, Junctions, Outlets

## Inlets

set  $P$  from data → set  $u, Q, A, c$  through linear extrapolation of characteristics

set  $Q$  from data → set  $u, A, c, P$  through linear extrapolation of characteristics

## Junctions

solve linear system of equations consisting of:

- conservation of mass,
- conservation of pressure,
- extrapolation of characteristics

## Outlets

0D-/ lumped parameter model: three element Windkessel (RCR) model

# Numerical Methods

## 1D-Model

FV method: MUSCL scheme with Lax-Friedrichs Flux

## Junctions & Outlets

Newton method

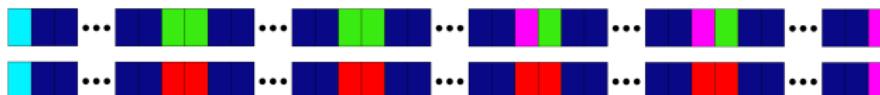
# Code Structure

```
1 def runSimulation(config_filename, J)
2     config = loadConfig(config_filename)
3     simulation_data = buildArterialNetwork(config)
4
5     P_t = [0]
6
7     converged = False
8     while not converged:
9         t = 0
10        i = 0
11        P_t_temp = P_t
12        while t < T:
13            dt = computeDt(simulation_data)
14            simulation_data = setBoundaryValues(simulation_data, dt)
15            simulation_data = muscl(simulation_data, dt)
16            P_t[i,:] = savePressure(simulation_data)
17            t = t + dt
18            i = i + 1
19            if i >= J:
20                break
21        converged = checkConv(P_t, P_t_temp)
```

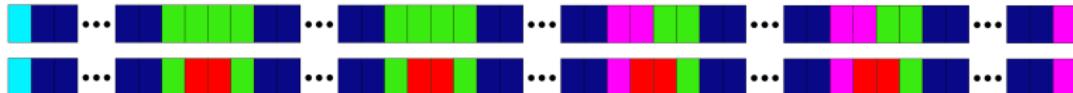
**Listing:**  $dt \hat{=} \text{timestep(CFL)}$ ,  $\text{setBoundaryValues} \hat{=} \text{inlet(from data), outlet(Windkessel)}$ ,  $\text{junctions(conservation laws)}$ ,  $\text{muscl} \hat{=} \text{Monotonic Upstream-centered Scheme for Conservation Laws(Finite Volume)}$

# Padding

without padding

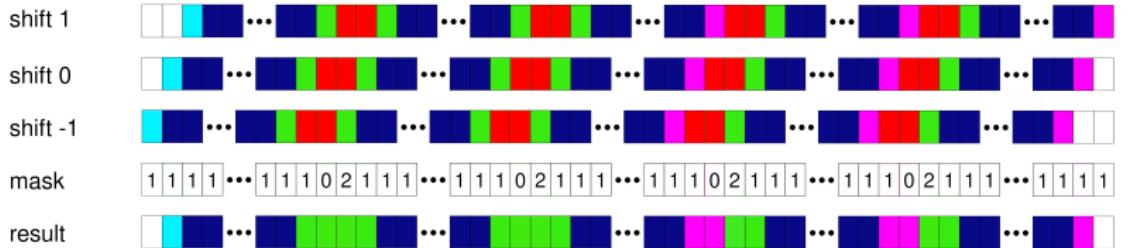


with padding



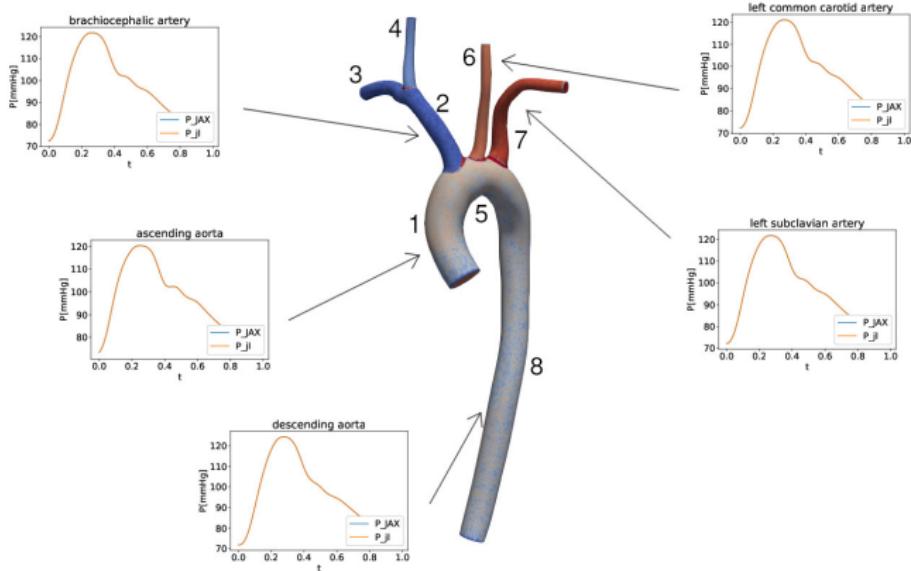
cyan = inlet, blue = vessel interior, green = junction, magenta = outlet, red = false value, white = zero, n = mask value

## Masking

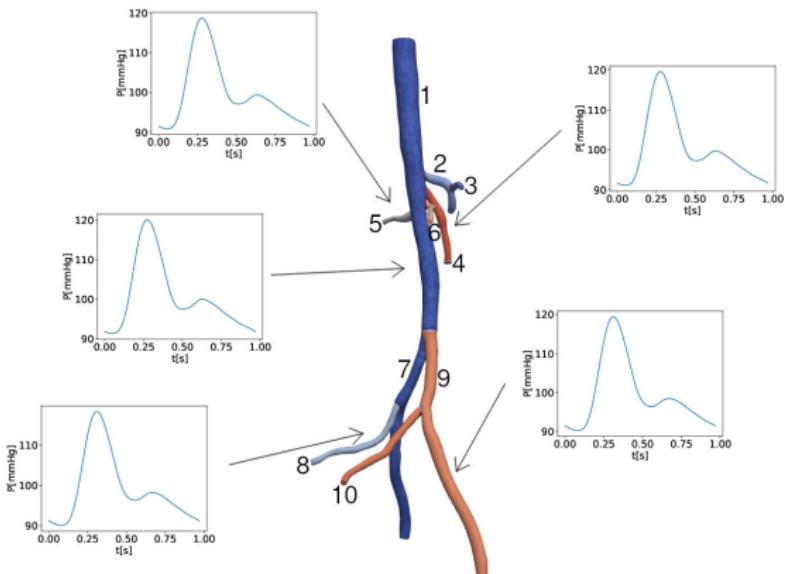


= inlet, = vessel interior, = junction, = outlet, = false value, = zero, = mask value

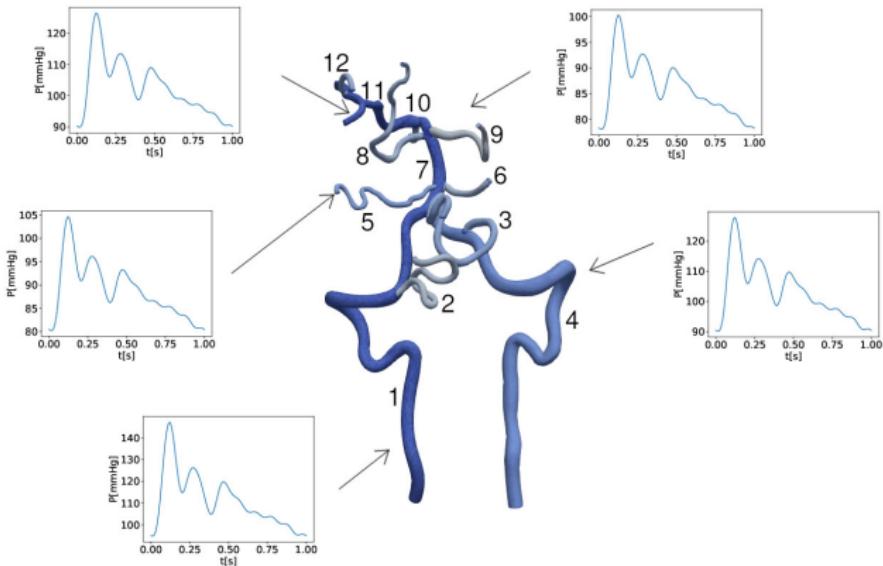
# Model: Aorta (0007\_H\_AO\_H)



# Model: Abdominal Arteries (0029\_H\_ABAO\_H)

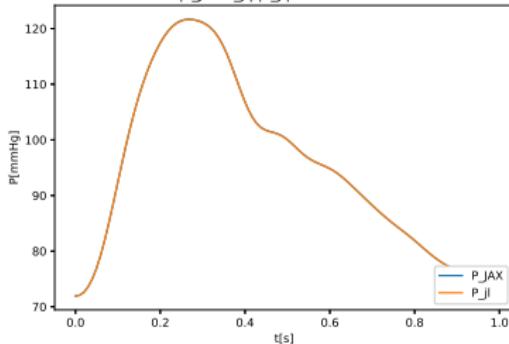


# Model: Cerebellar Arteries (0053\_H\_CERE\_H)



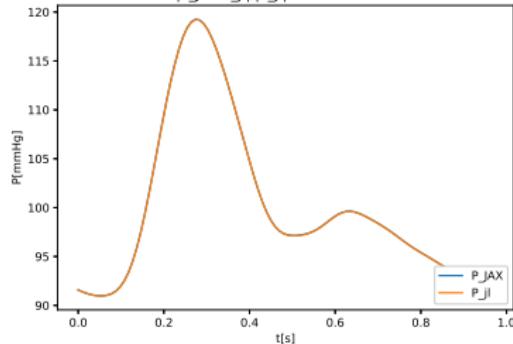
# Validation

network: 0007\_H\_AO\_H, # vessels: 9, vessel name: right subclavian arte  
relative error =  $|P_{JAX}-P_{jl}|/|P_{jl}| = 0.0003299442602849549\%$

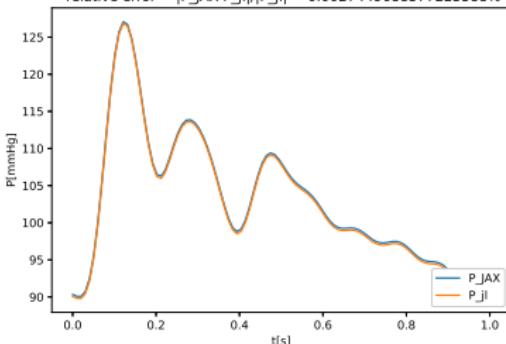


network: 0029\_H\_ABAO\_H, # vessels: 17, vessel name: celiac branch,  
relative error =  $|P_{JAX}-P_{jl}|/|P_{jl}| = 0.00038984287832215047\%$

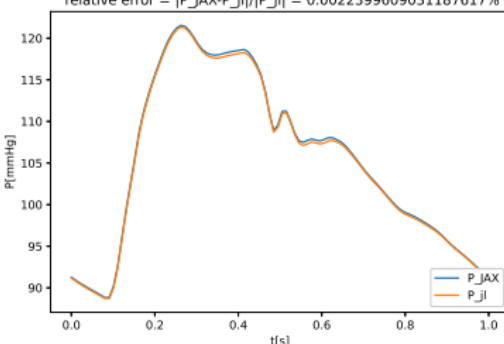
network: 0029\_H\_ABAO\_H, # vessels: 17, vessel name: celiac branch,  
relative error =  $|P_{JAX}-P_{jl}|/|P_{jl}| = 0.00038984287832215047\%$



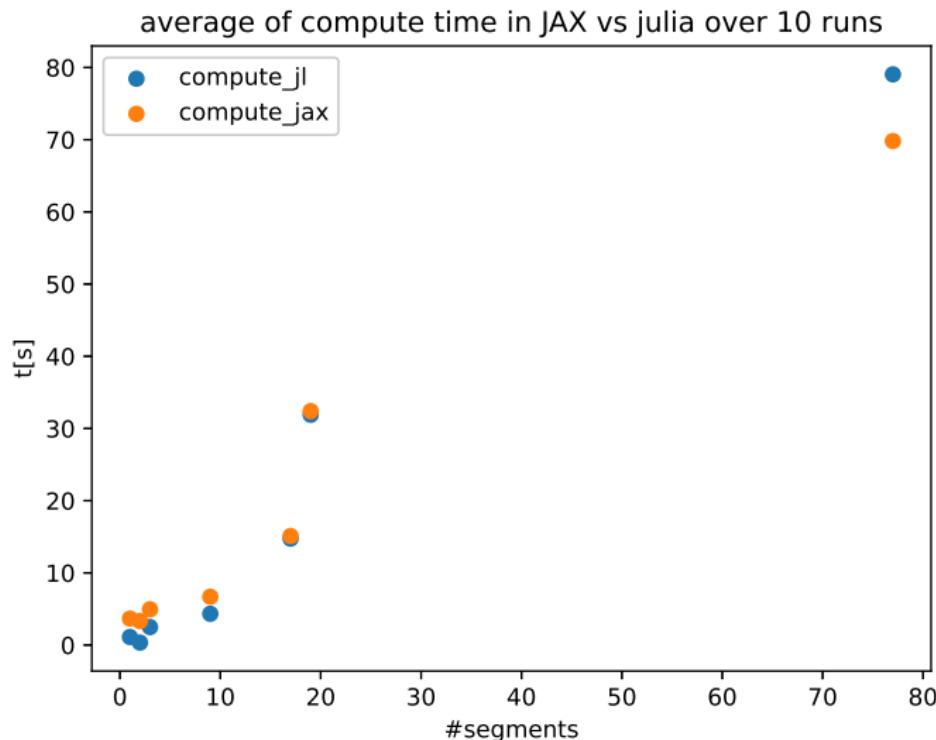
network: 0053\_H\_CERE\_H, # vessels: 19, vessel name: basilar artery IV  
relative error =  $|P_{JAX}-P_{jl}|/|P_{jl}| = 0.0027449688577225588\%$



network: adan56, # vessels: 77, vessel name: common hepatic,  
relative error =  $|P_{JAX}-P_{jl}|/|P_{jl}| = 0.0022399609031187617\%$



# Comparison



# Inference

## Demo

inferring an outlet resistance parameter from precomputed data

# Future Work

## Main Points of Interest

- improving performance (GPU optimization)
- fine tuning parameter inference
- sensitivity analysis

Introduction  
ooooo

Model  
ooooo

Implementation  
ooo

Results  
oooooo

Future Work  
oo•

# Questions?

Thank you for your attention!