Semiconductors: Band Gap of Germanium and the Hall Effect

Cameron Laedtke

April 24th 2019

PHYS 3605W

Overview

Determine the band gap of pure germanium (Ge) and to measure the carrier concentration and carrier type in doped Ge using the Hall Effect

General Notes

Germanium, a semiconductor, has unique current carrying properties that come from the way in which the electronic states of the material are filled. In particular, there is a gap E_g that separates the filled states in the valence band from the empty states of lowest energy in the conduction band. Current can be allowed to flow in the semiconductor either by adding electrons to the conduction band or removing electrons from the valence band.

Procedure

Bandgap of Germanium

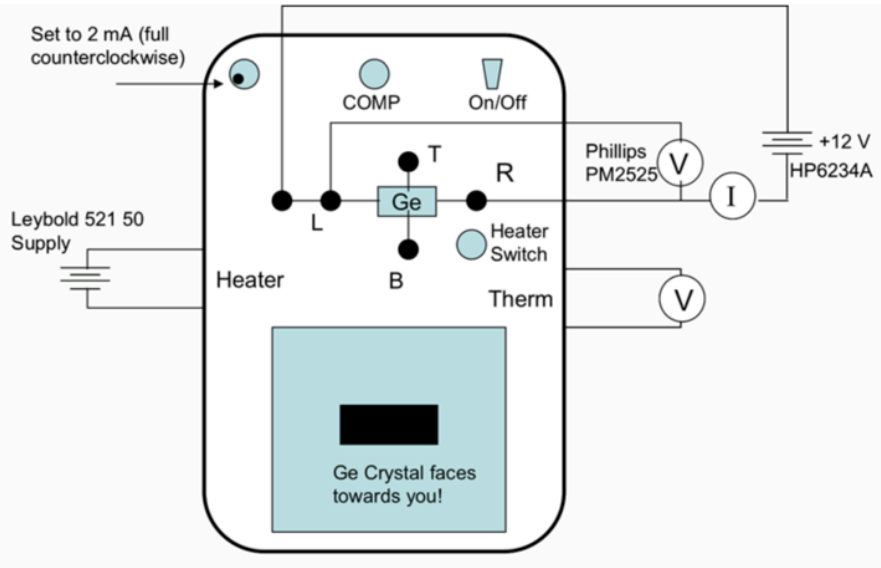
Our goal for the first part of the experiment was to obtain a value of the band gap E_g for Germanium by analyzing the relationship between temperature and conductivity of the crystal. The number of current carriers is proportional to temperature by the following relationship (derived from statistical mechanics).

$$n = f(T)e^{-\frac{E_g}{2k_BT}}$$

The conductivity of the crystal is proportional to the number of current cariers by the equation below.

$$\sigma = ne\mu \approx \sigma_0 e^{-\frac{E_g}{2k_BT}}$$

The lab setup, shown below in a diagram from the lab manual, included various instruments for measuring the current and voltage across the Germanium crystal as well as the thermistor.



After connecting the leads and turning on the equipment, we set a voltage of 4.77V flowing through the 6234A power supply. The heater power supply was set to 4.2V, with an initial current of 0.1 A. To measure temperature, we used a thermistor that was read out with a voltmeter using the calibration:

$$T(K) = 273 + 100 * V$$

The thermistor voltage had a tendancy to increase even when the current was held constant. To account for this, we waited between two and three minutes after increasing the current before taking measurements of the thermistor voltage as well as current and voltage across the Ge crystal. Additionally, the thermistor voltage was never completely stable. We took these temperature fluctuations into account by adding an uncertainty of $\pm 2K$ to our temperature measurements.

Increasing the heater current by small increments, we recorded the temperature, current and voltage across the Germanium crystal. Errors for current and voltage across the crystal are based on the minumum resoution of our recording instruments.

```
data=dlmread('Bandgap.csv',',',1,0);

temp = data(:,2)*100+273; % Kelvin
temp_err = 2; % Kelvin
I = data(:,4); % Amps
I_err = 0.00001;
V = data(:,1); % Volts
V_err = 0.001;
```

To determine the conductivity of the crystal for a given current and voltage, we derived the following equation from the equation for resistance $R = \frac{l}{\sigma A}$

$$\sigma = \frac{L}{\mathrm{RA}} = \frac{\mathrm{IL}}{\mathrm{Vwt}}$$

where w, L, and t are the width, length, and thickness of the crystal respectively, with errors based off the number of significant figures provided in the lab manual.

```
% Dimensions of the Ge crystal
l = 0.020; % meters
l_err = 0.001;
w = 0.010; % meters
w_err = 0.001;
t = 0.001; % meters
t_err = 0.0001;
```

Taking the natural log of the first equation for conductivity, we can experimentally obtain a band gap value by plotting $f(T) = \frac{1}{2k_B T}$ versus the log of the conductivity:

$$\ln(\sigma) = -E_g f(T) + \ln(\sigma)_0 \ ,$$

where error in $f(T)$ is obtained through standard error propagation procedure.

$$\Delta f(T) = \frac{\Delta T}{2k_B T^2}$$

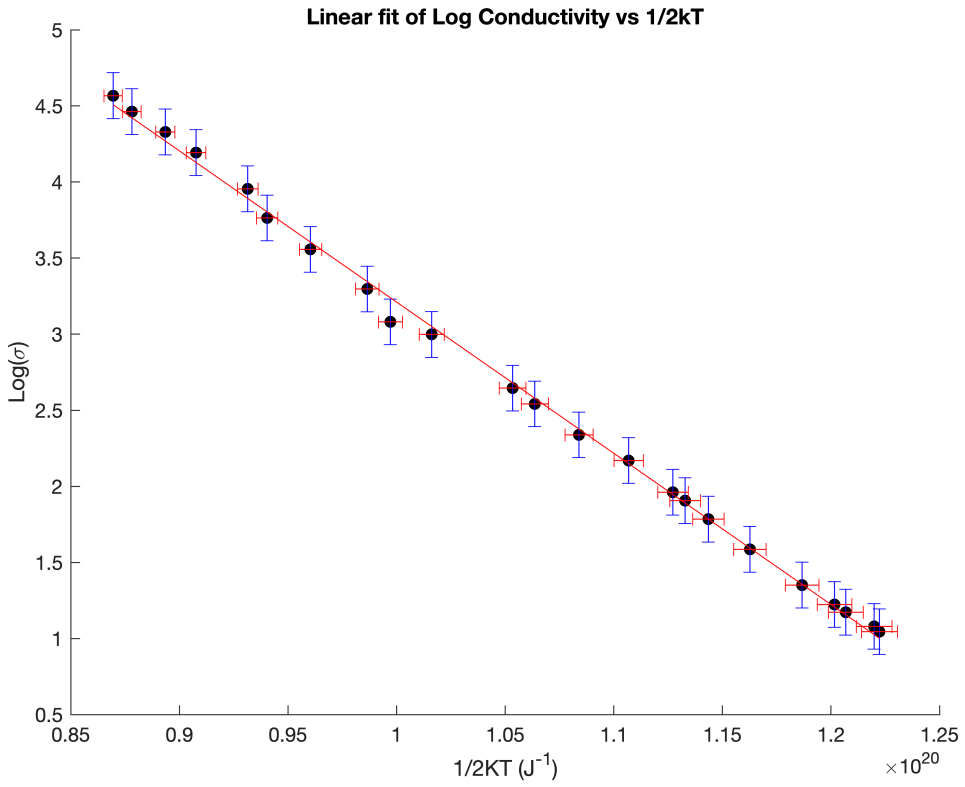
```
k = 1.381*10^(-23); % Joules/Kelvin
invKT = (2*k.*temp).^-1; % Joules^-1
invKT_err = temp_err./(2*k.*temp.^2); % Joules^-1
conductivity = (l*I)./(V*w*t); % Ohms^-1 * meters ^-1
log_conductivity = log(conductivity);
```

Error in the log of the conductivity is also obtained through standard error propagation procedure.

$$\Delta \ln(\sigma) = \frac{1}{\sigma}\sqrt{\Delta_L^2\left(\frac{I}{\mathrm{wtV}}\right)^2 + \Delta_I^2\left(\frac{L}{\mathrm{wtV}}\right)^2 + \Delta_w^2\left(\frac{\mathrm{IL}}{w^2\mathrm{tV}}\right)^2 + \Delta_t^2\left(\frac{\mathrm{IL}}{\mathrm{wt}^2 V}\right)^2 + \Delta_V^2\left(\frac{\mathrm{IL}}{\mathrm{wtV}^2}\right)^2}$$

```
log_conductivity_err = (1./conductivity).*((l_err.*I./(V*w*t)).^2 ...
    +(l*I_err./(V*w*t)).^2 ...
    +(l.*I*V_err./(w*t.*V.^2)).^2 ...
    +(l.*I*w_err./(w^2*t.*V)).^2 ...
    +(l.*I*t_err./(w.*V*t.^2)).^2).^.5;

scatter(invKT, log_conductivity, [], [0,0,0], 'filled')
xlabel('1/2KT (J^{-1})');
ylabel('Log(\sigma)');
title("Linear fit of Log Conductivity vs 1/2kT");

hold on;
vertErr = errorbar(invKT, log_conductivity, log_conductivity_err);
vertErr.LineStyle = 'none';
vertErr.Color = [0,0,1];
horizErr = errorbar(invKT,log_conductivity, invKT_err,'horizontal');
horizErr.LineStyle = 'none';
horizErr.Color = [1,0,0];
```

```
fit = myfit(invKT,log(conductivity),log_conductivity_err);
y_fit = invKT*fit(1,2) + fit(1,1);
plot(invKT, y_fit, 'r')

hold off;
```



The linear fit accomodates for the error bars well, with only a couple points falling outside the horizontal error bars on 1/2KT, and no points falling outside the error in the log of conductivity. Taking the (negative) slope of the linear fit, we obtain a final value for the band gap:

```
% converting it to eV
E_gap = -fit(1,2)./(1.6*10^-19)
```

```
E_gap = 0.6211
```

```
E_gap_err = fit(2,2)/(1.6*10^-19)
```

```
E_gap_err = 0.0168
```

```
room_temp_conductivity = exp(invKT(1:1)*fit(1,2) + fit(1,1))
```

```
room_temp_conductivity = 2.7222
```

$$E_g = 0.62 \pm 0.02\,\text{eV}$$

This value falls within 0.05 eV (3 sigma) from the correct value for Germanium (0.67 eV) [cited below]. Since the error bars accomodated for the linear fit quite well, this suggests some kind of systematic error

in our experiment rather than random error. It would be helpful to repeat such an experiment without the continual temperature drift that occured during our data taking. Ultimately, our experiment provided proof of the temperature dependance of conductivity in the case of Germanium.

At room temperature (300K), we obtained a value of conductivity of 2.72 $\Omega^{-1}m^{-1}$, which is eight orders of magnitude smaller than copper at room temperature.

Citation: Streetman, Ben G.; Sanjay Banerjee (2000). *Solid State electronic Devices* (5th ed.). New Jersey: Prentice Hall.

Doped Semi-Conductors and the Hall Effect

The next portion of our lab entailed measuring the current carrier density and sign of a Germanium crystal doped with impurities. An impurity that adds electrons to the conduction band (donor impurity), results in an n-type semiconfuctor. Adding an impurity that removes electrons from the valence band (acceptor impurity), results in a p-type semiconductor.

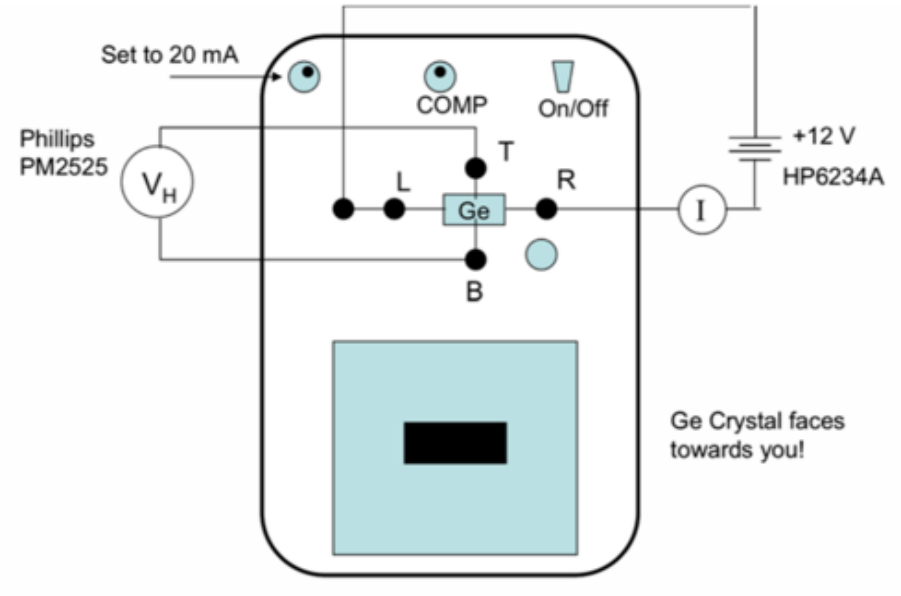
Donors and acceptors have different effects on charge transport across the crystal. In the case of donor impurities, current carrying electrons move across the crystal, whereas acceptor impurities cause positive charge carriers to move across the crystal in the opposite direction. When a magnetic field is applied around the crystal, the counteracting electrostatic and Lorentz forces cause a buildup of charge on one side of the crystal. This causes a voltage across the crystal in a direction perpendicular to the direction of the current, refered to as the Hall Voltage V_H.

Equating the electrostatic and Lorentz forces, and substiting the equation relating velocity and current, we obtain the following equations for n and p-type carrier densities:

$$n = \frac{\mathrm{IB}}{eV_H t} \qquad\qquad p = -\frac{\mathrm{IB}}{eV_H t}$$

where t is the thickness of the crystal. By applying a current and Hall voltage across the doped crystal in the precense of a magnetic field, we can experimentally obtain a value for carrier density and impurity type. With our experimental setup, we expect a positive relationship between Hall Voltage and magnetic field for negative charge carriers (n-type impurity) and a negative relationship for positive charge carriers (p-type).

We reconfigured our lab setup by placing a magnetic field sensor in front of the crystal, with magnets on both sides. We also rewired the leads for measurement of Hall voltage, and disconncted leads for the heater and thermistor. The new configuration is shown in the diagram below.



After the "turn on" procedure of the various devices, we set the current going through to 18.6 mA. The magnetic field sensor was set to "zero", and the COMP dial was adjusted so the Hall voltage was zero in the presence of no magnetic field. Additionally. we determined the direction of magnetic field which corresponded to a positive reading on the sensor.

Next, we varied the magnetic field between +1 and -1 kilo-Gauss, and recorded the corresponding Hall voltage. Error in the magnetic field, Hall voltage, and curent are based on the minimum resolution of the recording devices. Error in the crystal thickness is based on the number of significant figures given in the lab manual.

```
doped_data = dlmread('Doped.csv',',',1,0);
% device that gave us B field in Gauss
B = 10^(-4).*doped_data(:,1); % Tesla
B_err = 10^(-4).*doped_data(:,2);
% Read out on voltmeter
V_H = 10^(-3).*doped_data(:,3); % Volts
V_H_err = 10^(-3).*doped_data(:,4);
t_crystal = 0.001; % meters
t_crystal_err = 0.0001; % meters
% read out
I = 18.6*10^(-3); % Amps
I_err = 0.1*10^(-3) ;
sz = size(B);
I = I.*ones(sz);
I_err = I_err.*ones(sz);
e = 1.6*10^(-19); % Coulombs
```

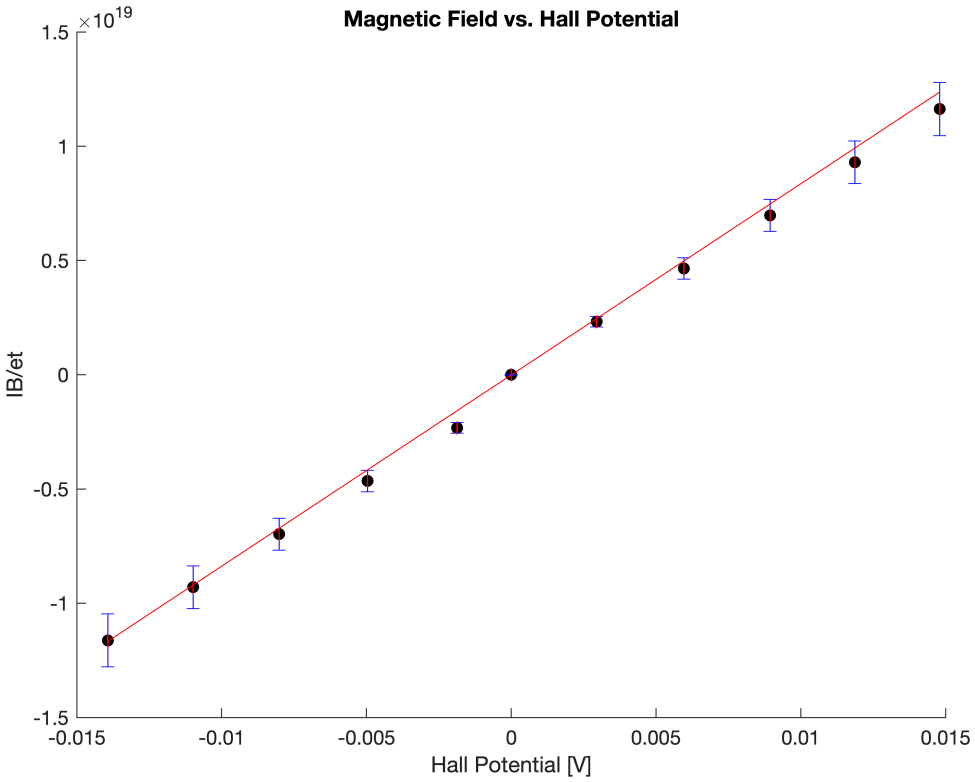
Since our data showed the relationship between magnetic field and Hall voltage to be positive, we conclude the impurity to be n-type. The carrier density can be calculated by manipulating the equation for n and performing a linear fit on the equation:

$$\frac{IB}{et} = nV_H + \left(\frac{IB}{et}\right)_0$$

Applying standard error propagation procedure, the error in $\frac{IB}{et}$ is the following:

$$\sigma_{\frac{IB}{et}} = \sqrt{\sigma_I^2\left(\frac{B}{et}\right)^2 + \sigma_B^2\left(\frac{I}{et}\right)^2 + \sigma_t^2\left(\frac{IB}{et^2}\right)^2}$$

```
IBet = (I.*B)./(e*t_crystal);
IBet_err = (((I.*B_err)./(e.*t_crystal)).^2 ...
    + ((I_err.*B)./(e*t_crystal)).^2 ...
    + ((I.*t_crystal_err.*B)./(e*t_crystal^2)).^2).^(0.5);

scatter(V_H, IBet, [], [0,0,0], 'filled');

xlabel("Hall Potential [V]");
ylabel("IB/et");
title('Magnetic Field vs. Hall Potential');

hold on;

y_err = errorbar(V_H, IBet, IBet_err);
y_err.LineStyle = 'none';
y_err.Color = [0,0,1];

x_err = errorbar(V_H, IBet, V_H_err, 'horizontal');
x_err.LineStyle = 'none';
x_err.Color = [1,0,0];

fit = myfit(V_H, IBet, IBet_err);
y_fit = V_H*fit(1,2) + fit(1,1);
plot(V_H, y_fit, 'r')
hold off;
```



The linear fit accomodates for the error bars well. Deviation from the linear fit at high magnetic fields is accomodated for in the error propagation, giving higher error bars for low and high magnetic fields. The positive relationship between Hall potential and magnetic field verifies that the charge carrier is indeed n-type.

```
n_ = fit(1,2)
```

```
n_ = 8.3685e+20
```

```
n_err = fit(2,2)
```

```
n_err = 2.6728e+19
```

From the slope of the fit, we obtained a carrier density value of

$$n = 8.37 \pm 0.27 \ * 10^{20} m^{-3}$$

To verify the difference between doped and undoped semi-conductors, we replaced our crystal with an undoped crystal. We recorded zero Hall voltage for all settings of magnetic field between -1 and +1 kilo-Gauss. This was expected, because a lack of impurities causes an equal number of charge carriers in the valence and conduction bands. As a result, Lorentz forces on positive and negative charge carriers flowing across the crystal cancel each other. This prevents a buildup of charge on either end of the crystal, thus preventing a Hall voltage from being induced.

```
function rval = myfit(x,y,ey)
    sx = sum(x ./ (ey .^ 2) );
    sy = sum(y ./ (ey .^ 2) );
    sxx = sum((x .* x) ./ (ey .^ 2) );
    sxy = sum((x .* y) ./ (ey .^ 2) );
    s = sum(1 ./ (ey .^ 2) );
    delta=sxx*s-sx*sx;
    a=(sxx*sy-sx*sxy)/delta;
    ea=sqrt(sxx/delta);
    b=(s*sxy-sx*sy)/delta;
    eb=sqrt(s/delta);
    rval=[ a, b ; ea, eb ];
end
```