Measurement of *e/m* for electrons

Purpose

The object of the experiment is to obtain a value of *e/m* for electrons from a measurement of their deflection in a uniform magnetic field. The first measurement of this quantity by J. J. Thomson, over 100 years ago, was crucial to acceptance of the idea of electrons as particles. Evaluation of the systematics is key to this experiment. Quantitatively examine the effect of the Earth's field and low anode voltage (< 200 V) on your results.

Key concepts

- Lorentz force
- Helmholtz coils
- Electron gun

Introduction

When a charged particle moving with velocity $\boldsymbol{v}$ passes through a magnetic field $\boldsymbol{B}$, it experiences a force $e\boldsymbol{v} \times \boldsymbol{B}$ where e is the particle's charge. Since this force is perpendicular to the direction of motion, it is a centripetal force and, if the magnetic field is uniform, the trajectory of the particle will be a helix, in general. If $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, the trajectory is circular and we have:

$$F = \frac{mv^2}{r} = evB\,,$$

where r is the radius of the circle.

Electrons originating from a heated filament and accelerated across a potential difference V between the filament and a positive plate will have a final kinetic energy given by $\frac{1}{2} mv^2 = eV$ assuming that they are emitted with negligible energy. Thus

$$v = \sqrt{\frac{2eV}{m}}\,.$$

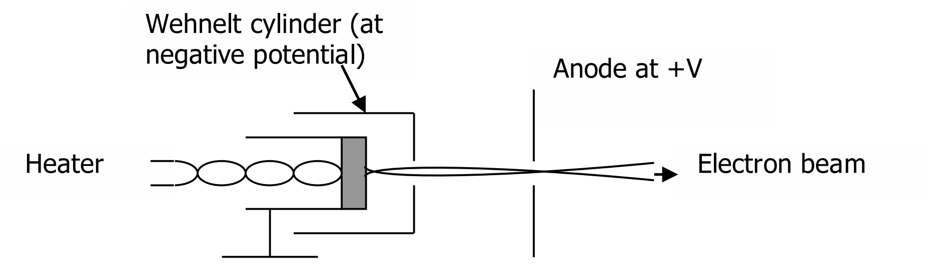
If these electrons are then allowed to enter a uniform magnetic field, the velocity v can be eliminated from these equations, giving a relation

$$V = \frac{e}{2m}\left(rB\right)^2.$$

The principle of this experiment is to measure V as a function of $(rB)^2$.

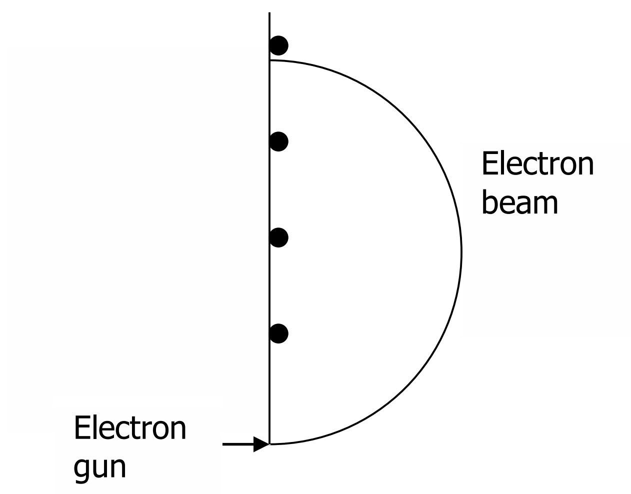
The apparatus

The apparatus consists of an evacuated spherical glass vessel containing an electron gun and acceleration electrodes. The electron gun consists of an indirectly heated barium oxide-coated cathode, a Wehnelt cylinder for focusing the beam and an anode with a hole in it, through which the electrons eventually emerge with energy eV.



After being accelerated, electrons emerge from a hole in the anode into a region which is free of electric fields. They travel at constant velocity in this region. To make the electron paths visible, the tube contains a small amount of a noble gas that becomes ionized and glows when electrons pass through it.

The diameters of the electron orbits resulting from application of a magnetic field can be determined by reference to a "ladder" which is placed vertically in the tube. The rungs on the ladder are separated by 20 mm.



The coils providing the deflecting magnetic field are configured as "Helmholtz coils", with coil separation equal to the coil radius, R_C. You should be able to show (using Biot-Savart) that the magnetic field at a distance x along the axis of a coil with radius R_C and carrying a current I is given by

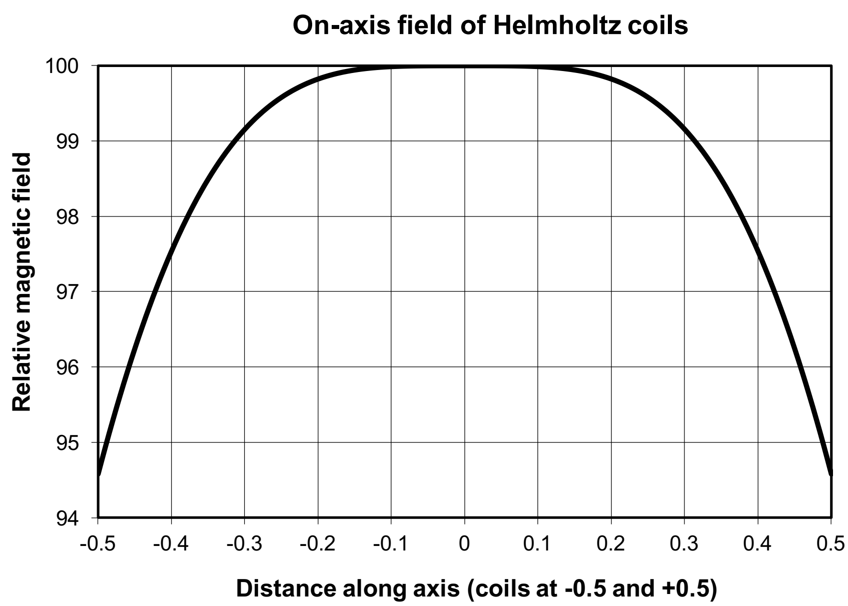
$$B = \frac{\mu_0}{2} \frac{NIR_C^2}{(R_C^2 + x^2)^{3/2}},$$

where N is the number of turns on the coil.

If two coils are separated by a distance *d,* the field at the center is then given by

$$B = \mu_0 NI \frac{R_C^2}{\left(R_C^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}.$$

When $d = R_C$ (the Helmholtz coil configuration), it is easy to show that $\partial B/\partial x = 0$ (obviously) and, more importantly, that the second derivative $\partial^2 B/\partial x^2 = 0$ at the center. The result is that the field is very nearly uniform over an extensive volume near the center, as shown in the diagram below. (Note the suppressed zero on the vertical axis).



As part of your preparations for the experiment, you should carry out error propagation for (e/m) in terms of V, r, I, and R_c.

Procedure

First measure the Helmholtz coils so that you can determine the magnetic field for a given current, i.e. determine the constant k in the relation $B=kI$. There are 124 turns on each coil. Given that the radius of the coils is equal to the separation, what is the best way to measure the diameter (prelab)?

Is the earth's magnetic field likely to affect your results (prelab)? According to NOAA geophysical data the magnitude of the field in Minneapolis is 0.56×10^{-4} Tesla, and points mostly downward with a small component north. Find the field components and write them in your logbook. Good information source is http://www.ngdc.noaa.gov/geomag-web/?id=declinationFormId#igrfwmm

Compare the maximum horizontal component of the earth's field with the field you expect when a current of several amperes is passed through the Helmholtz coils. Orient your apparatus to either minimize the effect or measure the effect. This could include taking some data with and against the field or calculating the effect of a sideways or downward pointing field. This is an important systematic error. You might want to do all this for the Pre-Lab and be prepared ahead of time.

The apparatus includes a unit that provides three of the power supplies that you will need: one that supplies the heater potential, one the negative (beam focusing) potential to the Wehnelt cylinder, and one that supplies up to 300 V to the anode. Connect all the grounds at the power supply and at the base of the apparatus (the cathode), then single wires should go to the appropriate connectors on the apparatus. There is a separate current supply for the Helmholtz coils. For maximum accuracy, use separate multimeters to measure the anode potential and coil current, rather than those attached to the supplies.

To begin, turn the anode supply to maximum (~300V) and then turn the heater supply to 6 – 7 V. You should see the heater light up and, after a short time the cathode should be hot enough for you to see an observable electron beam. **The cathode has a finite lifetime, and so the heater potential should always be kept at a minimum, only enough to produce an observable electron beam.**

Turn on the current to the Helmholtz coils (making sure that they are connected correctly) and align the tube so that the beam forms a circle, not a spiral which occurs when the beam and magnetic field are not precisely perpendicular. Vary the Wehnelt voltage to see the effect on the beam. Adjust the Wehnelt voltage in order to produce the narrowest beam possible.

Take a series of measurements of the anode potential V and coil current I for each rung of the ladder. Repeat some measurements to estimate errors. You should use parallax to determine where the beam actually passes a given rung, i.e. there are actually two rungs at each diameter, separated by a small distance. View directly along these rungs to ensure that your measurements have no dependence on viewing angle. You should estimate your precision on determining the position of the ring relative the rungs as an uncertainty in the measurement. You may also want to consider the possible uncertainty in the overall scale for the rungs and their placement relative to the gun.

Results and Systematics

Now determine *e/m* from your data in two different ways: (1) calculate e/m and its uncertainty for all measurements and form a weighted average. (2) Rewrite the formula for e/m so it represents a slope (or inverse slope), then fit using MATLAB, making sure that the largest errors correspond to the vertical axis. Look at your results and see if certain groups of points are systematically higher or lower, or seem to have a different slope. If they do, you should try to fit them separately and then quantify the differences. This is how we look for systematic errors, e.g do your individual values of *e/m* show any dependence on orbit radius *r*, or on the potential *V*? If groups of points seem to be different, then give the data points different symbols and try fitting their slopes separately. Reevaluate your assigned error and then combine the slopes in a weighted average. A plot of $\sqrt{V}$ *vs*. I can be revealing. Use this to determine if there are any background magnetic fields that might be affecting your measurement of e/m.