The Zeeman Effect: Normal vs. Anomalous

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PHYS 3605W

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Overview

Determine the splitting between energy levels in the cases of the longitudinal normal and anomalous Zeemen effect.

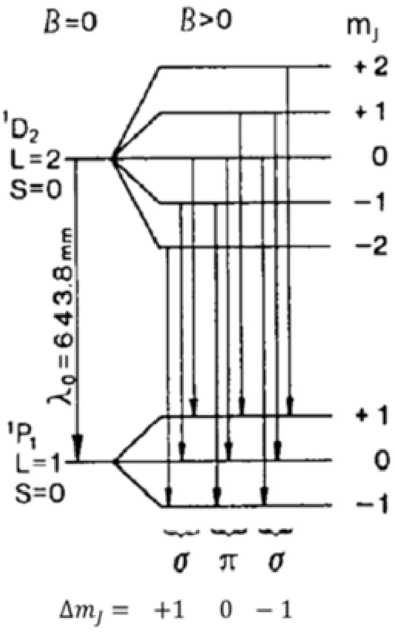
General Notes

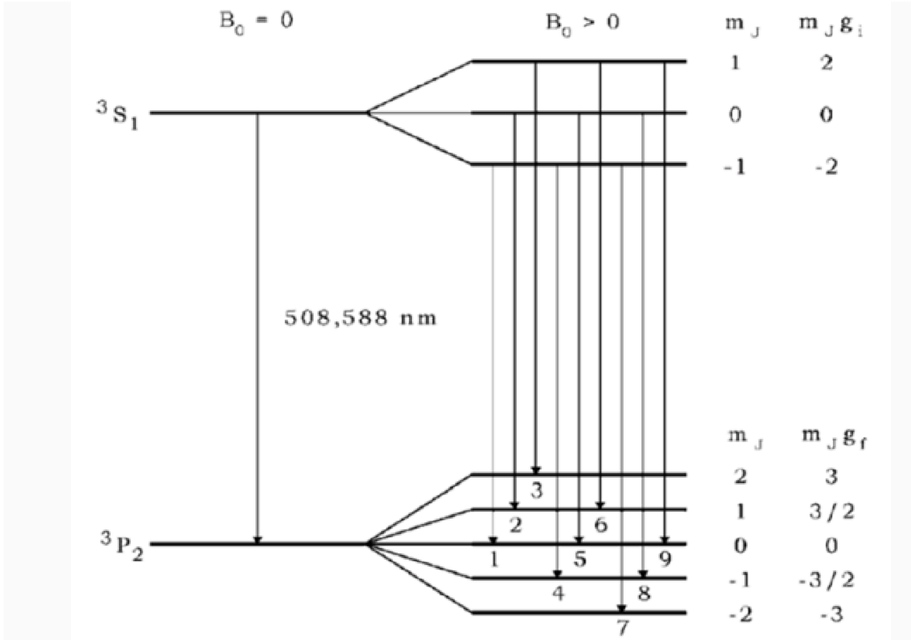
When placed in a megnetic field, the magnetic moment of the atom is quantized. Depending on the total, orbital, and spin angular momentum of the electronic state of the atom, each atomic level can be split into multiple components, known as *Zeeman splitting.* The z-component of the magnetic moment of an atom can be defined as $\mu_z = -g_J m_J \mu_B$ where g_J is the Lande g-factor. The Lande g-factor is calulated with the equation.

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

where J, L, and S are the total, orbital, and spin angular momentum of the electronic state of the atom. When placed in the presence of a magnetic field $B = B_0\hat{z}$, the magnetic energy of an atom is $E = -\mu * B = -\mu_z B_0 = -g_J m_J \mu_B B_O$. Each atomic level is split into 2*J* + 1 components, corresponding to the 2*J* + 1 possible values of m_J. The energy seperation between adjacent levels is $\Delta E_z = g_J \mu_B B_0$, where ΔE_z is known as the *Zeeman splitting*. The splitting can be obtained from observation of the optical radiation emitted by the transition between two different atomic states. Our experiment concerned the atomic transitions of Cadminium. Our goal was to observe and quantitatively confirm these atomic transitions in the cases of normal longitudinal and anomalous Zeeman effect.

Below are diagrams (from the lab manual) of transitions between two atomic states of Cadminium. The diagram on the left is for normal splitting, and the diagram on the right is for anomalous splitting.





The Normal Zeeman Effect is observed in transitions between two singlet states . As shown in the left figure, the higher energy $1D_2$ state splits into five seperate levels and lower energy $1P_1$ state splits into three. The nine allowed optical transitions consitutute three spectal energies corresponding to $\Delta m_J = +1, 0, \text{and} - 1$. The energy splitting between lines is thus

$$\Delta E = \mu_B B_0.$$

The Anomalous Zeeman Effect is observed in transitions between states where the total spin is not zero. In the case of Cd, the effect appears in the spectrum of the $3S_1$ to $3P_2$ transition. The anomalous effect produces nine distinct spectral energies corresponding to $\Delta m_J = +1, 0, -1$. Using the Lande g-factor $g(3S_1)$ = 2, and $g(3P_2)$ = 1.5, the energy splitting between lines is

$$\Delta E = \frac{\mu_B B_0}{2}$$

Quantitative measurements of normal and anomalous Zeeman splitting very much depend on the direction (relative to the magnetic field) along which light from each transition travels. Our experiment concerns the *longitudinal* Zeeman effect, where emitted light is paralell to the magnetic field. In this case only the $m = \pm 1$ transitions can be observed, because they emit circularily polarized light with a polarization vector in the y-z plane. This produces light propagation paralell to the magnetic field vector. The $m = 0$ transition emits light polarized paralell to the magnetic field, so the light propagates perpendicular to the magnetic vield vector. As a result of the $m = 0$ transition being absent, only two lines will appear for the normal Zeeman effect. Therefore, the energy splitting between the two lines is doubled, yeilding $\Delta E = 2\mu_B B_0$. With the anomalous effect, the $\Delta m_J = \pm 1$transitions are observable, producing six lines. The energy spitting is also doubled to $\Delta E = \mu_B B_0$.

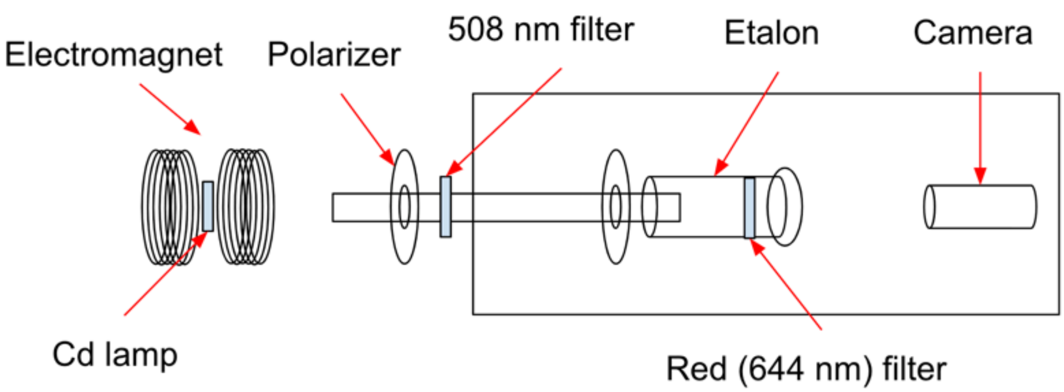
Since the difference in wavelengths between adjacent fringes is extremely small relative to the the difference in wavelengths between energy levels, a Fabry-Perot etalon was used to observe the fringes. When light seperated by small differences in wavelength pass through the etalon, the transmitted rays distinguish themselves as a series of circular interference patterns. Measurement of the different fringe radii allows us to experimentally confirm theoretical equations for energy splitting for the longitudinal normal and anomalous zeeman effect.

Procedure

Our lab setup included a two coil electromagnet with the coils wired in paralell. The leads of the wires were connected to a power supply through an aluminium box that contains a protection relay. The current going through the magnet was monitored by the voltage across a 0.050 Ω series resistor. The corresponding voltage (shunt voltage) was read out by a voltmeter. The magnetic field *B* (in Tesla) is related to the shunt voltage V (in Volts) by

$$B = 1.004V + 0.006 \rightarrow \text{for } V < 0.5V$$
$$B = 0.531V + 0.235 \rightarrow \text{for } 0.5 < V < 0.65V$$

The second equation is due to saturation of the magnet at very high fields. A Cd discharge lamp was mounted in the center of the magnet, which emitted light paralell to the magnetic field through holes bored in the pole pieces. All of the optics were mounted on an optical rail, some of which were inside a light-tight box. Outside the box was a polarizer, which was set to 45 degrees. Inside the box were two lenses, one of which contained the Fabry-Perot etalon. A lense placed after the etalon focused the light into a camera, where the interference pattens were imaged and analysed with the *ZeemanLab.vi.* software.



Follwing the "turn on" procedure of the relevant equipment, we began by first measuring the radii of interference fringes created from the normal Zeeman effect. The red (644 nm) band pass filter was placed into the a slot in the etalon housing. We turned the shunt voltage to 0.302 Volts, and recorded the radii of concentric fringes. The shunt voltage was increased to 0.477 volts, and additional data was recorded.

```
normal_1=dlmread('Shunt .302 Normal.csv',',',1,1);
normal_2=dlmread('Shunt .477 Normal.csv',',',1,1);
```

We calculated the average energy splitting experimentally using the equation below.

$$\Delta E = \frac{hc}{2nt}\frac{\delta}{\psi'}, \ \delta = r^2_{p,b} - r^2_{p,a} \quad \psi' = r^2_{p'+1,b} - r^2_{p',b}$$

where delta is the difference in the squares of the radii for any pair of *a* and *b* fringes of the *same* order *p*, and psi the difference in the squares of the radii of any two *a* fringes or any two *b* fringes of two consecutive orders *p' and p'+1.* n is the index of refraction inside the etalon, t is the thickness of the etalon, and *hc* comes from Plank's constant multiplied by the speed of light.

$r_{p,b}, r_{p,a}, r_{p'+1,b}$ and $r_{p',b}$ correspond to r1, r2, r3, and r4 respectively. The error associated with our recorded radius is based off our difficulty in resolving the fringes. Error in the thickness of the etalon is based off the number of significant figures provided in the lab manual.

```
r_err=1; % pixels
n=1.4519;
t=.003; % meters
hc=1.24*10^-6; % eV*meters
t_err = 0.0001; % meters

%Shunt .302 Normal
r1 = normal_1([2,4,6,8],:);
r2 = normal_1([1,3,5,7],:);
r3 = normal_1([3,4,5,6],:);
r4 = normal_1([1,2,3,4],:);

delta = r1.^2 - r2.^2;
psi = r3.^2 - r4.^2;
n_avg_energy(:,1)=(hc/(2*n*t)).*delta./psi;
```

Error in the average energy splitting was calculated from the standard procedure for propagating error based on the variables t, delta, and psi, and r each having their own inherent error.

$$\sigma_E = \sqrt{\sigma_t^2\left(\frac{\partial E}{\partial t}\right)^2 + \sigma_\delta^2\left(\frac{\partial E}{\partial \delta}\right)^2 + \sigma_\psi^2\left(\frac{\partial E}{\partial \psi}\right)^2}$$

The function *get_e_err()* at the bottom of the script defines each of the partial derivatives in the equation and computes the error.

```
n_avg_energy_err(:,1) = get_e_err(r1, r2, r3, r4, delta, psi);

%Shunt .477 Normal
r1 = normal_2([2,4,6,8],:);
r2 = normal_2([1,3,5,7],:);
r3 = normal_2([3,4,5,6],:);
r4 = normal_2([1,2,3,4],:);

delta = r1.^2 - r2.^2;
psi = r3.^2 - r4.^2;
n_avg_energy(:,2)=(hc/(2*n*t)).*delta./psi;
n_avg_energy_err(:,2) = get_e_err(r1, r2, r3, r4, delta, psi);
```

To measure the splitting due to the anomalous Zeeman effect, the red 644 nm filter was removed. We placed the 508 nm interference filter into on the rail before the light-box and after the polarizer. As the magnetic field increased, we observed each fringe split into two bands, similar to the normal Zeeman effect. Increasing the magnetic field even more, each of the bands split into three individual lines as expected by our qualitative predictions. We insured the upper band from the first order fringes did not overlap with the lower band from the second order fringe. Measurements were recored at shunt voltages V = 0.538V and V=0.562V. The average energy splitting for both shunt voltages are calculated below.

```
anomalous_1=dlmread('Shunt .538 Anomalous.csv',',',1,1);
anomalous_2=dlmread('Shunt .562 Anomalous.csv',',',1,1);

%Shunt .538 Anomalous
r1 = anomalous_1([4,5,6,10,11,12],:);
r2 = anomalous_1([1,2,3,7,8,9],:);
r3 = anomalous_1([7,8,9,10,11,12],:);
r4 = anomalous_1([1,2,3,4,5,6],:);

delta = r1.^2 - r2.^2;
psi = r3.^2 - r4.^2;

a_avg_energy(:,1)=(hc/(2*n*t)).*delta./psi;
a_avg_energy_err(:,1) = get_e_err(r1, r2, r3, r4, delta, psi);

%Shunt .562 Anomalous
r1 = anomalous_2([4,5,6,10,11,12],:);
r2 = anomalous_2([1,2,3,7,8,9],:);
r3 = anomalous_2([7,8,9,10,11,12],:);
r4 = anomalous_2([1,2,3,4,5,6],:);

delta = r1.^2 - r2.^2;
psi = r3.^2 - r4.^2;

a_avg_energy(:,2)=(hc/(2*n*t)).*delta./psi;
a_avg_energy_err(:,2) = get_e_err(r1, r2, r3, r4, delta, psi);
```

The experimentally obtained values for energy splitting can now be compared with the theoretical equations.

Normal splitting: $\Delta E = 2\mu_B B_0$

Anolagous splitting: $\Delta E = \mu_B B_0$

where $\mu_B = 5.79 * 10^{-5} \frac{\text{eV}}{T}$

The theoretical energy splits are calculated below for all shunt voltages.

```
%Theoretical Energy Splitting
mu_b = 5.79*10^-5; % eV/tesla
% Normal V=0.302V
E_theo(1)=mu_b*(1.004*.302+.006)*2; %eV
% Normal V=0.477V
E_theo(2)=mu_b*(1.004*.477+.006)*2; %eV
```

```
% Anomalous V=0.538V
E_theo(3)=mu_b*(.531*.538+.235); %eV
% Anomalous V=0.562V
E_theo(4)=mu_b*(.562*.562+.235); %eV
E_theo = E_theo.';
```

Since we are interested in the average energy seperation, we compute the weighted average energy for each set of measurements along with the error in the weighted average.

```
% weighted avg
E_avg(1)=sum(n_avg_energy(:,1)./n_avg_energy_err(:,1).^2)./sum(1./n_avg_energy_err(:,1
E_avg(2)=sum(n_avg_energy(:,2)./n_avg_energy_err(:,2).^2)./sum(1./n_avg_energy_err(:,2
E_avg(3)=sum(a_avg_energy(:,1)./a_avg_energy_err(:,1).^2)./sum(1./a_avg_energy_err(:,1
E_avg(4)=sum(a_avg_energy(:,2)./a_avg_energy_err(:,2).^2)./sum(1./a_avg_energy_err(:,2
E_avg = E_avg.';

% weighteg avg err
E_avg_err(1)=1/sqrt(sum(1./(n_avg_energy_err(:,1).^2))); %eV
E_avg_err(2)=1/sqrt(sum(1./(n_avg_energy_err(:,2).^2))); %eV
E_avg_err(3)=1/sqrt(sum(1./(a_avg_energy_err(:,1).^2))); %eV
E_avg_err(4)=1/sqrt(sum(1./(a_avg_energy_err(:,2).^2))); %eV
E_avg_err = E_avg_err.';
```

```
shunt = [0.302;0.477;0.538;0.562]; % Volts
t = table(shunt, E_avg, E_avg_err, energy_theo)
```

t = 4×4 table

	shunt	E_avg	E_avg_err	energy_theo
1	0.3020	4.7864e-05	5.8141e-06	3.5806e-05
2	0.4770	6.0092e-05	6.9964e-06	5.6152e-05
3	0.5380	3.9585e-05	2.5660e-06	3.0147e-05
4	0.5620	4.4801e-05	2.9706e-06	3.1894e-05

Zeeman Split	Shunt Voltage (V)	ΔE Avg. (10^-5 eV)	ΔE Theoretical (10^-5 eV)
Normal	0.302	4.79 ± 0.58	3.58
	0.477	6.01 ± 0.70	5.61
Anomalous	0.538	3.96 ± 0.26	3.01
	0.562	4.48 ± 0.97	3.19

The experimentally obtained values for average energy seperation vary in their consistancy with the theoreticall expected values. The energy splitting for normal splitting at shunt V = 0.477V was consistant its the theoretical value, and normal splitting for shunt V = 0.302V was around two sigma away from the theoretical value. For anomalous splitting, both the experimental and theoretical values differer by 3 to 4 sigma for shunt both shunt voltages. This suggests an underestimation of systematic errors in our calculations involving anomalous splitting. This makes sense because we ran into difficulty resolving the thin secondary secondary maxima. The experiment could have greatly benefitted from a better resolution camera.

Another source of systematic error to note is the possible uncertainty in the theoretical equation due to uncertainty in the magnetic field. A future experiments should take into account this uncertainty to obtain error bounds on theoretical equation. Finally, a notable the presence of spherical aberrations in the etalon could have caused errors. The aberrations cause the effective focal length to decrease for rays further from the optical axis. As a result, the measurements of circular fringes become less accurate further out from the center.

Overall, our measurements of the energy splitting of Cadminium in the precense of an electric field was consistent with what we expected qualitatively. The anomalous energy splitting is lower than the normal energy splitting. A future experiment would benefit from keeping the shunt voltage constant between measurments for both normal and anomalous splitting to verify that anomalous is indeed smaller by a factor of two as predicted theoretically.

Dispite the qualitative confirmation, our experiment was unable to obtain a rigorious quantitative confirmation of anomalous Zeeman splitting due to the possibilty of multiple overlooked systematic errors.

```
function e_err = get_e_err(r1, r2, r3, r4, delta, psi)
    r_err=2; % pixels
    n=1.4519;
    t=.003; % meters
    t_err = 0.0001; % meters
    hc=1.24*10^-6; % eV*meters

    delta_err = sqrt(r_err^2*(4.*r1.^2) + r_err^2*(4.*r2.^2));
    psi_err = sqrt(r_err^2*(4.*r3.^2) + r_err^2*(4.*r4.^2));
    e_partial_t = (-hc.*delta)./(2*n*t^2.*psi);
    e_partial_delta = (hc)./(2*n*t.*psi);
    e_partial_psi = (-hc.*delta)./(2*n*t.*psi.^2);

    e_err = sqrt(t_err^2.*(e_partial_t).^2 + delta_err.^2.*(e_partial_delta).^2 + psi_e

end
```