High-Resolution Spectroscopy of Hydrogen and Sodium

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PHYS 3605W

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Overview

Calibrate a diffraction grating spectrometer and use this for accurate measurements of visible lines of hydrogen and sodium spectra. We will analyze these spectra in terms of elementary atomic theory.

General Notes

Our experiment allowed us to investigate the structure Hydrogen and Sodium atoms by studying the emission and absorbtion of electromagnetic radiation. When in a gas at low presure, these atoms take on well defined descrete energy levels. When returning to a lower energy state, they emit electromagnetic radiation with well defined energies (and thus wavelengths). This is described by the equation

$$\Delta E = E_i - E_f = h\nu = \frac{hc}{\lambda}$$

The elementary atomic theory presented by Niels Bohr showed the energy of photons emitted from hydrogen can be related to integers representing their energy levels by the equation.

$$E = h\nu = \mathrm{ch}R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \qquad \text{where} \qquad R_H = \frac{M_{\mathrm{proton}}}{M_{\mathrm{proton}} + m_{\mathrm{electron}}} R_\infty \qquad \text{and} \qquad R_\infty = \frac{2\pi^2 m e^4}{ch^3}$$

Procedure

Our lab setup consisted of a spectrometer, photomultiplier tube, and an oscilloscope. The high voltage supply for the photomultiplier was set at -600 volts, and we used a signal frequency of 120 Hz on our oscilloscope to observe the output signal. The grating rotor was set to mm, and turned fully counterclockwise initially. High amplitude spectral lines could be observed from the eyepiece, and lower amplitude lines we observed by the digital readout.

Calibration of the spectrometer

A diffraction grating was used to seperate light waves of different wavelengths. The grating was a flat square of glass covered with a thin layer of plastic, with many fine, paralell grooves on the surface. The angle of the diffraction grating to the incident light beam was adjusted by means of a multi-turn dial. To calibrate the spectrometer, we needed to determine the relationship between wavelength and spectrometer dial reading. The figure below shows the basic function of the spectrometer.

Fig.1
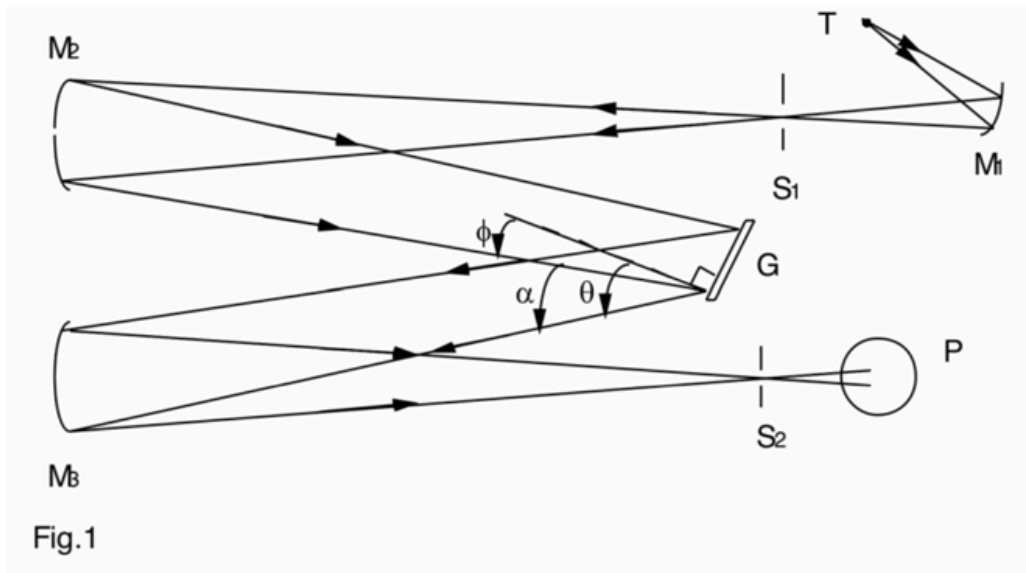
Constructive interference occurs when the relationship $\sin\theta + \sin\phi = m\frac{\lambda}{d}$ is satisfied, and our particular apparatus was constructed so that m=1.

A mercury lamp, which provides several sharp lines of well-known wavelengths, was used for calibration. The dial reading corresponding to the grating angle at which the line perpendicular to the grating is centered between M2 and M1 was recorded as D_0.

```
D_0 = 0.001*8.52; % meters
```

Next, we rotated the grating clockwise (using the multiturn dial), and recorded data on the mercury spectra lines. Although some of the lines were clearly visible from the eyepiece, there was significant difficulty in distinguishing some of the fainter mercury lines. There were various oscilloscope signals on the order of about 0.002 V, whereas the strongest spectral lines registered a signal of about 1.000 V.

Initial measurements were made of the dial readings corresponding to the strongest Hg spectrum lines, which had wavelengths (measured in air) given in the lab manual.

```
Data = dlmread('HRS_calibration.csv',',',0,0);
```

In order to connect any given dial reading with an estimated wavelength, we obtained a linear fit of our dial calibration data and recorded the fit coefficients. A quadratic fit would have been preferable, since the relationship between $\sin\phi$ (in the diagram above) and wavelength of light reaching the detector has a slight quadratic curve. However, the LSQfit_2D fitting program gave a quadratic fit that was much worse than a simple linear fit, with a χ^2 value of over 100,000. This could have been due to a bug in the program or an error in our data input. This was a major setback in our ability to obtain an accurate calibration of the spectrometer.

The error associated with the dial reading is based on the minimum resolution of the dial, with an added uncertainty from the difficulty to resolve certain spectral lines.

```
dial_err = Data(:,3); % mm
dial_err = dial_err.*0.001; % meters
dial = Data(:,1); % mm
dial = dial.*0.001; % meters
lambda = Data(:,2); % nm
% These reference wavelengths were obtained in air
lambda = lambda.*(10^-9); % meters

[a, b, ea, eb] = myfit(lambda, dial, dial_err);


errorbar(lambda, dial, dial_err, 's');
hold on
y_fit = lambda.*b + a;
plot(lambda, y_fit);
xlabel('Wavelength (meters)')
ylabel('Dial reading (meters)')
title('Linear fit of Wavelength vs. Dial Reading')
hold off
```

Linear fit of Wavelength vs. Dial Reading
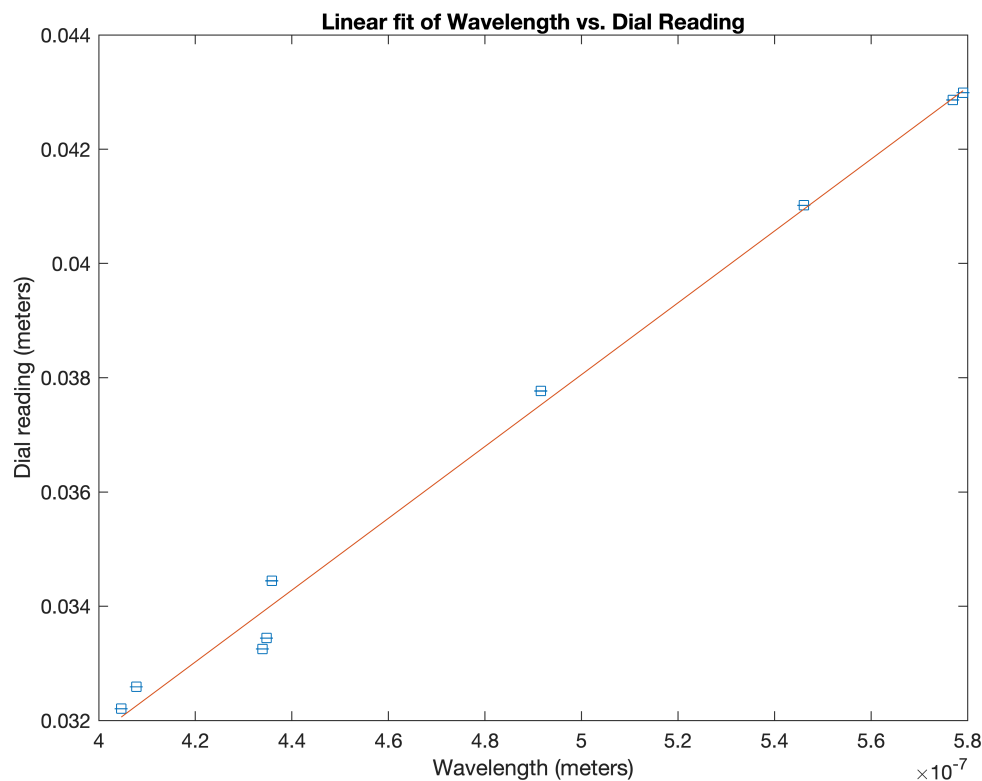
Visually inspecting the plot of dial reading as a function of wavelength, we see that the error bars accomodate for the linear fit reasonably well. Using the coefficients of the linear fit, we obtain an equation for wavelength with respect to dial reading. Standard procedure for error propagation was carried out based off the variables d, a, and b having their own inherent error.

$$\lambda = \frac{d-a}{b} \qquad \sigma_\lambda = \sqrt{\sigma_d^2\left(\frac{\partial\lambda}{\partial d}\right)^2 + \sigma_a^2\left(\frac{\partial\lambda}{\partial a}\right)^2 + \sigma_b^2\left(\frac{\partial\lambda}{\partial b}\right)^2} = \sqrt{\sigma_d^2\left(\frac{1}{b}\right)^2 + \sigma_a^2\left(\frac{1}{b}\right)^2 + \sigma_b^2\left(\frac{(a-d)}{b^2}\right)^2}$$

The Hydrogen Spectrum

We replaced the Hg lamp with a hydogen tube, and recorded the dial readings corresponding to four lines from the Hydrogen spectrum. The calibration information obtained above was then used to determine the wavelength for each line. Our goal was to determine an experimental value for hydrogenic Rydberg constant R_H using the equation

$$R_H = \frac{1}{\lambda_{\text{vac}}}\left(\frac{n_1^2\,n_2^2}{n_2^2 - n_1^2}\right) \text{ where } n_1 = 2 \text{ , and } n_2 = 3, 4, 5, \text{ and } 6 \qquad\qquad \sigma_{R_H} = \frac{\sigma_\lambda}{\lambda^2}\left(\frac{n_1^2\,n_2^2}{n_2^2 - n_1^2}\right)$$

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Since our calibration equation relied on wavelengths measured in air, our wavelengths were converted to vacuum using the equation $\lambda_{vac} = \lambda_{air}\,n$.

```
H_data = dlmread('HRS_H_spectrum.csv',',',[1,0,4,1]);
n2 = H_data(:,1); % energy level
n2_dial = 0.001.*H_data(:,2); % dial reading corresponding to energy level (meters)
sz = size(n2_dial);
n2_dial_err = 0.001.*0.005.*ones(sz);
n_air = 1.00029; % index of refraction for air
% fix for quadratic
H_lambda_air = (n2_dial - a)./b; % wavelength in air (meters)
% wavelength for four trainsitions of the hydrogen spectra n=3,4,5,6
H_lambda_vac = H_lambda_air.*n_air; % wavelength in vacuum (meters)
H_lambda_err = sqrt(n2_dial_err.^2.*(1./b).^2 + ea.^2.*(1./b).^2+eb.^2.*((a-n2_dial)./b
```

Next, we combined the values of R_H obtained from each energy level to compute a weighted average.

```
n1 = 2;
R_H = (1./H_lambda_vac).*((n1.^2.*n2.^2)./(n2.^2-n1.^2));
R_H_err = (H_lambda_err./H_lambda_vac.^2).*((n1.^2.*n2.^2)./(n2.^2-n1.^2));
R_H_weighted_avg = (sum(R_H./R_H_err.^2)./sum(1./R_H_err.^2))
```

```
R_H_weighted_avg = 8.7748e+06
```

```
R_H_weighted_avg_err = sqrt(1./sum(1./R_H_err.^2))
```

```
R_H_weighted_avg_err = 1.6919e+03
```

$R_H = 1.096776 * 10^{-7}\ m^{-1}$ (lab manual)

$R_H\ \text{(measured)} = 8.77 \pm 0.02 * 10^{-6}\ m^{-1}$

The value of R_H we obtained is within an order of magnitude of the expected value (provided in the lab manual), with an uncertainty that is unrealistically small. Our underestimation of uncertainty could have come from either a miscalculation of one of our error propagation equations, or a failure to take additional sources of error into account. Data on the fainter spectral lines for spectrometer calibration would have certainly helped provide a more accurate linear fit, and thus a more accurate estimate of R_H. Although this provides a rough qualitative confirmation of Bohr's elementary atomic theory, insufficient data makes it difficult to get a rigorous quantitative confirmation.

The Sodium Spectrum

For analysis of the sodium spectrum, we replaced the hydrogen tube with a sodium source, and let it warm for 10 minutes. Our goal was to measure the fine structure splitting of sodium due to the spin and corresponding magnetic moment of its valence electron. Since the magnetic moment has two different orientations ('spin up' or 'spin down'), each energy level is split into doublets. This causes the corresponding spectral lines to split into two seperate lines. We recorded the dial reading of each doublet spectral line for five different energy levels, and then calculated the difference in dial reading for each doublet.

```
Na_data = dlmread('HRS_Na_spectrum.csv',',',[1,0,10,0]);
Na_dial = 0.001.*Na_data(:,1);
Na_lambda = (Na_dial - a)./b; % wavelength in air (meters)
sz = size(Na_dial);
delta_dial_err = 0.001.*0.003.*ones(sz); % meters
delta_lambda_err = sqrt(delta_dial_err.^2.*(1./b).^2 + ea.^2.*(1./b).^2+eb.^2.*((a-Na_
delta_lambda_err = delta_lambda_err([1,3,5,7,9]);
delta_lambda = Na_lambda([2,4,6,8,10],:) - Na_lambda([1,3,5,7,9]);
```

The spectrometer calibration equation was then used to convert difference in dial readings to difference in wavelengths, giving us values for doulet seperation distance. Seperation distance is then used to calculate the energy seperation using the equation

$$\Delta E = hc\left(\frac{1}{\Delta\lambda}\right) \quad \sigma_{\Delta E} = hc\frac{\sigma_\lambda}{\Delta\lambda^2}$$

```
h = 6.626*10^-34; %Js
c = 3.0 * 10^8; %m/s
delta_E = h*c.*(1./delta_lambda);
delta_E_err = h*c.*delta_lambda_err./delta_lambda.^2;

delta_E_weighted_avg = (sum(delta_E./delta_E_err.^2)./sum(1./delta_E_err.^2))
```

```
delta_E_weighted_avg = 3.6622e-16
```

```
delta_E_weighted_avg_err = sqrt(1./sum(1./delta_E_err.^2))
```

```
delta_E_weighted_avg_err = 5.8393e-17
```

The experimentally obtained value for $\Delta E = 3.6 \pm 0.6\,\mathrm{meV}$ is slightly over two sigma away from the expected value of 2.1 meV. This is due to either an underestimation of uncertainty on ΔE or a systematic error causing a higher value than expected.

Determination of the grating groove spacing
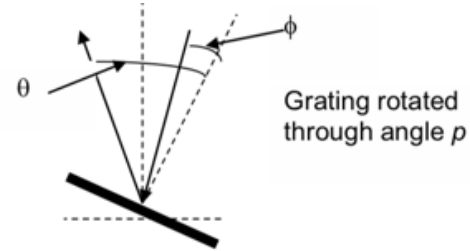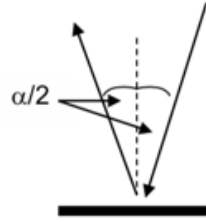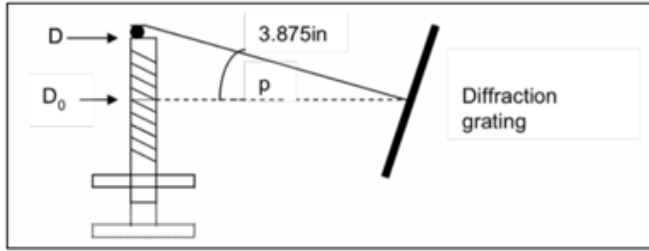
The geometry of the spectrometer grating, combined with the equation for constructive interference referenced earlier, allow us to derive an equation for the number n of grooves per meter.

The grating angle p for a given dial setting D is given by two equations

$$p = \sin^{-1}\left(\frac{D - D_0}{0.098425 \text{ m}}\right) \qquad p = \phi + \frac{\alpha}{2}$$

Combined, these equations can give an equation for angle of incidence ϕ

$$\phi = \sin^{-1}\left(\frac{D - D_0}{0.098425 \text{ m}}\right) - \frac{\alpha}{2} \qquad \begin{array}{l} \alpha = 8.40^o \\ D_0 = 8.52 \text{ mm} \end{array}$$

Finally, the value of ϕ is inserted into the diffraction equation, and we obtain.

$$n = \frac{1}{d} = \frac{\sin\phi(1 - \cos\alpha) + \cos\phi\sin\alpha}{\lambda}$$

Solving for groove spacing d, we can then obtain an estimated average distance between grooves.

```
alpha = 8.40;
psi = asin((dial-D_0)./(0.098425))-alpha/2;
n = (sin(psi).*(1-cos(alpha)) + cos(psi).*sin(alpha))./lambda;
d = 1./n; % meters
d_avg = sum(d)/9
```

```
d_avg = 1.1577e-06
```

The estimated average groove spacing is thus $d = 1.16 * 10^{-6} \, m$

```
function [a, b, ea, eb] = myfit(x,y,ey)
```

```
  sx = sum(x ./ (ey .^ 2) );
  sy = sum(y ./ (ey .^ 2) );
  sxx = sum((x .* x) ./ (ey .^ 2) );
  sxy = sum((x .* y) ./ (ey .^ 2) );
  s = sum(1 ./ (ey .^ 2) );
  delta=sxx*s-sx*sx;
  a=(sxx*sy-sx*sxy)/delta;
  ea=sqrt(sxx/delta);
  b=(s*sxy-sx*sy)/delta;
  eb=sqrt(s/delta);
end

function [chi, chi2] = getChi(y, fit_y, y_err)
   chi = (y - fit_y)./(y_err);
end
```