This is my title with equation: $x^2 + y^2 = c^2$

Zur Erlangung des akademischen Grades eines

DOKTORS DER NATURWISSENSCHAFTEN

von der KIT-Fakultät für Mathematik des
Karlsruher Instituts für Technologie (KIT)
genehmigte

DISSERTATION

von

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Ich versichere wahrheitsgemäß, die Arbeit selbstständig angefertigt, alle benutzten Hilfsmittel vollständig und genau angegeben und alles kenntlich gemacht zu haben, was aus Arbeiten anderer unverändert oder mit Abänderungen entnommen wurde.

I declare that I have developed and written the enclosed thesis completely by myself, and have not used sources or means without declaration in the text.

. . . .
Max Mustermann
Karlsruhe, 01.01.2021

Liste der noch zu erledigenden Punkte

Your first smart quote goes here. Of course, only if you want to

SOME NAME, Some Book Title

A second quote maybe?

SOME OTHER NAME, Some Other Book Title

Acknowledgments

Thank you so much everyone!

Preface

Place your preface content here.

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1 Transport equations

Imagine an infinitely large billiard table. A particle—represented by the white billiard ball—moves frictionlessly along straight lines until undergoing elastic, instantaneous collisions with background obstacles—the black billiard balls—that do not move due to their infinite weight and neglectable velocity. The situation is depicted in Figure 1.1.

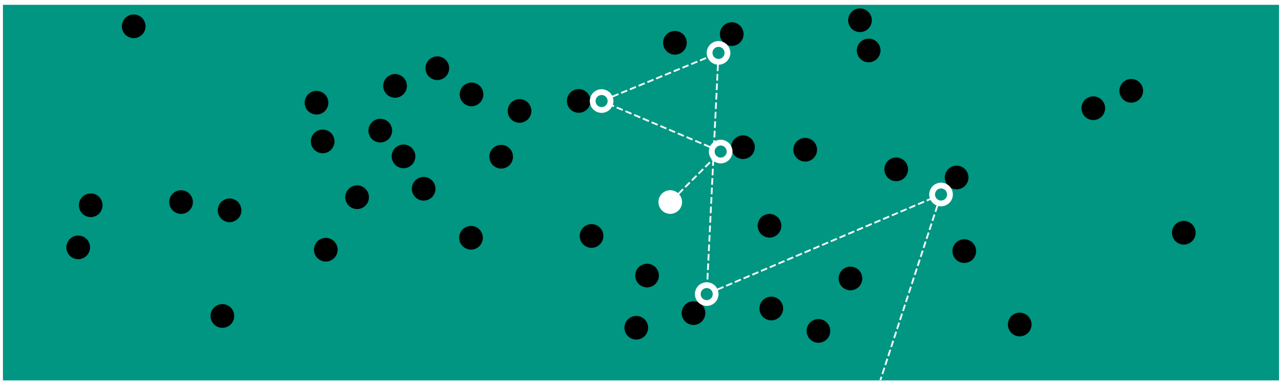

Figure 1.1: The white ball undergoes elastic collisions on the billiard table while the black balls' positions are fixed.

This in many ways simplified microscopic description of particle interactions will serve as the starting point for this thesis. Despite its simplicity, it allows the derivation of governing mesoscopic and macroscopic equations, as well as the construction of one of the most prominent numerical methods in transport theory—the Monte Carlo method. Furthermore, changing the microscopic picture will motivate non-classical transport, discussed in the second part of this thesis.

A magnitude of real-world phenomena and topics can be formulated in the language of kinetic theory. These include, but are not limited to, the theory of gases; radiation therapy for cancer treatment; modeling of nuclear reactors; or illumination in movies and computer games.

Transport equations try to achieve the following: Instead of describing a system by the behavior of every single molecule, every single particle, or every single light-ray, they seek a description in terms of statistical quantities that evolve in time. This is both necessary and often desirable. Necessary, since it already becomes computationally impossible to evolve the trajectory of each of the roughly $6.022 \cdot 10^{23}$ particles in one mole. Desirable, since it is generally of no interest to know each of these trajectories individually. Thus, transport equations are convenient tools that distill complex physical systems to manageable equations.

Throughout the scope of this thesis we are going to consider uncharged particle transport; that is, particles do not interact with another via long range interactions—like electrons do—but with the background medium or through direct collisions with another. Given these assumptions,

Weight function	Interval	Orthogonal polynomials
$w(x) = 1$	$[-1, 1]$	Legendre
$w(x) = 1/\sqrt{1-x^2}$	$(-1, 1)$	Chebyshev (1^{st} kind)
$w(x) = \sqrt{1-x^2}$	$[-1, 1]$	Chebyshev (2^{nd} kind)
$w(x) = e^{-x}$	$[0, \infty)$	Laguerre
$w(x) = e^{-x^2}$	$(-\infty, \infty)$	Hermite

Table 1.1: Table of weight functions and corresponding orthogonal polynomials.

particles move through phase space according to Newton's laws of motion

$$\begin{cases} \dot{\boldsymbol{x}}(t) & = \boldsymbol{v}(t), \\ \dot{\boldsymbol{v}}(t) & = \frac{1}{m}\mathbf{F}(t, \boldsymbol{x}(t), \boldsymbol{v}(t)), \end{cases} \tag{1.1}$$

with $\boldsymbol{x}(t), \boldsymbol{v}(t) \in \mathbb{R}^3$, $t \in \mathbb{R}^{\geq 0}$, $m \in \mathbb{R}^{>0}$, $\mathbf{F} : \mathbb{R}^{\geq 0} \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ sufficiently smooth, and subject to initial conditions $\boldsymbol{x}(0) = \boldsymbol{x}_0$ and $\boldsymbol{v}(0) = \boldsymbol{v}_0$. If we again assume that particles move with a much smaller mass and faster than background obstacles, and if we also neglect particle-particle interactions, it is easy to track particle trajectories through the obstacle field as already seen in Figure 1.1.

1.1 And here starts a new subsection

With some text.

1.1.1 And maybe a new subsection?

Some more text. A list of your todos—including the one here—can be found in the beginnig of your thesis. Remove that list before the final publication.

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1.2 Citations

You can cite people easily [1].

1.3 Table

For different weight functions w, some orthogonal polynomials are given in Table 1.1.

1.4 Algorithm

My favorite algorithm is Algorithm 1.2, not Algorithm 1.1.

Algorithm 1.1 The S_N method.

function $\mathrm{S}_N(\Delta t, t_{\mathrm{end}}, \mathrm{order}, n_x, n_y, \psi_0)$
    $t \leftarrow 0$
    $n_q \leftarrow \mathrm{order}^2$
    $P \leftarrow \mathrm{QPoints}(n_q) \in \mathbb{R}^{3\times n_q}$
    $W \leftarrow \mathrm{QWeights}(n_q) \in \mathbb{R}^{n_q}$
    $\psi \leftarrow \psi_0 \in \mathbb{R}^{n_q\times n_x\times n_y}$
    while $t < t_{\mathrm{end}}$ **do**
        $F \leftarrow \mathrm{ComputeFlux}(\psi, P, W)$
        $\psi \leftarrow \psi + \Delta t \cdot F$
        $t \leftarrow t + \Delta t$
    return ψ

Algorithm 1.2 The rS_N method.

function $\mathrm{rS}_N(\Delta t, t_{\mathrm{end}}, \mathrm{order}, n_x, n_y, \psi_0, \delta)$
    $t \leftarrow 0$
    $n_q \leftarrow 4 \cdot \mathrm{order}^2 - 8 \cdot \mathrm{order} + 6$
    $P \leftarrow \mathrm{NewQPoints}(n_q) \in \mathbb{R}^{3\times n_q}$
    $W \leftarrow \mathrm{NewQWeights}(n_q) \in \mathbb{R}^{n_q}$
    $\psi \leftarrow \psi_0 \in \mathbb{R}^{n_q\times n_x\times n_y}$
    while $t < t_{\mathrm{end}}$ **do**
        $F \leftarrow \mathrm{ComputeFlux}(\psi, P, W)$
        $\psi \leftarrow \psi + \Delta t \cdot F$
        $\alpha \leftarrow \delta \cdot \Delta t / n_q$
        $\psi, P \leftarrow \mathrm{RotateInterpolate}(\psi, P, \alpha)$
        $t \leftarrow t + \Delta t$
    return ψ

Bibliography

[1] Thomas Camminady, Martin Frank, and Edward W. Larsen. Nonclassical particle transport in heterogeneous materials. In *International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering*, 2017. 1.2