Summary of: Wireless Communication Using Unmanned Aerial Vehicles (UAVs): Optimal Transport Theory for Hover Time Optimization

Written by: Mohammad Mozaffari, Walid Saad, Mehdi Bennis, Mérouane Debbah
Presented by: Grant Stagg

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Outline

- Motivation
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Motivation

- UAVs are a promising solution for providing reliable wireless communication services
- UAVs can be used to expand coverage of existing networks during events
- UAVs can provide coverage during emergencies if existing networks are damaged
- Can be used as a cost effective way to extend existing networks
- Can establish line of site connections better than ground based networks

Relevant Background Literature

- Air-to-ground modeling in various propagation environments
  - A. Al-Hourani, S. Kandeepan, and A. Jamalipour, “Modeling air-to-ground path loss for low altitude platforms in urban environments,” in Proc. IEEE Global Commun. Conf. (GLOBECOM), Austin, TX, USA, Dec. 2014, pp. 2898–2904.
- Deployment for maximum coverage:
  - M. M. Azari, F. Rosas, K.-C. Chen, and S. Pollin, “Joint sum-rate and power gain analysis of an aerial base station,” in Proc. IEEE Global Commun. Conf. (GLOBECOM) Workshops, Dec. 2016, pp. 1–6.
  - E. Kalantari, H. Yanikomeroglu, and A. Yongacoglu, “On the number and 3D placement of drone base stations in wireless cellular networks,” in Proc. IEEE Veh. Technol. Conf., Sep. 2016, pp. 1–6.
- Optimal placement of UAVs to provide coverage for ground terminals:
  - J. Lyu, Y. Zeng, R. Zhang, and T. J. Lim, “Placement optimization of UAV-mounted mobile base stations,” IEEE Commun. Lett., vol. 21, no. 3, pp. 604–607, Mar. 2017

Relevant Background Literature - Cell (or user) association

- Area-to-UAV assignment for capacity enhancement
  - V. Sharma, M. Bennis, and R. Kumar, “UAV-assisted heterogeneous networks for capacity enhancement,” IEEE Commun. Lett., vol. 20, no. 6, pp. 1207–1210, Jun. 2016.
- Distance based association (Voronoi) and weighted Voronoi solutions:
  - A. Silva, H. Tembine, E. Altman, and M. Debbah, “Optimum and equilibrium in assignment problems with congestion: Mobile terminals association to base stations,” IEEE Trans. Autom. Control, vol. 58, no. 8, pp. 2018–2031, Aug. 2013.
  - H. Ghazzai. (Feb. 2015). Environment Aware Cellular Networks. [Online]. Available: http://repository.kaust.edu.sa/kaust/handle/10754/ 344436

Problem Statement

- Assume a distributions of users $f(x, y)$ in a geographical region $\mathcal{D} \subset \mathbb{R}^2$
- There are N users and M UAVs
- The positions of the UAVs are fixed and given as $\boldsymbol{s}_i = (x_i, y_i, h_i)$
- The goal is to assign partitions of the region for each UAV to provide wireless service maximizing the average service of each user
- The partition assigned to UAV i is $\mathcal{A}_i$

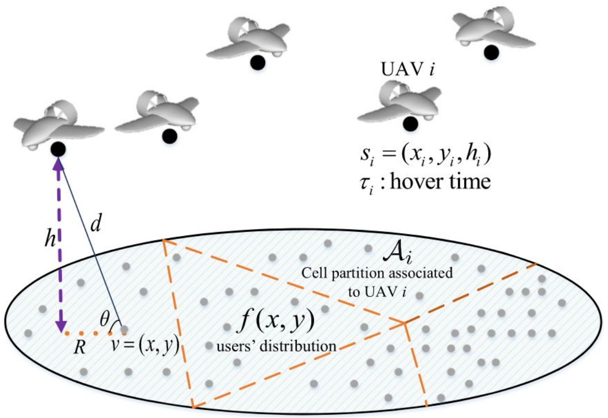


Fig. 1. System model.

Model

- We need to find the average data service at a location (x,y) and then maximize the average data service by correctly partitioning the region
- UAV parameters:
  - Bandwidth - B
  - Transmission power - P_i
  - Hover time - τ_i

Model

- Air-to-ground path loss model:
  - Probability of line-of-site connection depends on heights, locations and size of obstacles
  - In this paper they use the following model:

$$\Lambda_i(x, y) = \begin{cases} \left(\frac{4\pi f_c d_o}{c}\right)^2 (d_i(x, y)/d_o)^2 \mu_{\mathrm{LoS}}, & \text{LoS link,} \\ \left(\frac{4\pi f_c d_o}{c}\right)^2 (d_i(x, y)/d_o)^2 \mu_{\mathrm{NLoS}}, & \text{NLoS link,} \end{cases} \tag{1}$$

  - Carrier frequency - f_c
  - Free space reference distance - d_o (1 meter)
  - Distance between UAV i and location - $d_i(x, y)$

Model

- The probability of having a line-of-sight connection depends on the elevation angle between the user and UAV: $\theta_i = \sin^{-1}(\frac{h_i}{d_i(x,y)})$

$$P_{\mathrm{LoS},i} = b_1\left(\frac{180}{\pi}\theta_i - 15\right)^{b_2},$$

- Environmental impact factors - b_1 and b_2
- Probability of non-line-of-site connection $P_{\mathrm{NLoS},i} = 1 - P_{\mathrm{LoS},i}$

Model

- Using path loss model the average path loss from UAV i to location (x,y):

$$K_o {d_i}^2(x, y)[P_{\mathrm{LoS},i}\mu_{\mathrm{LoS}} + P_{\mathrm{NLoS},i}\mu_{\mathrm{NLoS}}]$$

$$K_o = \left(\frac{4\pi f_c}{c}\right)^2$$

- Average received power is:

$$\bar{P}_{r,i}(x, y) = \frac{P_i/\bar{N}_i}{K_o {d_i}^2(x, y)\left[P_{\mathrm{LoS},i}\mu_{\mathrm{LoS}} + P_{\mathrm{NLoS},i}\mu_{\mathrm{NLoS}}\right]},$$

- Where the average number of users in cell i is: $\bar{N}_i = N \int_{\mathcal{A}_i} f(x, y)\mathrm{d}x\mathrm{d}y$

Model

- Interference at location (x,y) from all UAVs but i:

$$I_i(x, y) = \beta \sum_{j \neq i} \bar{P}_{r,j}(x, y)$$

- Interference and noise will linearly decrease with the number of users in i
- Received SINR at location (x,y) is:

$$\gamma_i(x, y) = \frac{\bar{P}_{r,i}(x, y)}{I_i(x, y) + \sigma^2}$$

- Noise power - σ^2

Model

- Throughput of a user located at (x,y):

$$C_i(x, y) = W(x, y) \log_2 (1 + \gamma_i(x, y))$$

- Bandwidth allocated to user at location - $W(x, y)$
- Data service (number of bits transmitted) for user at (x,y):

$$L_i(x, y) = T_i C_i(x, y)$$

- Effective transmission time - T_i
-

Scenario 1 - Optimal cell partitioning for data service maximization under fairness constraints

- Goal: find partitions that maximizes the average data service to users on ground
- Hover time of UAV includes effective transmission time and control time

$$T_i = \tau_i - g_i\left(\int_{\mathcal{A}_i} f(x, y)\mathrm{d}x\mathrm{d}y\right)$$

- Control time is a function of the average number of users in the partition

Optimal cell partitioning for data service maximization under fairness constraints

- From previous equations $T_i B$ can be considered resources the UAV uses to service users
- The paper uses the following resource allocation scheme:

$$\frac{\alpha_i T_i B}{N \int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y} = \frac{\alpha_j T_j B}{N \int_{\mathcal{A}_j} f(x, y) \mathrm{d}x \mathrm{d}y}, \quad \forall i \neq j \in \mathcal{M}$$

- α_i is a resource allocation factor

Optimal cell partitioning for data service maximization under fairness constraints

- Using the fact that $\int_{\mathcal{D}} f(x, y)\mathrm{d}x\mathrm{d}y = \sum_{k=1}^{M} \int_{\mathcal{A}_k} f(x, y)\mathrm{d}x\mathrm{d}y = 1$

- The constraint on the number of users in each cell is:

$$\int_{\mathcal{A}_i} f(x, y)\mathrm{d}x\mathrm{d}y = \frac{\alpha_i T_i}{\sum_{k=1}^{M} \alpha_k T_k}$$

Optimal cell partitioning for data service maximization under fairness constraints

- Using the fact that $\int_{\mathcal{D}} f(x, y)\mathrm{d}x\mathrm{d}y = \sum_{k=1}^{M} \int_{\mathcal{A}_k} f(x, y)\mathrm{d}x\mathrm{d}y = 1$

- The constraint on the number of users in each cell is:

$$\int_{\mathcal{A}_i} f(x, y)\mathrm{d}x\mathrm{d}y = \frac{\alpha_i T_i}{\sum_{k=1}^{M} \alpha_k T_k} = \omega_i$$

Optimal cell partitioning for data service maximization under fairness constraints

- The average data service at location (x,y) is:

$$L_i(x,y) = \frac{T_i B}{N \int_{\mathcal{A}_i} f(x,y)\mathrm{d}x\mathrm{d}y} \log_2\left(1+\gamma_i(x,y)\right)$$
$$= \left(\frac{B}{N\alpha_i}\sum_{k=1}^{M} T_k\right) \log_2\left(1+\gamma_i(x,y)\right).$$

Optimal cell partitioning for data service maximization under fairness constraints

- The average data service at location (x,y) is:

$$
\begin{aligned}
L_i(x, y) &= \frac{T_i B}{N \int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y} \log_2 (1 + \gamma_i(x, y)) \\
&= \underbrace{\left(\frac{B}{N\alpha_i} \sum_{k=1}^{M} T_k \right)}_{\lambda_i} \log_2 (1 + \gamma_i(x, y)) .
\end{aligned}
$$

Optimal cell partitioning for data service maximization under fairness constraints

- Optimization problem: Maximize data service with the constraint that the number of users is ω_i

$$
\begin{aligned}
\min_{\mathcal{A}_i,\, i \in \mathcal{M}} \sum_{i=1}^{M} \int_{\mathcal{A}_i} & -\lambda_i \log_2 (1 + \gamma_i(x, y))\, f(x, y) \mathrm{d}x \mathrm{d}y \\
\text{s.t.} \quad & \int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y = \omega_i, \quad \forall i \in \mathcal{M}, \\
& \mathcal{A}_l \cap \mathcal{A}_m = \emptyset, \quad \forall l \neq m \in \mathcal{M}, \\
& \bigcup_{i \in \mathcal{M}} \mathcal{A}_i = \mathcal{D},
\end{aligned}
$$

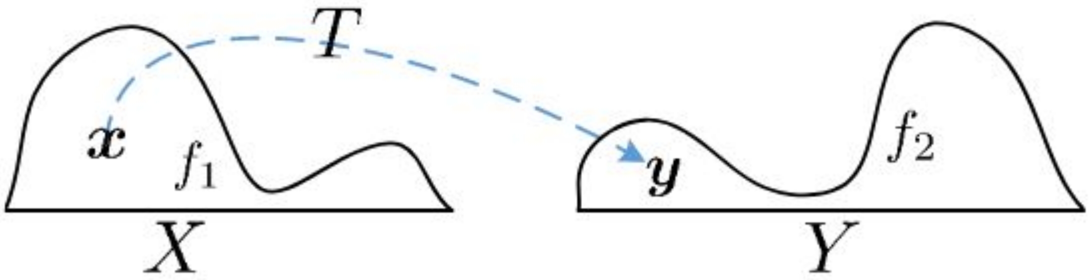
- This optimization problem is intractable
- Optimizing over arbitrarily shaped continuous areas

Optimal transport theory - Monge's problem

- Given two probability distributions, find the optimal transport map

$$\min_T \int_X c\left(\boldsymbol{x}, T(\boldsymbol{x})\right) f_1(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}; \quad T : X \to Y$$

- $c(\boldsymbol{x}, T(\boldsymbol{x}))$ Is the cost of transporting a unit mass from a location in X to a location in Y



Optimal transport theory - Monge-Kantorovich problem

- Monge's problem is difficult to solve because it is highly nonlinear and each point in the source distribution is mapped to a single location
- Kantorovich relaxed this problem by using a transport plans instead of maps where one point can go to multiple
- This is called the Monge-Kantorovich problem and is given as

$$\min_{\pi} \int_{X \times Y} c(\boldsymbol{x}, \boldsymbol{y}) \mathrm{d}\pi(\boldsymbol{x}, \boldsymbol{y}), \tag{16}$$

$$\text{s.t.} \int_{X} \mathrm{d}\pi(\boldsymbol{x}, \boldsymbol{y}) = f_1(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \quad \int_{Y} \mathrm{d}\pi(\boldsymbol{x}, \boldsymbol{y}) = f_2(\boldsymbol{y})\mathrm{d}\boldsymbol{y}, \tag{17}$$

- Where π is the transport plan which is a probability on $X \times Y$ whose marginals are f1 and f2

Optimal transport theory - Monge-Kantorovich problem

- *Kantorovich Duality Theorem:* The Monge-Kantorovich problem with two probability measures f_1 on $X \subset \mathbb{R}^n$ and f_2 on $Y \subset \mathbb{R}^n$ and a lower semicontinuous cost function the following equality holds:

$$\min_{\pi} \int_{X \times Y} c(\boldsymbol{x}, \boldsymbol{y}) \mathrm{d}\pi(\boldsymbol{x}, \boldsymbol{y}) = \max_{\varphi, \psi} \left\{ \int_X \varphi(\boldsymbol{x}) f_1(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} + \int_Y \psi(\boldsymbol{y}) f_2(\boldsymbol{y}) \mathrm{d}\boldsymbol{y}; \right. \left. \varphi(\boldsymbol{x}) + \psi(\boldsymbol{y}) \leq c(\boldsymbol{x}, \boldsymbol{y}), \ \forall (\boldsymbol{x}, \boldsymbol{y}) \in X \times Y \right\},$$

- Where $\varphi(\boldsymbol{x})$ and $\psi(\boldsymbol{y})$ are the Kantorovich potential functions

Optimal transport theory - Monge-Kantorovich problem

- When the source distribution and the cost function are continuous Monge's problem and the Monge-Kantorovich problem are equivalent and the transport map is given from the optimal Kantorovich potential functions:

$$T(\boldsymbol{x}) = \left\{ \boldsymbol{y} | \varphi^*(\boldsymbol{x}) + \psi^*(\boldsymbol{y}) = c(\boldsymbol{x}, \boldsymbol{y}) \right\}$$

How does this apply?

- The transport map goes from the continuous distribution of users to the discrete distribution of UAVs:

$$\left\{ T(\boldsymbol{v}) = \sum_{i \in \mathcal{M}} \boldsymbol{s}_i \mathbb{1}_{\mathcal{A}_i}(\boldsymbol{v}); \int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y = \omega_i \right\}$$

- Find the optimal transport map from the continuous distribution of user to the discrete distribution of UAVs using the cost function from the previous optimization problem

$$\int_{\mathcal{D}} J\left(\boldsymbol{v}, T(\boldsymbol{v})\right) f(x, y) \mathrm{d}x \mathrm{d}y$$

$$J(\boldsymbol{v}, \boldsymbol{s}_i) = J(x, y, \boldsymbol{s}_i) = -\lambda_i \log_2 \left(1 + \gamma_i(x, y)\right)$$

How does this apply?

- *Theorem 1:* The optimization problem in (11) is equivalent to:

$$\max_{\psi_i, i \in \mathcal{M}} \left\{ F(\boldsymbol{\psi}^T) = \sum_{i=1}^{M} \psi_i \omega_i + \int_{\mathcal{D}} \psi^c(x, y) f(x, y) \mathrm{d}x \mathrm{d}y \right\}, \tag{22}$$

- Where ψ_i are the optimization variables and $\psi^c(x, y) = \inf_i J(x, y, s_i) - \psi_i$
-

How does this apply?

- *Proof:* The source distribution is $f(x, y)$ and the destination is $\Gamma = \sum_{i \in \mathcal{M}} \omega_i \delta_{s_i}$

$$\min_T \int_{\mathcal{D}} J(\boldsymbol{v}, T(\boldsymbol{v})) f(x, y) \mathrm{d}x\mathrm{d}y$$
$$= \max_{\varphi, \psi} \left\{ \int_{\mathcal{D}} \varphi(\boldsymbol{v}) f(x, y) \mathrm{d}x\mathrm{d}y + \int_S \psi(s) \sum_{i \in \mathcal{M}} \omega_i \delta_{s - \boldsymbol{s}_i} \mathrm{d}s; \right.$$
$$\left. \varphi(\boldsymbol{v}) + \psi(\boldsymbol{s}) \leq J(\boldsymbol{v}, \boldsymbol{s}) \right\}$$
$$= \max_{\varphi, \psi} \left\{ \int_{\mathcal{D}} \varphi(x, y) f(x, y) \mathrm{d}x\mathrm{d}y + \sum_{i=1}^{M} \psi(\boldsymbol{s}_i) \omega_i; \right.$$
$$\left. \varphi(x, y) + \psi(\boldsymbol{s}_i) \leq J(x, y, s_i), \ \forall i \in \mathcal{M} \right\}. \qquad (23)$$

How does this apply?

- *Proof cont.:*
- To maximize the previous equation given any ψ the maximum value of φ needs to be chosen
- Given the inequality constraint this means:

$$\varphi(x, y) = \psi^c(x, y) = \inf_{\mathbf{s}_i} J(x, y, \mathbf{s}_i) - \psi(\mathbf{s}_i)$$

$$\min_T \int_{\mathcal{D}} J(\mathbf{v}, T(\mathbf{v})) f(x, y) \mathrm{d}x \mathrm{d}y$$

$$= \max_{\psi_i, i \in \mathcal{M}} \left\{ F(\boldsymbol{\psi}^T) = \sum_{i=1}^{M} \psi_i \omega_i + \int_{\mathcal{D}} \psi^c(x, y) f(x, y) \mathrm{d}x \mathrm{d}y \right\}$$

$$\psi^c(x, y) = \inf_i J(x, y, \mathbf{s}_i) - \psi_i .$$

How does this apply?

- *Proof cont.:*
- To maximize the previous equation given any ψ the maximum value of φ needs to be chosen
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$$= \max_{\psi_i, i \in \mathcal{M}} \left\{ F(\boldsymbol{\psi}^T) = \sum_{i=1}^{M} \psi_i \omega_i + \int_{\mathcal{D}} \psi^c(x, y) f(x, y) \mathrm{d}x \mathrm{d}y \right\}$$

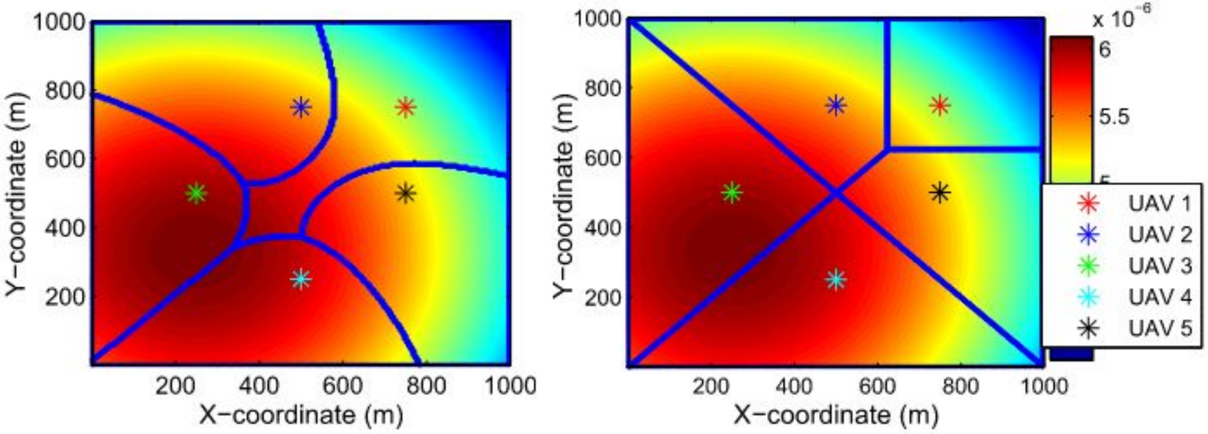
$$\psi^c(x, y) = \inf_i J(x, y, \mathbf{s}_i) - \psi_i .$$

How does this apply?

- The intractable optimization problem is now optimizing over variables
- The gradient of the optimization problem can also be found
- Using gradient descent, the optimal areas can be found using this optimization problem

Results - Scenario 1

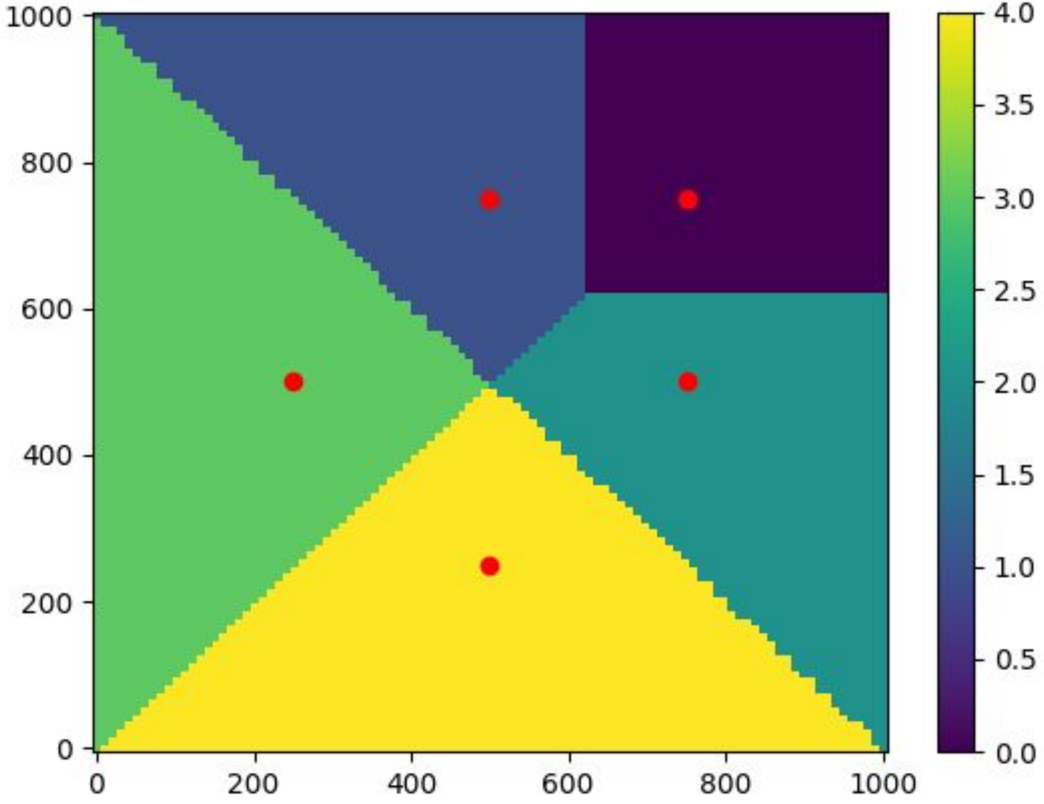
- The distribution of users is assumed to be a truncated gaussian.
- All UAVs have same bandwidth, hover time, and allocation factor so each UAV should be assigned the same number of users



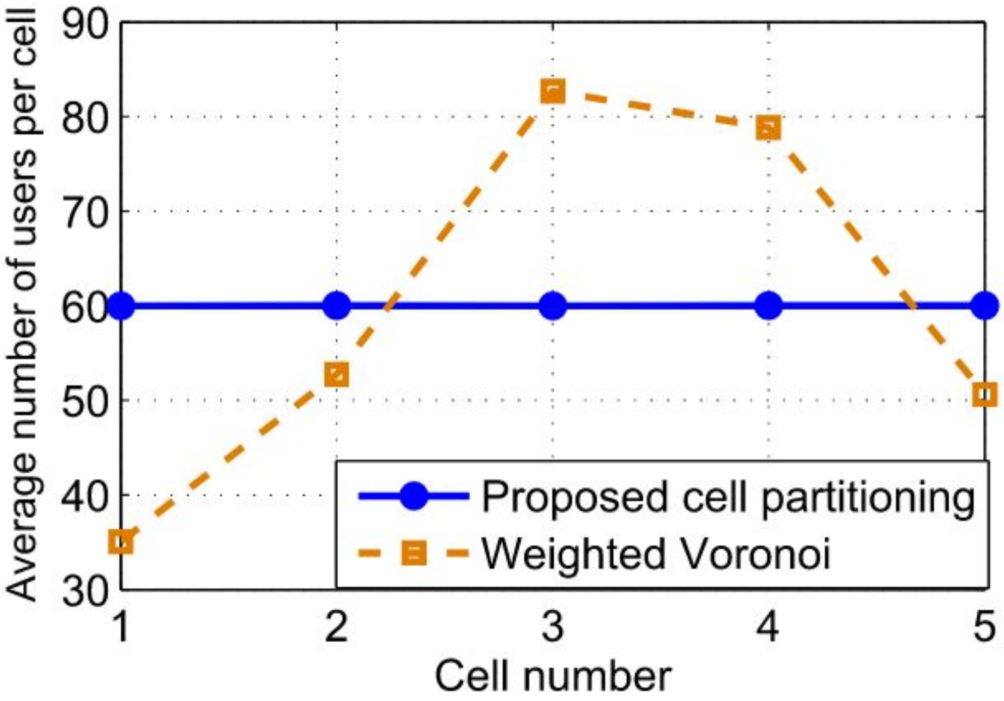
(a) Proposed optimal cell partitions.

(b) Weighted Voronoi diagram.

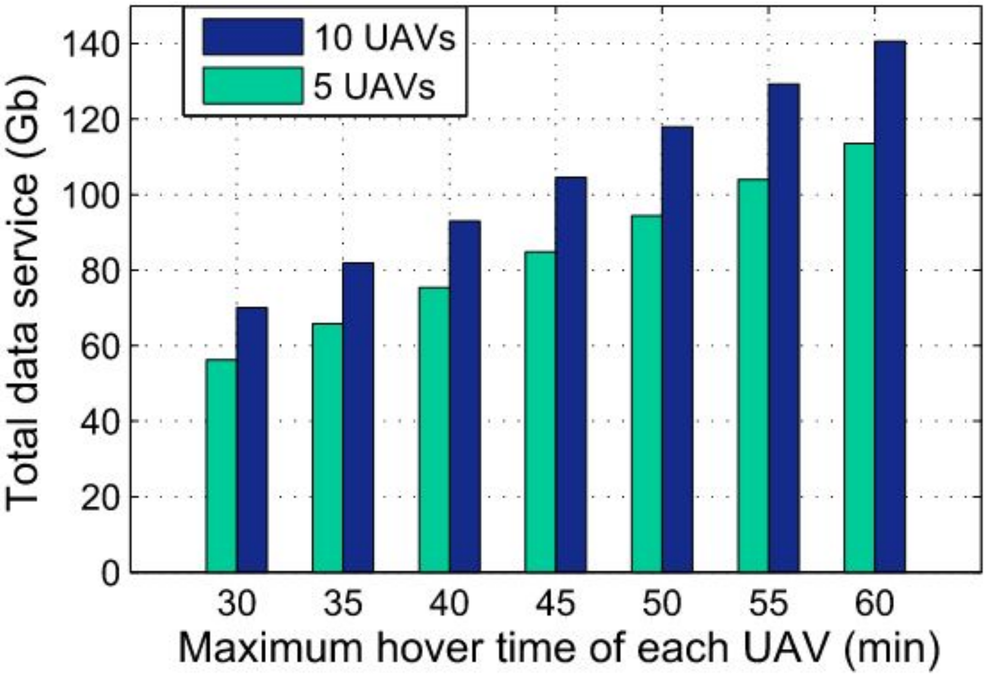
Results - Scenario 1



Results - Scenario 1



Results - Scenario 1



Scenario 2 - Minimum hover time for meeting load requirements

- Goal: meet load requirements of ground users while minimizing the average hover times of the UAVs
- Minimum average hover time of UAV i for serving the partition is given by:

$$\tau_i = \int_{\mathcal{A}_i} \frac{Nu}{C_i^B(x, y)} f(x, y) \mathrm{d}x \mathrm{d}y + g_i \left(\int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y \right), \tag{38}$$

- Load (in bits) of user at (x,y) - u
- Data service of user at (x,y) - $C_i^B = B \log_2(1 + \gamma_i(x, y))$

Minimum hover time for meeting load requirements

- *Proof:* Let $\mathcal{Y}$ be the set of users in a cell
- Time needed to service user r:

$$t_r = \frac{u_r}{W_r E_r}$$

- Bandwidth allocated to user r - W_r
- Spectral efficiency (bits/s/Hz) at user location - E_r
- Load (in bits) - u_r
- To serve all users in the cell, the hover time of the UAV must be $\max t_r + g_{\mathcal{Y}}$ where $g_{\mathcal{Y}}$ is the additional control time

Minimum hover time for meeting load requirements

- The hover time can be minimized with the following allocation scheme:

$$\min_{W_r,\, r=1,\dots,Y} \max \; t_r + g_{\mathcal{Y}},$$
$$\text{s.t.} \quad \sum_{r=1}^{Y} W_r = B.$$

- This can be transformed to:

$$\min Z + g_{\mathcal{Y}},$$
$$\text{s.t.} \quad Z \geq \frac{u_r}{W_r E_r}, \quad \forall r \in \mathcal{Y},$$
$$\sum_{r=1}^{Y} W_r = B.$$

Minimum hover time for meeting load requirements

- From previous we know:

$$Z \geq \frac{1}{B} \sum_{r=1}^{Y} \frac{u_r}{E_r}$$

- The minimum hover time under optimal bandwidth allocation:

$$\sum_{r=1}^{Y} \frac{u_r}{BE_r} + g\gamma$$

- Optimal bandwidth allocation:

$$W_r = \frac{Bu_r}{E_r} / \sum_{k=1}^{Y} \frac{u_k}{E_k}$$

- Total time needed for sequentially servicing all users in partition

$$\sum_{r=1}^{Y} \frac{u_r}{BE_r}$$

Minimum hover time for meeting load requirements

- Now that we know the average hover time needed we want to minimize that:

$$
\begin{aligned}
\min_{\mathcal{A}_i,\, i \in \mathcal{M}} \sum_{i=1}^{M} \Bigg[\int_{\mathcal{A}_i} \frac{Nu}{C_i^B(x, y)} f(x, y) \mathrm{d}x \mathrm{d}y \\
+ g_i \left(\int_{\mathcal{A}_i} f(x, y) \mathrm{d}x \mathrm{d}y \right) \Bigg] \\
\text{s.t. } \mathcal{A}_l \cap \mathcal{A}_m = \emptyset, \quad \forall l \neq m \in \mathcal{M}, \\
\bigcup_{i \in \mathcal{M}} \mathcal{A}_i = \mathcal{D},
\end{aligned}
$$

- Impossible to solve for same reasons as first scenario

Minimum hover time for meeting load requirements

- Use optimal transport theory - transport the distribution of users to distribution of UAVs while minimizing the data transport time (hover time)

$$\min_{T} \int_{\mathcal{D}} c(\boldsymbol{v}, \boldsymbol{s}) f(\boldsymbol{v}) \mathrm{d}\boldsymbol{v}, \quad \boldsymbol{s} = T(\boldsymbol{v})$$

$$c(\boldsymbol{v}, \boldsymbol{s}_i) = \frac{u}{C_i^B(x, y)} \qquad f(\boldsymbol{v}) = f(x, y)$$

$$\left\{ T(\boldsymbol{v}) = \sum_{i=1}^{M} \boldsymbol{s}_i \mathbb{1}_{\mathcal{A}_i}(\boldsymbol{v}); \int_{\mathcal{A}_i} f(\boldsymbol{v}) \mathrm{d}\boldsymbol{v} = a_i \right\}$$

Minimum hover time for meeting load requirements

- Using similar techniques to scenerio 1 the minmized hover time is given by:

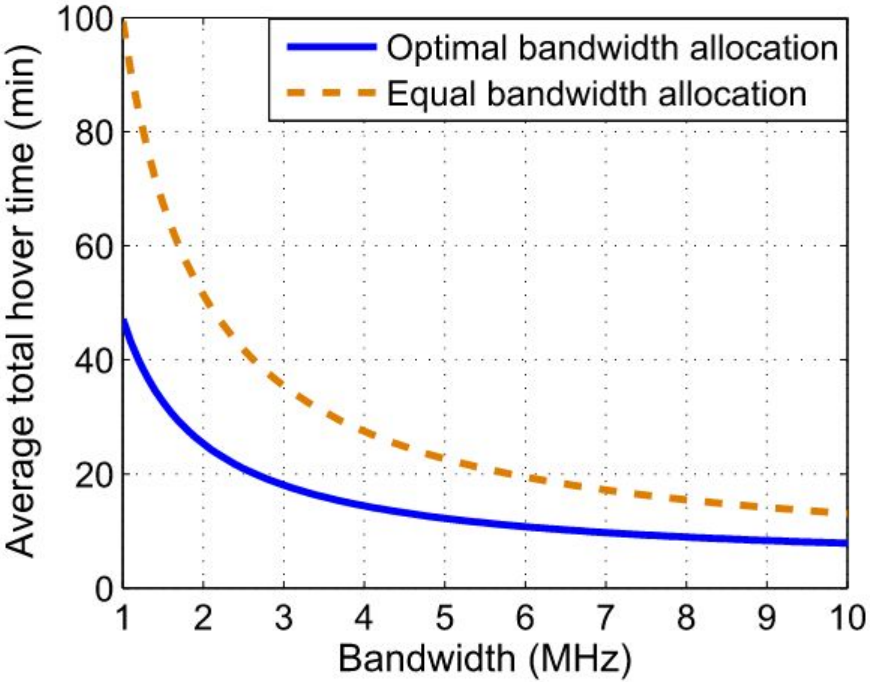
$$\tau_i^* = \int_{\mathcal{A}_i^*} \frac{Nu}{C_i^B(x,y)} f(x,y)\mathrm{d}x\mathrm{d}y + g_i\left(\int_{\mathcal{A}_i^*} f(x,y)\mathrm{d}x\mathrm{d}y\right), \tag{52}$$

$$\mathcal{A}_i^* = \left\{(x,y) \mid \frac{Nu}{C_i^B(x,y)} + g_i'(a_i) \leq \frac{Nu}{C_j^B(x,y)} + g_j'(a_j), \right.$$
$$\left. \forall j \neq i \in \mathcal{M}\right\}, \tag{53}$$

$$a_i = \int_{\mathcal{A}_i} f(x,y)\mathrm{d}x\mathrm{d}y$$

Results - Scenario 2

- 10 Mb data service requirement for each user



Future Work

- This paper assumes static UAV position, other papers have considered moving UAVs, but it would be interesting to use similar ideas as this paper with mobile UAVs
- Seeing how time varying distributions of users would effect the resource allocations