Summary of: Distributed multi-robot formation control in dynamic environments

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Outline

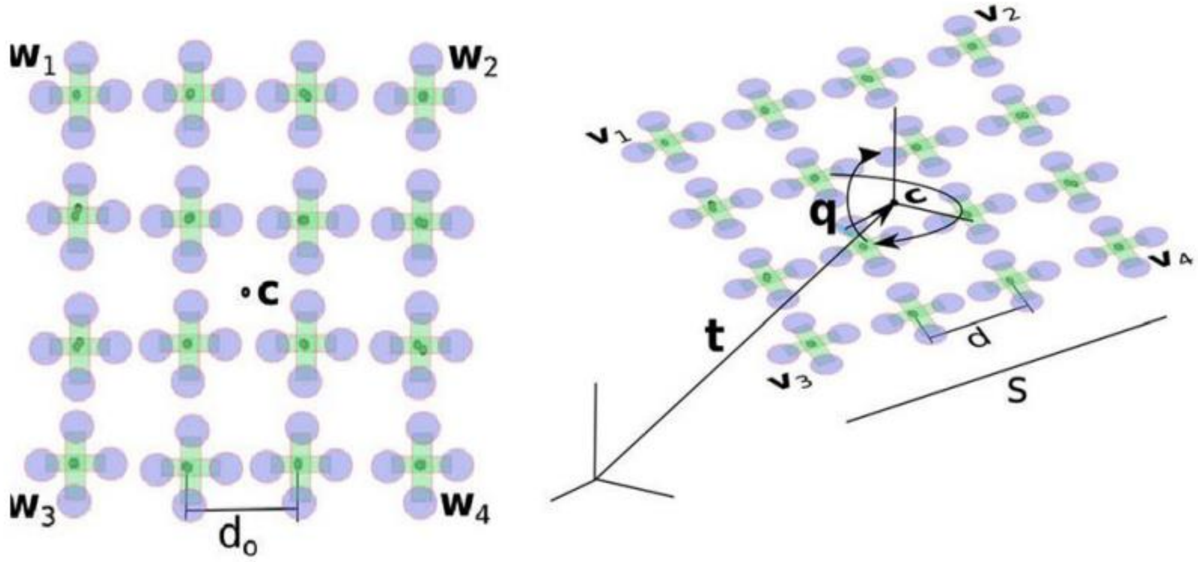
- Math review
- Problem formulation
- Prior work
- Methods and theoretical guarantees
- Simulation and hardware results
- Summary
- Future work

Math Review

- A compact set is closed and bounded
- Convex set S $:= \ \forall a, b \in S, \mathrm{line}_{[a,b]} \subset S$
- Convex function $f(\boldsymbol{x}) := \forall \boldsymbol{x_1}, \boldsymbol{x_2} \in D(X), \mathrm{line}_{[f(\boldsymbol{x_1}), f(\boldsymbol{x_2})]} \geq f(x)$
- N-polytope is an N dimensional closed shape formed by N-1 dimensional hyperplanes.

Problem Formulation

- Consider a team of *n* robots each with limited field of view and strongly connected communication graph $\mathcal{G}$.
- Robots prefer to fly in formation f, with *m* template formations.
- $\boldsymbol{v}_j^f = \boldsymbol{t} + sR(\boldsymbol{q})\boldsymbol{w}_j^f$
- $\boldsymbol{z} = [\boldsymbol{t}, s, \boldsymbol{q}] \in \mathbb{R}^3 \times \mathbb{R}_+ \times SO(3)$
- $\mathcal{V}(\boldsymbol{z}, f) = [\boldsymbol{v}_1^f \ \dots \ \boldsymbol{v}_n^f]$



Problem Formulation

- Problem 1: From a current time t_0, obtain a goal configuration $\boldsymbol{z}^*$ and formation index f^* for $t_0 + \tau$ s.t. deviation from desired configuration is minimized and $\mathcal{V}(\boldsymbol{z}^*, f^*) \times (t_0 + \tau) \subset \bigcup_{i \in \mathcal{I}} \mathcal{F}_i(t_0)$

- Problem 2: Given positions $\boldsymbol{p}_i$ at t_0 of all robots, ensure transition to new configuration at $t_0 + \tau$ is collision free, i.e.

  $\boldsymbol{p}_i(t) \times [t_0, t_0 + \tau] \subset \bigcup_{i \in \mathcal{I}} \mathcal{F}_i(t_0)$

Prior work

- Planar multi-robot navigation with obstacles (Ayanian and Kumar 2010 a,b)
- Distributed consensus algorithms (Ren and Beard 2008)
- Formation control (Oh et al. 2015) with holonomic constraints (Dong et al. 2015)
- Decentralized consensus with local sensing of obstacles (Mosteo 2008)
- Sequential convex optimization (Derenick et al. 2010)
- Centralized consensus and optimization of conflict free trajectories in formation with assigned formation positions (Alonso-Mora et al. 2017)

Method - Overview

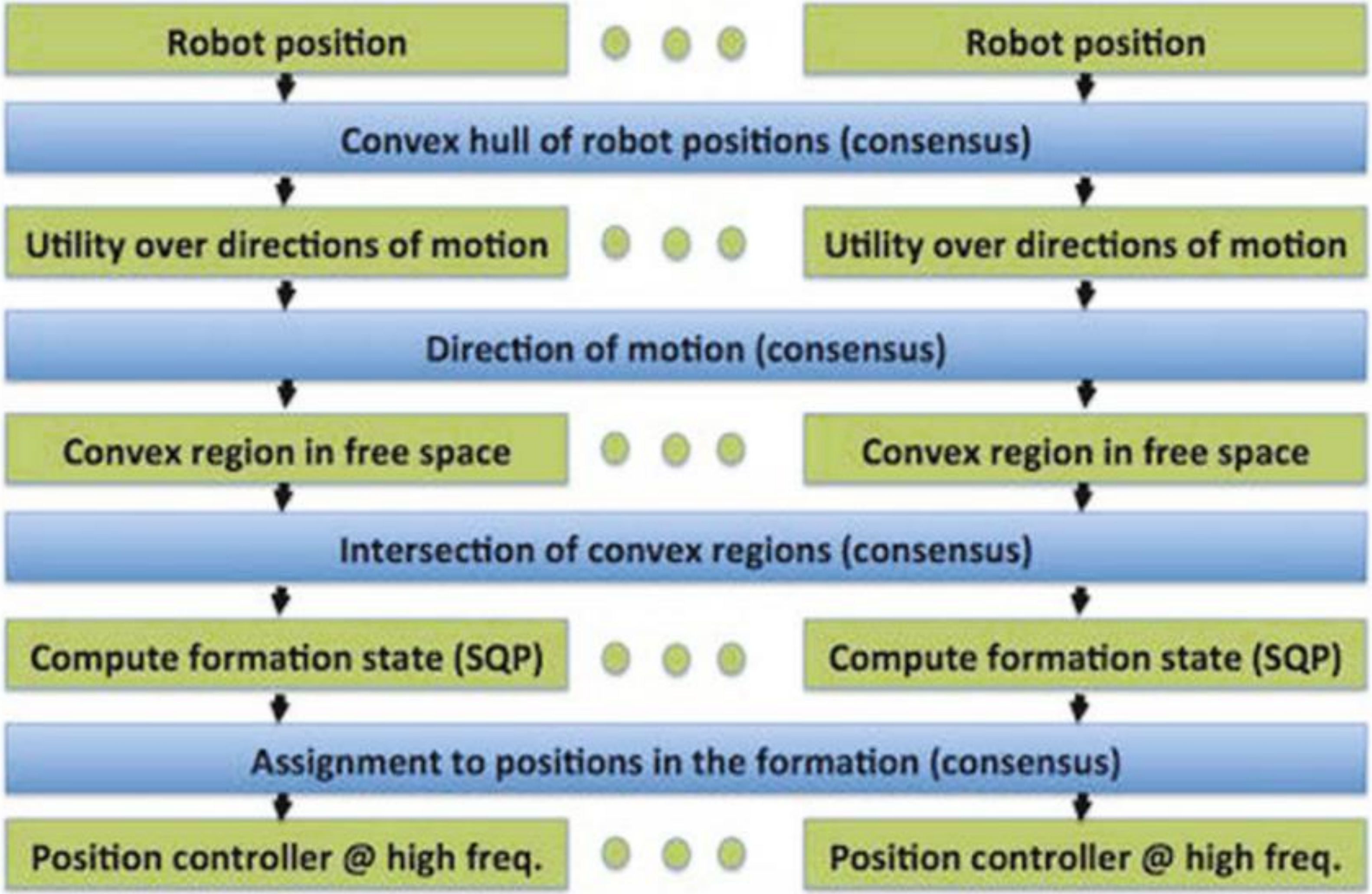


Figure 4, Alonso-Mora et al. 2018

Method – Distributed Convex Hull of Positions

Each robot independently calculates convex hull of formations:

1. $\mathcal{C}_i(0) = \{\boldsymbol{p}_i\}$
2. $for\ k = 0{:}\,d - 1$
3. Send $\mathcal{C}_i(k) \backslash \mathcal{C}_i(k-1)$ to all neighbors $j \in \mathcal{N}_i$
4. Receive $\mathcal{C}_j(k) \backslash \mathcal{C}_j(k-1)$ from all neighbors $j \in \mathcal{N}_i$
5. $\mathcal{C}_i(k+1) = \mathrm{ConvHull}(\mathcal{C}_i(k), \mathcal{C}_j(k) \backslash \mathcal{C}_j(k-1))$

Method – Distributed Preferred Direction of Motion

Given collection of headings $\Theta = \{\theta_1, \dots, \theta_k\}$ and individual robot utility function $u_i \colon \Theta \to \mathbb{R}^+$

1. $\boldsymbol{u}_i(0) = [u_i(\theta_1), \dots, u_i(\theta_k)]$
2. $for\ k = 0\colon d - 1$
3. $\quad$ Send $\boldsymbol{u}_i(k)$ to all neighbors $j \in \mathcal{N}_i$
4. $\quad$ Receive $\boldsymbol{u}_j(k)$from all neighbors $j \in \mathcal{N}_i$
5. $\quad \boldsymbol{u}_i(k+1) = \min_{j \in \mathcal{N}_i}(\boldsymbol{u}_i(k), \boldsymbol{u}_j(k))$
6. $\theta^* = \arg\max \boldsymbol{u}_i(d)$

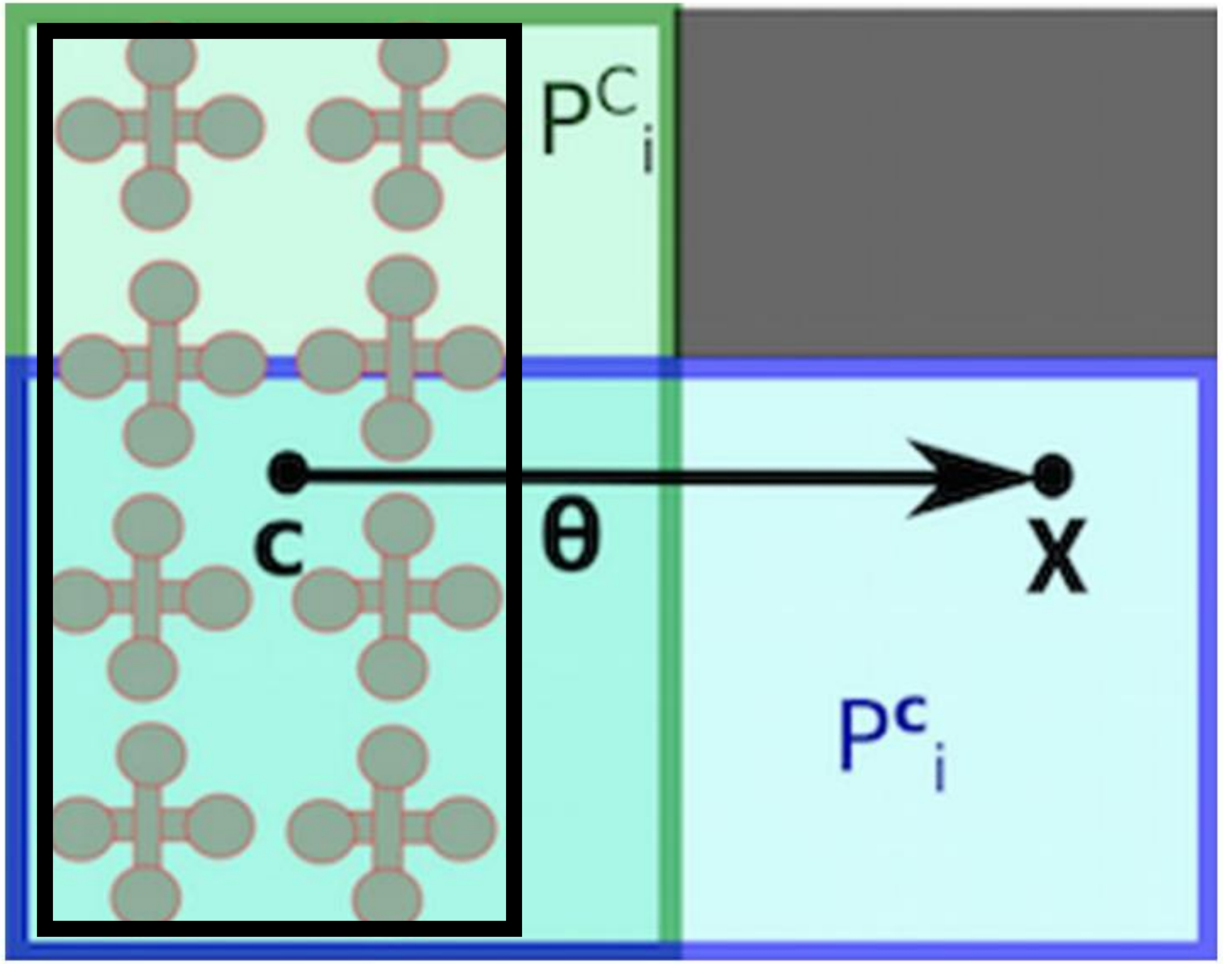
Method – Distributed Consensus on Convex Region in Free Space

For $\mathcal{X} = \boldsymbol{c} + \theta^* \tau$:

$\mathcal{P}_i^{\mathcal{C}} = \mathcal{P}_{\mathcal{C}\times 0}^{\mathcal{X}\times\tau}(\mathcal{F}_i(t_0))$ verifies robots can move to target configuration.

$\mathcal{P}_i^{\boldsymbol{c}} = \mathcal{P}_{\boldsymbol{c}\times 0}^{\mathcal{X}\times\tau}(\mathcal{F}_i(t_0))$ verifies robots can make progress towards goal.

$\mathcal{P}_i \;\; = \mathcal{P}_i^{\mathcal{C}} \cap \mathcal{P}_i^{\boldsymbol{c}}$ guarantees both conditions.



Method – Distributed Consensus on Convex Region in Free Space

1. $\mathcal{P}_i \ (0) = \mathcal{P}_i$
2. $for\ k = 0{:}\, d - 1$
3. Send $\mathcal{P}_i$ (k) to all neighbors $j \in \mathcal{N}_i$
4. Receive $\mathcal{P}_j \ (k)$from all neighbors $j \in \mathcal{N}_i$
5. $\mathcal{P}_i \ (k + 1) = \mathcal{P}_i \ \bigcap_{j \in \mathcal{N}_i} \mathcal{P}_j$

Method – Distributed Computation of Formation State

Each robot runs *m* optimizations to determine optimal-feasible formation state

$$\boldsymbol{z}_f^* = \arg\min_{\boldsymbol{z}} w_t \|\boldsymbol{t} - \boldsymbol{g}(t_1)\|^2 + w_s \|s - \bar{s}\|^2 + w_q \|\boldsymbol{q} - \overline{\boldsymbol{q}}\|^2 + c_f$$
$$s.t. \qquad \mathcal{V}(\boldsymbol{z}, f) \times t_1 \subset \mathcal{P}$$
$$s \geq 2 \frac{\max(r, h)}{d_f}$$

Not covered in paper. This problem formulation is especially well suited for sequential convex programming such as Scvx or SNOPT

Method – Distributed Position Assignment in Formation

If the formation configuration changes, individual robots must be assigned a position in the new formation:

Find the permutation matrix $\boldsymbol{\mathcal{X}}: \mathcal{I} \to \mathcal{I}$ such that

$$\min_{\boldsymbol{\mathcal{X}}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} x_{ij} \left\| \boldsymbol{p}_i - \boldsymbol{r}_j^* \right\|_2^2 .$$

Not covered in Paper. This is a distributed bounded linear simplex optimization problem with degeneracy. Suggested algorithm in (Burger et. al 2012).

Method – Position Control

Exact method not covered in paper:

- Authors use a high frequency position controller
- Collision avoidance with other robots handled by distributed dynamic-constrained velocity obstacle methods (Alonso-Mora et al. 2015b)

Simulation and Hardware Results

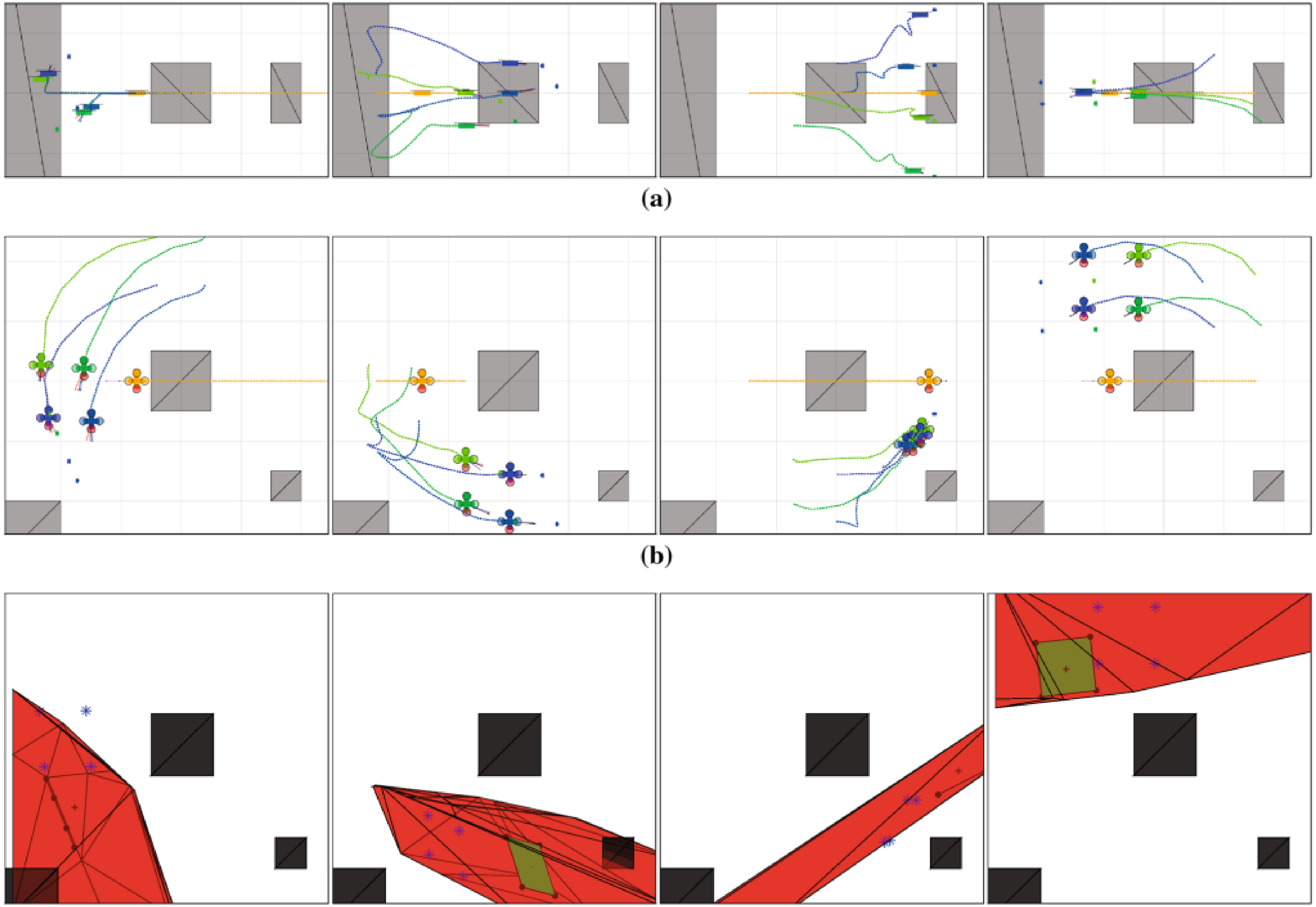
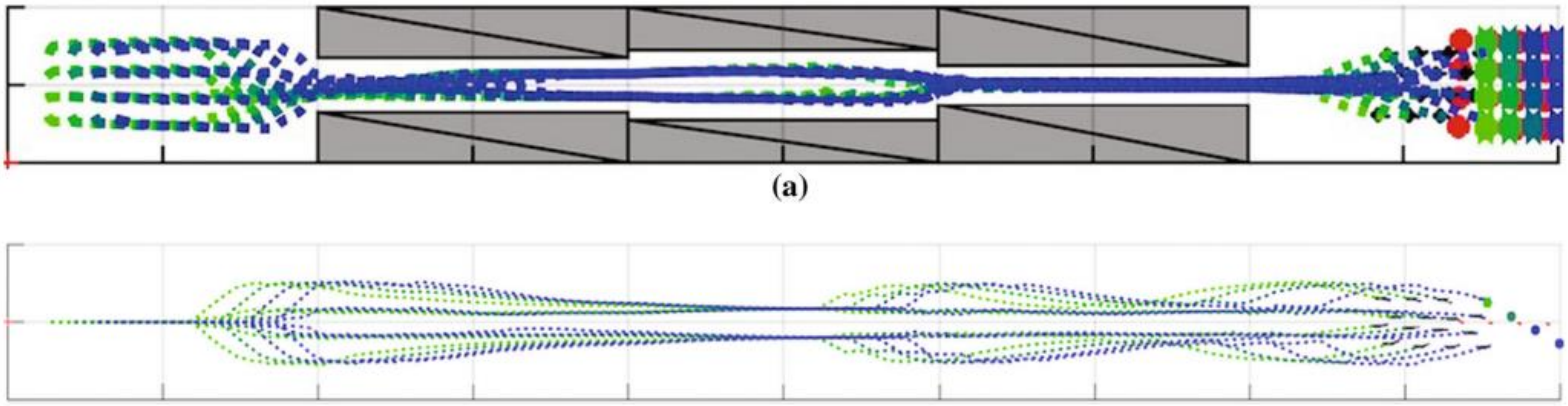


Figure 7, Alonso-Mora et al. 2018

Simulation and Hardware Results



(a)

(b)

Figure 8, Alonso-Mora et al. 2018

Simulation and Hardware Results

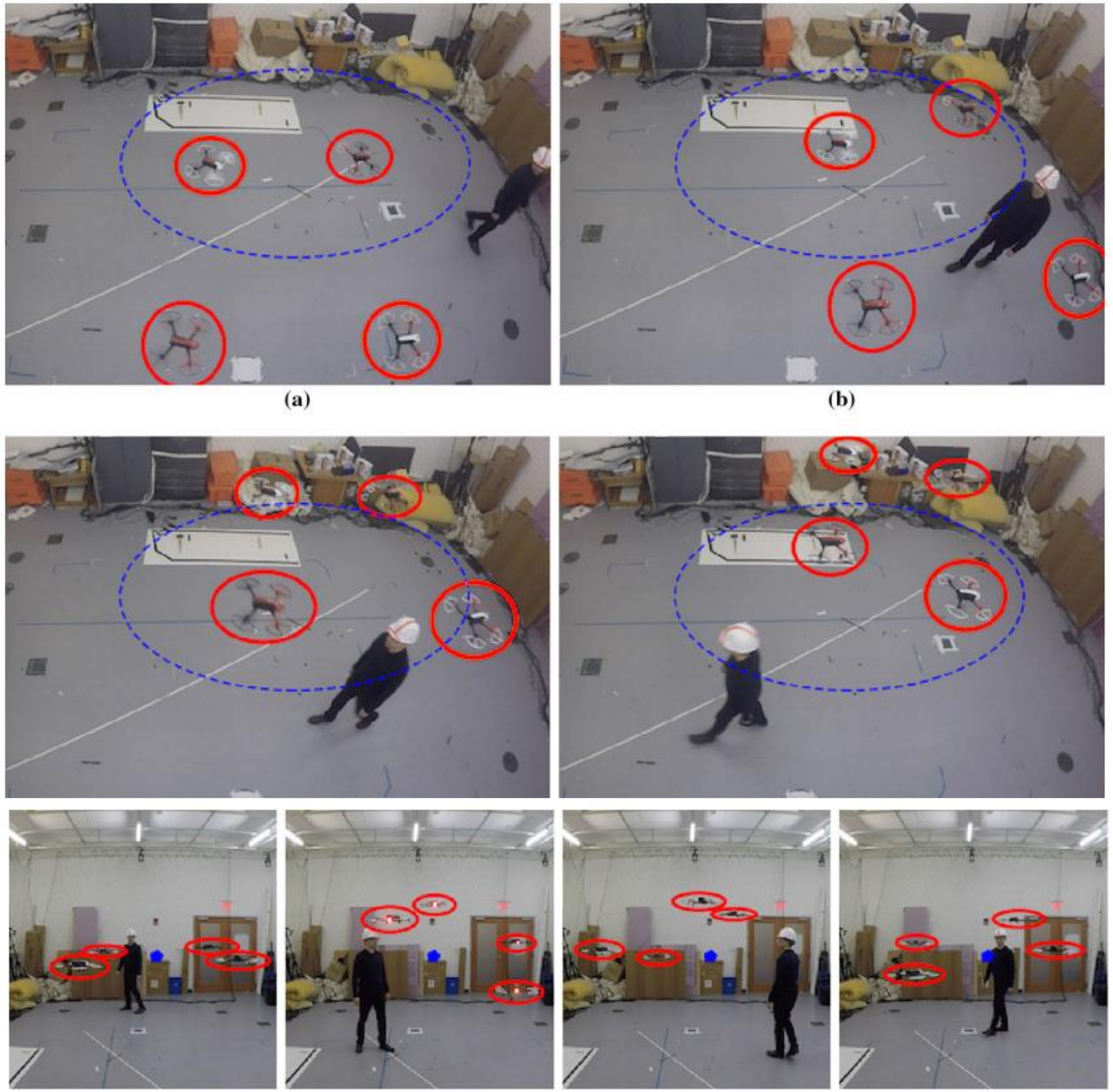
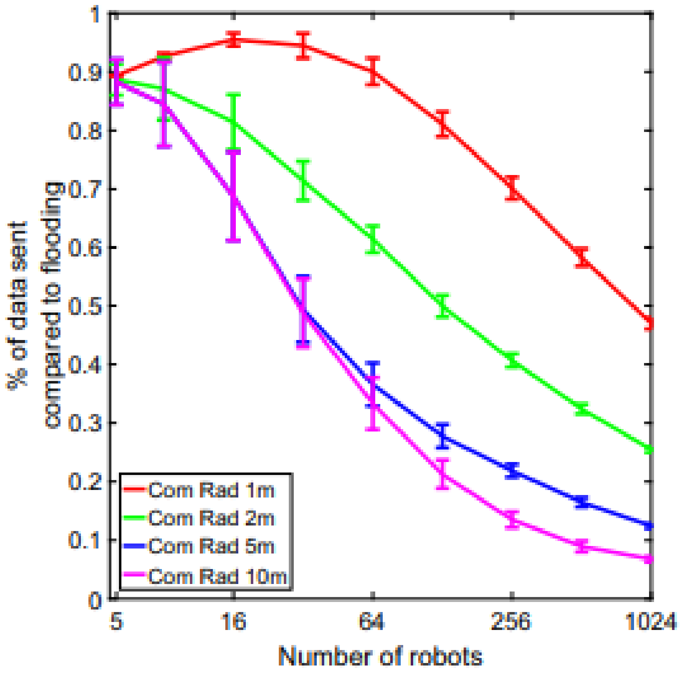
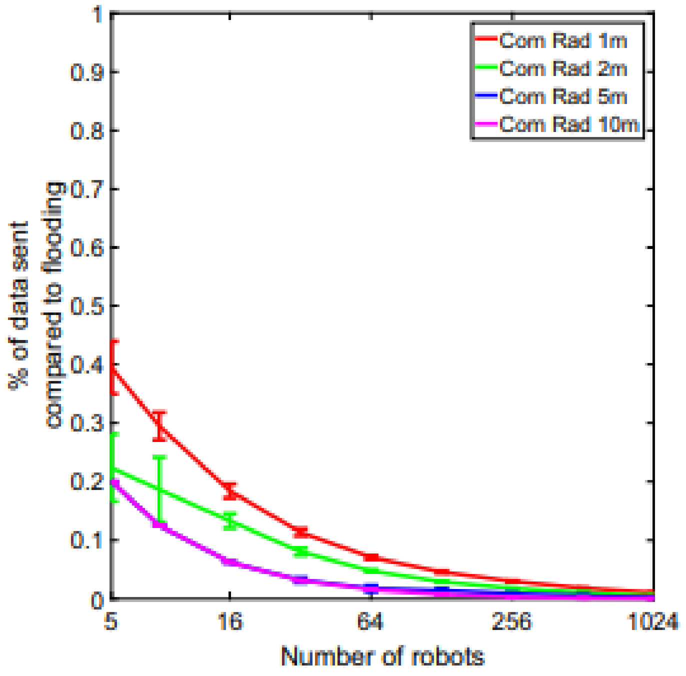


Figure 12-13, Alonso-Mora et al. 2018

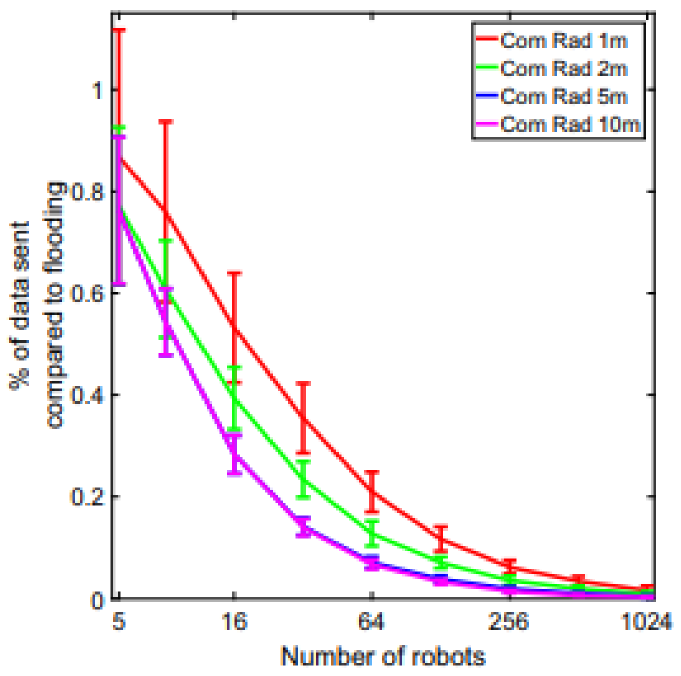
Simulation and Hardware Results



Distributed Convex Hull



Distributed Direction of Motion



Distributed Obstacle Free Region

Figure 6, Alonso-Mora et al. 2018

Summary

- Algorithm of multiple distributed consensus algorithms to ensure conflict free movement between formations.

Limitations Towards Future Work

- No guarantees on collision free movement.
- Locality of search space implies deadlocks may occur.
- Movement between formations is not optimal.
- Unable to handle rapid changes in dynamic obstacles.
- Obstacles require convex polytope approximation.