Summary of: A potential game approach to multiple UAV cooperative search and surveillance

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Outline

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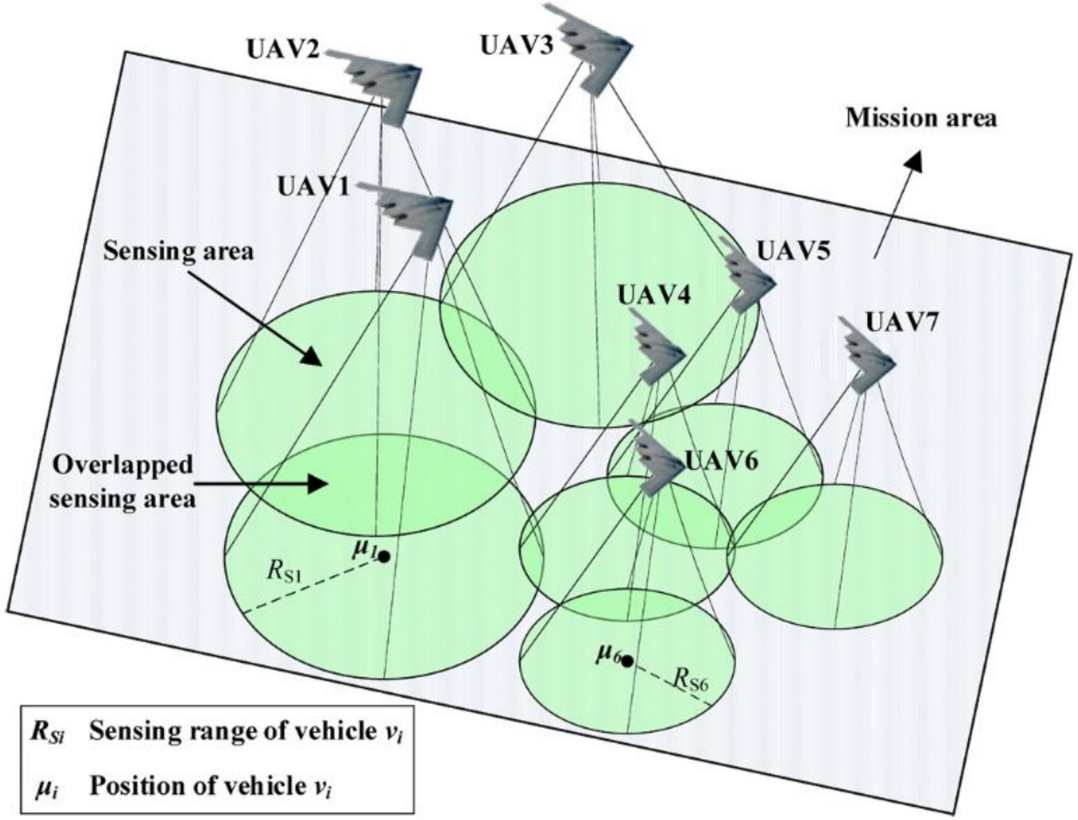
2. Relevant Literature
3. Problems that this paper is addressing
4. Problem formulation
5. Search & Surveillance Task
    a. Coordinate Motion
        i. Results
    b. Observations & Sensor Fusion
        i. Results
6. Summary & Future work



Motivation: Search & Surveillance

- Search & surveillance missions involve measuring and exploring an unknown region (e.g. target detection, environment monitoring, and map building)
- Using multiple vehicles have shown to perform such mission with greater efficiency
- Key problems that still need to be addressed using multiple vehicles
  - Individual agents must move and sense autonomously with limited communication and sensing capabilities
  - Network of agents needs to be robust to unexpected situations (i.e. dropout)
  - Differing global objectives and underlying constraints

Relevant Background Literature

- Assume homogeneous UAVs, added efforts to try to handle obstacle discontinuities
  - Provably convergent Kalman Filter: F. Zhang, N.E. Leonard, Cooperative filters and control for cooperative exploration, IEEE Trans. Autom. Control 55 (2010) 650–663.
  - Path Planning with Voronoi partitioning and consensus-based fusion: J. Hu, L. Xie, K.-Y. Lum, J. Xu, Multiagent information fusion and cooperative control in target search, IEEE Trans. Control Syst. Technol. 21 (2013) 1223–1235.
  - Cooperative search with various communication structures: P. Sujit, D. Ghose, Self assessment-based decision making for multiagent cooperative search, IEEE Trans. Autom. Sci. Eng. 8 (2011) 705–719.
- Potential game in cooperative control
  - J.R. Marden, G. Arslan, J.S. Shamma, Cooperative control and potential games, IEEE Trans. Syst. Man Cybern., Part B, Cybern. 39 (2009) 1393–1407
  - L.M. De Campos, J.M. Fernandez-Luna, J.A. Gámez, J.M. Puerta, Ant colony optimization for learning Bayesian networks, Int. J. Approx. Reason. 31 (2002) 291–311.
- Improved Potential Game with Multi-UAVs
  - Ni, J., Tang, G., Mo, Z., Cao, W., & Yang, S. X. (2020). An Improved Potential Game Theory Based Method for Multi-UAV Cooperative Search. IEEE Access, 8, 47787–47796.

Problems being addressed

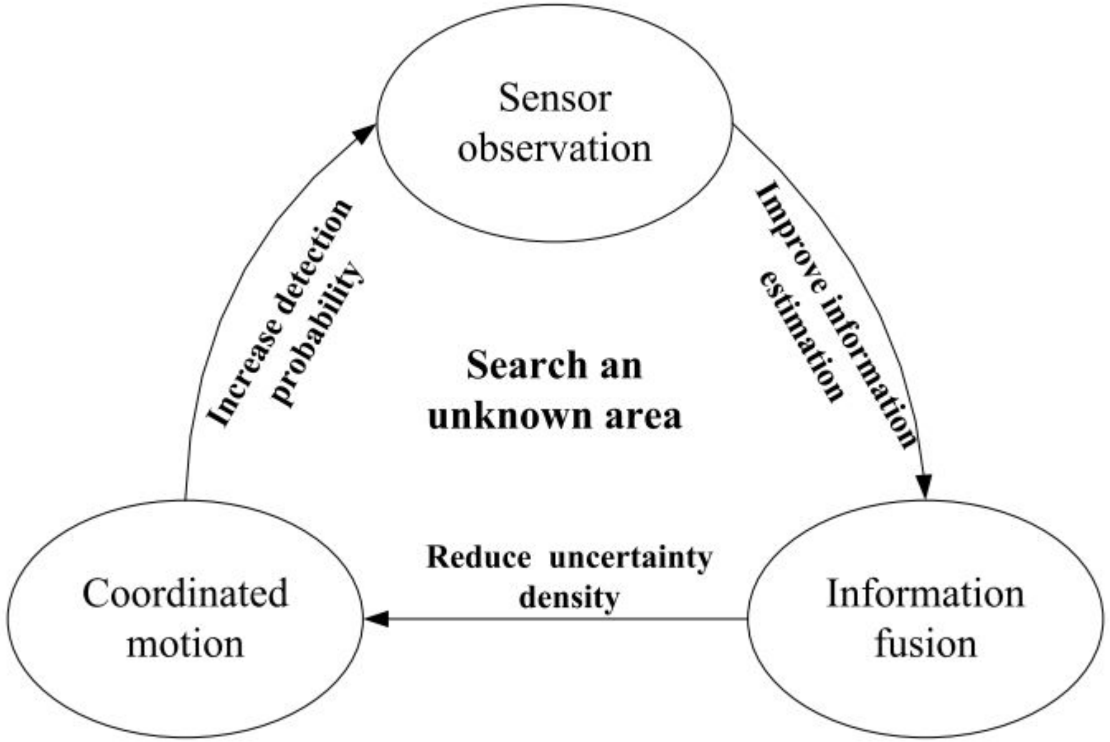
1. Remove problems associated with the discontinuities imposed from obstacles
2. UAVs are no longer required to be homogeneous (i.e. same sensing and navigation capabilities)
3. The potential function **guarantees** utilities of individuals are localized to themselves yet aligned with the global objective

THE MAIN CONTRIBUTION: “the development of a potential game formulation for cooperative search.”

Problem Formulation: Search & Surveillance

Sequential Tasks of Cooperative Search

1. Coordinated Motion
   a. UAVs move towards locations with high uncertainty
2. Sensor Observations
3. Cooperative Information Fusion with Neighbors



Problem Formulation: Definitions

- $V = \{v_1, \ldots, v_N\}$
- $\Omega \in \mathbb{R}^2$ (Mission Space)
- $g \in \mathbb{R}^2$ (Center of a cell in Ω)
- R_{Ci} (Communication range of v_i)
- Z_{igt} (Measurement of v_i on cell g at time t)
  - Assume if center of cell, g, is within sensing range, then entire cell can be observed
- Dynamic Net: (E,V)
  - $E = \{(v_i,v_j): v_i,v_j \in V;\ ||\mu_i - \mu_j|| \leq R_{Ci}\}$
- Neighbor set of v_i:
  - $N_i = \{v_j \in V \mid (v_i,v_j) \in E\} \cup \{v_i\}$
  - Assume a vehicle is a neighbor with itself

- Def 1: (Exact) Potential Game

Definition 1 *(Exact Potential Games [28]).* A game $G(S, \{A_i, i \in S\}, \{U_i, i \in S\})$ is called an exact potential game if there exists a global function $\Phi : A \to \mathbf{R}$ such that,

$$U_i(a_i', a_{-i}) - U_i(a_i, a_{-i}) = \Phi(a_i', a_{-i}) - \Phi(a_i, a_{-i}),$$
$$\forall i \in S,\ a_i, a_i' \in A_i,\ a_{-i} \in A_{-i}, \quad (1)$$

where the global function Φ is known as the potential function of game G. Following the common practice, we will henceforth refer to exact potential games simply as potential games.

- Def 2: Nash Equilibrium

Definition 2 *(Nash Equilibrium [29]).* A strategy $a^* = (a_i^*, a_{-i}^*)$ is called a Nash Equilibrium of game $G(S, \{A_i, i \in S\}, \{U_i, i \in S\})$, if and only if

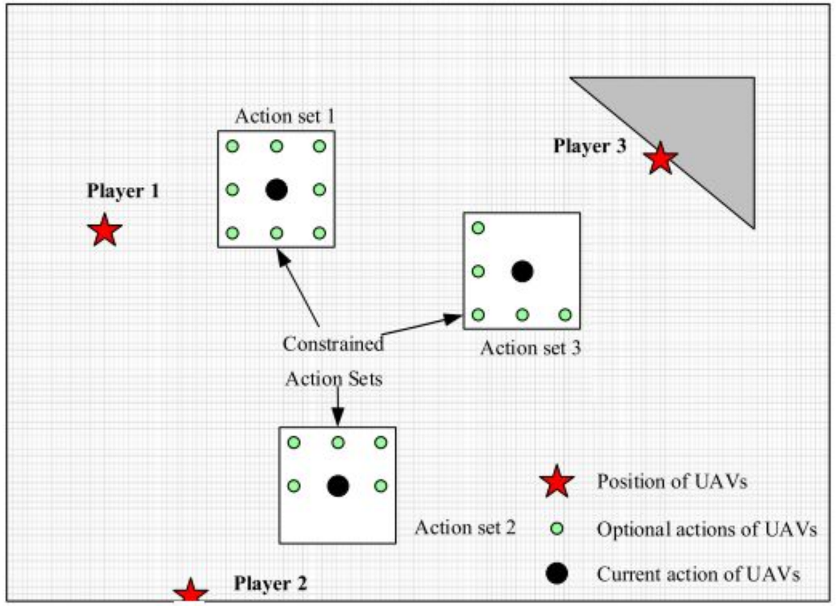
$$U_i(a_i^*, a_{-i}^*) \leq U_i(a_i, a_{-i}^*), \quad \forall i \in S,\ \forall a_i \in A_i. \quad (2)$$

1. Coordinated Motion as a Potential Game

Density Map: $\eta(g): \Omega \longrightarrow \mathbb{R}_+$

Action of v_i: $a_i \in A_i$

Constrained action set: $C_{ai(t-1)} \subseteq A_i$



UAV Performance/Potential function:

$$\Phi(a) = \Phi(\mu_1, \mu_2, \ldots, \mu_n) = \int_{\Omega} f\Big(\min_{i \in \{1,2,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg \tag{3}$$

where

$$f(\|g - \mu_i\|) = \begin{cases} e^{-\|g-\mu_i\|} & \|g - \mu_i\| \le R_{S_i} \\ 0 & \text{otherwise} \end{cases}$$

UAV Utility Function:

$$U_i(a_i, a_{-i}) = \int_{\Omega} f\Big(\min_{i \in \{1,2,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg - \int_{\Omega} f\Big(\min_{i \in \{1,2,\ldots,i-1,i+1,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg. \tag{4}$$

1. Coordinated Motion as a Potential Game (cont)

$$\Phi(a) = \Phi(\mu_1, \mu_2, \ldots, \mu_n) = \int_{\Omega} f\Big(\min_{i\in\{1,2,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg \tag{3}$$

where

$$f(\|g-\mu_i\|) = \begin{cases} e^{-\|g-\mu_i\|} & \|g-\mu_i\| \le R_{S_i} \\ 0 & \text{otherwise} \end{cases}$$

$$U_i(a_i, a_{-i}) = \int_{\Omega} f\Big(\min_{i\in\{1,2,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg - \int_{\Omega} f\Big(\min_{i\in\{1,2,\ldots,i-1,i+1,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg. \tag{4}$$

Lemma 1. *Consider a coverage problem formulated as a multiplayer game, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of players, and $A = A_1 \times A_2 \times \cdots \times A_n$ is the set of joint actions. If player v_i takes $U_i(a_i, a_{-i})$ defined by Eq. (4) as its individual utility, then it constitutes a potential game with the potential function $\Phi(a)$ defined by Eq. (3).*

Proof. Let $a_i = \mu_i$ and $a_i' = \mu_i'$ be two possible actions for v_i, and let a_{-i} denote actions of the remaining players. Note that Eq. (4) can be rewritten as

$$U_i(a_i, a_{-i}) = \Phi(a_i, a_{-i}) - \int_{\Omega} f\Big(\min_{i\in\{1,2,\ldots,i-1,i+1,\ldots,n\}} \|g - \mu_i\|\Big)\eta(g)dg. \tag{5}$$

Using Eq. (5), we get,

$$\begin{aligned} &U_i(a_i', a_{-i}) - U_i(a_i, a_{-i}) \\ &= \Phi(a_i', a_{-i}) - \Phi(a_{-i}) - \big(\Phi(a_i, a_{-i}) - \Phi(a_{-i})\big) \\ &= \Phi(a_i', a_{-i}) - \Phi(a_i, a_{-i}). \end{aligned} \tag{6}$$

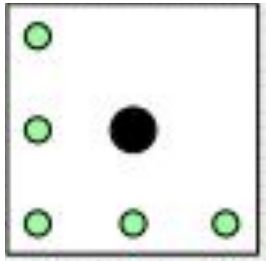
1. Coordinate Motion: Binary Log-Linear Learning (BLLL) for Optimal Coverage

- Log-linear learning guarantees convergence to the optimal Nash equilibrium, but computationally expensive b/c UAVs need to compute utility of **all possible actions**
- Binary Log-Linear Learning (BLLL) allows agent to choose between a trial action and its previous action given the utility of both:

$$\begin{cases} P\big(a_i(t) = a_i(t-1)\big) = \dfrac{e^{\frac{1}{\tau}U_i(a(t-1))}}{e^{\frac{1}{\tau}U_i(a(t-1))} + e^{\frac{1}{\tau}U_i(a_i',a_{-i}(t-1))}} \\ P\big(a_i(t) = a_i'\big) = \dfrac{e^{\frac{1}{\tau}U_i(a_i',a_{-i}(t-1))}}{e^{\frac{1}{\tau}U_i(a(t-1))} + e^{\frac{1}{\tau}U_i(a_i',a_{-i}(t-1))}} \end{cases}$$

where

$$\boxed{\begin{cases} P\big(a_i' = a_i\big) = 1/z_i, \quad \text{for } a_i \in C_{a_i(t-1)} \backslash a_i(t-1) \\ P\big(a_i' = a_i(t-1)\big) = 1 - \big(|C_{a_i(t-1)}| - 1\big)/z_i \end{cases}}$$



Example:

$z_i = 9$; $|C_{a_i(t-1)}| = 7$

$P(a_i') =$ [1/9,1/9,3/9,1/9,1/9,1/9]

Let tau=0.2,

$U_i(a(t-1))=0.1$,

$U_i(a',a_{-i}(t-1))=0.8$

then $P(a_i(t))=[0.03,0.97]$

1. Coordinate Motion: Binary Log-Linear Learning (BLLL) for Optimal Coverage

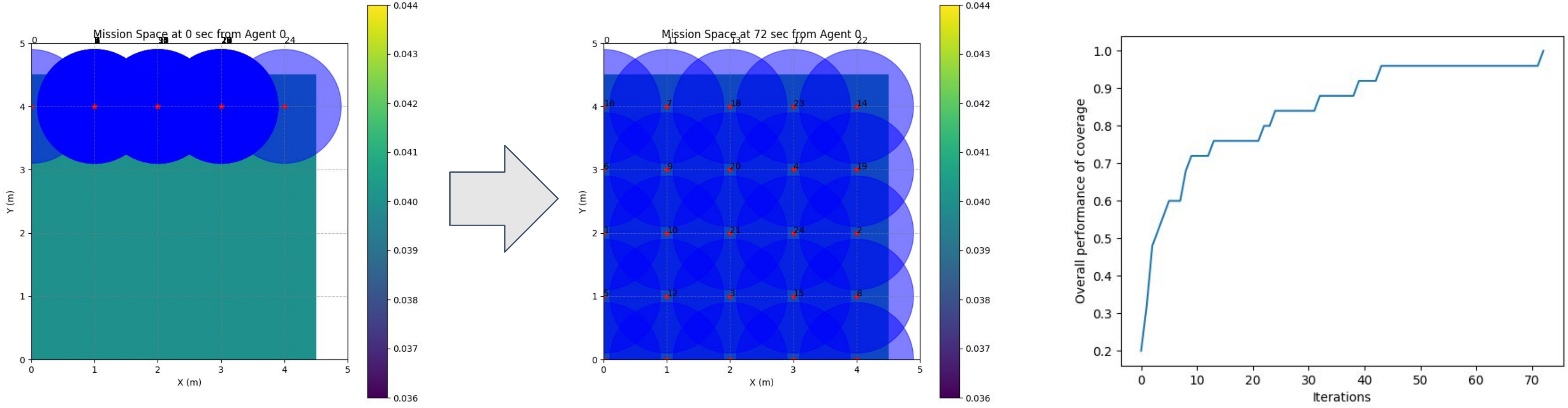
- How does BLLL guarantee optimal coverage?

Proposition 1 *(Reachability). For any vehicle $v_i \in V$ and any action pair $a_i(0), a_i(m) \in A_i$, there exists a sequence of actions $a_i(0) \rightarrow a_i(1) \rightarrow \ldots \rightarrow a_i(m)$ such that $a_i(t) \in C_{a_i(t-1)}$ for all $t \in \{1, 2, \ldots, m\}$.*

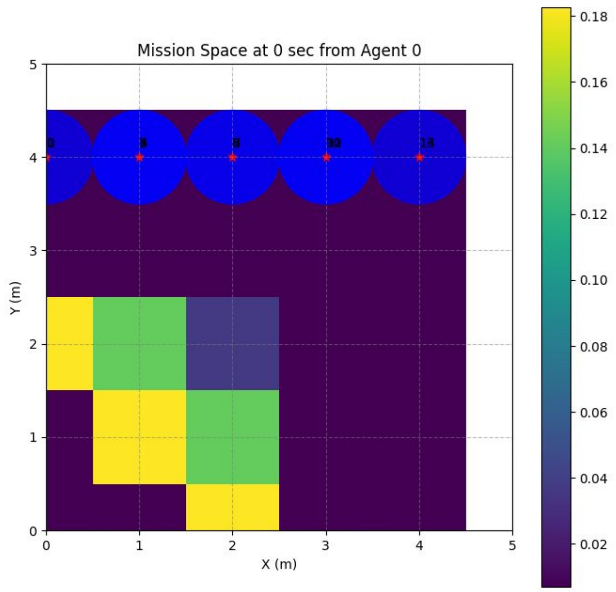
Proposition 2 *(Reversibility). For any vehicle $v_i \in V$ and any action pair $a_i, a_i' \in A_i, a_i' \in C_{a_i} \Leftrightarrow a_i \in C_{a_i'}$.*

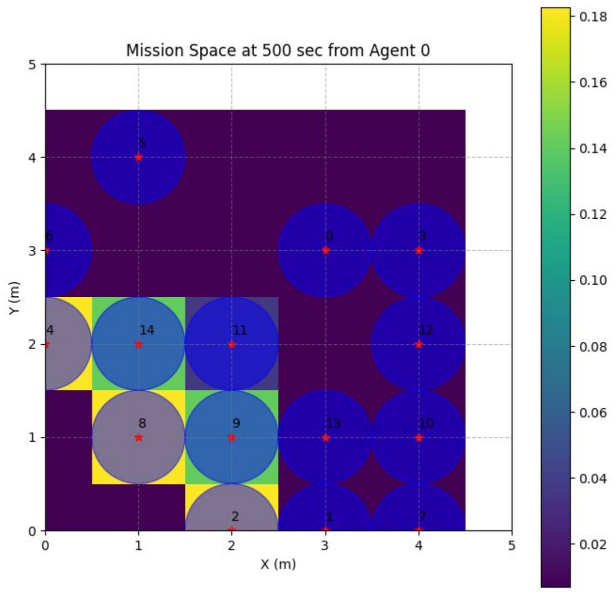
Theorem 1. *Consider the coordinated motion problem for optimal coverage formulated as a potential game with the potential function defined by Eq. (3), where all the players adhere to binary log-linear learning. If the designed constrained action sets meet the requirements of Proposition 1 and Proposition 2, the coverage performance $\Phi(a)$ will be maximized asymptotically with sufficient large time t, provided that $\tau \rightarrow 0$, i.e., $\lim_{\tau \rightarrow 0, t \rightarrow \infty} P(a = \arg\max_{\tilde{a} \in A} \Phi(\tilde{a})) = 1$.*

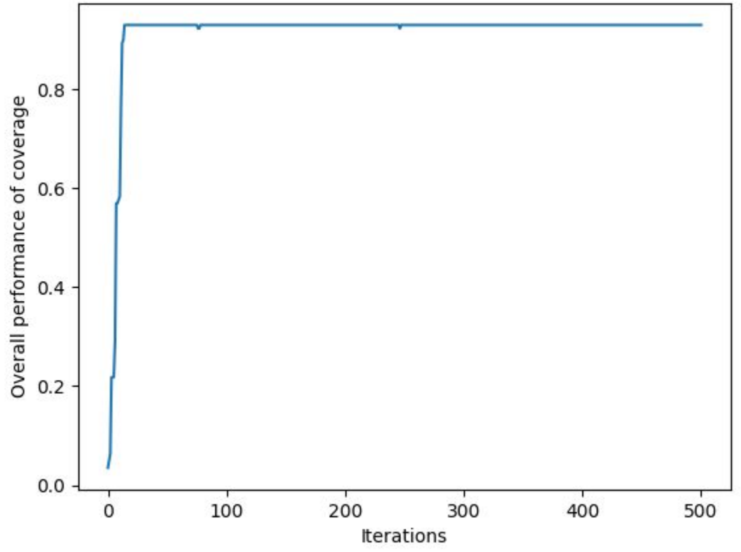
1. Coordinate Motion: Results ($N = |\Omega|$)



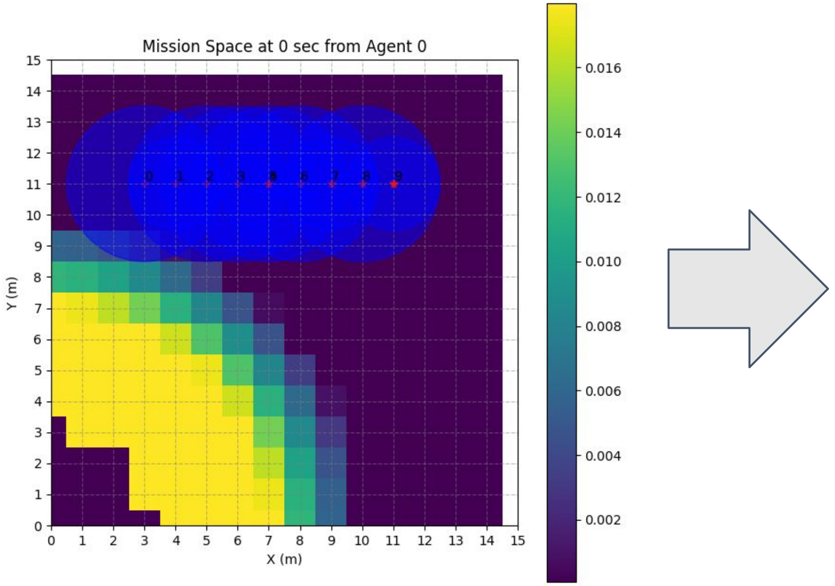
1. Coordinate Motion: Results (N < |Ω|)

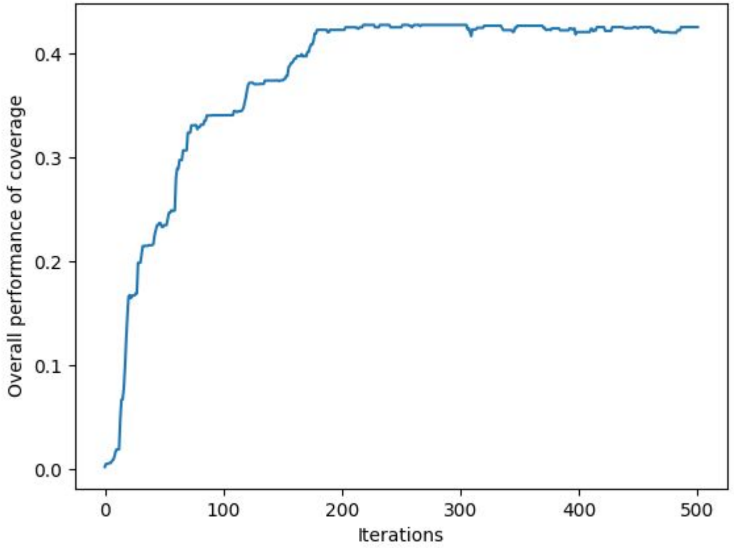






1. Coordinate Motion: Results (Heterogeneous, Complex Prob Map)





2. Cooperative Search and Surveillance (don't know probability map beforehand)

1. Perform Observations Z_{igt} for $\forall v_i \in V$

2. Cooperative Search and Surveillance (don't know probability map beforehand)

1. Perform Observations Z_{igt} for $\forall v_i \in V$
2. Update individual agent's probability map, H_{igt}

$$P_{i,g,t} = P(\theta_g = 1 \mid Z_{i,g,t})$$
$$= \frac{P(Z_{i,g,t} \mid \theta_g = 1) P_{i,g,t-1}}{P(Z_{i,g,t} \mid \theta_g = 1) P(\theta_g = 1) + P(Z_{i,g,t} \mid \theta_g = 0) P(\theta_g = 0)}$$
$$= \begin{cases} \frac{p_c P_{i,g,t-1}}{p_c P_{i,g,t-1} + p_f (1 - P_{i,g,t-1})} & \text{if } Z_{i,g,t} = 1 \\ \frac{(1-p_c) P_{i,g,t-1}}{(1-p_c) P_{i,g,t-1} + (1-p_f)(1 - P_{i,g,t-1})} & \text{if } Z_{i,g,t} = 0 \\ P_{i,g,t-1} & \text{otherwise.} \end{cases} \quad (24)$$

p_c=Probability of detection
p_f=Probability of false alarm
(predefined numbers between 0,1)

$$H_{i,g,t} = \ln\left(\frac{1}{P_{i,g,t}} - 1\right). \quad (25)$$

Consequently, Eq. (24) can be transformed into

$$H_{i,g,t} = \begin{cases} H_{i,g,t-1} + \ln \frac{p_f}{p_c} & \text{if } Z_{i,g,t} = 1 \\ H_{i,g,t-1} + \ln \frac{1-p_f}{1-p_c} & \text{if } Z_{i,g,t} = 0 \\ H_{i,g,t-1} & \text{otherwise.} \end{cases} \quad (26)$$

2. Cooperative Search and Surveillance (don't know probability map beforehand)

1. Perform Observations Z_{igt} for $\forall v_i \in V$
2. Update individual agent's probability map, H_{igt}
3. Transmit updated probability map, H_{igt}, to neighbors, N_i

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1. Perform Observations Z_{igt} for $\forall v_i \in V$
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4. Each agent fuses probability maps together from neighbors and self

$$Q_{i,g,t} = \sum_{j=1}^{n} \omega_{i,j,t} H_{j,g,t}.$$

$$\boxed{\omega_{i,j,t} = \begin{cases} \frac{1}{1+\max\{\kappa_i,\kappa_j\}} & \text{if } \{v_i, v_j\} \in E \\ 1 - \sum_{\{v_i,v_k\}\in E} \omega_{i,k,t} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}}$$

2. Cooperative Search and Surveillance (don't know probability map beforehand)

1. Perform Observations Z_{igt} for $\forall v_i \in V$
2. Update individual agent's probability map, H_{igt}
3. Transmit updated probability map, H_{igt}, to neighbors, N_i
4. Each agent fuses probability maps together from neighbors and self
5. Update uncertainty map, η_{igt}, of each agent

$$\eta_{i,g,t} = e^{-k_\eta \| Q_{i,g,t} \|}$$

where, k_n=positive gain (default=1)

Full Pipeline

Table 1
Procedures of multiple UAV cooperative search using the proposed potential game approach.

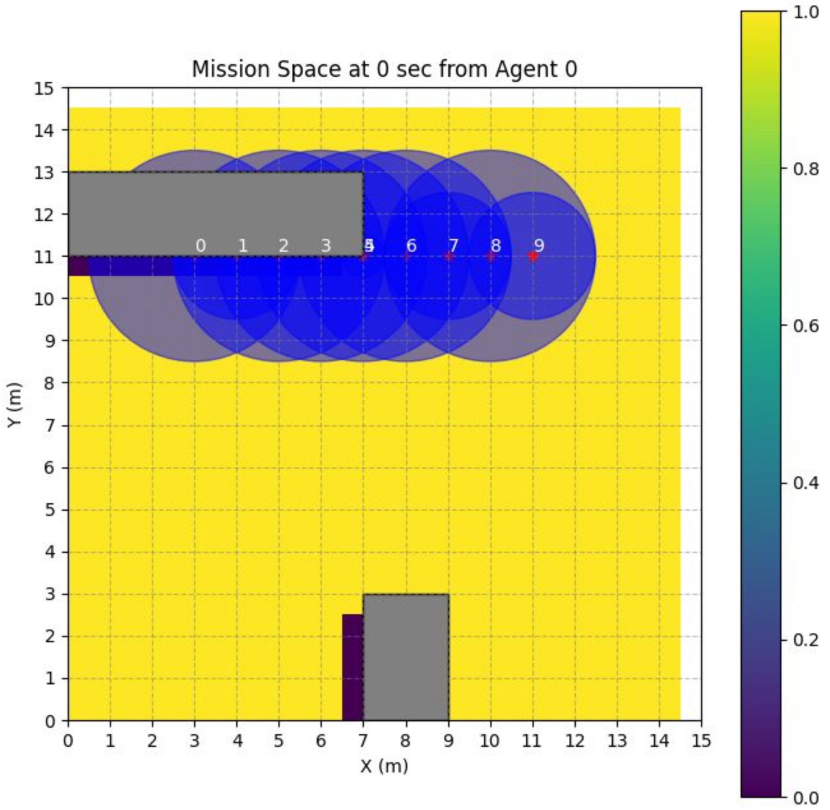
Name:	Cooperative search using multiple UAVs
Goal:	Maximize area coverage and data collection
Requires:	Limited sensing and communication capabilities

The mission region is uniformly portioned into M cells and each vehicle is initialized with an individual map Q
While the mission space Ω is not fully understood **do**
  /* **Optimal coverage using binary log-linear learning** */
  For all UAVs in the player set V **do**
    Randomly select a vehicle $v_i \in V$ from the player set
    Choose a trial action $a_i'(t)$ from the constrained action set, $a_i'(t) \in C_{a_i(t-1)}$
    The selected vehicle v_i computes its current utility $U_i(a(t-1))$ and the expected utility $U_i(a_i', a_{-i}(t-1))$ according to Eq. (5)
    Vehicle v_i choose an action according to Eq. (8) utilizing the calculated utilities
    Vehicle v_i decides which direction to move toward based on $a_i(t)$
  End For
  /* **Sensor observations and information fusion** */
  For each vehicle in the player set $v_i \in V$ **do**
    Performs observations $Z_{i,g,t}$ over regions within its sensing range
    Updates its individual map $H_{i,g,t}$ using observed sensor readings according to Eq. (26)
    Transmits its updated map $H_{i,g,t}$ to its neighbors determined by its communication range
    Performs information fusion based on Eq. (28)
    Update the uncertainty map $\eta_{i,g,t}$ according to the fused information to direct the motion control
  End For
  $t = t + 1$
End While

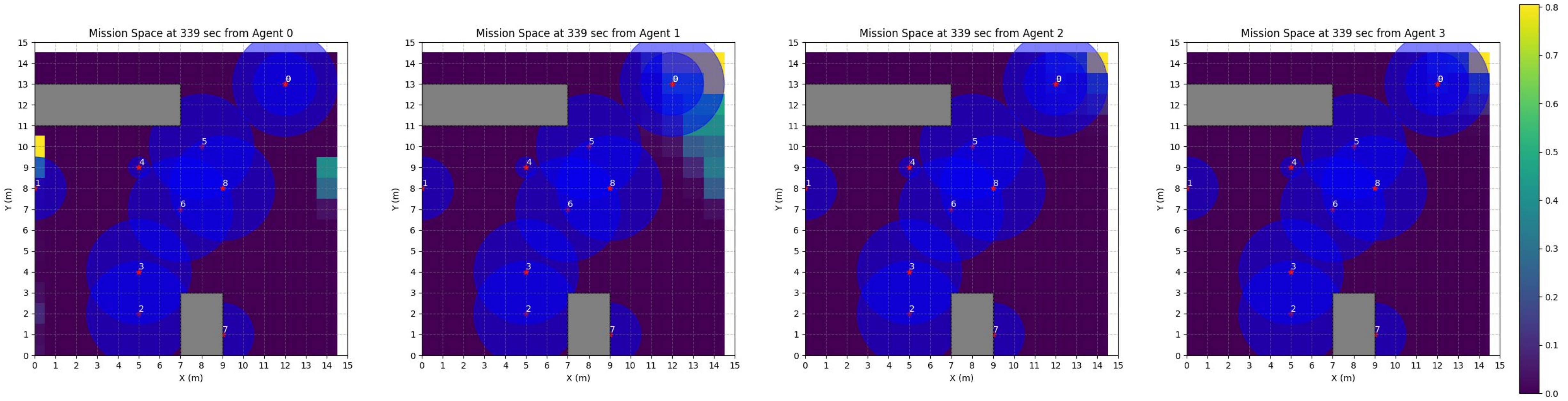
Theoretical Results

- What techniques were used? Provide a basic explanation of this if we haven’t already covered it in class.
- What guarantees were the authors able to provide?
- What assumptions did they have to make in order to provide those guarantees?
- Walk through one or two of the major theoretical results and show how they were proven.

Full pipeline results - N=10, Heterogeneous Agents, Obstacles, Colorbar=Uncertainty



Full pipeline results



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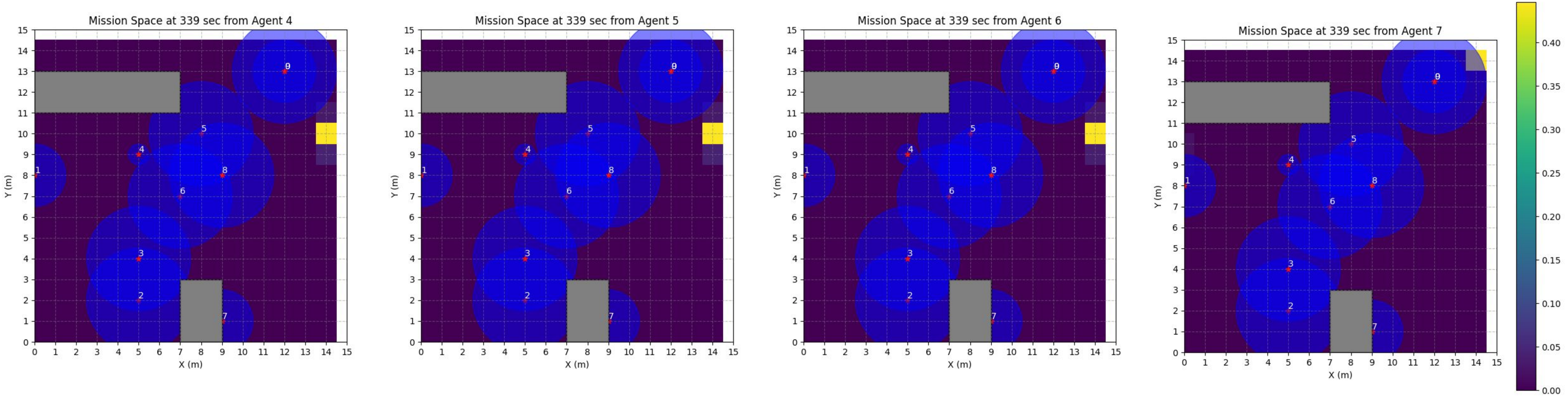

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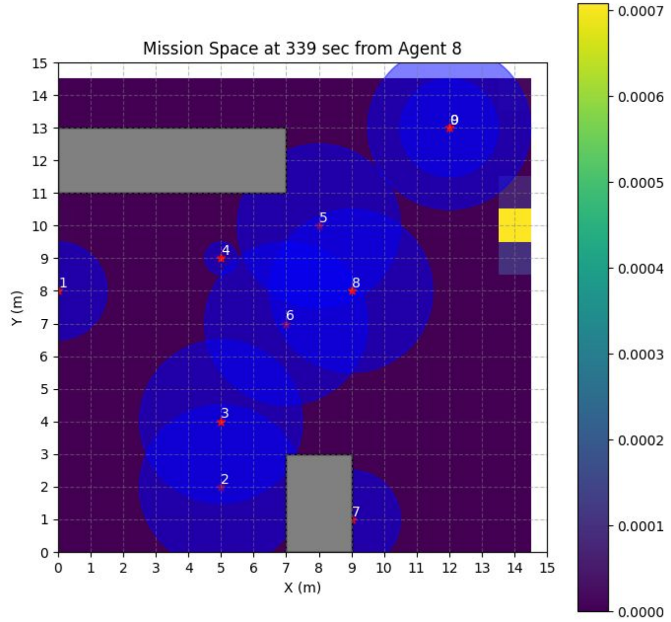

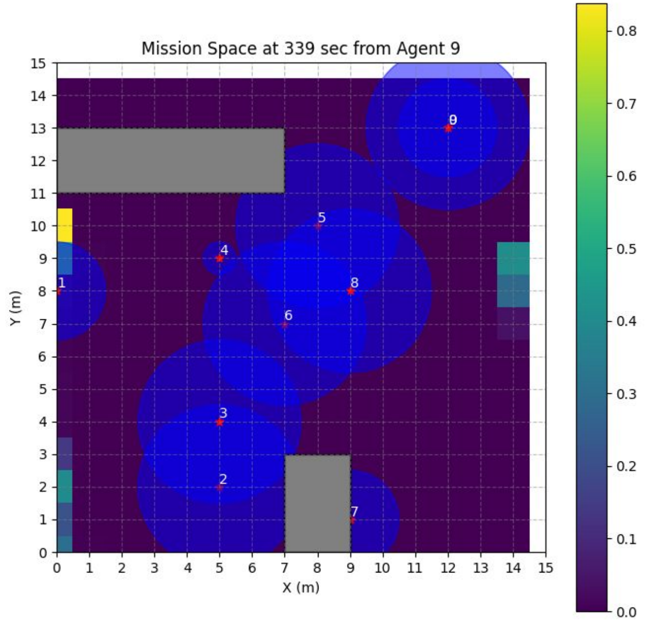

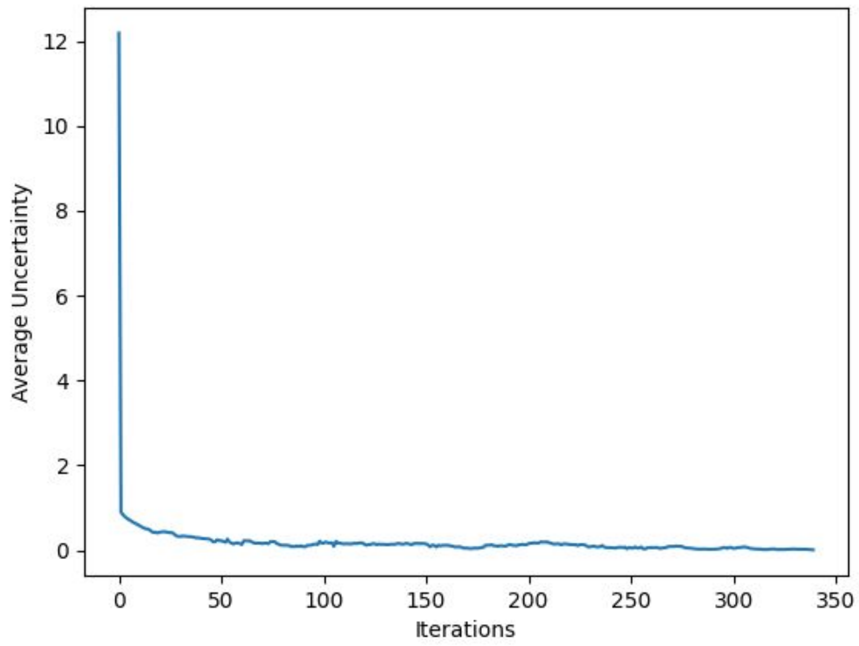

Summary

Potential Game formulation of Cooperative Search & Surveillance missions provides:

- Guarantees in mission map coverage asymptotically
- Capabilities for heterogeneous multi-UAVs
- Computational efficiency utilizing nonlinear transforms (i.e. $P_{igt} \longrightarrow H_{igt}$)
- Sensor fusion formulation will reach average consensus if either
  - UAV communication topology constitutes a connected network
  - UAV communication topology constitutes a connected network with independent link failures
- Clear metrics to evaluate coverage and uncertainty

Future Work

- Add in actual targets to see how the group survey’s the target overtime

- Add in different agent interests. For example agent i might want to track target k and agent j might want to track target l.

- Use Gaussian Processes for a continuous uncertainty map