

DECENTRALIZED METHODS FOR WEAPON-TARGET ASSIGNMENT

Implementation of [1] by Landon Shumway

EE 682R – Multi-agent Systems

4/15/24

[1] K. Volle, J. Rogers, and K. Brink, “Decentralized Cooperative Control Methods for the Modified Weapon–Target Assignment Problem,” *Journal of Guidance, Control, and Dynamics*, Jul. 2016, doi: [10.2514/1.G001752](https://doi.org/10.2514/1.G001752).

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Outline

- Motivation of Problem
- Background Literature
- Set up
- Methods
- Results
- Conclusion/Future Work/Questions

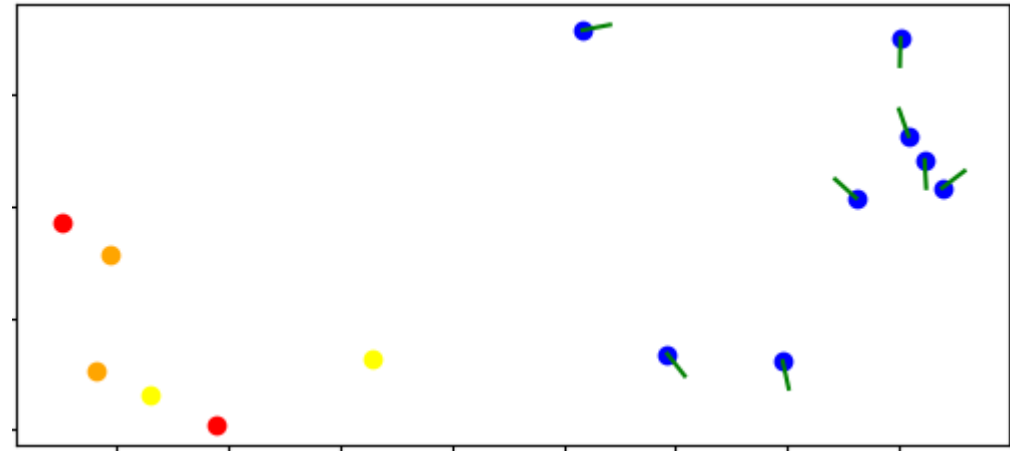
Motivation of Problem

- Weapon-Target Assignment Problem
 - Set of mobile agents/weapons
 - Set of stationary targets
 - Find the optimal weapon-target pairing Q :

$$Q = \{(i, j) | i \in I, j \in J\}$$

$$I = \{1, \dots, N\}$$

$$J \subseteq \{1, \dots, M\}$$



Background Literature

Paper implementation:

- K. Volle, J. Rogers, and K. Brink, “Decentralized Cooperative Control Methods for the Modified Weapon–Target Assignment Problem,” *Journal of Guidance, Control, and Dynamics*, Jul. 2016, doi: 10.2514/1.G001752.

WTA first explored (linear approximation of nonlinear aspects):

- A. S. Manne, “A Target-Assignment Problem,” *Operations Research*, vol. 6, no. 3, pp. 346–351, Jun. 1958, doi: 10.1287/opre.6.3.346.

Integer programming problem:

- R. K. Ahuja, A. Kumar, K. C. Jha, and J. B. Orlin, “Exact and Heuristic Algorithms for the Weapon-Target Assignment Problem,” *Operations Research*, vol. 55, no. 6, pp. 1136–1146, Dec. 2007, doi: 10.1287/opre.1070.0440.

Game Theory:

- G. Arslan, J. R. Marden, and J. S. Shamma, “Autonomous Vehicle-Target Assignment: A Game-Theoretical Formulation,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 129, no. 5, pp. 584–596, Apr. 2007, doi: 10.1115/1.2766722.
- A. Chapman, R. A. Micillo, R. Kota, and N. Jennings, “Decentralised Dynamic Task Allocation: A Practical Game–Theoretic Approach,” presented at the The Eighth International Conference on Autonomous Agents and Multiagent Systems (AAMAS ’09) (10/05/09 - 15/05/09), May 2009, pp. 915–922. Accessed: Apr. 10, 2024. [Online]. Available: <https://eprints.soton.ac.uk/267066/>

Simulated Annealing:

- E. E. Witte, R. D. Chamberlain, and M. A. Franklin, “Task assignment by parallel simulated annealing,” in *Proceedings., 1990 IEEE International Conference on Computer Design: VLSI in Computers and Processors*, Sep. 1990, pp. 74–77. doi: 10.1109/ICCD.1990.130165.
- S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, “Optimization by Simulated Annealing,” *Science*, vol. 220, no. 4598, pp. 671–680, May 1983, doi: 10.1126/science.220.4598.671.

Hybrid Solutions:

- Z.-J. Lee and W.-L. Lee, “A Hybrid Search Algorithm of Ant Colony Optimization and Genetic Algorithm Applied to Weapon-Target Assignment Problems,” in *Intelligent Data Engineering and Automated Learning*, J. Liu, Y. Cheung, and H. Yin, Eds., Berlin, Heidelberg: Springer, 2003, pp. 278–285. doi: 10.1007/978-3-540-45080-1-37.
- M. Alighanbari and J. P. How, “Decentralized Task Assignment for Unmanned Aerial Vehicles,” in *Proceedings of the 44th IEEE Conference on Decision and Control*, Dec. 2005, pp. 5668–5673. doi: 10.1109/CDC.2005.1583066.
- O. Shehory and S. Kraus, “Methods for task allocation via agent coalition formation,” *Artificial Intelligence*, vol. 101, no. 1, pp. 165–200, May 1998, doi: 10.1016/S0004-3702(98)00045-9.

Newer Methods:

- J. Guo, G. Hu, Z. Guo, and M. Zhou, “Evaluation Model, Intelligent Assignment, and Cooperative Interception in Multimissile and Multitarget Engagement,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 58, no. 4, pp. 3104–3115, Aug. 2022, doi: 10.1109/TAES.2022.3144111.
- B. Gaudet, K. Drozd, and R. Furfaro, “Deep Reinforcement Learning for Weapons to Targets Assignment in a Hypersonic strike.” *arXiv*, Oct. 27, 2023. doi: 10.48550/arXiv.2310.18509.

Novel Contribution

- When defined as an optimization problem, there are many ways to define what “optimal” means
 - Most destroyed targets, most destroyed high-priority targets, etc.
- This paper proposes three new cost functions (beyond the traditional cost function), that induce different behaviors depending on the goal of the scenario

Set up – Agent Model

- 2DOF (constant velocity and altitude)

- Commanded heading:

$$\psi_{i,com} = \arctan \left(\frac{e_j - e_i}{n_j - n_i} \right)$$

- Control law:

$$\dot{\psi} = k_{\psi}(\psi_{i,com} - \psi_i)$$

- Determining if a target is within range:

$$\dot{d}_i = \frac{v_h}{\sqrt{(n_i - n_j)^2 + (e_i - e_j)^2}} d_i \quad \frac{v_h}{\dot{d}_i} \leq \gamma_{\max}$$

Set up – Target Kill Probability

- Each target has a desired kill probability:

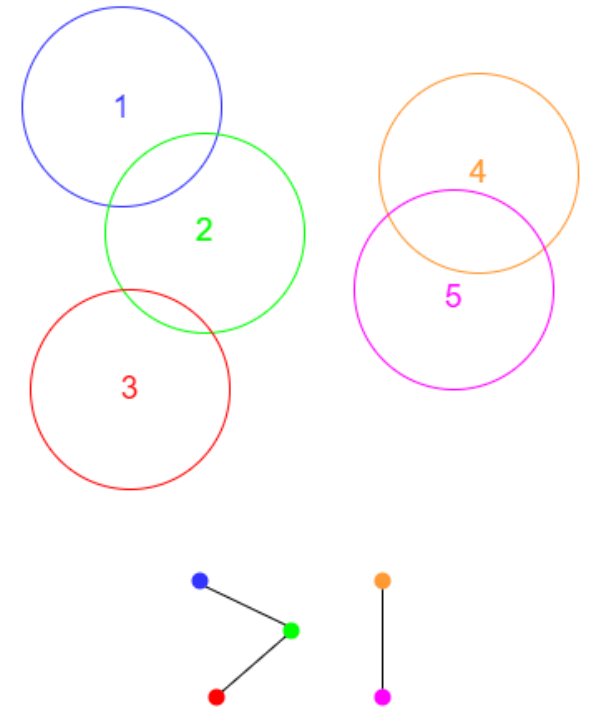
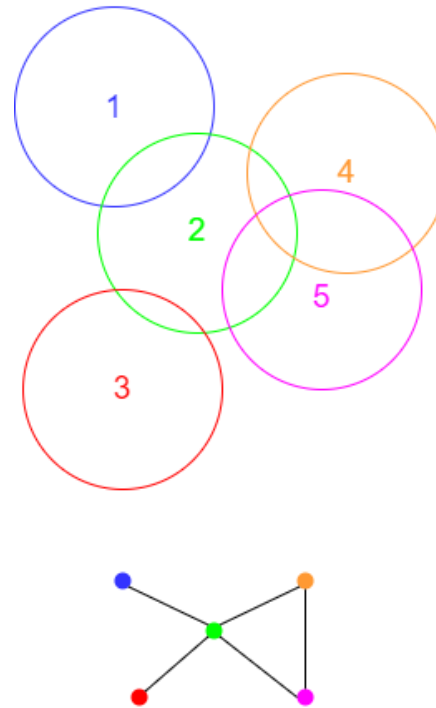
$$Pk_{\text{des},j} \text{ (where } 0 \leq Pk_{\text{des},j} \leq 1)$$

- Calculated kill probability:

$$Pk_{\Sigma,j} = 1 - \prod_{i=1, q_i=j}^N \underbrace{\left(1 - \overbrace{Pk_{i,j}}^{\text{Weapon effectiveness of agent } i} + Pk_{i,j} \overbrace{Pa_{i,j}}^{\text{Probability that agent } i \text{ gets attrited}} \right)}_{\substack{\text{Probability of agent } i \\ \text{NOT destroying target } j}}$$

Set up - Communication

- Agents select targets in a turn-based auction algorithm and broadcast their decision to all agents within its communication range
- This decision gets re-broadcast in a daisy-chain network
- Communication is assumed to occur much faster than decision-making rounds, which are assumed to be on a much shorter timescale than the arrival time from agents to targets
- Each agent maintains a current “estimate” with the most up-to-date information it has on weapon-target pairs. It uses this estimate when deciding which target to pursue
- Broken connections in the network result in out-of-date estimates
 - Not explicitly accounted for in the methods of this paper, but recognized



Set up - Optimization

- Greedy search
- Using its estimate of the current assignment set Q , each agent i selects the target j that minimizes some cost function the most:

$$q_i = \arg \min_{j \in J} C(Q_j)$$

- Q_j is the resulting global assignment set if weapon i selects target j
- Greedy search does not always find the optimal solution, but consistently finds a local minimum
- Each agent uses its own estimate and executes its own greedy search, making this method fully decentralized

Set up – Traditional Cost Function

- Cost function is the sum of the the probability that each target is *not* destroyed multiplied by its desired kill probability (“value” term)

$$C_T(Q) = \sum_{j=1}^M (1 - Pk_{\Sigma,j}) Pk_{\text{des},j}$$

- The higher the calculated kill probability is, the smaller the resulting cost is
- The desired kill probability weights the importance of each target
- Problem: when there is a large disparity in target values, there is nothing stopping weapons from over-assigning a high-priority target even when its desired kill probability has already been reached
 - Waste of weapon resources

Methods – Sufficiency Threshold CF

- Provides no incentive to engage a target whose desired kill probability has already been met

$$C_{ST}(Q) = \sum_{j=1}^M \begin{cases} 0 & Pk_{\Sigma,j} > Pk_{des,j} \\ \frac{Pk_{des,j} - Pk_{\Sigma,j}}{(1 - Pk_{des,j})^\alpha} & Pk_{\Sigma,j} \leq Pk_{des,j} \end{cases}$$

- Denominator term sets the target “value” as a nonlinear function of $Pk_{des,j}$
 - Alpha is a tuning parameter (higher alpha = more weight to $Pk_{des,j}$)

Methods – Enforced Tiering CF

- Divide the targets into τ tiers (typical to group by $Pk_{des,j}$ values):

$$T \in 1, \dots, \tau \quad \text{Lower value for } T \text{ denotes a higher priority tier}$$

- Cost function sums over tiers:

$$C_{ET}(Q) = \sum_{T=1}^{\tau} \begin{cases} C_{ST}(Q_T) & (Pk_{\Sigma,j} \geq Pk_{des,j} \forall j \in t, \forall t < T) \\ P_T & \text{else} \end{cases}$$

- In regular terms: if a higher tier contains targets whose $Pk_{des,j}$ have not been met, enforce some penalty P_T . Else, compute the sufficiency threshold cost function for targets within the current tier
- Penalty must be higher than $C_{ST,max}$

Methods – Completion CF

- Encourages reaching the $Pk_{des,j}$ for as many targets as possible:

$$C_{C,i}(Q_j) = \frac{\ln(1 - Pk_{des,j}) - \ln(1 - Pk_{\Sigma,j})}{\ln(1 - Pk_{i,j} + Pk_{i,j}Pa_{i,j})}$$

- Results in targets with lower $Pk_{des,j}$ being targeted first
- Derivation is too complex to cover in this presentation, but the basic idea is that agent i calculates how many additional weapons of its own effectiveness would be needed to reach $Pk_{des,j}$ for target j . This calculation is done for all targets, and the target that requires the least amount of additional weapons is selected.

Results – Scenario

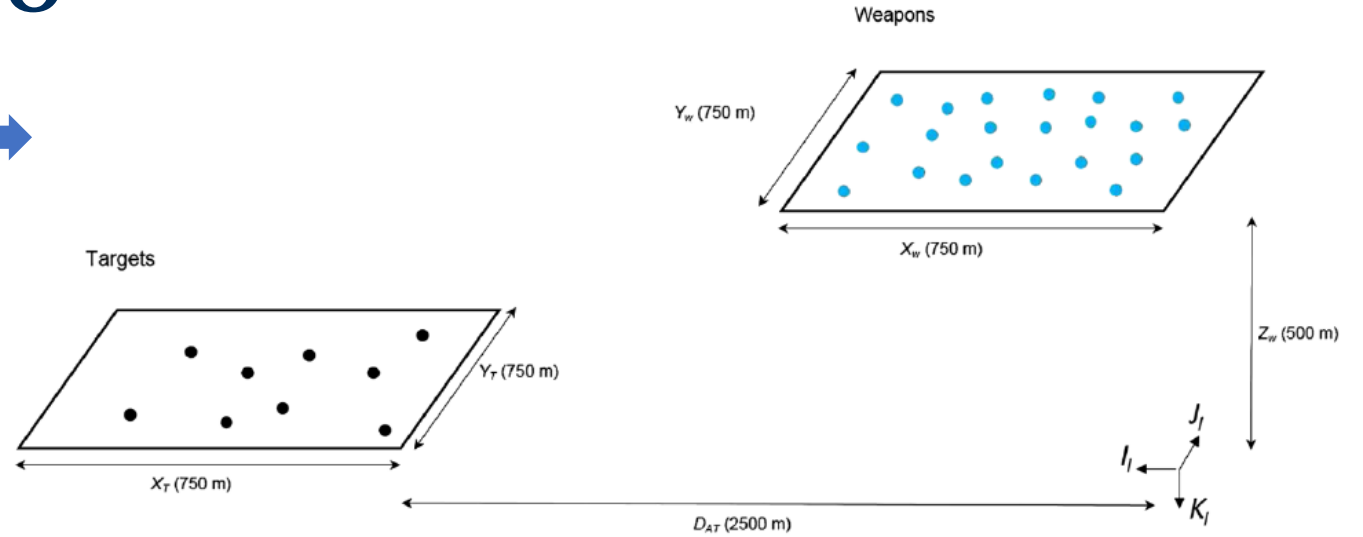
- Engagement geometry ➡

- 6 Targets

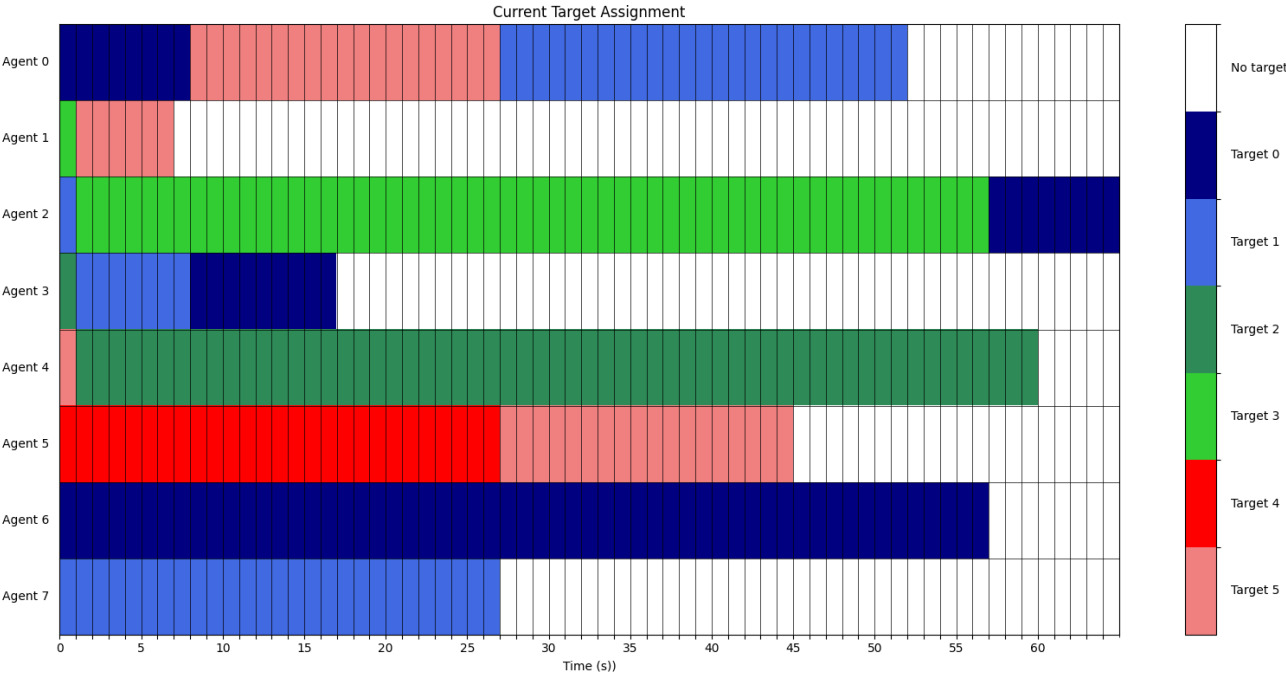
- Targets 0, 1: $Pk_{des,j} = 0.9$
- Targets 2, 3: $Pk_{des,j} = 0.8$
- Targets 4, 5: $Pk_{des,j} = 0.7$

- 8 Weapons

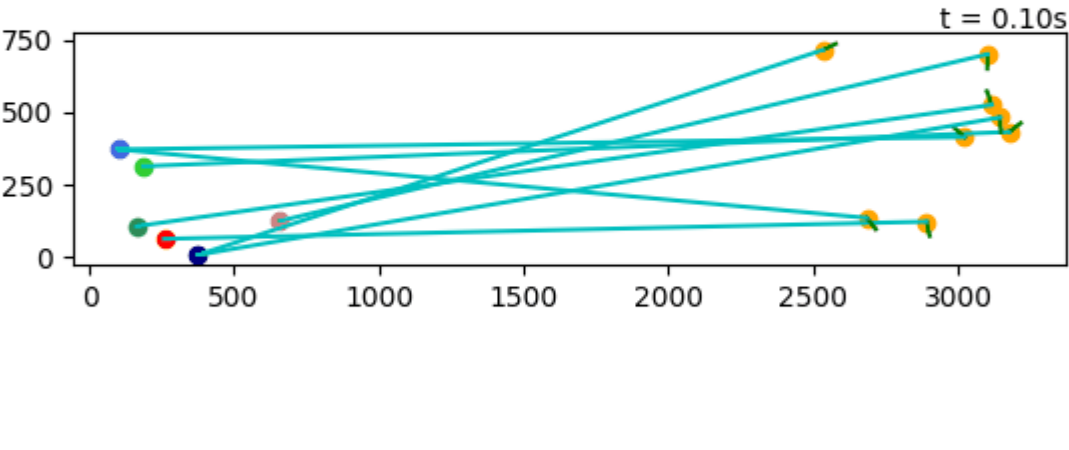
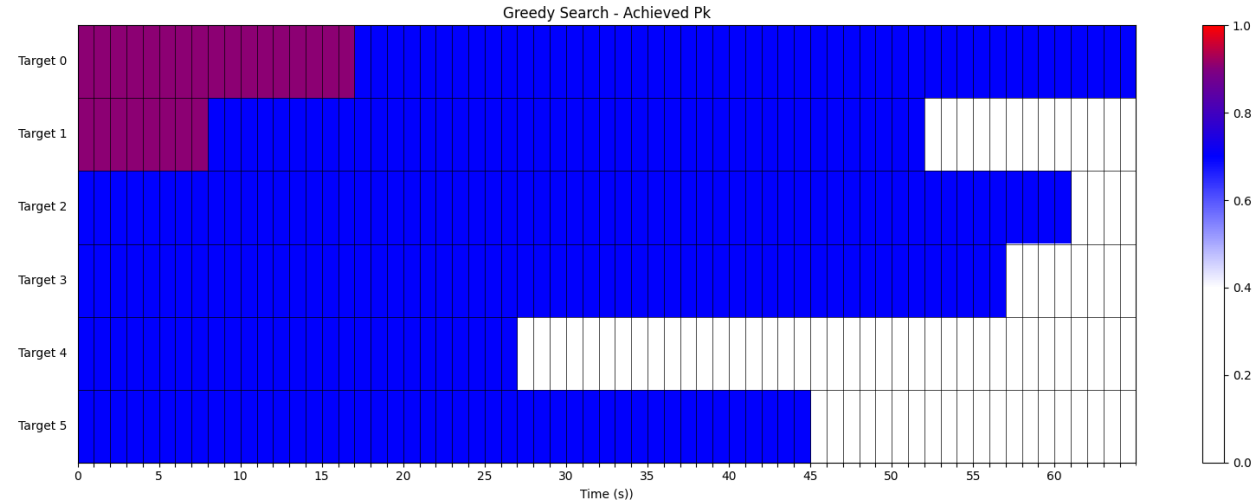
- All have weapon effectiveness of 0.7
 - Upon collision, the agent is destroyed and the algorithm randomly decides if the target is destroyed with a probability equal to the weapon effectiveness of the agent
 - 0.7 is quite low (0.9 is more accurate/reasonable), but having targets fail to be destroyed induces switching agent/target assignments more, which better illustrates the differences between cost functions



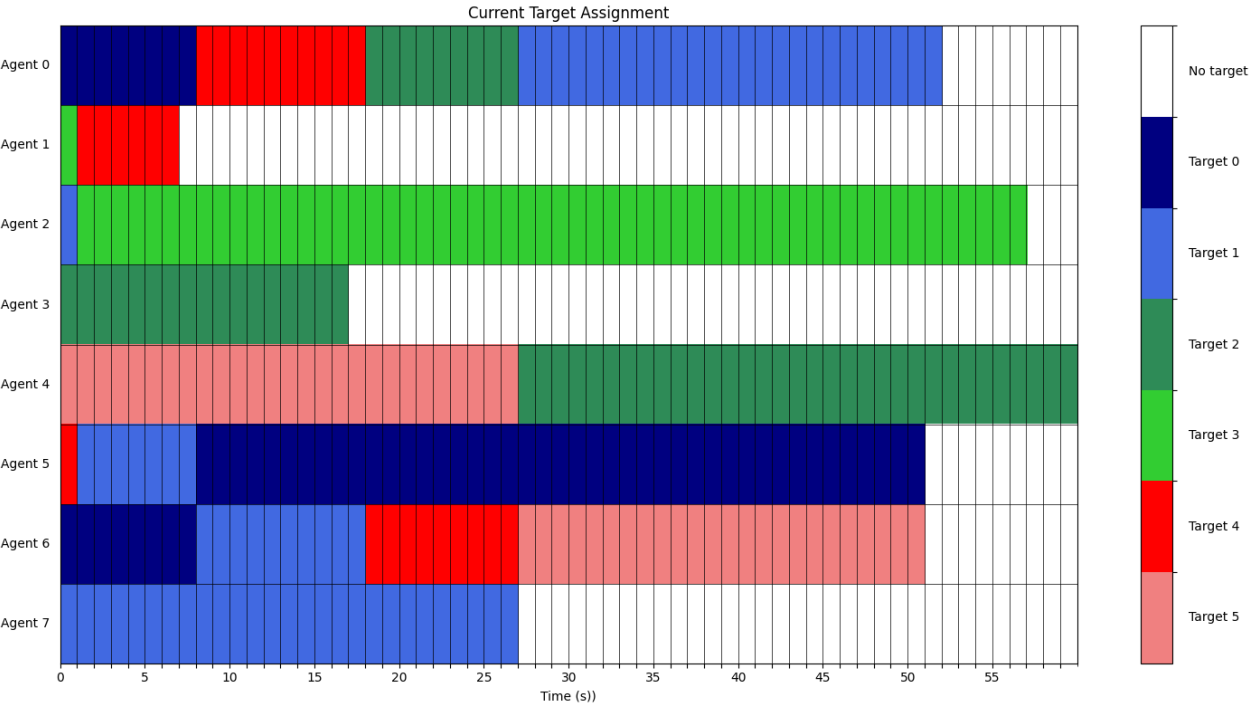
Results – Traditional CF



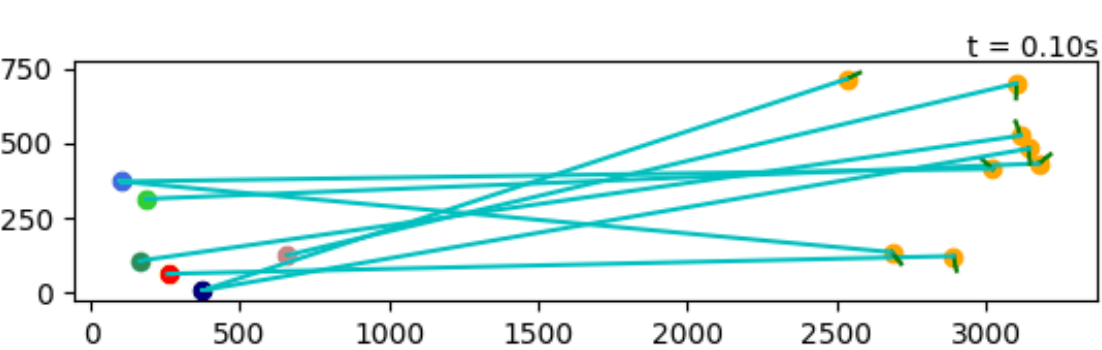
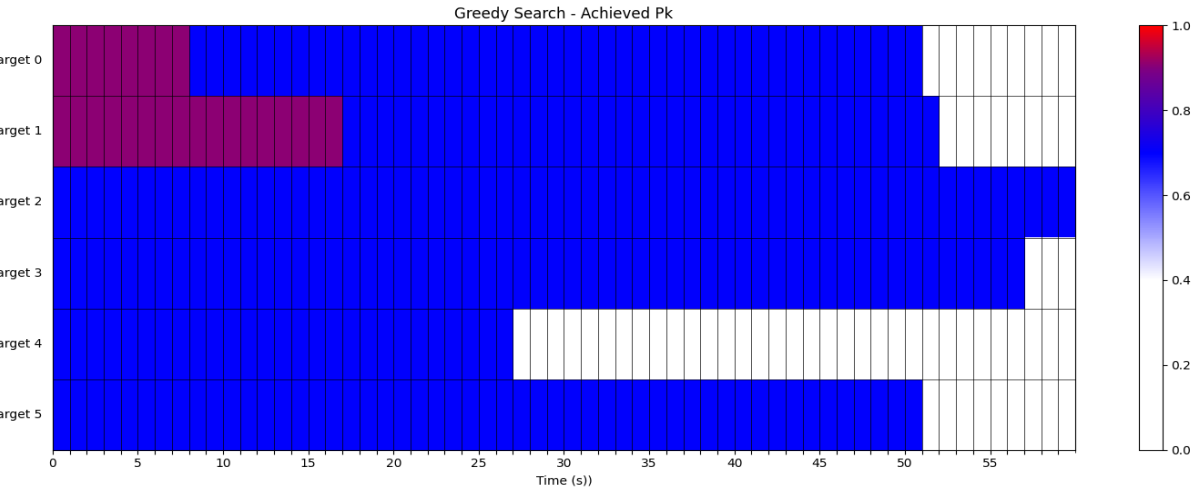
Time (s)	Event
7.1	Agent 1 gets attrited
17.1	Agent 3 gets attrited
26.9	Agent 7 gets attrited
45.4	Agent 5 collides with Target 5; Target 5 is destroyed
52.0	Agent 0 collides with Target 1; Target 1 is destroyed
56.6	Agent 6 collides with Target 0; Target 0 survives
60.3	Agent 4 collides with Target 2; Target 2 survives
64.6	Agent 2 collides with Target 0; Target 0 survives



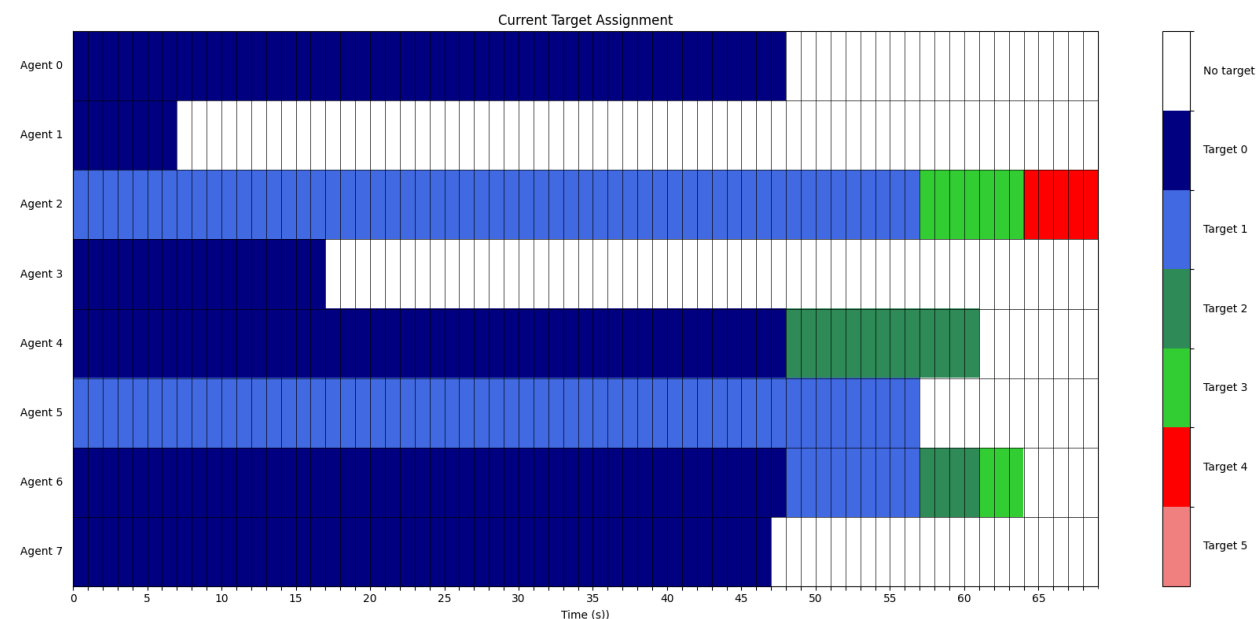
Results – Sufficiency Threshold CF



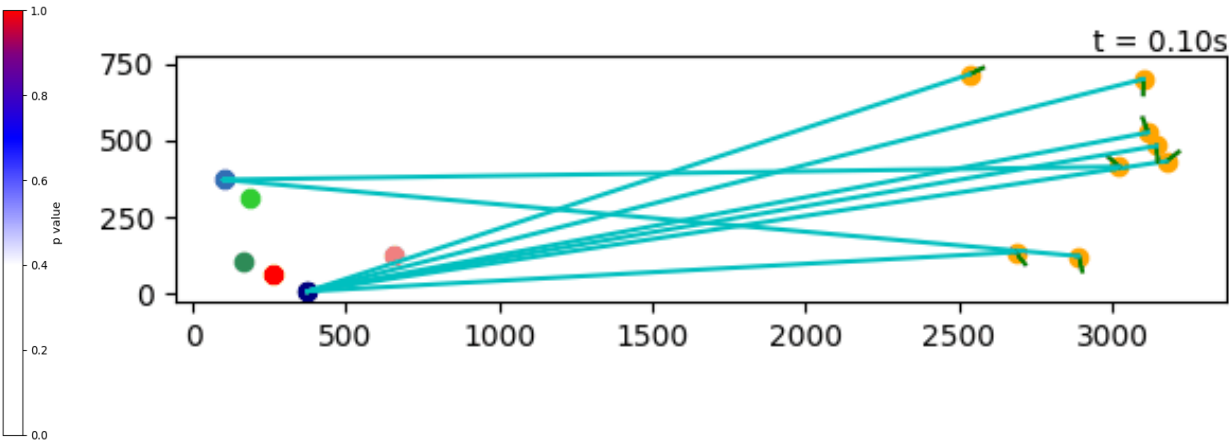
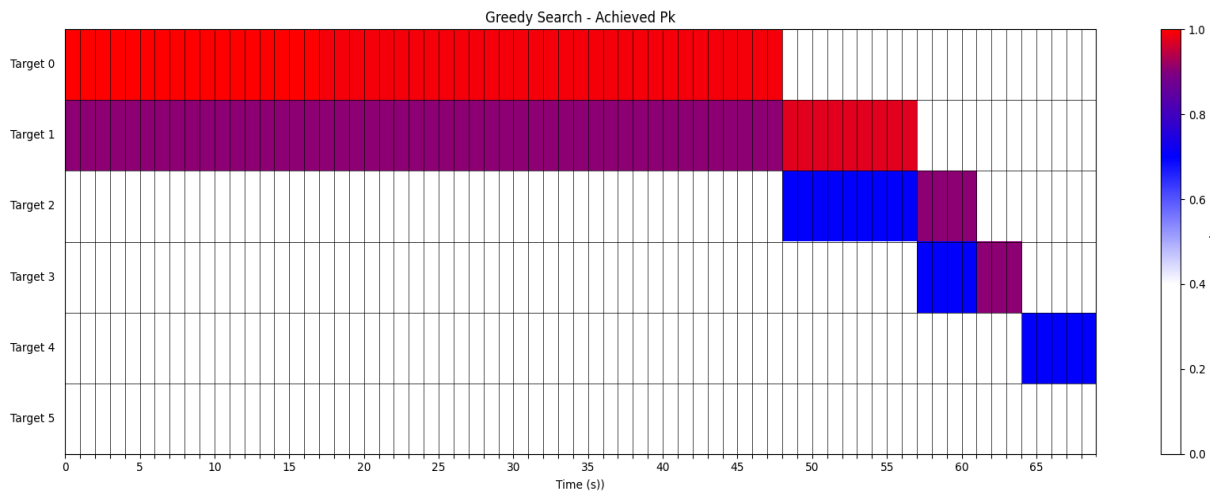
Time (s)	Event
7.1	Agent 1 gets attrited
17.1	Agent 3 gets attrited
26.9	Agent 7 gets attrited
50.8	Agent 6 collides with Target 5; Target 5 survives
51.1	Agent 5 collides with Target 0; Target 0 is destroyed
51.6	Agent 0 collides with Target 1; Target 1 is destroyed
56.9	Agent 2 collides with Target 3; Target 3 survives
63.0	Agent 4 collides with Target 2; Target 2 survives



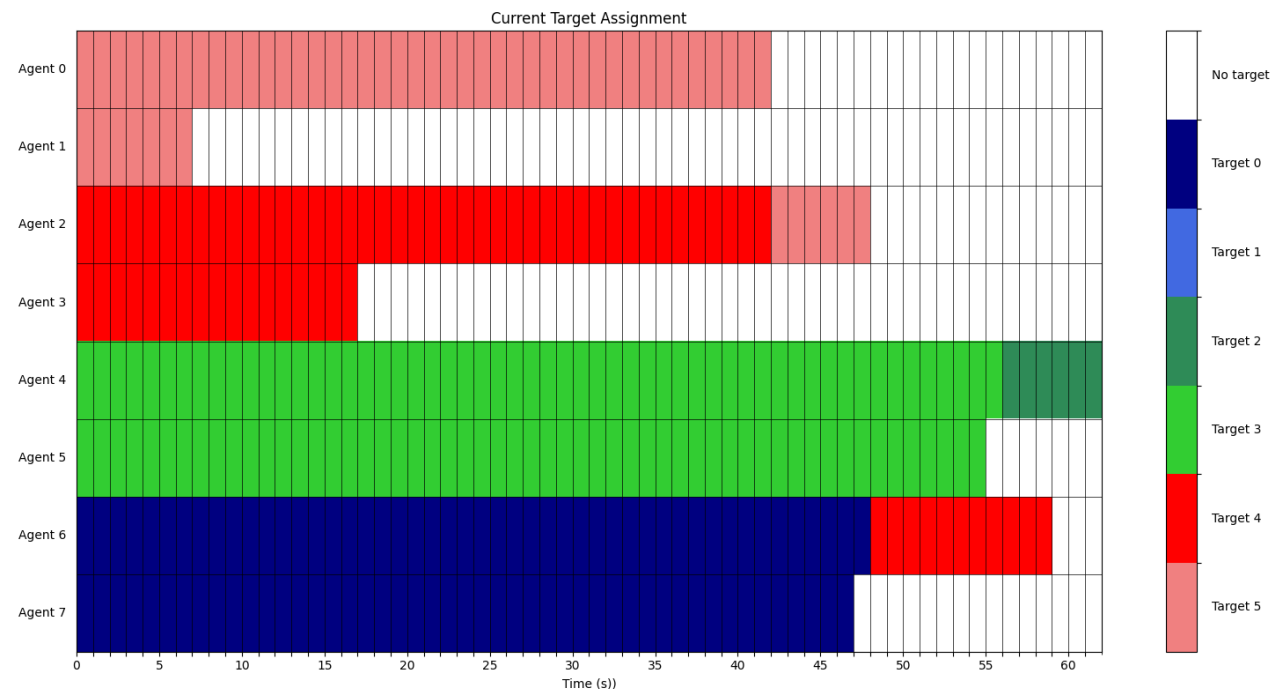
Results – Enforced Tiering CF



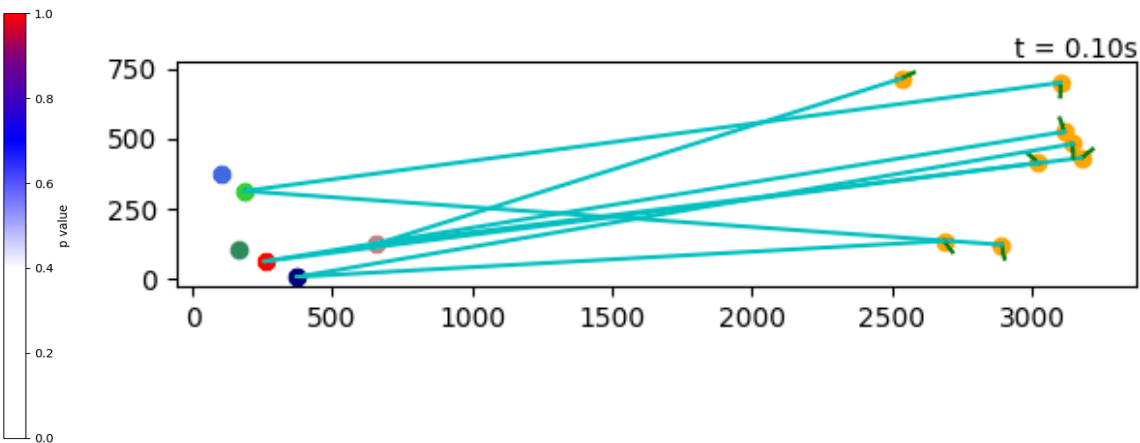
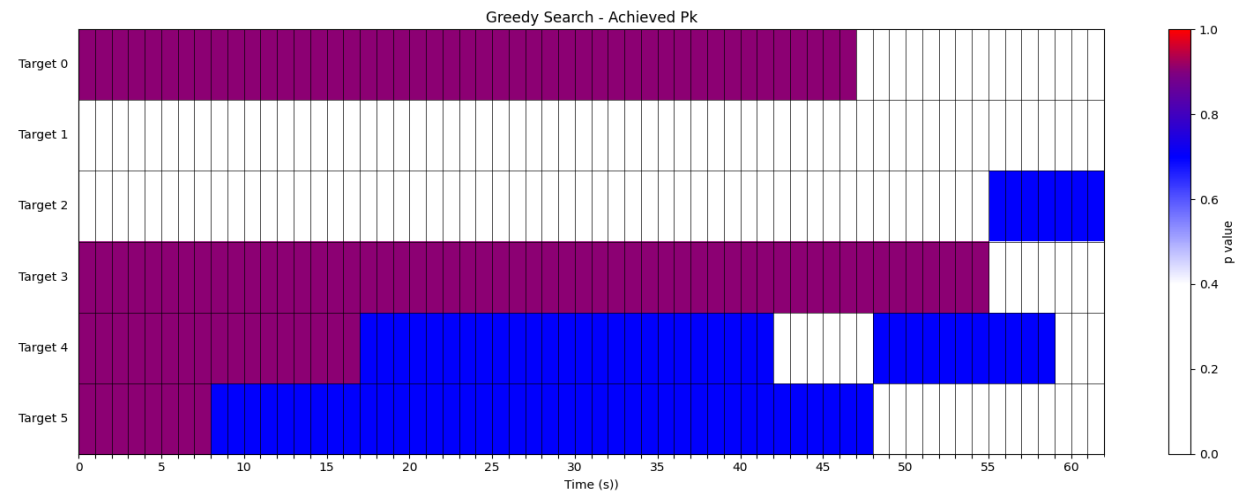
Time (s)	Event
7.1	Agent 1 gets attrited
17.0	Agent 3 gets attrited
47.4	Agent 7 collides with Target 0; Target 0 survives
47.8	Agent 0 collides with Target 0; Target 0 is destroyed
56.9	Agent 5 collides with Target 1; Target 1 is destroyed
60.6	Agent 4 collides with Target 2; Target 2 is destroyed
63.9	Agent 6 collides with Target 3; Target 3 is destroyed
69.0	Agent 2 collides with Target 4; Target 4 survives



Results – Completion CF



Time (s)	Event
7.1	Agent 1 gets attrited
17.0	Agent 3 gets attrited
47.4	Agent 7 collides with Target 0; Target 0 survives
47.8	Agent 0 collides with Target 0; Target 0 is destroyed
56.9	Agent 5 collides with Target 1; Target 1 is destroyed
60.6	Agent 4 collides with Target 2; Target 2 is destroyed
63.9	Agent 6 collides with Target 3; Target 3 is destroyed
69.0	Agent 2 collides with Target 4; Target 4 survives



Conclusion/Future Work

- These three novel cost functions induce different behaviors in target assignment as expected and work reliably within the decentralized framework of the problem

Future Work

- Implement in 3D simulation environment
 - 3DOF and 6DOF dynamic models
 - Midcourse/terminal phase guidance laws
 - Defending agents replace random attrition
- Methods for estimating attrition probability and weapons effectiveness

Questions?