Summary of: Multi-Agent Coordination by Decentralized Estimation and Control [1]

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[1] Yang, Peng, Randy A. Freeman, and Kevin M. Lynch. "Multi-agent coordination by decentralized estimation and control." IEEE Transactions on Automatic Control 53.11 (2008): 2480-2496.

Outline

- Motivation and Purpose
- Background Literature
- Design Methodology
- Formation Control: Modeling and Setup
- High-Pass Estimators: Theorem 2 and 3
  - Simulation results
- PI Estimators: Theorem 4
  - Simulation results
- Kinematic Agents

Motivation

- Intelligent behavior doesn't necessarily need global properties of the system
  - Schooling and flocking
  - Only applies to a specific subset of problems
- Not many tools to help design local controllers that achieve desired global conditions
- Reactive "memoryless" controllers are not often sufficient
  - Good only for a certain class of systems

Purpose

- To design a distributed control law to control global properties of the system.
  - Includes estimation of those properties as well as the local controller
- Shows that "the estimate-and-control approach provides guarantees on the convergence of the swarm to desired formation statistics in the face of changing communication networks and the addition and deletion of agents."

Relevant Background Literature - Average Estimation Algorithms

- R. Olfati-Saber and R. Murray, 2004
  - $\dot{x} = -Lx$ will converge to average of initial conditions if graph is strongly connected and balanced **even with switching network topologies and bounded uniform communication delays**.
  - Graph has to stay strongly connected and balanced
- D. P. Spanos, R. Olfati-Saber, and R. M. Murray, 2005
  - High-pass dynamic consensus filter proposed to track the average of changing inputs
- R. A. Freeman, P. Yang, and K. M. Lynch, 2006
  - Proposed dynamic consensus filter with better noise rejection and robustness to addition/deletion of agents
  - These filters are used in this paper

Relevant Backgound Literature - Expanding Beyond Reactive Controllers

The goal is to design a controller for a broader class of tasks.

- J. Cortés, 2006; and J. Cortés, S. Martíez, and F. Bullo, 2005
  - Spatially distributed objective function J (gradient of objective is a function only of the positions of the agents)
  - These objective functions suggest a control law
  - Sensing graphs have to be designed for the objective, since not all functions are spatially distributed over interaction graphs.
- M. Porfiri, D. G. Roberson, and D. J. Stilwell, 2006
  - Proposed decentralized estimate-and-control method, but causes steady-state error if the graph isn't fully connected

4-Step Design Methodology

1. Choose a cost functional J
2. Design an initial local controller $K^{initial}$.
   - This may use global information (can't implement it directly).
   - Often based on gradient descent.
3. Design a signal generator G, a global state estimator Q and R.
   - Must be sufficient so agent can use these estimators to calculate everything $K^{initial}$ needs.
4. Replace the global variables in $K^{initial}$ with the local estimates. Add terms to maintain convergence.

Example: Formation Control - Model and Formulation

- Swarm has n mobile agents, $p_1, \ldots, p_n \in \mathbb{R}^m$, and combined vector $p = [p_1^T \ldots p_n^T]^T \in \mathbb{R}^{mn}$, where m is the number of dimensions in the state vector
- $m = 2$, so in this problem, $p = [p_x, p_y]^T$
- Dynamics are $\ddot{p}_i = u_i$
- Communication is modeled as bidirectional
  - p_i communicates with p_j if and only if $|p_i - p_j| \geq r$
  - Each p represents a proximity graph that can change in time

Example: Formation Control

Step 1: Define cost functional J

- Define a goal function and goal vector

$$f(p) = \frac{1}{n(p)} \sum_{i=1}^{n(p)} \phi(p_i),\ f^* \in \mathbb{R}^\ell$$

- Where

$$\phi(p_i) = \begin{bmatrix} p_i \\ uds(p_i p_i^T) \end{bmatrix} \in \mathbb{R}^\ell$$

- Define

$$J(p) = [f(p) - f^*]^T \Gamma [f(p) - f^*]$$

Notes on Cost Function J

Controller will use the gradient ∇J.

Let $\text{Crit}(J) \triangleq p \in \mathcal{B} : \nabla J(p) = 0$

- "Bad" critical points: $f(p) \neq f^*$
  - Some bad critical points can be a stable equilibrium of $\dot{p} = \nabla J(p)$ even if it is a strict local maxima.
  - To rule this out, we can assume J is locally constant on $\text{Crit}(J)$. This will be the case for J (as defined) if f is subanalytic.
- "Good" critical points: $f(p) = f^*$
- We want bad critical points to be unstable:
  - Strongly Unsteady
    - Small perturbation will cause it to leave neighborhood of point forever
  - Weakly Unsteady
    - Lyapunov unstable and unattractive

Theoretical Results: Theorem 2

Let ϕ be defined as before, let $\mathcal{D} = \bigcup_{n=m+1}^{\infty} \mathbb{R}^{mn}$, and let $f^* \in f(\mathcal{D})$. Then there exists a symmetric matrix $\Gamma > 0$ s.t. for every bad critical point $p \in \mathcal{D}$ of J, the Hessian matrix $\mathcal{H}J(p)$ has at least one strictly negative eigenvalue.

- This means that for every bad critical point in D, there exists a Γ such that the bad critical point is not a local minimum (i.e., is a saddle point or local maxima).

Nonlinear Gradient Control with High-Pass Estimators

Step 2: Design an initial local controller $K^{initial}$

$$u_i = -B\dot{p}_i - [\mathcal{J}\phi(p_i)]^T \Gamma [f(p) - f^*]$$

Where

Agent State: $x_i = [p_i^T \dot{p}_i^T]^T$

Dynamics: $\dot{x}_i = \begin{bmatrix} \dot{p}_i \\ u_i \end{bmatrix}$

Damping matrix $B \in \mathbb{R}^{m \times m}$

Jacobian of ϕ, $\mathcal{J}\phi(\cdot)$

Nonlinear Gradient Control with High-Pass Estimators

Step 3: Design signal generator G, and global state estimator Q and R.

$$s_i = G(x_i, z_i, \eta_i, y_i, S_i) = \begin{bmatrix} p_i \\ y_i \end{bmatrix}$$

$$\dot{\eta}_i = Q(x_i, z_i, \eta_i, y_i, S_i) = -\gamma\eta_i - \sum_{j \neq i} \mathbf{a}(\mathbf{p_i}, \mathbf{p_j})[y_i - y_j]$$

$$y_i = R(x_i, z_i, \eta_i, y_i, S_i) = \eta_i + \phi(p_i)$$

Nonlinear Gradient Control with High-Pass Estimators

Step 4: Replace the global variables in $K^{initial}$ with local estimates

$$
\begin{aligned}
u_i &= K(x_i, z_i, \eta_i, y_i, S_i) \\
&= -B\dot{p}_i \\
&- [\mathcal{J}\phi(p_i)]^T \Gamma [y_i - f^*] \\
&- [\mathcal{J}\phi(p_i)]^T \Lambda [\mathcal{J}\phi(p_i)] \dot{p}_i
\end{aligned}
$$

Where $\Lambda \in \mathbb{R}^{\ell \times \ell}$ is a damping gain matrix

Limitations of High-Pass Estimators

- Controllers guarantee passivity from $e_i \to z_i$, where $e_i = f(p) - y_i$ and $z_i = [\mathcal{J}\phi(p_i)]\dot{p}_i$
- Estimators guarantee passivity from $z_i \to e_i$
- Dynamics of the estimator are $\dot{\chi} = -\gamma\chi$, where $\chi = \sum_{i=1}^{n} \eta_i$

Theoretical Results: Theorem 3

Suppose:

- ϕ is C^2 and proper
- n and $f^* \in f(\mathcal{C})$ are fixed
- $B + B^T > 0$
- $\Lambda + \Lambda^T \geq 0$
- $a(\cdot, \cdot)$ is C^1, and symmetric
- Graph is connected
- One of the following:
  (i) $\gamma = 0$ and $\sum_{i=1}^{n} \eta_t(t_0) = 0$, or
  (ii) $\gamma > 0$, $\Lambda + \Lambda^T > 0$, and $a(\cdot, \cdot)$ is constant

Theoretical Results: Theorem 3

Then:

- Each trajectory of the swarm system is bounded in forward time
- Positive limit set L^+ consists of equilibria

If (in addition to (i)):

- ϕ is subanalytic
- There exists $\mathcal{D} \subset \mathcal{B}$ s.t. $\Gamma \in \mathcal{G}(f^*, \mathcal{D})$ and $p(t) \in \mathcal{D}$ for all $t \geq t_0$

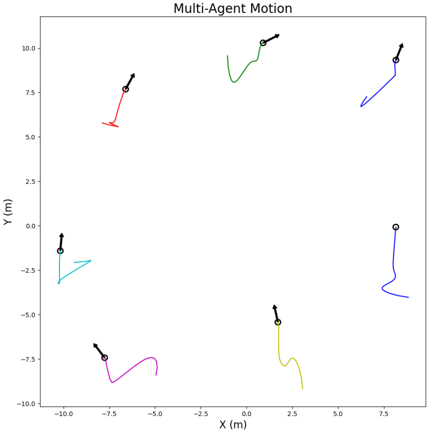
Then every positive limit set that contains a bad equilibrium is strongly unsteady.

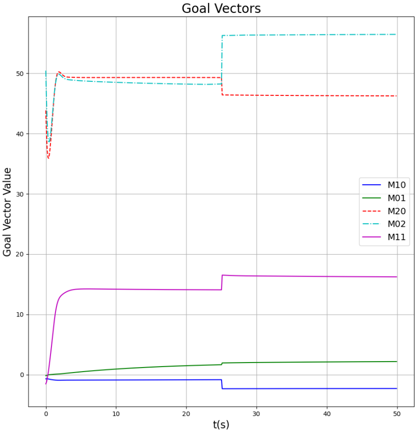
Note on Theorem 3

- Theorem 3 is valid if B is a C^1 function and a function of states and internal estimator states, and if $B(\cdot) + B(\cdot)^T > 0$.
- We can use B as another source of control.

Simulation Results: High-Pass Estimators

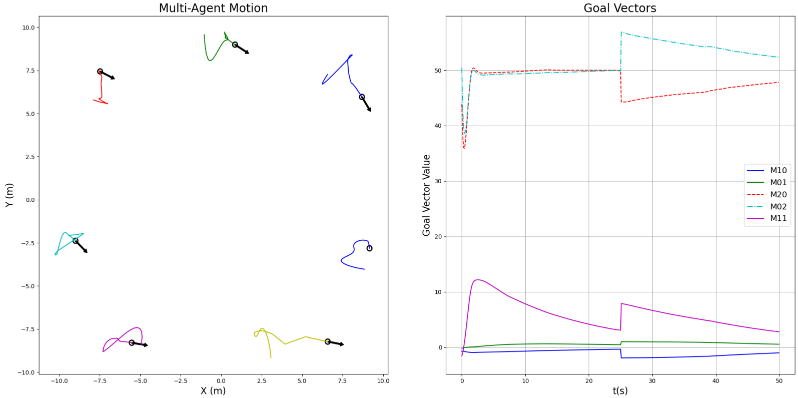
$$f^* = [0, 0, 50, 50, 0]^T, \ \gamma = 0.0$$





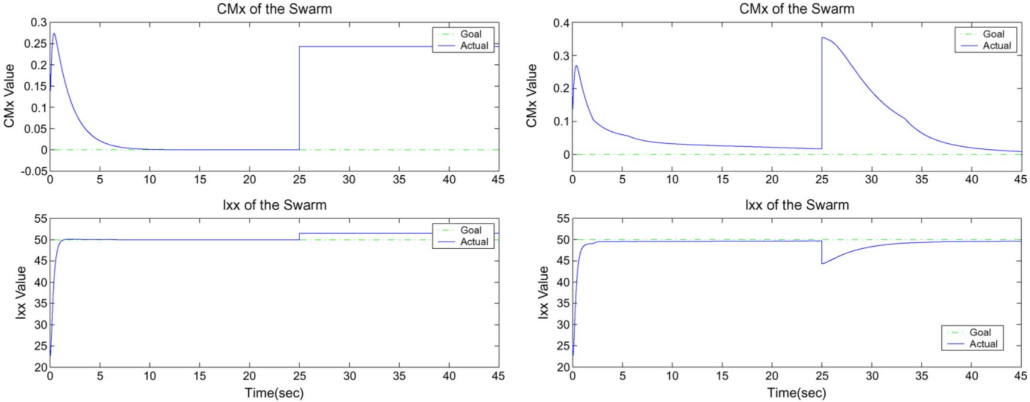
Simulation Results: High-Pass Estimators with forgetting factor

$$f^* = [0, 0, 50, 50, 0]^T, \ \gamma = 0.0$$



Simulation Results: High-Pass Estimators

These are the results from the paper using the same parameters as before.



Nonlinear Gradient Control With PI Estimators

Graph must stay connected!

$$s_i = \begin{bmatrix} p_i \\ \eta_i \end{bmatrix}, \text{ where } \eta_i = \begin{bmatrix} v_i \\ w_i \end{bmatrix}$$

$$\dot{v}_i = -\gamma v_i - \sum_{j \neq i} a(p_i, p_j)[v_i - v_j] + \sum_{j \neq i} b(p_i, p_j)[w_i - w_j] + \gamma \phi(p_i)$$

$$\dot{w}_i = -\sum_{j \neq i} b(p_i, p_j)[v_i - v_j]$$

$$y_i = v_i$$

Nonlinear Gradient Control with PI Estimators

$$
\begin{aligned}
u_i = &- B\dot{p}_i \\
&- [\mathcal{J}\phi(p_i)]^T \Gamma [y_i - f^*] \\
&- [\mathcal{J}\phi(p_i)]^T \Lambda [\mathcal{J}\phi(p_i)] \dot{p}_i \\
&- \mathbf{c}\zeta(\mathbf{p_i})\dot{\mathbf{p}}_\mathbf{i}
\end{aligned}
$$

Where $c > 0$ is a scalar nonlinear damping gain and $\zeta(p_i) : \mathbb{R}^m \to \mathbb{R}$ is a C^1 function.

Theoretical Results: Theorem 4

Suppose:

- ϕ is C^2 and proper
- n and $f^* \in f(\mathcal{C})$ are fixed
- $B + B^T > 0$
- $\Lambda + \Lambda^T \geq 0$
- $a(\cdot,\cdot)$ **and** $b(\cdot,\cdot)$ **are** C^1**, symmetric, and bounded, and** b **has bounded 1st order partial derivatives**
- Graph is connected
- One of the following:
  - (i) $\gamma = 0$ and $\sum_{i=1}^{n} \eta_t(t_0) = 0$, or
  - (ii) $\gamma > 0$, $\Lambda + \Lambda^T > 0$, and $a(\cdot,\cdot)$ is constant

Theoretical Results: Theorem 4

$$\lambda_{max}(\Gamma) < \delta_i \lambda_{min}(\Lambda + \Lambda^T) \leq \delta_2 c$$

Theoretical Results: Theorem 4

Then:

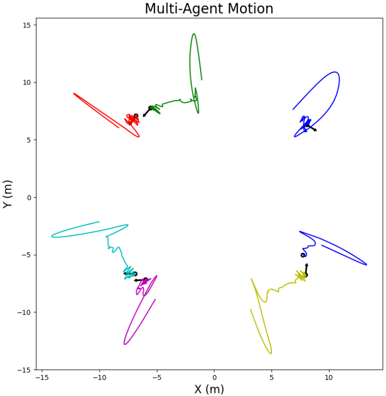
- Each trajectory of the swarm system is bounded in forward time
- Positive limit set L^+ consists of equilibria

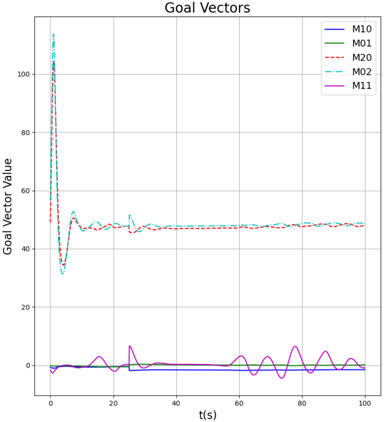
If (in addition to (i)):

- ϕ is subanalytic
- There exists $\mathcal{D} \subset \mathcal{B}$ s.t. $\Gamma \in \mathcal{G}(ff^*, \mathcal{D})$ and $p(t) \in \mathcal{D}$ for all $t \geq t_0$

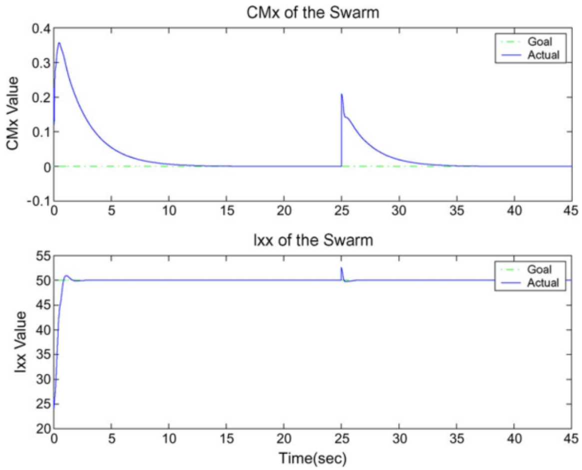
Then every positive limit set that contains a bad equilibrium is strongly unsteady.

Simulation Results: PI Estimators





Simulation Results: PI Estimators



Kinematic Agents

We can also control velocity ($u_i = \dot{p}_i$) instead of acceleration, using either the high-pass or the PI estimators. The control law becomes

$$u_i = -\left[B + [\mathcal{J}\phi(p_i)]^T \Lambda [\mathcal{J}\phi(p_i)]\right]^{-1} [\mathcal{J}\phi(p_i)]^T \Gamma [y_i - f^*]$$

The convergence properties are the same as before.

Assumptions to keep in mind

- Bidirectional communication
- Communication network can change, but must stay connected