Assignment 6

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March 19, 2019

1 Question 1

1. Suppose that $X_1, \cdots, X_{15}$ are from a distribution with an unknown variance σ^2. We are using the sample variance S^2 to estimate σ^2. Implement the bootstrap method to estimate $\text{Var}(S^2)$ if the data are 5,4,9,6,21,17,11,20,7,12,21,15,13,16,8.

1.1 Codes

```
#include<iostream>
#include<cmath>
using namespace std;
double samVar(){
    int count=15;
    int index;
    double u;     //u(0,1)
    double mean,mean2,samVar;
    double data[15]={5,4,9,6,21,17,11,20,7,12,21,15,13,16,8};

    mean = 0;
    mean2 = 0;
    for(int i = 0; i < count; i++)
    {
        u = ((double)rand()/(RAND_MAX));
        index = (int) (u*15);
        mean = mean + data[index];
        mean2 = mean2 + data[index]*data[index];
    }
    samVar = (mean2-mean*mean/15.0)/14.0;
    return samVar;
}

int main(){
    double theta,mean,mean2,variance;
    int count = 100000;
    srand(1) ;    // set up random seed

    mean = 0;
    mean2 = 0;
    for(int i = 0; i < count; i++)
    {
        theta = samVar();
        mean = mean + theta;
        mean2 = mean2 + theta*theta;
    }
    variance = (mean2 - mean*mean/count)/count;
    cout<<"Variance(S^2) is "<<variance<<endl;

}
```

1.2 Results

```
Variance(S^2) is 58.6956
```

2 Question 2

2. In Homeworks 3 and 5, you were asked to continually roll a pair of fair dice until all possible outcomes $2, 3, \cdots, 12$ had occurred at least once and conduct a simulation study to approximate thee expected number of dice rolls that are needed. Implement the bootstrap to approximate the variance your results when using $N = 1000$ realizations.

2.1 Codes

```
#include<iostream>
#include<cmath>
using namespace std;
int twoDices(){
    double u1,u2 ; //u1,u2~u(0,1)
    int n1,n2,outcome;//n1,n2~{1,2,3,4,5,6};n1+n2
    int outComes[11] = {2,3,4,5,6,7,8,9,10,11,12}; //outcomes
    int outComeSum;//indicator whether all possible outcomes are shown up
    int const maxSum = 77; // sum of all outcomes
    int i;
    outComeSum = 0;
    i = 0;
    while(outComeSum != maxSum){
        u1 = ((double) rand() / (RAND_MAX));
        u2 = ((double) rand() / (RAND_MAX));
        n1 = (int)(u1 * 6.0) + 1 ;
        n2 = (int)(u2 * 6.0) + 1 ;
        outcome = n1 + n2;
        if( outcome == outComes[outcome-2]){
            outComes[outcome-2] = 0;
            outComeSum = outComeSum + outcome;
        }
        i = i + 1;
    }
    return i;
}
double samMean(int data[1000]){
    int count=1000;
    int index;
    double u;     //u(0,1)
    double mean;
    mean = 0;
    for(int i = 0; i < count; i++)
    {
        u = ((double)rand()/(RAND_MAX));
        index = (int) (u*1000);
        mean = mean + data[index];
    }
    mean = mean /count;
    return mean;
}
 int main(){
     double x;          //realization
     double mean, mean2, variance;  //mean, variance of estimator
     int count = 1000;
     int data[count];  //store generated data
     srand(1);// set up random seed

     for(int i = 0; i < count; i++)
     {
        data[i] = twoDices();
     }
     mean = 0;
     mean2 = 0;
     for(int i = 0; i < count; i++)
     {
         x = samMean(data);
         mean = mean + x;
         mean2 = mean2 + x*x;
     }
    variance = (mean2 - mean*mean/count)/(count-1);
    cout<<"Variance is "<<variance<<endl;
 }
```

2.2 Results

```
Variance is 1.21473
```

3 Question 3

3. Busses arrive at a bus terminal according to a Poisson process with rate 15 per hour. Each bus is equally likely to discharge 20, 21,$\cdots$,40 passengers with the numbers in the different busses being independent. Simulate the arrival of passengers over a one hour period. Find the mean number of passengers arriving over a one hour period. What is the probability more than 500 passengers arrive over a one hour period?

3.1 Codes

```
#include<iostream>
#include<cmath>
using namespace std;

int busComing(){
    int numPassengers;//total number of passengers
    double t;//current time
    double u1,u2;//u(0,1),exp(15)
    t = 0;
    numPassengers = 0;
    while(t < 1.0){
        u1 = ((double)rand()/(RAND_MAX));
        u2 = ((double)rand()/(RAND_MAX));
        t = t - log(u1)/15;
        numPassengers = numPassengers + 20 + ((int) u2*21);
    }
    return numPassengers;
}

int main(){
    int N = 100000;
    int comer;
    double mean; //number of total arrivers in a hour
    int count;
    double prob;//prob. of total >500
    mean = 0;
    count = 0;
    srand(1);  //set up random seed
    for(int i = 0; i < N; i++)
    {
        comer = busComing();
        if (comer > 500) {
            count = count + 1;
        }
        mean =  mean + comer;
    }
    prob = ((double) count / N);
    mean = mean / N;
    cout<<"Mean number of passengers arriving over a hour is "<<mean<<endl;
    cout<<"Probability of customers greater than 500 is "<<prob<<endl;
}
```

3.2 Results

```
Mean number of passengers arriving over a hour is 320.285
Probability of customers greater than 500 is 0.01098
```

4 Question 4

4. Customers arrive at a store according to a nonhomogeneous Poisson process with intensity

$$\lambda(t) = 3 + \frac{4}{t+1}$$

Simulate the arrival of customers over a ten hour period. Find the mean number of customers arriving over a ten hour period.

4.1 Codes

```
#include<iostream>
#include<cmath>
using namespace std;
double possionProcess(){
    double numCustmers;//total number of custmers
    double t;//current time
    double u1,u2;//u(0,1)
    double lambdat;
    t = 0;
    numCustmers = 0;
    while(t < 10.0){
        u1 = ((double)rand()/(RAND_MAX));
        t = t - log(u1)/7;
        if (t>10.0){
            break;
        }
        u2 = ((double)rand()/(RAND_MAX));
        lambdat = 3.0 + 4.0/(t+1.0);
        if (u2 < lambdat/7) {
            numCustmers = numCustmers + 1;
        }
    }
    return numCustmers;
}
int main(){
    int N = 100000;//total realizations
    double mean;
    double custmers;
    srand(1);
    for(int i = 0; i < N; i++)
    {
        custmers = possionProcess();
        mean = mean + custmers;
    }
    mean = mean / N;
    cout<<"Mean␣Number␣of␣custmers␣arriving␣over␣ten␣hours␣is␣"<<mean<<endl;
}
```

4.2 Results

```
Mean Number of custmers arriving over ten hours is 39.5686
```