6) En el caso 3D tenemos que si $\{e^i\}$, define un sistema de coordenadas (dextrogiro) no necesariamente ortogonal, entonces, demuestre que:

a) $$e^i = \frac{e_j \times e_k}{e_i \cdot (e_j \times e_k)}, \quad i, j, k = 1, 2, 3$$ y sus permutaciones cíclicas:

Sea $\vec{a} = a^i |e_i\rangle$, de manera que:

$\vec{a} = a^i e_i \quad (i = 1, 2, 3)$, y sabemos que: $e_i e^j = \delta_i^j$

$\Rightarrow$ $e_1 e^2 = 0, \; e_1 e^3 = 0, \; e_2 e^1 = 0, \; e_2 e^3 = 0, \; e_3 e^1 = 0, \; e_3 e^2 = 0$

y ademas: $e_1 e^1 = 1, \; e_2 e^2 = 1, \; e_3 e^3 = 1.$ (*)

Como e^1 es perpendicular a e_2 y e_3 $\Rightarrow e^1 = \alpha_1 (e_2 \times e_3)$

Por ende $\rightarrow$ $e^2 = \alpha_2 (e_3 \times e_1)$ y $e^3 = \alpha_3 (e_1 \times e_2)$. (**)

Entonces, reemplazando los e^i en (*) tenemos

$e_1 [\alpha_1 (e_3 \times e_2)] = 1 \Rightarrow \alpha_1 e_1 \cdot (e_3 \times e_2) = 1$

$$\therefore \alpha_1 = \frac{1}{e_1 \cdot (e_3 \times e_2)}$$

y así;

$$\alpha_2 = \frac{1}{e_2 \cdot (e_1 \times e_3)}$$

$$\alpha_3 = \frac{1}{e_3 \cdot (e_2 \times e_1)}$$

Por lo tanto, al reemplazar los α_i en $(**)$, tenemos que:

$$\left.\begin{aligned} e^1 &= \frac{(e_2 \times e_3)}{e_1 (e_2 \times e_3)} \\ e^2 &= \frac{(e_3 \times e_1)}{e_2 \cdot (e_3 \times e_1)} \\ e^3 &= \frac{(e_1 \times e_2)}{e_3 \cdot (e_1 \times e_2)} \end{aligned}\right\} e^i$$

$\Rightarrow$

$$e^i = \frac{(e_j \times e_k)}{e_i \cdot (e_j \times e_k)} \quad //$$

b) Si los volumenes $V = e_1 \cdot (e_2 \times e_3)$ y $\tilde{V} = e^1 (e^2 \times e^3)$, entonces $V \cdot \tilde{V} = 1$

tenemos que:

$$V \tilde{V} = [e_1 \cdot (e_2 \times e_3)][e^1 (e^2 \times e^3)]$$

$$= [\cancel{e_1 (e_2 \times e_3)}]\left[\frac{(e_2 \times e_3)}{\cancel{e_1 (e_2 \times e_3)}} (e^2 \times e^3)\right]$$

$$= (e_2 \times e_3) \cdot (e^2 \times e^3)$$

$$= (e_2 e^2)(e_3 e^3) - \overset{0}{\cancel{(e_2 e^3)}}\overset{0}{\cancel{(e_3 e^2)}}$$

$$= (e_2 e^2)(e_3 e^3) = 1 \cdot 1 = 1$$

$$\Rightarrow V \cdot \tilde{V} = 1 \quad //$$

d) $\vec{w}_1 = 4\hat{i} + 3\hat{j} + \hat{k}$, $\vec{w}_2 = 3\hat{i} + 3\hat{j}$ y $\vec{w}_3 = 2\hat{k}$

1)

$$e^i = \frac{(w_j \times w_k)}{w_i \cdot (w_j \times w_k)} =$$

$\Rightarrow$

$$e^1 = \frac{(w_2 \times w_3)}{w_1 \cdot (w_2 \times w_3)} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix}}{w_1 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix}}$$

$$\Rightarrow e^1 = \frac{6\hat{i}}{24} = \frac{1}{4}\hat{i}$$

$$e^2 = \frac{(w_3 \times w_1)}{w_2 \cdot (w_3 \times w_1)} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 4 & 3 & 1 \end{vmatrix}}{w_2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 4 & 3 & 1 \end{vmatrix}} = \frac{-6\hat{i} + 8\hat{j}}{6}$$

$$\Rightarrow e^2 = -\hat{i} + \frac{4}{3}\hat{j}$$

$$e^3 = \frac{(w_1 \times w_2)}{w_3 \cdot (w_1 \times w_2)} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ 3 & 3 & 0 \end{vmatrix}}{w_3 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ 3 & 3 & 0 \end{vmatrix}} = \frac{-3\hat{i} + 3\hat{j} + 3\hat{k}}{6}$$

$$\Rightarrow e^3 = \frac{1}{2}(-\hat{i} + \hat{j} + \hat{k})$$