Solución: Pendes 2a, 2d, sección 1.5.9

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- $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} = x^i \hat{\imath}_i$
- $\vec{a} = \vec{a}(\vec{r}) = \vec{a}(x,y,z) = a^i(x,y,z)\hat{\imath}_i$

  $\vec{b} = \vec{b}(\vec{r}) = \vec{b}(x,y,z) = b^i(x,y,z)\hat{\imath}_i$
- $\Phi = \Phi(\vec{r}) = \Phi(x,y,z) \wedge \varphi = \varphi(\vec{r}) = \varphi(x,y,z)$

Utilizar la notación de índices y demostrar:

a) $\vec{\nabla}(\Phi\varphi) = \Phi\vec{\nabla}(\varphi) + \varphi\vec{\nabla}(\Phi)$

En efecto:

$$\vec{\nabla}(\Phi\varphi) = \vec{\nabla}(\Phi(\vec{r})\varphi(\vec{r})) = \vec{\nabla}(\Phi(x^i)\varphi(x^i))$$

$$= \frac{\partial}{\partial x^j}(\Phi(x^i)\varphi(x^i))\hat{\imath}_j = \left(\Phi(x^i)\frac{\partial\varphi(x^i)}{\partial x^j} + \varphi(x^i)\frac{\partial\Phi(x^i)}{\partial x^j}\right)\hat{\imath}_j$$

$$= \Phi(\vec{r})\frac{\partial\varphi(\vec{x})}{\partial x^i}\hat{\imath}_j + \varphi(\vec{r})\frac{\partial\Phi(\vec{r})}{\partial x^i}\hat{\imath}_j = \Phi(\vec{r})\vec{\nabla}\varphi(\vec{r}) + \varphi(\vec{r})\vec{\nabla}\Phi(\vec{r}).$$

d) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{a})$ ¿Qué puede decir de $\vec{\nabla}\times(\vec{\nabla}\cdot\vec{a})$?

Tenemos que: $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{a}) = \varepsilon_{ijk}\partial^i\partial^j a^k(x,y,z)$ → Por Levi-Civita.

$$= \partial^1\partial^2 a^3(x,y,z) - \partial^1\partial^3 a^2(x,y,z)$$
$$-\partial^2\partial^1 a^3(x,y,z) + \partial^2\partial^3 a^1(x,y,z)$$
$$+\partial^3\partial^1 a^2(x,y,z) - \partial^3\partial^2 a^1(x,y,z)$$

Por el teorema de Schwartz tenemos que

$$\partial^1\partial^2 a^3 = \partial^2\partial^1 a^3$$

$$\partial^1\partial^3 a^2 = \partial^3\partial^1 a^2$$

$$\partial^2\partial^3 a^1 = \partial^3\partial^2 a^1$$

Por lo tanto podemos decir que:

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{a}) = \varepsilon^{ijk}\partial_i\partial_j a_k(x,y,z) = 0$$

• de $\vec{\nabla}\times(\vec{\nabla}\cdot\vec{a})$ Podemos decir que al ser $\vec{\nabla}\cdot\vec{a}$ un valor numerico (escalar) no es posible aplicar el rotacional por ende $\vec{\nabla}\times(\vec{\nabla}\cdot\vec{a})$ es cero o no tiene algun resultado.

f) $\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - \vec{\nabla}^2\vec{a}$

En efecto: $\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) = \varepsilon^{ijk}\partial_j(\varepsilon^{kmn}\partial_m a_n)\hat{i}_i$

$$= \partial_2(\partial_1 a_2 - \partial_2 a_1)\hat{i}_1 - \partial_3(\partial_3 a_1 - \partial_1 a_3)\hat{i}_1 - \partial_1(\partial_1 a_2 - \partial_2 a_1)\hat{i}_2$$

$$+ \partial_3(\partial_2 a_3 - \partial_3 a_2)\hat{i}_2 + \partial_1(\partial_3 a_1 - \partial_1 a_3)\hat{i}_3 - \partial_2(\partial_2 a_3 - \partial_3 a_2)\hat{i}_3$$

$$= \partial_3(\partial_1 a_1)\hat{i}_3 - \partial_1\partial_1 a_3\hat{i}_3 + \partial_1(\partial_2 a_2)\hat{i}_1 - \partial_2\partial_2 a_1\hat{i}_1$$

$$+ \partial_2(\partial_3 a_3)\hat{i}_2 - \partial_3\partial_3 a_2\hat{i}_2 + \partial_2(\partial_1 a_1)\hat{i}_2 - \partial_1\partial_1 a_2\hat{i}_2$$

$$+ \partial_3(\partial_2 a_2)\hat{i}_3 - \partial_2\partial_2 a_3\hat{i}_3 + \partial_1(\partial_3 a_3)\hat{i}_1 - \partial_3\partial_3 a_1\hat{i}_1$$

$$= \partial^i(\partial^j a_j)\hat{i}_i - (\partial^j\partial_j)a_i\hat{i}_i = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - (\vec{\nabla}\cdot\vec{\nabla})\vec{a} =$$