1.6.7

② Demuestre:

a) $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha)$

- Partimos de la formula de De Moivre

$$[\cos(\theta) + i\,\text{sen}(\theta)]^n = \cos(n\theta) + i\,\text{sen}(n\theta), n \in \mathbb{Z}$$

con $n = 3$ tenemos:

$$\cos(3\theta) + i\,\text{sen}(3\theta) = [\cos(\theta) + i\,\text{sen}(\theta)]^3$$

- resolviendo el cubo:

$$\cos(3\theta) + i\,\text{sen}(3\theta) = \cos^3(\theta) - 3\cos(\theta)\,\text{sen}^2\theta + i[3\cos^2(\theta)\,\text{sen}\,\theta - \text{sen}^3\theta]$$

Igualando la parte real:

$$\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\,\text{sen}^2\theta$$

$\theta = \alpha$

$$\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2(\alpha)$$

b) $\operatorname{sen}(3\alpha) = 3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)$

- Partiendo de la demortración 2a e igualando la parte imaginaria tenemos:

$$\cos(3\theta) + i\operatorname{sen}(3\theta) = \cos^3(\theta) - 3\cos(\theta)\operatorname{sen}^2(\theta) + i\left[3\cos^2(\theta)\operatorname{sen}(\theta) - \operatorname{sen}^3\theta\right]$$

$$\operatorname{sen}(3\theta) = 3\cos^2(\theta)\operatorname{sen}(\theta) - \operatorname{sen}^3(\theta)$$

$$\theta = \alpha$$

$$\operatorname{sen}(3\alpha) = 3\cos^2(\alpha)\operatorname{sen}(\alpha) - \operatorname{sen}^3(\alpha)$$