

Math 4190 Spring 2024 - Dr. Neil Calkin

Properties of sets

1. Counting and Sets

$$A = (A - B) \cup (A \cap B)$$

$$B = (B - A) \cup (A \cap B)$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

formulas for cardinalities of unions/intersections of sets

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cup B \cup C|$$

general formula for set union intersection

$$|A_1 \cup A_2 \cup \dots \cup A_n| = \sum_{i=1}^n |A_i| - \sum_{i < j} |A_i \cap A_j| + \sum_{i < j < k} |A_i \cap A_j \cap A_k| - \dots$$

Pigeonhole principle

if you place $n+1$ pieces of mail into n mailboxes, then at least one mailbox will have more than one piece of mail

Generalised Pigeonhole Principle. if

$$|A| > k * |B|$$

then for every total function $f : A \rightarrow B$ maps at least $k+1$ different elements to the same element of B .

chess problem

- claim: for any coloring of a chessboard with different colors, we can find a rectangle so that the squares in the corners of the rectangle are all the same color.

six people at a party problem

- among any six people some have shaken hands (red edge) some have not shaken hands (blue edge)

Combinations and Permutations

I am a combinatorist by training, my phd is in combinatorics. I have never said the word k-comb/perm in anger except to say that it makes em angry. Given a finite set of n values $\{a_1, a_2, a_3, \dots, a_n\}$ a k -permutation of n objects is a list with or without repetition of k values from the set.

a k -combination is the unordered version of the same if repetition is not allowed it is a k -subset.

- if repetition is not allowed it is a k -subset
- if repetition is allowed it is a k -multisubset

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

- Birthday Paradox
- with 23 people in a room, you have a 50% chance of having 2 people with the same birthday.

$$\frac{365!}{365^k(365-k)!}$$

who was pingala? where did he live?

Jan 29 2024

Binomial Coefficients, Pascal's Triangle,

- Recall: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$$(1+x)^{n+1} = (1+x)(1+x^n) = \sum_{k=0}^n 1 + x \binom{n}{k} x^k$$

-Binomial Theorem

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k$$

Jan 31 2024

Combinations and Permutations

- How many ways to write, with repetition, k numbers from 1 to n if
 1. The order we write them is irrelevant
 2. The order we write them is relevant
- We regard 122, 212, 221 as the same object.
- Stars and bars approach
 - any string of k stars and $n-1$ bars | will convert to a string of k 1's, 2's, ..., n 's
 - There are $n+k-1$ positions in which to place k stars and $n-1$ |'s
 - There are $\binom{n+k-1}{k}$ ways to pick the k positions where the stars should be.
 - More formally.... Hopefully.... There are $n-1$ positions we need to pick to place the |'s so $\binom{n+k-1}{n-1}$ ways to do it
 - $\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!} = \binom{n+k-1}{n-1}$

N Choose K formula

- $\binom{n}{k} = \frac{n!}{(n-k)!k!}$
 - How many anagrams of MAIM MAIM MAMI AMMI MMAI IMMA IMAM IAMM MIMA AMIM
 - How many anagrams of MISSISSIPI? 11! permutations for distinguishing between MISSISSIPI (order matters)

Multinomial Coefficient

- numbers of the form

$$\frac{n!}{i_1!i_2! \dots i_k!}$$

are sometimes written $\binom{n}{i_1 i_2 \dots i_k}$

and are known as multinomial coefficients.

- $(x_1 + x_2 + \dots + x_n)^n = \sum_{(k)}$
- 4^11 know the difference

Feb 5 2024

- Something something, thomas jefferson cryptography.
- william tut in something park britian who gives a shit

Proofs.... Finally

- How do we prove things?
- Write down careful definitions
- keep track of axioms/given information/assumptions
- Build sequence of deductions to arrivev at the desired result.
- quest of the peacock: good book, lots of epic virtue signaling