Math 4190 Spring 2024 - Dr. Neil Calkin

Properties of sets

1. Counting and Sets

$$A = (A - B) \cup (A \cap B)$$

$$B = (B - A) \cup (A \cap B)$$

$|A \cup B| = |A| + |B| - |A \cap B|$

formulas for cardinalities of unions/intersections of sets

$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cup B \cup C|$

general formula for set union intersection

$| \ A_1 \cup A_2 \cup ..... \cup A_n \ | = \sum_{i=1}^{n} |A_i| - \sum_{i<j} | \ A_i \cap A_j \ | + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - ...$

Pidgeonhole principle

if you place n+1 pieces of mail into n mailboxes, then at least one mailbox will have more than one piece of mail

Generalised Pigeonhole Principle. if

$$|A| > k * |B|$$

then for every total function $f : A \to B$ maps at least $k + 1$ different elements to the same element of B.

chess problem

- claim: for any coloring of a chessboard with different colors, we can find a rectangle so that the squares in the corners of the rectangle are all the same color.

six people at a party problem

- among any six people some have shaken hands (red edge) some have not shaken hands (blue edge)

Combinations and Permutations

I am a combinatorist by training, my phd is in combinatorics. I have never said the word k-comb/perm in anger except to say that it makes em angry.

Given a finite set of n values $\{a_1, a_2, a_3, ..., a_n\}$

a k-permutation of n objects is a list with or without repetition of k values from the set.

a k-combination is the unordered version of the same if repetition if not allowed it is a k-subset.
- if repetition is not allowed it is a k-subset
- if repetition is allowed it is a k-multisubset

$\binom{n}{k} = \frac{n!}{(n-k)!k!}$

- Birthday Paradox
- with 23 people in a room, you have a 50% chance of having 2 people with the same birthday.

$\frac{365!}{365^{k(365-k)!}}$

who was pingala? where did he live?

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Binomial Coefficients, Pascal's Triangle,

- Recall: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$(1+x)^{n+1} = (1+x)(1+x^n) = \sum_{k=0}^{n} 1 + x\binom{n}{k}x^k$

-Binomial Theorem

$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k}x^k$