Math 4190 Spring 2024 - Dr. Neil Calkin

Properties of sets

1. Counting and Sets

$$A = (A - B) \cup (A \cap B)$$

$$B = (B - A) \cup (A \cap B)$$

$|A \cup B| = |A| + |B| - |A \cap B|$

formulas for cardinalities of unions/intersections of sets

$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cup B \cup C|$

general formula for set union intersection

$|\ A_1 \cup A_2 \cup ..... \cup A_n\ | = \sum_{i=1}^{n} |A_i| - \sum_{i<j} |\ A_i \cap A_j\ | + \sum_{i<j<k} |A_i \cap A_j \cap A_k| - ...$

Pidgeonhole principle

if you place n+1 pieces of mail into n mailboxes, then at least one mailbox will have more than one piece of mail

Generalised Pigeonhole Principle. if

$$|A| > k * |B|$$

then for every total function $f : A \to B$ maps at least $k + 1$ different elements to the same element of B.

chess problem

- claim: for any coloring of a chessboard with different colors, we can find a rectangle so that the squares in the corners of the rectangle are all the same color.

six people at a party problem

- among any six people some have shaken hands (red edge) some have not shaken hands (blue edge)

Combinations and Permutations

I am a combinatorist by training, my phd is in combinatorics. I have never said the word k-comb/perm in anger except to say that it makes em angry. Given a finite set of n values $\{a_1, a_2, a_3, ..., a_n\}$ a k-permutation of n objects is a list with or without repetition of k values from the set.

a k-combination is the unordered version of the same if repetition if not allowed it is a k-subset.
- if repetition is not allowed it is a k-subset
- if repetition is allowed it is a k-multisubset

$\binom{n}{k} = \frac{n!}{(n-k)!k!}$

- Birthday Paradox
- with 23 people in a room, you have a 50% chance of having 2 people with the same birthday.

$\frac{365!}{365^{k(365-k)!}}$

who was pingala? where did he live?

Jan 29 2024 – Binomial Coefficients, Pascal's Triangle,

- Recall: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$

$(1+x)^{n+1} = (1+x)(1+x^n) = \sum_{k=0}^{n} 1 + x\binom{n}{k}x^k$

-Binomial Theorem

$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k}x^k$

Jan 31 2024 – Combinations and Permutations

- How many way to write, with repetition, k numbers from 1 to n if
  1. The order we write them is irreleveant
  2. The order we write them is releveant
- We regard 122, 212, 221 as the same object.
- Stars and bar approach
  - any string of k stars and n-1 bars | will convert to a strings of k 1′s, 2′1, … n's
  - There are n+k-1 positions in which to place k stars and n-1 |'s
  - There are $\binom{n+k-1}{k}$ ways to pick the k positions where the stars should be.
  - More formally….. Hopefully…. There are n-1 positions we need to pick to plave the |'s so

  $\binom{n+k-1}{n-1}$ ways to do it
  - $\binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1!)} = \binom{n+k-1}{n-1}$

N Choose K formula

- $\binom{n}{k} = \frac{n!}{(n-k!)k!}$
  - How many anagrams of MAIM MAIM MAMI AMMI MMAI IMMA IMAM IAMM MIMA AMIM
  - How many anagrams of MISSISSIPPI? 11! permutations for distinguishing between MISSISSIPPI (order matters)

Multinomial Coeficcient

- numbers of the form

$$\frac{n!}{i_1!i_2!....i_k!}$$

  are sometimes written $\binom{n}{i_1 i_2 ... i_k}$

and are known as multinomial coeficcients.

- $(x_1 + x_2 + ... + x_n)^n = \sum_k$
- 4^11 know the difference

Feb 5 2024

- Something something, thomas jefferson cryptography.
- william tut in something park britian who gives a shit

Proofs…. Finally

- How do we prove things?
- Write down careful definitions
- keep track of axioms/given information/assumptions
- Build sequence of deductions to arrivev at the desired result.
- quest of the peacock: good book, lots of epic virtue signaling

Feb 12 2024 – Proofs part 2 (3 technically)

- Hypothesis: Every integer greater than 2 is divisible by a prime number.

Feb 16 2024 – Proof part 4

- $\left(\frac{1}{10^5} \sum_{-\infty}^{\infty} e^{-\frac{n^2}{10^{10}}}\right)^2 \sim = \pi$
- What about quantified statements?
- Consider the statement $\forall x, P(x) \to Q(x)$
- Converse: $\forall x, Q(x) \to P(x)$
- inverse: $\forall x, \neg P(x) \to \neg Q(x)$
- contrapositive: $\forall x \neg Q(x) \to \neg(x)$

math club: 5:00 pm m105? floor one of M all the way down on the left.

Exam:
- True or False about predicate value of P(1,1), P(2,2), solve if there is a value of P
- Question aboout sets A = some set
- (i) is {3} element of A
- (ii) is {3} element of A
- Draw a venn diagram for some union or intersection of sets
- STATEMENT: for any horse H, if H is a palomino, h has a saddle. logic, modus ponens modus tollens
  - be able to write down the converse, contrapositive, and negation of the statement.
  - Consider the integers a, b: if a > 7 and b > 10 then a+b > 18.
    - let P = "a > 7", q = "b > 10" and r = "a+b > 18"
    - express the statement using p, q, r.
    - convert the statement using OR, write down the negation of a statement as an english sentence
    - calculation using binomial theorem: coefficient or otherwise.

      $$x^{10}$$

      in $(x^2 - 1)^8$
    - simple answer, no expansion.
    - an inclusion - exclusion question (dealing with sets and intersections and things)
    - Question about using truth tables to determine whether two statements involving p, q. r are logically equivalent.
    - counting questions, how many ways can you select 10 pizzas from 4 choices.
    - pidgeonhole principle

- selecting 5 students to represent the class at some presentation, anagram of mississippi
- Basic logical laws

Feb 26 2024

- low down triple dealing
- permutations involving dealing cards

Feb 28 2024

- Relations, binary relations.
- properties that relations on a set A can have. i
- aside we aer looking to develop the idea of "sameness" which we will call an "equivalence relation." Clearly we alwyasy want x to be the same as x.
- reflexive: x $\sim$ x
- Symmetric: we'll say that $\sim$ is symmetric if whenever a $\sim$ b then b $\sim$ a as well
- transitivity: we call a relation transitive if whenever aRb and bRc, we must have aRc as well.
- a symmetric transitive relation on A will have the property that for any a, if there is a

b elementof a so that

- our notion of sameness will be an equivalence relation if it is (i) reflexive, (ii) symmetric, (iii) transitive.

March 1 2024

- A function $F : A \to B$ is a particular kind of realtion on A x B it has the property that for every a, there is exactly one value f(a) $\in$ b
- some particular types of function are quire useful:
  - 1-to-1 injection
  - onto or surjection
  - both, bijection.
  - we say $|A| \leq |B|$ if there exists a 1-1 functions $f : A \to B$ if onto, then $|A| \geq |B|$
  - cantor turned the world of mathematics upside down when he proved that there are different sizes of infinity.

Cardinalities of sets

- There are sets bigger than |N|, proven by cantor
- Proof: take any function

  $$f : S \to P(S)$$

  we can construct a subset of s so that for any $a \in S, T \neq f(a)$ if $a \in f(a), a \notin T$
- Another proof: take any listing of the reals, we construct a number not in the list.
- Okay there are infinitely many different sizes of infinite sets.
- $$|\mathbb{Q}| = |\mathbb{N}|$$