# Recursive Methods

Sometimes when solving a problem, we can compute the solution of a simpler version of the *same problem*. Eventually we reach the most basic version, for which the answer is trivial.

Manager: Recorder:

Presenter: Reflector:

## Content Learning Objectives

*After completing this activity, students should be able to:*

- Identify the base case and recursive step of the factorial method.
- Trace a recursive method by hand and predict its final output.
- Explain what happens in memory when a method calls itself.

## Process Skill Goals

*During the activity, students should make progress toward:*

- Evaluating mathematical sequences to gain insight on recursion. (Information Processing)

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# Model 1 Factorial

"In mathematics, the ***factorial*** of a non-negative integer $n$, denoted by $n!$, is the product of all positive integers less than or equal to $n$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$."

Source: https://en.wikipedia.org/wiki/Factorial

| $n$ | $n!$ |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 2 |
| 3 | 6 |
| 4 | 24 |
| 5 | 120 |

## Questions (25 min) Start time:

**1**. Consider how to calculate $4! = 24$.

a) Write out all the numbers that need to be multiplied:

$4! =$

b) Rewrite the expression using $3!$ instead of $3 \times 2 \times 1$:

$4! =$

**2**. Write an expression similar to #1b showing how each factorial can be calculated in terms of a simpler factorial.

a) $3! =$

b) $2! =$

c) $100! =$

d) $n! =$

**3**. What is the value of $0!$ based on Model 1? Does it make sense to define $0!$ in terms of a simpler factorial? Why or why not?

*If we repeatedly break down a problem into smaller versions of itself, we eventually reach a basic problem that can't be broken down any further. Such a problem, like* 0!*, is referred to as the* ***base case****.*

**4**. Assume you already have a working method named `factorial(int n)` that returns $n!$ for any positive integer.

a) Review your answer to #2c that shows how to compute 100! using a simpler factorial. Convert this expression to Java by using the `factorial` method instead of the `!` operator.

b) Now rewrite your answer to #2d in Java using the variable `n`.

**5**. Here is a `factorial` method that includes output for debugging:

```
1  public static int factorial(int n) {
2      System.out.println("n is " + n);
3      if (n == 0) {
4          return 1;  // base case
5      } else {
6          System.out.printf("need factorial of %d\n", n - 1);
7          int answer = factorial(n - 1);
8          System.out.printf("factorial of %d is %d\n", n - 1, answer);
9          return n * answer;
10     }
11 }
12
13 public static void main(String[] args) {
14     System.out.println(factorial(3));
15 }
```

a) What specific method is invoked on line 7?

b) Why is the `if` statement required on line 3?

**6**. A method that invokes itself is called **recursive**. What two steps were necessary to define the `factorial` method? How were these steps implemented in Java?

**7**. How many distinct method calls would be made to `factorial` to compute the factorial of 3? Identify the value of the parameter *n* for each of these separate calls.

**8**. Here is the complete output from the program in #5. Identify which distinct method call printed each line. In other words, which lines were printed by `factorial(3)`, which lines were printed by `factorial(2)`, and so on.

```
n is 3
need factorial of 2
n is 2
need factorial of 1
n is 1
need factorial of 0
n is 0
factorial of 0 is 1
factorial of 1 is 1
factorial of 2 is 2
6
```

**9**. What happens if you try to calculate the factorial of a negative number? How could you prevent this behavior in the `factorial` method?

**10**. Trivia question: What is the largest factorial you can compute in Java when using `int` as the data type? If you don't know, how could you find out?

# Model 2 Summation

"In mathematics, ***summation*** (capital Greek sigma symbol: Σ) is the addition of a sequence of numbers; the result is their sum or total."

$$\sum_{i=1}^{100} i = 1 + 2 + 3 + \ldots + 100 = 5050$$

Source: https://en.wikipedia.org/wiki/Summation

## Questions (20 min) Start time:

**11**. Consider how to calculate $\sum_{i=1}^{4} i = 10$.

a) Write out all the numbers that need to be added:

$\sum_{i=1}^{4} i =$

b) Show how this sum can be calculated in terms of a smaller summation.

$\sum_{i=1}^{4} i =$

**12**. Write an expression similar to #11b showing how any summation of $n$ integers can be calculated in terms of a smaller summation.

$$\sum_{i=1}^{n} i =$$

**13**. What is the base case of the summation? (Write the complete formula, not just the value.)

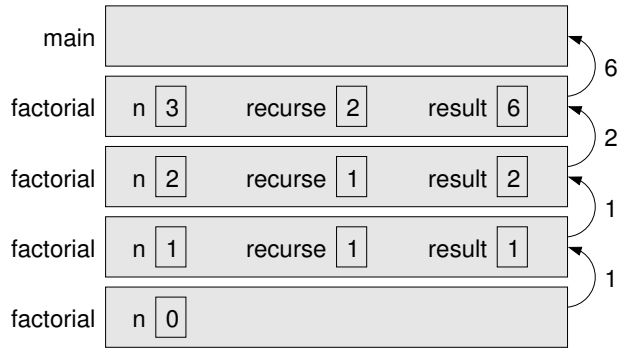
**14**. Implement a recursive method that takes a single parameter `n` and returns the sum $1 + 2 + \ldots + n$. It should only have an `if` statement and two `return` statements.

```
public static int summation(int n) {



}
```

**15.** Discuss how the `factorial` method below uses temporary variables. What lines would you have to change to implement the `summation` method instead?

```
public static int factorial(int n) {
    if (n == 0) {
        return 1;  // base case
    }
    int recurse = factorial(n - 1);
    int result = n * recurse;
    return result;
}
```

**16.** Here is a stack diagram of `factorial(3)` when invoked from `main`. Draw a similar diagram for `summation(3)` as described in the previous question.




**17.** Why are there no values for `recurse` and `result` in the stack diagram for the last call to `factorial` (when `n == 0`)?

**18.** Looking at the stack diagram, how is it possible that the parameter `n` can have multiple values in memory at the same time?