Maximum flow

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Let $G = (V, E), s, t)$ be a graph $G = (V, E)$ with source s and sink t. We will formulate the max-flow problem as a linear program.
Let $c : E \to \mathbb{R}^+$ be our cost function. We will make a slight modification by adding an edge from t to s that has infinite capacity. The objective is to maximize the flow along this edge. We have a variable f_{uv} for every edge $(u, v) \in E$. This is the *flow* over (u, v). The problem then becomes:

$$\begin{aligned}
&\text{maximize } f_{ts} \\
&\text{subject to } f_{ij} \leq c_{ij}, && (i,j) \in E \\
&\qquad \sum_{j:(j,i)\in E} f_{ji} - \sum_{j:(i,j)\in E} f_{ij} \leq 0, && i \in V \\
&\qquad f_{ij} \geq 0, && (i,j) \in E.
\end{aligned}$$