

# Analysis Of Algorithms

Assignment 1

Cameron Perdue

2020-09-17

1.  $2^{2n+1}$   
 $2^{2^n}$   
 $(n+1)!$   
 $n!$   
 $e^n$   
 $n * 2^n$   
 $2^n$   
 $(3/2)^n$   
 $n^{\lg(\lg(n))}, \lg(n)^{\lg(n)}$   
 $(\lg(n))!$   
 $n^3$   
 $n^2, 4^{\lg(n)}$   
 $n \lg(n), \lg(n!)$   
 $n, 2^{\lg(n)}$   
 $(\sqrt{2})^{\lg(n)}$   
 $2^{\sqrt{2} \lg(n)}$   
 $(\lg(n))^2$   
 $\ln(n)$   
 $\sqrt{\lg(n)}$   
 $\ln(\ln(n))$   
 $1, n^{1/\lg(n)}$

2.  $A^x, x^c, \text{kthrt}(x), \log_b(x)$

3. (a) Lemma  $n = O(n^2)$

Consider  $N \geq 1, c = 1$

Suppose  $n \geq 1$

$1 \leq n \Rightarrow n \leq n^2$  - By definition both sides are positive

$|n| \leq 1 * |n^2|$

(b) Lemma  $n^2 = O(n^2)$

Consider  $N \geq 1, c = 1$

Suppose  $n \geq 1$

$1 \leq n$

$0 \leq 0$

$n^2 \leq n^2$  - Multiply both sides by 0 then add  $n^2$

$|n^2| \leq 1 * |n^2|$

(c) Lemma  $3n^2 + 5n = n^2$

Consider  $N \geq 1$ ,  $c = 8$

Suppose  $n \geq 1 \Rightarrow 1 \leq n$

$$n \leq n^2$$

$$5n \leq 5n^2$$

$$3n^2 + 5n \leq 8n^2$$

$$|3n^2 + 5n| \leq 8 * |n^2|$$

4.  $\ln(n) - \ln(1)$  is  $O(\ln(n))$  because:

$\lim_{n \rightarrow \infty} (\ln(n) - \ln(1)) / \ln(n)$  is  $1 - 0$ , nonzero constant means that  $\ln(n) - \ln(1)$  is  $\Theta(\ln(n))$  which by definition makes it big  $O$  of  $n$ . If  $H_n$  is less than  $\ln(n) - \ln(1)$  then it is also  $O(\ln(n))$

6. (a) Base Case:

$$f(2; a, b) = f(1; b, a+b)$$

$$= a + b$$

$$f(2; a, b) = f(1; a, b) + f(0; a, b)$$

$$= a + b$$

Assume:

$$f(k; a, b) = f(k-1; a, b) + f(k-2; a, b)$$

Show:

$$f(k+1; a, b) = f(k; a, b) + f(k-1; a, b)$$

$$f(k+1; a, b) = f(k; b, a+b)$$

$$= f(k-1; b, a+b) + f(k-2; b, a+b)$$

$$= f(k; a, b) + f(k-1; a, b)$$

$$f(k+1; a, b) = f(k; a, b) + f(k-1; a, b)$$