Analysis of Algorithms Homework 2

1. $T_f(0) = 0$

$T_f(1) = 0$

$T_f(n) = 1 + T_f(n-1)$

Time complexity of $f(n; a, b) = n - 1$

$T_f(n) = 1 + (1 + T_f(n-2))$

$T_f(n) = k + T_f(n-k)$ let $k = n-1$

$= n - 1 + T_f(n-(n-1))$

$= n - 1 + T_f(1)$

$T_f(n) = n - 1 + 0$

2. Base case $n = 2$

$f(2; a, b) = f(1; b, a+b) = a + b$

$f(1; a, b) = b$

$L^1(a, b) = (b, a+b)$ ✓

<u>Assume</u>

$L^k(a, b) = (f(k; a, b) + f(k+1; a, b))$

<u>Show</u> $L^{k+1}(a, b) = (f(k+1; a, b) + f(k+2; a, b))$

$L^{k+1}(a, b) = L^k(b, a+b) = (f(k; b, b+a), f(k+1, b, b+a))$

$f(k+1; a, b) = f(k; b, b+a)$

$f(k+2; a, b) = f(k+1; b, b+a)$

By the induction hypothesis

$\square$

4. (a) Pseudo-Polynomial time is a function that takes time according to a polynomial function based on the length of the input

(b) ~~fib~~ it is not a pseudo polynomial time algorithm. Doubling the input value, 1 more bit, increases the amount of additions exponentially

(c) ~~fib~~ It is pseudo polynomial. doubling the value also doubles the the amount of calculations

5. (a) $T(n+1) = T(n) + 5$

$T(x) = T(x-1) + 5$

$= (T(x-2) + 5) + 5$

$= ((T(x-3)+5)+5)+5$

$= T(n-k) + 5k$

$T(n) = 5n$

$k = n$

$T(n) = T(0) + 5n$

$= 0 + 5n$

(b) $T(n+1) = n + T(n)$

$T(x) = (x-1) + T(x-1)$

$= x - 1 + (x - 2 + T(x-2))$

$k = n$

$\frac{n^2 - n}{2} = T(n)$

$T(n) = \sum_{i=0}^{n-1} i = \frac{(n-1)(n)}{2}$

6. (a) $T(n+1) = 2T(n)$

$T(x) = 2T(x-1)$

$= 2(2T(x-1))$

$= 2(2(2T(x-3)))$

$k = n$

$T(n) = 2^n$

$T(n) = 2^k T(n-k)$

(b) $T(n+1) = 2^{n+1} + T(n)$

$k = n$

$T(x) = 2^x + T(x-1)$

$T(n) = n2^n + 1$

$T(n) = 2^n + T(n-1)$

$= 2^n + (2^{n-1} + T(n-2))$

$= 2^n + (2^{n-1} + (2^{n-2} + T(n-3)))$

$2^{n-(k-1)} + T(n-k)$

$2^{n-(n-1)}$

$2^{n-n+1} + T(0)$

$2^1 + 1 = 3$

$T(n) = 3$

7. (a) $T(n) = n + T(n/2)$

$n = 2^m$

$k = m$

$$\frac{2^m}{2^{m-1}} = 2$$

$$\begin{aligned}
T(n) &= n + T(n/2) \\
&= n + (n/2 + T(n/4)) \\
&= n + n/2 + n/4 + T(n/8) \\
&= \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \frac{n}{2^3} \dots \frac{n}{2^{k-1}} + T(n/2^k)
\end{aligned}$$

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$2^m \dots 2^{m-1} + 2^{m-3} \dots 8 + 4 + 2$

$$\sum_{i=1}^{m} 2^i + T(n/2^k) = \frac{2^{m+1} - 1}{2-1} + 1$$

$$= 2^{m+1} - 1 + 1$$

$$= 2^{m+1}$$

$$T(n) = 2n$$

(b) $T(n) = 1 + T(n/3)$

$3^m = n$

$k = m$

$$\begin{aligned}
&= 1 + 1 + T(n/9) \\
&= k + T(n/3^k) \\
&= m + T(1)
\end{aligned}$$

$m\,lg(2) + 1\,lg(2)$

$lg(2^m) + 1$

$$T(n) = lg(n) + 1$$