task

April 15, 2021

1 Tarefa 2: Álgebra Linear e Otimização para ML - MO431A

Universidade Estadual de Campinas (UNICAMP), Instituto de Computação (IC)

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[1]:
```python
# RA & Name
print('265673: ' + 'Gabriel Luciano Gomes')
print('192880: ' + 'Lucas Borges Rondon')
print('265674: ' + 'Paulo Júnio Reis Rodrigues')
```

```
265673: Gabriel Luciano Gomes
192880: Lucas Borges Rondon
265674: Paulo Júnio Reis Rodrigues
```

1.0.1 Imports necessários

[2]:
```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LogNorm
import tensorflow as tf
```

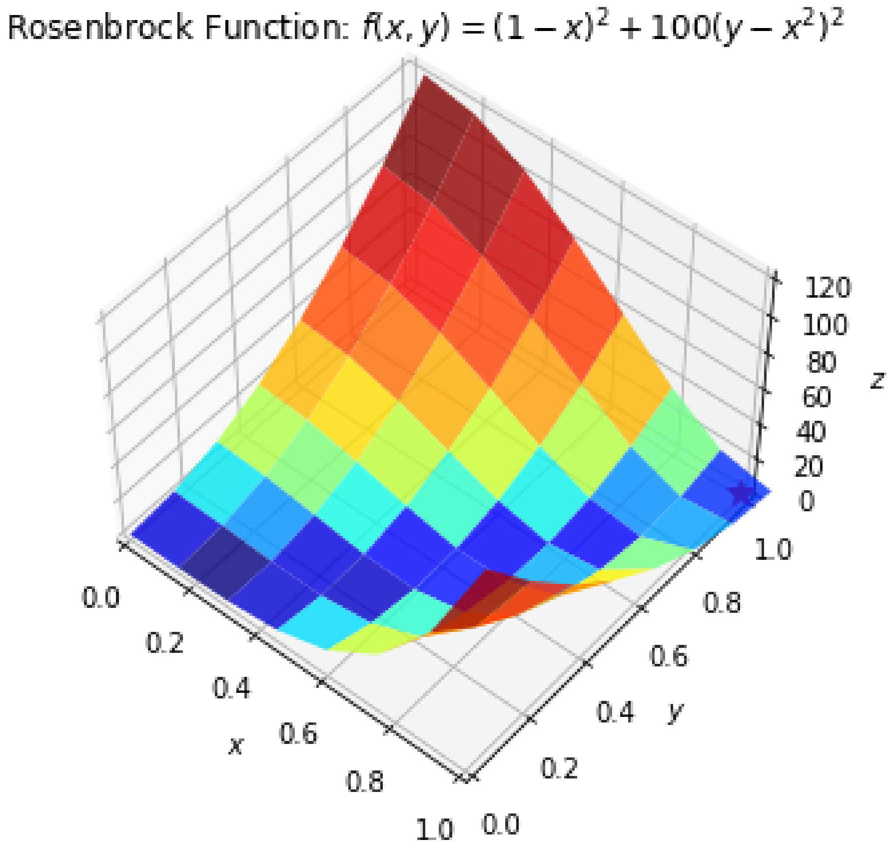
1.1 Equação Rosenbrock 2D

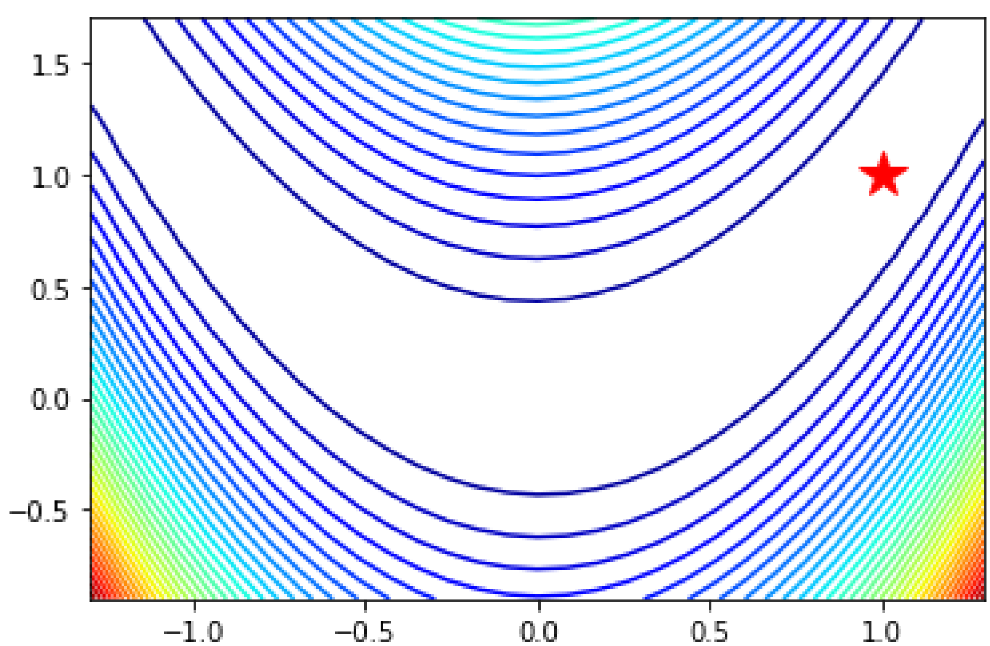
[3]:
```python
# função fx de Rosenbrock em 2D
def fx(x1, x2):
    return ((1 - x1)**2) + 100*((x2 - x1**2)**2)
```

1.1.1 Formato da função

[4]:
```python
xmin, xmax, xstep = 0, 1, .15
ymin, ymax, ystep = 0, 1, .15

x, y = np.meshgrid(np.arange(xmin, xmax + xstep, xstep), np.arange(ymin, ymax +
 ↪ystep, ystep))

z = fx(x, y)

minima = np.array([1, 1])
```

```python
minima_ = minima.reshape(-1, 1)

fig = plt.figure(figsize=(8, 5))
ax = plt.axes(projection='3d', elev=50, azim=-50)

plt.title(r'Rosenbrock Function: $f(x,y) = (1-x)^2 + 100(y-x^2)^2$ ')

ax.plot_surface(x, y, z, norm=LogNorm(), rstride=1, cstride=1,
                edgecolor='none', alpha=.8, cmap=plt.cm.jet)
ax.plot(*minima_, fx(*minima_), 'r*', markersize=10)

ax.set_xlabel('$x$')
ax.set_ylabel('$y$')
ax.set_zlabel('$z$')

ax.set_xlim((xmin, xmax))
ax.set_ylim((ymin, ymax))

plt.show()

_, ax = plt.subplots(1, 1)
# make a contour plot of the rosenbrock function surface.
X, Y = np.meshgrid(np.linspace(-1.3, 1.3, 31), np.linspace(-0.9, 1.7, 31))
Z = fx(X, Y)
ax.plot(*minima_, 'r*', markersize=18)
ax.contour(X, Y, Z, 40, cmap=plt.cm.jet)

ax.set_xlim(-1.3, 1.3)
ax.set_ylim(-0.9, 1.7)
plt.show()
```


Rosenbrock Function: $f(x, y) = (1 - x)^2 + 100(y - x^2)^2$
x
y
z
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
120


1.5
1.0
0.5
0.0
−0.5
−1.0
−0.5
0.0
0.5
1.0

1.1.2 Função gradiente de f

[5]:
```
# derivada parcial de fx em x
def dx(x1, x2):
    return 2 * (200*x1**3 - 200*x1*x2 + x1 -1)

# derivada parcial de fx em y
def dy(x1, x2):
    return 200*(x2 - x1**2)
```

1.1.3 Plot das informações

[6]:
```
def plotInfo(points, fx_values, steps):
    newList = [(elem1, elem2) for elem1, elem2 in points]
    plt.title(r'Values of $f(x)$ X # of iterations')
    plt.plot(fx_values)
    plt.ylabel(r'values of $f(x)$')
    plt.xlabel(r'# of iterations')
    plt.show()

    path = np.array(newList).T

    _, ax = plt.subplots(1, 1)
    # make a contour plot of the rosenbrock function surface.
    X, Y = np.meshgrid(np.linspace(-1.3, 1.3, 31), np.linspace(-0.9, 1.7, 31))
    Z = fx(X, Y)
    ax.plot(*minima_, 'r*', markersize=18)
    ax.contour(X, Y, Z, 40, cmap=plt.cm.jet)
    ax.quiver(path[0,:-1], path[1,:-1], path[0,1:]-path[0,:-1], path[1,1:
↪]-path[1,:-1],
        scale_units='xy', angles='xy', scale=2, color='k')

    ax.set_xlim(-1.3, 1.3)
    ax.set_ylim(-0.9, 1.7)
    plt.show()


    print(f'Best value of x: {points[len(points)-1]}')
    print(f'Number os steps: {steps}')
```

1.1.4 L.r = 1.e-3

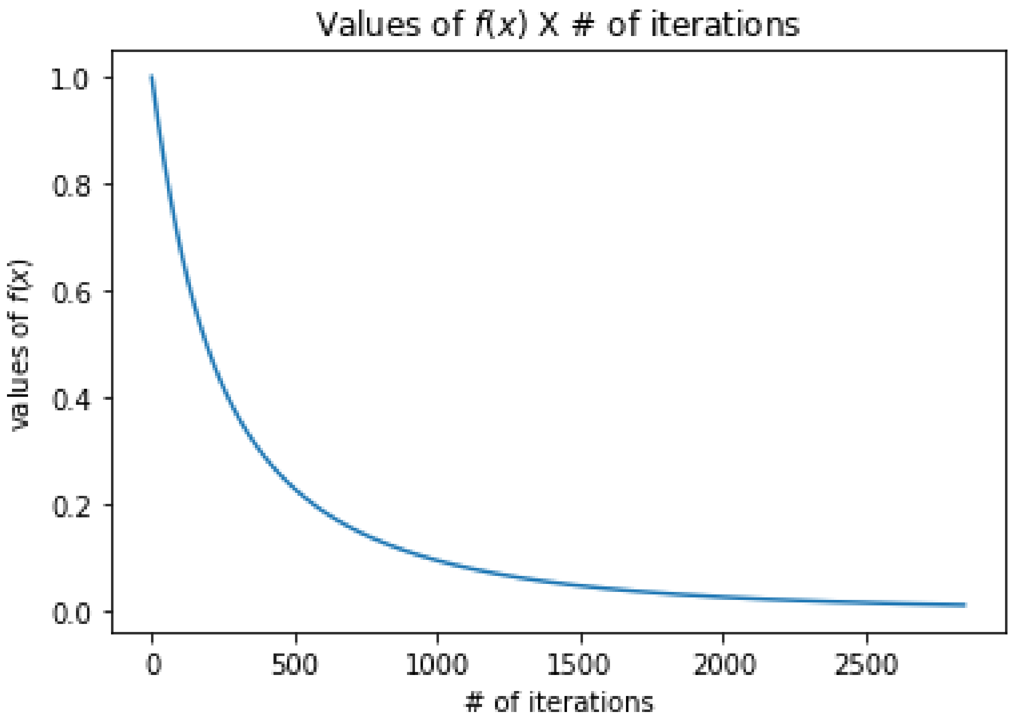
[7]:
```
x = 0.0
y = 0.0
points = [np.array([x, y])]
```

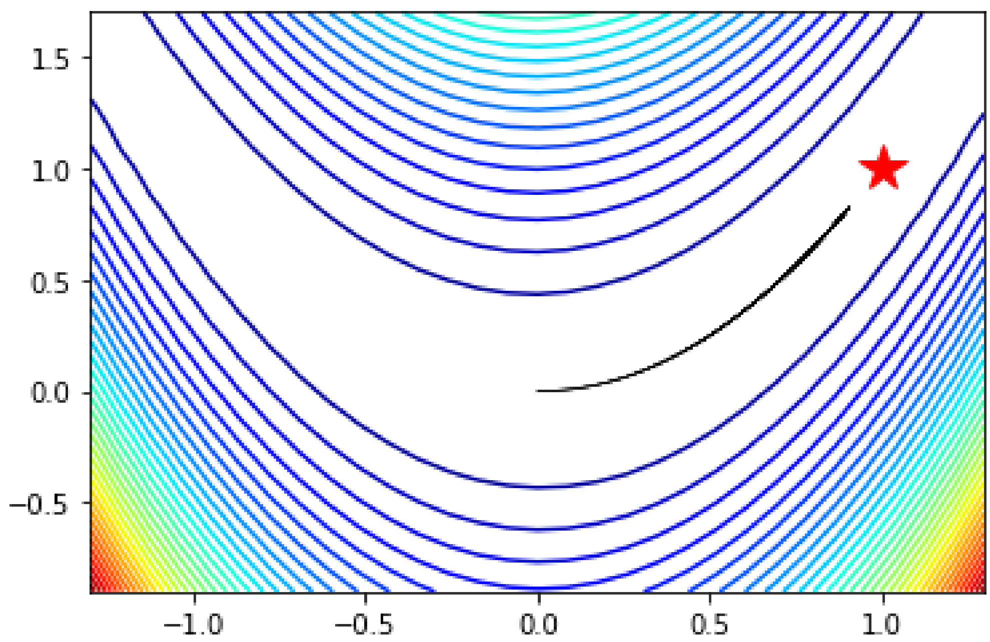
```
fx_values = [fx(x,y)]

tol = 1
steps = 0
lr = 10**-3

while (tol > 10**-5) and (steps < 50000):
    # Compute function to old point
    f_old = fx(x, y)

    # Compute gradient and new point
    x -= lr * dx(x, y)
    y -= lr * dy(x, y)

    # Compute function to new point
    f_new = fx(x, y)

    # Compute tolerancy
    tol = np.abs(f_new - f_old)

    # Append current point to list
    points.append(np.array([x, y]))
    fx_values.append(f_new)

    steps+=1

print(f'Valor da tolerância: {tol}, Valor de passos: {steps}')
```

```
Valor da tolerância: 9.998940970952844e-06, Valor de passos: 2842
```

[8]:

```
plotInfo(points, fx_values, steps)
```





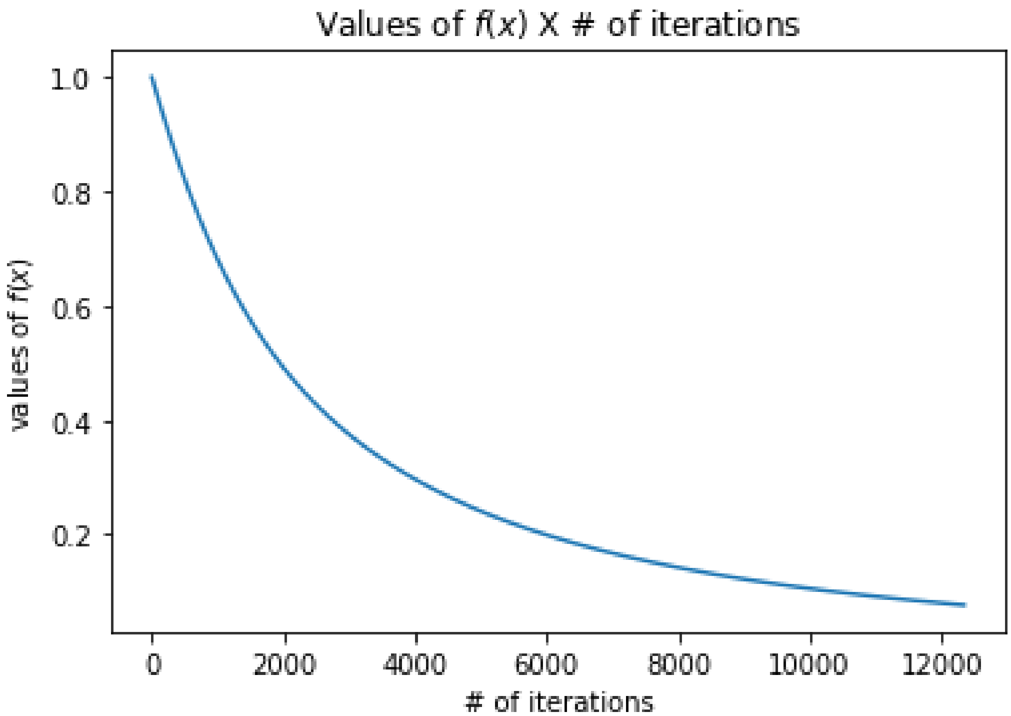

```
Best value of x: [0.90489266 0.81845007]
Number os steps: 2842
```

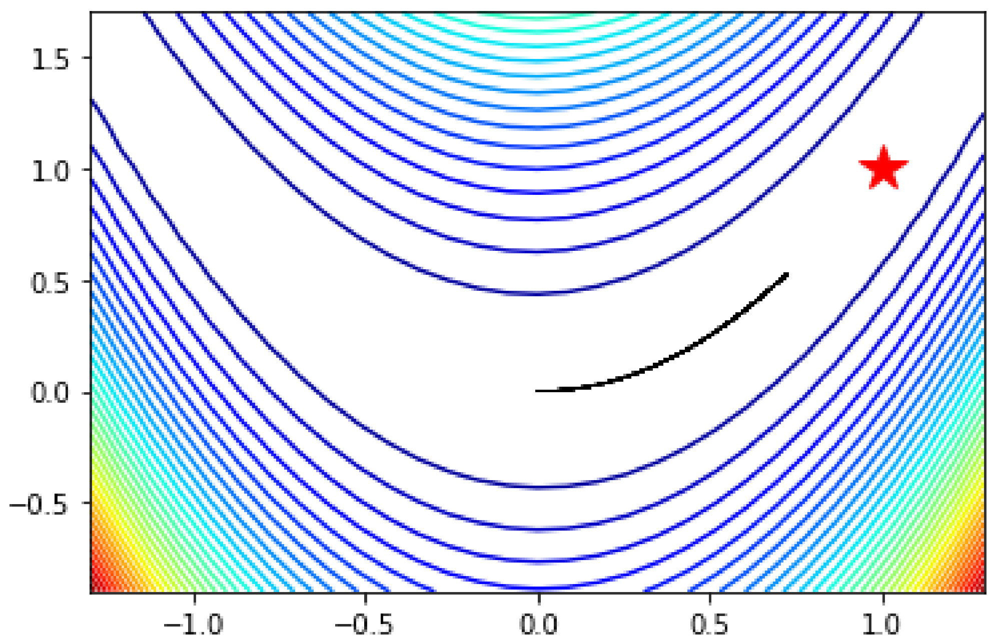
1.1.5 L.r = 1.e-4

[9]:
```
x = 0.0
y = 0.0
points = [np.array([x, y])]
fx_values = [fx(x,y)]

tol = 1
steps = 0
lr = 10**-4

while (tol > 10**-5) and (steps < 50000):
    # Compute function to old point
    f_old = fx(x, y)

    # Compute gradient and new point
    x -= lr * dx(x, y)
    y -= lr * dy(x, y)

    # Compute function to new point
    f_new = fx(x, y)

    # Compute tolerancy
    tol = np.abs(f_new - f_old)

    # Append current point to list
    points.append(np.array([x, y]))
    fx_values.append(f_new)

    steps+=1

print(f'Valor da tolerância: {tol}, Valor de passos: {steps}')
```

```
Valor da tolerância: 9.999994764961495e-06, Valor de passos: 12346
```

[10]:
```
plotInfo(points, fx_values, steps)
```






```
Best value of x: [0.72373005 0.5225001 ]
Number os steps: 12346
```

1.1.6 L.r grande

[11]:
```python
x = 0.0
y = 0.0
points = [np.array([x, y])]
fx_values = [fx(x,y)]

tol = 1
steps = 0
lr = 8**-2

while (tol > 10**-5) and (steps < 50000):
    # Compute function to old point
    f_old = fx(x, y)

    # Compute gradient and new point
    x -= lr * dx(x, y)
    y -= lr * dy(x, y)

    # Compute function to new point
    f_new = fx(x, y)

    # Compute tolerancy
    tol = np.abs(f_new - f_old)

    # Append current point to list
    points.append(np.array([x, y]))
    fx_values.append(f_new)

    steps+=1

print(f'Valor da tolerância: {tol}, Valor de passos: {steps}')
```

```
---------------------------------------------------------------------------
OverflowError                             Traceback (most recent call last)
<ipython-input-11-c56dd3c1f90d> in <module>
     17
     18     # Compute function to new point
---> 19     f_new = fx(x, y)
     20
     21     # Compute tolerancy

<ipython-input-3-a71a6fa93933> in fx(x1, x2)
      1 # função fx de Rosenbrock em 2D
      2 def fx(x1, x2):
----> 3     return ((1 - x1)**2) + 100*((x2 - x1**2)**2)
```

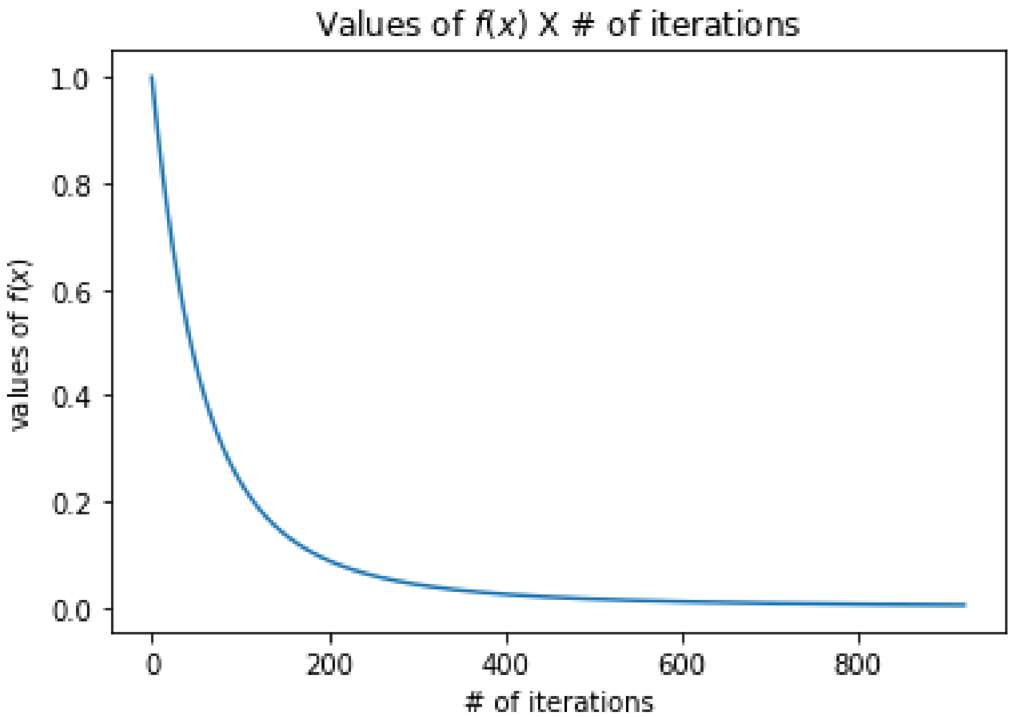
```
OverflowError: (34, 'Result too large')
```

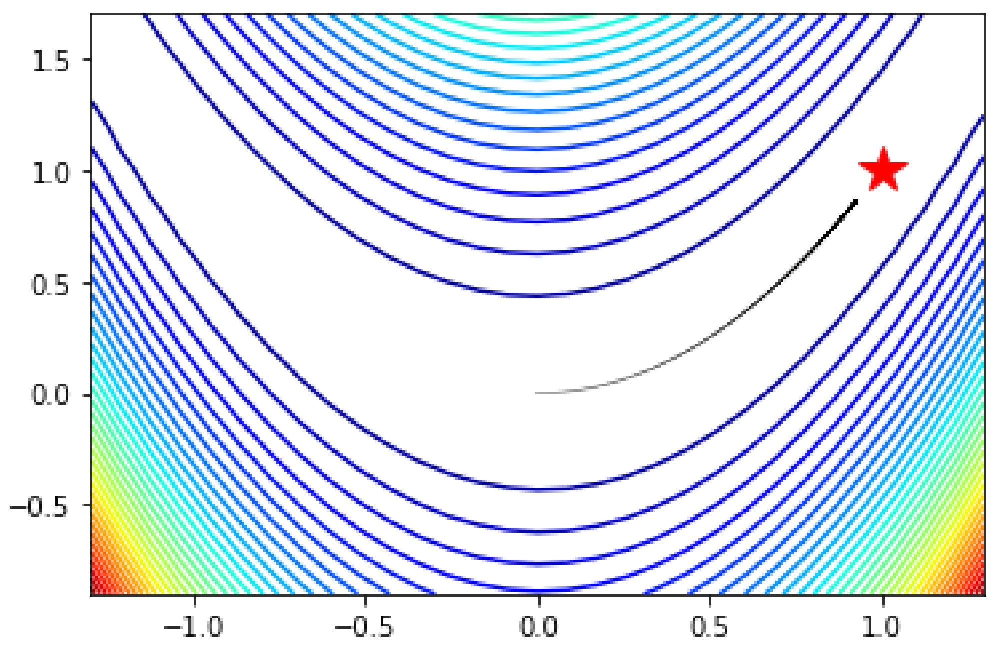
1.1.7 Politica de redução do l.r

[12]:
```
x = 0.0
y = 0.0
points = [np.array([x, y])]
fx_values = [fx(x,y)]

tol = 1
steps = 0
lr = 3**-5

while (tol > 10**-5) and (steps < 50000):
    # Compute function to old point
    f_old = fx(x, y)

    # Compute gradient and new point
    x -= lr * dx(x, y)
    y -= lr * dy(x, y)

    # Compute function to new point
    f_new = fx(x, y)

    # Compute tolerancy
    tol = np.abs(f_new - f_old)

    # Append current point to list
    points.append(np.array([x, y]))
    fx_values.append(f_new)

    # Updating learning rate
    lr = lr * 0.999

    steps+=1

print(f'Valor da tolerância: {tol}, Valor de passos: {steps}')
```

```
Valor da tolerância: 9.985739491158961e-06, Valor de passos: 919
```

[13]:
```
plotInfo(points, fx_values, steps)
```



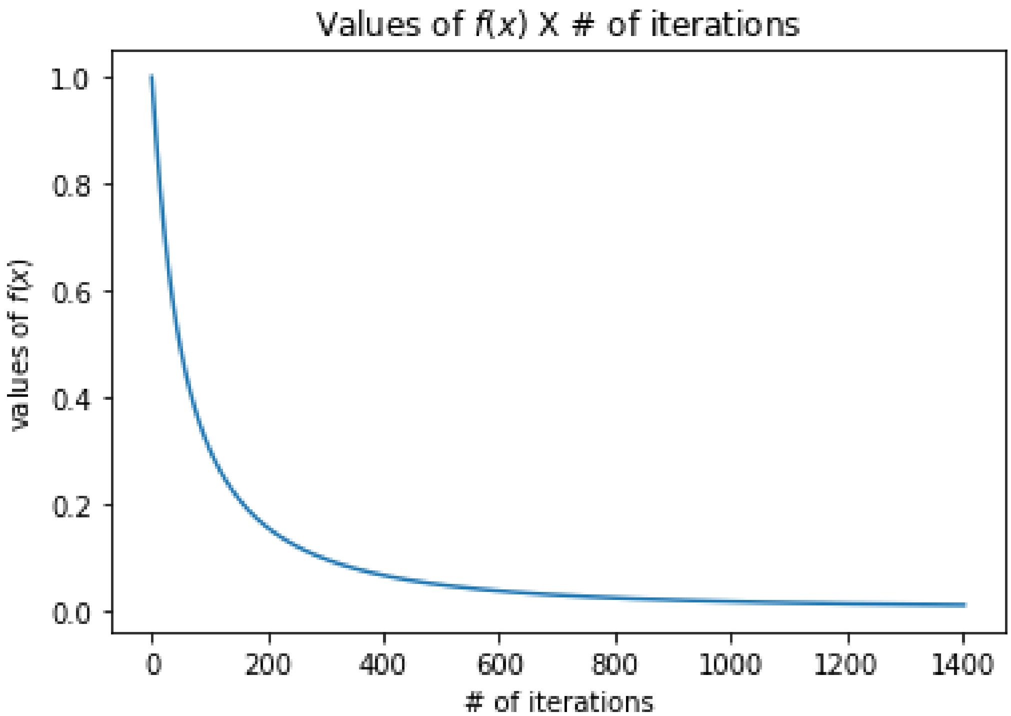


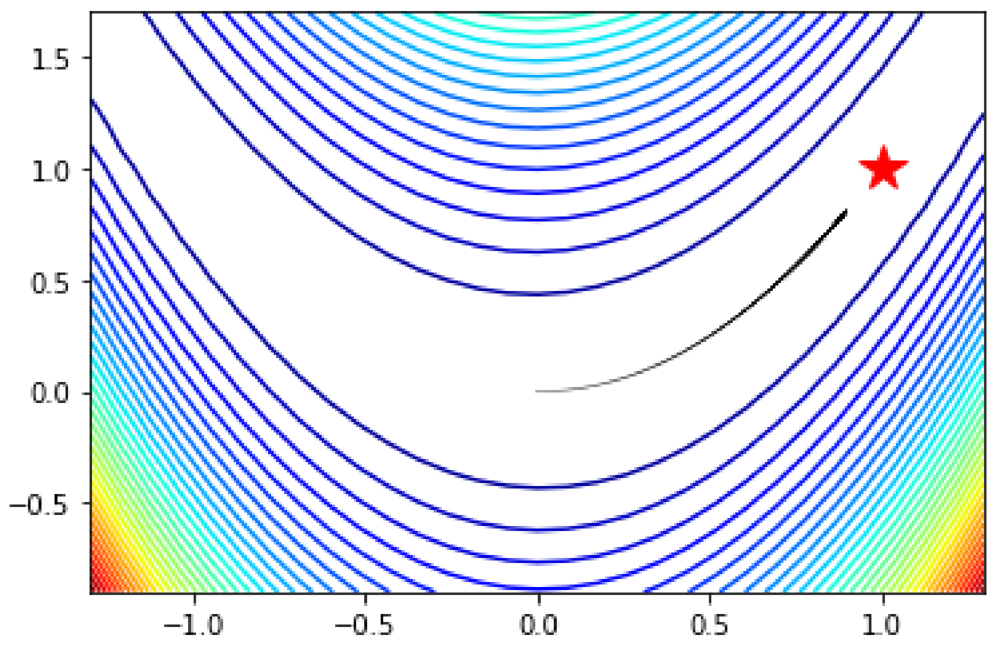

```
Best value of x: [0.92901798 0.86280725]
Number os steps: 919
```

1.1.8 Utilizando Tensorflow para calcular o gradiente

[14]:
```
points = [np.array([0.0, 0.0])]


x = tf.Variable(0.0)
y = tf.Variable(0.0)

fx_values = [fx(x,y).numpy()]
tol = 1
steps = 0

lr = 3**-5

while (tol > 10**-5) and (steps < 50000):

    # Compute function to old point
    f_old = fx(x, y).numpy()

    # Computing gradient with TensorFlow
    with tf.GradientTape(persistent=True) as g:
      g.watch([x,y])
      # Rosenbrock function
      z = (1-x)**2 + 100*((y-x**2)**2)

    # Generatin new point
    # Multiply gradient by learning rate
    x.assign_sub(lr * g.gradient(z, x).numpy())
    y.assign_sub(lr * g.gradient(z, y).numpy())

    # Compute function to new point
    f_new = fx(x, y).numpy()

    # Compute tolerancy
    tol = abs(f_new - f_old)

    # Append current point to list
    points.append(np.array([x, y]))
    fx_values.append(f_new)

    # Updating learning rate
    lr = lr * 0.999

    steps += 1
```

[15]:
```
plotInfo(points, fx_values, steps)
```






```
Best value of x: [0.8979067 0.8058023]
Number os steps: 1403
```