Homework 07 - Network Science

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1 Rate Equation

1.1 Directed Barabasi-Albert Model

For this exercise, we will follow the growth and preferential attachment rules. At each step in time, a new node is added to the network with m links, each being incoming or outgoing with probability 0.5. The probability that a node i already in the networks is the other extremity of one of these new links depends on its in-degree k_i^{in} and is given by

$$\Pi(k_i^{in}) = \frac{k_i^{in}}{\sum_j k_i^{in}}$$

Since the book give us the same definition for a undirected network, this graph will follow the same ideas. However, as it has incoming and outcoming links, we are going to adapt the formula and stick with:

$$\Pi(k_i^{in}) = \frac{k_i^{in}}{\sum_j k_i^{in}} = \frac{k}{mt}$$

1.1.1 Obtain the degree distribution for in-degrees in such a network. Does it follow a power law?

In order to obtain the degree distribution, the same steps used at Homework 6. So, by saying that, we have:

$$\Pi(k_i^{in}) = \frac{k}{mt} \text{ (5.31)} \tag{1}$$

$$\frac{k}{mt} \times Np_k \times m = kp_k \text{ (5.32 - N = t, so the cancel is possible)} \tag{2}$$

$$kp_k \text{ (5.33)} \tag{3}$$

$$(k-1)p_{k-1} \text{ (5.34)} \tag{4}$$

$$(N+1)p_k = Np_k + (k-1)p_{k-1} - kp_k \text{ (5.35)} \tag{5}$$

$$Np_k + p_k = Np_k + (k-1)p_{k-1} - kp_k \tag{6}$$

$$p_k = (k-1)p_{k-1} - kp_k \text{ (5.42)} \tag{7}$$

$$p_k = kp_{k-1} - p_{k-1} - kp_k = -p_{k-1} - k[p_k - p_{k-1}] \text{ (5.43)} \tag{8}$$

$$p_k = -p_{k-1} - k\frac{[p_k - p_{k-1}]}{k-(k-1)} \approx -p_{k-1} - k\frac{\partial p_k}{\partial k} \text{ (5.44)} \tag{9}$$

$$2p_k = -k\frac{\partial p_k}{\partial k} \text{ (5.45 - for a high value of k we have that } p_{k-1} = p_k\text{)} \tag{10}$$

$$1 = -k\frac{\partial p_k}{2p_k \partial k} \ (\div 2p_k) \tag{11}$$

$$\frac{\partial k}{k} = -\frac{\partial p_k}{2p_k} \ (\times \partial k \ \& \ \div k) \tag{12}$$

$$\int \frac{dk}{k} = \int \frac{dp_k}{2p_k} \text{ (integrate both sides)} \tag{13}$$

$$\ln(k) = -\frac{\ln(p_k)}{2} \tag{14}$$

$$-2\ln(k) = \ln(p_k) \ (\times 2) \rightarrow \ln(k)^{-2} = \ln(p_k) \tag{15}$$

$$exp(\ln(k)^{-2}) = exp(\ln(p_k)) \text{ (exponentiate both sides)} \tag{16}$$

$$p_k = k^{-2} \tag{17}$$

Finally, the book gave us at section 4.2 that a power law is approximately defined by

$$p_k \sim k^{-\gamma}$$

Furthermore, from step 17, we got the degree distribution which has the same form ($\gamma = 2$). Thus, the obtained degree distribution does follow a power law.

1.1.2 Obtain the degree distribution for the out-degree in such a network. Does it follow a power law?

Each link in the network has 50% of being a incoming or outcoming link. By saying so, both degree distributions are exactly the same, since they have same probability of appearence. Hence, it is the same as defined at 1.1.1.

$$p_k \sim k^{-\gamma}$$