Homework 11 - Network Science

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1 Classic Epidemics on Bipartite Networks

For this assignment, we're going to consider a bipartite network with the following properties:

- Two types of nodes - male (M) and female (F)
- The same number N of nodes of each type
- Rate of transmission from M to F is β_1
- Rate of transmission from F to M is β_2

It is asked to write the equations of the corresponding Susceptible-Infected (SI) model, assuming a classical approach with homogeneous mixing. Also, to write the differential equations governing the growth over time of both $f(t)$ and $m(t)$.

1.1 Differential equation of $f(t)$

To achieve the equation, we're going to base on SI model equation defined by Barabási in his book at section 10.2. The formula is given by:

$$\frac{dI(t)}{dt} = \beta \langle k \rangle \frac{S(t)I(t)}{N}$$

where, $\beta\langle k \rangle$ is the transmission rate (in this case β_1), $S(t)$ is the susceptible individuals who haven't yet contacted the pathogen at a time t, and $I(t)$ the contagious individuals who have contacted the pathogen at a time t. Since the pathogen can be transmitted only from a node of a type to a node of other type, we can conclude that females only are infected by males. So, in this case, the infected group is going to be the males.

As we have two different groups, we can write the two sets of S and I as:

$$i_m = \frac{I_m}{N} \quad s_m = \frac{S_m}{N} \quad i_f = \frac{I_f}{N} \quad s_f = \frac{S_f}{N}$$

Thus, replacing the values, the following formula will be generated:

$$
\begin{aligned}
\frac{dI_f(t)}{dt} &= \beta_1 \frac{S_f(t)I_m(t)}{N} \\
\frac{d(i_f N)}{dt} &= \beta_1 \frac{(s_f N)(i_m N)}{N} && \text{(replacing } S_f \text{ and } I_m \text{ by } s_f \text{ and } i_m\text{)} \\
\frac{d(i_f N)}{dt} &= \beta_1 s_f (i_m N) \\
\frac{di_f}{dt} &= \beta_1 s_f i_m && \text{(divide both sides by N)} \\
\frac{di_f}{dt} &= \beta_1 i_m (1 - i_f) && \text{(replacing } s_f \text{ by } (1 - i_f)\text{)} \\
\frac{di_f}{i_m(1 - i_f)} &= \beta_1 dt
\end{aligned}
$$

$$
\frac{di_f}{i_m} + \frac{di_f}{(1 - i_f)} = \beta_1 dt \quad \blacksquare
$$

1.2 Differential equation of $m(t)$

This equation follows the same idea as presented in 1.1. The only difference is that $\beta\langle k\rangle$ now is β_2 that leads to females infecting males. Thus, we can write the formula as following:

$$
\begin{aligned}
\frac{dI_m(t)}{dt} &= \beta_2 \frac{S_m(t)I_f(t)}{N} \\
\frac{d(i_m N)}{dt} &= \beta_2 \frac{(s_m N)(i_f N)}{N} && \text{(replacing } S_m \text{ and } I_f \text{ by } s_m \text{ and } i_f\text{)} \\
\frac{d(i_m N)}{dt} &= \beta_2 s_m (i_f N) \\
\frac{di_m}{dt} &= \beta_2 s_m i_f && \text{(divide both sides by N)} \\
\frac{di_m}{dt} &= \beta_2 i_f (1 - i_m) && \text{(replacing } s_m \text{ by } (1 - i_m)\text{)} \\
\frac{di_m}{i_f(1 - i_m)} &= \beta_2 dt
\end{aligned}
$$

$$
\frac{di_m}{i_f} + \frac{di_m}{(1 - i_m)} = \beta_2 dt \quad \blacksquare
$$