Homework 08 - Network Science

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1 Star network

Considering a star network, with N nodes (N much larger than 1), compute the following:

1.1 Degree distribution

As it is a star-shepd network, it has one (N-1)-degree node and the rest are 1-degree nodes. Moreover, it is necessary to compute the degree distribution (p_k) for both possible degrees. Thus, we have:

$$p_k = \frac{N_k}{N}, \text{ where } N_k \text{ is the number of nodes with degree } k$$

Accordingly, both distributions are $p_1 = \frac{N-1}{N}$ and $p_{(N-1)} = \frac{1}{N}$.

1.2 Probability q_k that moving along a randomly chosen link we find at its end a node with degree k

To compute the q_k probability, the following formula (described in Barabási's book at chapter 7), will be used:

$$q_k = \frac{kp_k}{\langle k \rangle}$$

Additionally, the average degree of this network can be obtained from

$$\begin{aligned}
\langle k \rangle &= \sum_{1}^{N-1} kpk = 1 * p_1 + (N-1) * p_{(N-1)} \\
&= 1 * \frac{N-1}{N} + (N-1) * \frac{1}{N} \\
&= \frac{2(N-1)}{N}
\end{aligned}$$

Using the obatained $\langle k \rangle$ formula into q_k, we have that

$$q_1 = \frac{1 * \frac{N-1}{N}}{\frac{2(N-1)}{N}} = \frac{N-1}{N} * \frac{N}{2(N-1)} = \frac{1}{2}$$

$$q_{(N-1)} = \frac{(N-1) * \frac{1}{N}}{\frac{2(N-1)}{N}} = \frac{N-1}{N} * \frac{N}{2(N-1)} = \frac{1}{2}$$

1.3 Compute the degree correlation coefficient r for this network

Barabási also described on his book at chapter 7 the formula to compute this coefficient, so we have that

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}, \qquad \sigma^2 = \sum_k k^2 q_k - \left[\sum_k k q_k\right]^2$$

whereas e_{jk} is the probability to find a node with degree j which has a node with degree k at the end of its link. So, as the only possible values for jk are $(N-1)(1)$ and $(1)(N-1)$, for this network we have:

$$\sum_{jk} e_{jk} = 1, \qquad e_{jk} = \frac{1}{2}, \quad \text{and } e_{jk} = 0, \quad \forall jk \neq (N-1)(1) \text{ or } (1)(N-1)$$

$$\begin{aligned}
\sigma^2 &= \sum_k k^2 q_k - \left[\sum_k k q_k\right]^2 \\
&= \left(1^2 * \frac{1}{2} + (N-1)^2 * \frac{1}{2}\right) - \left(1 * \frac{1}{2} + (N-1) * \frac{1}{2}\right)^2 \\
&= \left(\frac{1}{2} + \frac{N^2 - 2N + 1}{2}\right) - \left(\frac{1}{2} + \frac{N-1}{2}\right)^2 \\
&= \frac{N^2 - 2N + 2}{2} - \frac{N^2}{4} = \frac{2N^2 - 4N + 4 - N^2}{4} \\
&= \frac{N^2 - 4N + 4}{4} = \frac{(N-2)^2}{4}
\end{aligned}$$

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2} = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma^2}$$

$$\begin{aligned}
\sum_{jk} jk(e_{jk} - q_j q_k) &= 1 * 1\left(0 - \frac{1}{4}\right) + 1 * (N-1)\left(\frac{1}{2} - \frac{1}{4}\right) + (N-1) * 1\left(\frac{1}{2} - \frac{1}{4}\right) + (N-1) * (N-1)\left(0 - \frac{1}{4}\right) \\
&= -\frac{1}{4} + \frac{N-1}{4} + \frac{N-1}{4} + \frac{-N^2 + 2N - 1}{4} \\
&= \frac{-N^2 + 4N - 4}{4} = -\frac{(N-2)^2}{4} \\
r &= \frac{-\frac{(N-2)^2}{4}}{\frac{(N-2)^2}{4}} = -\frac{(N-2)^2}{4} * \frac{4}{(N-2)^2} \\
&= -1
\end{aligned}$$

1.4 Is the network assortative or dissasortative? How can we tell?

The analysed network is dissasortative. We can infer this information based on the r coefficient obtained at 1.3. As the value is negative, the hub tend to connect to small degree nodes (edge nodes), which results in the given star-shaped network.