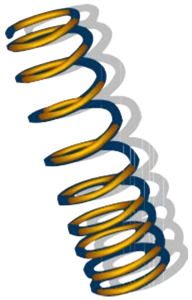
8b – High-Level Synthesis: Force-Directed Scheduling

ECE 474A/57A
COMPUTER-AIDED LOGIC DESIGN

Force-Directed Scheduling (FDS)

- Heuristic scheduling algorithms
  - Consider the unscheduled CDFG under a physics-based spring model
  - Operators are subjected to physical 'forces', both repelling and attracting them to particular time slices
    - Larger the force, the larger the concurrency
  - Goal is to find the optimal placement of vertices into a schedule, when subject to these 'forces'



- Minimum latency under resource-constraint
  - Force directed list scheduling
  - Extension of list scheduling algorithms

- Minimum resource under latency-constraint
  - Force directed scheduling

This is the one we will consider

Force-Directed Scheduling (FDS)

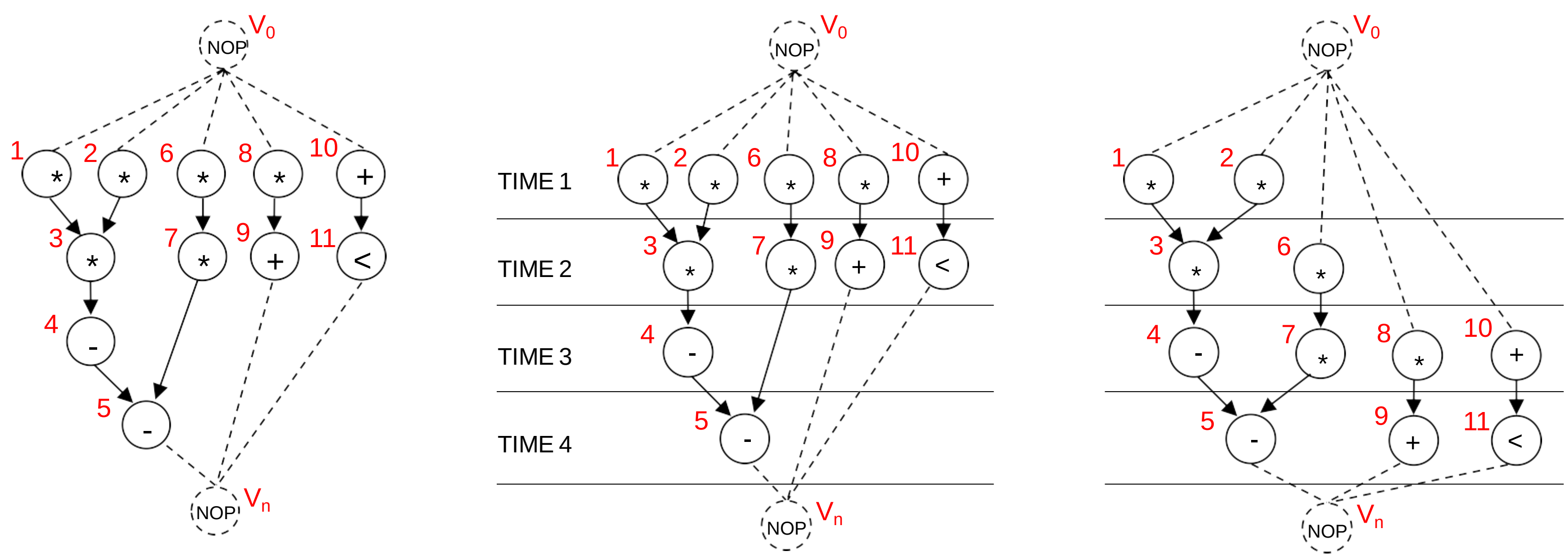
- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ̄ ){
    repeat {
        Compute the time frames;
        Compute the operations and type probabilities;
        Compute the self-forces, predecessor/successor forces and total forces;
        Schedule the operation with least force and update its time-frame;
    } until (all operations scheduled);
    return (t);
}
```

Force-Directed Scheduling (FDS)

Time Frames

- Time frame of an operation is the time interval where it can be scheduled
  - Denoted by $\{[t_i^S, t_i^L]; i = 0, 1, \ldots, n\}$
  - Earliest and latest start times can be computed by ASAP and ALAP algorithms

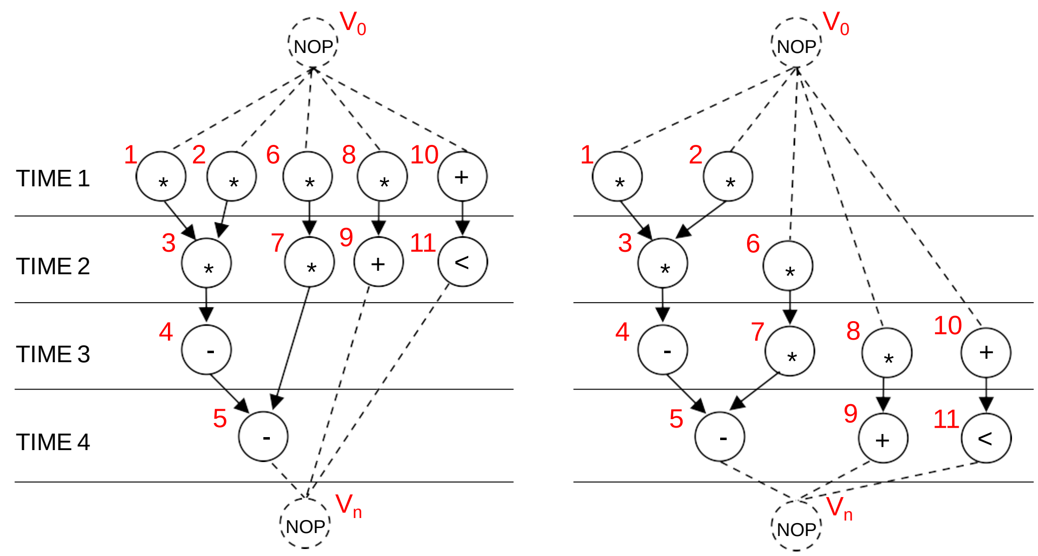


- Width of time frame of an operation is equal to its mobility plus 1

Force-Directed Scheduling (FDS)

Example 2

- Time frames for various operation assuming a latency bound of 4
  - Latency bound needed for ALAP scheduling



operation v_1

ASAP time = 1

ALAP time = 1

time frame = [1, 1]

operation v_2

ASAP time = 1

ALAP time = 1

time frame = [1, 1]

operation v_6

ASAP time = 1

ALAP time = 2

time frame = [1, 2]

operation v_8

ASAP time = 1

ALAP time = 3

time frame = [1, 3]

Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

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}
```

Force-Directed Scheduling (FDS)

Operation Probability

- Operation Probability is a function
  - Equal to zero outside of the corresponding time frame
  - Equal to reciprocal of the frame width inside the time frame

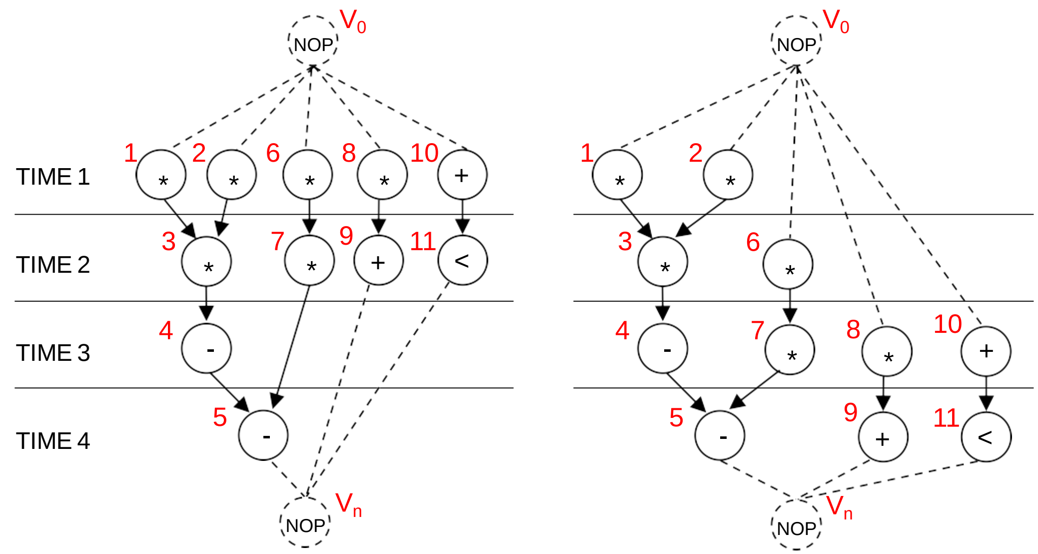
- Denoted the probability of the operations at time l by $\{p_i(l);\ i = 0, 1, \ldots, n\}$

- What is the significance?
  - Operations whose time frame is one unit wide are bound to start in one specific time
  - For remaining operations, the larger the width, the lower the probability that the operation is scheduled in any given step inside the corresponding time frame

Force-Directed Scheduling (FDS)

Example 3

- Operation Probability for various operations
  - Equal to zero outside of the corresponding time frame
  - Equal to reciprocal of the frame width inside the time frame



operation v_1

time frame = [1, 1]

frame width = 1

$p_1(1) = 1$, $p_1(2) = 0$

$p_1(3) = 0$, $p_1(4) = 0$

operation v_2

time frame = [1, 1]

frame width = 1

$p_2(1) = 1$, $p_2(2) = 0$

$p_2(3) = 0$, $p_2(4) = 0$

operation v_6

time frame = [1, 2]

frame width = 2

$p_6(1) = 0.5$, $p_6(2) = 0.5$

$p_6(3) = 0$, $p_6(4) = 0$

operation v_8

time frame = [1, 3]

frame width = 3

$p_8(1) = 0.3$, $p_8(2) = 0.3$

$p_8(3) = 0.3$, $p_8(4) = 0$

Force-Directed Scheduling (FDS)

Type Distribution

- Type Distribution is the sum of probabilities of the operations implemented by a specific resource at any time step of interest
  - Denote distribution at time l by $\{q_k(l);\ k = 1, 2, \ldots, n_{res}\}$

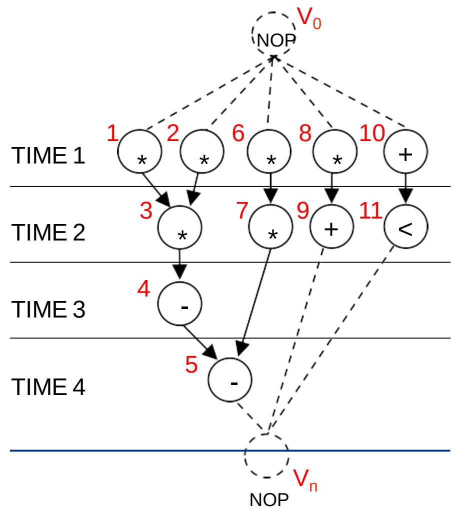
- Distribution graph is a plot of any operation-type distribution over the scheduled steps
  - Shows likelihood that a resource is used at each scheduled step
  - Uniform plot in a distribution graph means that a type is evenly scattered in the schedule and a good measure of utilization

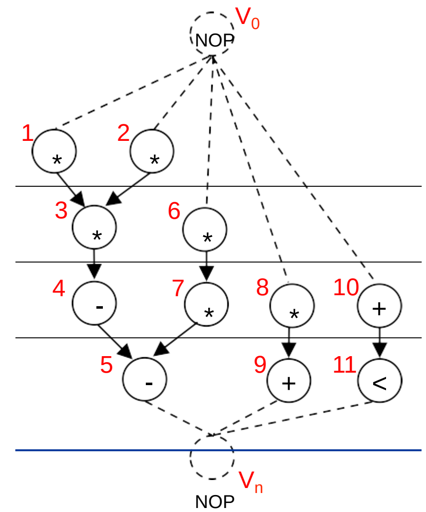
Force-Directed Scheduling (FDS)

Example 4

- Distribution graph for ALU
  - Sum of probabilities of the operations implemented by a specific resource at any time step of interest

	p(1)	p(2)	p(3)	p(4)
v_4 = [3, 3], width = 1	0	0	1	0
v_5 = [4, 4], width = 1	0	0	0	1
v_9 = [2, 4], width = 3	0	0.3	0.3	0.3
v_{10} = [1, 3], width = 3	0.3	0.3	0.3	0
v_{11} = [2, 4], width = 3	0	0.3	0.3	0.3



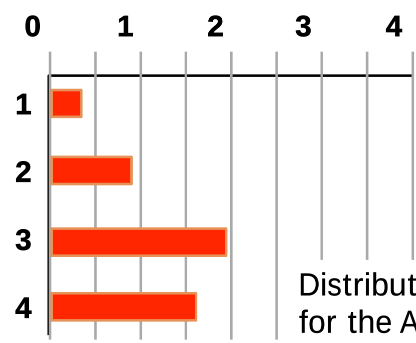


$$q_2(1) = 0 + 0 + 0 + 0.3 + 0 = 0.3$$

$$q_2(2) = 0 + 0 + 0.3 + 0.3 + 0.3 = 0.9$$

$$q_2(3) = 1 + 0 + 0.3 + 0.3 + 0.3 = 1.9$$

$$q_2(4) = 0 + 1 + 0.3 + 0 + 0.3 = 1.6$$



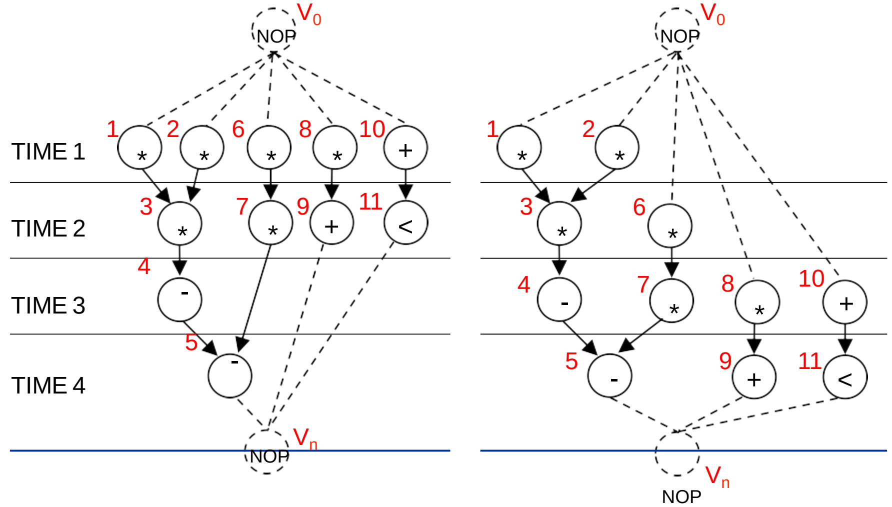
Distribution graph for the ALU

Force-Directed Scheduling (FDS)

Example 5

- Distribution graph for Multiplier
  - Sum of probabilities of the operations implemented by a specific resource at any time step of interest

	p(1)	p(2)	p(3)	p(4)
v_1 = [1, 1], width = 1	1	0	0	0
v_2 = [1, 1], width = 1	1	0	0	0
v_3 = [2, 2], width = 1	0	1	0	0
v_6 = [1, 2], width = 2	0.5	0.5	0	0
v_7 = [2, 3], width = 2	0	0.5	0.5	0
v_8 = [1, 3], width = 3	0.3	0.3	0.3	0

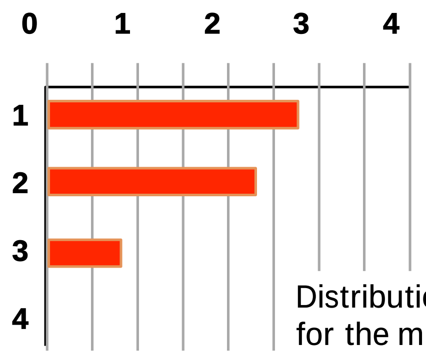


$q_2(1) = 1 + 1 + 0 + 0.5 + 0 + 0.3 = 2.8$

$q_2(2) = 0 + 0 + 1 + 0.5 + 0.5 + 0.3 = 2.3$

$q_2(3) = 0 + 0 + 0 + 0 + 0.5 + 0.3 = 0.8$

$q_2(4) = 0 + 0 + 0 + 0 + 0 + 0 = 0$



Distribution graph for the multiplier

Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ̄ ){
    repeat {
        Compute the time frames;
        Compute the operations and type probabilities;
        Compute the self-forces, predecessor/successor forces and total forces;
        Schedule the operation with least force and update its time-frame;
    } until (all operations scheduled);
    return (t);
}
```

Force-Directed Scheduling (FDS)

Self Force

- Self Force
  - Scheduling an operation will effect overall concurrency
  - Every operation has "self force" for every C-step of its time frame
  - Desirable scheduling will have negative self force

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

$$\textbf{Self Force(j)} = \sum_{i=t}^{b} \textbf{Force(i)}$$

Force-Directed Scheduling (FDS)

Example 6

- Calculate Self Force for v_6
  - Assignment of v6 to time step 1
  - Assignment of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$

<u>Assuming v6 assigned to time step 1</u>

Self force = 2.8(1-0.5) + 2.3(0-0.5)

Distribution graph values to time step 1 and 2

1 indicates that v6 schedule in time 1, minus the operator probability in time 1

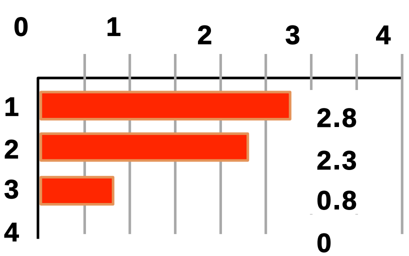
0 indicates that v6 is NOT scheduled in time 1, minus the operator probability in time 2

Time frame and operation probability for v_6

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

Distribution graph for the multiplier



Force-Directed Scheduling (FDS)

Example 6

- Calculate Self Force for v_6
  - Assignment of v6 to time step 1
  - Assignment of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

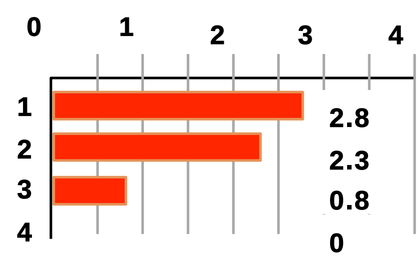
$$\text{Self Force(j)} = \sum_{i=t}^{b} \text{Force(i)}$$

Time frame and operation probability for v_6

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

Distribution graph for the multiplier



<u>Assuming v6 assigned to time step 1</u>

Self force = 2.8(1-0.5) + 2.3(0-0.5)

= 0.25

<u>Assuming v6 assigned to time step 2</u>

Self force = 2.8(0-0.5) + 2.3(1-0.5)

= -0.25

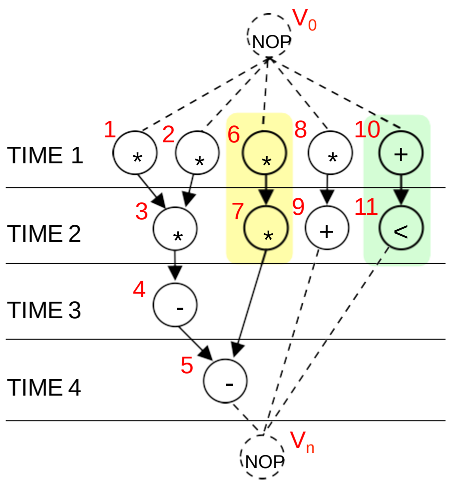
Want to reduce force (concurrency), time step 2 looks better

How does this impact other operations?

Force-Directed Scheduling (FDS)

Predecessor/Successor Forces

- Predecessor/Successor Force
  - Scheduling an operation may affect the time frames of other linked operations
  - This may negate the benefits of the desired assignment
  - Predecessor/Successor Forces = Sum of Self Forces of any implicitly scheduled operations



If v_6 scheduled in time 2, then v_7 has to be scheduled in time 3

If v_{11} scheduled in time 3, then v_{10} has to be scheduled in time 1 or 2

Force-Directed Scheduling (FDS)

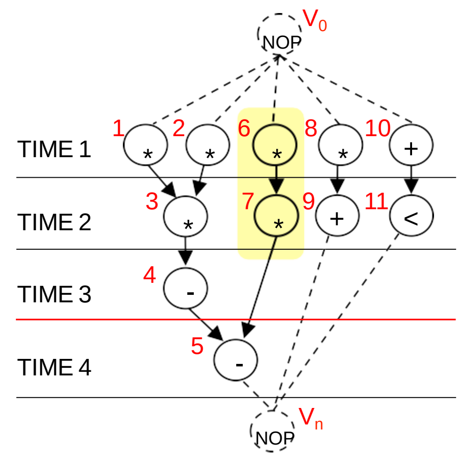
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$



<u>**Assuming v6 assigned to time step 1**</u>

no predecessor effected

Predecessor force = 0

no successor effected

v_7 can be scheduled at time 2 or 3

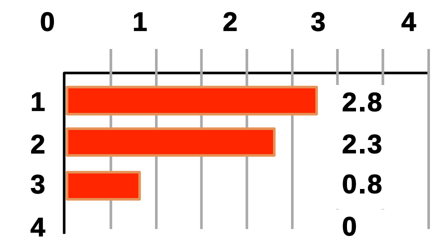
Successor force = 0

Total force = Self Force + Predecessor force + Successor force

= 0.25 + 0 + 0

= 0.25

Distribution graph for the multiplier



Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2

p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Force-Directed Scheduling (FDS)

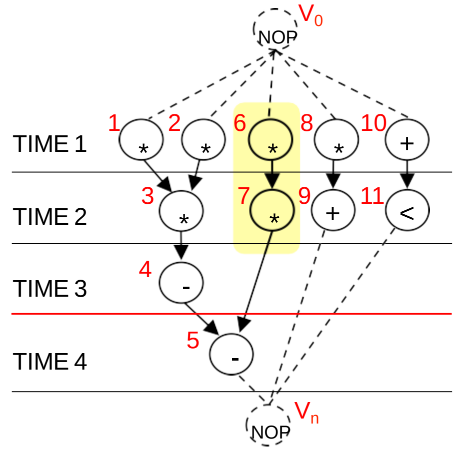
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

$$\textbf{Self Force(j)} = \sum_{i=t}^{b} \textbf{Force(i)}$$



<u>Assuming v6 assigned to time step 2</u>

no predecessor effected

Predecessor force = 0

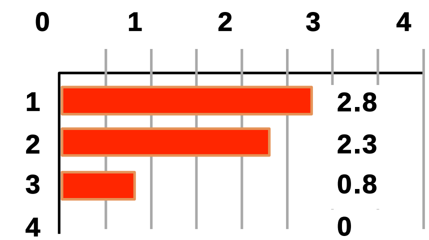
v_7 can only be scheduled at time 3

Successor force = sum of self forces of implicitly scheduled operations

= 2.3(0-0.5) + 0.8(1-0.5)

= -0.75

Distribution graph for the multiplier



Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2

p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Total force = Self Force + Predecessor force + Successor force

= -0.25 + 0 + -0.75

= -1

Force-Directed Scheduling (FDS)

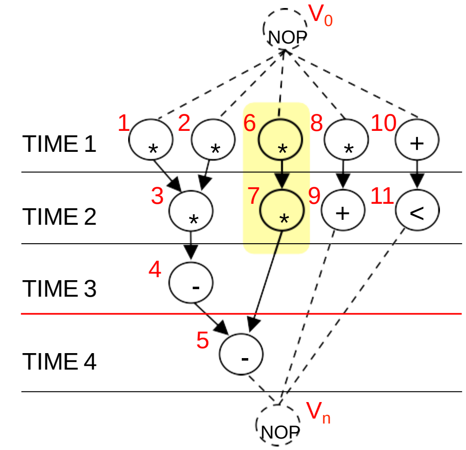
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

Self Force(j) = $\sum_{i=t}^{b}$ **Force(i)**



<u>**Assuming v6 assigned to time step 1**</u>

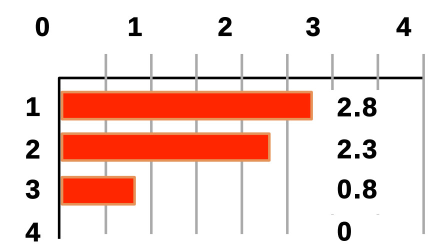
Total force = 0.25

<u>**Assuming v6 assigned to time step 2**</u>

Total force = -1

Better choice – want to reduce force in the minimum resource under latency-constraint

Distribution graph for the multiplier



Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2

p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
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}
```

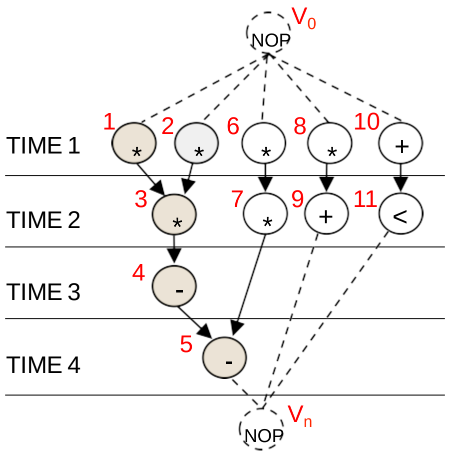
At each iteration time frame, probabilities, and forces need to be recalculated

Forces relate to concurrency – we choose lowest force so we can minimize number of resources

Results have shown FDS superior to list scheduling, but run time are long for larger graph (limited usage)

Force-Directed Scheduling (FDS)

- Previous example only looked at v6
- Algorithm tells us to calculate ALL unscheduled nodes, then schedule operation assignment with smallest force



Conclusion

- Considered several types of scheduling algorithms
  - Unconstrained Scheduling - ASAP
  - Latency-Constrained Scheduling – ALAP
  - Resource-Constrained Scheduling – Hu's Algorithm

- Practical Scheduling problems possibly include multiple-cycle operations with different types
  - Minimum-Latency, Resource-Constrained and Minimum-Resource, Latency-Constrained problems become difficult to solve efficiently
  - Heuristics developed
    - *List Scheduling (LIST_L)*
    - *List Scheduling (LIST_R)*
    - *Force-directed Scheduling*
    - *Trace Scheduling*
    - *Percolation Scheduling*