

# Learning with **Temporal Point Processes**



## Models & Inference

**Isabel Valera**  
Max Planck Institute for Intelligent Systems

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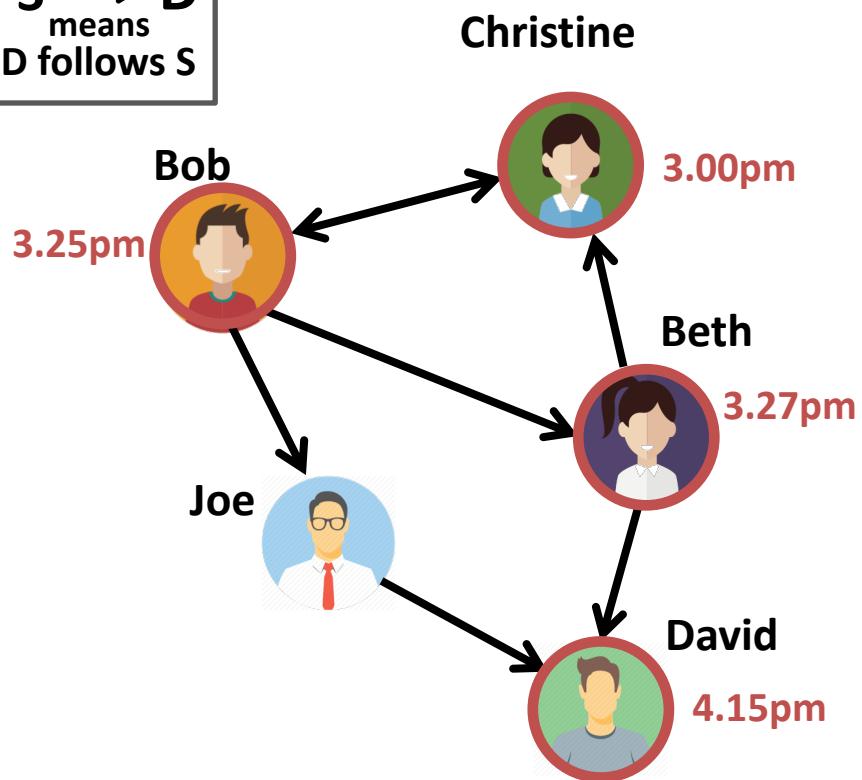
# Models & Inference

- 1. Modeling event sequences**
2. Clustering event sequences
3. Capturing complex dynamics
4. Causal reasoning on event sequences

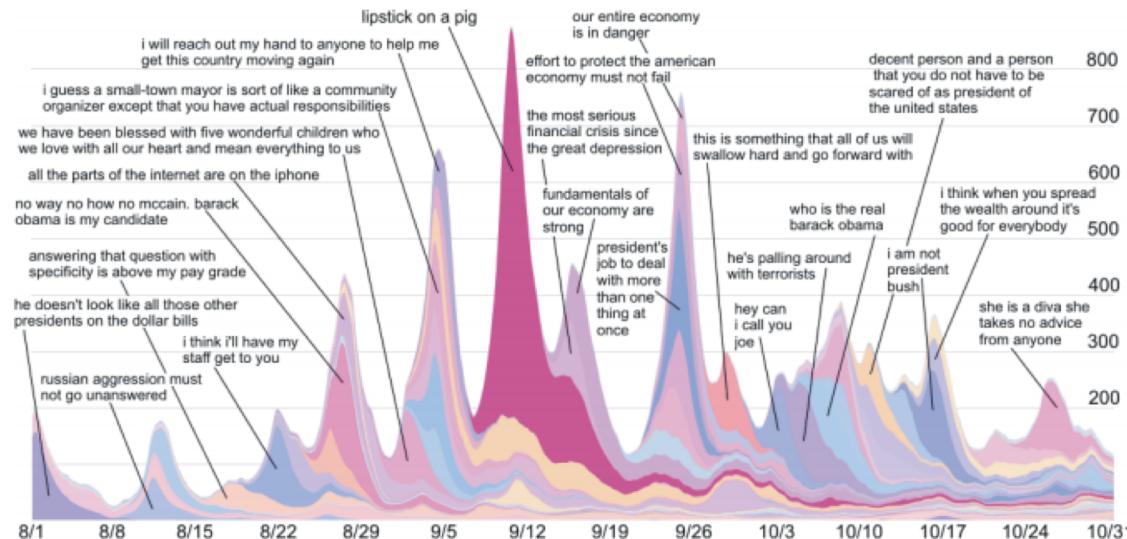
# Event sequences as cascades

$S \rightarrow D$

means  
D follows S

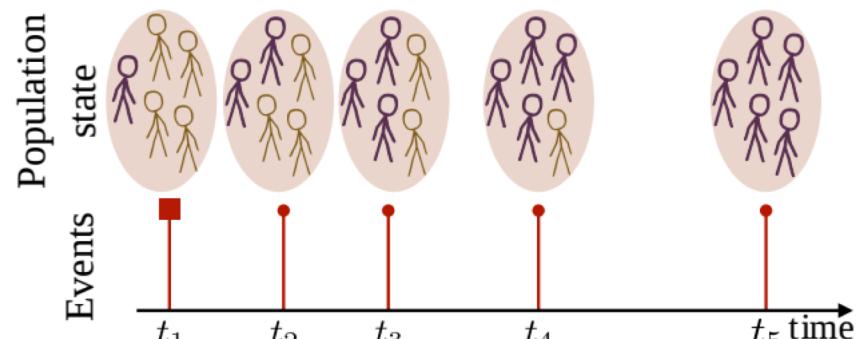


## Information Diffusion



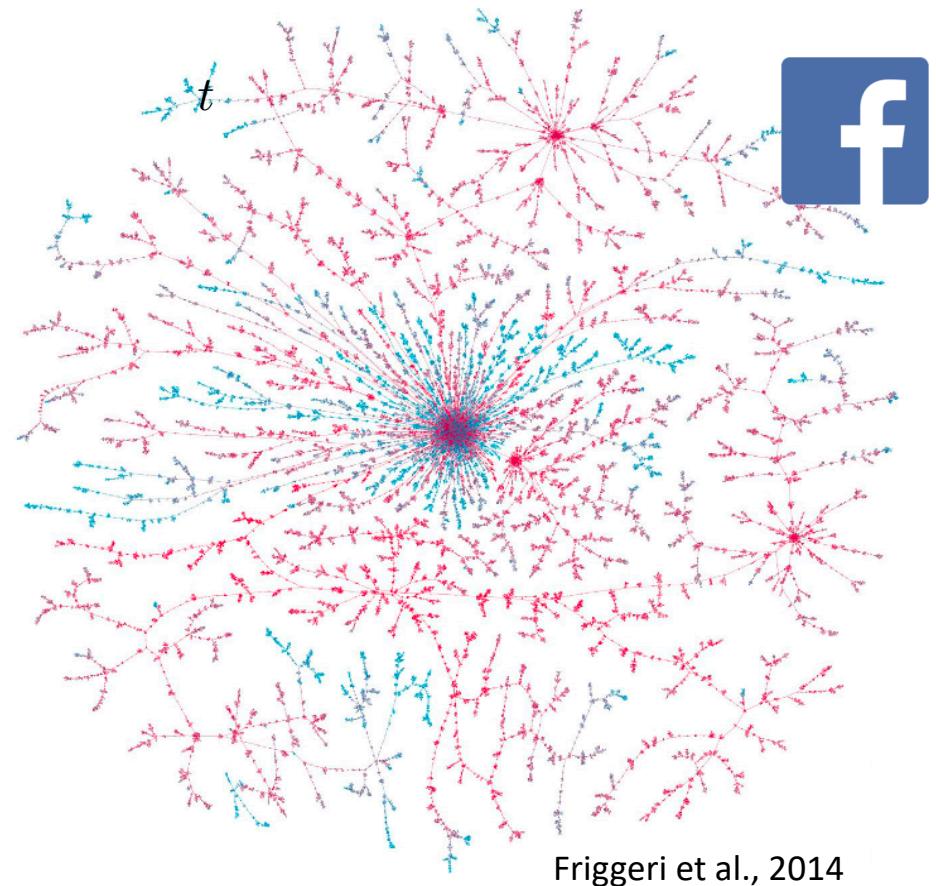
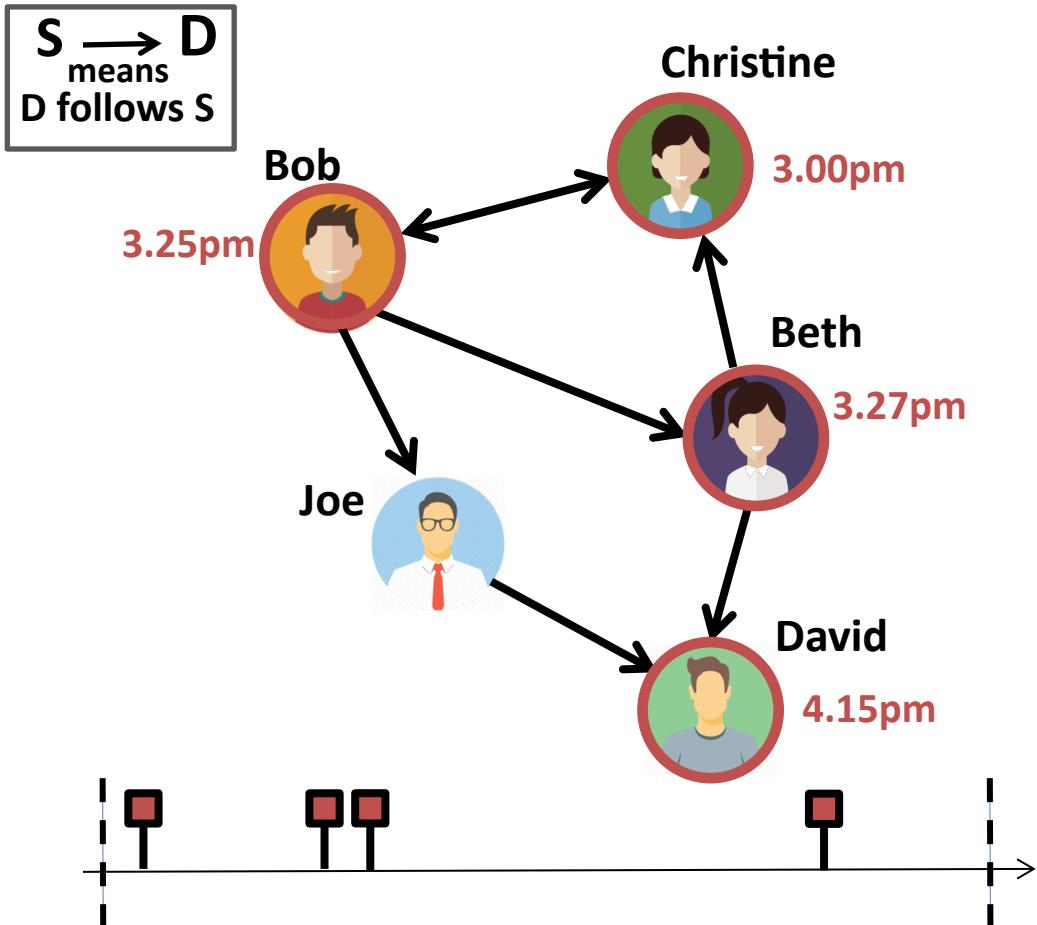
[Leskovec et al., 2009]

## Disease Diffusion



[Rizoiu et al., 2018]

# An example: idea adoption



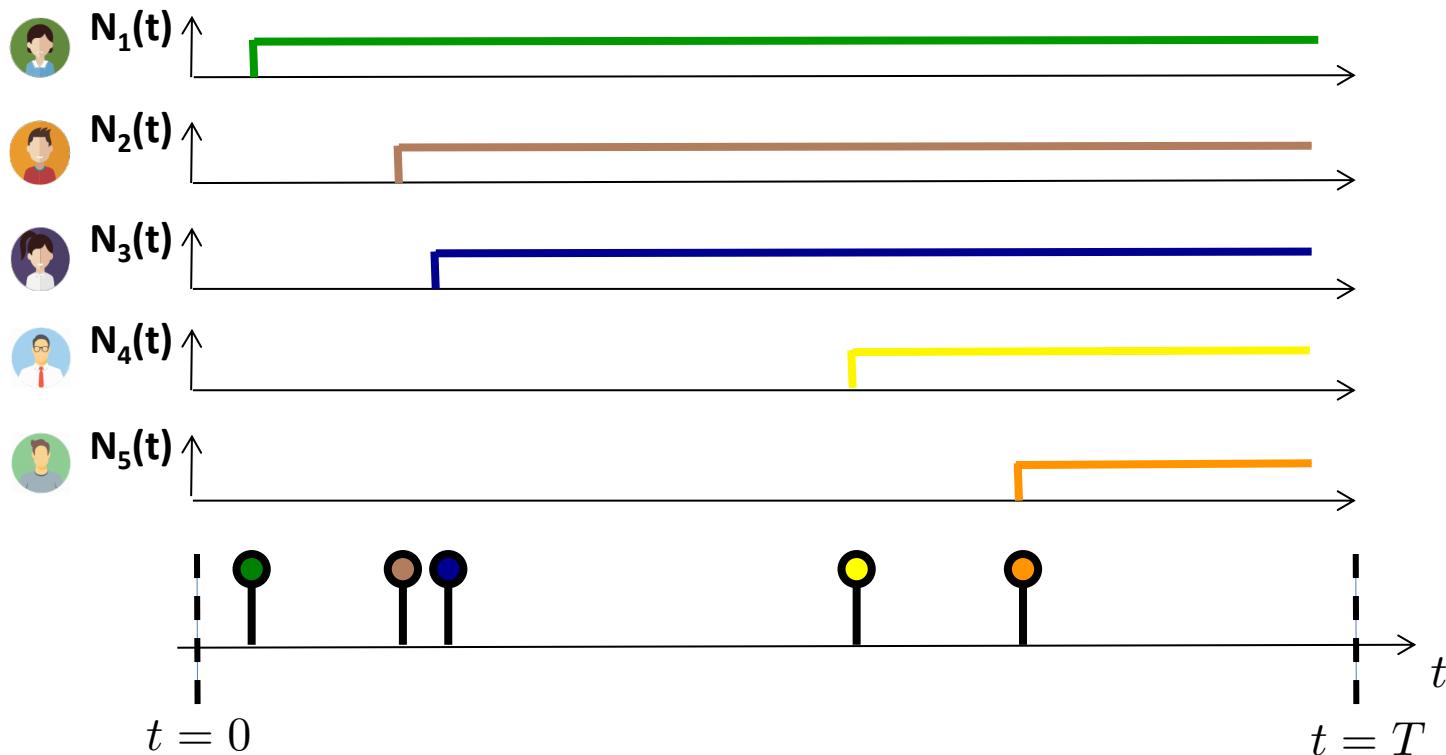
**They can have an impact  
in the off-line world**

**theguardian**

Click and elect: how fake news helped Donald Trump win a real election

# Infection cascade representation

We represent an infection cascade using terminating temporal point processes:

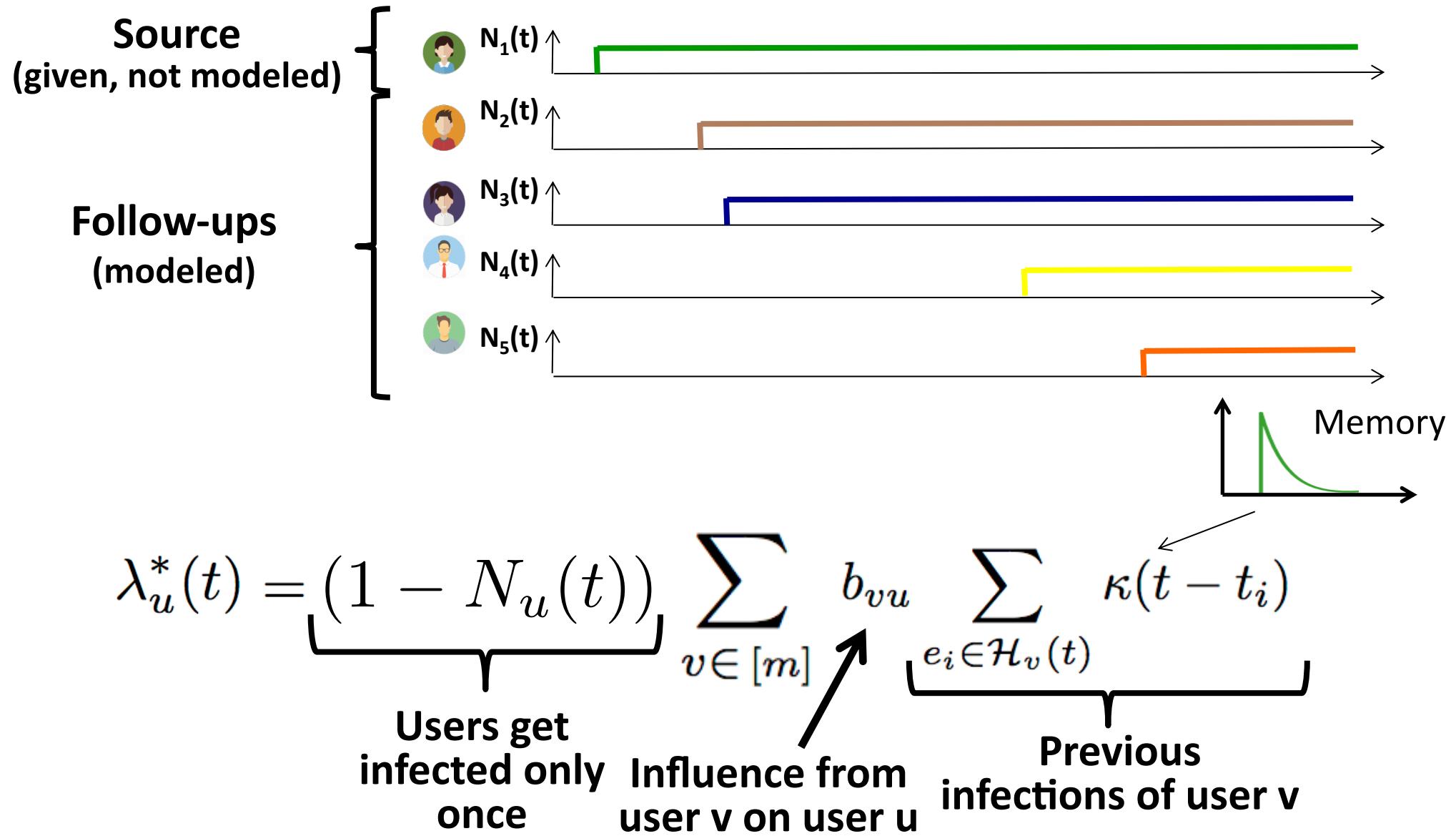


**Infection event:**

$(u_i, m_i, t_i)$

User      ↓  
Cascade      Time

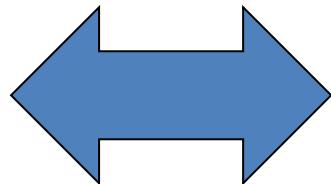
# Infection intensity



# Model inference from multiple cascades

Conditional  
intensities

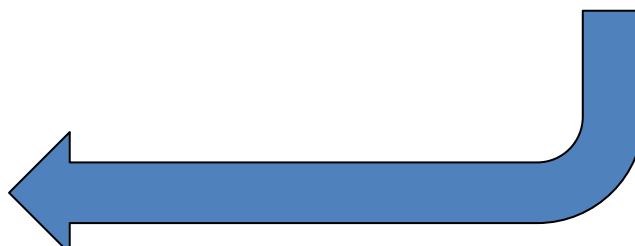
$$\lambda_u^*(t)$$



Diffusion log-likelihood

$$\mathcal{L} = \sum_{u=1}^n \log \lambda_u^*(t_u) - \int_0^T \lambda_u^*(\tau) d\tau$$

Maximum likelihood  
approach to find  
model parameters!



Sum up log-likelihoods  
of multiple cascades!

**Theorem.** For any choice of parametric memory,  
the **maximum likelihood** problem is **convex in B**.

# Example of real-world diffusion process

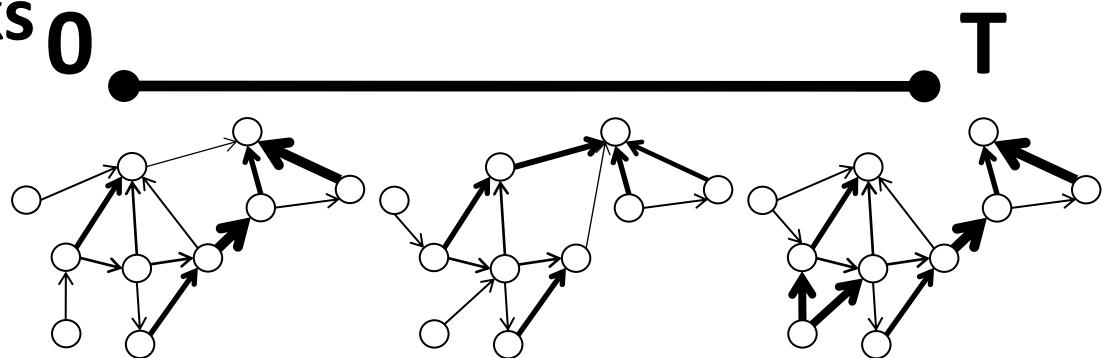
Youtube video: <http://youtu.be/hBeaSfRCU4c>

# Dynamic influence

In some cases, influence change over time:



Propagation over networks with variable influence



# Recurrent events: beyond cascades

Up to this point, each user is only infected once, and event sequences can be seen as cascades.

In general, users perform recurrent events over time. E.g., people repeatedly express their opinion online:



How social media is revolutionizing debates

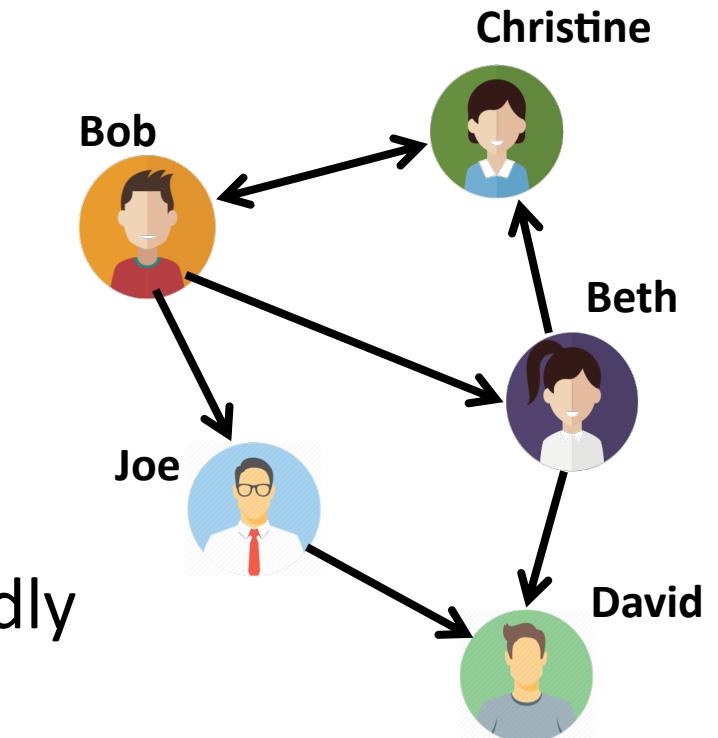
The New York Times

Campaigns Use Social Media to Lure Younger Voters

The New York Times  
Social Media Are Giving a Voice to Taste Buds

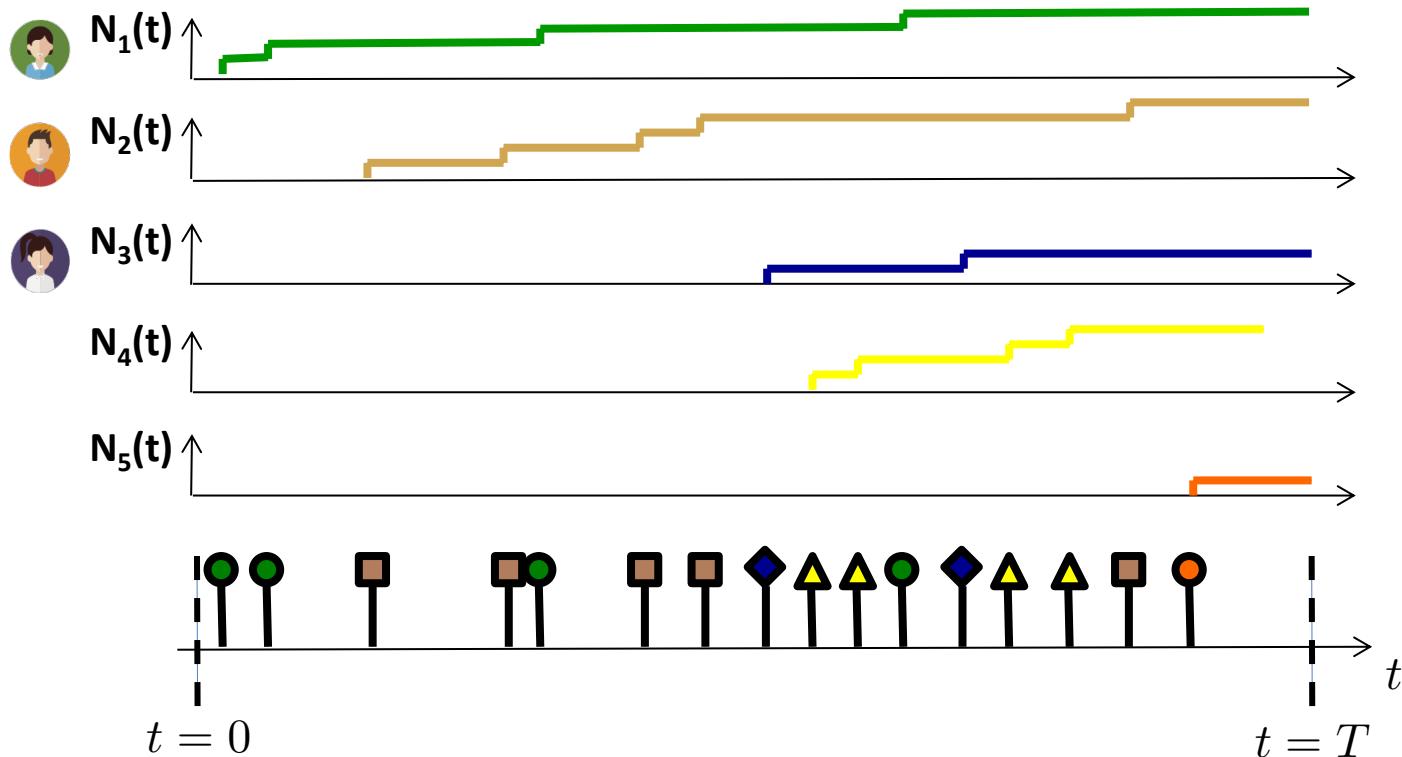


Twitter Unveils A New Set Of Brand-Centric Analytics



# Recurrent events representation

We represent messages using **nonterminating temporal point processes**:

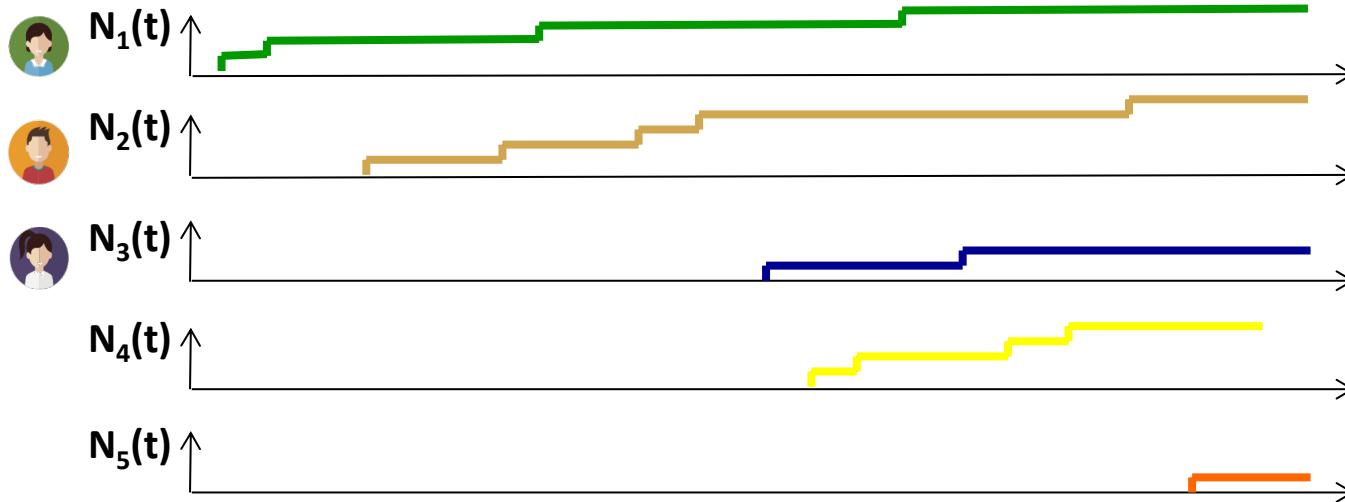


**Recurrent event:**

$$(u_i, t_i)$$

User  $\nwarrow$  Time  $\searrow$

# Recurrent events intensity



**Cascade sources!**

$$\lambda_u^*(t) = \mu_u + \sum_{v \in [m]} b_{vu} \sum_{e_i \in \mathcal{H}_v(t)} \kappa(t - t_i)$$

User's intensity      Events on her own initiative      Influence from user v on user u      Previous messages by user v

Memory

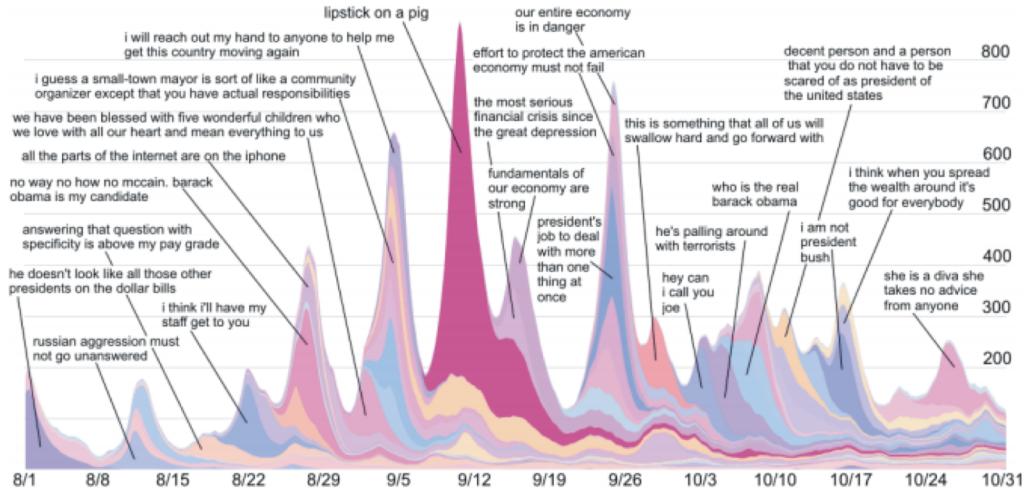
**Hawkes process**

# Models & Inference

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4. Causal reasoning on event sequences

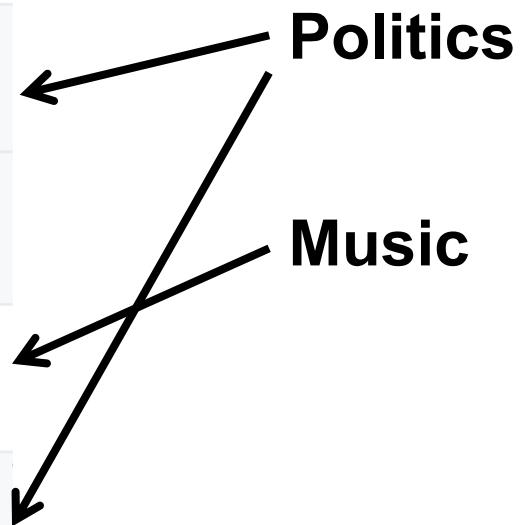
# Event sequences

So far, we have assumed the cascade (topic, meme, etc.) that each event belongs to was known.



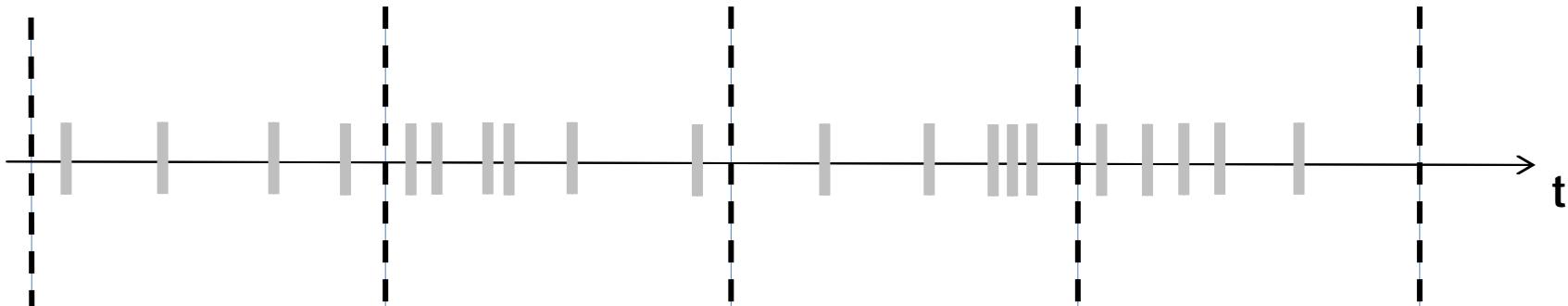
Often, the cluster (topic, meme, etc.) that each event in a sequence belongs to is not known:

-  BBC News (World)  @BBCWorld · 4m  
Turkey election: Erdogan win ushers in new presidential era
-  BBC News (World)  @BBCWorld · 46m  
Dublin church: Seven injured as car hits pedestrians
-  BBC News (World)  @BBCWorld · 2h  
Nigerian music star D'banj's son 'drowns at home'
-  BBC News (World)  @BBCWorld · 2h  
Turkey election: Country's heart split over Erdogan victory

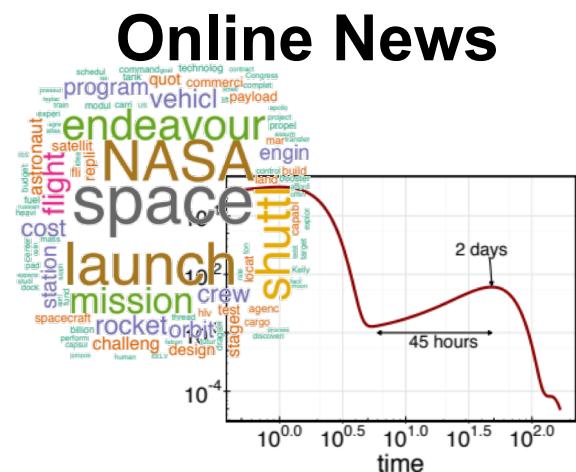
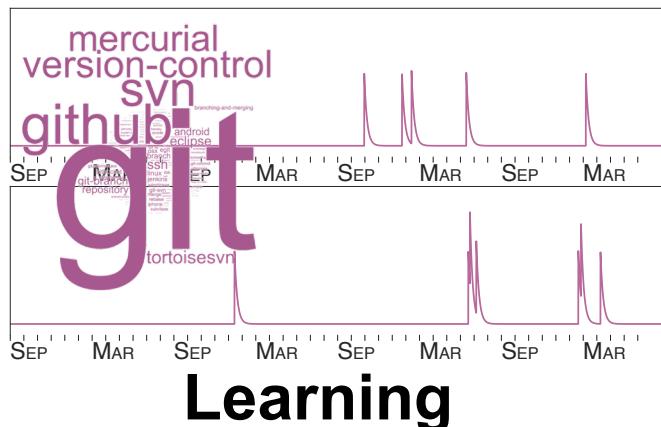


# Clustering event sequences

Assume the event **cluster** to be **hidden** and aim to automatically **learn the cluster assignments** from the data:



# Bayesian methods to cluster event sequences in the context of: Earthquakes



Health care	
Method	DMHP
ICU Patient	<b>0.3778</b>
IPTV User	<b>0.2004</b>

[Du et al., 2015; Mavroforakis et al., 2017; Xu & Zha, 2017]

# Hierarchical Dirichlet Hawkes process

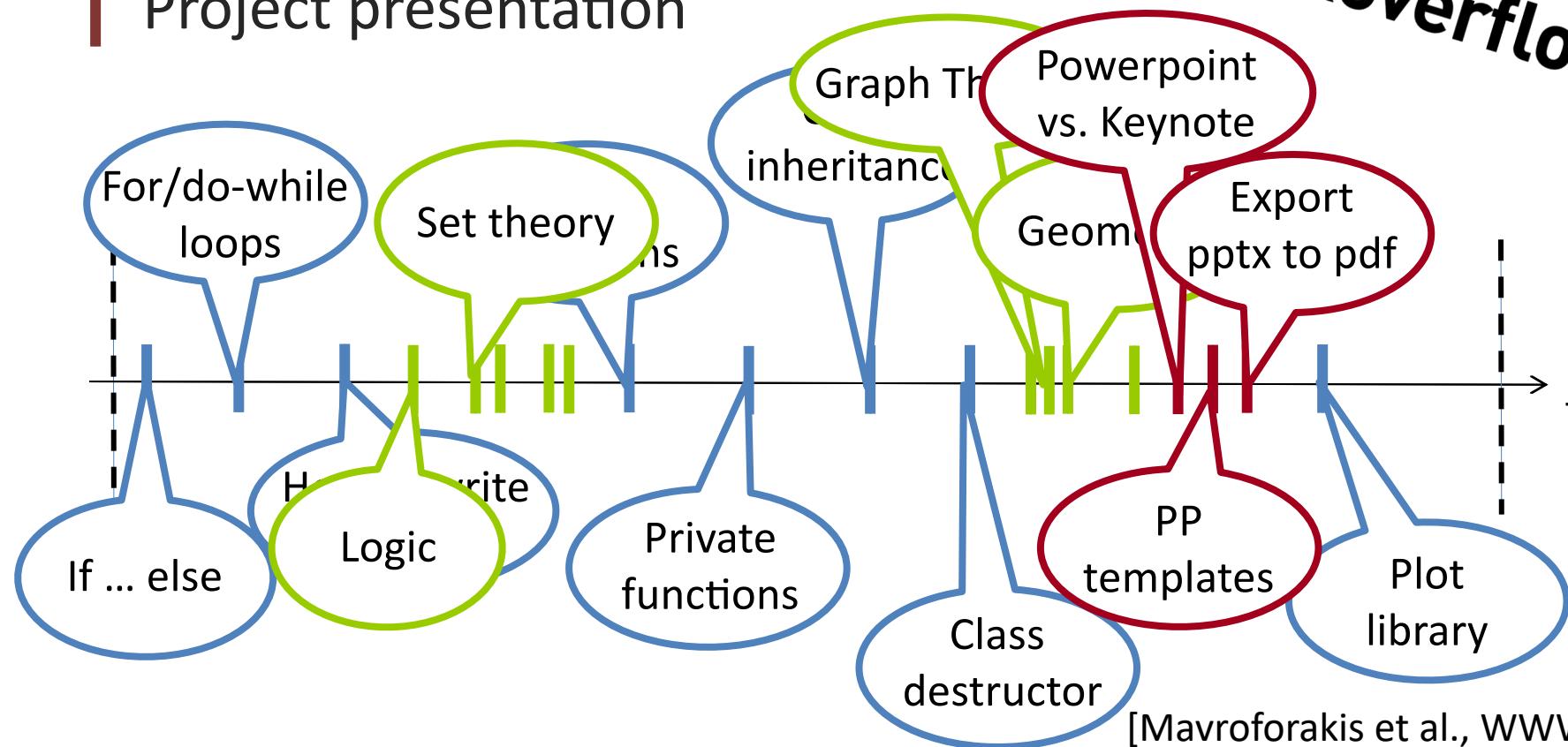


## 1st year computer science student

Introduction to programming

Discrete math

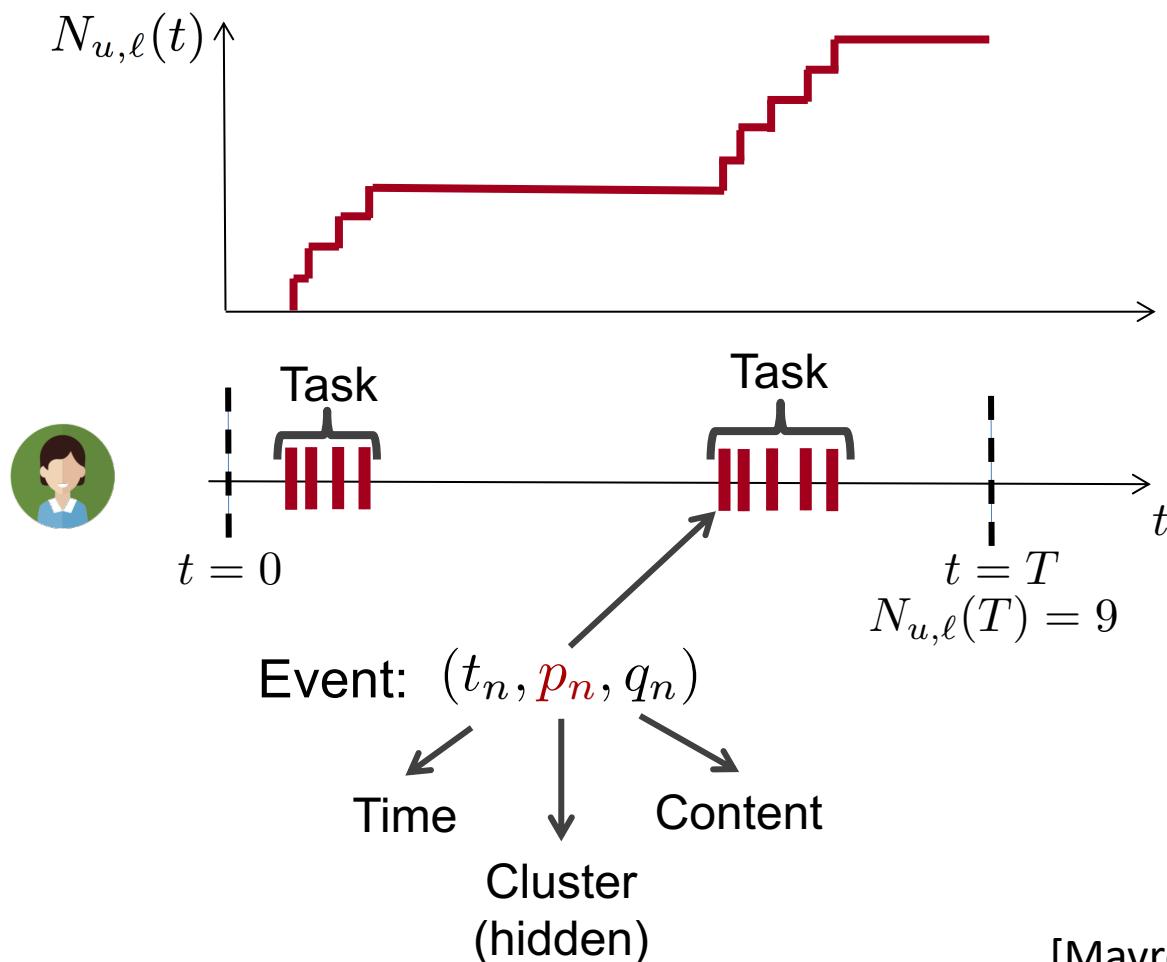
Project presentation



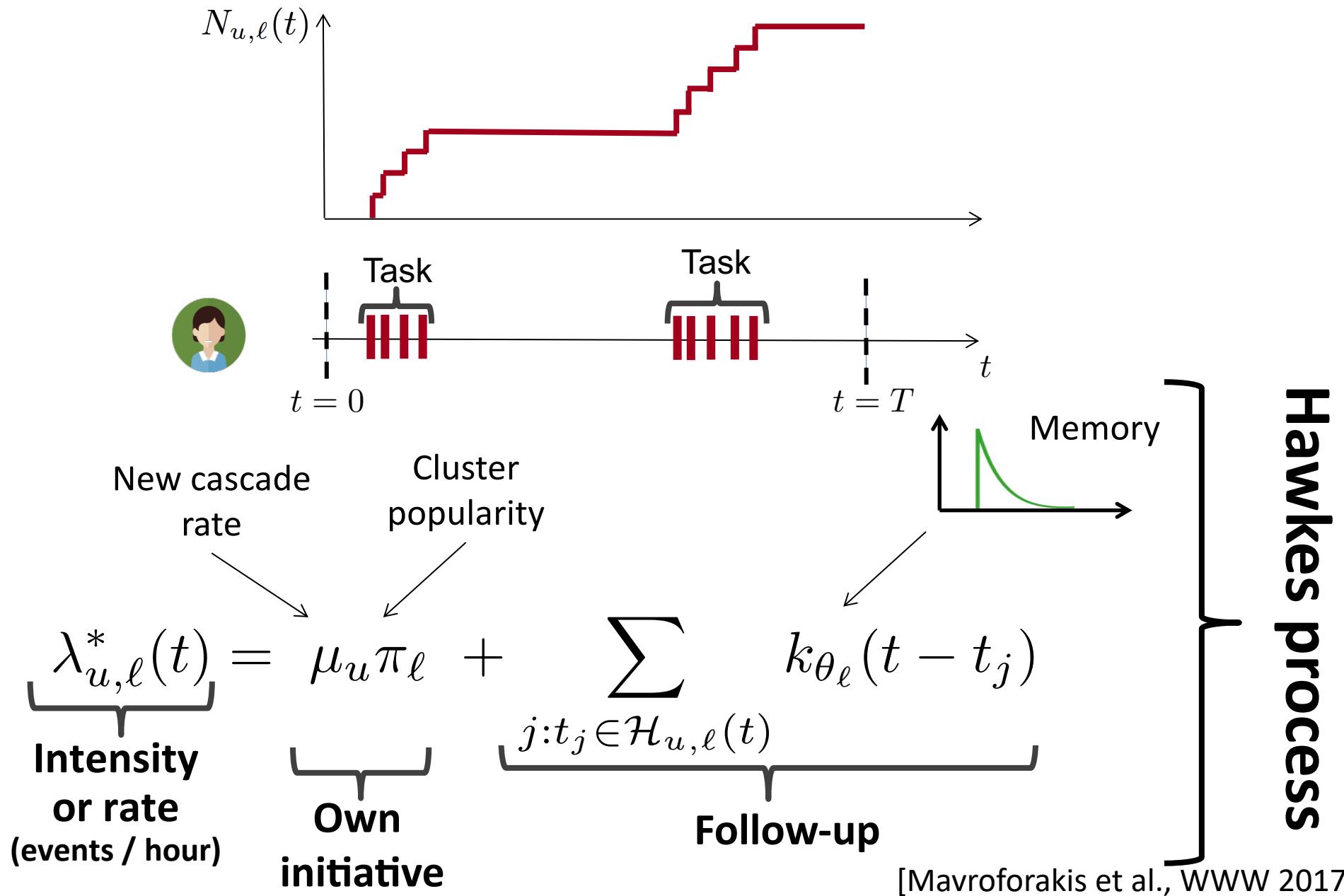
[Mavroforakis et al., WWW 2017]

# Events representation

We represent the events using **marked temporal point processes**:

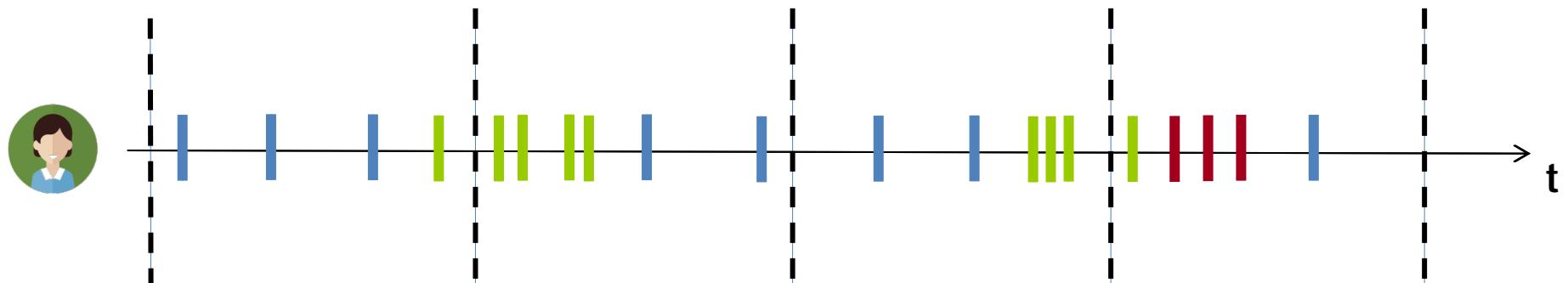


# Cluster intensity



# User events intensity

Users adopt more than one cluster:



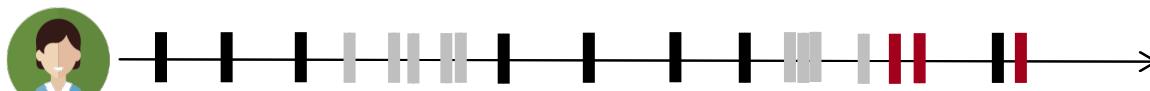
A user's learning events as a multidimensional Hawkes:

$$\text{Time } \downarrow \quad \text{cluster } \downarrow \quad (t_n, p_n) \sim \text{Hawkes} \begin{pmatrix} \lambda_{u,1}^*(t) \\ \vdots \\ \lambda_{u,\infty}^*(t) \end{pmatrix}$$

$$\text{Content} \rightarrow q_n = \omega \quad \omega_j \sim \text{Multinomial}(\theta_p)$$

# People share same clusters

*Different users adopt same clusters*



Efficient model inference using  
Sequential Montecarlo!

Clusters

- Shared parameters across users.

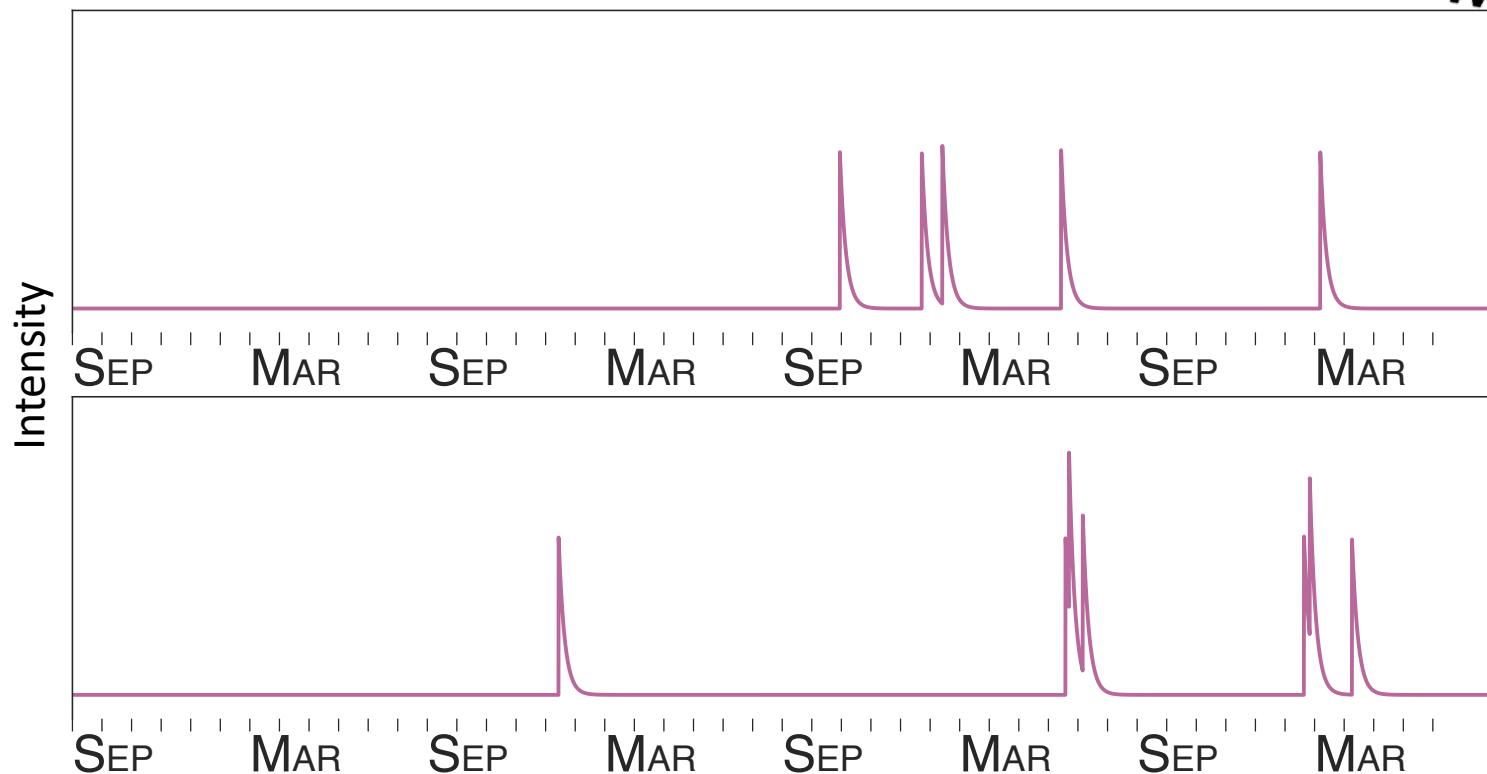
Details in the  
reference below!

# Learning cluster (I): Version Control

# Content



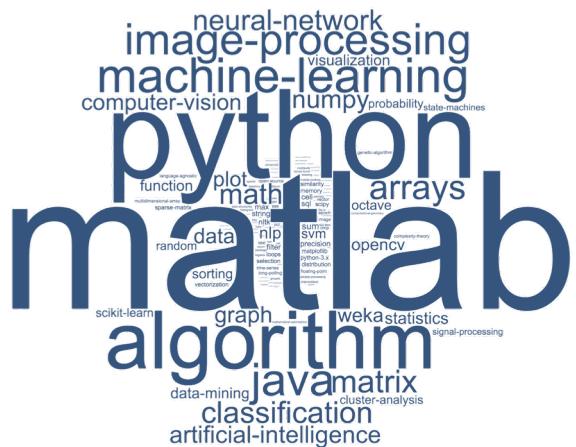
# Intensities



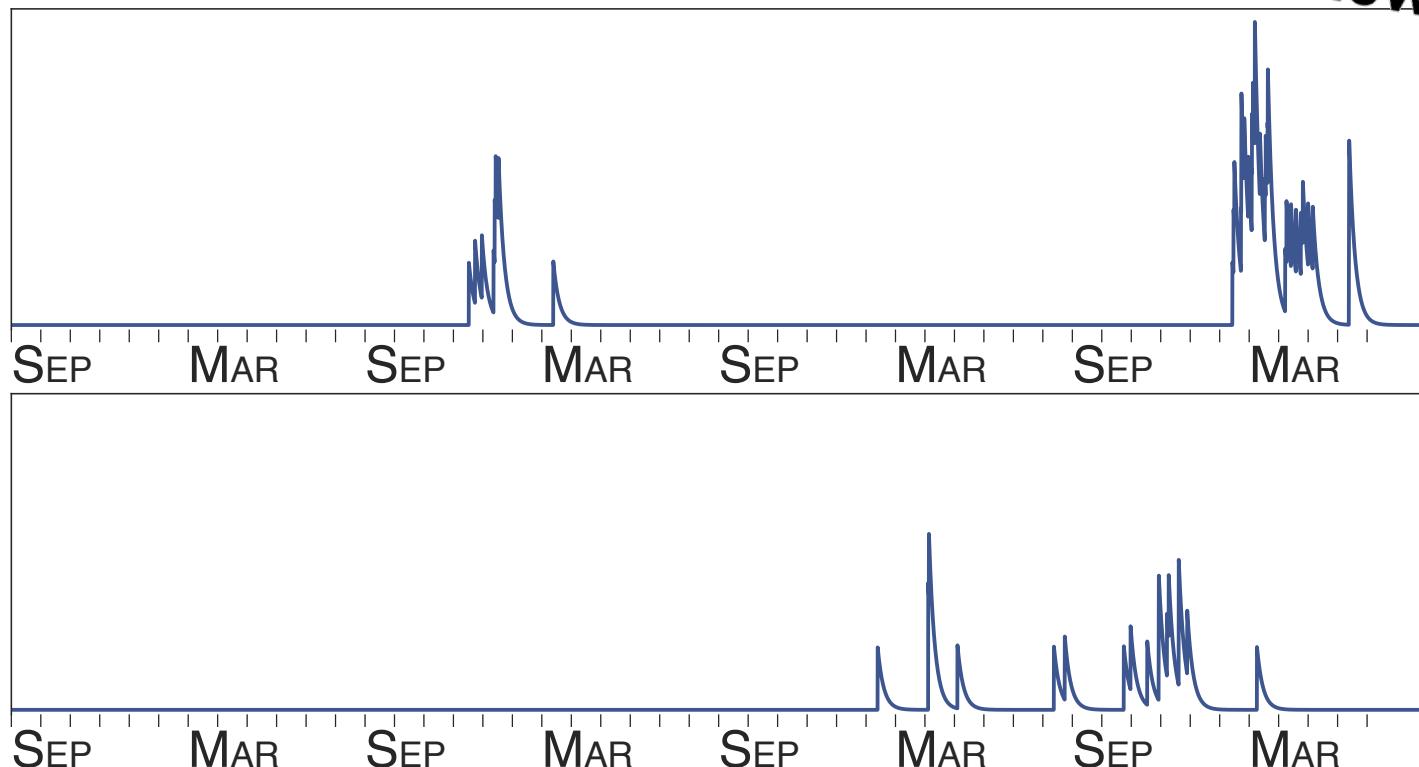
**Version control tasks tend to be specific,  
quickly solved after performing few questions**

# Learning cluster (II): Machine learning

# Content



# Intensities



**Machine learning tasks tend to be more complex and require asking more questions**

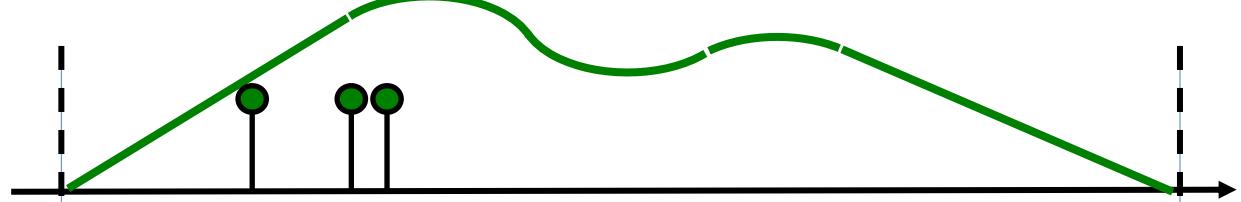
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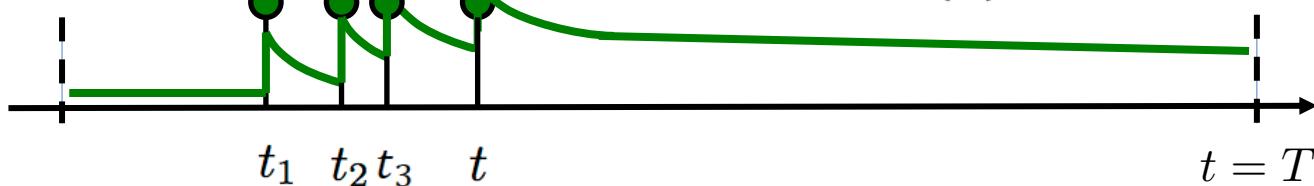
# Towards real-world temporal dynamics

Up to now, we have focused on simple temporal dynamics (and intensity functions):

$$\lambda^*(t) = \mu$$



$$\lambda^*(t) = \sum_j \alpha_j k(t - t_j)$$



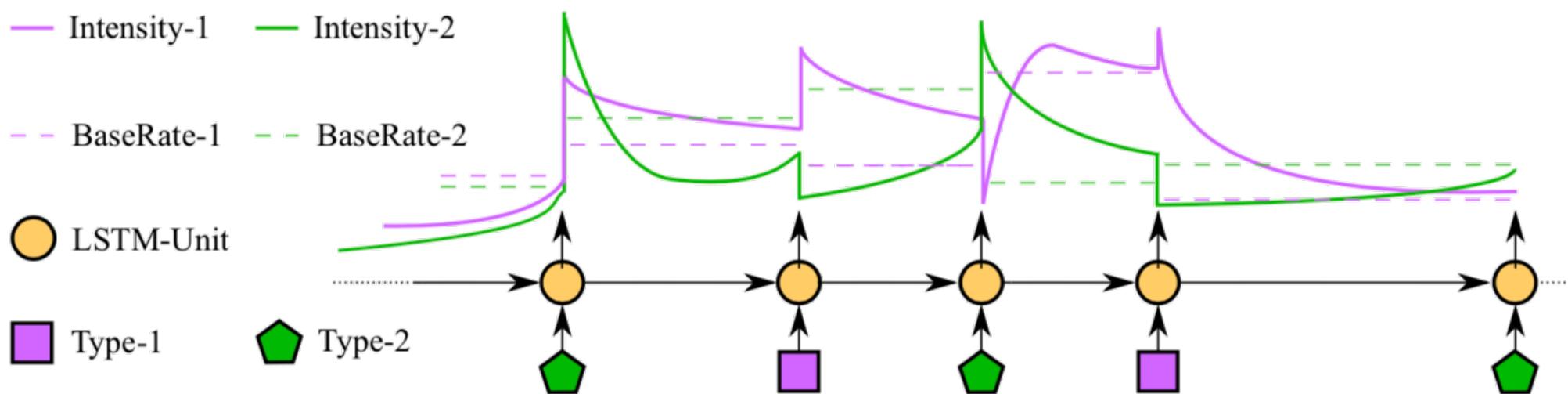
$$\lambda^*(t) = \mu + \alpha \sum_{t_i \in \mathcal{H}(t)} \kappa_\omega(t - t_i)$$

Recent works make use of **RNNs** to capture more complex dynamics

[Du et al., 2016; Dai et al., 2016; Mei & Eisner, 2017; Jing & Smola, 2017;  
Trivedi et al., 2017; Xiao et al., 2017a; 2018]

# Neural Hawkes process

- 1) History effect does not need to be additive
- 2) Allows for complex memory effects  
(such as delays)



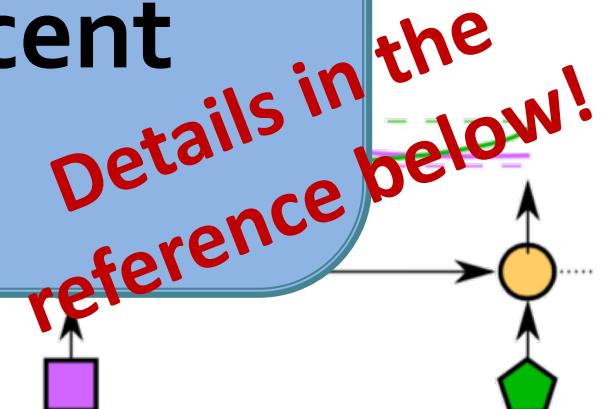
# Neural Hawkes process

$$\lambda_u(t) = f_u(\mathbf{w}_u^\top \mathbf{h}(t))$$

$$\mathbf{h}(t) = \text{DNN}(\mathcal{U}(t))$$

Memory

Parameter learning using  
stochastic gradient descent

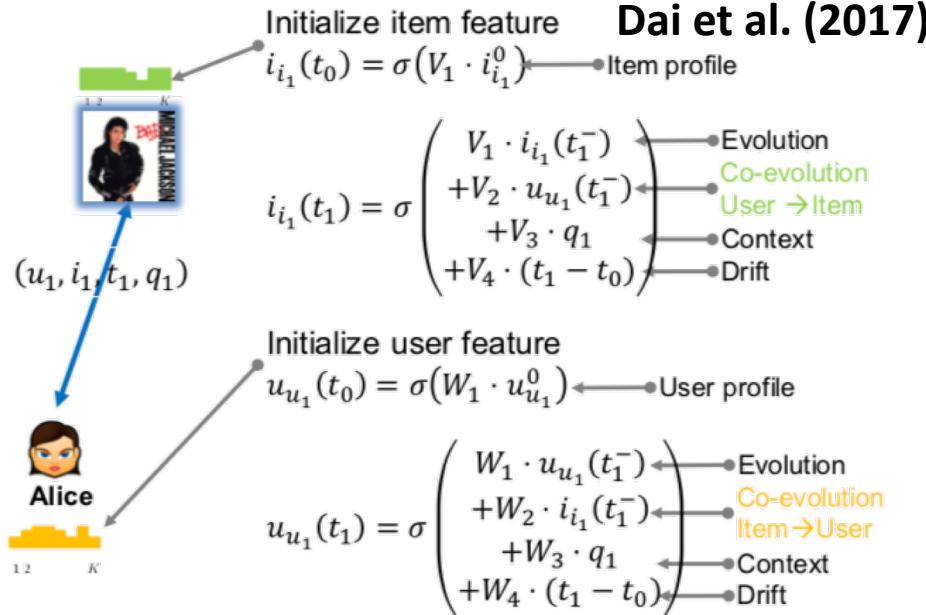


# Applications (I): Predictive Models

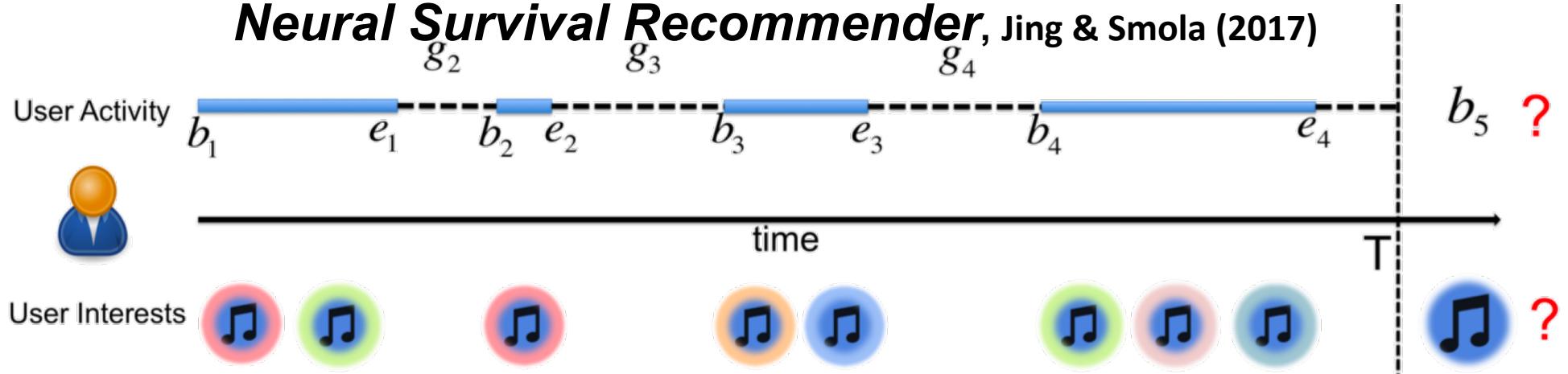
## *Know-Evolve*, Trivedi et al. (2017)



## *Coevolutionary Embedding*, Dai et al. (2017)

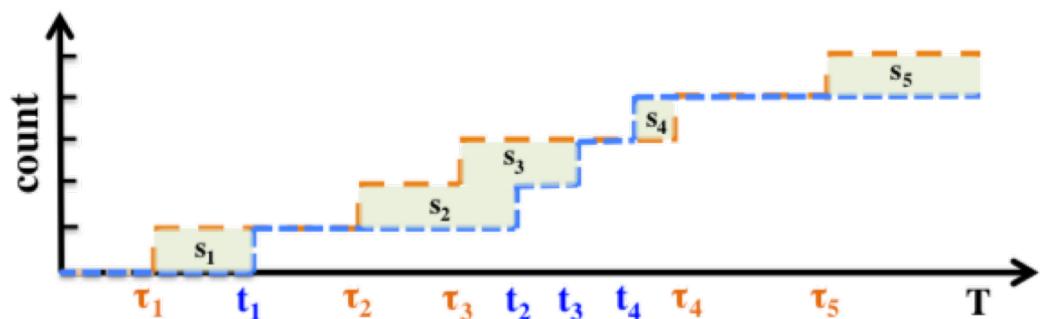


## *Neural Survival Recommender*, Jing & Smola (2017)



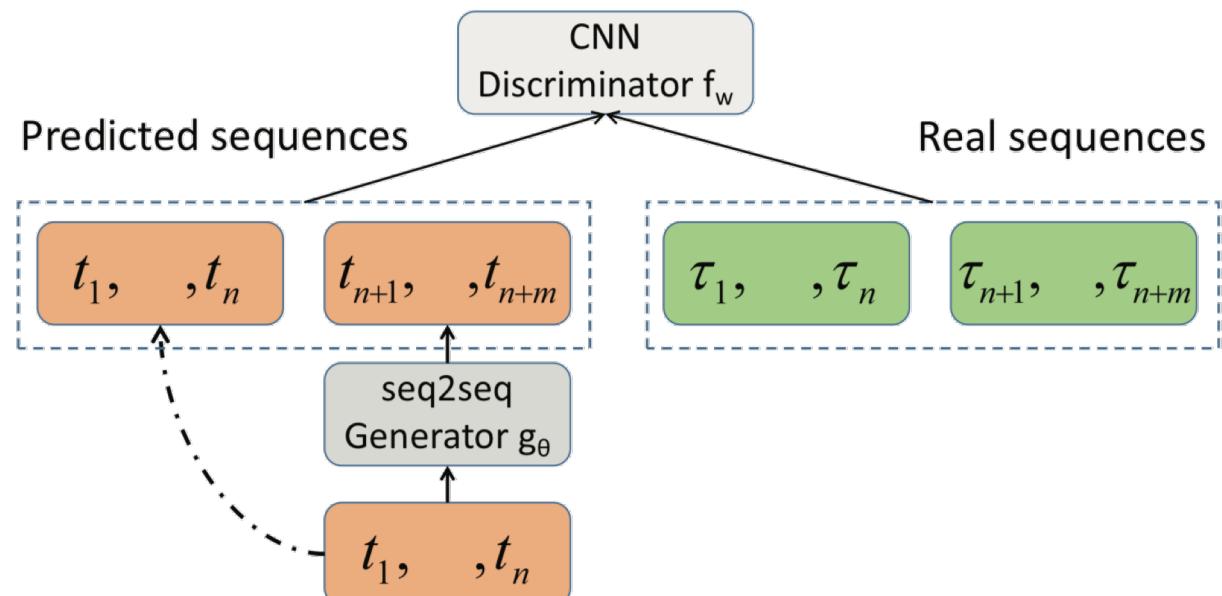
# Applications (II): Generative Models

**Key idea:** Intensity- and likelihood-free models



Wasserstein-Distance for  
Temporal Point Processes

GAN architecture



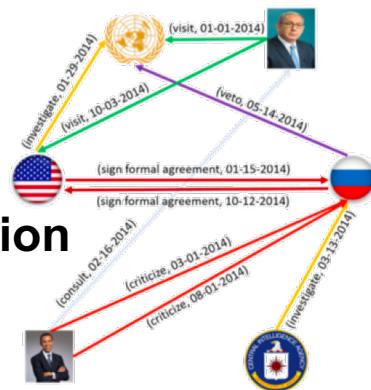
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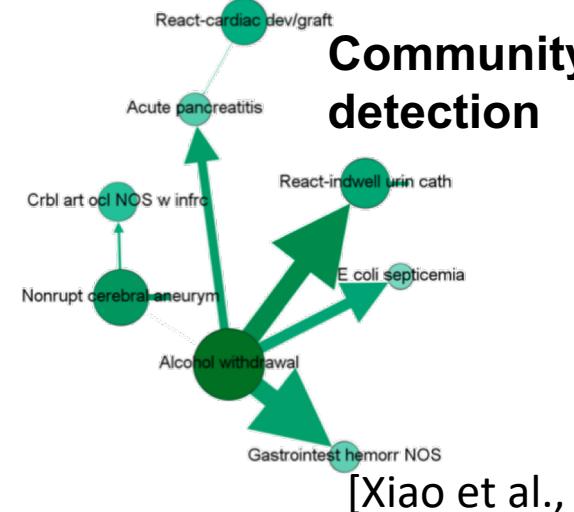
# Temporal point processes beyond prediction

So far, we have focused on models that improve predictions:

## Link prediction

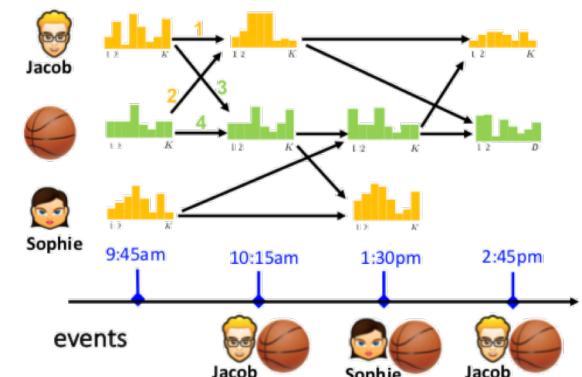


[Trivedi et al., 2017]



[Xiao et al., 2017]

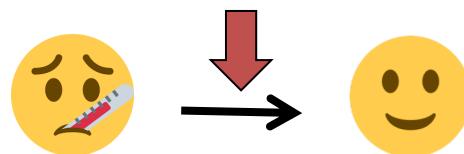
## Recommendations



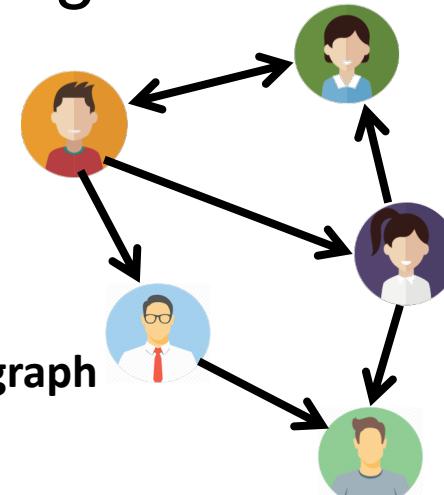
[Dai et al., 2017]

Recent works have focused on performing **causal inference using event sequences**:

## Treatment effect



## Granger causality graph



[Xu et al., 2016; Achab et al., 2017; Kuśmierczyk & Gomez-Rodriguez, 2018]

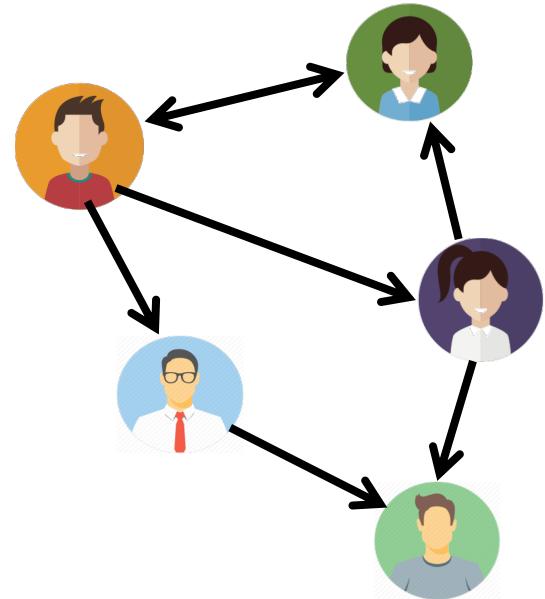
# Uncovering Causality from Hawkes Processes

## Multivariate Hawkes process:

$$N(t) = \sum_{u \in \mathcal{U}} N_u(t)$$

$$\lambda_u(t) = \mu_u + \sum_{v \in \mathcal{U}} \int_0^t k_{u,v}(t - t') dN_v(t')$$

Effect of v's past events on u



## Granger causality:

“X causes Y in the sense of Granger causality if forecasting future values of Y is more successful while taking X past values into account”

[Granger, 1969]

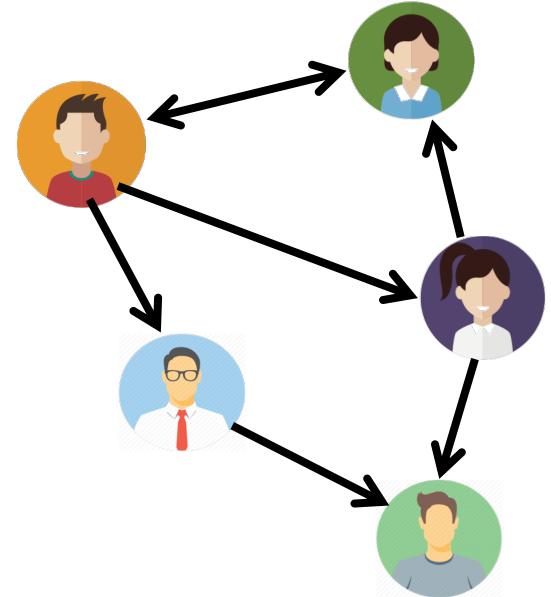
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Effect of v's past events on u



## Granger causality on multivariate Hawkes processes:

“  $N_v(t)$  does not Granger-cause  $N_u(t)$  w.r.t.  $N(t)$  if and only if  $k_{u,v}(\tau) = 0$  for  $\tau \in \mathbb{R}^+$  ”

[Eichler et al., 2016]

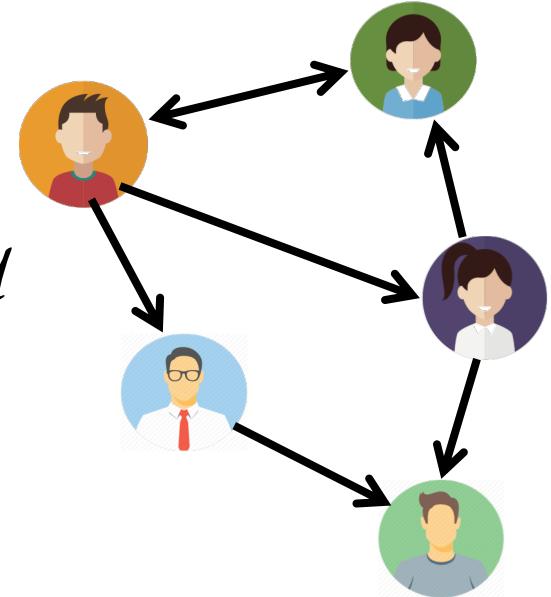
[Achab et al., ICML 2017]

# Uncovering Causality from Hawkes Processes

Goal is to estimate  $G = [g_{uv}]$ , where:

$$g_{uv} = \int_0^{+\infty} k_{u,v}(\tau) d\tau \geq 0 \text{ for all } u, v \in \mathcal{U}$$

Average total # of events of node  $u$  whose *direct* ancestor is an event by node  $v$



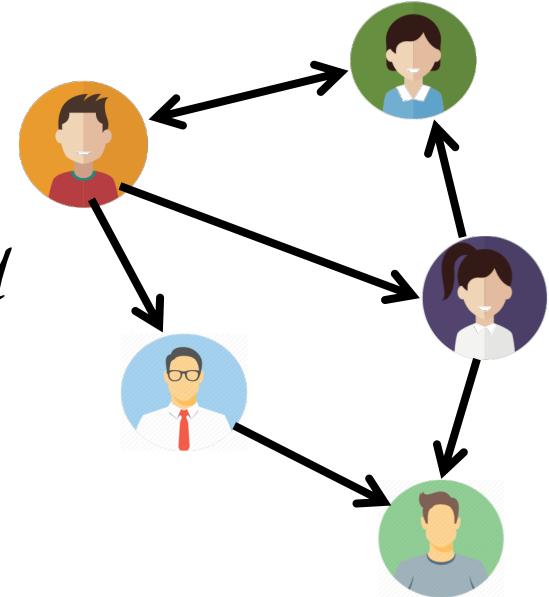
Then,  $G = [g_{uv}]$  quantifies the *direct causal relationship* between nodes.

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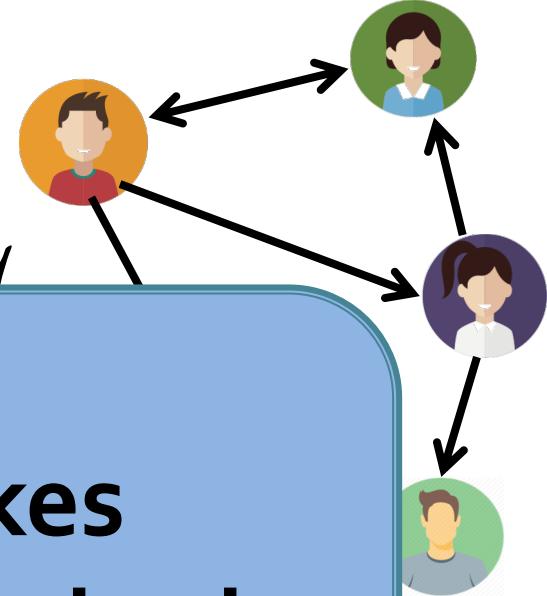
Then,  $G = [g_{uv}]$  quantifies the *direct causal relationship* between nodes.

**Key idea:** Estimate  $G$  using the cumulants  $dN(t)$  of the Hawkes process.

# Uncovering Causality from Hawkes Processes

Goal is to estimate  $G = [g_{uv}]$ , where:

$$g_{uv} = \int_{-\infty}^{+\infty} k_u(\tau) d\tau > 0 \text{ for all } u, v \in \mathcal{U}$$



**Non parametric Hawkes  
cumulant estimation method**

(with TensorFlow implementation)

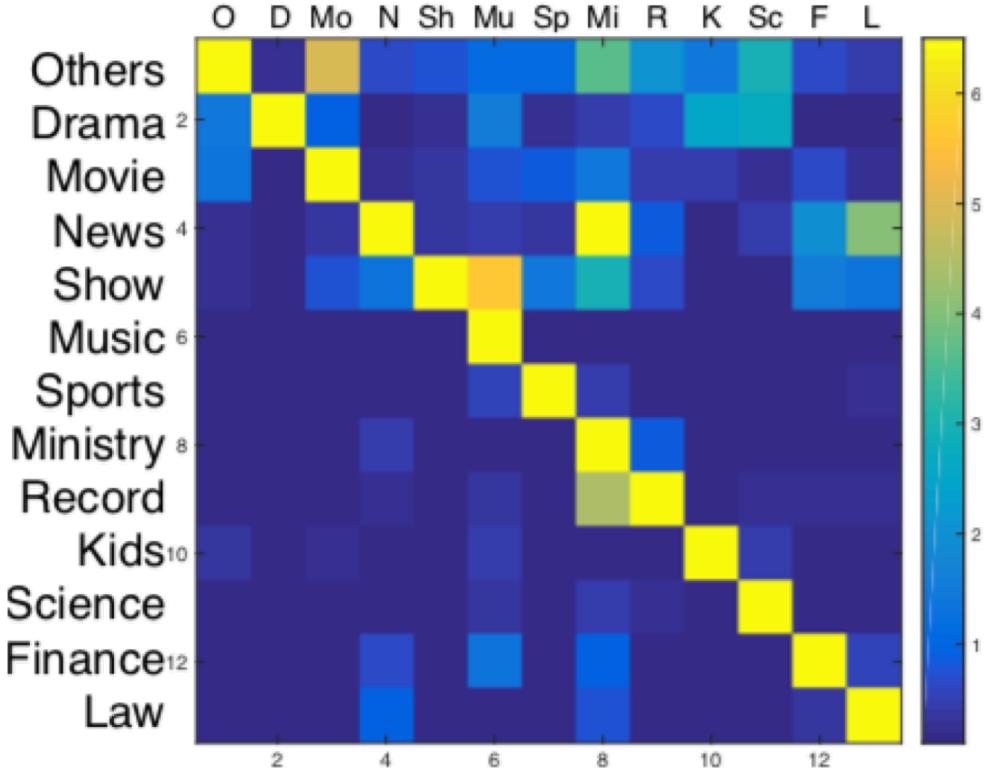
*Details in the **hip**  
reference below!*

The  
bet

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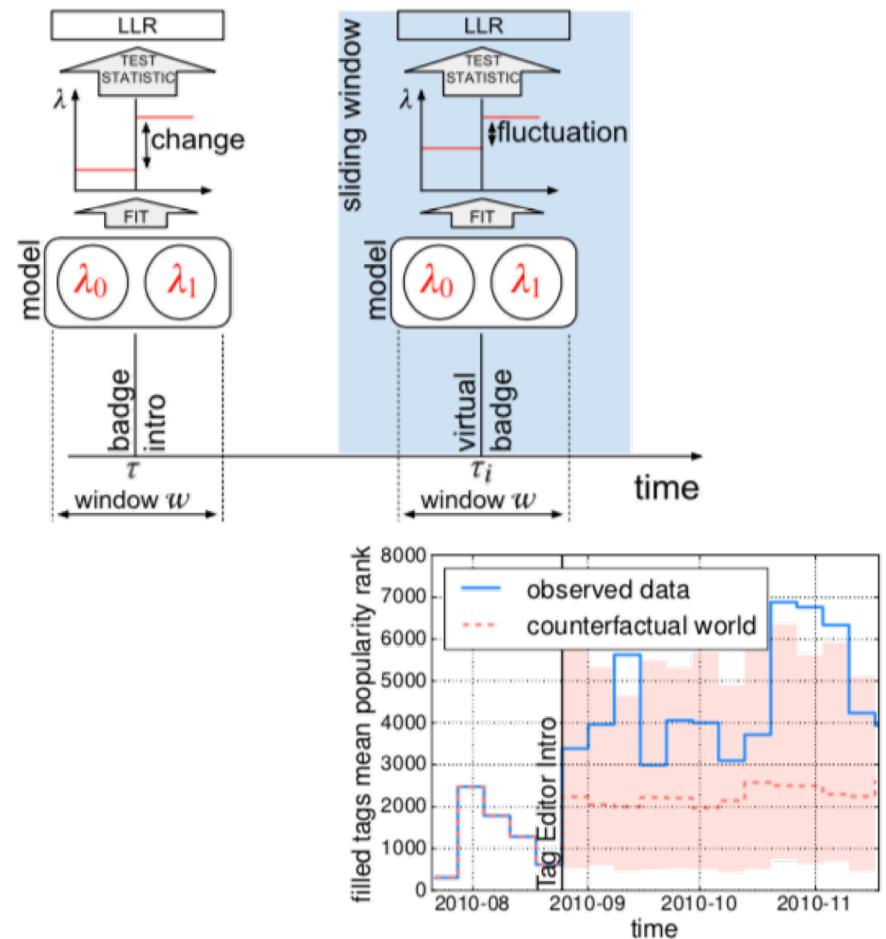
# Causal reasoning: Applications

## Infectivity matrix estimation



[Xu et al., 2016]

## Effect of Badges



[Kuśmierczyk & Gomez-Rodriguez, 2018]

# Outline of the Seminar

## TEMPORAL POINT PROCESSES (TPPs): INTRO

1. Intensity function
2. Basic building blocks
3. Superposition
4. Marks and SDEs with jumps

## MODELS & INFERENCE

1. Modeling event sequences
2. Clustering event sequences
3. Capturing complex dynamics
4. Causal reasoning on event sequences

## RL & CONTROL

1. Marked TPPs: a new setting
2. Stochastic optimal control
3. Reinforcement learning

This  
lecture

Next  
lecture