

[illegible]

(b) (5) DPP, (b) (5) ACP

(b) (7)(C), (b) (7)(D)

Learning Outcomes Assessed

1. Define a generic strategy for solving problems
2. Represent problems using appropriate description languages
3. Apply selection and repetition
4. Solve problems using working storage
5. Apply sequence and iteration to solving problems
6. Integrate problem solving with program design

START OF MAIN SECTION - TOTAL 100 MARKS

There are fifteen (15) questions in this section

1. The mutants are giving trouble again and this time it is up to you (an ordinary human) to assist. You are a transporter on a secret island mutant prison where your job is to transport mutant prisoners, one at a time, from the quarantine centre to the interrogation centre across a dangerous river.

The mutant prisoners are never restrained in any way and always behave when you are present around them since they are really just a bunch of misunderstood nice guys.

Explain in detail how you would transport Deadpool, Sabretooth and Wolverine from the quarantine center to the interrogation centre with the following constraints:

- a) You are only allowed to use a single boat to transport the mutant prisoners.
- b) The single boat used for transportation can only hold you and one other mutant prisoner.
- c) Sabretooth and Wolverine hate each other and when left alone, they will start fighting which will get you fired.
- d) Deadpool and Wolverine hate each other and when left alone, they will start fighting which will get you fired.
- e) Sabretooth and Deadpool are buddies that will never fight but instead enjoy planning how they will beat Wolverine the next time they meet him when you are not present.
- f) You can travel across the river as many times as you want, with or without a passenger, once Deadpool, Sabretooth and Wolverine get transferred from the quarantine center to the interrogation centre without any fights breaking out in your absence.

[6 Marks]

2. Obito has been given a new mission - to package party bags of blue, green and red marbles for the upcoming party for the kids. His teacher Minato gave him 6533 blue marbles, 7097 green marbles and 6157 red marbles and gave the following instructions:

- (a) Every marble must be placed in a party bag.
- (b) Every party bag must have the same number of blue marbles.
- (c) Every party bag must have the same number of green marbles.
- (d) Every party bag must have the same number of red marbles.
- (e) The maximum number of party bags must be used.

Unfortunately, Obito has been daydreaming about his crush Rin the whole time and has not done anything and now its mere minutes before the party starts! Help Obito determine the maximum number of party bags to purchase as well as the number of each type of marble in a party bag.

[6 Marks]

3. What is the smallest number of people that must be present in a room to guarantee that seven people have the same *uppercase two letter initials* using their first name and last name only?

[3 Marks]

4. Several 1x1x1 white cubes are assembled to make a single 7x7x7 white cube. If the outside surface of the 7x7x7 white cube is painted red, how many 1x1x1 cubes will have:

- a) All faces painted white?
- b) Exactly five faces painted white?
- c) Exactly four faces painted white?

[9 Marks]

5. You are given a bag filled with squares made out of wire. No two squares have the same size. The bag contains a single square of length equal to every possible integer between 39 and 93 inclusive. You must discard from the bag only those squares that have an odd area.

- (a) Avoiding manual addition, what the the total length of wire used to make all the remaining squares in the bag?
 (b) Write a pseudo code algorithm entitled SUM_SQUARES that solves this problem using variables and a loop.

[8 Marks]

6. You are given two (2) empty buckets, X and Y . Bucket X holds a volume of 4 litres when completely filled and bucket Y holds a volume of 9 litres when completely filled. You are also given a single basin which is initially empty and holds a volume of 10 litres when completely filled. You must come up with four separate algorithms as follows:

Use only buckets X and Y to measure out the following volumes of water into the basin:

- a) 2 litres. b) 3 litres. c) 6 litres. d) 7 litres.

You must adhere to the following constraints:

- C1) There are no measurement markings on any basin or bucket which would indicate the volume of water present in the basin or bucket when it is not full.
 C2) You are allowed to transfer water from one bucket to another bucket.
 C3) You must transfer the required water into a basin all at once from a single bucket. You are not allowed to fill a basin in parts.
 C4) You are not allowed to transfer water from the basin into any bucket.
 C5) In each algorithm, the basin and buckets X and Y are all initially empty and buckets X and Y must both be empty at the end.

You are only allowed to use any number of one or more of the following instructions to implement your algorithms:

- A) Fill bucket X from the main pipe.
 B) Fill bucket Y from the main pipe.
 C) Transfer water from bucket X to bucket Y until bucket X is empty.
 D) Transfer water from bucket Y to bucket X until bucket Y is empty.
 E) Transfer water from bucket X to bucket Y until bucket Y is filled.
 F) Transfer water from bucket Y to bucket X until bucket X is filled.
 G) Empty bucket X into the drain.
 H) Empty bucket Y into the drain.
 I) Empty bucket X into the basin.
 J) Empty bucket Y into the basin.

Each algorithm must be specified as a sequence of uppercase letters horizontally from left to right where each letter (A to J) represents one of the instructions above. For example, the algorithm: **Measure out 1 litre of water into the basin** can be specified as: **B F G F G J**

[8 Marks]

7. Shikamaru wants to unlock Timari's smartphone but does not know the password. Timari tells Shikamaru that her smartphone's password, P , which is a base ten positive integer, consists of four (4) positive base ten integers (W , X , Y and Z) written one after the other in that order with no characters or spaces between them. So if $W = 29_{10}$, $X = 57_{10}$, $Y = 83_{10}$ and $Z = 14_{10}$ then Timari's password $P = 29578314_{10}$. Help Shikamaru find Timari's smartphone password if Timari says that W , X , Y and Z are given by:

$$W = 1089_{10} \text{ OR } 22_{16} \quad X = (0011\ 0001_2 \times 2A_{16}) - (0101\ 0000_2 \times 31_8)$$

$$Y = 1175_8 \div 31_{16} \quad Z = 189_{10} \text{ AND } 57_{16}$$

[10 Marks]

8. Tobirama has a sock drawer filled with 6 pairs of black socks, 5 pairs of grey socks, 4 pairs of red socks, 7 pairs of green socks and 5 pairs of blue socks. He must get up early to go to the airport to catch a plane and leave for his vacation and it is very dark outside. Electricity has been cut off and he has no light sources so he cannot see the socks in his sock drawer. What is the least number of socks that Tobirama must choose from his sock drawer such that:
- (a) He obtains 6 pairs of socks that match? (The pairs can be different colors.)
 (b) He obtains 6 pairs of socks, all of the same color? (e.g 6 pairs of all black or 6 pairs of all green).

[4 Marks]

9. Nooshem the alien has one hand and does arithmetic in base N where N is a single digit base ten integer. Nooshem says that the values of the sums $33_N + 44_N$ and $23_N + 34_N$ each have different number of digits in base N . How many fingers does Nooshem have?

[6 Marks]

10. Ino and Choji are playing a game involving squares and rectangles only. Ino gives Choji two positive integers: the first positive integer is the length and the second positive integer is the sum of the length and width. Ino wants Choji to tell her three things: Whether the shape is a square or a rectangle and the perimeter and area of the shape. Write a pseudo code algorithm entitled SQUARE_OR_RECTANGLE that helps Choji as he is hungry and cannot concentrate to answer Ino's questions. Your algorithm must work as shown below:

```
console> Enter length: 2
console> Enter sum of length and width: 4
This shape is a Square with perimeter 8 and area 4
```

```
console> Enter length: 2
console> Enter sum of length and width: 6
This shape is a Rectangle with perimeter 12 and area 8
```

[8 Marks]

11. How would you vertically stack several rectangular shoeboxes on top of each other such that the stack has the shortest height? Write a pseudo code algorithm entitled SHOEBOXES that reads four positive integers: the three dimensions of the rectangular shoebox and the number of rectangular shoeboxes to stack vertically. Your algorithm must determine how to best stack the rectangular shoeboxes vertically so that they have the minimum possible height. Your algorithm must work as shown below:

```
console> Enter length of shoebox: 3
console> Enter width of shoebox: 5
console> Enter height of shoebox: 2
console> Enter number of shoeboxes: 7
The minimum height of 7 shoeboxes of dimensions 3x5x2 is 14
```

```
console> Enter length of shoebox: 9
console> Enter width of shoebox: 5
console> Enter height of shoebox: 7
console> Enter number of shoeboxes: 4
The minimum height of 4 shoeboxes of dimensions 9x5x7 is 20
```

```
console> Enter length of shoebox: 3
console> Enter width of shoebox: 5
console> Enter height of shoebox: 8
console> Enter number of shoeboxes: 5
The minimum height of 5 shoeboxes of dimensions 3x5x8 is 15
```

[8 Marks]

12. There are 89 people in a room. What is the largest value of n such that the statement '*At least n people in this room have birthdays falling in the same month*' is always true? [2 Marks]
13. X_{10} is a base ten 3 digit prime number with the left most digits being 1 then 6. Y_{10} is a base ten 2 digit prime number with the left most digit being 8. P_2 and Q_2 are both binary numbers given by $P_2 = X_{10} \text{ AND } Y_{10}$ and $Q_2 = X_{10} \text{ OR } Y_{10}$. If P_2 has a single 1 bit and Q_2 has a single 0 bit, find X_{10} and Y_{10} . [8 Marks]
14. Write a pseudo code algorithm entitled PERCENTAGE_TO_GRADE that reads in a single positive integer between 0 and 100 inclusive that describes the percentage score in an exam. Your program must print the equivalent letter grade obtained by the person according to the ranges specified below:
- A grade of **A** corresponds to a percentage mark between 100 to 80 inclusive.
 - A grade of **B** corresponds to a percentage mark between 79 to 60 inclusive.
 - A grade of **C** corresponds to a percentage mark between 59 to 40 inclusive.
 - A grade of **F** corresponds to a percentage mark between 39 to 0 inclusive.

Your algorithm must work as shown below:

```
console> Please enter your percentage mark: 88
Your grade is: A
```

```
console> Please enter your percentage mark: 67
Your grade is: B
```

```
console> Please enter your percentage mark: 45
Your grade is: C
```

```
console> Please enter your percentage mark: 12
Your grade is: F
```

[6 Marks]

15. Hashirama is buying his favourite snacks at the grocery where there is a single cashier with an unlimited supply of 5 dollar bills and 1 dollar bills only. Hashirama pays the cashier for his snacks and instructs the cashier to give him his change using the least number of bills. Hashirama is bad at Math and wants you help him check his change that he got back from the cashier.

Write a pseudo code algorithm entitled MAKE_CHANGE that reads in a single non-negative integer value representing the total change Hashirama should get back. Your algorithm must print the least number of bills that he should get back and the number of bills of each kind. Your algorithm should work as shown below:

```
console> Please enter the amount of change in dollars: 19
7 bills in total : 3 $5 bills and 4 $1 bills
```

```
console> Please enter the amount of change in dollars: 20
4 bills in total : 4 $5 bills and 0 $1 bills
```

```
console> Please enter the amount of change in dollars: 3
3 bills in total : 0 $5 bills and 3 $1 bills
```

[8 Marks]

END OF MAIN SECTION

START OF BONUS SECTION - TOTAL 100 MARKS**There are fifteen (15) questions in this section**

- Hanzo is spending his vacation in a beach house that is located very close to the seashore. In the night time after he has dinner, he spends only a few minutes on the upstairs porch where he enjoys watching the flickering lights given off by three (3) lighthouses located some distance away in the ocean - Lighthouses **Jiraiya**, **Orochimaru** and **Tsunade**. Most of the time, he would see a flash from only one lighthouse with each lighthouse taking turns flashing as time goes on. Sometimes, if he stays out on the porch longer, he would see a flash from only two lighthouses simultaneously. Over time, each possible pair of lighthouses would flash together simultaneously except the other lighthouse. Last night he stayed out on the porch even longer and he eventually saw a flash from all three lighthouses simultaneously! This piqued his interest and he decided to observe the timing of the flashes of light from each lighthouse. Hanzo notices that each lighthouse has a '*period*', which is the *constant* time taken between successive flashes of light from that lighthouse.

Lighthouse **Jiraiya** has a period of 35 seconds. Lighthouse **Orochimaru** has a period of 45 seconds. Lighthouse **Tsunade** has a period of 63 seconds. Tonight, Hanzo first notices a simultaneous triple flash at 07:37:15 pm (37 minutes and 15 seconds past 7 o'clock).

Help Hanzo determine how many more simultaneous triple flashes will occur until midnight inclusive because he does not want to stay up past midnight counting simultaneous triple flashes.

[10 Marks]
- Forog and Gorof are both one-handed aliens of different species. Both of them have between 2 and 9 fingers inclusive. Both Forog and Gorof do arithmetic in different bases as they each have a different number of fingers. They each have a favourite number that is the largest three digit number in their respective bases.

Forog's and Gorof's favourite numbers can be represented in base ten as F_{10} and G_{10} respectively. The digits of F_{10} are added in base ten to give X_{10} . The digits of G_{10} are added in base ten to give Y_{10} .

 - Deduce how many fingers each alien has if F_{10} and G_{10} are both even.
 - Deduce how many fingers each alien has if, F_{10} is not a multiple of X_{10} and G_{10} is not a multiple of Y_{10} and X_{10} and Y_{10} are both odd.

[10 Marks]
- In a box there are red and blue balls. If you select a handful of them with eyes closed, you have to grab at least 5 of them to make sure at least one of them is red and you have to grab at least 10 of them to make sure both colors appear among the balls selected. How many balls are there in the box?

[4 Marks]
- You are given a bag filled with *equilateral* triangles made out of wire. No two *equilateral* triangles have the same size. The bag contains a single *equilateral* triangle with a side of length equal to every possible integer between 28 and 82 inclusive. You must discard from the bag only those *equilateral* triangles that have an even perimeter.

 - What the the total length of wire used to make all the remaining *equilateral* triangles in the bag?
 - Write a pseudo code algorithm entitled SUM_TRIANGLES that solves this problem using variables and a loop.

[10 Marks]
- A survey was done at a local ComicCon festival where 110 cosplay attendees were asked which of the Avengers characters they liked. 100 attendees liked Thor, 20 attendees liked Iron Man and 15 attendees liked Captain America. Also, 12 attendees liked Thor and Iron Man, 10 attendees liked Thor and Captain America and 2 attendees liked Thor and Captain America only. There were attendees that liked all three of the aforementioned Avengers. No attendee disliked all three of the aforementioned Avengers.

 - Find out how many attendees liked Iron Man only.
 - Find out how many attendees liked Captain America only.
 - Find out how many attendees *disliked* Thor but liked *both* Iron Man *and* Captain America.

[10 Marks]
- A large bag consists of identical balls of different colors. Each ball could be one of 7 colors and for each color, there are 77 balls in the bag. At least how many balls are needed to be picked out to ensure that one can obtain 7 groups of 7 balls each such that in each group the balls are monochromatic?

Note: Monochromatic means that all balls in the group are same in color. The balls in different groups can have the same color. For example, if we had 49 balls of the first color, then we are done.

[4 Marks]

7. A math contest is made up of 52 multiple choice questions each worth either 0 (if wrong or no answer) or 1 (if correct). How many students must write the test to be sure that at least 39 have the same final score?

[4 Marks]

8. Danzo works a minimum of 0 hours and a maximum of 12 hours only on Mondays, Wednesdays and Fridays.

Danzo works for a fixed rate of \$10 per hour for the first 10 hours of the week, \$15 per hour for the second 10 hours of the week, \$20 per hour for the third 10 hours of the week and \$25 per hour for the last 6 hours of the week.

Danzo receives a bonus of \$50 whenever he works for more than 10 hours on exactly 1 day only. Danzo receives a bonus of \$75 whenever he works for more than 10 hours on exactly 2 days only. Danzo receives a bonus of \$100 whenever he works for more than 10 hours on all 3 days.

Write a pseudo code algorithm entitled DANZO_WAGE_CALCULATOR that reads in three integers between 0 and 12 inclusive that represent the hours worked on Monday, Wednesday and Friday of a given week and prints Danzo's total salary for that week after all adjustments. Your algorithm must work as shown below:

```
console> Enter number of hours worked on Monday: 10
console> Enter number of hours worked on Wednesday: 10
console> Enter number of hours worked on Friday: 10
Total Weekly Wage: $450
```

```
console> Enter number of hours worked on Monday: 11
console> Enter number of hours worked on Wednesday: 0
console> Enter number of hours worked on Friday: 9
Total Weekly Wage: $300
```

```
console> Enter number of hours worked on Monday: 11
console> Enter number of hours worked on Wednesday: 0
console> Enter number of hours worked on Friday: 11
Total Weekly Wage: $365
```

```
console> Enter number of hours worked on Monday: 11
console> Enter number of hours worked on Wednesday: 11
console> Enter number of hours worked on Friday: 11
Total Weekly Wage: $625
```

[10 Marks]

9. A box contains 100 balls of the following colours: 28 red, 17 blue, 21 green, 10 white, 12 yellow and 12 black. What is the smallest number n such that any n balls drawn from the box will contain at least 15 balls of the same colour?

[4 Marks]

10. P is a 6 digit binary number that contains only 2 one bits. Q is the 2 digit octal representation of P . R is the 2 digit hexadecimal representation of P . S is the decimal representation of P . Either Q or R does not contain any zero digits. The sum of the digits of S is an odd decimal number. If the one bits in P are consecutive, find P .

[5 Marks]

11. There are 11 objects that have to be placed in n slots. Find the number of maximum possible slots n such that there exists at least a single slot in which 3 objects are placed?

[3 Marks]

12. Itachi asks his friends to guess the three digit password to his smart phone. Hidan guesses 682 and Itachi says only one number is correct and correctly placed. Kakuzu guesses 614 and Itachi says only one number is correct but incorrectly placed. Sasori guesses 206 and Itachi says two numbers are correct but incorrectly placed. Deidara guesses 738 and Itachi says no number is correct. Kisame guesses 780 and Itachi says only one number is correct but incorrectly placed. Help Itachi's friends find the password to his smart phone from his feedback above.

[8 Marks]

13. Write an algorithm entitled IS_MULTIPLE that reads in two positive integers from the user and works as follows:

```
console> Enter 1st integer: 6
console> Enter 2nd integer: 3
6 is a multiple of 3
```

```
console> Enter 1st integer: 4
console> Enter 2nd integer: 8
8 is a multiple of 4
```

```
console> Enter 1st integer: 4
console> Enter 2nd integer: 9
No integer is a multiple of the other
```

[6 Marks]

14. Write an algorithm entitled COST_PRICE_CALCULATOR that reads in the current selling price and the percentage profit made on the cost price (two positive floating-point numbers) and calculates the original cost price.

```
console> Enter the current selling price: $25
console> Enter the percentage profit: 25
Original Cost Price of Item : $20
```

```
console> Enter the current selling price: $11.20
console> Enter the percentage profit: 12
Original Cost Price of Item : $10.00
```

```
console> Enter the current selling price: $30.22
console> Enter the percentage profit: 18.50
Original Cost Price of item : $25.50
```

```
console> Enter the current selling price: $21.74
console> Enter the percentage profit: 17.50
Original Cost Price of item : $18.50
```

[6 Marks]

15. Write an algorithm entitled SUM_OF_FACTORS that reads a single positive integer from the user and displays the sum of all the positive integer factors of the integer. Your algorithm must work as follows:

```
console> Enter a positive integer: 5
Sum of all positive integer factors of 5 is 6
```

```
console> Enter a positive integer: 6
Sum of all positive integer factors of 6 is 12
```

[6 Marks]

END OF BONUS SECTION