Main_4

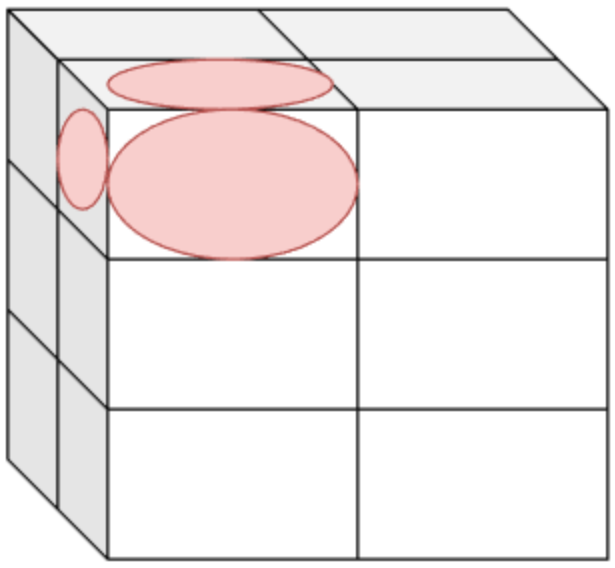
How many 1x1x1 cubes to make a single 7x7x7?

Since the cubes have the same length, width and height creating a 7^3 cube is just creating $1^3 \times 7$

Area of a cube is $Side^2$

We can identify 3 position types in cubes made up of smaller cubes.

Corner Position

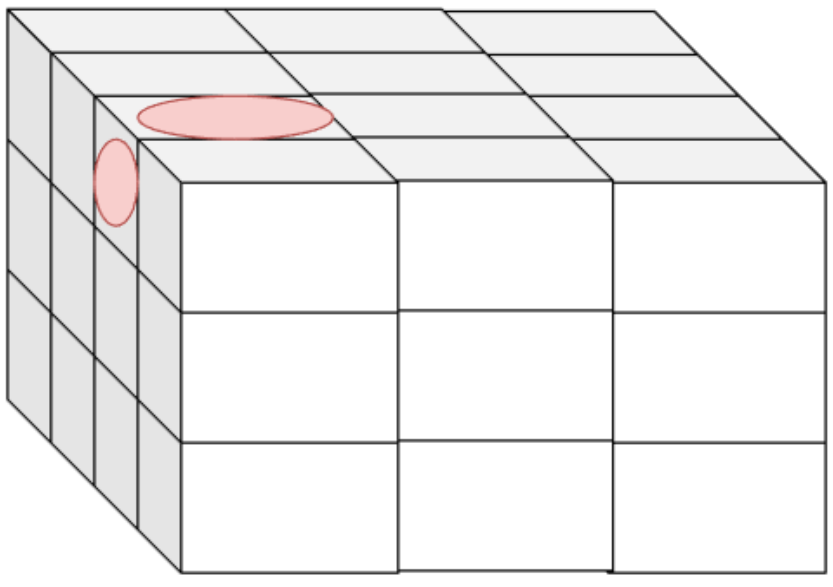


This is where **three (3) faces** of the smaller cube are on the outside layer of the large cube.

This type will have **3 red faces** and 3 white faces.

The 3 red faces will be exposed to the red paint while the 3 faces inside will still be white.

Edge Position

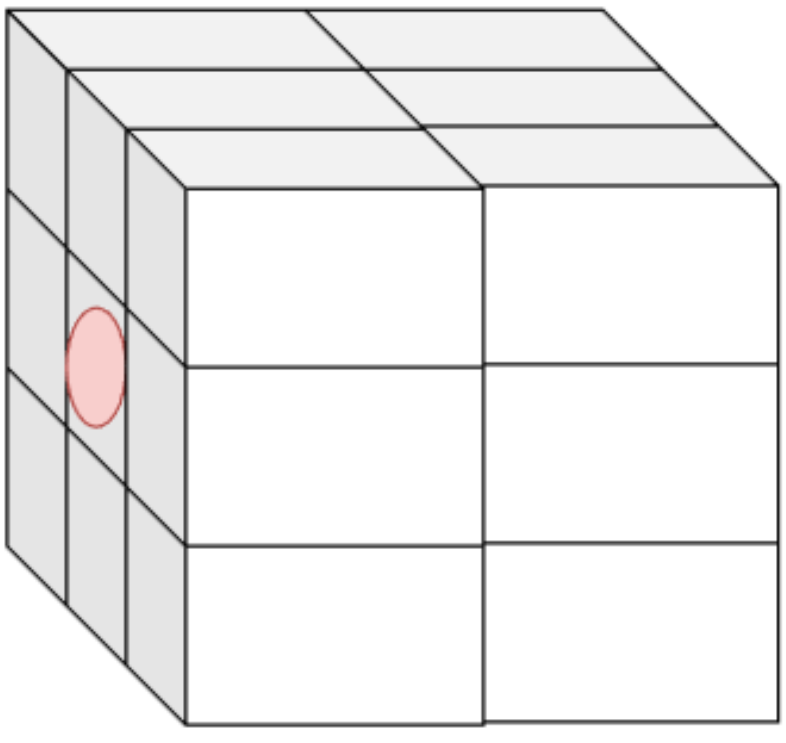


This is where **two (2) faces** of the smaller cube are on the outside layer of the large cube.

This type will have 2 red faces and 4 white faces.

The 2 red faces will be exposed to the red paint while the 4 faces inside will still be white.

Middle Position



This is where **one (1) face** of the smaller cube are on the outside layer of the large cube.

This type will have 1 red face and 4 white faces.

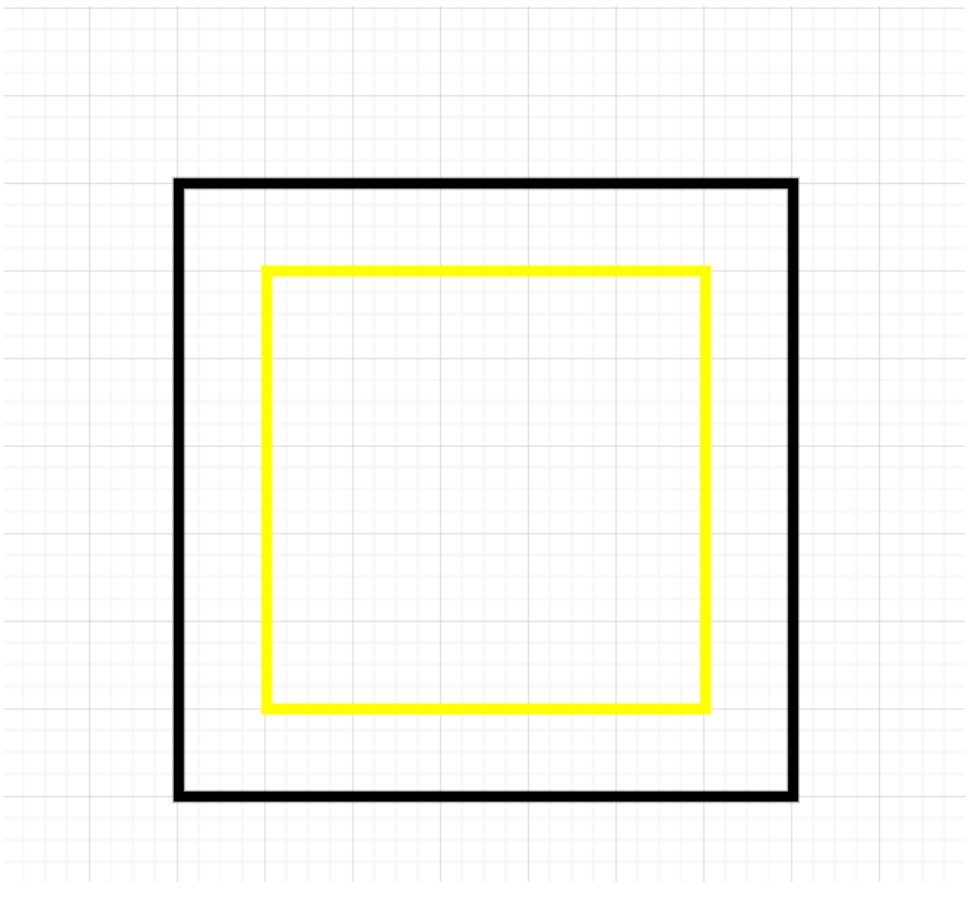
The 2 red faces will be exposed to the red paint while the 4 faces inside will still be white.

Part A - All faces painted white

The only cubes that will have all faces painted white are those that have **NO** faces exposed to the outer layer of the 7x7x7 layer cube.

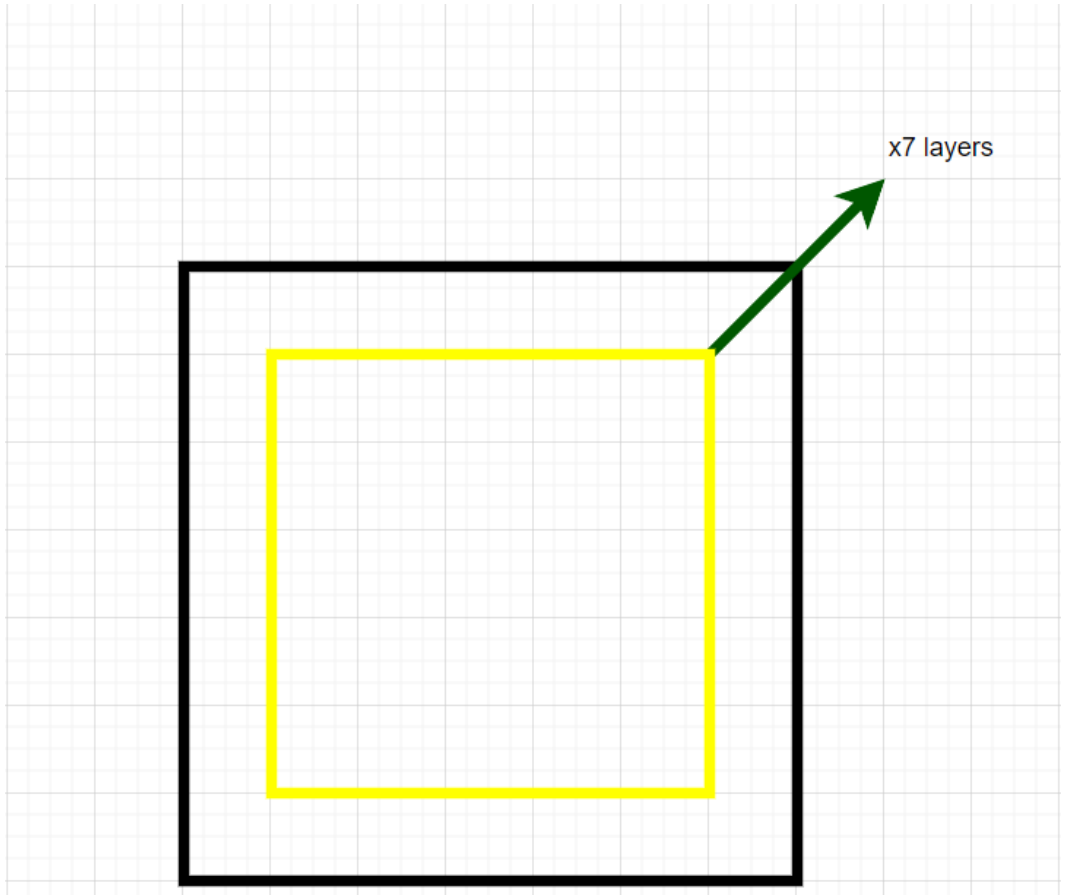
We can think of just one face at a time right now, a 7x7 square.

If we cut out the outermost edges of the square i.e. we remain with a 5x5 square (indicated with yellow)



This inner square does not touch the outside 2 dimentional world *i.e. 7x7 square*

If we extend this yellow square into three dimentions *i.e. 7x7x7 cube* We get almost the same result just multipled by 7 layers(the width of the cube) .



Calculations

$7 \times 7 = 49 \rightarrow 49 \times 7 = 343$ total cubes

$5 \times 5 = 25 \rightarrow 25 \times 5 = 125$ ***inner*** cubes

Answer

> There are 125 cubes with ALL faces painted white

Part B - Exactly Five Faces Painted White?

Note: The 7x7x7 cube is made up of 343 1x1x1 cubes

$7 \times 7 = 49 \rightarrow 49 \times 7 = 343$ total cubes

VERY IMPORTANT *- The inner part of the 7x7x7 cube do not count in the following discussion!* ***The cubes inside the large 7x7x7 are completely white cubes.***

To have 5 white faces you must have 1 red face. This means only ONE (1) face may be exposed to the outside of the 7x7x7 cube.

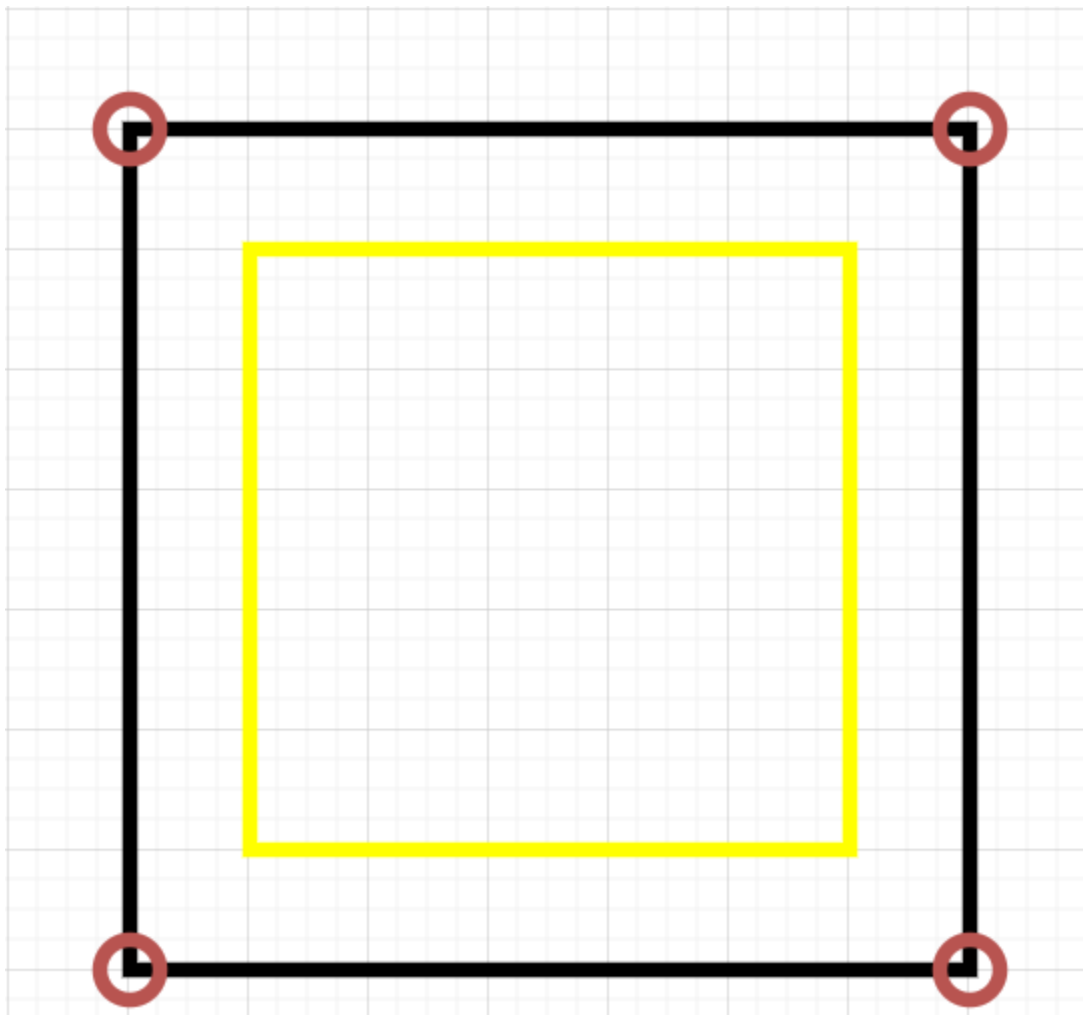
If we refer back to the “Middle Position” diagram earlier in the document we can see where in the 7x7x7 structure only ONE face of a 1x1x1 cube may be exposed.

But how many of these occur?

This is very simple. This only occurs where the other two types do not occur. A cube always has 8 corners.

We can rule out these 8 cubes now $\rightarrow 343 - 8 = 335$

We can again look to the 2 dimentional world for some ease of understanding.



The yellow portion is neither touching the corners of the 7x7 square (red) nor is it touching the edges of the 7x7 square (yellow).

The dimensions of this cube is 5x5.

No back to the more complicated 3 dimensions.

Since the cube has 6 faces…the inner cubes don’t count (since they are fully white) we must multiply the number of “Middle Position” cubes by the number of faces the 7x7x7 cube has (6).

Answer and Calculations

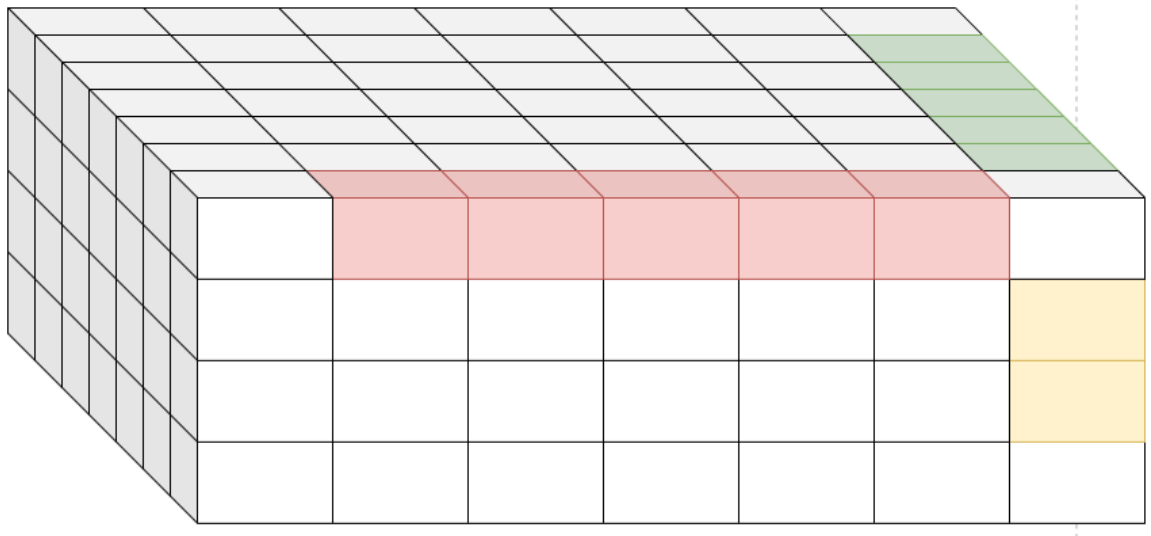
1 middle face → $5 \times 5 = 25$

6 middle faces → $25 \times 6 = 150$

Part C - Exactly Four Faces Painted White

We have to think about this one in 3d sadly.

If we think about a cube (or look at the diagra of one I’ll place below). We can see that it has edges on three (3) axes. There are edges on the length, width and height of the cube.



The red cubes illustrate the length edges, the green cubes indicate the width edges, and the yellow cubes indicate the height edges.

We can visualize(with all powerful imagination (or math)) how many of these edge cubes exist in the larger 7x7x7 cube.

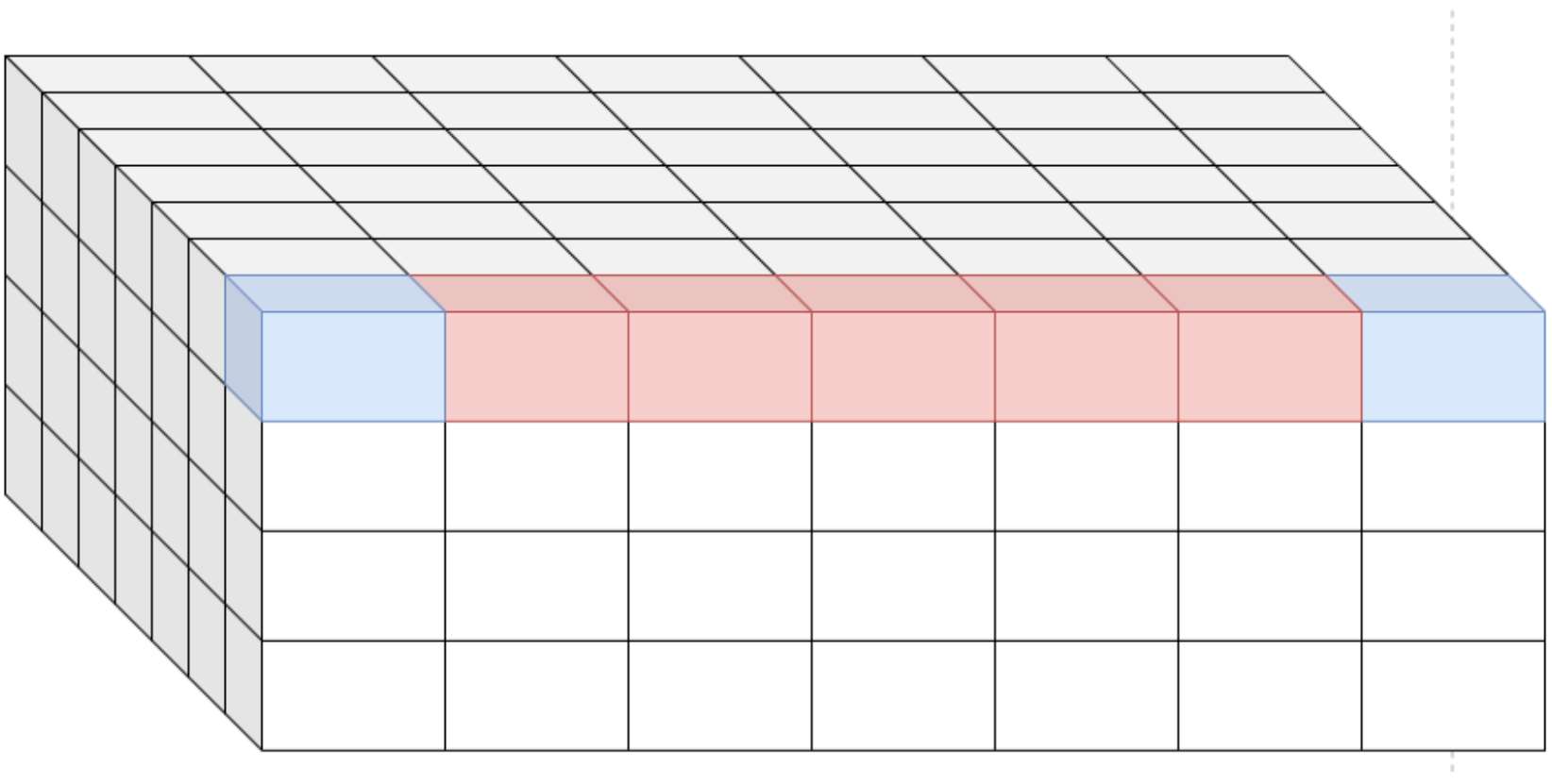
*Fact: A cube **always** has 12 edges.*

Since cubes always have 12 edges we now need to identify how many 1x1x1 cubes are present in a single edge.

Keep in mind; the length of all the edges are equal.

The entire edge of the large 7x7x7 cube is obviously 7 (7 1x1x1 cubes) **BUT** this number includes the “Corner Positions” aswell. In 1 length of a cube there are 2 corner pieces

- *as indicated by the blue corner pieces surrounding the red edge pieces in the diagram below*



This one edge now has 5 1x1x1 cubes (red)when ignoring the corners (blue).

Now lets remember how many edges the entire stucture has…12

Calculations

$$5 \times 12 = 60$$

5 1x1x1 blocks in an edge* 12 edges

Answer

There are 60 cubes with exactly 60 faces painted white.