Integral Indefinida – Aula 1

01. $\int \partial x \quad = \quad x+c$

02. $\int x^p \partial x \quad = \quad \frac{x^{p+1}}{p+1}+c \rightarrow p \neq -1$

03. $\int e^x \partial x \quad = \quad e^x+c$

04. $\int \frac{\partial x}{x} \quad = \quad \ln|x|+c$

05. $\int u^p \partial u \quad = \quad \frac{u^{p+1}}{p+1}+c \rightarrow p \neq -1$

06. $\int e^u \partial u \quad = \quad e^u+c$

07. $\int \frac{\partial u}{u} \quad = \quad \ln|u|+c$

08. $\int a^u \partial u \quad = \quad \frac{a^u}{\ln|a|}+c$

Exercício I

$$\int \partial x = x+c$$
$$\int x^3 \partial x = \frac{x^4}{4}+c \tag{1}$$

Exercício II

$$\int 3x^4 \partial x = 3\int x^4 \partial x = 3\frac{x^5}{5}+c = \frac{3x^5}{5}+c \tag{2}$$

Exercício III

$$\int (4x^5+7)\partial x = \int 4x^5 \partial x + \int 7 \partial x = 4\int x^5 \partial x + 7\int \partial x = 4\frac{x^6}{6}+7x+c = \frac{2x^6}{3}+7x+c \tag{3}$$

Exercício IV

$$\int 3\partial x = 3\int \partial x = 3x+c \tag{4}$$

Exercício V

$$\int 5x^7 \partial x = 5\int x^7 \partial x = \frac{5x^8}{8}+c \tag{5}$$

Exercício VI

$$\int\left(5+3x^2-7x^3\right)\partial x=5\int\partial x+3\int x^2\partial x-7\int x^3\partial x=5x+\frac{3x^3}{3}-\frac{7x^4}{4}+c=\frac{-7x^4}{4}+x^3+5x+c \tag{6}$$

Integral Indefinida – Aula 2

Exercício I

$$\int\frac{\partial x}{x^4}=\int x^{-4}\partial x=\frac{x^{-3}}{-3}+c=\frac{-1}{3x^3}+c \tag{7}$$

Exercício II

$$\int\sqrt{x^3}\partial x=\int x^{\frac{3}{2}}\partial x=\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+c=\sqrt{x^5}\cdot\frac{2}{5}=\frac{2\sqrt{x^5}}{5}+c \tag{8}$$

Exercício II

$$\int\left(7\sqrt[5]{x^2}+\frac{3}{x^3}\right)\partial x=\int\left(7x^{\frac{2}{5}}+3x^{-3}\right)\partial x=7\int x^{\frac{2}{5}}\partial x+3\int x^{-3}\partial x=7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)}+3\frac{x^{-2}}{-2}+c=5\sqrt[5]{x^7}-\frac{3}{2x^2}+c \tag{9}$$

Integral indefinida – Aula 3

Exercício I

$$\int\left(3x^{-4}-3x+4\right)\partial x=3\int x^{-4}\partial x-3\int x\partial x+4\int\partial x=3\frac{x^{-3}}{(-3)}-3\frac{x^2}{2}+4x+c=\frac{-1}{x^3}-\frac{3x^2}{2}+4x+c \tag{10}$$

Exercício II

$$\int\frac{2}{3}\sqrt{x}\partial x=\frac{2}{3}\int x^{\frac{1}{2}}\partial x=\frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{4\sqrt{x^3}}{9}+c \tag{11}$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}x^3}{\sqrt[3]{x^2}}\partial x=\int \frac{x^{\frac{1}{2}}x^3}{x^{\frac{2}{3}}}\partial x=\int x^{\frac{1}{2}+3-\frac{2}{3}}\partial x=\int x^{\frac{3+18-4}{6}}\partial x=\int x^{\frac{17}{6}}\partial x=\frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)}+c=\frac{6\sqrt[6]{x^{23}}}{23}+c \quad (12)$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3}=\int x^{-3}\partial x=\frac{x^{-2}}{-2}+c=\frac{-1}{2x^2}+c \quad (13)$$

Exercício II

$$\int \frac{\partial x}{x}=\ln x+c \quad (14)$$

Exercício III

$$\int \sqrt{2x+1}\partial x=\int \sqrt{u}\frac{\partial u}{2}=\frac{1}{2}\int u^{\frac{1}{2}}\partial u=\frac{1}{2}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{1}{2}\frac{2}{3}\sqrt{u^3}+c=\frac{\sqrt{u^3}}{3}+c=\frac{\sqrt{(2x+1)^3}}{3}+c$$

$$u=2x+1\rightarrow\frac{\partial u}{2}=\partial x \quad (15)$$

$$\frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+c\right)}{\partial x}=\frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1}\cdot 2+0=(2x+1)^{\frac{1}{2}}=\sqrt{2x+1}$$

Exercício IV

$$\int \frac{5x^3+2x+3}{x}\partial x=\int\left(\frac{5x^3}{x}+\frac{2x}{x}+\frac{3}{x}\right)\partial x=\int\left(5x^2+2+\frac{3}{x}\right)\partial x=$$

$$5\int x^2\partial x+2\int\partial x+3\int\frac{\partial x}{x}=5\frac{x^3}{3}+2x+3\ln|x|+c=\frac{5x^3}{3}+2x+3\ln|x|+c \quad (16)$$

$$\frac{\partial\left(\frac{5x^3}{3}+2x+3\ln|x|+c\right)}{\partial x}=\frac{5}{3}3x^2+2+3\frac{1}{x}+0=5x^2+2+\frac{3}{x}=\frac{5x^3+2x+3}{x}$$

Exercício V

$$\int\left(2x^4+3+5e^x+\frac{7}{x}\right)\partial x=2\int x^4\partial x+3\int\partial x+5\int e^x\partial x+7\int\frac{\partial x}{x}=$$
$$2\frac{x^5}{5}+3x+5e^x+7\ln|x|+c=\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c \tag{17}$$
$$\frac{\partial\left(\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c\right)}{\partial x}=\frac{2}{5}5x^4+3+5e^x+7\frac{1}{x}+0=2x^4+3+5e^x+\frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int\frac{5t^2+7}{\sqrt[3]{t^4}}\partial t=\int\frac{5t^2+7}{t^{\frac{4}{3}}}\partial t=\int t^{\frac{-4}{3}}\left(5t^2+7\right)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\partial t=$$
$$5\int t^{\frac{2}{3}}\partial t+7\int t^{\frac{-4}{3}}\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot 3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c \tag{18}$$
$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$
$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int\frac{\partial x}{\sqrt{x}}=\int\frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c \tag{19}$$

Exercício II

$$\int\left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int\frac{\partial x}{x}=3e^x+2\ln|x|+c \tag{20}$$

Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \int u^3 \frac{\partial u}{4} = \frac{1}{4} \int u^3 \partial u = \frac{1}{4}\frac{u^4}{4} + c = \frac{u^4}{16} + c = \frac{(2x^2+1)^4}{16} + c =$$
$$\frac{(2x^2+1)^4}{2^4} + c = \left(\frac{2x^2+1}{2}\right)^4 + c = \left(x^2 + \frac{1}{2}\right)^4 + c$$
$$u = 2x^2+1 \rightarrow \frac{\partial u}{4} = x \partial x \tag{21}$$
$$\frac{\partial\left[\left(x^2+\frac{1}{2}\right)^4 + c\right]}{\partial x} = 4\left(x^2+\frac{1}{2}\right)^3 \cdot 2x + 0 = 8x\left(x^2+\frac{1}{2}\right)^3 = 8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2 =$$
$$(8x^3+4x)\left(x^4+x^2+\frac{1}{4}\right) = 8x^7+8x^5+2x^3+4x^5+4x^3+x = 8x^7+12x^5+6x^3+x$$
$$(2x^2+1)^3 x = (2x^2+1)^2(2x^2+1)x = (4x^4+4x^2+1)(2x^3+x) = 8x^7+4x^5+8x^5+4x^3+2x^3+x =$$
$$8x^7+12x^5+6x^3+x$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x \partial x = 3\int e^x \partial x = 3e^x + c$$
$$\int e^{x^2+1} x \partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2}\int e^u \partial u = \frac{1}{2}e^u + c = \frac{e^{x^2+1}}{2} + c$$
$$u = x^2+1 \rightarrow \frac{\partial u}{2} = x \partial x \tag{22}$$
$$\frac{\partial\left(\frac{e^{x^2+1}}{2} + c\right)}{\partial x} = \frac{1}{2}e^{x^2+1} 2x + 0 = e^{x^2+1} x$$

Exercício II

$$\int e^{x^4+1} x^3 \partial x = \int e^u \frac{\partial u}{4} = \frac{1}{4}\int e^u \partial u = \frac{1}{4}e^u + c = \frac{e^{x^4+1}}{4} + c$$
$$u = x^4+1 \rightarrow \frac{\partial u}{4} = x^3 \partial x \tag{23}$$
$$\frac{\partial\left(\frac{e^{x^4+1}}{4} + c\right)}{\partial x} = \frac{1}{4}e^{x^4+1} 4x^3 + 0 = e^{x^4+1} x^3$$

Exercício III

$$\int \frac{x}{\left(2x^2-1\right)^3}\partial x=\int \left(2x^2-1\right)^{-3} x\,\partial x=\int u^{-3}\frac{\partial u}{4}=\frac{1}{4}\int u^{-3}\partial u=\frac{1}{4}\frac{u^{-2}}{(-2)}+c=\frac{-1}{8u^2}+c=$$
$$\frac{-1}{8\left(2x^2-1\right)^2}+c$$
$$u=2x^2-1\rightarrow\frac{\partial u}{4}=x\,\partial x \tag{24}$$
$$\frac{\partial\left(\frac{-1}{8\left(2x^2-1\right)^2}+c\right)}{\partial x}=\frac{\partial\left(\frac{-\left(2x^2-1\right)^{-2}}{8}+c\right)}{\partial x}=\frac{-1}{8}(-2)\left(2x^2-1\right)^{-3}4x+0=\left(2x^2-1\right)^{-3}x=$$
$$\frac{x}{\left(2x^2-1\right)^3}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\partial x=\int \left(2x^2-1\right)^{-1} x\,\partial x=\int u^{-1}\frac{\partial u}{4}=\frac{1}{4}\int u^{-1}\partial u=\frac{1}{4}\ln|u|+c=\frac{\ln\left|2x^2-1\right|}{4}+c$$
$$u=2x^2-1\rightarrow\frac{\partial u}{4}=x\,\partial x \tag{25}$$
$$\frac{\partial\left(\frac{\ln\left|2x^2-1\right|}{4}+c\right)}{\partial x}=\frac{1}{4}\frac{1}{2x^2-1}4x+0=\frac{x}{2x^2-1}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u-1)^2\sqrt{u}\,\partial u=\int (u-1)^2 u^{\frac{1}{2}}\,\partial u=\int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u=$$
$$\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u=\int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u=\int u^{\frac{5}{2}}\partial u-2\int u^{\frac{3}{2}}\partial u+\int u^{\frac{1}{2}}\partial u=$$
$$\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)}-2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^7}}{7}-\frac{4\sqrt{u^5}}{5}+\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c$$
$$u=1+x\rightarrow x=u-1\rightarrow \partial u=\partial x \tag{26}$$
$$\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{3}{2}}}{3}+c\right)}{\partial x}=$$
$$\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}}-\frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}}+\frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}}+0=(1+x)^{\frac{5}{2}}-2(1+x)^{\frac{3}{2}}+(1+x)^{\frac{1}{2}}=$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}}-2(1+x)^{\frac{3}{2}-\frac{1}{2}}+1\right)=(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}}-2(1+x)^{\frac{2}{2}}+1\right)=$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^2-2(1+x)+1\right)=(1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1)=(1+x)^{\frac{1}{2}}x^2=x^2\sqrt{1+x}$$

Exercício II

$$\int x^2\sqrt{1+x}\,\partial x\rightarrow \int (u^2-1)^2 u\,2u\,\partial u=2\int (u^2-1)^2u^2\,\partial u=2\int (u^4-2u^2+1)u^2\,\partial u=$$
$$2\int \left(u^6-2u^4+u^2\right)\partial u=2\int u^6\partial u-4\int u^4\partial u+2\int u^2\partial u=2\frac{u^7}{7}-4\frac{u^5}{5}+2\frac{u^3}{3}+c=$$
$$\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c \tag{27}$$
$$u=\sqrt{1+x}\rightarrow u^2=1+x\rightarrow x=u^2-1\rightarrow \partial x=2u\,\partial u$$

O que é uma Integral Definida – Aula 10

Exercício I

$$\int_1^2 x^3\,\partial x=\left.\frac{x^4}{4}\right]_1^2=\frac{(2)^4}{4}-\frac{(1)^4}{4}=\frac{16}{4}-\frac{1}{4}=\frac{16-1}{4}=\frac{15}{4}=3{,}75 \tag{28}$$

Integral Definida – Aula 10a

Exercício I

$$\int_0^2 (6x^2-4x+5)\partial x = 6\int_0^2 x^2\partial x - 4\int_0^2 x\partial x + 5\int_0^2 \partial x = 6\frac{x^3}{3} - 4\frac{x^2}{2} + 5x\Big]_0^2 = 2x^3-2x^2+5x\Big]_0^2 = \\ x(2x^2-2x+5)\Big]_0^2 = [(2)(2(2)^2-2(2)+5)] - [(0)(2(0)^2-2(0)+5)] = 2(8-4+5) = 2\cdot 9 = 18 \tag{29}$$

Exercício II

$$\int_{-1}^0 (2x-e^x)\partial x = 2\int_{-1}^0 x\partial x - \int_{-1}^0 e^x\partial x = 2\frac{x^2}{2} - e^x\Big]_{-1}^0 = x^2-e^x\Big]_{-1}^0 = ((0)^2-e^{(0)}) - ((-1)^2-e^{(-1)}) = \\ -1-\left(1-\frac{1}{e}\right) = -1-1+\frac{1}{e} = -2+\frac{1}{e} \tag{30}$$

Integral definida – Aula 11

Exercício I

$$\frac{5\pi}{4}\int_0^2 \frac{r\partial r}{1+r^2} = \frac{5\pi}{4}\int_0^2 (1+r^2)^{-1} r\partial r = \frac{5\pi}{4}\int_0^2 u^{-1}\frac{\partial u}{2} = \frac{5\pi}{4}\frac{1}{2}\int_0^2 u^{-1}\partial u = \frac{5\pi}{8}\int_0^2 u^{-1}\partial u = \\ \frac{5\pi}{8}\ln|u|\Big]_0^2 = \frac{5\pi}{8}\ln|1+r^2|\Big]_0^2 = \left[\frac{5\pi}{8}\ln|1+(2)^2|\right] - \left[\frac{5\pi}{8}\ln|1+(0)^2|\right] = \frac{5\pi}{8}\ln|5| - \frac{5\pi}{8}\ln|1| = \\ \frac{5\pi}{8}(\ln|5|-\ln|1|) = \frac{5\pi}{8}(\ln|5|-0) = \frac{5\pi}{8}\ln|5| \\ u=1+r^2 \to \frac{\partial u}{2} = r\partial r \\ \ln|1| = x \to e^x = 1 = e^0 \to x=0 \\ \ln|5| = x \to e^x = 5 \tag{31}$$

Exercício II

$$2\pi\int_0^2 r^2\partial r = 2\pi\frac{r^3}{3}\Big]_0^2 = \frac{2\pi r^3}{3}\Big]_0^2 = \left(\frac{2\pi(2)^3}{3}\right) - \left(\frac{2\pi(0)^3}{3}\right) = \frac{16\pi}{3} \tag{32}$$

Exercício II

$$2\pi\int_0^{\sqrt{2}} (4r-2r^3)\partial r = 8\pi\int_0^{\sqrt{2}} r\partial r - 4\pi\int_0^{\sqrt{2}} r^3\partial r = 8\pi\frac{r^2}{2} - 4\pi\frac{r^4}{4}\Big]_0^{\sqrt{2}} = 4\pi r^2 - \pi r^4\Big]_0^{\sqrt{2}} = \\ \pi r^2(4-r^2)\Big]_0^{\sqrt{2}} = [\pi(\sqrt{2})^2(4-(\sqrt{2})^2)] - [\pi(0)^2(4-(0)^2)] = 2\pi(4-2) = 4\pi \tag{33}$$

Exercício III

$$\pi\int_0^2 x^2\,\partial x=\pi\frac{x^3}{3}\Big]_0^2=\frac{\pi x^3}{3}\Big]_0^2=\left(\frac{\pi(2)^3}{3}\right)-\left(\frac{\pi(0)^3}{3}\right)=\frac{8\pi}{3} \tag{34}$$

Exercício IV

$$\frac{\pi}{16}\int_1^4 x^4\,\partial x=\frac{\pi}{16}\frac{x^5}{5}\Big]_1^4=\frac{\pi x^5}{80}\Big]_1^4=\left(\frac{\pi(4)^5}{80}\right)-\left(\frac{\pi(1)^5}{80}\right)=\frac{4^5\pi}{80}-\frac{\pi}{80}=\frac{1024\pi-\pi}{80}=\frac{1023\pi}{80} \tag{35}$$

Exercício V

$$\pi\int_1^2 (x^2)^2\,\partial x=\pi\int_1^2 x^4\,\partial x=\pi\frac{x^5}{5}\Big]_1^2=\frac{\pi x^5}{5}\Big]_1^2=\left(\frac{\pi(2)^5}{5}\right)-\left(\frac{\pi(1)^5}{5}\right)=\frac{32\pi}{5}-\frac{\pi}{5}=\frac{32\pi-\pi}{5}=\frac{31\pi}{5} \tag{36}$$

Exercício VI

$$\begin{gathered}\pi\int_{-1}^2 (-x^4-x^2+6x+8)\,\partial x=-\pi\int_{-1}^2 x^4\,\partial x-\pi\int_{-1}^2 x^2\partial x+6\pi\int_{-1}^2 x\,\partial x+8\pi\int_{-1}^2 \partial x=\\ -\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\Big]_{-1}^2=\frac{-\pi x^5}{5}-\frac{\pi x^3}{3}+3\pi x^2+8\pi x\Big]_{-1}^2=-\pi x\left(\frac{x^4}{5}+\frac{x^2}{3}-3x-8\right)\Big]_{-1}^2\\ \left[-\pi(2)\left(\frac{(2)^4}{5}+\frac{(2)^2}{3}-3(2)-8\right)\right]-\left[-\pi(-1)\left(\frac{(-1)^4}{5}+\frac{(-1)^2}{3}-3(-1)-8\right)\right]=\\ -2\pi\left(\frac{16}{5}+\frac{4}{3}-6-8\right)-\pi\left(\frac{1}{5}+\frac{1}{3}+3-8\right)=-2\pi\left(\frac{48+20-210}{15}\right)-\pi\left(\frac{3+5-75}{15}\right)=\\ 2\pi\left(\frac{142}{15}\right)+\pi\left(\frac{67}{15}\right)=\pi\left(\frac{284}{15}+\frac{67}{15}\right)=\frac{351\pi}{15}=\frac{3^3\cdot 13\pi}{3\cdot 5}=\frac{3^2\cdot 13\pi}{5}=\frac{117\pi}{5}\end{gathered} \tag{37}$$

Exercício VII

$$\begin{gathered}\pi\int_0^8 (\sqrt[3]{y})^2\,\partial y=\pi\int_0^8 \left(y^{\frac{1}{3}}\right)^2\partial y=\pi\int_0^8 y^{\frac{2}{3}}\partial y=\pi\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\Bigg]_0^8=\frac{3\pi\sqrt[3]{y^5}}{5}\Big]_0^8=\\ \left(\frac{3\pi\sqrt[3]{(8)^5}}{5}\right)-\left(\frac{3\pi\sqrt[3]{(0)^5}}{5}\right)=\frac{3\pi\sqrt[3]{8^3\cdot 8^2}}{5}=\frac{3\pi 2^3\sqrt[3]{(2^3)^2}}{5}=\frac{3\pi 2^3 2^2}{5}=\frac{3\pi 2^5}{5}=\frac{96\pi}{5}\end{gathered} \tag{38}$$

Integral definida – Aula 12

Exercício I

$$\int_1^2 2x\,\partial x=2\int_1^2 x\,\partial x=2\frac{x^2}{2}\Big]_1^2=x^2]_1^2=((2)^2)-((1)^2)=4-1=4-1=3 \tag{39}$$

Exercício II

$$\int_1^4 2\sqrt{x}\,\partial x=\int_1^4 2x^{\frac{1}{2}}\,\partial x=2\int_1^4 x^{\frac{1}{2}}\,\partial x=2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_1^4=\frac{4\sqrt{x^3}}{3}\Bigg]_1^4=\left(\frac{4\sqrt{(4)^3}}{3}\right)-\left(\frac{4\sqrt{(1)^3}}{3}\right)=$$
$$\frac{4\sqrt{4^2 2^2}}{3}-\frac{4}{3}=\frac{32}{3}-\frac{4}{3}=\frac{32-4}{3}=\frac{28}{3} \tag{40}$$

Exercício III

$$\int_1^2 4x^2\,\partial x=4\int_1^2 x^2\,\partial x=4\frac{x^3}{3}\Bigg]_1^2=\frac{4}{3}x^3\Bigg]_1^2=\frac{4}{3}\left(2^3-1^3\right)=\frac{4}{3}7=\frac{28}{3} \tag{41}$$

Integrais definidas e indefinidas – Aula 13

Exercício I

$$\int\left(\frac{3}{x^4}+\frac{2x^2}{3}-2x+7\right)\partial x=\int\left(3x^{-4}+\frac{2}{3}x^2-2x+7\right)\partial x=$$
$$3\int x^{-4}\,\partial x+\frac{2}{3}\int x^2\,\partial x-2\int x\,\partial x+7\int \partial x=3\frac{x^{-3}}{-3}+\frac{2}{3}\frac{x^3}{3}-2\frac{x^2}{2}+7x+c=$$
$$\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c$$
$$\frac{\partial\left(\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c\right)}{\partial x}=\frac{\partial\left(-x^{-3}+\frac{2}{9}x^3-x^2+7x+c\right)}{\partial x}=3x^{-4}+\frac{2}{9}3x^2-2x+7+0=$$
$$\frac{3}{x^4}+\frac{2x^2}{3}-2x+7 \tag{42}$$

Exercício II

$$\int 5\sqrt[3]{x^2}\,\partial x=\int 5x^{\frac{2}{3}}\,\partial x=5\int x^{\frac{2}{3}}\,\partial x=5\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+c=3\sqrt[3]{x^5}+c$$
$$\frac{\partial\left(3\sqrt[3]{x^5}+c\right)}{\partial x}=\frac{\partial\left(3x^{\frac{5}{3}}+c\right)}{\partial x}=3\frac{5}{3}x^{\frac{2}{3}}+0=5\sqrt[3]{x^2} \tag{43}$$

Exercício III

$$\int_2^4 2x^3\partial x=2\int_2^4 x^3\partial x=2\frac{x^4}{4}\Big]_2^4=\frac{1}{2}x^4\Big]_2^4=\frac{1}{2}\left(4^4-2^4\right)=\frac{1}{2}\left((2\cdot 2)^4-2^4\right)=\frac{1}{2}\left(2^4\cdot 2^4-2^4\right)=$$
$$\frac{2^4}{2}\left(2^4-1\right)=2^3(16-1)=8\cdot 15=120 \tag{44}$$

Exercício IV

$$\int_1^2\left(3x^2-2x\right)\partial x=3\int_1^2 x^2\partial x-2\int_1^2 x\,\partial x=3\frac{x^3}{3}-2\frac{x^2}{2}\Big]_1^2=x^3-x^2\Big]_1^2=x^2(x-1)\Big]_1^2=$$
$$\left[2^2(2-1)\right]-\left[1^2(1-1)\right]=4 \tag{45}$$

Integral definida pelo método da substituição – U du – Aula 14

Exercício I

$$\int_0^2\sqrt{2x^2+1}\,x\,\partial x=\int_0^2\sqrt{u}\frac{\partial u}{4}=\frac{1}{4}\int_0^2 u^{\frac{1}{2}}\partial u=\frac{1}{4}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_0^2=\frac{1}{4}\frac{2}{3}\sqrt{u^3}\Big]_0^2=\frac{1}{6}\sqrt{u^3}\Big]_0^2=\frac{1}{6}\sqrt{\left(2x^2+1\right)^3}\Big]_0^2=$$
$$\frac{1}{6}\left[\sqrt{\left(2\cdot 2^2+1\right)^3}-\sqrt{\left(2\cdot 0^2+1\right)^3}\right]=\frac{1}{6}\left(\sqrt{9^3}-\sqrt{1^3}\right)=\frac{1}{6}\left(\sqrt{9^2 3^2}-1\right)=\frac{1}{6}(27-1)=\frac{1}{6}26=\frac{13}{3} \tag{46}$$
$$u=2x^2+1\rightarrow\frac{\partial u}{4}=x\,\partial x$$

Integral Método da Substituição – Aula 15

Exercício I

$$\int\left(x^3-1\right)^4x^2\partial x=\int u^4\frac{\partial u}{3}=\frac{1}{3}\int u^4\partial u=\frac{1}{3}\frac{u^5}{5}+c=\frac{\left(x^3-1\right)^5}{15}+c \tag{47}$$
$$u=x^3-1\rightarrow\frac{\partial u}{3}=x^2\partial x$$

Exercício II

$$\int\frac{x}{\left(x^2-1\right)^3}\partial x=\int\left(x^2-1\right)^{-3}x\,\partial x=\int u^{-3}\frac{\partial u}{2}=\frac{1}{2}\int u^{-3}\partial u=\frac{1}{2}\frac{u^{-2}}{(-2)}+c=\frac{-1}{4u^2}+c=$$
$$\frac{-1}{4\left(x^2-1\right)^2}+c \tag{48}$$
$$u=x^2-1\rightarrow\frac{\partial u}{2}=x\,\partial x$$

Exercício III

$$\int \frac{x}{\left(x^2-1\right)} \partial x = \int \left(x^2-1\right)^{-1} x \partial x = \int u^{-1} \frac{\partial u}{2} = \frac{1}{2} \int u^{-1} \partial u = \frac{1}{2} \ln|u| + c = \frac{\ln\left|x^2-1\right|}{2} + c$$
$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x \partial x \tag{49}$$

Exercício IV

$$\int e^{x^2-1} x \partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2} e^u \partial u = \frac{1}{2} e^u + c = \frac{e^{x^2-1}}{2} + c$$
$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x \partial x \tag{50}$$

Exercício V

$$\int \sqrt{x^3-4}\, x^2 \partial x = \int u^{\frac{1}{2}} \frac{\partial u}{3} = \frac{1}{3} \int u^{\frac{1}{2}} \partial u = \frac{1}{3} \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{1}{3} \frac{2}{3} \sqrt{u^3} + c = \frac{2\sqrt{u^3}}{9} + c =$$
$$\frac{2\sqrt{\left(x^3-4\right)^3}}{9} + c \tag{51}$$
$$u = x^3-4 \rightarrow \frac{\partial u}{3} = x^2 \partial x$$

Exercício VI

$$\int e^{\sqrt{x}} \frac{\partial x}{\sqrt{x}} = \int e^{x^{\frac{1}{2}}} \frac{\partial x}{x^{\frac{1}{2}}} = \int e^{x^{\frac{1}{2}}} x^{\frac{-1}{2}} \partial x = \int e^u 2 \partial u = 2 \int e^u \partial u = 2 e^u + c = 2 e^{\sqrt{x}} + c$$
$$u = \sqrt{x} = x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2} x^{\frac{-1}{2}} \partial x \rightarrow 2 \partial u = x^{\frac{-1}{2}} \partial x \tag{52}$$

Exercício VII

$$\int \frac{x \partial x}{\sqrt[5]{x^2-1}} = \int \frac{x}{\left(x^2-1\right)^{\frac{1}{5}}} \partial x = \int \left(x^2-1\right)^{\frac{-1}{5}} x \partial x = \int u^{\frac{-1}{5}} \frac{\partial u}{2} = \frac{1}{2} \int u^{\frac{-1}{5}} \partial u = \frac{1}{2} \frac{u^{\frac{4}{5}}}{\left(\frac{4}{5}\right)} + c =$$
$$\frac{1}{2} \frac{5}{4} \sqrt[5]{u^4} + c = \frac{5 \sqrt[5]{u^4}}{8} + c = \frac{5 \sqrt[5]{\left(x^2-1\right)^4}}{8} + c \tag{53}$$
$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x \partial x$$

Exercício VIII

$$\int \frac{e^t \partial t}{e^t+4} = \int \left(e^t+4\right)^{-1} e^t \partial t = \int u^{-1} \partial u = \ln|u| + c = \ln\left|e^t+4\right| + c \tag{54}$$
$$u = e^t+4 \rightarrow \partial u = e^t \partial t$$

Integral de uma Função Exponencial Qualquer – Aula 16

Exercício I

$$\int \sqrt{10^{3x}}\partial x = \int 10^{\frac{3x}{2}} \partial x = \int 10^u \frac{2}{3}\partial u = \frac{2}{3}\int 10^u \partial u = \frac{2}{3}\frac{10^u}{\ln|10|} + c = \frac{2\cdot 10^{\frac{3x}{2}}}{3\ln|10|} + c = \frac{2\sqrt{10^{3x}}}{3\ln|10|} + c$$
$$u = \frac{3x}{2} \rightarrow \partial u = \frac{3}{2}\partial x = \frac{2}{3}\partial u = \partial x$$

$$\frac{\partial\left(\frac{2\sqrt{10^{3x}}}{3\ln|10|} + c\right)}{\partial x} = \frac{\partial\left(\frac{2}{3\ln|10|}10^{\frac{3x}{2}} + c\right)}{\partial x} = \frac{\cancel{2}}{\cancel{3\ln|10|}}\frac{\cancel{3\ln|10|}\sqrt{10^{3x}}}{\cancel{2}} + 0 = \sqrt{10^{3x}} \tag{55}$$
$$y = 10^{\frac{3x}{2}} \rightarrow \ln|y| = \ln\left|10^{\frac{3x}{2}}\right| = \frac{3x}{2}\ln|10| = \frac{3\ln|10|}{2}x$$
$$\frac{\partial(\ln|y|)}{\partial y} = \frac{\partial\left(\frac{3\ln|10|}{2}x\right)}{\partial x} \rightarrow \frac{1}{y}\partial y = \frac{3\ln|10|}{2}\partial x \rightarrow \partial y = y\frac{3\ln|10|}{2}\partial x \rightarrow \frac{\partial y}{\partial x} =$$
$$10^{\frac{3x}{2}}\frac{3\ln|10|}{2} = \frac{3\ln|10|\sqrt{10^{3x}}}{2}$$

Integral de função marginal – Aula 17

Exercício I

O custo marginal por unidade x é dado pela expressão $\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5}$. Determine a função custo total c(x) da produção, sabendo-se que o custo fixo para x = 0 é de R$2000,00.

$$\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5} \rightarrow \cancel{\int \partial c(x)} = \int\left(20 - \frac{4x^3}{5}\right)\partial x \rightarrow \int\left(20 - \frac{4x^3}{5}\right)\partial x = 20\int \partial x - \frac{4}{5}\int x^3 \partial x =$$
$$20x - \frac{4}{5}\frac{x^4}{4} + c = 20x + \frac{x^4}{5} + c$$
$$20\cdot 0 + \frac{0^4}{5} + c = 2000 \rightarrow c = 2000 \tag{56}$$
$$c(x) = 20x + \frac{x^4}{5} + 2000$$

Exercício II

O rendimento marginal de um bem em quantidade (x) é dado pela expressão $Rm=800-2x^2$. Ache o rendimento total para x = 6, sabendo que quando x = 0, R = 0.

$$\frac{\partial r(x)}{\partial x}=800-2x^2 \rightarrow \cancel{\int \partial r(x)}=\int (800-2x^2)\partial x \rightarrow \int (800-2x^2)\partial x=800\int \partial x-2\int x^2 \partial x$$
$$800x-2\frac{x^3}{3}+c=800x-\frac{2x^3}{3}+c$$
$$800\cdot 0-\frac{2\cdot 0^2}{3}+c=0 \rightarrow c=0 \tag{57}$$
$$r(x)=800x-\frac{2x^3}{3}$$
$$r(6)=800\cdot 6-\frac{2\cdot 6^3}{3}=4800-\frac{2\cdot 2^3\cdot 3^3}{3}=4800-2^4\cdot 3^2=4800-16\cdot 9=4800-144=4656$$

Método da substituição com sen(x) e cos(x) – Aula 18

01.	$\int sen(u)\partial u$	$=-\cos(u)+c$
02.	$\int \cos(u)\partial u$	$= sen(u)+c$
03.	$\int tg(u)\partial u$	$= \ln\lvert sec(u)\rvert+c$
04.	$\int cotg(u)\partial u$	$= \ln\lvert sen(u)\rvert+c$
05.	$\int sec(u)\partial u$	$= \ln\lvert sec(u)+tg(u)\rvert+c$
06.	$\int cossec(u)\partial u$	$= \ln\lvert cossec(u)-cotg(u)\rvert+c$
07.	$\int sec^2(u)\partial u$	$= tg(u)+c$
08.	$\int cossec^2(u)\partial u$	$=-cotg(u)+c$
09.	$\int sec(u)tg(u)\partial u$	$= sec(u)+c$
10.	$\int cossec(u)cotg(u)\partial u$	$=-cossec(u)+c$

Exercício I

$$\int sen(2x^2-1)x\partial x=\int sen(u)\frac{\partial u}{4}=\frac{1}{4}sen(u)\partial u=\frac{1}{4}(-\cos(u))+c=\frac{-\cos(u)}{4}+c=$$
$$\frac{-\cos(2x^2-1)}{4}+c$$
$$u=2x^2-1 \rightarrow \frac{\partial u}{4}=x\partial x \tag{58}$$

$$\frac{\partial\left(\frac{-\cos(2x^2-1)}{4}+c\right)}{\partial x}=\frac{-1}{4}[-sen(2x^2-1)]4x+0=sen(2x^2-1)x$$

Exercício II

$$\int \cos(3x^3+4)x^2\partial x=\int \cos(u)\frac{\partial u}{9}=\frac{1}{9}\int \cos(u)\partial u=\frac{1}{9}sen(u)+c=\frac{sen(3x^3+4)}{9}+c$$
$$u=3x^3+4\rightarrow\frac{\partial u}{9}=x^2\partial x \tag{59}$$
$$\frac{\partial\left(\frac{sen(3x^3+4)}{9}+c\right)}{\partial x}=\frac{1}{9}\cos(3x^3+4)9x^2+0=\cos(3x^3+4)x^2$$

Exercício III

$$\int sen(\sqrt{x})\frac{\partial x}{\sqrt{x}}=\int sen\left(x^{\frac{1}{2}}\right)\frac{\partial x}{x^{\frac{1}{2}}}=\int sen\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int sen(u)2\partial u=2\int sen(u)\partial u=$$
$$2(-\cos(u))+c=-2\cos\sqrt{x}+c \tag{60}$$
$$u=\sqrt{x}=x^{\frac{1}{2}}\rightarrow\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

Exercício IV

$$\int sen(x)\cos(x)\partial x=\int u\partial u=\frac{u^2}{2}+c=\frac{sen^2(x)}{2}+c$$
$$u=sen(x)\rightarrow\partial u=\cos(x)\partial x \tag{61}$$
$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x}=\frac{1}{2}2sen(x)\cos(x)+0=sen(x)\cos(x)$$

Exercício V

$$\int sen(\cos(x))sen(x)\partial x=\int sen(u)(-\partial u)=-\int sen(u)\partial u=-(-\cos(u))+c=$$
$$\cos(\cos(x))+c$$
$$u=\cos(x)\rightarrow-\partial u=sen(x)\partial x \tag{62}$$
$$\frac{\partial[\cos(\cos(x))+c]}{\partial x}=-sen(\cos(x))(-sen(x))+0=sen(\cos(x))sen(x)$$

Exercício VI

$$\int\sqrt{sen(\theta)}\cos(\theta)\partial\theta=\int u^{\frac{1}{2}}\partial u=\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{sen^3(\theta)}}{3}+c \tag{63}$$
$$u=sen(\theta)\rightarrow\partial u=\cos(\theta)\partial\theta$$

Exercício VI

$$\int \ln|x|\frac{\partial x}{x}=\int \ln|x|x^{-1}\partial x=\int u\partial u=\frac{u^2}{2}+c=\frac{\ln^2|x|}{2}+c$$
$$u=\ln|x|\rightarrow\partial u=\frac{1}{x}\partial x\rightarrow\partial u=x^{-1}\partial x \tag{64}$$
$$\frac{\partial\left(\frac{\ln^2|x|}{2}+c\right)}{\partial x}=\frac{1}{2}2\ln|x|\frac{1}{x}+0=\frac{\ln|x|}{x}$$

Exercício VII

$$\int \frac{\partial x}{x\ln^2|x|}=\int \left(x\ln^2|x|\right)^{-1}\partial x=\int \ln^{-2}|x|x^{-1}\partial x=\int u^{-2}\partial u=\frac{u^{-1}}{(-1)}=\frac{-1}{u}+c=\frac{-1}{\ln|x|}+c$$
$$u=\ln|x|\rightarrow\partial u=\frac{1}{x}\partial x\rightarrow\partial u=x^{-1}\partial x \tag{65}$$
$$\frac{\partial\left(\frac{-1}{\ln|x|}+c\right)}{\partial x}=\frac{\partial\left(-\ln^{-1}|x|+c\right)}{\partial x}=\ln^{-2}\frac{1}{x}+0=\frac{1}{x\ln^2|x|}$$

Exercício VIII

$$\int \frac{sen(\theta)\partial\theta}{(5-\cos(\theta))^3}=\int (5-\cos(\theta))^{-3}sen(\theta)\partial\theta=\int u^{-3}\partial u=\frac{u^{-2}}{(-2)}+c=\frac{-1}{2u^2}+c=$$
$$\frac{-1}{2(5-\cos(\theta))^2}+c$$
$$u=5-\cos(\theta)\rightarrow\partial u=-(-sen(\theta))\partial x\rightarrow\partial u=sen(\theta)\partial\theta \tag{66}$$
$$\frac{\partial\left(\frac{-1}{2(5-\cos(\theta))^2}+c\right)}{\partial\theta}=\frac{\partial\left(\frac{-1}{2}(5-\cos(\theta))^{-2}+c\right)}{\partial\theta}=\frac{-1}{2}(-2)(5-\cos(\theta))^{-3}(-(-sen(\theta)))=$$
$$(5-\cos(\theta))^{-3}sen(\theta)=\frac{sen(\theta)}{(5-\cos(\theta))^3}$$

Integração de funções trigonométricas – Aula 19

Exercício I

$$\int tg(x)\partial x=\int \frac{sen(x)}{\cos(x)}\partial x=\int \cos^{-1}(x)sen(x)\partial x=\int u^{-1}(-\partial u)=-\int u^{-1}\partial u=$$
$$-\ln|u|+c=\ln|u^{-1}|+c=\ln\left|\frac{1}{u}\right|+c=\ln\left|\frac{1}{\cos(x)}\right|+c=\ln|sec(x)|+c$$
$$u=\cos(x)\rightarrow-\partial u=sen(x)\partial x$$
$$\frac{\partial(\ln|sec(x)|+c)}{\partial x}=\frac{\partial\left(\ln\left|\frac{1}{\cos(x)}\right|+c\right)}{\partial x}=\frac{\partial(\ln|\cos^{-1}(x)|+c)}{\partial x}=\frac{\partial(-\ln|\cos(x)|+c)}{\partial x}=$$
$$\frac{-1}{\cos(x)}(-sen(x))+0=\frac{sen(x)}{\cos(x)}=tg(x) \tag{67}$$

Exercício II

$$\int cotg(x)\partial x=\int \frac{\cos(x)}{sen(x)}\partial x=\int sen^{-1}(x)\cos(x)\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sen(x)|+c$$
$$u=sen(x)\rightarrow\partial u=\cos(x)\partial x$$
$$\frac{\partial(\ln|sen(x)|+c)}{\partial x}=\frac{1}{sen(x)}\cos(x)+0=\frac{\cos(x)}{sen(x)}=cotg(x) \tag{68}$$

Exercício III

$$\int sec(x)\partial x=\int sec(x)\left(\frac{sec(x)+tg(x)}{sec(x)+tg(x)}\right)\partial x=\int \frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}\partial x=$$
$$\int [sec(x)+tg(x)]^{-1}[sec(x)tg(x)+sec^2(x)]\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sec(x)+tg(x)|+c$$
$$u=sec(x)+tg(x)\rightarrow\partial u=[sec(x)tg(x)+sec^2(x)]\partial x$$
$$\frac{\partial(\ln|sec(x)+tg(x)|+c)}{\partial x}=\frac{1}{sec(x)+tg(x)}[sec(x)tg(x)+sec^2(x)]+0=$$
$$\frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}=\frac{sec(x)\cancel{[sec(x)+tg(x)]}}{\cancel{sec(x)+tg(x)}}=sec(x) \tag{69}$$

Exercício IV

$$\int cossec(x)\partial x \int cossec(x)\left(\frac{cossec(x)-cotg(x)}{cossec(x)-cotg(x)}\right)\partial x=$$
$$\int \frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}\partial x=$$
$$\int [cossec(x)-cotg(x)]^{-1}\left[-cossec(x)cotg(x)+cossec^2(x)\right]\partial x=\int u^{-1}\partial u=$$
$$\ln|u|+c=\ln|cossec(x)-cotg(x)|+c$$
$$u=cossec(x)-cotg(x)\rightarrow \partial u=\left[-cossec(x)cotg(x)-(-cossec^2(x))\right]\partial x\rightarrow$$
$$\partial u=\left[-cossec(x)cotg(x)+cossec^2(x)\right]\partial x \tag{70}$$

$$\frac{\partial(\ln|cossec(x)-cotg(x)|+c)}{\partial x}=$$
$$\frac{1}{cossec(x)-cotg(x)}\left[-cossec(x)cotg(x)-(-cossec^2(x))\right]+0=$$
$$\frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}=\frac{cossec(x)\cancel{[cossec(x)-cotg(x)]}}{\cancel{cossec(x)-cotg(x)}}=cossec(x)$$

Exercício V

$$\int tg(3x)\partial x=\int tg(u)\frac{\partial u}{3}=\frac{1}{3}tg(u)\partial u=\frac{1}{3}\ln|sec(u)|+c=\frac{\ln|sec(u)|}{3}+c=\frac{\ln|sec(3x)|}{3}+c$$
$$u=3x\rightarrow\frac{\partial u}{3}=\partial x \tag{71}$$

Exercício VI

$$\int\frac{\partial x}{sen(2x)}=\int sen^{-1}(2x)\partial x=\int cossec(2x)\partial x=\int cossec(u)\frac{\partial u}{2}=\frac{1}{2}\int cossec(u)\partial u=$$
$$\frac{1}{2}\ln|cossec(u)-cotg(u)|+c=\frac{\ln|cossec(u)-cotg(u)|}{2}+c=\frac{\ln|cossec(2x)-cotg(2x)|}{2}+c$$
$$u=2x\rightarrow\frac{\partial u}{2}=\partial x \tag{72}$$

Integração de funções trigonométricas – Aula 20

Exercício I

$$\int\frac{tg(\sqrt{x})\partial x}{\sqrt{x}}=\int\frac{tg\left(x^{\frac{1}{2}}\right)\partial x}{x^{\frac{1}{2}}}=\int tg\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int tg(u)2\partial u=2\int tg(u)\partial u=$$
$$2\ln|sec(u)|+c=2\ln|sec(\sqrt{x})|+c$$
$$u=\sqrt{x}=x^{\frac{1}{2}}\rightarrow\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x \tag{73}$$

Exercício II

$$\int \frac{cotg(\ln|x|)\partial x}{x} = \int cotg(\ln|x|)x^{-1}\partial x = \int cotg(u)\partial u = \ln|sen(u)| + c = \ln|sen(\ln|x|)| + c$$
$$u = \ln|x| \rightarrow \partial u = \frac{1}{x}\partial x \rightarrow \partial u = x^{-1}\partial x \tag{74}$$

Exercício III

$$\int sec(5x-\pi)\partial x = \int sec(u)\frac{\partial u}{5} = \frac{1}{5}\int sec(u)\partial u = \frac{1}{5}\ln|sec(u)+tg(u)| + c =$$
$$\frac{\ln|sec(5x-\pi)+tg(5x-\pi)|}{5} + c \tag{75}$$
$$u = 5x - \pi \rightarrow \frac{\partial u}{5} = \partial x$$

Integral de potência sen(x) ou cos(x) – Aula 21

$\int \cos^n(x)\partial x$

$\int sen^n(x)\partial x$

$n \rightarrow$ ímpar $\quad sen^2(x)+\cos^2(x)=1$

$n \rightarrow$ par $\quad sen^2(x) = \frac{1-\cos(2x)}{2}$

$\cos^2(x) = \frac{1+\cos(2x)}{2}$

$\int sen^m(x)\cos^n(x)\partial x$

$n \vee m \rightarrow$ ímpar $\quad$ Separa o $\partial u \begin{cases} sen(x)\partial x \\ \cos(x)\partial x \end{cases}$

Transfoma em $sen^p(x)$ ou $\cos^p(x)$ através de $sen^2(x)+\cos^2(x)=1$

$n \wedge m \rightarrow$ par $\quad sen^2(x) = \frac{1-\cos(2x)}{2}$

$\cos^2(x) = \frac{1+\cos(2x)}{2}$

Exercício I

$$\int \cos^5(x)\partial x=\int \cos^4(x)\cos(x)\partial x=\int \left(\cos^2(x)\right)^2\cos(x)\partial x=\int \left(1-sen^2(x)\right)^2\cos(x)\partial x=$$
$$\int \left[1-2sen^2(x)+sen^4(x)\right]\cos(x)\partial x=\int \left[\cos(x)-2sen^2(x)\cos(x)+sen^4(x)\cos(x)\right]\partial x=$$
$$\int \cos(x)\partial x-2\int sen^2(x)\cos(x)\partial x+\int sen^4(x)\cos(x)\partial x=\int \partial u-2\int u^2\partial u+\int u^4\partial u=$$
$$u-2\frac{u^3}{3}+\frac{u^5}{5}+c=sen(x)-\frac{2sen^3(x)}{3}+\frac{sen^5(x)}{5}+c$$
$$u=sen(x)\rightarrow\partial u=\cos(x)\partial x$$

$$\frac{\partial\left(sen(x)-\frac{2sen^3(x)}{3}+\frac{sen^5(x)}{5}+c\right)}{\partial x}=\frac{\partial\left(sen(x)-\frac{2}{3}sen^3(x)+\frac{1}{5}sen^5(x)+c\right)}{\partial x}= \tag{76}$$
$$\cos(x)-\frac{2}{\cancel{3}}\cancel{3}sen^2(x)\cos(x)+\frac{1}{\cancel{5}}\cancel{5}sen^4(x)\cos(x)+0=\cos(x)\left[1-2sen^2(x)+sen^4(x)\right]=$$
$$\cos(x)\left[1-\left(sen^2(x)+sen^2(x)\right)+\left(sen^2(x)\right)^2\right]=$$
$$\cos(x)\left[\left(1-sen^2(x)\right)-sen^2(x)+\left(1-\cos^2(x)\right)^2\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+\left(1-2\cos^2(x)+\cos^4(x)\right)\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+1-\left(\cos^2(x)+\cos^2(x)\right)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}+\left(1-sen^2(x)\right)\cancel{-\cos^2(x)}-\cos^2(x)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)-\cos^2(x)}+\cos^4(x)\right]=\cos(x)\cos^4(x)=\cos^5(x)$$

Exercício II

$$
\begin{gathered}
\int sen^4(x)\partial x=\int \left(sen^2(x)\right)^2\partial x=\int \left(\frac{1-\cos(2x)}{2}\right)^2\partial x=\int \frac{1-2\cos(2x)+\cos^2(2x)}{4}\partial x=\\
\frac{1}{4}\int \left[1-2\cos(2x)+\frac{1+\cos(2\cdot 2x)}{2}\right]\partial x=\frac{1}{4}\int \partial x-\frac{1}{2}\int \cos(2x)\partial x+\frac{1}{8}\int [1+\cos(4x)]\partial x=\\
\frac{1}{4}\int \partial x-\frac{1}{2}\int \cos(2x)\partial x+\frac{1}{8}\int \partial x+\frac{1}{8}\int \cos(4x)\partial x=\\
\frac{1}{4}\int \frac{\partial u}{2}-\frac{1}{2}\int \cos(u)\frac{\partial u}{2}+\frac{1}{8}\int \frac{\partial u}{2}+\frac{1}{8}\int \cos(2u)\frac{\partial u}{2}=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{16}\int \cos(2u)\partial u=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{16}\int \cos(v)\frac{\partial v}{2}=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{32}\int \cos(v)\partial v=\\
\frac{1}{8}u-\frac{1}{4}sen(u)+\frac{1}{16}u+\frac{1}{32}sen(v)+c=\frac{2u+u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\\
\frac{3u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\frac{3\cdot 2x}{16}-\frac{sen(2x)}{4}+\frac{sen(2\cdot 2x)}{32}+c=\\
\frac{3x}{8}-\frac{sen(2x)}{4}+\frac{sen(4x)}{32}+c\\
u=2x\rightarrow\frac{\partial u}{2}=\partial x\\
v=2u\rightarrow\frac{\partial v}{2}=\partial u
\end{gathered}
\qquad (77)
$$

Integral do produto de potência entre sen(x) e cos(x) – Aula 22

Exercício I

$$\int sen^5(x)\cos^2(x)\partial x=\int sen^4(x)\cos^2(x)sen(x)\partial x=\int \left(sen^2(x)\right)^2\cos^2(x)sen(x)\partial x=$$
$$\int \left(1-\cos^2(x)\right)^2\cos^2(x)sen(x)\partial x=\int \left[1-2\cos^2(x)+\cos^4(x)\right]\cos^2(x)sen(x)\partial x=$$
$$\int \left[\cos^2(x)-2\cos^4(x)+\cos^6(x)\right]sen(x)\partial x=$$
$$\int \cos^2(x)sen(x)\partial x-2\int \cos^4(x)sen(x)\partial x+\int \cos^6(x)sen(x)\partial x=$$
$$\int u^2(-\partial u)-2\int u^4(-\partial u)+\int u^6(-\partial u)=-\int u^2\partial u+2\int u^4\partial u-\int u^6\partial u=$$
$$\frac{-u^3}{3}+2\frac{u^5}{5}-\frac{u^7}{7}+c=\frac{-\cos^3(x)}{3}+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}+c$$
$$u=\cos(x)\rightarrow-\partial u=sen(x)\partial x \tag{78}$$

$$\frac{\partial\left(\frac{-\cos^3(x)}{3}+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}+c\right)}{\partial x}=\frac{\partial\left(-\frac{1}{3}\cos^3(x)+\frac{2}{5}\cos^5(x)-\frac{1}{7}\cos^7(x)+c\right)}{\partial x}=$$
$$-\frac{1}{3}3\cos^2(x)(-sen(x))+\frac{2}{5}5\cos^4(x)(-sen(x))-\frac{1}{7}7\cos^6(x)(-sen(x))+0=$$
$$\cos^2(x)sen(x)-2\cos^4(x)sen(x)+\cos^6(x)sen(x)=sen(x)\left[\cos^2(x)-2\cos^4(x)+\cos^6(x)\right]=$$
$$sen(x)\cos^2(x)\left[1-2\cos^2(x)+\cos^4(x)\right]=\left(1-\cos^2(x)\right)^2\cos^2(x)sen(x)=$$
$$\left(sen^2(x)\right)^2\cos^2(x)sen(x)=sen^4(x)\cos^2(x)sen(x)=sen^5(x)\cos^2(x)$$

Exercício II

$$\int sen^2(x)\cos^4(x)\partial x=\int \frac{1-\cos(2x)}{2}\left(\cos^2(x)\right)^2\partial x=\frac{1}{2}\int \left(1-\cos(2x)\right)\left(\frac{1+\cos(2x)}{2}\right)^2\partial x$$

$$\frac{1}{2}\int \left(1-\cos(2x)\right)\left(\frac{1+2\cos(2x)+\cos^2(2x)}{4}\right)\partial x=$$

$$\frac{1}{8}\int \left(1-\cos(2x)\right)\left(1+2\cos(2x)+\cos^2(2x)\right)\partial x=$$

$$\frac{1}{8}\int \left(1+\cancel{2}\cos(2x)\cancel{+\cos^2(2x)}\cancel{-\cos(2x)}-\cancel{2}\cos^2(2x)-\cos^3(2x)\right)\partial x=$$

$$\frac{1}{8}\int \left(sen^2(2x)+\cos(2x)-\cos^2(2x)\cos(2x)\right)\partial x=$$

$$\frac{1}{8}\int sen^2(2x)\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \cos^2(2x)\cos(2x)\partial x=$$

$$\frac{1}{8}\int \frac{1-\cos(4x)}{2}\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \left(1-sen^2(2x)\right)\cos(2x)\partial x= \qquad (79)$$

$$\frac{1}{16}\int \left(1-\cos(4x)\right)\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \left(\cos(2x)-sen^2(2x)\cos(2x)\right)\partial x=$$

$$\frac{1}{16}\int \partial x-\frac{1}{16}\int \cos(4x)\partial x\cancel{+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \cos(2x)\partial x}+\frac{1}{8}\int sen^2(2x)\cos(2x)\partial$$

$$\frac{1}{16}\int \partial x-\frac{1}{32}\int \cos(2y)\partial y+\frac{1}{16}\int sen^2(y)\cos(y)\partial y=$$

$$\frac{1}{16}\int \partial x-\frac{1}{64}\int \cos(z)\partial z+\frac{1}{16}\int u^2\partial u=\frac{1}{16}x-\frac{1}{64}sen(z)+\frac{1}{16}\frac{u^3}{3}+c=$$

$$\frac{x}{16}-\frac{sen(2y)}{64}+\frac{sen^3(y)}{48}+c=\frac{x}{16}-\frac{sen(4x)}{64}+\frac{sen^3(2x)}{48}+c$$

$$y=2x\rightarrow\frac{\partial y}{2}=\partial x;z=2y\rightarrow\frac{\partial z}{2}=\partial y;u=sen(y)\rightarrow\partial u=\cos(y)\partial y$$

Integral de Funções Trigonométricas – Aula 23

$$tg(x)=\frac{sen(x)}{\cos(x)},cotg(x)=\frac{\cos(s)}{sen(x)}$$

$$sec(x)=\frac{1}{\cos(x)},cossec(x)=\frac{1}{sen(x)}$$

$$sen^2(x)+\cos^2(x)=1$$

$$\frac{sen^2(x)}{sen^2(x)}+\frac{\cos^2(x)}{sen^2(x)}=\frac{1}{sen^2(x)}\rightarrow 1+cotg^2(x)=cossec^2(x)\rightarrow cossec^2(x)-cotg^2(x)=1$$

$$\frac{sen^2(x)}{\cos^2(x)}+\frac{\cos^2(x)}{\cos^2(x)}=\frac{1}{\cos^2(x)}\rightarrow tg^2(x)+1=sec^2(x)\rightarrow sec^2(x)-tg^2(x)=1$$

Exercício I

$$\int [tg(2x)+cotg(2x)]^2 \partial x = \int \left[tg^2(2x)+2\,tg(2x)cotg(2x)+cotg^2(2x)\right]\partial x =$$
$$\int tg^2(2x)\partial x + 2\int tg(2x)cotg(2x)\partial x + \int cotg^2(2x)\partial x =$$
$$\int \left(sec^2(2x)-1\right)\partial x + 2\int \frac{\cancel{sen(2x)}}{\cancel{\cos(2x)}}\frac{\cancel{\cos(2x)}}{\cancel{sen(2x)}}\partial x + \int \left(cossec^2(2x)-1\right)\partial x =$$
$$\int sec^2(2x)\partial x - \int \partial x + 2\int \partial x + \int cossec^2(2x)\partial x - \int \partial x =$$
$$\frac{1}{2}\int sec^2(u)\partial u - \int \partial x + 2\int \partial x + \frac{1}{2}\int cossec^2(u)\partial u - \int \partial x =$$
$$\frac{1}{2}tg(u)\cancel{-x}\cancel{+2x}+\frac{1}{2}(-cotg(u))\cancel{-x}+c=\frac{tg(2x)}{2}-\frac{cotg(2x)}{2}+c \tag{80}$$
$$u=2x \rightarrow \frac{\partial u}{2}=\partial x$$

$$\frac{\partial\left(\frac{tg(2x)}{2}-\frac{cotg(2x)}{2}+c\right)}{\partial x}=\frac{1}{2}sec^2(2x)2-\frac{1}{2}\left(-cossec^2(2x)\right)2+0=$$
$$sec^2(2x)+cossec^2(2x)=\left(1+tg^2(2x)\right)+\left(1+cossec^2(2x)\right)=2+tg^2(2x)+cotg^2(2x)=$$
$$tg^2(2x)+2\cancel{tg(2x)cotg(2x)}+cotg^2(2x)=[tg(2x)+cotg(2x)]^2$$

Integral Definida com Seno e Cosseno – Aula 24

	$0(0^o)$	$\frac{\pi}{6}(30^o)$	$\frac{\pi}{4}(45^o)$	$\frac{\pi}{3}(60^o)$	$\frac{\pi}{2}(90^o)$
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

Exercício I

$$\int_0^{\frac{\pi}{4}}\left(\frac{1+\cos^2(\theta)}{\cos^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{4}}\left(\frac{1}{\cos^2(\theta)}+\frac{\cos^2(\theta)}{\cos^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{4}}\left(sec^2(\theta)+1\right)\partial\theta=$$
$$\int_0^{\frac{\pi}{4}} sec^2(\theta)\partial\theta+\int_0^{\frac{\pi}{4}}\partial\theta=tg(\theta)\Big]_0^{\frac{\pi}{4}}+\theta\Big]_0^{\frac{\pi}{4}}=\left(tg\left(\frac{\pi}{4}\right)-tg(0)\right)+\left(\frac{\pi}{4}-0\right)=(1-0)+\left(\frac{\pi}{4}-0\right)=1+\frac{\pi}{4}= \tag{81}$$
$$\frac{4+\pi}{4}$$

Exercício II

$$\int_0^{\pi}(4\,sen(\theta)-3\cos(\theta))\partial\theta=4\int_0^{\pi}sen(\theta)\partial\theta-3\int_0^{\pi}\cos(\theta)\partial\theta=4(-\cos(\theta))\Big]_0^{\pi}-3\,sen(\theta)\Big]_0^{\pi}=$$
$$-4\cos(\theta)\Big]_0^{\pi}-3\,sen(\theta)\Big]_0^{\pi}=-4(\cos(\pi)-\cos(0))-3(sen(\pi)-sen(0))= \tag{82}$$
$$-4(-1-1)-3(0-0)=-4(-2)=8$$

Integral Definida para funções trigonométricas – Aula 25

Exercício I

$$\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)+sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)}{sec^2(\theta)}+\frac{sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\left(sen(\theta)\frac{1}{\left(\frac{1}{\cos^2(\theta)}\right)}+sen(\theta)\frac{sen^2(\theta)}{\cancel{\cos^2(\theta)}}\frac{1}{\left(\frac{1}{\cancel{\cos^2(\theta)}}\right)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\left(\cos^2(\theta)sen(\theta)+sen^3(\theta)\right)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}sen^2(\theta)sen(\theta)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}\left(1-\cos^2(\theta)\right)sen(\theta)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}\left(sen(\theta)-\cos^2(\theta)sen(\theta)\right)\partial\theta= \quad (83)$$
$$\cancel{\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta}+\int_0^{\frac{\pi}{3}}sen(\theta)\partial\theta-\cancel{\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta}=$$
$$-\cos(\theta)]_0^{\frac{\pi}{3}}=-\left(\cos\left(\frac{\pi}{3}\right)-\cos(0)\right)=-\left(\frac{1}{2}-1\right)=-\left(\frac{-1}{2}\right)=\frac{1}{2}$$

$$\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)+sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\frac{sen(\theta)\left(1+tg^2(\theta)\right)}{sec^2(\theta)}\partial\theta=\int_0^{\frac{\pi}{3}}\frac{sen(\theta)\cancel{sec^2(\theta)}}{\cancel{sec^2(\theta)}}\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}sen(\theta)\partial\theta=-\cos(\theta)]_0^{\frac{\pi}{3}}=-\left(\cos\left(\frac{\pi}{3}\right)-\cos(0)\right)=-\left(\frac{1}{2}-1\right)=-\left(\frac{-1}{2}\right)=\frac{1}{2}$$

Exercício II

$$\int_0^{\pi}sec^2\left(\frac{t}{4}\right)\partial t=4\int_0^{\pi}sec^2(u)\partial u=4\,tg(u)]_0^{\pi}=4\,tg\left(\frac{t}{4}\right)\Big]_0^{\pi}=4\left(tg\left(\frac{\pi}{4}\right)-tg(0)\right)=4(1-0)=4 \quad (84)$$
$$u=\frac{t}{4}\rightarrow 4\partial u=\partial t$$

Integral envolvendo Funções Seno e Cosseno – Aula 26

$sen(2x)=2sen(x)\cos(x)$

Exercício I

$$\int \frac{sen(x)}{1-sen^2(x)}\partial x=\int \frac{sen(x)}{\cos^2(x)}\partial x=\int \cos^{-2}(x)sen(x)\partial x=-\int u^{-2}\partial u=\frac{-u^{-1}}{(-1)}+c= \\ u^{-1}+c=\frac{1}{u}+c=\frac{1}{\cos(x)}+c=sec(x)+c \\ u=\cos(x)\rightarrow-\partial u=sen(x)\partial x \tag{85}$$

Exercício II

$$\int \frac{sen(2x)}{\cos(x)}\partial x=\int \frac{2sen(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\partial x=2\int sen(x)\partial x=2(-\cos(x))+c=-2\cos(x)+c \tag{86}$$

Integral de uma função exponencial de seno – Aula 27

Exercício I

$$\int e^{sen(\theta)}\cos(\theta)\partial\theta=\int e^u\partial u=e^u+c=e^{sen(\theta)}+c \\ u=sen(\theta)\rightarrow\partial u=\cos(\theta)\partial\theta \tag{87}$$

Exercício II

$$\int sen(\pi t)\partial t=\frac{1}{\pi}\int sen(u)\partial u=\frac{1}{\pi}(-\cos(u))+c=\frac{-\cos(u)}{\pi}+c=\frac{-\cos(\pi t)}{\pi}+c \\ u=\pi t\rightarrow\frac{\partial u}{\pi}=\partial t \tag{88}$$

Integral Definida do Produto de secante e tangente – Aula 28

Exercício I

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\cos(\theta)}\frac{sen(\theta)}{\cos(\theta)}\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{sen(\theta)}{\cos^2(\theta)}\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\cos^{-2}(\theta)sen(\theta)\partial\theta= \\ -\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}u^{-2}\partial u=\left[\frac{-u^{-1}}{(-1)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=[u^{-1}]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{u}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{\cos\left(\frac{\pi}{3}\right)}-\frac{1}{\cos\left(\frac{\pi}{4}\right)}=\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{\left(\frac{\sqrt{2}}{2}\right)}= \\ 2-\frac{2}{\sqrt{2}}=2-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}=2-\sqrt{2} \\ u=\cos(\theta)\rightarrow-\partial u=sen(\theta)\partial\theta \tag{89}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\partial\theta=[sec(\theta)]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{\cos\left(\frac{\pi}{3}\right)}-\frac{1}{\cos\left(\frac{\pi}{4}\right)}=\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{\left(\frac{\sqrt{2}}{2}\right)}=2-\frac{2}{\sqrt{2}}= \\ 2-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}=2-\sqrt{2}$$

Integral de seno pelo Método da Substituição – Aula 29

Exercício I

$$\int \sqrt{x}\, sen\left(1+\sqrt{x^3}\right)\partial x=\int sen\left(1+x^{\frac{3}{2}}\right)x^{\frac{1}{2}}\partial x=\frac{2}{3}sen(u)\partial u=\frac{2}{3}(-\cos(u))+c=$$
$$\frac{-2\cos(1+\sqrt{x^3})}{3}+c \qquad (90)$$
$$u=1+\sqrt{x^3}=1+x^{\frac{3}{2}}\rightarrow\partial u=\frac{3}{2}x^{\frac{1}{2}}\partial x=\frac{2\partial u}{3}=x^{\frac{1}{2}}\partial x$$

Integral Trigonométrica – Método da Substituição – Aula 30

Exercício I

$$\int (1+tg(\theta))^5 sec^2(\theta)\partial\theta=\int u^5\partial u=\frac{u^6}{6}+c=\frac{(1+tg(\theta))^6}{6}+c \qquad (91)$$
$$u=1+tg(\theta)\rightarrow\partial u=sec^2(\theta)\partial\theta$$

Exercício II

$$\int sec(2\theta)tg(2\theta)\partial\theta=\frac{1}{2}\int sec(u)tg(u)\partial u=\frac{1}{2}sec(u)+c=\frac{sec(2\theta)}{2}+c \qquad (92)$$
$$u=2\theta\rightarrow\frac{\partial u}{2}=\partial\theta$$

Integral Trigonométrica – Método da Substituição – Aula 31

Exercício I

$$\int \sqrt{cotg(x)}cossec^2(x)\partial x=\int cotg^{\frac{1}{2}}(x)cossec^2(x)\partial x=-\int u^{\frac{1}{2}}\partial u=\frac{-u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{-2\sqrt{u^3}}{3}+c=$$
$$\frac{-2\sqrt{cotg^3(x)}}{3}+c \qquad (93)$$
$$u=cotg(x)\rightarrow-\partial u=cossec^2(x)$$

Exercício II

$$\int sec^3(x)tg(x)\partial x=\int sec^2(x)sec(x)tg(x)\partial x=\int u^2\partial u=\frac{u^3}{3}+c=\frac{sec^3(x)}{3}+c \qquad (94)$$
$$u=sec(x)\rightarrow\partial u=sec(x)tg(x)\partial x$$

Integral de Cosseno pelo Método da Substituição – Aulа 32

Exercício I

$$\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\partial x=\int \cos\left(\pi x^{-1}\right)x^{-2}\partial x=\frac{-1}{\pi}\int \cos(u)\partial u=\frac{-1}{\pi}sen(u)+c=\frac{-sen\left(\frac{\pi}{x}\right)}{\pi}+c$$
$$u=\frac{\pi}{x}=\pi x^{-1}\rightarrow \partial u=-\pi x^{-2}\rightarrow \frac{-\partial u}{\pi}=x^{-2}\partial x \tag{95}$$

Integral de Potência de Tangente – Aula 33

Exercício I

$$\int tg^3(x)\partial x=\int tg^2(x)tg(x)\partial x=\int \left(sec^2(x)-1\right)tg(x)\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int tg(x)\partial x=\int tg(x)sec^2(x)\partial x-\int \frac{sen(x)}{\cos(x)}\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int \cos^{-1}(x)sen(x)\partial=\int u\partial u+\int v^{-1}\partial v=\frac{u^2}{2}+\ln|v|+c=$$
$$\frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$
$$u=tg(x)\rightarrow \partial u=sec^2(x)\partial x\,;v=\cos(x)\rightarrow -\partial v=sen(x)\partial x \tag{96}$$
$$\int tg^3(x)\partial x=\int tg^2(x)tg(x)\partial x=\int \left(sec^2(x)-1\right)tg(x)\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int tg(x)\partial x=\int u\partial u-\int tg(x)\partial x=\frac{u^2}{2}-\left(\ln|sec(x)|\right)+c=$$
$$\frac{u^2}{2}-\left(\ln\left|\cos^{-1}(x)\right|\right)+c=\frac{u^2}{2}-\left(-\ln|\cos(x)|\right)+c=\frac{u^2}{2}+\ln|\cos(x)|+c=\frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$
$$u=tg(x)\rightarrow \partial u=sec^2(x)\partial x$$

Integral de Potência de Cotangente – Aula 34

Exercício I

$$\int cotg^4(3x)\partial x=\int cotg^2(3x)cotg^2(3x)\partial x=\int \left(cossec^2(3x)-1\right)cotg^2(3x)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int cotg^2(3x)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int \left(cossec^2(3x)-1\right)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int cossec^2(3x)\partial x+\int \partial x=$$
$$\frac{1}{3}\int cotg^2(u)cossec^2(u)\partial u-\frac{1}{3}\int cossec^2(u)\partial u+\int \partial x=$$
$$\frac{-1}{3}\int v^2\partial v+\frac{1}{3}\int \partial v+\int \partial x=\frac{-1}{3}\frac{v^3}{3}+\frac{1}{3}v+x+c=\frac{-cotg^3(u)}{9}+\frac{cotg(u)}{3}+x+c= \tag{97}$$
$$\frac{-cotg^3(3x)}{9}+\frac{cotg(3x)}{3}+x+c$$
$$u=3x\rightarrow \frac{\partial u}{3}=\partial x\,;v=cotg(u)\rightarrow -\partial v=cossec^2(u)$$

Integral da Potência de Secante – Aula 35

Exercício I

$$\int cossec^6(x)\partial x=\int cossec^4(x)cossec^2(x)\partial x=\int \left(cossec^2(x)\right)^2 cossec^2(x)\partial x=$$
$$\int \left(cotg^2(x)+1\right)^2 cossec^2(x)\partial x=\int \left(cotg^4(x)+2cotg^2(x)+1\right)cossec^2(x)\partial x=$$
$$\int cotg^4(x)cossec^2(x)\partial x+2\int cotg^2(x)cossec^2(x)\partial x+\int cossec^2(x)\partial x=$$
$$-\int u^4\partial u-2u^2\partial u-\int \partial u=\frac{-u^5}{5}-2\frac{u^3}{3}-u+c=\frac{-cotg^5(x)}{5}-\frac{2cotg^3(x)}{3}-cotg(x)+c \tag{98}$$
$$u=cotg(x)\rightarrow -\partial u=cossec^2(x)\partial x$$

Integral de Secante ao Cubo – Aula 36

$$\int u\partial v=uv-\int v\partial u$$

Exercício I

$$\int sec^3(x)\partial x=\int sec(x)sec^2(x)\partial x=sec(x)tg(x)-\int tg(x)sec(x)tg(x)\partial x=$$
$$sec(x)tg(x)-\int tg^2(x)sec(x)\partial x=sec(x)tg(x)-\int \left(sec^2(x)-1\right)sec(x)\partial x=$$
$$sec(x)tg(x)-\int sec^3(x)\partial x+\int sec(x)\partial x\rightarrow$$
$$\int sec^3(x)\partial x+\int sec^3(x)\partial x=sec(x)tg(x)+\int sec(x)\partial x\rightarrow$$
$$2\int sec^3(x)\partial x=sec(x)tg(x)+\ln|sec(x)+tg(x)|+c\rightarrow$$
$$\int sec^3(x)\partial x=\frac{sec(x)tg(x)+\ln|sec(x)+tg(x)|+c}{2}=\frac{sec(x)tg(x)}{2}+\frac{\ln|sec(x)+tg(x)|}{2}+\frac{c}{2}= \tag{99}$$
$$\frac{sec(x)tg(x)}{2}+\frac{\ln|sec(x)+tg(x)|}{2}+c$$
$$u=sec(x)\rightarrow\partial u=sec(x)tg(x)\partial x$$
$$v=tg(x)\rightarrow\partial v=sec^2(x)\partial x$$

Integral de Potência de Secante e Tangente – Aula 37

$$\int tg^m(x)sec^n(x)\partial x$$

n	$\rightarrow$	par	Guarde $sec^2(x)$ Use $sec^2(x)=tg^2(x)+1$ $u=tg(x)$
m	$\rightarrow$	impar	Guarde $sec(x)tg(x)$ Use $tg^2(x)=sec^2(x)-1$ $u=sec(x)$
m	$\rightarrow$	par	
n	$\rightarrow$	impar	Use $tg^2(x)=sec^2(x)-1$ para reduzir somente a $sec(x)$ Após aplique integral por partes

Exercício I

$$
\begin{gathered}
\int tg^5(x)sec^4(x)\partial x=\int tg^5(x)sec^2(x)sec^2(x)\partial x=\int tg^5(x)\left(tg^2(x)+1\right)sec^2(x)\partial x=\\
\int tg^5(x)\left(tg^2(x)sec^2(x)+sec^2(x)\right)\partial x=\int tg^7(x)sec^2(x)\partial x+\int tg^5(x)sec^2(x)\partial x=\\
\int u^7\partial u+\int u^5\partial u=\frac{u^8}{8}+\frac{u^6}{6}+c=\frac{tg^8(x)}{8}+\frac{tg^6(x)}{6}+c\\
u=tg(x)\rightarrow\partial u=sec^2(x)\partial x
\end{gathered}
\tag{100}
$$

Integral de Potência de Secante e Tangente – Aula 38

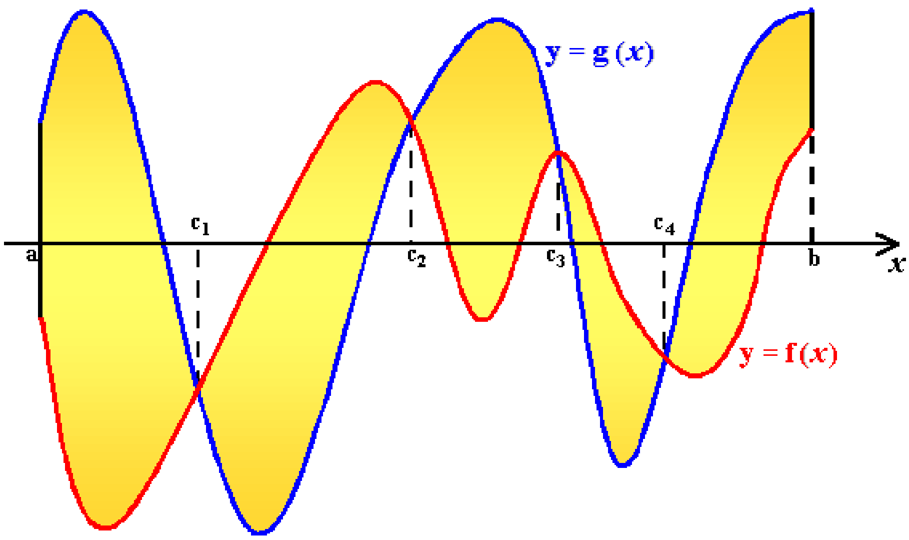
Exercício I

$$
\begin{gathered}
\int tg^5(x)sec^7(x)\partial x=\int tg^4(x)sec^6(x)sec(x)tg(x)\partial x=\\
\int \left(tg^2(x)\right)^2 sec^6(x)sec(x)tg(x)\partial x=\int \left(sec^2(x)-1\right)^2 sec^6(x)sec(x)tg(x)\partial x=\\
\int \left(sec^4(x)-2sec^2(x)+1\right)sec^6(x)sec(x)tg(x)\partial x=\\
\int sec^{10}(x)sec(x)tg(x)\partial x-2\int sec^8(x)sec(x)tg(x)\partial x+\int sec^6(x)sec(x)tg(x)\partial x=\\
\int u^{10}\partial u-2\int u^8\partial u+\int u^6\partial u=\frac{u^{11}}{11}-2\frac{u^9}{9}+\frac{u^7}{7}+c=\frac{sec^{11}(x)}{11}-\frac{2sec^9(x)}{9}+\frac{sec^7(x)}{7}+c\\
u=sec(x)\rightarrow\partial u=sec(x)tg(x)\partial x
\end{gathered}
\tag{101}
$$

Exercício II

$$
\begin{gathered}
\int tg^2(x)sec(x)\partial x=\int \left(sec^2(x)-1\right)sec(x)\partial x=\int sec(x)sec^2(x)\partial x-\int sec(x)\partial x\rightarrow\\
\int u\partial v=uv-\int v\partial u\rightarrow\int sec(x)sec^2(x)\partial x=sec(x)tg(x)-\int tg(x)sec(x)tg(x)\partial x=\\
sec(x)tg(x)-\int tg^2(x)sec(x)\partial x=sec(x)tg(x)-\int \left(sec^2(x)-1\right)sec(x)\partial x=\\
sec(x)tg(x)-\int sec(x)sec^2(x)\partial x+\int sec(x)\partial x\rightarrow\\
\int sec(x)sec^2(x)\partial x+\int sec(x)sec^2(x)\partial x=sec(x)tg(x)+\int sec(x)\partial x\rightarrow\\
2\int sec(x)sec^2(x)\partial x=sec(x)tg(x)+\ln|sec(x)+tg(x)|+c\rightarrow\\
\int sec(x)sec^2(x)\partial x=\frac{sec(x)tg(x)+\ln|sec(x)+tg(x)|+c}{2}=\\
\int tg^2(x)sec(x)\partial x=\left(\frac{sec(x)tg(x)+\ln|sec(x)+tg(x)|+c}{2}\right)-\int sec(x)\partial x=\\
\frac{sec(x)tg(x)+\ln|sec(x)+tg(x)|+c}{2}-\ln|sec(x)+tg(x)|+c=\\
\frac{sec(x)tg(x)\cancel{+\ln|sec(x)+tg(x)|}-\cancel{2}\ln|sec(x)+tg(x)|+c}{2}=\\
\frac{sec(x)tg(x)}{2}-\frac{\ln|sec(x)+tg(x)|}{2}+c\\
u=sec(x)\rightarrow\partial u=sec(x)tg(x)\partial x\\
v=tg(x)\rightarrow\partial v=sec^2(x)\partial x
\end{gathered}
\tag{102}
$$

Cálculo de área com integrais – Aula 1



$$\int_{c_1}^{c_2}[f(x)-g(x)]\partial x+\int_{c_2}^{c_3}[g(x)-f(x)]\partial x+\int_{c_3}^{c_4}[f(x)-g(x)]\partial x$$

Exercício I

$$f(x)=x^2\,;g(x)=x$$
$$f(x)=g(x)\rightarrow x^2=x\rightarrow x^2-x=0\rightarrow x(x-1)=0\rightarrow$$
$$x=0\rightarrow x_1=0\,;x-1=0\rightarrow x=1\rightarrow x_2=1$$
$$\int_0^1[g(x)-f(x)]\partial x=\int_0^1\left(x-x^2\right)\partial x=\int_0^1 x\,\partial x-\int_0^1 x^2\partial x=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=\left(\frac{1^2}{2}-\frac{1^3}{3}\right)-\left(\frac{0^2}{2}-\frac{0^3}{3}\right)=$$
$$\frac{1}{2}-\frac{1}{3}-0=\frac{3-2}{6}=\frac{1}{6}=0{,}166 \tag{103}$$

Exercício II

$$f(x)=x^2\,;g(x)=1$$
$$\int_0^1[g(x)-f(x)]\partial x+\int_1^2[f(x)-g(x)]\partial x=\int_0^1\left(1-x^2\right)\partial x+\int_1^2\left(x^2-1\right)\partial x=$$
$$\int_0^1\partial x-\int_0^1 x^2\partial x+\int_1^2 x^2\partial x-\int_1^2\partial x=\left[x-\frac{x^3}{3}\right]_0^1+\left[\frac{x^3}{3}-x\right]_1^2=\left[\frac{3x-x^3}{3}\right]_0^1+\left[\frac{x^3-3x}{3}\right]_1^2= \tag{104}$$
$$\left[\frac{x\left(3-x^2\right)}{3}\right]_0^1+\left[\frac{x\left(x^2-3\right)}{3}\right]_1^2=\left[\frac{1\left(3-1^2\right)}{3}\cancel{-\frac{0\left(3-0^2\right)}{3}}\right]+\left[\frac{2\left(2^2-3\right)}{3}-\frac{1\left(1^2-3\right)}{3}\right]=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=$$
$$\frac{6}{3}=2$$

Exercício III

$$f(x)=2x-2;\, g(x)=0$$
$$\int_0^1[g(x)-f(x)]\partial x+\int_1^3[f(x)-g(x)]\partial x=\int_0^1(0-(2x-2))\partial x+\int_1^3(2x-2-0)\partial x=$$
$$-2\int_0^1 x\partial x+2\int_0^1\partial x+2\int_1^3 x\partial x-2\int_1^3\partial x=\left[-2\frac{x^2}{2}+2x\right]_0^1+\left[2\frac{x^2}{2}-2x\right]_1^3= \tag{105}$$
$$[-x^2+2x]_0^1+[x^2-2x]_1^3=[-x(x-2)]_0^1+[x(x-2)]_1^3=$$
$$[-1(1-2)\cancel{-(-0(0-2))}]+[3(3-2)-1(1-2)]=1+3+1=5$$

Cálculo de área com integrais – Aula 2

Exercício I

$$y^2=4x\rightarrow y=\pm\sqrt{4x}=\pm2\sqrt{x};\, x_1=1;\, x_2=4$$
$$y_1(1)=2\sqrt{1}=2;\, y_1(4)=2\sqrt{4}=2\cdot2=4$$
$$y_2(1)=-2\sqrt{1}=-2;\, y_2(4)=-2\sqrt{4}=-2\cdot2=-4$$
$$f(x)=2\sqrt{x}=2x^{\frac{1}{2}};\, g(x)=0$$
$$\int_1^4[f(x)-g(x)]\partial x=\int_1^4[2\sqrt{x}-0]\partial x=2\int_1^4 x^{\frac{1}{2}}\partial x=\left[2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_1^4=\left[\frac{4\sqrt{x^3}}{3}\right]_1^4=\frac{4}{3}[\sqrt{x^3}]_1^4= \tag{106}$$
$$\frac{4}{3}[\sqrt{4^3}-\sqrt{1^3}]=\frac{4}{3}\left[\sqrt{(2^2)^3}-1\right]=\frac{4}{3}[2^3-1]=\frac{4}{3}[2^3-1]=\frac{4}{3}7=\frac{28}{3}=9{,}33$$

Exercício II

$$f(x)=x^2;\, g(x)=2x+3$$
$$f(x)=g(x)\rightarrow x^2=2x+3\rightarrow x^2-(2x+3)=0\rightarrow x^2-2x-3=0\rightarrow x^2-2x-3+1-1=0\rightarrow$$
$$x^2-2x+1-4=0\rightarrow(x-1)^2-4=0\rightarrow(x-1)^2=4\rightarrow x-1=\pm\sqrt{4}\rightarrow x=\pm2+1\rightarrow$$
$$x=2+1\rightarrow x_1=3;\, x=-2+1\rightarrow x_2=-1$$
$$f(0)=(0)^2=0;\, g(0)=2(0)+3=3$$
$$\int_{-1}^3[g(x)-f(x)]\partial x=\int_{-1}^3(2x+3-x^2)\partial x=2\int_{-1}^3 x\partial x+3\int_{-1}^3\partial x-\int_{-1}^3 x^2\partial x= \tag{107}$$
$$\left[2\frac{x^2}{2}+3x-\frac{x^3}{3}\right]_{-1}^3=\left[x^2+3x-\frac{x^3}{3}\right]_{-1}^3=\left[\frac{3x^2+9x-x^3}{3}\right]_{-1}^3=\frac{1}{3}[x(3x+9-x^2)]_{-1}^3=$$
$$\frac{1}{3}[3(3\cdot3+9-3^2)-(-1)(3(-1)+9-(-1)^2)]=\frac{1}{3}[3\cdot9+5]=\frac{1}{3}32=\frac{32}{3}=10{,}66$$

Cálculo de área com integrais – Aula 3

Exercício I

$$f(x)=-x^2+4x-3;g(x)=0$$
$$f(x)=g(x)\rightarrow -x^2+4x-3=0\rightarrow x^2-4x+3+(2)^2-(2)^2=0\rightarrow x^2-4x+4-1=0\rightarrow$$
$$(x-2)^2-1=0\rightarrow x-2=\pm\sqrt{1}\rightarrow x=\pm 1+2\rightarrow$$
$$x=1+2\rightarrow x_1=3;x=-1+2\rightarrow x_2=1$$
$$f(2)=-(2)^2+4(2)-3=-4+8-3=1;g(2)=0$$
$$\int_1^3[f(x)-g(x)]\partial x=\int_1^3\left(-x^2+4x-3-0\right)\partial x=-\int_1^3 x^2\partial x+4\int_1^3 x\partial x-3\int_1^3\partial x\rightarrow \tag{108}$$
$$\left[\frac{-x^3}{3}+4\frac{x^2}{2}-3x\right]_1^3=\left[\frac{-x^3}{3}+2x^2-3x\right]_1^3=\left[\frac{-x^3+6x^2-9x}{3}\right]_1^3=\frac{1}{3}\left[-x\left(x^2-6x+9\right)\right]_1^3=$$
$$\frac{1}{3}\left[-3\left(3^2-6\cdot 3+9\right)-\left(-1\left(1^2-6\cdot 1+9\right)\right)\right]=\frac{1}{3}[-3\cdot 0+4]=\frac{1}{3}4=\frac{4}{3}=1{,}33$$

Exercício II

$$f(x)=-x^2+4;g(x)=-5$$
$$f(x)=g(x)\rightarrow -x^2+4=-5\rightarrow -x^2=-5-4\rightarrow x^2=9\rightarrow x=\pm\sqrt{9}\rightarrow x=\pm 3$$
$$x_1=-3;x_2=3$$
$$f(0)=-(0)^2+4=4;g(0)=-5$$
$$\int_{-3}^3[f(x)-g(x)]\partial x=\int_{-3}^3\left(-x^2+4+5\right)\partial x=-\int_{-3}^3 x^2\partial x+4\int_{-3}^3\partial x+5\int_{-3}^3\partial x= \tag{109}$$
$$\left[\frac{-x^3}{3}+4x+5x\right]_{-3}^3=\left[\frac{-x^3+12x+15x}{3}\right]_{-3}^3=\frac{1}{3}\left[-x\left(x^2-12-15\right)\right]_{-3}^3=\frac{1}{3}\left[-x\left(x^2-27\right)\right]_{-3}^3=$$
$$\frac{1}{3}\left[-3\left(3^2-27\right)+(-3)\left((-3)^2-27\right)\right]=\frac{1}{3}[-3(-18)-3(-18)]=\frac{1}{3}[54+54]=\frac{1}{3}108=36$$

Cálculo de áreas com integrais – Aula 4

Exercício I

$$g(x)=x+2;f(x)=x^2-x+2$$
$$f(x)=g(x)\rightarrow x^2-x+2=x+2\rightarrow x^2-x+2-x-2=0\rightarrow x^2-2x=0\rightarrow x(x-2)=0\rightarrow$$
$$x=0\rightarrow x_1=0;x-2=0\rightarrow x=2\rightarrow x_2=2$$
$$g(1)=1+2=3;f(1)=1^2-1+2=2$$
$$\int_0^2[g(x)-f(x)]\partial x=\int_0^2\left[x+2-\left(x^2-x+2\right)\right]\partial x=\int_0^2\left(x+2-x^2+x-2\right)\partial x=\int_0^2\left(-x^2+2x\right)\partial x= \tag{110}$$
$$-\int_0^2 x^2\partial x+2\int_0^2 x\partial x=\left[\frac{-x^3}{3}+2\frac{x^2}{2}\right]_0^2=\left[\frac{-x^3}{3}+x^2\right]_0^2=\left[\frac{-x^3+3x^2}{3}\right]_0^2=\frac{1}{3}\left[-x\left(x^2-3x\right)\right]_0^2=$$
$$\frac{1}{3}\left[-2\left(2^2-3\cdot 2\right)+0\left(0^2-3\cdot 0\right)\right]=\frac{1}{3}4=\frac{4}{3}=1{,}33$$

Cálculo de áreas com integrais – Aula 5

Exercício I

$$f(x)=4x\,;\,g(x)=x^3+3x^2$$

$$f(x)=g(x)\rightarrow 4x=x^3+3x^2\rightarrow x^3+3x^2-4x=0\rightarrow x(x^2+3x-4)=0\rightarrow$$

$$x=0\rightarrow x_1=0\,;\,x^2+3x-4=0\rightarrow x^2+3x-4+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=0\rightarrow x^2+3x-4+\frac{9}{4}-\frac{9}{4}=0\rightarrow$$

$$x^2+3x+\frac{9}{4}-\frac{25}{4}=0\rightarrow\left(x+\frac{3}{2}\right)^2=\frac{25}{4}\rightarrow x+\frac{3}{2}=\pm\sqrt{\frac{25}{4}}\rightarrow x=\frac{\pm 5}{2}-\frac{3}{2}\rightarrow$$

$$x=\frac{-5}{2}-\frac{3}{2}=\frac{-5-3}{2}=\frac{-8}{2}=-4\rightarrow x_2=-4\,;\,x=\frac{5}{2}-\frac{3}{2}=\frac{5-3}{2}=\frac{2}{2}=1\rightarrow x_3=1$$

$$f(-1)=4(-1)=-4\,;\,g(-1)=(-1)^3+3(-1)^2=-1+3=2$$

$$f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)=2\,;\,g\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2=\frac{1}{8}+\frac{3}{4}=\frac{1+6}{8}=\frac{7}{8}$$

$$\int_{-4}^{0}[g(x)-f(x)]\partial x+\int_{0}^{1}[f(x)-g(x)]\partial x=\int_{-4}^{0}(x^3+3x^2-4x)\partial x+\int_{0}^{1}(4x-x^3-3x^2)\partial x= \quad (111)$$

$$\int_{-4}^{0}x^3\partial x+3\int_{-4}^{0}x^2\partial x-4\int_{-4}^{0}x\partial x+4\int_{0}^{1}x\partial x-\int_{0}^{1}x^3\partial x-3\int_{0}^{1}x^2\partial x=$$

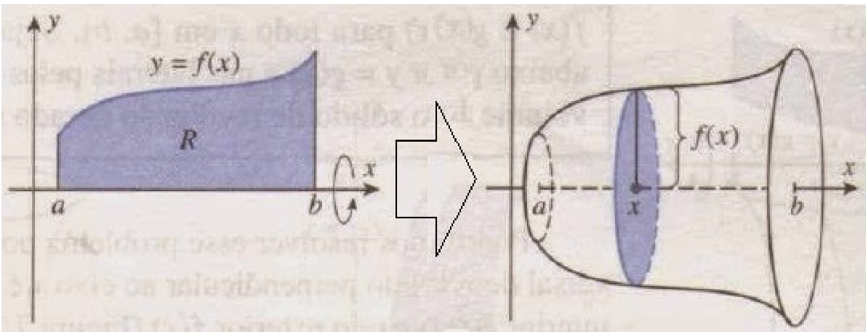
$$\left[\frac{x^4}{4}+3\frac{x^3}{3}-4\frac{x^2}{2}\right]_{-4}^{0}+\left[4\frac{x^2}{2}-\frac{x^4}{4}-3\frac{x^3}{3}\right]_{0}^{1}=\left[\frac{x^4}{4}+x^3-2x^2\right]_{-4}^{0}+\left[2x^2-\frac{x^4}{4}-x^3\right]_{0}^{1}=$$

$$\left[\frac{x^4+4x^3-8x^2}{4}\right]_{-4}^{0}+\left[\frac{8x^2-x^4-4x^3}{4}\right]_{0}^{1}=\frac{1}{4}\left[x^2(x^2+4x-8)\right]_{-4}^{0}+\frac{1}{4}\left[x^2(-x^2-4x+8)\right]_{0}^{1}=$$

$$\frac{1}{4}\left[\cancel{0^2(0^2+4\cdot 0-8)}-(-4)^2((-4)^2+4(-4)-8)\right]+\frac{1}{4}\left[1^2(-1^2-4\cdot 1+8)\cancel{-0^2(-0^2-4\cdot 0+8)}\right]=$$

$$\frac{1}{4}[-16(\cancel{16-16}-8)]+\frac{1}{4}[-1-4+8]=\frac{1}{4}(128+3)=\frac{131}{4}=32{,}75$$

Cálculo de volume com integrais simples – Aula 1

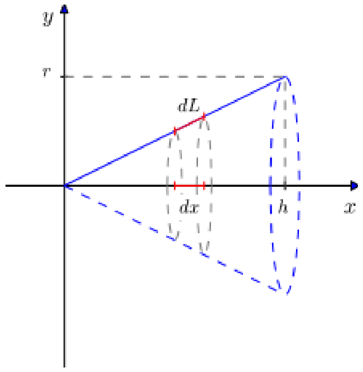


Volume do cilindro: $v=\pi r^2 h\,;\,r=y=f(x)\,;\,h=\partial x\,;\,\partial v=\pi[f(x)]^2\partial x$

Volume do sólido: $v=\int_a^b\partial v=\int_a^b\pi[f(x)]^2\partial x\rightarrow v=\pi\int_a^b[f(x)]^2\partial x$

Exercício I

Determine o volume do sólido de revolução gerado pela rotação em torno do eixo dos x da região R delimitado pelo gráfico das equações: $y=x\,;y=0\,;x=0\,;x=2$



$$f(x)=x\,;x_1=0\,;x_2=2$$

$$v=\pi\int_0^2 x^2\partial x=\pi\left[\frac{x^3}{3}\right]_0^2=\frac{\pi}{3}[x^3]_0^2=\frac{\pi}{3}[2^3-0^3]=\frac{\pi}{3}8=\frac{8\pi}{3}=2{,}667\pi \tag{112}$$

Exercício II

$$f(x)=\frac{x^2}{4}\,;x_1=1\,;x_2=4$$

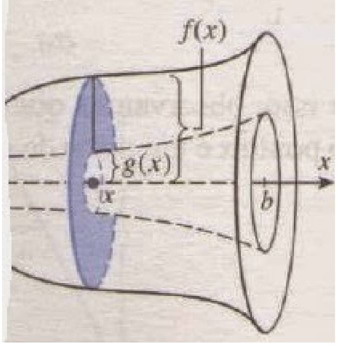
$$v=\pi\int_1^4\left(\frac{x^2}{4}\right)^2\partial x=\pi\int_1^4\frac{x^4}{16}\partial x=\frac{\pi}{16}\int_1^4 x^4\partial x=\frac{\pi}{16}\left[\frac{x^5}{5}\right]_1^4=\frac{\pi}{80}[x^5]_1^4=\frac{\pi}{80}[4^5-1^5]=\frac{1023\pi}{80}= \tag{113}$$

$$12{,}7875\pi$$

Exercício III

$$f(x)=x^2\,;x_1=1\,;x_2=2$$

$$v=\pi\int_1^2 x^4\partial x=\pi\left[\frac{x^5}{5}\right]_1^2=\frac{\pi}{5}[x^5]_1^2=\frac{\pi}{5}[2^5-1^5]=\frac{31\pi}{5}=6{,}2\pi \tag{114}$$



Volume do cilindro: $v=\pi r_1^2 h-\pi r_2^2 h=\pi\left(r_1^2-r_2^2\right)h; r_1=f(x); r_2=g(x); h=\partial x$

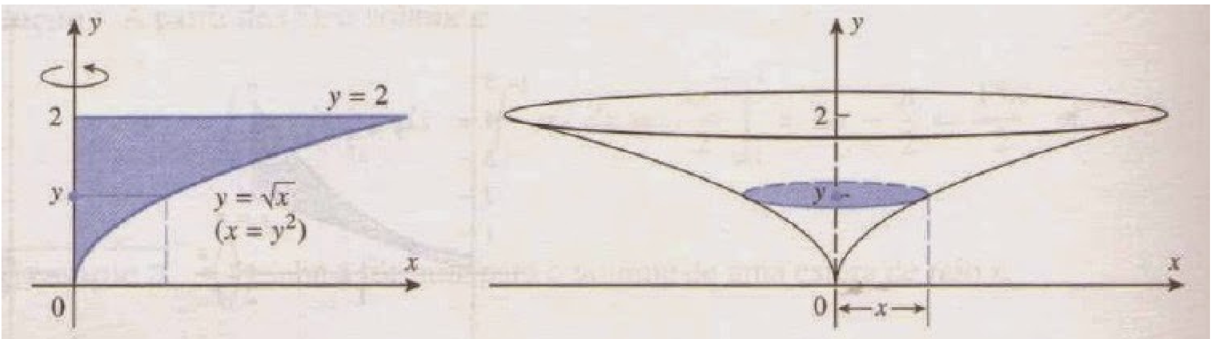
$$\partial v=\pi\left[(f(x))^2-(g(x))^2\right]\partial x$$

Volume do sólido: $v=\int_a^b \partial v=\int_a^b \pi\left[(f(x))^2-(g(x))^2\right]\partial x \rightarrow v=\pi\int_a^b\left[(f(x))^2-(g(x))^2\right]\partial x$

Exercício IV

$$\begin{gathered}
f(x)=x^2+1; g(x)=x+3\\
f(x)=g(x)\rightarrow x^2+1=x+3\rightarrow x^2+1-x-3=0\rightarrow x^2-x-2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0\rightarrow\\
x^2-x+\frac{1}{4}-\frac{9}{4}=0\rightarrow\left(x-\frac{1}{2}\right)^2=\frac{9}{4}\rightarrow x=\pm\sqrt{\frac{9}{4}}+\frac{1}{2}\rightarrow x=\frac{\pm 3}{2}+\frac{1}{2}\\
x=\frac{-3}{2}+\frac{1}{2}=\frac{-2}{2}=-1\rightarrow x_1=-1; x=\frac{3}{2}+\frac{1}{2}=\frac{3+1}{2}=\frac{4}{2}=2\rightarrow x_2=2\\
f(0)=0^2+1=1; g(0)=0+3=3\\
v=\pi\int_{-1}^{2}\left[(g(x))^2-(f(x))^2\right]\partial x=\pi\int_{-1}^{2}\left[(x+3)^2-\left(x^2+1\right)^2\right]\partial x=\\
\pi\int_{-1}^{2}\left[\left(x^2+6x+9\right)-\left(x^4+2x^2+1\right)\right]\partial x=\pi\int_{-1}^{2}\left(x^2+6x+9-x^4-2x^2-1\right)\partial x=\\
\pi\int_{-1}^{2}\left(-x^4-x^2+6x+8\right)\partial x=-\pi\int_{-1}^{2}x^4\partial x-\pi\int_{-1}^{2}x^2\partial x+6\pi\int_{-1}^{2}x\partial x+8\pi\int_{-1}^{2}\partial x=\\
\left[-\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\right]_{-1}^{2}=\pi\left[\frac{-x^5}{5}-\frac{x^3}{3}+3x^2+8x\right]_{-1}^{2}=\\
\pi\left[\frac{-3x^5-5x^3+45x^2+120x}{15}\right]_{-1}^{2}=\frac{\pi}{15}\left[-x\left(3x^4+5x^2-45x-120\right)\right]_{-1}^{2}=\\
\frac{\pi}{15}\left[-2\left(3\cdot 2^4+5\cdot 2^2-45\cdot 2-120\right)+(-1)\left(3(-1)^4+5(-1)^2-45(-1)-120\right)\right]=\\
\frac{\pi}{15}\left[-2(48+20-90-120)-(3+5+45-120)\right]=\frac{\pi}{15}\left[-2(-142)-(-67)\right]=\frac{\pi}{15}[284+67]=\\
\frac{\pi}{15}351=\frac{117\pi}{5}=23{,}4\pi
\end{gathered}\tag{115}$$

Cálculo de volume com integrais simples – Aula 2



Volume do cilindro: $v=\pi r^2 h;r=x=f(y);h=\partial y;\partial v=\pi[f(y)]^2\partial y$

Volume do sólido: $v=\int_a^b \partial v=\int_a^b \pi[f(y)]^2\partial y \rightarrow v=\pi\int_a^b [f(y)]^2\partial y$

Exercício I

A região limitada pelas funções $y=x^3$ e $y=8$, gira em torno do eixo y. Determine o volume do sólido de revolução obtido.

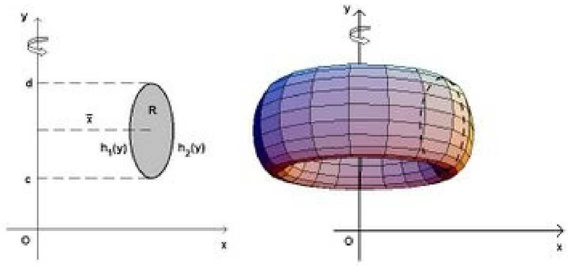
Volume do sólido em torno do eixo y:

$$y=f(x)=x^3 \rightarrow x=f(y)=\sqrt[3]{y}$$
$$f(x)=0\rightarrow x^3=0\rightarrow x=\sqrt[3]{0}=0\rightarrow x_1=0;f(x)=8\rightarrow x^3=8\rightarrow x=\sqrt[3]{8}=2\rightarrow x_2=2$$
$$f(y)=0\rightarrow \sqrt[3]{y}=0\rightarrow y=0^3=0\rightarrow y_1=0;f(y)=2\rightarrow \sqrt[3]{y}=2\rightarrow y=2^3=8\rightarrow y_2=8$$
$$v=\pi\int_0^8 [f(y)]^2\partial y=\pi\int_0^8 \left(\sqrt[3]{y}\right)^2\partial y=\pi\int_0^8 y^{\frac{2}{3}}\partial y=\pi\left[\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\right]_0^8=\pi\left[\frac{3\sqrt[3]{y^5}}{5}\right]_0^8=\frac{3\pi}{5}\left[\sqrt[3]{y^5}\right]_0^8=$$
$$\frac{3\pi}{5}\left[\sqrt[3]{8^5}-\sqrt[3]{0^5}\right]=\frac{3\sqrt[3]{\left(2^3\right)^5}\pi}{5}=\frac{3\cdot 2^5\pi}{5}=\frac{96\pi}{5}=19{,}2\pi \tag{116}$$

Volume do sólido em torno do eixo x:

$$v=\pi\int_0^2 [f(x)]^2\partial x=\pi\int_0^2 \left[x^3\right]^2\partial x=\pi\int_0^2 x^6\partial x=\pi\left[\frac{x^7}{7}\right]_0^2=\frac{\pi}{7}\left[x^7\right]_0^2=\frac{\pi}{7}\left[2^7-0^7\right]=\frac{128\pi}{7}=$$
$$18{,}2857142\pi$$

$$\lim_{x\rightarrow\infty} f(x)\rightarrow \lim_{x\rightarrow\infty}\left[x^3\right]=\infty^3=\infty;\ \lim_{x\rightarrow-\infty} f(x)\rightarrow \lim_{x\rightarrow-\infty}\left[x^3\right]=(-\infty)^3=-\infty$$



Volume do cilindro: $v=\pi r_1^2 h-\pi r_2^2 h=\pi\left(r_1^2-r_2^2\right)h; r_1=f(y); r_2=g(y); h=\partial y$

$$\partial v=\pi\left[(f(y))^2-(g(y))^2\right]\partial y$$

Volume do sólido: $v=\int_a^b \partial v=\int_a^b \pi\left[(f(y))^2-(g(y))^2\right]\partial y \rightarrow v=\pi\int_a^b \left[(f(y))^2-(g(y))^2\right]\partial y$

Integral por Partes – Aula 1

$$\int u\partial v=uv-\int v\partial u$$

Regras: LIATE ou ILATE

L = Logaritmica: $\ln|x|;\ln|x+1|$
I = Inversa da trigonométrica: $arc\,tg(x); arc\,sen(x)$
A = Aritmética: $x^2; x^{-3}$
T = Trigométrica: $sen(x);\cos(x)$
E = Exponencial: $e^x; e^{x+1}$

Exercício I

$$\begin{gathered}
\int x^2 sen(x)\partial x=x^2(-\cos(x))-\int(-\cos(x))2x\partial x=-x^2\cos(x)+2\int x\cos(x)\partial x= \\
-x^2\cos(x)+2\left(x\,sen(x)+\cos(x)+c\right)=-x^2\cos(x)+2x\,sen(x)+2\cos(x)+c \\
u=x^2\rightarrow\partial u=2x\partial x \\
v=-\cos(x)\rightarrow\partial v=sen(x)\partial x \\
\int x\cos(x)\partial x=x\,sen(x)-\int sen(x)\partial=x\,sen(x)-(-\cos(x))+c=x\,sen(x)+\cos(x)+c \\
u=x\rightarrow\partial u=\partial x \\
v=sen(x)\rightarrow\partial v=\cos(x)\partial x
\end{gathered}\tag{117}$$

$$\frac{\partial\left(-x^2\cos(x)+2x\,sen(x)+2\cos(x)+c\right)}{\partial x}=$$

$$\left[-\left(2x\cos(x)+x^2(-sen(x))\right)\right]+\left(2sen(x)+2x\cos(x)\right)+2(-sen(x))+0=$$

$$\cancel{-2x\cos(x)}+x^2 sen(x)+\cancel{2sen(x)}+\cancel{2x\cos(x)}\cancel{-2sen(x)}=x^2 sen(x)$$

Exercício II

$$
\begin{gathered}
\int \ln|x| tg(x)\,\partial x = \ln|x| \ln|sec(x)| - \int \ln|sec(x)| x^{-1}\,\partial x = \\
\ln|x| \ln|\cos^{-1}(x)| - \int \ln|\cos^{-1}(x)| x^{-1}\,\partial x = -\ln|x| \ln|\cos(x)| + \int \ln|\cos(x)| x^{-1}\,\partial x = \\
\cancel{-\ln|x| \ln|\cos(x)| + \ln|\cos(x)| \ln|x|} + \int \ln|x| tg(x)\,\partial x \\
u = \ln|x| \rightarrow \partial u = \frac{1}{x}\partial x \\
v = \ln|sec(x)| \rightarrow \partial v = tg(x)\,\partial x \\
\int \ln|\cos(x)| x^{-1}\,\partial x = \ln|\cos(x)| \ln|x| + \int \ln|x| tg(x)\,\partial x \\
u = \ln|\cos(x)| \rightarrow \partial u = \frac{1}{\cos(x)}(-sen(x))\,\partial x = \frac{-sen(x)}{\cos(x)}\partial x = -tg(x)\,\partial x \\
v = \ln|x| \rightarrow \partial v = \frac{1}{x}\partial x
\end{gathered}
\tag{118}
$$

Exercício III

$$
\begin{gathered}
\int x\, sen(x)\,\partial x = -x\cos(x) + \int \cos(x)\,\partial x = -x\cos(x) + sen(x) + c \\
u = x \rightarrow \partial u = \partial x \\
v = -\cos(x) \rightarrow \partial v = sen(x)\,\partial x \\
\frac{\partial(-x\cos(x) + sen(x) + c)}{\partial x} = -[\cos(x) + x(-sen(x))] + \cos(x) + 0 = \\
\cancel{-\cos(x)} + x\, sen(x) \cancel{+\cos(x)} = x\, sen(x)
\end{gathered}
\tag{119}
$$

Exercício IV

$$
\begin{gathered}
\int x e^x\,\partial x = x e^x - \int e^x\,\partial x = x e^x - e^x + c \\
u = x \rightarrow \partial u = \partial x \\
\partial v = e^x\,\partial x \rightarrow v = \int \partial v = e^x \\
\frac{\partial(x e^x - e^x + c)}{\partial x} = \cancel{e^x} + x e^x \cancel{-e^x} + 0 = x e^x
\end{gathered}
\tag{120}
$$

Exercício V

$$
\begin{gathered}
\int \ln|x|\,\partial x = \ln|x| x - \int x \frac{1}{x}\partial x = x\ln|x| - \int \partial x = x\ln|x| - x + c \\
u = \ln|x| \rightarrow \partial u = \frac{1}{x}\partial x \\
\partial v = \partial x \rightarrow v = \int \partial v = x \\
\frac{\partial(x\ln|x| - x + c)}{\partial x} = \ln|x| + x\frac{1}{x} - 1 + 0 = \ln|x| + 1 - 1 = \ln|x|
\end{gathered}
\tag{121}
$$

Exercício VI

$$\begin{gathered}
\int x^2\cos(x)\partial x = x^2 sen(x) - 2\int sen(x)\,x\,\partial x = x^2 sen(x) - 2\left(-x\cos(x) + sen(x)\right) + c = \\
x^2 sen(x) + 2x\cos(x) - 2\,sen(x) + c \\
u = x^2 \rightarrow \partial u = 2x\,\partial x \\
\partial v = \cos(x)\partial x \rightarrow v = \int \partial v = sen(x) \\
\int sen(x)\,x\,\partial x = -x\cos(x) + \int \cos(x)\partial x = -x\cos(x) + sen(x) + c \\
u = x \rightarrow \partial u = \partial x \\
\partial v = sen(x)\partial x \rightarrow v = \int \partial v = -\cos(x)
\end{gathered} \tag{122}$$

$$\begin{gathered}
\frac{\partial\left(x^2 sen(x) + 2x\cos(x) - 2\,sen(x) + c\right)}{\partial x} = \\
2x\,sen(x) + x^2\cos(x) \cancel{+2\cos(x)} + 2x(-sen(x)) \cancel{-2\cos(x)} + 0 = \\
\cancel{2x\,sen(x)} + x^2\cos(x) \cancel{-2x\,sen(x)} = x^2\cos(x)
\end{gathered}$$

Exercício VII

$$\begin{gathered}
\int \frac{x}{\sqrt{x+1}}\partial x = \int \frac{x}{(x+1)^{\frac{1}{2}}}\partial x = \int (x+1)^{\frac{-1}{2}} x\,\partial x = 2x\sqrt{x+1} - 2\int \sqrt{x+1}\,\partial x = \\
2x\sqrt{x+1} - 2\int (x+1)^{\frac{1}{2}}\partial x = 2x\sqrt{x+1} - 2\frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = 2x\sqrt{x+1} - \frac{4\sqrt{(x+1)^3}}{3} + c \\
u = x \rightarrow \partial u = \partial x \\
\partial v = (x+1)^{\frac{-1}{2}}\partial x \rightarrow v = \int \partial v = \frac{(x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = 2\sqrt{x+1}
\end{gathered} \tag{123}$$

$$\begin{gathered}
\frac{\partial\left(2x\sqrt{x+1} - \frac{4\sqrt{(x+1)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(2x(x+1)^{\frac{1}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}} + c\right)}{\partial x} = \\
2(x+1)^{\frac{1}{2}} + 2x\frac{1}{\cancel{2}}(x+1)^{\frac{-1}{2}} - \frac{4}{3}\frac{\cancel{3}}{2}(x+1)^{\frac{1}{2}} + 0 = \cancel{2\sqrt{x+1}} + \frac{x}{\sqrt{x+1}} \cancel{-2\sqrt{x+1}} = \frac{x}{\sqrt{x+1}}
\end{gathered}$$

Integral por Partes – Aula 2

Exercício I

$$\int (2x+1)sen(x)\partial x=-(2x+1)\cos(x)+2\int \cos(x)\partial x=-(2x+1)\cos(x)+2sen(x)+c=$$
$$-2x\cos(x)-\cos(x)+2sen(x)+c$$
$$u=2x+1\to\partial u=2\partial x$$
$$\partial v=sen(x)\partial x\to v=\int \partial v=-\cos(x) \tag{124}$$
$$\frac{\partial(-(2x+1)\cos(x)+2sen(x)+c)}{\partial x}=-[2\cos(x)+(2x+1)(-sen(x))]+2\cos(x)+0=$$
$$\cancel{-2\cos(x)}+(2x+1)sen(x)+\cancel{2\cos(x)}=(2x+1)sen(x)$$

Exercício II

$$\int \ln|x|\frac{\partial x}{x^2}=\int \ln|x|x^{-2}\partial x=\ln|x|\left(\frac{-1}{x}\right)+\int \frac{1}{x}\frac{1}{x}\partial x=\frac{-\ln|x|}{x}+\int x^{-2}\partial x=\frac{-\ln|x|}{x}+\frac{x^{-1}}{(-1)}+c$$
$$\frac{-\ln|x|}{x}-\frac{1}{x}+c=\frac{-\ln|x|-1}{x}+c=\frac{\ln|x^{-1}|-1}{x}+c=\frac{\ln\left|\frac{1}{x}\right|-1}{x}+c$$
$$u=\ln|x|\to\partial u=\frac{1}{x}\partial x$$
$$\partial v=x^{-2}\partial x\to v=\int \partial v=\frac{x^{-1}}{(-1)}=\frac{-1}{x} \tag{125}$$
$$\frac{\partial\left(\frac{\ln\left|\frac{1}{x}\right|-1}{x}+c\right)}{\partial x}=\frac{\partial\left(\frac{\ln|x^{-1}|-1}{x}+c\right)}{\partial x}=\frac{\frac{1}{x^{-1}}(-x^{-2})x-(\ln|x^{-1}|-1)}{x^2}+0=$$
$$\frac{\frac{-1}{\left(\frac{1}{\cancel{x}}\right)}\left(\frac{1}{\cancel{x^2}}\right)\cancel{x}-\left(\ln\left|\frac{1}{x}\right|-1\right)}{x^2}=\frac{\cancel{-1}-\ln\left|\frac{1}{x}\right|\cancel{+1}}{x^2}=\frac{-\ln|x^{-1}|}{x^2}=\frac{\ln|x|}{x^2}$$

Exercício III

$$\int sen(x)\cos(x)\partial x=sen(x)sen(x)-\int sen(x)\cos(x)\partial x\to$$
$$\int sen(x)\cos(x)\partial x+\int sen(x)\cos(x)\partial x=sen^2(x)\to 2\int sen(x)\cos(x)\partial x=sen^2(x)\to$$
$$\int sen(x)\cos(x)\partial x=\frac{sen^2(x)}{2}+c$$
$$u=sen(x)\to\partial u=\cos(x)\partial x$$
$$\partial v=\cos(x)\partial x\to v=\int \partial v=sen(x) \tag{126}$$
$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{2}sen^2(x)+c\right)}{\partial x}=\frac{1}{\cancel{2}}\cancel{2}sen(x)\cos(x)+0=sen(x)\cos(x)$$

Exercício IV

$$\int \sqrt{x}\ln|x|\partial x=\int \ln|x|x^{\frac{1}{2}}\partial x=\ln|x|\frac{2\sqrt{x^3}}{3}-\int \frac{2\sqrt{x^3}}{3}\frac{1}{x}\partial x=\frac{2\sqrt{x^3}\ln|x|}{3}-\frac{2}{3}\int x^{\frac{3}{2}}x^{-1}\partial x=$$
$$\frac{2\sqrt{x^3}\ln|x|}{3}-\frac{2}{3}\int x^{\frac{1}{2}}\partial x=\frac{2\sqrt{x^3}\ln|x|}{3}-\frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{x^3}\ln|x|}{3}-\frac{4\sqrt{x^3}}{9}+c=$$
$$\frac{2\sqrt{x^2x}\ln|x|}{3}-\frac{4\sqrt{x^2x}}{9}+c=\frac{2x\sqrt{x}\ln|x|}{3}-\frac{4x\sqrt{x}}{9}+c$$
$$u=\ln|x|\rightarrow\partial u=\frac{1}{x}\partial x$$
$$\partial v=x^{\frac{1}{2}}\partial x\rightarrow v=\int \partial v=\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}=\frac{2\sqrt{x^3}}{3} \tag{127}$$

$$\frac{\partial\left(\frac{2x\sqrt{x}\ln|x|}{3}-\frac{4x\sqrt{x}}{9}+c\right)}{\partial x}=\frac{\partial\left(\frac{2}{3}x^{\frac{3}{2}}\ln|x|-\frac{4}{9}x^{\frac{3}{2}}+c\right)}{\partial x}=\frac{\cancel{2}}{\cancel{3}}\frac{\cancel{3}}{\cancel{2}}x^{\frac{1}{2}}\ln|x|+\frac{2}{3}x^{\frac{3}{2}}\frac{1}{x}-\frac{4}{9}\frac{3}{2}x^{\frac{1}{2}}+0=$$
$$x^{\frac{1}{2}}\ln|x|+\frac{2}{3}x^{\frac{3}{2}-1}-\frac{2}{3}x^{\frac{1}{2}}=x^{\frac{1}{2}}\ln|x|\frac{\cancel{+2\sqrt{x}}}{\cancel{3}}\frac{\cancel{-2\sqrt{x}}}{\cancel{3}}=\sqrt{x}\ln|x|$$

Integral por Partes – Aula 3

Exercício I

$$\int arc\,sen(x)\partial x=x\,arc\,sen(x)-\int x\frac{1}{\sqrt{1-x^2}}\partial x=x\,arc\,sen(x)-\int (1-x^2)^{\frac{-1}{2}}x\partial x=$$
$$x\,arc\,sen(x)+\sqrt{1-x^2}+c$$
$$u=arc\,sen(x)\rightarrow\partial u=\frac{1}{\sqrt{1-x^2}}\partial x$$
$$\partial v=\partial x\rightarrow v=\int \partial v=x$$
$$\int (1-x^2)^{\frac{-1}{2}}x\partial x=\int u^{\frac{-1}{2}}\left(\frac{-\partial u}{2}\right)=\frac{-1}{2}\int u^{\frac{-1}{2}}\partial u=\frac{-1}{\cancel{2}}\frac{u^{\frac{1}{2}}}{\left(\frac{1}{\cancel{2}}\right)}+c=-\sqrt{u}+c=-\sqrt{1-x^2}+c$$
$$u=1-x^2\rightarrow\partial u=-2x\partial x\rightarrow\frac{-\partial u}{2}=x\partial x \tag{128}$$

$$\frac{\partial\left(x\,arc\,sen(x)+\sqrt{1-x^2}+c\right)}{\partial x}=\frac{\partial\left(x\,arc\,sen(x)+(1-x^2)^{\frac{1}{2}}+c\right)}{\partial x}=$$
$$arc\,sen(x)+x\frac{1}{\sqrt{1-x^2}}+\frac{1}{2}(1-x^2)^{\frac{-1}{2}}(-2x)+0=arc\,sen(x)+\frac{x}{\sqrt{1-x^2}}-\frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}=$$
$$arc\,sen(x)\frac{\cancel{+x}}{\cancel{\sqrt{1-x^2}}}\frac{\cancel{-x}}{\cancel{\sqrt{1-x^2}}}=arc\,sen(x)$$

Exercício II

$$\int arc\,tg(x)\partial x = x\,arc\,tg(x) - \int x\frac{1}{1+x^2}\partial x = x\,arc\,tg(x) - \int (1+x^2)^{-1}x\,\partial x =$$
$$x\,arc\,tg(x) - \frac{\ln|1+x^2|}{2} + c$$
$$u = arc\,tg(x) \rightarrow \partial u = \frac{1}{1+x^2}\partial x$$
$$\partial v = \partial x \rightarrow v = \int \partial v = x$$
$$\int (1+x^2)^{-1}x\,\partial x = \int u^{-1}\frac{\partial u}{2} = \frac{1}{2}\int u^{-1}\partial u = \frac{1}{2}\ln|u| + c = \frac{\ln|u|}{2} + c = \frac{\ln|1+x^2|}{2} + c \tag{129}$$
$$u = 1+x^2 \rightarrow \partial u = 2x\,\partial x \rightarrow \frac{\partial u}{2} = x\,\partial x$$

$$\frac{\partial\left(x\,arc\,tg(x) - \frac{\ln|1+x^2|}{2} + c\right)}{\partial x} = \frac{\partial\left(x\,arc\,tg(x) - \frac{1}{2}\ln|1+x^2| + c\right)}{\partial x} =$$
$$arc\,tg(x) + x\frac{1}{1+x^2} - \frac{1}{\cancel{2}}\frac{1}{1+x^2}\cancel{2}x + 0 = arc\,tg(x)\frac{\cancel{+x}}{\cancel{1+x^2}}\frac{\cancel{-x}}{\cancel{1+x^2}} = arc\,tg(x)$$

Exercício III

$$\int x\,arc\,tg(x)\partial x = \frac{x^2}{2}arc\,tg(x) - \int \frac{x^2}{2}\frac{1}{1+x^2}\partial x = \frac{x^2 arc\,tg(x)}{2} - \frac{1}{2}\int (1+x^2)^{-1}x^2\partial x =$$
$$\frac{x^2 arc\,tg(x)}{2} - \frac{1}{2}(x - arc\,tg(x)) + c = \frac{x^2 arc\,tg(x)}{2} - \frac{x}{2} + \frac{arc\,tg(x)}{2} + c =$$
$$\frac{x^2 arc\,tg(x) + arc\,tg(x) - x}{2} + c$$
$$u = arc\,tg(x) \rightarrow \partial u = \frac{1}{1+x^2}\partial x$$
$$\partial v = x\,\partial x \rightarrow v = \int \partial v = \frac{x^2}{2}$$
$$\int (1+x^2)^{-1}x^2\partial x = \int \frac{x^2}{1+x^2}\partial x = \int \left(1 - \frac{1}{1+x^2}\right)\partial x = \int \partial x - \int \frac{1}{1+x^2}\partial x = x - arc\,tg(x) + c \tag{130}$$

$$\frac{\partial\left(\frac{x^2 arc\,tg(x) + arc\,tg(x) - x}{2} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{2}x^2 arc\,tg(x) + \frac{1}{2}arc\,tg(x) - \frac{1}{2}x + c\right)}{\partial x} =$$
$$\frac{1}{2}\left(2x\,arc\,tg(x) + x^2\frac{1}{1+x^2}\right) + \frac{1}{2}\frac{1}{1+x^2} - \frac{1}{2} + 0 = x\,arc\,tg(x) + \frac{x^2}{2(1+x^2)} + \frac{1}{2(1+x^2)} - \frac{1}{2} =$$
$$\frac{2(1+x^2)x\,arc\,tg(x) + x^2 + 1 - (1+x^2)}{2(1+x^2)} = \frac{\cancel{2(1+x^2)}x\,arc\,tg(x)\cancel{+x^2+1-1-x^2}}{\cancel{2(1+x^2)}} = x\,arc\,tg(x)$$

Integral por Partes – Aula 4

Exercício I

$$
\begin{gathered}
\int e^x sen(x)\partial x = sen(x)e^x - \int e^x \cos(x)\partial x = sen(x)e^x - \left(\cos(x)e^x + \int e^x sen(x)\partial x\right) = \\
sen(x)e^x - \cos(x)e^x - \int e^x sen(x)\partial x \rightarrow \\
\int e^x sen(x)\partial x + \int e^x sen(x)\partial x = sen(x)e^x - \cos(x)e^x \rightarrow \\
2\int e^x sen(x)\partial x = sen(x)e^x - \cos(x)e^x \rightarrow \int e^x sen(x)\partial x = \frac{sen(x)e^x - \cos(x)e^x}{2} + c = \\
\frac{e^x(sen(x) - \cos(x))}{2} + c \\
u = sen(x) \rightarrow \partial u = \cos(x)\partial x \\
\partial v = e^x \partial x \rightarrow v = \int \partial v = e^x \\
\int e^x \cos(x)\partial x = \cos(x)e^x + \int e^x sen(x)\partial x \\
u = \cos(x) \rightarrow \partial u = -sen(x)\partial x \\
\partial v = e^x \partial x \rightarrow v = \int \partial v = e^x
\end{gathered} \tag{131}
$$

$$
\begin{gathered}
\frac{\partial\left(\frac{e^x(sen(x)-\cos(x))}{2} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{2}e^x sen(x) - \frac{1}{2}e^x \cos(x) + c\right)}{\partial x} = \\
\frac{1}{2}\left(e^x sen(x) + e^x \cos(x)\right) - \frac{1}{2}\left[e^x \cos(x) + e^x(-sen(x))\right] + 0 = \\
\frac{e^x sen(x)}{2}\cancel{\frac{+e^x \cos(x)}{2}}\cancel{\frac{-e^x \cos(x)}{2}} + \frac{e^x sen(x)}{2} = \frac{\cancel{2}e^x sen(x)}{\cancel{2}} = e^x sen(x)
\end{gathered}
$$

Exercício II

$$
\begin{gathered}
\int \ln|x| x^5 \partial x = \frac{x^6}{6}\ln|x| - \int \frac{x^6}{6}\frac{1}{x}\partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6}\int x^5 \partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6}\frac{x^6}{6} + c = \\
\frac{x^6 \ln|x|}{6} - \frac{x^6}{36} + c = \frac{6x^6 \ln|x| - x^6}{36} + c = \frac{x^6(6\ln|x| - 1)}{36} + c \\
u = \ln|x| \rightarrow \partial u = \frac{1}{x}\partial x \\
\partial v = x^5 \partial x \rightarrow v = \int \partial v = \frac{x^6}{6}
\end{gathered} \tag{132}
$$

$$
\begin{gathered}
\frac{\partial\left(\frac{x^6(6\ln|x|-1)}{36} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{6}x^6 \ln|x| - \frac{1}{36}x^6 + c\right)}{\partial x} = \frac{1}{6}\left(6x^5 \ln|x| + x^6 \frac{1}{x}\right) - \frac{1}{36}6x^5 + 0 = \\
x^5 \ln|x| + \frac{x^6}{6x} - \frac{x^5}{6} = \frac{\cancel{6}x^{\cancel{6}5}\ln|x| \cancel{+x^6 - x^6}}{\cancel{6x}} = x^5 \ln|x|
\end{gathered}
$$

Exercício III

$$\begin{gathered}\int sec^3(x)\partial x=\int sec(x)sec^2(x)\partial x=sec(x)tg(x)-\int tg(x)sec(x)tg(x)\partial x=\\ sec(x)tg(x)-\int tg^2(x)sec(x)\partial x=sec(x)tg(x)-\int\left(sec^2(x)-1\right)sec(x)\partial x=\\ sec(x)tg(x)-\int sec^3(x)\partial x+\int sec(x)\partial x\rightarrow\\ \int sec^3(x)\partial x+\int sec^3(x)\partial x=sec(x)tg(x)+\int sec(x)\partial x\rightarrow\\ 2\int sec^3(x)\partial x=sec(x)tg(x)+\ln|sec(x)+tg(x)|+c\rightarrow\\ \int sec^3(x)\partial x=\frac{sec(x)tg(x)+\ln|sec(x)+tg(x)|}{2}+c\\ u=sec(x)\rightarrow\partial u=sec(x)tg(x)\partial x\\ \partial v=sec^2(x)\partial x\rightarrow v=\int\partial v=tg(x)\end{gathered}\tag{133}$$

Integral por Partes – Aula 5

Exercício I

$$\begin{gathered}\int x\,sec^2(x)\partial x=x\,tg(x)-\int tg(x)\partial x=x\,tg(x)-\ln|sec(x)|+c\\ u=x\rightarrow\partial u=\partial x\\ \partial v=sec^2(x)\partial x\rightarrow v=\int\partial v=tg(x)\\ \\ \frac{\partial\left(x\,tg(x)-\ln|sec(x)|+c\right)}{\partial x}=\frac{\partial\left(x\,tg(x)-\ln|\cos^{-1}(x)|+c\right)}{\partial x}=\frac{\partial\left(x\,tg(x)+\ln|\cos(x)|+c\right)}{\partial x}=\\ tg(x)+x\,sec^2(x)+\frac{1}{\cos(x)}(-sen(x))+0=tg(x)+x\,sec^2(x)-\frac{sen(x)}{\cos(x)}=\\ \cancel{tg(x)}+x\,sec^2(x)-\cancel{tg(x)}=x\,sec^2(x)\\ \frac{\partial\left(x\,tg(x)-\ln|sec(x)|+c\right)}{\partial x}=tg(x)+x\,sec^2(x)-\frac{1}{\cancel{sec(x)}}\cancel{sec(x)}tg(x)+0=\\ \cancel{tg(x)}+x\,sec^2(x)-\cancel{tg(x)}=x\,sec^2(x)\end{gathered}\tag{134}$$

Integral por Frações Parciais – Aula 1

$$\int\frac{(x-2)\partial x}{(x-1)(x+1)(x-3)}=\int\left(\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-3}\right)\partial x$$

$$\int\frac{(x-2)\partial x}{(x-1)^3(x+1)}=\int\left(\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{D}{x+1}\right)\partial x$$

$$\int\frac{(x-2)\partial x}{\left(x^2-2x+1\right)(x+3)}=\int\frac{(Ax+B)\partial x}{x^2-2x+1}+\int\frac{C\partial x}{x+3}$$

Exercício I

$$\int \frac{\partial x}{x^2-4} = \int \frac{\partial x}{(x-2)(x+2)} = \int \left(\frac{A}{x-2}+\frac{B}{x+2}\right)\partial x = \int \frac{\partial x}{x^2-4} = \int \left(\frac{\left(\frac{1}{4}\right)}{x-2}+\frac{\left(\frac{-1}{4}\right)}{x+2}\right)\partial x =$$
$$\int \left(\frac{1}{4(x-2)}-\frac{1}{4(x+2)}\right)\partial x = \frac{1}{4}\int (x-2)^{-1}\partial x - \frac{1}{4}\int (x+2)^{-1}\partial x =$$
$$\frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + c = \frac{1}{4}(\ln|x-2| - \ln|x+2|) + c$$

$$\begin{gathered} \frac{\frac{1}{(x-2)(x+2)} = \frac{A}{x-2}+\frac{B}{x+2}}{(x-2)(x+2)} \rightarrow \\ 1 = A(x+2)+B(x-2) \\ x=-2 \rightarrow 1 = A(-2+2)+B(-2-2) \rightarrow 1 = 0-4B \rightarrow B = \frac{-1}{4} \\ x=2 \rightarrow 1 = A(2+2)+B(2-2) \rightarrow 1 = 4A+0 \rightarrow A = \frac{1}{4} \end{gathered} \tag{135}$$

$$\frac{\partial\left[\frac{1}{4}(\ln|x-2| - \ln|x+2|) + c\right]}{\partial x} = \frac{1}{4}\left(\frac{1}{x-2}-\frac{1}{x+2}\right) = \frac{1}{4}\left(\frac{x+2-(x-2)}{x^2-4}\right) = \frac{1}{4}\left(\frac{\cancel{x}+2\cancel{-x}+2}{x^2-4}\right) =$$
$$\frac{1}{\cancel{4}}\frac{\cancel{4}}{x^2-4} = \frac{1}{x^2-4}$$

Exercício II

$$\int \frac{(x-1)\partial x}{x^3-x^2-2x}=\int \frac{(x-1)\partial x}{x(x^2-x-2)}=\int \frac{(x-1)\partial x}{x(x-2)(x+1)}=\int \left(\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+1}\right)\partial x=$$

$$\int \left(\frac{\left(\frac{1}{2}\right)}{x}+\frac{\left(\frac{1}{6}\right)}{x-2}+\frac{\left(\frac{-2}{3}\right)}{x+1}\right)\partial x=\frac{1}{2}\int x^{-1}\partial x+\frac{1}{6}\int (x-2)^{-1}\partial x-\frac{2}{3}(x+1)^{-1}=$$

$$\frac{1}{2}\ln|x|+\frac{1}{6}\ln|x-2|-\frac{2}{3}\ln|x+1|+c$$

$$x^2-x-2=0\rightarrow x^2-x-2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0\rightarrow x^2-x+\frac{1}{4}-\frac{9}{4}=0\rightarrow \left(x-\frac{1}{2}\right)^2=\frac{9}{4}\rightarrow x=$$

$$\pm\sqrt{\frac{9}{4}}+\frac{1}{2}=\frac{\pm 3}{2}+\frac{1}{2}$$

$$x_1=\frac{3+1}{2}=\frac{4}{2}=2;\, x_2=\frac{-3+1}{2}=\frac{-2}{2}=-1$$

$$ax^2+bx+c=a(x-x_1)(x-x_2)\rightarrow x^2-x-2=(x-2)(x-(-1))=(x-2)(x+1)$$

$$\frac{\frac{x-1}{x(x-2)(x+1)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+1}}{x(x-2)(x+1)}\rightarrow x-1=A(x-2)(x+1)+Bx(x+1)+Cx(x-2) \tag{136}$$

$$x=-1\rightarrow -1-1=\cancel{A(-1-2)(-1+1)}+\cancel{B(-1)(-1+1)}+C(-1)(-1-2)\rightarrow -2=3C\rightarrow C=\frac{-2}{3}$$

$$x=2\rightarrow 2-1=\cancel{A(2-2)(2+1)}+B(2)(2+1)\cancel{+C(2)(2-2)}\rightarrow 1=6B\rightarrow B=\frac{1}{6}$$

$$x=0\rightarrow 0-1=A(0-2)(0+1)\cancel{+B(0)(0+1)}\cancel{+C(0)(0-2)}\rightarrow -1=-2A\rightarrow A=\frac{1}{2}$$

$$\frac{\partial\left(\frac{1}{2}\ln|x|+\frac{1}{6}\ln|x-2|-\frac{2}{3}\ln|x+1|+c\right)}{\partial x}=\frac{1}{2}\frac{1}{x}+\frac{1}{6}\frac{1}{x-2}-\frac{2}{3}\frac{1}{x+1}+0=$$

$$\frac{1}{2x}+\frac{1}{6(x-2)}-\frac{2}{3(x+1)}=\frac{18(x-2)(x+1)+6x(x+1)-24x(x-2)}{36x(x-2)(x+1)}=$$

$$\frac{6(3(x-2)(x+1)+x(x+1)-4x(x-2))}{36x(x-2)(x+1)}=\frac{\cancel{3x^2}-3x-6\cancel{+x^2}+x\cancel{-4x^2}+8x}{6x(x-2)(x+1)}=$$

$$\frac{6x-6}{6x(x-2)(x+1)}=\frac{\cancel{6}(x-1)}{\cancel{6}x(x-2)(x+1)}=\frac{x-1}{x(x^2-x-2)}=\frac{x-1}{x^3-x^2-2x}$$

Integral por Frações Parciais – Aula 2

Exercício I

$$\int \frac{(x-2)\partial x}{x^3-3x^2-x+3}=\int \frac{(x-2)\partial x}{x(x^2-1)-3(x^2-1)}=\int \frac{(x-2)\partial x}{(x^2-1)(x-3)}=\int \frac{(x-2)\partial x}{(x+1)(x-1)(x-3)}=$$

$$\int\left(\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x-3}\right)\partial x=\int\left(\frac{\left(\frac{-3}{8}\right)}{x+1}+\frac{\left(\frac{1}{4}\right)}{x-1}+\frac{\left(\frac{1}{8}\right)}{x-3}\right)\partial x=$$

$$\frac{-3}{8}\int (x+1)^{-1}\partial x+\frac{1}{4}\int (x-1)^{-1}\partial x+\frac{1}{8}\int (x-3)^{-1}\partial x=$$

$$\frac{-3}{8}\ln|x+1|+\frac{1}{4}\ln|x-1|+\frac{1}{8}\ln|x-3|+c=\frac{-3\ln|x+1|}{8}+\frac{\ln|x-1|}{4}+\frac{\ln|x-3|}{8}+c$$

$$\frac{\frac{x-2}{(x+1)(x-1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{x-3}}{(x+1)(x-1)(x-3)}\rightarrow$$

$$x-2=A(x-1)(x-3)+B(x+1)(x-3)+C(x+1)(x-1)$$

$$x=1\rightarrow 1-2=\cancel{A(1-1)(1-3)}+B(1+1)(1-3)\cancel{+C(1+1)(1-1)}\rightarrow -1=-4B\rightarrow B=\frac{1}{4}$$

$$x=3\rightarrow 3-2=\cancel{A(3-1)(3-3)+B(3+1)(3-3)}+C(3+1)(3-1)\rightarrow 1=8C\rightarrow C=\frac{1}{8} \tag{137}$$

$$x=-1\rightarrow -1-2=A(-1-1)(-1-3)\cancel{+B(-1+1)(-1-3)+C(-1+1)(-1-1)}\rightarrow -3=8A\rightarrow$$

$$A=\frac{-3}{8}$$

$$\frac{\partial\left(\frac{-3\ln|x+1|}{8}+\frac{\ln|x-1|}{4}+\frac{\ln|x-3|}{8}+c\right)}{\partial x}=\frac{\partial\left(\frac{-3}{8}\ln|x+1|+\frac{1}{4}\ln|x-1|+\frac{1}{8}\ln|x-3|+c\right)}{\partial x}=$$

$$\frac{-3}{8}\frac{1}{x+1}+\frac{1}{4}\frac{1}{x-1}+\frac{1}{8}\frac{1}{x-3}+0=\frac{-3}{8(x+1)}+\frac{1}{4(x-1)}+\frac{1}{8(x-3)}=$$

$$\frac{-96(x-1)(x-3)+64(x+1)(x-3)+32(x+1)(x-1)}{256(x+1)(x-1)(x-3)}=$$

$$\frac{-32[3(x^2-4x+3)-2(x^2-2x-3)-(x^2-1)]}{256(x+1)(x-1)(x-3)}=\frac{-[\cancel{3x^2}-12x+9\cancel{-2x^2}+4x+6\cancel{-x^2}+1]}{8(x+1)(x-1)(x-3)}=$$

$$\frac{-\cancel{8}[-x+2]}{\cancel{8}(x+1)(x-1)(x-3)}=\frac{x-2}{(x^2-1)(x-3)}=\frac{x-2}{x^3-3x^2-x+3}$$

Integração por Frações Parciais – Aula 3

Exercício I

$$\begin{gathered}
\int \frac{-4x^3\,\partial x}{2x^3+x^2-2x-1}=\int\left(-2+\frac{2x^2-4x-2}{2x^3+x^2-2x-1}\right)\partial x=-2\int\partial x+\int\frac{2x^2-4x-2}{2x^3+x^2-2x-1}\partial x=\\
-2\int\partial x+\int\frac{2x^2-4x-2}{x^2(2x+1)-(2x+1)}\partial x=-2\int\partial x+\int\frac{2x^2-4x-2}{(2x+1)(x^2-1)}\partial x=\\
-2\int\partial x+\int\frac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}\partial x=-2x-\frac{\ln|2x+1|}{3}+2\ln|x+1|-\frac{2\ln|x-1|}{3}+c\\
\int\frac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}\partial x=\int\left(\frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{x-1}\right)\partial x=\int\left(\frac{\left(\frac{-2}{3}\right)}{2x+1}+\frac{2}{x+1}+\frac{\left(\frac{-2}{3}\right)}{x-1}\right)\partial x=\\
\frac{-2}{3}\int(2x+1)^{-1}\partial x+2\int(x+1)^{-1}\partial x+\frac{-2}{3}\int(x-1)^{-1}\partial x=\\
\frac{-\cancel{2}}{3}\int(u)^{-1}\frac{\partial u}{\cancel{2}}+2\int(x+1)^{-1}\partial x+\frac{-2}{3}\int(x-1)^{-1}\partial x=\frac{-1}{3}\ln|u|+2\ln|x+1|-\frac{2}{3}\ln|x-1|+c=\\
\frac{-\ln|2x+1|}{3}+2\ln|x+1|-\frac{2\ln|x-1|}{3}+c\\
\frac{\dfrac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x-1}}{(2x+1)(x+1)(x-1)}\rightarrow\\
2x^2-4x-2=A(x+1)(x-1)+B(2x+1)(x-1)+C(2x+1)(x+1)\\
x=-1\rightarrow 2(-1)^2-4(-1)-2=\\
\cancel{A(-1+1)(-1-1)}+B(2(-1)+1)(-1-1)\cancel{+C(2(-1)+1)(-1+1)}\rightarrow\\
\cancel{2}+4\cancel{-2}=B(-2+1)(-2)\rightarrow 4=2B\rightarrow B=2\\
x=1\rightarrow 2\cdot1^2-4\cdot1-2=\cancel{A(1+1)(1-1)+B(2\cdot1+1)(1-1)}+C(2\cdot1+1)(1+1)\rightarrow\\
\cancel{2}-4\cancel{-2}=6C\rightarrow C=\frac{-4}{6}\rightarrow C=\frac{-2}{3}\\
x=\frac{-1}{2}\rightarrow 2\left(\frac{-1}{2}\right)^2-4\left(\frac{-1}{2}\right)-2=\\
A\left(\frac{-1}{2}+1\right)\left(\frac{-1}{2}-1\right)\cancel{+B\left[2\left(\frac{-1}{2}\right)+1\right]\left(\frac{-1}{2}-1\right)+C\left[2\left(\frac{-1}{2}\right)+1\right]\left(\frac{-1}{2}+1\right)}\rightarrow\\
2\left(\frac{1}{4}\right)\cancel{+2-2}=A\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right)\rightarrow\frac{1}{2}=\frac{-3A}{4}\rightarrow A=\frac{1}{2}\left(\frac{4}{-3}\right)\rightarrow A=\frac{-2}{3}\\
\frac{\partial\left(-2x-\frac{1}{3}\ln|2x+1|+2\ln|x+1|-\frac{2}{3}\ln|x-1|+c\right)}{\partial x}=-2-\frac{1}{3}\frac{1}{2x+1}2+2\frac{1}{x+1}-\frac{2}{3}\frac{1}{x-1}+0=\\
-2-\frac{2}{3(2x+1)}+\frac{2}{x+1}-\frac{2}{3(x-1)}=\\
\frac{-18(2x+1)(x+1)(x-1)-6(x+1)(x-1)+18(2x+1)(x-1)-6(2x+1)(x+1)}{9(2x+1)(x+1)(x-1)}=\\
\frac{-6[3(2x+1)(x+1)(x-1)+(x+1)(x-1)-3(2x+1)(x-1)+(2x+1)(x+1)]}{9(2x+1)(x+1)(x-1)}=\\
\frac{-2[\cancel{6}2x^3\cancel{+3x^2}\cancel{-6x}\cancel{-3}\cancel{+x^2}\cancel{-1}\cancel{-6x^2}\cancel{+3x}\cancel{+3}\cancel{+2x^2}\cancel{+3x}\cancel{+1}]}{\cancel{3}(2x+1)(x+1)(x-1)}=\frac{-4x^3}{2x^3+x^2-2x-1}
\end{gathered}\tag{138}$$

Integração por Frações Parciais – Aula 4

Exercício I

$$\int \frac{3x\,\partial x}{(2x+1)(x-1)^2}=\int\left(\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}\right)\partial x=\int\left(\frac{\left(\frac{-2}{3}\right)}{2x+1}+\frac{\left(\frac{1}{3}\right)}{x-1}+\frac{1}{(x-1)^2}\right)\partial x=$$

$$\frac{-2}{3}\int\frac{\partial x}{2x+1}+\frac{1}{3}\int\frac{\partial x}{x-1}+\int\frac{\partial x}{(x-1)^2}=\frac{-\cancel{2}}{3}\int u^{-1}\frac{\partial u}{\cancel{2}}+\frac{1}{3}\int(x-1)^{-1}\partial x+\int(x-1)^{-2}\partial x=$$

$$\frac{-1}{3}\ln|u|+\frac{1}{3}\ln|x-1|+\frac{(x-1)^{-1}}{(-1)}+c=\frac{-\ln|2x+1|}{3}+\frac{\ln|x-1|}{3}-\frac{1}{x-1}+c$$

$$u=2x+1\rightarrow\frac{\partial u}{2}=\partial x$$

$$\frac{\frac{3x}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}}{(2x+1)(x-1)^2}\rightarrow 3x=A(x-1)^2+B(2x+1)(x-1)+C(2x+1)$$

$$x=1\rightarrow 3\cdot1=\cancel{A(1-1)^2+B(2\cdot1+1)(1-1)}+C(2\cdot1+1)\rightarrow 3=3C\rightarrow C=1$$

$$x=\frac{-1}{2}\rightarrow 3\left(\frac{-1}{2}\right)=A\left(\frac{-1}{2}-1\right)^2\cancel{+B\left[2\left(\frac{-1}{2}\right)+1\right]\left(\frac{-1}{2}-1\right)+C\left[2\left(\frac{-1}{2}\right)+1\right]}\rightarrow$$

$$\frac{-3}{2}=A\left(\frac{-3}{2}\right)^2\rightarrow\frac{-3}{2}=\frac{9A}{4}\rightarrow A=\frac{-3}{2}\frac{4}{9}\rightarrow A=\frac{-2}{3} \qquad (139)$$

$$x=0\rightarrow 3\cdot0=A(0-1)^2+B(2\cdot0+1)(0-1)+C(2\cdot0+1)\rightarrow 0=A-B+C\rightarrow B=A+C=\frac{1}{3}$$

$$\frac{\partial\left(\frac{-\ln|2x+1|}{3}+\frac{\ln|x-1|}{3}-\frac{1}{x-1}+c\right)}{\partial x}=\frac{\partial\left(\frac{-1}{3}\ln|2x+1|+\frac{1}{3}\ln|x-1|-(x-1)^{-1}+c\right)}{\partial x}=$$

$$\frac{-1}{3}\frac{1}{2x+1}2+\frac{1}{3}\frac{1}{x-1}+(x-1)^{-2}+0=\frac{-2}{3(2x+1)}+\frac{1}{3(x-1)}+\frac{1}{(x-1)^2}=$$

$$\frac{-6(x-1)^3+3(2x+1)(x-1)^2+9(2x+1)(x-1)}{9(2x+1)(x-1)^3}=$$

$$\frac{-\cancel{3(x-1)}\left[2(x-1)^2-(2x+1)(x-1)-3(2x+1)\right]}{\cancel{9}3(2x+1)(x-1)^{\cancel{3}2}}=$$

$$\frac{-\left[2(x^2-2x+1)-(2x^2-x-1)-3(2x+1)\right]}{3(2x+1)(x-1)^2}=\frac{-(\cancel{2x^2}-4x+\cancel{2}\cancel{-2x^2}+x\cancel{+1}-6x\cancel{-3})}{3(2x+1)(x-1)^2}=$$

$$\frac{\cancel{9}3x}{\cancel{3}(2x+1)(x-1)^2}=\frac{3x}{(2x+1)(x-1)^2}$$

Exercício II

$$\int \frac{(x+2)\partial x}{x^3(x-1)} = \int \frac{(x+2)\partial x}{(x+0)^3(x-1)} = \int \left(\frac{A}{x+0} + \frac{B}{(x+0)^2} + \frac{C}{(x+0)^3} + \frac{D}{x-1}\right)\partial x =$$
$$\int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}\right)\partial x = A\int x^{-1}\partial x + B\int x^{-2}\partial x + C\int x^{-3}\partial x + D\int (x-1)^{-1}\partial x =$$
$$A\ln|x| + B\frac{x^{-1}}{(-1)} + C\frac{x^{-2}}{(-2)} + D\ln|x-1| + c = A\ln|x| - B\frac{1}{x} - C\frac{1}{2x^2} + D\ln|x-1| + c =$$
$$-3\ln|x| + 3\frac{1}{x} + \cancel{2}\frac{1}{\cancel{2}x^2} + 3\ln|x-1| + c = -3\ln|x| + \frac{3}{x} + \frac{1}{x^2} + 3\ln|x-1| + c$$

$$\frac{\frac{x+2}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}}{x^3(x-1)} \to x+2 = Ax^2(x-1) + Bx(x-1) + C(x-1) + Dx^3$$
$$x=1 \to 1+2 = \cancel{A\cdot 1^2(1-1) + B\cdot 1(1-1) + C(1-1)} + D\cdot 1^3 \to D=3$$
$$x=0 \to 0+2 = \cancel{A\cdot 0^2(0-1) + B\cdot 0(0-1)} + C(0-1) + \cancel{D\cdot 0^3} \to 2 = -C \to C=-2$$
$$x+2 = Ax^2(x-1) + Bx(x-1) - 2(x-1) + 3x^3 \to x+2 = Ax^2(x-1) + Bx(x-1) - 2x + 2 + 3x^3 \to \tag{140}$$
$$x\cancel{+2} + 2x \cancel{-2} - 3x^3 = Ax^2(x-1) + Bx(x-1) \to -3x^3 + 3x = Ax^2(x-1) + Bx(x-1) \to$$
$$-3x\left(x^2-1\right) = Ax^2(x-1) + Bx(x-1) \to -3x(x+1)\cancel{(x-1)} = Ax^2\cancel{(x-1)} + Bx\cancel{(x-1)} \to$$
$$-3x(x+1) = x(Ax+B) \to \frac{-3\cancel{x}(x+1)}{\cancel{x}} = Ax+B \to -3(x+1) = Ax+B$$
$$x=0 \to -3(0+1) = \cancel{A\cdot 0} + B \to B=-3$$
$$x=1 \to -3(1+1) = A\cdot 1 - 3 \to -6 = A-3 \to A = -6+3 \to A=-3$$

$$\frac{\partial\left(-3\ln|x| + \frac{3}{x} + \frac{1}{x^2} + 3\ln|x-1| + c\right)}{\partial x} = \frac{\partial\left(-3\ln|x| + 3x^{-1} + x^{-2} + 3\ln|x-1| + c\right)}{\partial x} =$$
$$-3\frac{1}{x} - 3x^{-2} - 2x^{-3} + 3\frac{1}{x-1} + 0 = \frac{-3}{x} - \frac{3}{x^2} - \frac{2}{x^3} + \frac{3}{x-1} =$$
$$\frac{-3x^2(x-1) - 3x(x-1) - 2(x-1) + 3x^3}{x^3(x-1)} = \frac{\cancel{-3x^3} \cancel{+3x^2} \cancel{-3x^2} + 3x - 2x + 2 \cancel{+3x^3}}{x^3(x-1)} = \frac{x+2}{x^3(x-1)}$$

Integração por Frações Parciais – Aula 5

Exercício I

$$\int \frac{x^2 \partial x}{(x-4)(x^2+1)} = \int \left(\frac{A}{x-4}+\frac{Bx+C}{x^2+1}\right)\partial x = \int \left(\frac{\left(\frac{16}{17}\right)}{x-4}+\frac{\left(\frac{1}{17}x+\frac{4}{17}\right)}{x^2+1}\right)\partial x =$$

$$\frac{16}{17}\int (x-4)^{-1}\partial x + \int \frac{\frac{1}{17}(x+4)}{x^2+1}\partial x = \frac{16}{17}\ln|x-4| + \frac{1}{17}\int \frac{x+4}{x^2+1}\partial x =$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\int\left(\frac{x}{x^2+1}+\frac{4}{x^2+1}\right)\partial x = \frac{16\ln|x-4|}{17}+\frac{1}{17}\int \frac{x}{x^2+1}\partial x + \frac{1}{17}\int \frac{4}{x^2+1}\partial x =$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\int (x^2+1)^{-1}x\,\partial x + \frac{4}{17}\int (x^2+1)^{-1}\partial x =$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\frac{\ln|x^2+1|}{2}+\frac{4}{17}arc\,tg(x)+c = \frac{16\ln|x-4|}{17}+\frac{\ln|x^2+1|}{34}+\frac{4\,arc\,tg(x)}{17}+c$$

$$\int (x^2+1)^{-1}x\,\partial x = \int u^{-1}\frac{\partial u}{2} = \frac{1}{2}\int u^{-1}\partial u = \frac{1}{2}\ln|u|+c = \frac{\ln|x^2+1|}{2}+c$$

$$u = x^2+1 \rightarrow \frac{\partial u}{2} = x\,\partial x$$

$$\int (x^2+1)^{-1}\partial x = \int \frac{1}{1+x^2}\partial x = arc\,tg(x)+c$$

(141)

$$\frac{\frac{x^2}{(x-4)(x^2+1)} = \frac{A}{x-4}+\frac{Bx+C}{x^2+1}}{(x-4)(x^2+1)} \rightarrow x^2 = A(x^2+1)+(Bx+C)(x-4)$$

$$x=4 \rightarrow 4^2 = A(4^2+1)+\cancel{(4B+C)(4-4)} \rightarrow 16 = 17A \rightarrow A = \frac{16}{17}$$

$$x^2 = \frac{16}{17}(x^2+1)+(Bx+C)(x-4) \rightarrow x^2 - \frac{16}{17}(x^2+1) = (Bx+C)(x-4) \rightarrow$$

$$\frac{\left(\frac{17x^2-16(x^2+1)}{17}\right)}{x-4} = Bx+C \rightarrow \frac{17x^2-16(x^2+1)}{17(x-4)} = Bx+C$$

$$x=0 \rightarrow \frac{\cancel{17\cdot 0^2}\cancel{-16}\,4(\cancel{0^2}+1)}{17\cancel{(0-4)}} = B\cdot 0 + C \rightarrow C = \frac{4}{17}$$

$$\frac{17x^2-16(x^2+1)}{17(x-4)} = Bx + \frac{4}{17} \rightarrow \frac{17x^2-16(x^2+1)}{17(x-4)} - \frac{4}{17} = Bx \rightarrow$$

$$\rightarrow \frac{17x^2-16(x^2+1)-4(x-4)}{17(x-4)} = Bx \rightarrow B = \frac{17x^2-16(x^2+1)-4(x-4)}{17x(x-4)} \rightarrow$$

$$\frac{17\cdot 1^2-16(1^2+1)-4(1-4)}{17\cdot 1(1-4)} = \frac{17-32+12}{(-51)} = \frac{-3}{(-51)} = \frac{1}{17}$$

Exercício II

$$\int \frac{3x^4+4x^3+16x^2+20x+9}{(x+2)(x^2+3)^2}\partial x=\int\left(\frac{A}{x+2}+\frac{Bx+C}{x^2+3}+\frac{Dx+E}{(x^2+3)^2}\right)\partial x=$$

$$\int\left(\frac{1}{x+2}+\frac{2x+0}{x^2+3}+\frac{4x+0}{(x^2+3)^2}\right)\partial x=\int (x+2)^{-1}\partial x+2\int (x^2+3)^{-1}x\,\partial x+4\int (x^2+3)^{-2}x\,\partial x=$$

$$\int (x+2)^{-1}\partial x+\cancel{2}\int u^{-1}\frac{\partial u}{\cancel{2}}+\cancel{4}2\int u^{-2}\frac{\partial u}{\cancel{2}}=\ln|x+2|+\ln|u|+2\frac{u^{-1}}{(-1)}+c=$$

$$\ln|x+2|+\ln|u|-\frac{2}{u}+c=\ln|x+2|+\ln|x^2+3|-\frac{2}{x^2+3}+c$$

$$u=x^2+3\rightarrow\frac{\partial u}{2}=x\,\partial x$$

$$\frac{\dfrac{3x^4+4x^3+16x^2+20x+9}{(x+2)(x^2+3)^2}=\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+3}+\dfrac{Dx+E}{(x^2+3)^2}}{(x+2)(x^2+3)^2}\rightarrow$$

$$3x^4+4x^3+16x^2+20x+9=A(x^2+3)^2+(Bx+C)(x+2)(x^2+3)+(Dx+E)(x+2)$$

$$x=-2\rightarrow 3(-2)^4+4(-2)^3+16(-2)^2+20(-2)+9=$$

$$A((-2)^2+3)^2\cancel{+(B(-2)+C)(-2+2)((-2)^2+3)+(D(-2)+E)(-2+2)}\rightarrow$$

$$48-32+64-40+9=A(4+3)^2\rightarrow 49=49A\rightarrow A=\frac{49}{49}=1 \tag{142}$$

$$\frac{-(x^2+3)^2+3x^4+4x^3+16x^2+20x+9}{x+2}=(Bx+C)(x^2+3)+Dx+E$$

$$x=0\rightarrow\frac{-(0^2+3)^2\cancel{+3\cdot 0^4+4\cdot 0^3+16\cdot 0^2+20\cdot 0}+9}{0+2}=(\cancel{B\cdot}0+C)(0^2+3)\cancel{+D\cdot}0+E\rightarrow$$

$$\frac{-9+9}{2}=3C+E\rightarrow 3C+E=0\rightarrow E=-3C=0$$

$$x=-1\rightarrow\frac{-((-1)^2+3)^2+3(-1)^4+4(-1)^3+16(-1)^2+20(-1)+9}{-1+2}=$$

$$(B(-1)+C)((-1)^2+3)+D(-1)-3C\rightarrow\cancel{-16}+3-4\cancel{+16}-20+9=(-B+C)4-D-3C\rightarrow$$

$$-12=-4B+4C-D-3C\rightarrow -12=-4B+C-D\rightarrow D=-4B+C+12=4$$

$$x=1\rightarrow\frac{-(1^2+3)^2+3\cdot 1^4+4\cdot 1^3+16\cdot 1^2+20\cdot 1+9}{1+2}=(B\cdot 1+C)(1^2+3)-4B+C+12-3C\rightarrow$$

$$\frac{\cancel{-16}+3+4\cancel{+16}+20+9}{3}=\cancel{4B}+4C\cancel{-4B}+C+12-3C\rightarrow 12-12=2C\rightarrow C=0$$

$$x=2\rightarrow\frac{-(2^2+3)^2+3\cdot 2^4+4\cdot 2^3+16\cdot 2^2+20\cdot 2+9}{2+2}=(B\cdot 2+0)(2^2+3)+(-4B+0+12)2+0\rightarrow$$

$$\frac{\cancel{-49}+48+32+64\cancel{+40+9}}{4}=14B-8B+24\rightarrow 36-24=6B\rightarrow B=\frac{12}{6}=2$$

Integral por Frações Parciais – Aula 6

Exercício I

$$\int \frac{(11x+17)\partial x}{2x^2+7x-4} = \int \frac{(11x+17)\partial x}{(x+4)(2x-1)} = \int \left(\frac{A}{x+4}+\frac{B}{2x-1}\right)\partial x = \int \left(\frac{3}{x+4}+\frac{5}{2x-1}\right)\partial x$$

$$3\int (x+4)^{-1}\partial x + 5\int (2x-1)^{-1}\partial x = 3\int (x+4)^{-1}\partial x + 5\int u^{-1}\frac{\partial u}{2} =$$

$$3\int (x+4)^{-1}\partial x + \frac{5}{2}\int u^{-1}\partial u = 3\ln|x+4| + \frac{5}{2}\ln|u| + c = 3\ln|x+4| + \frac{5\ln|2x-1|}{2} + c$$

$$u = 2x-1 \rightarrow \frac{\partial u}{2} = \partial x$$

$$2x^2+7x-4=0 \rightarrow 2x^2+7x-4+\left(\frac{7}{2\sqrt{2}}\right)^2-\left(\frac{7}{2\sqrt{2}}\right)^2=0 \rightarrow$$

$$2x^2+7x-4+\left(\frac{7\sqrt{2}}{4}\right)^2-\left(\frac{7\sqrt{2}}{4}\right)^2 \rightarrow 2x^2+7x+\frac{49}{8}-\frac{81}{8}=0 \rightarrow \left(x\sqrt{2}+\frac{7\sqrt{2}}{4}\right)^2=\frac{81}{8} \rightarrow$$

$$x\sqrt{2}=\pm\sqrt{\frac{81}{8}}-\frac{7\sqrt{2}}{4} \rightarrow x=\frac{\left(\frac{\pm 9}{2\sqrt{2}}-\frac{7\sqrt{2}}{4}\right)}{\sqrt{2}}=\frac{\left(\frac{\pm 18-7\sqrt{2}\sqrt{2}}{4\sqrt{2}}\right)}{\sqrt{2}}=\frac{\pm 18-14}{8}$$

$$x=\frac{-18-14}{8}=\frac{-32}{8}=-4 \rightarrow x_1=-4\,;\,x=\frac{18-14}{8}=\frac{4}{8}=\frac{1}{2}=x_2=\frac{1}{2} \qquad (143)$$

$$a(x-x_1)(x-x_2)=2(x+4)\left(x-\frac{1}{2}\right)=(x+4)(2x-1)$$

$$\frac{\dfrac{11x+17}{(x+4)(2x-1)}=\dfrac{A}{x+4}+\dfrac{B}{2x-1}}{(x+4)(2x-1)} \rightarrow 11x+17=A(2x-1)+B(x+4)$$

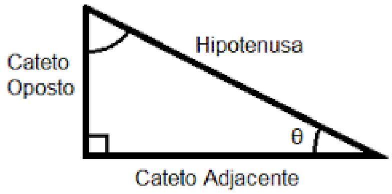
$$x=\frac{1}{2} \rightarrow 11\frac{1}{2}+17=\cancel{A(2\frac{1}{2}-1)}+B(\frac{1}{2}+4) \rightarrow \frac{11+34}{\cancel{2}}=B\left(\frac{1+8}{\cancel{2}}\right) \rightarrow B=\frac{45}{9}=5$$

$$x=-4 \rightarrow 11(-4)+17=A(2(-4)-1)\cancel{+B(-4+4)} \rightarrow -44+17=-9A \rightarrow A=\frac{-27}{-9}=3$$

$$\frac{\partial\left(3\ln|x+4|+\frac{5\ln|2x-1|}{2}+c\right)}{\partial x}=3\frac{1}{x+4}+\frac{5}{\cancel{2}}\frac{1}{2x-1}\cancel{2}+0=\frac{3}{x+4}+\frac{5}{2x-1}=$$

$$\frac{3(2x-1)+5(x+4)}{(x+4)(2x-1)}=\frac{6x-3+5x+20}{(x+4)(2x-1)}=\frac{11x+17}{(x+4)(2x-1)}=\frac{11x+17}{2x^2+7x-4}$$

Integração por Substituição Trigonométrica – Aula 1



$$\begin{array}{lll} 1^o & \sqrt{a^2-x^2} \rightarrow & x=a\cdot sen(t) \\ 2^o & \sqrt{a^2+x^2} \rightarrow & x=a\cdot tg(t) \\ 3^o & \sqrt{x^2-a^2} \rightarrow & x=a\cdot sec(t) \end{array}$$

$$sen^2(t)+\cos^2(t)=1$$
$$\cos^2(t)=\frac{1+\cos(2t)}{2}$$
$$sen(2t)=2\,sen(t)\cos(t)$$
$$sen(t)=x\rightarrow t=arc\,sen(x)$$

t: angulo ; c_o: cateto oposto ; c_a: cateto adjacente ; h: hipotenusa

$$sen(t)=\frac{c_0}{h};\cos(t)=\frac{c_a}{h};tg(t)=\frac{c_o}{c_a}$$
$$h^2=c_o^2+c_a^2$$

Exercício I

$$\begin{gathered}
\int\sqrt{1-x^2}\partial x=\int\sqrt{1-sen^2(t)}\cos(t)\partial t=\int\sqrt{\cos^2(t)}\cos(t)\partial t=\int\cos^2(t)\partial t=\\
\int\frac{1+\cos(2t)}{2}\partial t=\frac{1}{2}\int(1+\cos(2t))\partial t=\frac{1}{2}\int\partial t+\frac{1}{2}\int\cos(2t)\partial t=\\
\frac{1}{2}t+\frac{1}{2}\int\cos(u)\frac{\partial u}{2}=\frac{t}{2}+\frac{1}{4}\int\cos(u)\partial u=\frac{t}{2}+\frac{1}{4}sen(u)+c=\frac{t}{2}+\frac{sen(2t)}{4}+c=\\
\frac{t}{2}+\frac{2\,sen(t)\cos(t)}{4}+c=\frac{t}{2}+\frac{sen(t)\cos(t)}{2}+c=\frac{arc\,sen(x)}{2}+\frac{x\sqrt{1-x^2}}{2}+c\\
x=1\cdot sen(t)=sen(t)\rightarrow\partial x=\cos(t)\partial t;u=2t\rightarrow\frac{\partial u}{2}=\partial t\\
sen(t)=\frac{c_o}{h}=\frac{x}{1}=x;c_0=x;h=1;\cos(t)=\frac{c_a}{h}=\frac{\sqrt{1-x^2}}{1}=\sqrt{1-x^2}\\
h^2=c_o^2+c_a^2\rightarrow 1=x^2+c_a^2\rightarrow c_a=\sqrt{1-x^2}\\
\frac{\partial\left(\frac{arc\,sen(x)}{2}+\frac{x\sqrt{1-x^2}}{2}+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{2}arc\,sen(x)+\frac{1}{2}x(1-x^2)^{\frac{1}{2}}+c\right)}{\partial x}=\\
\frac{1}{2}\frac{1}{\sqrt{1-x^2}}+\frac{1}{2}\left[(1-x^2)^{\frac{1}{2}}+x\frac{1}{\cancel{2}}(1-x^2)^{\frac{-1}{2}}(-\cancel{2}x)\right]+0=\frac{1}{2\sqrt{1-x^2}}+\frac{1}{2}\left(\sqrt{1-x^2}-\frac{x^2}{\sqrt{1-x^2}}\right)=\\
\frac{1}{2\sqrt{1-x^2}}+\frac{1-2x^2}{2\sqrt{1-x^2}}=\frac{2-2x^2}{2\sqrt{1-x^2}}=\frac{\cancel{2}(1-x^2)}{\cancel{2}\sqrt{1-x^2}}=\sqrt{1-x^2}
\end{gathered}\tag{144}$$

Integração por Substituição Trigonométrica – Aula 2

Exercício I

$$\int \sqrt{x^2+36}\,\partial x=\int \sqrt{6^2+x^2}\,\partial x=\int \sqrt{36+[6\,tg(t)]^2}\,6\,sec^2(t)\,\partial t=$$
$$6\int \sqrt{36+36\,tg^2(t)}\,sec^2(t)\,\partial t=6\int \sqrt{36[1+tg^2(t)]}\,sec^2(t)\,\partial t=6\int \sqrt{36\,sec^2(t)}\,sec^2(t)\,\partial t=$$
$$6\int [6\,sec(t)]\,sec^2(t)\,\partial t=36\int sec^2(t)\,sec(t)\,\partial t=36\,\frac{sec(t)\,tg(t)+\ln|sec(t)+tg(t)|}{2}+c=$$
$$18[sec(t)\,tg(t)+\ln|sec(t)+tg(t)|]+c=18\left[\frac{\sqrt{x^2+36}}{6}\,\frac{x}{6}+\ln\left|\frac{\sqrt{x^2+36}}{6}+\frac{x}{6}\right|\right]+c=$$
$$\frac{x\sqrt{x^2+36}}{2}+18\ln\left|\frac{x+\sqrt{x^2+36}}{6}\right|+c$$
$$x=6\,tg(t)\rightarrow\partial x=6\,sec^2(t)\,\partial t$$
$$\int sec^2(t)\,sec(t)\,\partial t=sec(t)\,tg(t)-\int tg^2(t)\,sec(t)\,\partial t=$$
$$sec(t)\,tg(t)-\int \left[sec^2(t)-1\right]sec(t)\,\partial t=sec(t)\,tg(t)-\int sec^3(t)\,\partial t+\int sec(t)\,\partial t\rightarrow$$
$$2\int sec^3(t)\,\partial t=sec(t)\,tg(t)+\int sec(t)\,\partial t\rightarrow\int sec^3(t)\,\partial t=$$
$$\frac{sec(t)\,tg(t)+\ln|sec(t)+tg(t)|}{2}+c$$
$$u=sec(t)\rightarrow\partial u=sec(t)\,tg(t)\,\partial t$$
$$\partial v=sec^2(t)\,\partial t\rightarrow v=\int \partial v=tg(t)$$

$$x=6\,tg(t)\rightarrow tg(t)=\frac{c_o}{c_a}=\frac{x}{6};\,c_o=x;\,c_a=6$$
$$h^2=c_o^2+c_a^2=x^2+36\rightarrow h=\sqrt{x^2+36} \tag{145}$$
$$sec(t)=\frac{1}{\cos(t)}\rightarrow\cos(t)=\frac{c_a}{h}=\frac{6}{\sqrt{x^2+36}}\rightarrow sec(t)=\frac{1}{\left(\frac{6}{\sqrt{x^2+36}}\right)}=\frac{\sqrt{x^2+36}}{6}$$

$$\frac{\partial\left(\frac{x\sqrt{x^2+36}}{2}+18\ln\left|\frac{x+\sqrt{x^2+36}}{6}\right|+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{2}x(x^2+36)^{\frac{1}{2}}+18\ln\left|\frac{1}{6}x+\frac{1}{6}(x^2+36)^{\frac{1}{2}}\right|+c\right)}{\partial x}=$$
$$\frac{1}{2}\left((x^2+36)^{\frac{1}{2}}+x\frac{1}{\cancel{2}}(x^2+36)^{\frac{-1}{2}}\cancel{2}x\right)+18\frac{1}{\left(\frac{x+\sqrt{x^2+36}}{6}\right)}\left(\frac{1}{6}+\frac{1}{6}\frac{1}{\cancel{2}}(x^2+36)^{\frac{-1}{2}}\cancel{2}x\right)+0=$$
$$\frac{1}{2}\left(\sqrt{x^2+36}+\frac{x^2}{\sqrt{x^2+36}}\right)+\frac{108}{x+\sqrt{x^2+36}}\left(\frac{1}{6}+\frac{x}{6\sqrt{x^2+36}}\right)=$$
$$\frac{1}{2}\,\frac{x^2+36+x^2}{\sqrt{x^2+36}}+\frac{108}{\cancel{x+\sqrt{x^2+36}}}\,\frac{\cancel{x+\sqrt{x^2+36}}}{6\sqrt{x^2+36}}=\frac{1}{2}\,\frac{2x^2+36}{\sqrt{x^2+36}}+\frac{108}{6\sqrt{x^2+36}}=\frac{1}{\cancel{2}}\,\frac{\cancel{2}(x^2+18)}{\sqrt{x^2+36}}+\frac{18}{\sqrt{x^2+36}}=$$
$$\frac{x^2+36}{\sqrt{x^2+36}}\left(\frac{\sqrt{x^2+36}}{\sqrt{x^2+36}}\right)=\frac{\cancel{(x^2+36)}\sqrt{x^2+36}}{\cancel{x^2+36}}=\sqrt{x^2+36}$$

Integração por Substituição Trigonométrica – Aula 3

Exercício I

$$\int \frac{\partial x}{\sqrt{x^2-16}}=\int \frac{\partial x}{\sqrt{x^2-4^2}}=\int \frac{4\sec(t)tg(t)\partial t}{\sqrt{(4\sec(t))^2-16}}=\int \frac{4\sec(t)tg(t)\partial t}{\sqrt{16\sec^2(t)-16}}=\int \frac{4\sec(t)tg(t)\partial t}{\sqrt{16\left(\sec^2(t)-1\right)}}=$$
$$\int \frac{4\sec(t)tg(t)\partial t}{\sqrt{16tg^2(t)}}=\int \frac{\cancel{4}\sec(t)\cancel{tg(t)}\partial t}{\cancel{4tg(t)}}=\ln|\sec(t)+tg(t)|+c=\ln\left|\frac{x}{4}+\frac{\sqrt{x^2-16}}{4}\right|+c=$$
$$\ln\left|\frac{x+\sqrt{x^2-16}}{4}\right|+c=\ln\left|x+\sqrt{x^2-16}\right|-\ln 4+c=\ln\left|x+\sqrt{x^2-16}\right|+c$$
$$x=4\sec(t)\to\partial x=4\sec(t)tg(t)\partial t$$
$$x=4\sec(t)\to\sec(t)=\frac{x}{4}=\frac{1}{\cos(t)}=\frac{1}{\left(\frac{c_a}{h}\right)}=\frac{h}{c_a};h=x;c_a=4 \tag{146}$$
$$h^2=c_o^2+c_a^2\to x^2=c_o^2+16\to c_o=\sqrt{x^2-16}$$
$$tg(t)=\frac{c_o}{c_a}=\frac{\sqrt{x^2-16}}{4}$$

$$\frac{\partial\left(\ln\left|x+\sqrt{x^2-16}\right|+c\right)}{\partial x}=\frac{\partial\left(\ln\left|x+\left(x^2-16\right)^{\frac{1}{2}}\right|+c\right)}{\partial x}=\frac{1}{x+\sqrt{x^2-16}}\left(1+\frac{1}{\cancel{2}}\left(x^2-16\right)^{\frac{-1}{2}}\cancel{2}x\right)+0=$$
$$\frac{1}{x+\sqrt{x^2-16}}\left(1+\frac{x}{\sqrt{x^2-16}}\right)=\frac{1}{\cancel{x+\sqrt{x^2-16}}}\frac{\cancel{x+\sqrt{x^2-16}}}{\sqrt{x^2-16}}=\frac{1}{\sqrt{x^2-16}}$$

Exercício II

$$\int \frac{\sqrt{x^2-25}}{x}\partial x=\int \frac{\sqrt{x^2-5^2}}{x}\partial x=\int \frac{\sqrt{[5\sec(t)]^2-25}\,\cancel{5\sec(t)}tg(t)\partial t}{\cancel{5\sec(t)}}=$$
$$\int\sqrt{25\sec^2(t)-25}\,tg(t)\partial t=\int\sqrt{25\left(\sec^2(t)-1\right)}\,tg(t)\partial t=\int\sqrt{25tg^2(t)}\,tg(t)\partial t=$$
$$\int[5tg(t)]tg(t)\partial t=5\int tg^2(t)\partial t=5\int\left(\sec^2(t)-1\right)\partial t=5\int\sec^2(t)\partial t-5\int\partial t=$$
$$5tg(t)-5t+c=5\frac{\sqrt{x^2-25}}{5}-5\cdot 5\,arc\sec\left(\frac{x}{5}\right)+c=\sqrt{x^2-25}-25\,arc\sec\left(\frac{x}{5}\right)+c$$
$$x=5\sec(t)\to\partial x=5\sec(t)tg(t)\partial t$$
$$\sec(t)=\frac{x}{5}=\frac{1}{\cos(t)}=\frac{1}{\left(\frac{c_a}{h}\right)}=\frac{h}{c_a};h=x;c_a=5 \tag{147}$$
$$h^2=c_o^2+c_a^2\to x^2=c_o^2+25\to c_o=\sqrt{x^2-25}$$
$$tg(t)=\frac{c_o}{c_a}=\frac{\sqrt{x^2-25}}{5}$$
$$\sec(t)=\frac{x}{5}\to t=5\,arc\sec\left(\frac{x}{5}\right)$$

Integral por Substituição Trigonométrica – [Aula 4]

Exercício I

$$
\begin{gathered}
\int \sqrt{4-x^2}\,\partial x=\int \sqrt{2^2-x^2}\,\partial x=\int \sqrt{4-(2\,sen(t))^2}\,2\cos(t)\,\partial t= \\
\int \sqrt{4-4\,sen^2(t)}\,2\cos(t)\,\partial t=\int \sqrt{4(1-sen^2(t))}\,2\cos(t)\,\partial t= \\
\int \sqrt{4\cos^2(t)}\,2\cos(t)\,\partial t=\int 2\cos(t)\,2\cos(t)\,\partial t=4\int \cos^2(t)\,\partial t= \\
\cancel{4}\,2\int \frac{1+\cos(2t)}{\cancel{2}}\partial t=2\int (1+\cos(2t))\partial t=2\int \partial t+2\int \cos(2t)\partial t=2t+\cancel{2}\int \cos(u)\frac{\partial u}{\cancel{2}}= \\
2t+sen(u)+c=2t+sen(2t)+c=2t+2\,sen(t)\cos(t)+c=2\,arc\,sen\left(\frac{x}{2}\right)+\cancel{2}\,\frac{x}{\cancel{2}}\,\frac{\sqrt{4-x^2}}{2}+c= \\
2\,arc\,sen\left(\frac{x}{2}\right)+\frac{x\sqrt{4-x^2}}{2}+c \\
x=2\,sen(t)\rightarrow \partial x=2\cos(t)\,\partial t \\
u=2t\rightarrow \frac{\partial u}{2}=\partial t \\
sen(t)=\frac{x}{2}=\frac{c_o}{h}\rightarrow t=arc\,sen\left(\frac{x}{2}\right);\,c_o=x;\,h=2\rightarrow 4=x^2+c_a\rightarrow c_a=\sqrt{4-x^2} \\
\cos(t)=\frac{c_a}{h}=\frac{\sqrt{4-x^2}}{2}
\end{gathered}
\tag{148}
$$

$$
\begin{gathered}
\frac{\partial\left(2\,arc\,sen\left(\frac{x}{2}\right)+\frac{x\sqrt{4-x^2}}{2}+c\right)}{\partial x}=\frac{\partial\left(2\,arc\,sen\left(\frac{1}{2}x\right)+\frac{1}{2}x(4-x^2)^{\frac{1}{2}}+c\right)}{\partial x}= \\
\cancel{2}\,\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,\frac{1}{\cancel{2}}+\frac{1}{2}\left((4-x^2)^{\frac{1}{2}}+x\,\frac{1}{\cancel{2}}(4-x^2)^{\frac{-1}{2}}(-\cancel{2}x)\right)+0=\frac{1}{\sqrt{1-\frac{x^2}{4}}}+\frac{1}{2}\left(\sqrt{4-x^2}-\frac{x^2}{\sqrt{4-x^2}}\right)= \\
\frac{1}{\left(\frac{\sqrt{4-x^2}}{2}\right)}+\frac{1}{2}\left(\frac{4-x^2-x^2}{\sqrt{4-x^2}}\right)=\frac{2}{\sqrt{4-x^2}}+\frac{4-2x^2}{2\sqrt{4-x^2}}=\frac{2}{\sqrt{4-x^2}}+\frac{\cancel{2}(2-x^2)}{\cancel{2}\sqrt{4-x^2}}=\frac{4-x^2}{\sqrt{4-x^2}}=\sqrt{4-x^2}
\end{gathered}
$$

Integral Imprópria – [Aula]

Exercício I

$$
\begin{gathered}
f(x)=\frac{1}{x^3}\rightarrow \partial f(x)=\frac{\partial x}{x^3} \\
\lim_{t\rightarrow\infty}\int_1^t \partial f(x)=\lim_{t\rightarrow\infty}\int_1^t \frac{\partial x}{x^3}=\lim_{t\rightarrow\infty}\int_1^t x^{-3}\,\partial x=\lim_{t\rightarrow\infty}\left[\frac{x^{-2}}{(-2)}\right]_1^t=\lim_{t\rightarrow\infty}\left[\frac{-1}{2x^2}\right]_1^t=\frac{-1}{2}\lim_{t\rightarrow\infty}\left[\frac{1}{x^2}\right]_1^t= \\
\frac{-1}{2}\lim_{t\rightarrow\infty}\left[\frac{1}{t^2}-\frac{1}{1^2}\right]=\frac{-1}{2}\lim_{t\rightarrow\infty}\left[\frac{1}{t^2}-1\right]=\frac{-1}{2}\left(\frac{1}{\infty^2}-1\right)=\frac{-1}{2}(0^+-1)=\frac{1}{2} \\
\int_1^\infty \partial f(x)=\frac{1}{2}
\end{gathered}
\tag{149}
$$

Exercício II

$$
\begin{gathered}
f(x)=\frac{1}{x} \rightarrow \partial f(x)=\frac{\partial x}{x} \\
\lim_{t\rightarrow\infty}\int_{1}^{t}\partial f(x)=\lim_{t\rightarrow\infty}\int_{1}^{t}\frac{\partial x}{x}=\lim_{t\rightarrow\infty}[\ln|x|]_{1}^{t}=\lim_{t\rightarrow\infty}[\ln|t|-\ln|1|]=\ln|\infty|-\ln|1|=\infty-0=\infty \\
\int_{1}^{\infty}\partial f(x)=\infty \\
e^{x}=\infty\rightarrow e^{x}=e^{\infty}\rightarrow x=\infty\rightarrow \ln|\infty|=\infty \\
e^{x}=1\rightarrow e^{x}=e^{0}\rightarrow x=0\rightarrow \ln|1|=0
\end{gathered} \tag{150}
$$

Exercício III

$$
\begin{gathered}
f(x)=\frac{1}{x} \rightarrow \partial f(x)=\frac{\partial x}{x} \\
\lim_{t\rightarrow 0}\int_{t}^{1}\partial f(x)=\lim_{t\rightarrow 0}\int_{t}^{1}\frac{\partial x}{x}=\lim_{t\rightarrow 0}[\ln|x|]_{t}^{1}=\lim_{t\rightarrow 0}[\ln|1|-\ln|t|]=\ln|1|-\ln|0|=0-(-\infty)=\infty \\
\int_{0}^{1}\partial f(x)=\infty \\
e^{x}=1\rightarrow e^{x}=e^{0}\rightarrow x=0\rightarrow \ln|1|=0 \\
e^{x}=0\rightarrow e^{x}=\frac{1}{e^{\infty}}=e^{-\infty}\rightarrow x=-\infty\rightarrow \ln|0|=-\infty
\end{gathered} \tag{151}
$$