Integral Indefinida – <u>Aula 1</u>

01. $\int \partial x \quad = \quad x + c$

02. $\int x^p \partial x \quad = \quad \dfrac{x^{p+1}}{p+1} + c \quad \rightarrow \quad p \neq -1$

03. $\int e^x \partial x \quad = \quad e^x + c$

04. $\int \dfrac{\partial x}{x} \quad = \quad \ln|x| + c$

05. $\int u^p \partial u \quad = \quad \dfrac{u^{p+1}}{p+1} + c \quad \rightarrow \quad p \neq -1$

06. $\int e^u \partial u \quad = \quad e^u + c$

07. $\int \dfrac{\partial u}{u} \quad = \quad \ln|u| + c$

08. $\int a^u \partial u \quad = \quad \dfrac{a^u}{\ln|a|} + c$

Exercício I

$$\int \partial x = x + c$$
$$\int x^3 \partial x = \frac{x^4}{4} + c \tag{1}$$

Exercício II

$$\int 3x^4 \partial x = 3 \int x^4 \partial x = 3 \frac{x^5}{5} + c = \frac{3x^5}{5} + c \tag{2}$$

Exercício III

$$\int (4x^5 + 7) \partial x = \int 4x^5 \partial x + \int 7 \partial x = 4 \int x^5 \partial x + 7 \int \partial x = 4 \frac{x^6}{6} + 7x + c = \frac{2x^6}{3} + 7x + c \tag{3}$$

Exercício IV

$$\int 3 \partial x = 3 \int \partial x = 3x + c \tag{4}$$

Exercício V

$$\int 5x^7 \partial x = 5 \int x^7 \partial x = \frac{5x^8}{8} + c \tag{5}$$

Exercício VI

$$\int \left(5+3x^2-7x^3\right)\partial x=5\int \partial x+3\int x^2\partial x-7\int x^3\partial x=5x+\frac{3x^3}{3}-\frac{7x^4}{4}+c=$$
$$\frac{-7x^4}{4}+x^3+5x+c \tag{6}$$

Integral Indefinida – Aula 2

Exercício I

$$\int \frac{\partial x}{x^4}=\int x^{-4}\partial x=\frac{x^{-3}}{-3}+c=\frac{-1}{3x^3}+c \tag{7}$$

Exercício II

$$\int \sqrt{x^3}\partial x=\int x^{\frac{3}{2}}\partial x=\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+c=\sqrt{x^5}\cdot\frac{2}{5}=\frac{2\sqrt{x^5}}{5}+c \tag{8}$$

Exercício II

$$\int \left(7\sqrt[5]{x^2}+\frac{3}{x^3}\right)\partial x=\int \left(7x^{\frac{2}{5}}+3x^{-3}\right)\partial x=7\int x^{\frac{2}{5}}\partial x+3\int x^{-3}\partial x=7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)}+3\frac{x^{-2}}{-2}+c=$$
$$5\sqrt[5]{x^7}-\frac{3}{2x^2}+c \tag{9}$$

Integral indefinida – Aula 3

Exercício I

$$\int \left(3x^{-4}-3x+4\right)\partial x=3\int x^{-4}\partial x-3\int x\partial x+4\int \partial x=3\frac{x^{-3}}{(-3)}-3\frac{x^2}{2}+4x+c=$$
$$\frac{-1}{x^3}-\frac{3x^2}{2}+4x+c \tag{10}$$

Exercício II

$$\int \frac{2}{3}\sqrt{x}\partial x=\frac{2}{3}\int x^{\frac{1}{2}}\partial x=\frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{4\sqrt{x^3}}{9}+c \tag{11}$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}\,x^3}{\sqrt[3]{x^2}}\,\partial x = \int \frac{x^{\frac{1}{2}}x^3}{x^{\frac{2}{3}}}\,\partial x = \int x^{\frac{1}{2}+3-\frac{2}{3}}\,\partial x = \int x^{\frac{3+18-4}{6}}\,\partial x = \int x^{\frac{17}{6}}\,\partial x = \frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)}+c = \frac{6\sqrt[6]{x^{23}}}{23}+c \tag{12}$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3} = \int x^{-3}\,\partial x = \frac{x^{-2}}{-2}+c = \frac{-1}{2x^2}+c \tag{13}$$

Exercício II

$$\int \frac{\partial x}{x} = \ln x + c \tag{14}$$

Exercício III

$$\int \sqrt{2x+1}\,\partial x = \int \sqrt{u}\,\frac{\partial u}{2} = \frac{1}{2}\int u^{\frac{1}{2}}\,\partial u = \frac{1}{2}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c = \frac{1}{2}\frac{2}{3}\sqrt{u^3}+c = \frac{\sqrt{u^3}}{3}+c = \frac{\sqrt{(2x+1)^3}}{3}+c$$

$$u = 2x+1 \rightarrow \frac{\partial u}{2} = \partial x \tag{15}$$

$$\frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3}+c\right)}{\partial x} = \frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+c\right)}{\partial x} = \frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1}\cdot 2+0 = (2x+1)^{\frac{1}{2}} = \sqrt{2x+1}$$

Exercício IV

$$\int \frac{5x^3+2x+3}{x}\,\partial x = \int \left(\frac{5x^3}{x}+\frac{2x}{x}+\frac{3}{x}\right)\partial x = \int \left(5x^2+2+\frac{3}{x}\right)\partial x =$$

$$5\int x^2\,\partial x + 2\int \partial x + 3\int \frac{\partial x}{x} = 5\frac{x^3}{3}+2x+3\ln|x|+c = \frac{5x^3}{3}+2x+3\ln|x|+c \tag{16}$$

$$\frac{\partial\left(\frac{5x^3}{3}+2x+3\ln|x|+c\right)}{\partial x} = \frac{5}{3}3x^2+2+3\frac{1}{x}+0 = 5x^2+2+\frac{3}{x} = \frac{5x^3+2x+3}{x}$$

Exercício V

$$\int \left(2x^4+3+5e^x+\frac{7}{x}\right)\partial x=2\int x^4\,\partial x+3\int \partial x+5\int e^x\,\partial x+7\int \frac{\partial x}{x}=$$

$$2\frac{x^5}{5}+3x+5e^x+7\ln|x|+c=\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c$$

(17)

$$\frac{\partial\left(\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c\right)}{\partial x}=\frac{2}{5}5x^4+3+5e^x+7\frac{1}{x}+0=2x^4+3+5e^x+\frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int \frac{5t^2+7}{\sqrt[3]{t^4}}\,\partial t=\int \frac{5t^2+7}{t^{\frac{4}{3}}}\,\partial t=\int t^{\frac{-4}{3}}\left(5t^2+7\right)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\,\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\,\partial t=$$

$$5\int t^{\frac{2}{3}}\,\partial t+7\int t^{\frac{-4}{3}}\,\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c$$

(18)

$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$

$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int \frac{\partial x}{\sqrt{x}}=\int \frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\,\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c$$

(19)

Exercício II

$$\int \left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int \frac{\partial x}{x}=3e^x+2\ln|x|+c$$

(20)

Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \int u^3 \frac{\partial u}{4} = \frac{1}{4} \int u^3 \partial u = \frac{1}{4} \frac{u^4}{4} + c = \frac{u^4}{16} + c = \frac{(2x^2+1)^4}{16} + c =$$

$$\frac{(2x^2+1)^4}{2^4} + c = \left(\frac{2x^2+1}{2}\right)^4 + c = \left(x^2+\frac{1}{2}\right)^4 + c$$

$$u = 2x^2+1 \rightarrow \frac{\partial u}{4} = x \partial x$$

(21)

$$\frac{\partial \left[\left(x^2+\frac{1}{2}\right)^4 + c\right]}{\partial x} = 4\left(x^2+\frac{1}{2}\right)^3 \cdot 2x + 0 = 8x\left(x^2+\frac{1}{2}\right)^3 = 8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2 =$$

$$\left(8x^3+4x\right)\left(x^4+x^2+\frac{1}{4}\right) = 8x^7+8x^5+2x^3+4x^5+4x^3+x = 8x^7+12x^5+6x^3+x$$

$$(2x^2+1)^3 x = (2x^2+1)^2(2x^2+1)x = (4x^4+4x^2+1)(2x^3+x) = 8x^7+4x^5+8x^5+4x^3+2x^3+x =$$

$$8x^7+12x^5+6x^3+x$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x \partial x = 3 \int e^x \partial x = 3e^x + c$$

$$\int e^{x^2+1} x \partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2} \int e^u \partial u = \frac{1}{2} e^u + c = \frac{e^{x^2+1}}{2} + c$$

$$u = x^2+1 \rightarrow \frac{\partial u}{2} = x \partial x$$

(22)

$$\frac{\partial \left(\frac{e^{x^2+1}}{2} + c\right)}{\partial x} = \frac{1}{2} e^{x^2+1} 2x + 0 = e^{x^2+1} x$$

Exercício II

$$\int e^{x^4+1} x^3 \partial x = \int e^u \frac{\partial u}{4} = \frac{1}{4} \int e^u \partial u = \frac{1}{4} e^u + c = \frac{e^{x^4+1}}{4} + c$$

$$u = x^4+1 \rightarrow \frac{\partial u}{4} = x^3 \partial x$$

(23)

$$\frac{\partial \left(\frac{e^{x^4+1}}{4} + c\right)}{\partial x} = \frac{1}{4} e^{x^4+1} 4x^3 + 0 = e^{x^4+1} x^3$$

Exercício III

$$\int \frac{x}{\left(2x^2-1\right)^3}\,\partial x = \int \left(2x^2-1\right)^{-3} x\,\partial x = \int u^{-3}\frac{\partial u}{4} = \frac{1}{4}\int u^{-3}\,\partial u = \frac{1}{4}\frac{u^{-2}}{(-2)}+c = \frac{-1}{8u^2}+c =$$

$$\frac{-1}{8\left(2x^2-1\right)^2}+c$$

$$u=2x^2-1 \rightarrow \frac{\partial u}{4}=x\,\partial x$$

(24)

$$\frac{\partial\left(\frac{-1}{8\left(2x^2-1\right)^2}+c\right)}{\partial x} = \frac{\partial\left(\frac{-\left(2x^2-1\right)^{-2}}{8}+c\right)}{\partial x} = \frac{-1}{8}(-2)\left(2x^2-1\right)^{-3}4x+0 = \left(2x^2-1\right)^{-3}x =$$

$$\frac{x}{\left(2x^2-1\right)^3}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\,\partial x = \int \left(2x^2-1\right)^{-1} x\,\partial x = \int u^{-1}\frac{\partial u}{4} = \frac{1}{4}\int u^{-1}\,\partial u = \frac{1}{4}\ln|u|+c = \frac{\ln\left|2x^2-1\right|}{4}+c$$

$$u=2x^2-1 \rightarrow \frac{\partial u}{4}=x\,\partial x$$

(25)

$$\frac{\partial\left(\frac{\ln\left|2x^2-1\right|}{4}+c\right)}{\partial x} = \frac{1}{4}\frac{1}{2x^2-1}4x+0 = \frac{x}{2x^2-1}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$\int x^2 \sqrt{1+x}\,\partial x \rightarrow \int (u-1)^2 \sqrt{u}\,\partial u = \int (u-1)^2 u^{\frac{1}{2}}\,\partial u = \int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u =$$

$$\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u = \int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u = \int u^{\frac{5}{2}}\partial u - 2\int u^{\frac{3}{2}}\partial u + \int u^{\frac{1}{2}}\partial u =$$

$$\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)}-2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c = \frac{2\sqrt{u^7}}{7}-\frac{4\sqrt{u^5}}{5}+\frac{2\sqrt{u^3}}{3}+c = \frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c$$

$$u = 1+x \rightarrow x = u-1 \rightarrow \partial u = \partial x$$

(26)

$$\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\right)}{\partial x} = \frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{3}{2}}}{3}+c\right)}{\partial x} =$$

$$\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}}-\frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}}+\frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}}+0 = (1+x)^{\frac{5}{2}}-2(1+x)^{\frac{3}{2}}+(1+x)^{\frac{1}{2}} =$$

$$(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}}-2(1+x)^{\frac{3}{2}-\frac{1}{2}}+1\right) = (1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}}-2(1+x)^{\frac{2}{2}}+1\right) =$$

$$(1+x)^{\frac{1}{2}}\left((1+x)^2-2(1+x)+1\right) = (1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1) = (1+x)^{\frac{1}{2}}x^2 = x^2\sqrt{1+x}$$

Exercício II

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u^2-1)^2 u\,2u\,\partial u = 2\int (u^2-1)^2 u^2\,\partial u = 2\int (u^4-2u^2+1)u^2\,\partial u =$$

$$2\int \left(u^6-2u^4+u^2\right)\partial u = 2\int u^6\partial u - 4\int u^4\partial u + 2\int u^2\partial u = 2\frac{u^7}{7}-4\frac{u^5}{5}+2\frac{u^3}{3}+c =$$

$$\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c$$

$$u = \sqrt{1+x} \rightarrow u^2 = 1+x \rightarrow x = u^2-1 \rightarrow \partial x = 2u\,\partial u$$

(27)

O que é uma Integral Definida – Aula 10

Exercício I

$$\int_1^2 x^3\partial x = \frac{x^4}{4}\Big]_1^2 = \frac{(2)^4}{4}-\frac{(1)^4}{4} = \frac{16}{4}-\frac{1}{4} = \frac{16-1}{4} = \frac{15}{4} = 3{,}75$$

(28)

Integral Definida – <u>Aula 10a</u>

Exercício I

$$\int_0^2 \left(6x^2 - 4x + 5\right)\partial x = 6\int_0^2 x^2\partial x - 4\int_0^2 x\partial x + 5\int_0^2 \partial x = 6\frac{x^3}{3} - 4\frac{x^2}{2} + 5x\bigg]_0^2 = 2x^3 - 2x^2 + 5x\big]_0^2 =$$

$$x\left(2x^2 - 2x + 5\right)\big]_0^2 = \left[(2)\left(2(2)^2 - 2(2) + 5\right)\right] - \left[(0)\left(2(0)^2 - 2(0) + 5\right)\right] = 2(8 - 4 + 5) = 2\cdot 9 = 18 \tag{29}$$

Exercício II

$$\int_{-1}^0 \left(2x - e^x\right)\partial x = 2\int_{-1}^0 x\partial x - \int_{-1}^0 e^x\partial x = 2\frac{x^2}{2} - e^x\bigg]_{-1}^0 = x^2 - e^x\big]_{-1}^0 = \left((0)^2 - e^{(0)}\right) - \left((-1)^2 - e^{(-1)}\right) =$$

$$-1 - \left(1 - \frac{1}{e}\right) = -1 - 1 + \frac{1}{e} = -2 + \frac{1}{e} \tag{30}$$

Integral definida – <u>Aula 11</u>

Exercício I

$$\frac{5\pi}{4}\int_0^2 \frac{r\partial r}{1 + r^2} = \frac{5\pi}{4}\int_0^2 \left(1 + r^2\right)^{-1} r\partial r = \frac{5\pi}{4}\int_0^2 u^{-1}\frac{\partial u}{2} = \frac{5\pi}{4}\frac{1}{2}\int_0^2 u^{-1}\partial u = \frac{5\pi}{8}\int_0^2 u^{-1}\partial u =$$

$$\frac{5\pi}{8}\ln|u|\bigg]_0^2 = \frac{5\pi}{8}\ln\left|1 + r^2\right|\bigg]_0^2 = \left[\frac{5\pi}{8}\ln\left|1 + (2)^2\right|\right] - \left[\frac{5\pi}{8}\ln\left|1 + (0)^2\right|\right] = \frac{5\pi}{8}\ln|5| - \frac{5\pi}{8}\ln|1| =$$

$$\frac{5\pi}{8}\left(\ln|5| - \ln|1|\right) = \frac{5\pi}{8}\left(\ln|5| - 0\right) = \frac{5\pi}{8}\ln|5| \tag{31}$$

$$u = 1 + r^2 \rightarrow \frac{\partial u}{2} = r\partial r$$

$$\ln|1| = x \rightarrow e^x = 1 = e^0 \rightarrow x = 0$$

$$\ln|5| = x \rightarrow e^x = 5$$

Exercício II

$$2\pi\int_0^2 r^2\partial r = 2\pi\frac{r^3}{3}\bigg]_0^2 = \frac{2\pi r^3}{3}\bigg]_0^2 = \left(\frac{2\pi(2)^3}{3}\right) - \left(\frac{2\pi(0)^3}{3}\right) = \frac{16\pi}{3} \tag{32}$$

Exercício II

$$2\pi\int_0^{\sqrt{2}} \left(4r - 2r^3\right)\partial r = 8\pi\int_0^{\sqrt{2}} r\partial r - 4\pi\int_0^{\sqrt{2}} r^3\partial r = 8\pi\frac{r^2}{2} - 4\pi\frac{r^4}{4}\bigg]_0^{\sqrt{2}} = 4\pi r^2 - \pi r^4\big]_0^{\sqrt{2}} =$$

$$\pi r^2\left(4 - r^2\right)\big]_0^{\sqrt{2}} = \left[\pi\left(\sqrt{2}\right)^2\left(4 - \left(\sqrt{2}\right)^2\right)\right] - \left[\pi(0)^2\left(4 - (0)^2\right)\right] = 2\pi(4 - 2) = 4\pi \tag{33}$$

Exercício III

$$\pi\int_{0}^{2}x^2\,\partial x=\pi\frac{x^3}{3}\bigg]_{0}^{2}=\frac{\pi x^3}{3}\bigg]_{0}^{2}=\left(\frac{\pi(2)^3}{3}\right)-\left(\frac{\pi(0)^3}{3}\right)=\frac{8\pi}{3} \tag{34}$$

Exercício IV

$$\frac{\pi}{16}\int_{1}^{4}x^4\,\partial x=\frac{\pi}{16}\frac{x^5}{5}\bigg]_{1}^{4}=\frac{\pi x^5}{80}\bigg]_{1}^{4}=\left(\frac{\pi(4)^5}{80}\right)-\left(\frac{\pi(1)^5}{80}\right)=\frac{4^5\pi}{80}-\frac{\pi}{80}=\frac{1024\pi-\pi}{80}=\frac{1023\pi}{80} \tag{35}$$

Exercício V

$$\pi\int_{1}^{2}\left(x^2\right)^2\partial x=\pi\int_{1}^{2}x^4\,\partial x=\pi\frac{x^5}{5}\bigg]_{1}^{2}=\frac{\pi x^5}{5}\bigg]_{1}^{2}=\left(\frac{\pi(2)^5}{5}\right)-\left(\frac{\pi(1)^5}{5}\right)=\frac{32\pi}{5}-\frac{\pi}{5}=\frac{32\pi-\pi}{5}=\frac{31\pi}{5} \tag{36}$$

Exercício VI

$$\pi\int_{-1}^{2}\left(-x^4-x^2+6x+8\right)\partial x=-\pi\int_{-1}^{2}x^4\,\partial x-\pi\int_{-1}^{2}x^2\,\partial x+6\pi\int_{-1}^{2}x\,\partial x+8\pi\int_{-1}^{2}\partial x=$$

$$-\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\bigg]_{-1}^{2}=\frac{-\pi x^5}{5}-\frac{\pi x^3}{3}+3\pi x^2+8\pi x\bigg]_{-1}^{2}=-\pi x\left(\frac{x^4}{5}+\frac{x^2}{3}-3x-8\right)\bigg]_{-1}^{2}$$

$$\left[-\pi(2)\left(\frac{(2)^4}{5}+\frac{(2)^2}{3}-3(2)-8\right)\right]-\left[-\pi(-1)\left(\frac{(-1)^4}{5}+\frac{(-1)^2}{3}-3(-1)-8\right)\right]= \tag{37}$$

$$-2\pi\left(\frac{16}{5}+\frac{4}{3}-6-8\right)-\pi\left(\frac{1}{5}+\frac{1}{3}+3-8\right)=-2\pi\left(\frac{48+20-210}{15}\right)-\pi\left(\frac{3+5-75}{15}\right)=$$

$$2\pi\left(\frac{142}{15}\right)+\pi\left(\frac{67}{15}\right)=\pi\left(\frac{284}{15}+\frac{67}{15}\right)=\frac{351\pi}{15}=\frac{3^3\cdot13\pi}{3\cdot5}=\frac{3^2\cdot13\pi}{5}=\frac{117\pi}{5}$$

Exercício VII

$$\pi\int_{0}^{8}\left(\sqrt[3]{y}\right)^2\partial y=\pi\int_{0}^{8}\left(y^{\frac{1}{3}}\right)^2\partial y=\pi\int_{0}^{8}y^{\frac{2}{3}}\partial y=\pi\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\bigg]_{0}^{8}=\frac{3\pi\sqrt[3]{y^5}}{5}\bigg]_{0}^{8}=$$

$$\left(\frac{3\pi\sqrt[3]{(8)^5}}{5}\right)-\left(\frac{3\pi\sqrt[3]{(0)^5}}{5}\right)=\frac{3\pi\sqrt[3]{8^3\cdot8^2}}{5}=\frac{3\pi2^3\sqrt[3]{(2^3)^2}}{5}=\frac{3\pi2^3 2^2}{5}=\frac{3\pi2^5}{5}=\frac{96\pi}{5} \tag{38}$$

Integral definida – Aula 12

Exercício I

$$\int_{1}^{2}2x\,\partial x=2\int_{1}^{2}x\,\partial x=2\frac{x^2}{2}\bigg]_{1}^{2}=x^2\big]_{1}^{2}=\left((2)^2\right)-\left((1)^2\right)=4-1=4-1=3 \tag{39}$$

Exercício II

$$\int_1^4 2\sqrt{x}\,\partial x = \int_1^4 2x^{\frac{1}{2}}\partial x = 2\int_1^4 x^{\frac{1}{2}}\partial x = 2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_1^4 = \frac{4\sqrt{x^3}}{3}\Bigg]_1^4 = \left(\frac{4\sqrt{(4)^3}}{3}\right) - \left(\frac{4\sqrt{(1)^3}}{3}\right) =$$

$$\frac{4\sqrt{4^2 2^2}}{3} - \frac{4}{3} = \frac{32}{3} - \frac{4}{3} = \frac{32-4}{3} = \frac{28}{3}$$

(40)

Exercício III

$$\int_1^2 4x^2 \partial x = 4\int_1^2 x^2 \partial x = 4\frac{x^3}{3}\Bigg]_1^2 = \frac{4}{3}x^3\Bigg]_1^2 = \frac{4}{3}\left(2^3 - 1^3\right) = \frac{4}{3}7 = \frac{28}{3}$$

(41)

Integrais definidas e indefinidas – Aula 13

Exercício I

$$\int\left(\frac{3}{x^4} + \frac{2x^2}{3} - 2x + 7\right)\partial x = \int\left(3x^{-4} + \frac{2}{3}x^2 - 2x + 7\right)\partial x =$$

$$3\int x^{-4}\partial x + \frac{2}{3}\int x^2 \partial x - 2\int x\,\partial x + 7\int \partial x = 3\frac{x^{-3}}{-3} + \frac{2}{3}\frac{x^3}{3} - 2\frac{x^2}{2} + 7x + c =$$

$$\frac{-1}{x^3} + \frac{2x^3}{9} - x^2 + 7x + c$$

(42)

$$\frac{\partial\left(\frac{-1}{x^3} + \frac{2x^3}{9} - x^2 + 7x + c\right)}{\partial x} = \frac{\partial\left(-x^{-3} + \frac{2}{9}x^3 - x^2 + 7x + c\right)}{\partial x} = 3x^{-4} + \frac{2}{9}3x^2 - 2x + 7 + 0 =$$

$$\frac{3}{x^4} + \frac{2x^2}{3} - 2x + 7$$

Exercício II

$$\int 5\sqrt[3]{x^2}\,\partial x = \int 5x^{\frac{2}{3}}\partial x = 5\int x^{\frac{2}{3}}\partial x = 5\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} + c = 3\sqrt[3]{x^5} + c$$

(43)

$$\frac{\partial\left(3\sqrt[3]{x^5} + c\right)}{\partial x} = \frac{\partial\left(3x^{\frac{5}{3}} + c\right)}{\partial x} = 3\frac{5}{3}x^{\frac{2}{3}} + 0 = 5\sqrt[3]{x^2}$$

Exercício III

$$\int_2^4 2x^3 \partial x = 2\int_2^4 x^3 \partial x = 2\frac{x^4}{4}\bigg]_2^4 = \frac{1}{2}x^4\bigg]_2^4 = \frac{1}{2}\left(4^4 - 2^4\right) = \frac{1}{2}\left((2\cdot 2)^4 - 2^4\right) = \frac{1}{2}\left(2^4\cdot 2^4 - 2^4\right) =$$
$$\frac{2^4}{2}\left(2^4 - 1\right) = 2^3\left(16 - 1\right) = 8\cdot 15 = 120 \tag{44}$$

Exercício IV

$$\int_1^2 \left(3x^2 - 2x\right)\partial x = 3\int_1^2 x^2\partial x - 2\int_1^2 x\partial x = 3\frac{x^3}{3} - 2\frac{x^2}{2}\bigg]_1^2 = x^3 - x^2\big]_1^2 = x^2(x-1)\big]_1^2 =$$
$$\left[2^2(2-1)\right] - \left[1^2(1-1)\right] = 4 \tag{45}$$

Integral definida pelo método da substituição – U du – <u>Aula 14</u>

Exercício I

$$\int_0^2 \sqrt{2x^2+1}\, x\,\partial x = \int_0^2 \sqrt{u}\,\frac{\partial u}{4} = \frac{1}{4}\int_0^2 u^{\frac{1}{2}}\partial u = \frac{1}{4}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\bigg]_0^2 = \frac{1}{4}\frac{2}{3}\sqrt{u^3}\bigg]_0^2 = \frac{1}{6}\sqrt{u^3}\bigg]_0^2 = \frac{1}{6}\sqrt{\left(2x^2+1\right)^3}\bigg]_0^2 =$$
$$\frac{1}{6}\left[\sqrt{\left(2\cdot 2^2+1\right)^3} - \sqrt{\left(2\cdot 0^2+1\right)^3}\right] = \frac{1}{6}\left(\sqrt{9^3} - \sqrt{1^3}\right) = \frac{1}{6}\left(\sqrt{9^2 3^2} - 1\right) = \frac{1}{6}(27-1) = \frac{1}{6}26 = \frac{13}{3} \tag{46}$$
$$u = 2x^2 + 1 \rightarrow \frac{\partial u}{4} = x\,\partial x$$

Integral Método da Substituição – <u>Aula 15</u>

Exercício I

$$\int \left(x^3-1\right)^4 x^2\,\partial x = \int u^4\,\frac{\partial u}{3} = \frac{1}{3}\int u^4\,\partial u = \frac{1}{3}\frac{u^5}{5} + c = \frac{\left(x^3-1\right)^5}{15} + c \tag{47}$$
$$u = x^3 - 1 \rightarrow \frac{\partial u}{3} = x^2\,\partial x$$

Exercício II

$$\int \frac{x}{\left(x^2-1\right)^3}\,\partial x = \int \left(x^2-1\right)^{-3} x\,\partial x = \int u^{-3}\frac{\partial u}{2} = \frac{1}{2}\int u^{-3}\partial u = \frac{1}{2}\frac{u^{-2}}{(-2)} + c = \frac{-1}{4u^2} + c =$$
$$\frac{-1}{4\left(x^2-1\right)^2} + c \tag{48}$$
$$u = x^2 - 1 \rightarrow \frac{\partial u}{2} = x\,\partial x$$

Exercício III

$$\int \frac{x}{(x^2-1)}\,\partial x = \int (x^2-1)^{-1} x\,\partial x = \int u^{-1}\frac{\partial u}{2} = \frac{1}{2}\int u^{-1}\,\partial u = \frac{1}{2}\ln|u|+c = \frac{\ln|x^2-1|}{2}+c \tag{49}$$

$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x\,\partial x$$

Exercício IV

$$\int e^{x^2-1} x\,\partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2}e^u\,\partial u = \frac{1}{2}e^u+c = \frac{e^{x^2-1}}{2}+c \tag{50}$$

$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x\,\partial x$$

Exercício V

$$\int \sqrt{x^3-4}\, x^2 \,\partial x = \int u^{\frac{1}{2}}\frac{\partial u}{3} = \frac{1}{3}\int u^{\frac{1}{2}}\,\partial u = \frac{1}{3}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c = \frac{1}{3}\frac{2}{3}\sqrt{u^3}+c = \frac{2\sqrt{u^3}}{9}+c = $$

$$\frac{2\sqrt{(x^3-4)^3}}{9}+c \tag{51}$$

$$u = x^3-4 \rightarrow \frac{\partial u}{3} = x^2\,\partial x$$

Exercício VI

$$\int e^{\sqrt{x}}\frac{\partial x}{\sqrt{x}} = \int e^{x^{\frac{1}{2}}}\frac{\partial x}{x^{\frac{1}{2}}} = \int e^{x^{\frac{1}{2}}} x^{\frac{-1}{2}}\,\partial x = \int e^u 2\,\partial u = 2\int e^u\,\partial u = 2e^u+c = 2e^{\sqrt{x}}+c \tag{52}$$

$$u = \sqrt{x} = x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2}x^{\frac{-1}{2}}\,\partial x \rightarrow 2\partial u = x^{\frac{-1}{2}}\,\partial x$$

Exercício VII

$$\int \frac{x\,\partial x}{\sqrt[5]{x^2-1}} = \int \frac{x}{(x^2-1)^{\frac{1}{5}}}\,\partial x = \int (x^2-1)^{\frac{-1}{5}} x\,\partial x = \int u^{\frac{-1}{5}}\frac{\partial u}{2} = \frac{1}{2}\int u^{\frac{-1}{5}}\,\partial u = \frac{1}{2}\frac{u^{\frac{4}{5}}}{\left(\frac{4}{5}\right)}+c = $$

$$\frac{1}{2}\frac{5}{4}\sqrt[5]{u^4}+c = \frac{5\sqrt[5]{u^4}}{8}+c = \frac{5\sqrt[5]{(x^2-1)^4}}{8}+c \tag{53}$$

$$u = x^2-1 \rightarrow \frac{\partial u}{2} = x\,\partial x$$

Exercício VIII

$$\int \frac{e^t \partial t}{e^t+4} = \int \left(e^t+4\right)^{-1} e^t \partial t = \int u^{-1} \partial u = \ln|u|+c = \ln\left|e^t+4\right|+c \tag{54}$$

$$u = e^t+4 \rightarrow \partial u = e^t \partial t$$

Integral de uma Função Exponencial Qualquer – Aula 16

Exercício I

$$\int \sqrt{10^{3x}}\,\partial x = \int 10^{\frac{3x}{2}} \partial x = \int 10^u \frac{2}{3} \partial u = \frac{2}{3}\int 10^u \partial u = \frac{2}{3}\frac{10^u}{\ln|10|}+c = \frac{2\cdot 10^{\frac{3x}{2}}}{3\ln|10|}+c = \frac{2\sqrt{10^{3x}}}{3\ln|10|}+c$$

$$u = \frac{3x}{2} \rightarrow \partial u = \frac{3}{2}\partial x = \frac{2}{3}\partial u = \partial x$$

$$\frac{\partial\left(\frac{2\sqrt{10^{3x}}}{3\ln|10|}+c\right)}{\partial x} = \frac{\partial\left(\frac{2}{3\ln|10|}10^{\frac{3x}{2}}+c\right)}{\partial x} = \frac{2}{3\ln|10|}\frac{3\ln|10|\sqrt{10^{3x}}}{2}+0 = \sqrt{10^{3x}} \tag{55}$$

$$y = 10^{\frac{3x}{2}} \rightarrow \ln|y| = \ln\left|10^{\frac{3x}{2}}\right| = \frac{3x}{2}\ln|10| = \frac{3\ln|10|}{2}x$$

$$\frac{\partial(\ln|y|)}{\partial y} = \frac{\partial\left(\frac{3\ln|10|}{2}x\right)}{\partial x} \rightarrow \frac{1}{y}\partial y = \frac{3\ln|10|}{2}\partial x \rightarrow \partial y = y\frac{3\ln|10|}{2}\partial x \rightarrow \frac{\partial y}{\partial x} = $$

$$10^{\frac{3x}{2}}\frac{3\ln|10|}{2} = \frac{3\ln|10|\sqrt{10^{3x}}}{2}$$

Integral de função marginal – Aula 17

Exercício I

O custo marginal por unidade x é dado pela expressão $\frac{\partial c(x)}{\partial x} = 20-\frac{4x^3}{5}$. Determine a função custo total c(x) da produção, sabendo-se que o custo fixo para x = 0 é de R\$2000,00.

$$\frac{\partial c(x)}{\partial x} = 20-\frac{4x^3}{5} \rightarrow \int \partial c(x) = \int \left(20-\frac{4x^3}{5}\right)\partial x \rightarrow \int \left(20-\frac{4x^3}{5}\right)\partial x = 20\int \partial x - \frac{4}{5}\int x^3 \partial x = $$

$$20x-\frac{4}{5}\frac{x^4}{4}+c = 20x+\frac{x^4}{5}+c$$

$$20\cdot 0 + \frac{0^4}{5}+c = 2000 \rightarrow c = 2000 \tag{56}$$

$$c(x) = 20x+\frac{x^4}{5}+2000$$

Exercício II

O rendimento marginal de um bem em quantidade (x) é dado pela expressão $Rm = 800 - 2x^2$. Ache o rendimento total para x = 6, sabendo que quando x = 0, R = 0.

$$\frac{\partial r(x)}{\partial x} = 800 - 2x^2 \rightarrow \int \partial r(x) = \int \left(800 - 2x^2\right)\partial x \rightarrow \int \left(800 - 2x^2\right)\partial x = 800\int \partial x - 2\int x^2 \partial x$$

$$800x - 2\frac{x^3}{3} + c = 800x - \frac{2x^3}{3} + c$$

$$800\cdot 0 - \frac{2\cdot 0^2}{3} + c = 0 \rightarrow c = 0 \tag{57}$$

$$r(x) = 800x - \frac{2x^3}{3}$$

$$r(6) = 800\cdot 6 - \frac{2\cdot 6^3}{3} = 4800 - \frac{2\cdot 2^3 \cdot 3^3}{3} = 4800 - 2^4\cdot 3^2 = 4800 - 16\cdot 9 = 4800 - 144 = 4656$$

Método da substituição com sen(x) e cos(x) – <u>Aula 18</u>

01. $\int sen(u)\partial u$ $= -\cos(u) + c$
02. $\int \cos(u)\partial u$ $= sen(u) + c$
03. $\int tg(u)\partial u$ $= \ln|sec(u)| + c$
04. $\int cotg(u)\partial u$ $= \ln|sen(u)| + c$
05. $\int sec(u)\partial u$ $= \ln|sec(u) + tg(u)| + c$
06. $\int cossec(u)\partial u$ $= \ln|cossec(u) - cotg(u)| + c$
07. $\int sec^2(u)\partial u$ $= tg(u) + c$
08. $\int cossec^2(u)\partial u$ $= -cotg(u) + c$
09. $\int sec(u)tg(u)\partial u$ $= sec(u) + c$
10. $\int cossec(u)cotg(u)\partial u$ $= -cossec(u) + c$

Exercício I

$$\int sen\left(2x^2 - 1\right)x\,\partial x = \int sen(u)\frac{\partial u}{4} = \frac{1}{4}sen(u)\partial u = \frac{1}{4}(-\cos(u)) + c = \frac{-\cos(u)}{4} + c =$$

$$\frac{-\cos\left(2x^2 - 1\right)}{4} + c$$

$$u = 2x^2 - 1 \rightarrow \frac{\partial u}{4} = x\,\partial x \tag{58}$$

$$\frac{\partial\left(\dfrac{-\cos\left(2x^2 - 1\right)}{4} + c\right)}{\partial x} = \frac{-1}{4}\left[-sen\left(2x^2 - 1\right)\right]4x + 0 = sen\left(2x^2 - 1\right)x$$

Exercício II

$$\int \cos\left(3x^3+4\right)x^2\partial x = \int \cos(u)\frac{\partial u}{9} = \frac{1}{9}\int \cos(u)\partial u = \frac{1}{9}sen(u)+c = \frac{sen\left(3x^3+4\right)}{9}+c$$

$$u=3x^3+4 \rightarrow \frac{\partial u}{9}=x^2\partial x$$

(59)

$$\frac{\partial\left(\frac{sen\left(3x^3+4\right)}{9}+c\right)}{\partial x}=\frac{1}{9}\cos\left(3x^3+4\right)9x^2+0=\cos\left(3x^3+4\right)x^2$$

Exercício III

$$\int sen(\sqrt{x})\frac{\partial x}{\sqrt{x}} = \int sen\left(x^{\frac{1}{2}}\right)\frac{\partial x}{x^{\frac{1}{2}}} = \int sen\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x = \int sen(u)2\partial u = 2\int sen(u)\partial u =$$

$$2\left(-\cos(u)\right)+c = -2\cos\sqrt{x}+c$$

(60)

$$u=\sqrt{x}=x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2}x^{\frac{-1}{2}}\partial x \rightarrow 2\partial u = x^{\frac{-1}{2}}\partial x$$

Exercício IV

$$\int sen(x)\cos(x)\partial x = \int u\partial u = \frac{u^2}{2}+c = \frac{sen^2(x)}{2}+c$$

$$u=sen(x) \rightarrow \partial u = \cos(x)\partial x$$

(61)

$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x}=\frac{1}{2}2\,sen(x)\cos(x)+0=sen(x)\cos(x)$$

Exercício V

$$\int sen(\cos(x))sen(x)\partial x = \int sen(u)(-\partial u) = -\int sen(u)\partial u = -(-\cos(u))+c =$$

$$\cos(\cos(x))+c$$

$$u=\cos(x) \rightarrow -\partial u = sen(x)\partial x$$

(62)

$$\frac{\partial[\cos(\cos(x))+c]}{\partial x}=-sen(\cos(x))(-sen(x))+0=sen(\cos(x))sen(x)$$

Exercício VI

$$\int \sqrt{sen(\theta)}\cos(\theta)\partial\theta = \int u^{\frac{1}{2}}\partial u = \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c = \frac{2\sqrt{u^3}}{3}+c = \frac{2\sqrt{sen^3(\theta)}}{3}+c$$

(63)

$$u=sen(\theta) \rightarrow \partial u = \cos(\theta)\partial\theta$$

Exercício VI

$$\int \ln|x| \frac{\partial x}{x} = \int \ln|x| x^{-1} \partial x = \int u \partial u = \frac{u^2}{2} + c = \frac{\ln^2|x|}{2} + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x$$

(64)

$$\frac{\partial\left(\frac{\ln^2|x|}{2} + c\right)}{\partial x} = \frac{1}{2} 2\ln|x| \frac{1}{x} + 0 = \frac{\ln|x|}{x}$$

Exercício VII

$$\int \frac{\partial x}{x \ln^2|x|} = \int \left(x \ln^2|x|\right)^{-1} \partial x = \int \ln^{-2}|x| x^{-1} \partial x = \int u^{-2} \partial u = \frac{u^{-1}}{(-1)} = \frac{-1}{u} + c = \frac{-1}{\ln|x|} + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x$$

(65)

$$\frac{\partial\left(\frac{-1}{\ln|x|} + c\right)}{\partial x} = \frac{\partial\left(-\ln^{-1}|x| + c\right)}{\partial x} = \ln^{-2} \frac{1}{x} + 0 = \frac{1}{x \ln^2|x|}$$

Exercício VIII

$$\int \frac{sen(\theta)\partial\theta}{(5-\cos(\theta))^3} = \int (5-\cos(\theta))^{-3} sen(\theta) \partial\theta = \int u^{-3} \partial u = \frac{u^{-2}}{(-2)} + c = \frac{-1}{2u^2} + c =$$

$$\frac{-1}{2(5-\cos(\theta))^2} + c$$

$$u = 5 - \cos(\theta) \rightarrow \partial u = -(-sen(\theta))\partial x \rightarrow \partial u = sen(\theta)\partial\theta$$

(66)

$$\frac{\partial\left(\frac{-1}{2(5-\cos(\theta))^2} + c\right)}{\partial\theta} = \frac{\partial\left(\frac{-1}{2}(5-\cos(\theta))^{-2} + c\right)}{\partial\theta} = \frac{-1}{2}(-2)(5-\cos(\theta))^{-3}(-(-sen(\theta))) =$$

$$(5-\cos(\theta))^{-3} sen(\theta) = \frac{sen(\theta)}{(5-\cos(\theta))^3}$$

Integração de funções trigonométricas – <u>Aula 19</u>

Exercício I

$$\int tg(x)\partial x = \int \frac{sen(x)}{\cos(x)}\partial x = \int \cos^{-1}(x)sen(x)\partial x = \int u^{-1}(-\partial u) = -\int u^{-1}\partial u =$$

$$-\ln|u|+c = \ln|u^{-1}|+c = \ln\left|\frac{1}{u}\right|+c = \ln\left|\frac{1}{\cos(x)}\right|+c = \ln|sec(x)|+c$$

$$u = \cos(x) \rightarrow -\partial u = sen(x)\partial x$$

(67)

$$\frac{\partial(\ln|sec(x)|+c)}{\partial x} = \frac{\partial\left(\ln\left|\frac{1}{\cos(x)}\right|+c\right)}{\partial x} = \frac{\partial(\ln|\cos^{-1}(x)|+c)}{\partial x} = \frac{\partial(-\ln|\cos(x)|+c)}{\partial x} =$$

$$\frac{-1}{\cos(x)}(-sen(x))+0 = \frac{sen(x)}{\cos(x)} = tg(x)$$

Exercício II

$$\int cotg(x)\partial x = \int \frac{\cos(x)}{sen(x)}\partial x = \int sen^{-1}(x)\cos(x)\partial x = \int u^{-1}\partial u = \ln|u|+c = \ln|sen(x)|+c$$

$$u = sen(x) \rightarrow \partial u = \cos(x)\partial x$$

(68)

$$\frac{\partial(\ln|sen(x)|+c)}{\partial x} = \frac{1}{sen(x)}\cos(x)+0 = \frac{\cos(x)}{sen(x)} = cotg(x)$$

Exercício III

$$\int sec(x)\partial x = \int sec(x)\left(\frac{sec(x)+tg(x)}{sec(x)+tg(x)}\right)\partial x = \int \frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}\partial x =$$

$$\int [sec(x)+tg(x)]^{-1}[sec(x)tg(x)+sec^2(x)]\partial x = \int u^{-1}\partial u = \ln|u|+c = \ln|sec(x)+tg(x)|+c$$

$$u = sec(x)+tg(x) \rightarrow \partial u = [sec(x)tg(x)+sec^2(x)]\partial x$$

(69)

$$\frac{\partial(\ln|sec(x)+tg(x)|+c)}{\partial x} = \frac{1}{sec(x)+tg(x)}[sec(x)tg(x)+sec^2(x)]+0 =$$

$$\frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)} = \frac{sec(x)\cancel{[sec(x)+tg(x)]}}{\cancel{sec(x)+tg(x)}} = sec(x)$$

Exercício IV

$$\int cossec(x)\partial x \int cossec(x)\left(\frac{cossec(x)-cotg(x)}{cossec(x)-cotg(x)}\right)\partial x=$$

$$\int \frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}\partial x=$$

$$\int [cossec(x)-cotg(x)]^{-1}[-cossec(x)cotg(x)+cossec^2(x)]\partial x=\int u^{-1}\partial u=$$

$$\ln|u|+c=\ln|cossec(x)-cotg(x)|+c$$

$$u=cossec(x)-cotg(x)\rightarrow \partial u=[-cossec(x)cotg(x)-(-cossec^2(x))]\partial x\rightarrow$$

$$\partial u=[-cossec(x)cotg(x)+cossec^2(x)]\partial x$$

(70)

$$\frac{\partial(\ln|cossec(x)-cotg(x)|+c)}{\partial x}=$$

$$\frac{1}{cossec(x)-cotg(x)}[-cossec(x)cotg(x)-(-cossec^2(x))]+0=$$

$$\frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}=\frac{cossec(x)[\cancel{cossec(x)-cotg(x)}]}{\cancel{cossec(x)-cotg(x)}}=cossec(x)$$

Exercício V

$$\int tg(3x)\partial x=\int tg(u)\frac{\partial u}{3}=\frac{1}{3}tg(u)\partial u=\frac{1}{3}\ln|sec(u)|+c=\frac{\ln|sec(u)|}{3}+c=\frac{\ln|sec(3x)|}{3}+c$$

$$u=3x\rightarrow\frac{\partial u}{3}=\partial x$$

(71)

Exercício VI

$$\int \frac{\partial x}{sen(2x)}=\int sen^{-1}(2x)\partial x=\int cossec(2x)\partial x=\int cossec(u)\frac{\partial u}{2}=\frac{1}{2}\int cossec(u)\partial u=$$

$$\frac{1}{2}\ln|cossec(u)-cotg(u)|+c=\frac{\ln|cossec(u)-cotg(u)|}{2}+c=\frac{\ln|cossec(2x)-cotg(2x)|}{2}+c$$

$$u=2x\rightarrow\frac{\partial u}{2}=\partial x$$

(72)

Integração de funções trigonométricas – Aula 20

Exercício I

$$\int \frac{tg(\sqrt{x})\partial x}{\sqrt{x}}=\int \frac{tg\left(x^{\frac{1}{2}}\right)\partial x}{x^{\frac{1}{2}}}=\int tg\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int tg(u)2\partial u=2\int tg(u)\partial u=$$

$$2\ln|sec(u)|+c=2\ln|sec(\sqrt{x})|+c$$

$$u=\sqrt{x}=x^{\frac{1}{2}}\rightarrow\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

(73)

Exercício II

$$\int \frac{cotg(\ln|x|)\partial x}{x} = \int cotg(\ln|x|)x^{-1}\partial x = \int cotg(u)\partial u = \ln|sen(u)|+c = \ln|sen(\ln|x|)|+c$$

$$u=\ln|x| \rightarrow \partial u = \frac{1}{x}\partial x \rightarrow \partial u = x^{-1}\partial x$$

(74)

Exercício III

$$\int sec(5x-\pi)\partial x = \int sec(u)\frac{\partial u}{5} = \frac{1}{5}\int sec(u)\partial u = \frac{1}{5}\ln|sec(u)+tg(u)|+c =$$

$$\frac{\ln|sec(5x-\pi)+tg(5x-\pi)|}{5}+c$$

$$u=5x-\pi \rightarrow \frac{\partial u}{5}=\partial x$$

(75)

Integral de potência sen(x) ou cos(x) – Aula 21

$\int \cos^n(x)\partial x$
$\int sen^n(x)\partial x$

$n \rightarrow$ ímpar $\quad sen^2(x)+\cos^2(x)=1$

$$sen^2(x)=\frac{1-\cos(2x)}{2}$$

$n \rightarrow$ par

$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$

$\int sen^m(x)\cos^n(x)\partial x$

$n \vee m \rightarrow$ ímpar $\quad$ Separa o $\partial u \begin{cases} sen(x)\partial x \\ \cos(x)\partial x \end{cases}$

Transfoma em $sen^p(x)$ ou $\cos^p(x)$ através de $sen^2(x)+\cos^2(x)=1$

$$sen^2(x)=\frac{1-\cos(2x)}{2}$$

$n \wedge m \rightarrow$ par

$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$

Exercício I

$$\int \cos^5(x)\,\partial x = \int \cos^4(x)\cos(x)\,\partial x = \int \left(\cos^2(x)\right)^2 \cos(x)\,\partial x = \int \left(1-sen^2(x)\right)^2 \cos(x)\,\partial x =$$
$$\int \left[1-2\,sen^2(x)+sen^4(x)\right]\cos(x)\,\partial x = \int \left[\cos(x)-2\,sen^2(x)\cos(x)+sen^4(x)\cos(x)\right]\partial x =$$
$$\int \cos(x)\,\partial x - 2\int sen^2(x)\cos(x)\,\partial x + \int sen^4(x)\cos(x)\,\partial x = \int \partial u - 2\int u^2\,\partial u + \int u^4\,\partial u =$$
$$u - 2\frac{u^3}{3} + \frac{u^5}{5} + c = sen(x) - \frac{2\,sen^3(x)}{3} + \frac{sen^5(x)}{5} + c$$
$$u = sen(x) \rightarrow \partial u = \cos(x)\,\partial x$$

$$\frac{\partial\left(sen(x)-\dfrac{2\,sen^3(x)}{3}+\dfrac{sen^5(x)}{5}+c\right)}{\partial x} = \frac{\partial\left(sen(x)-\dfrac{2}{3}sen^3(x)+\dfrac{1}{5}sen^5(x)+c\right)}{\partial x} = \qquad (76)$$

$$\cos(x)-\frac{2}{3}3\,sen^2(x)\cos(x)+\frac{1}{5}5\,sen^4(x)\cos(x)+0 = \cos(x)\left[1-2\,sen^2(x)+sen^4(x)\right]=$$
$$\cos(x)\left[1-\left(sen^2(x)+sen^2(x)\right)+\left(sen^2(x)\right)^2\right]=$$
$$\cos(x)\left[\left(1-sen^2(x)\right)-sen^2(x)+\left(1-\cos^2(x)\right)^2\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+\left(1-2\cos^2(x)+\cos^4(x)\right)\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+1-\left(\cos^2(x)+\cos^2(x)\right)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}+\left(1-sen^2(x)\right)-\cancel{\cos^2(x)}-\cos^2(x)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}-\cancel{\cos^2(x)}+\cos^4(x)\right]=\cos(x)\cos^4(x)=\cos^5(x)$$

Exercício II

$$\int sen^4(x)\partial x = \int \left(sen^2(x)\right)^2 \partial x = \int \left(\frac{1-\cos(2x)}{2}\right)^2 \partial x = \int \frac{1-2\cos(2x)+\cos^2(2x)}{4}\partial x =$$

$$\frac{1}{4}\int \left[1-2\cos(2x)+\frac{1+\cos(2\cdot 2x)}{2}\right]\partial x = \frac{1}{4}\int \partial x - \frac{1}{2}\int \cos(2x)\partial x + \frac{1}{8}\int \left[1+\cos(4x)\right]\partial x =$$

$$\frac{1}{4}\int \partial x - \frac{1}{2}\int \cos(2x)\partial x + \frac{1}{8}\int \partial x + \frac{1}{8}\int \cos(4x)\partial x =$$

$$\frac{1}{4}\int \frac{\partial u}{2} - \frac{1}{2}\int \cos(u)\frac{\partial u}{2} + \frac{1}{8}\int \frac{\partial u}{2} + \frac{1}{8}\int \cos(2u)\frac{\partial u}{2} =$$

$$\frac{1}{8}\int \partial u - \frac{1}{4}\int \cos(u)\partial u + \frac{1}{16}\int \partial u + \frac{1}{16}\int \cos(2u)\partial u =$$

$$\frac{1}{8}\int \partial u - \frac{1}{4}\int \cos(u)\partial u + \frac{1}{16}\int \partial u + \frac{1}{16}\int \cos(v)\frac{\partial v}{2} =$$

$$\frac{1}{8}\int \partial u - \frac{1}{4}\int \cos(u)\partial u + \frac{1}{16}\int \partial u + \frac{1}{32}\int \cos(v)\partial v =$$

(77)

$$\frac{1}{8}u - \frac{1}{4}sen(u) + \frac{1}{16}u + \frac{1}{32}sen(v) + c = \frac{2u+u}{16} - \frac{sen(u)}{4} + \frac{sen(2u)}{32} + c =$$

$$\frac{3u}{16} - \frac{sen(u)}{4} + \frac{sen(2u)}{32} + c = \frac{3\cdot 2x}{16} - \frac{sen(2x)}{4} + \frac{sen(2\cdot 2x)}{32} + c =$$

$$\frac{3x}{8} - \frac{sen(2x)}{4} + \frac{sen(4x)}{32} + c$$

$$u = 2x \rightarrow \frac{\partial u}{2} = \partial x$$

$$v = 2u \rightarrow \frac{\partial v}{2} = \partial u$$

Integral do produto de potência entre sen(x) e cos(x) – <u>Aula 22</u>

Exercício I

$$\int sen^5(x)\cos^2(x)\,\partial x = \int sen^4(x)\cos^2(x)sen(x)\,\partial x = \int \left(sen^2(x)\right)^2\cos^2(x)sen(x)\,\partial x =$$

$$\int \left(1-\cos^2(x)\right)^2\cos^2(x)sen(x)\,\partial x = \int \left[1-2\cos^2(x)+\cos^4(x)\right]\cos^2(x)sen(x)\,\partial x =$$

$$\int \left[\cos^2(x)-2\cos^4(x)+\cos^6(x)\right]sen(x)\,\partial x =$$

$$\int \cos^2(x)sen(x)\,\partial x - 2\int \cos^4(x)sen(x)\,\partial x + \int \cos^6(x)sen(x)\,\partial x =$$

$$\int u^2(-\partial u) - 2\int u^4(-\partial u) + \int u^6(-\partial u) = -\int u^2\,\partial u + 2\int u^4\,\partial u - \int u^6\,\partial u =$$

$$\frac{-u^3}{3} + 2\frac{u^5}{5} - \frac{u^7}{7} + c = \frac{-\cos^3(x)}{3} + \frac{2\cos^5(x)}{5} - \frac{\cos^7(x)}{7} + c$$

$$u = \cos(x) \rightarrow -\partial u = sen(x)\,\partial x$$

(78)

$$\frac{\partial\left(\dfrac{-\cos^3(x)}{3} + \dfrac{2\cos^5(x)}{5} - \dfrac{\cos^7(x)}{7} + c\right)}{\partial x} = \frac{\partial\left(-\dfrac{1}{3}\cos^3(x) + \dfrac{2}{5}\cos^5(x) - \dfrac{1}{7}\cos^7(x) + c\right)}{\partial x} =$$

$$-\frac{1}{3}3\cos^2(x)(-sen(x)) + \frac{2}{5}5\cos^4(x)(-sen(x)) - \frac{1}{7}7\cos^6(x)(-sen(x)) + 0 =$$

$$\cos^2(x)sen(x) - 2\cos^4(x)sen(x) + \cos^6(x)sen(x) = sen(x)\left[\cos^2(x) - 2\cos^4(x) + \cos^6(x)\right] =$$

$$sen(x)\cos^2(x)\left[1 - 2\cos^2(x) + \cos^4(x)\right] = \left(1-\cos^2(x)\right)^2\cos^2(x)sen(x) =$$

$$\left(sen^2(x)\right)^2\cos^2(x)sen(x) = sen^4(x)\cos^2(x)sen(x) = sen^5(x)\cos^2(x)$$

Exercício II

$$\int sen^2(x)\cos^4(x)\partial x = \int \frac{1-\cos(2x)}{2}\left(\cos^2(x)\right)^2\partial x = \frac{1}{2}\int (1-\cos(2x))\left(\frac{1+\cos(2x)}{2}\right)^2\partial x$$

$$\frac{1}{2}\int (1-\cos(2x))\left(\frac{1+2\cos(2x)+\cos^2(2x)}{4}\right)\partial x =$$

$$\frac{1}{8}\int (1-\cos(2x))(1+2\cos(2x)+\cos^2(2x))\partial x =$$

$$\frac{1}{8}\int \left(1+2\cos(2x)+\cos^2(2x)-\cos(2x)-2\cos^2(2x)-\cos^3(2x)\right)\partial x =$$

$$\frac{1}{8}\int \left(sen^2(2x)+\cos(2x)-\cos^2(2x)\cos(2x)\right)\partial x =$$

$$\frac{1}{8}\int sen^2(2x)\partial x + \frac{1}{8}\int \cos(2x)\partial x - \frac{1}{8}\int \cos^2(2x)\cos(2x)\partial x =$$

$$\frac{1}{8}\int \frac{1-\cos(4x)}{2}\partial x + \frac{1}{8}\int \cos(2x)\partial x - \frac{1}{8}\int (1-sen^2(2x))\cos(2x)\partial x = \qquad (79)$$

$$\frac{1}{16}\int (1-\cos(4x))\partial x + \frac{1}{8}\int \cos(2x)\partial x - \frac{1}{8}\int \left(\cos(2x)-sen^2(2x)\cos(2x)\right)\partial x =$$

$$\frac{1}{16}\int \partial x - \frac{1}{16}\int \cos(4x)\partial x + \frac{1}{8}\int \cos(2x)\partial x - \frac{1}{8}\int \cos(2x)\partial x + \frac{1}{8}\int sen^2(2x)\cos(2x)\partial$$

$$\frac{1}{16}\int \partial x - \frac{1}{32}\int \cos(2y)\partial y + \frac{1}{16}\int sen^2(y)\cos(y)\partial y =$$

$$\frac{1}{16}\int \partial x - \frac{1}{64}\int \cos(z)\partial z + \frac{1}{16}\int u^2\partial u = \frac{1}{16}x - \frac{1}{64}sen(z) + \frac{1}{16}\frac{u^3}{3} + c =$$

$$\frac{x}{16} - \frac{sen(2y)}{64} + \frac{sen^3(y)}{48} + c = \frac{x}{16} - \frac{sen(4x)}{64} + \frac{sen^3(2x)}{48} + c$$

$$y=2x \to \frac{\partial y}{2} = \partial x \,;\, z=2y \to \frac{\partial z}{2} = \partial y \,;\, u=sen(y) \to \partial u=\cos(y)\partial y$$

Integral de Funções Trigonométricas – <u>Aula 23</u>

$$tg(x) = \frac{sen(x)}{\cos(x)}, cotg(x) = \frac{\cos(s)}{sen(x)}$$

$$sec(x) = \frac{1}{\cos(x)}, cossec(x) = \frac{1}{sen(x)}$$

$$sen^2(x) + \cos^2(x) = 1$$

$$\frac{sen^2(x)}{sen^2(x)} + \frac{\cos^2(x)}{sen^2(x)} = \frac{1}{sen^2(x)} \to 1+cotg^2(x) = cossec^2(x) \to cossec^2(x)-cotg^2(x) = 1$$

$$\frac{sen^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} \to tg^2(x)+1 = sec^2(x) \to sec^2(x)-tg^2(x) = 1$$

Exercício I

$$\int \left[tg(2x) + cotg(2x) \right]^2 \partial x = \int \left[tg^2(2x) + 2\,tg(2x)\,cotg(2x) + cotg^2(2x) \right] \partial x =$$
$$\int tg^2(2x)\partial x + 2\int tg(2x)\,cotg(2x)\partial x + \int cotg^2(2x)\partial x =$$
$$\int \left(sec^2(2x) - 1 \right)\partial x + 2\int \frac{\cancel{sen(2x)}}{\cancel{cos(2x)}}\frac{\cancel{cos(2x)}}{\cancel{sen(2x)}}\partial x + \int \left(cossec^2(2x) - 1 \right)\partial x =$$
$$\int sec^2(2x)\partial x - \int \partial x + 2\int \partial x + \int cossec^2(2x)\partial x - \int \partial x =$$
$$\frac{1}{2}\int sec^2(u)\partial u - \int \partial x + 2\int \partial x + \frac{1}{2}\int cossec^2(u)\partial u - \int \partial x =$$
$$\frac{1}{2}tg(u) \cancel{-x} + \cancel{2x} + \frac{1}{2}(-cotg(u)) \cancel{-x} + c = \frac{tg(2x)}{2} - \frac{cotg(2x)}{2} + c \qquad (80)$$
$$u = 2x \rightarrow \frac{\partial u}{2} = \partial x$$

$$\frac{\partial \left(\dfrac{tg(2x)}{2} - \dfrac{cotg(2x)}{2} + c \right)}{\partial x} = \frac{1}{2}sec^2(2x)2 - \frac{1}{2}\left(-cossec^2(2x)\right)2 + 0 =$$
$$sec^2(2x) + cossec^2(2x) = \left(1 + tg^2(2x)\right) + \left(1 + cossec^2(2x)\right) = 2 + tg^2(2x) + cotg^2(2x) =$$
$$tg^2(2x) + 2\cancel{tg(2x)cotg(2x)} + cotg^2(2x) = \left[tg(2x) + cotg(2x) \right]^2$$

Integral Definida com Seno e Cosseno – Aula 24

	$0\,(0^o)$	$\frac{\pi}{6}\,(30^o)$	$\frac{\pi}{4}\,(45^o)$	$\frac{\pi}{3}\,(60^o)$	$\frac{\pi}{2}\,(90^o)$
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

Exercício I

$$\int_0^{\frac{\pi}{4}} \left(\frac{1 + cos^2(\theta)}{cos^2(\theta)} \right)\partial\theta = \int_0^{\frac{\pi}{4}} \left(\frac{1}{cos^2(\theta)} + \frac{cos^2(\theta)}{cos^2(\theta)} \right)\partial\theta = \int_0^{\frac{\pi}{4}} \left(sec^2(\theta) + 1 \right)\partial\theta =$$
$$\int_0^{\frac{\pi}{4}} sec^2(\theta)\partial\theta + \int_0^{\frac{\pi}{4}} \partial\theta = tg(\theta)\Big]_0^{\frac{\pi}{4}} + \theta\Big]_0^{\frac{\pi}{4}} = \left(tg\left(\frac{\pi}{4}\right) - tg(0) \right) + \left(\frac{\pi}{4} - 0 \right) = (1 - 0) + \left(\frac{\pi}{4} - 0 \right) = 1 + \frac{\pi}{4} = \qquad (81)$$
$$\frac{4 + \pi}{4}$$

Exercício II

$$\int_0^{\pi} \left(4\,sen(\theta) - 3\cos(\theta) \right)\partial\theta = 4\int_0^{\pi} sen(\theta)\partial\theta - 3\int_0^{\pi} cos(\theta)\partial\theta = 4(-\cos(\theta))\Big]_0^{\pi} - 3\,sen(\theta)\Big]_0^{\pi} =$$
$$-4\cos(\theta)\Big]_0^{\pi} - 3\,sen(\theta)\Big]_0^{\pi} = -4\left(\cos(\pi) - \cos(0)\right) - 3\left(sen(\pi) - sen(0)\right) = \qquad (82)$$
$$-4(-1-1) - 3(0-0) = -4(-2) = 8$$

Integral Definida para funções trigonométricas – <u>Aula 25</u>

Exercício I

$$\int_0^{\frac{\pi}{3}} \left(\frac{sen(\theta) + sen(\theta) tg^2(\theta)}{sec^2(\theta)} \right) \partial\theta = \int_0^{\frac{\pi}{3}} \left(\frac{sen(\theta)}{sec^2(\theta)} + \frac{sen(\theta) tg^2(\theta)}{sec^2(\theta)} \right) \partial\theta =$$

$$\int_0^{\frac{\pi}{3}} \left(sen(\theta) \frac{1}{\left(\frac{1}{\cos^2(\theta)} \right)} + sen(\theta) \frac{sen^2(\theta)}{\cos^2(\theta)} \frac{1}{\left(\frac{1}{\cos^2(\theta)} \right)} \right) \partial\theta = \int_0^{\frac{\pi}{3}} \left(\cos^2(\theta) sen(\theta) + sen^3(\theta) \right) \partial\theta =$$

$$\int_0^{\frac{\pi}{3}} \cos^2(\theta) sen(\theta) \partial\theta + \int_0^{\frac{\pi}{3}} sen^2(\theta) sen(\theta) \partial\theta =$$

$$\int_0^{\frac{\pi}{3}} \cos^2(\theta) sen(\theta) \partial\theta + \int_0^{\frac{\pi}{3}} \left(1 - \cos^2(\theta) \right) sen(\theta) \partial\theta =$$

(83)

$$\int_0^{\frac{\pi}{3}} \cos^2(\theta) sen(\theta) \partial\theta + \int_0^{\frac{\pi}{3}} \left(sen(\theta) - \cos^2(\theta) sen(\theta) \right) \partial\theta =$$

$$\int_0^{\frac{\pi}{3}} \cos^2(\theta) sen(\theta) \partial\theta + \int_0^{\frac{\pi}{3}} sen(\theta) \partial\theta - \int_0^{\frac{\pi}{3}} \cos^2(\theta) sen(\theta) \partial\theta =$$

$$-\cos(\theta) \Big]_0^{\frac{\pi}{3}} = -\left(\cos\left(\frac{\pi}{3} \right) - \cos(0) \right) = -\left(\frac{1}{2} - 1 \right) = -\left(\frac{-1}{2} \right) = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{3}} \left(\frac{sen(\theta) + sen(\theta) tg^2(\theta)}{sec^2(\theta)} \right) \partial\theta = \int_0^{\frac{\pi}{3}} \frac{sen(\theta) \left(1 + tg^2(\theta) \right)}{sec^2(\theta)} \partial\theta = \int_0^{\frac{\pi}{3}} \frac{sen(\theta) sec^2(\theta)}{sec^2(\theta)} \partial\theta =$$

$$\int_0^{\frac{\pi}{3}} sen(\theta) \partial\theta = -\cos(\theta) \Big]_0^{\frac{\pi}{3}} = -\left(\cos\left(\frac{\pi}{3} \right) - \cos(0) \right) = -\left(\frac{1}{2} - 1 \right) = -\left(\frac{-1}{2} \right) = \frac{1}{2}$$

Exercício II

$$\int_0^{\pi} sec^2\left(\frac{t}{4} \right) \partial t = 4 \int_0^{\pi} sec^2(u) \partial u = 4 \, tg(u) \Big]_0^{\pi} = 4 \, tg\left(\frac{t}{4} \right) \Big]_0^{\pi} = 4 \left(tg\left(\frac{\pi}{4} \right) - tg(0) \right) = 4 (1 - 0) = 4$$

(84)

$$u = \frac{t}{4} \rightarrow 4 \, \partial u = \partial t$$

Integral envolvendo Funções Seno e Cosseno – <u>Aula 26</u>

$$sen(2x) = 2 sen(x) \cos(x)$$

Exercício I

$$\int \frac{sen(x)}{1-sen^2(x)}\partial x = \int \frac{sen(x)}{\cos^2(x)}\partial x = \int \cos^{-2}(x)\,sen(x)\,\partial x = -\int u^{-2}\partial u = \frac{-u^{-1}}{(-1)}+c =$$
$$u^{-1}+c = \frac{1}{u}+c = \frac{1}{\cos(x)}+c = sec(x)+c \tag{85}$$
$$u = \cos(x) \rightarrow -\partial u = sen(x)\,\partial x$$

Exercício II

$$\int \frac{sen(2x)}{\cos(x)}\partial x = \int \frac{2\,sen(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\partial x = 2\int sen(x)\,\partial x = 2(-\cos(x))+c = -2\cos(x)+c \tag{86}$$

Integral de uma função exponencial de seno – Aula 27

Exercício I

$$\int e^{sen(\theta)}\cos(\theta)\,\partial\theta = \int e^u \partial u = e^u + c = e^{sen(\theta)}+c \tag{87}$$
$$u = sen(\theta) \rightarrow \partial u = \cos(\theta)\,\partial\theta$$

Exercício II

$$\int sen(\pi t)\,\partial t = \frac{1}{\pi}\int sen(u)\,\partial u = \frac{1}{\pi}(-\cos(u))+c = \frac{-\cos(u)}{\pi}+c = \frac{-\cos(\pi t)}{\pi}+c \tag{88}$$
$$u = \pi t \rightarrow \frac{\partial u}{\pi} = \partial t$$

Integral Definida do Produto de secante e tangente – Aula 28

Exercício I

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\,\partial\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos(\theta)}\frac{sen(\theta)}{\cos(\theta)}\partial\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{sen(\theta)}{\cos^2(\theta)}\partial\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^{-2}(\theta)\,sen(\theta)\,\partial\theta =$$
$$-\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} u^{-2}\partial u = \left[\frac{-u^{-1}}{(-1)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[u^{-1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[\frac{1}{u}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} - \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} =$$
$$2 - \frac{2}{\sqrt{2}} = 2 - \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = 2-\sqrt{2} \tag{89}$$
$$u = \cos(\theta) \rightarrow -\partial u = sen(\theta)\,\partial\theta$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\,\partial\theta = \left[sec(\theta)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} - \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 2 - \frac{2}{\sqrt{2}} =$$
$$2 - \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = 2-\sqrt{2}$$

Integral de seno pelo Método da Substituição – <u>Aula 29</u>

Exercício I

$$\int \sqrt{x}\,sen\left(1+\sqrt{x^3}\right)\partial x=\int sen\left(1+x^{\frac{3}{2}}\right)x^{\frac{1}{2}}\partial x=\frac{2}{3}sen(u)\partial u=\frac{2}{3}(-\cos(u))+c=$$

$$\frac{-2\cos\left(1+\sqrt{x^3}\right)}{3}+c \tag{90}$$

$$u=1+\sqrt{x^3}=1+x^{\frac{3}{2}}\Rightarrow\partial u=\frac{3}{2}x^{\frac{1}{2}}\partial x=\frac{2\partial u}{3}=x^{\frac{1}{2}}\partial x$$

Integral Trigonométrica – Método da Substituição – <u>Aula 30</u>

Exercício I

$$\int (1+tg(\theta))^5 sec^2(\theta)\partial\theta=\int u^5\partial u=\frac{u^6}{6}+c=\frac{(1+tg(\theta))^6}{6}+c \tag{91}$$

$$u=1+tg(\theta)\Rightarrow\partial u=sec^2(\theta)\partial\theta$$

Exercício II

$$\int sec(2\theta)tg(2\theta)\partial\theta=\frac{1}{2}\int sec(u)tg(u)\partial u=\frac{1}{2}sec(u)+c=\frac{sec(2\theta)}{2}+c \tag{92}$$

$$u=2\theta\Rightarrow\frac{\partial u}{2}=\partial\theta$$

Integral Trigonométrica – Método da Substituição – <u>Aula 31</u>

Exercício I

$$\int \sqrt{cotg(x)}\,cossec^2(x)\partial x=\int cotg^{\frac{1}{2}}(x)cossec^2(x)\partial x=-\int u^{\frac{1}{2}}\partial u=\frac{-u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{-2\sqrt{u^3}}{3}+c=$$

$$\frac{-2\sqrt{cotg^3(x)}}{3}+c \tag{93}$$

$$u=cotg(x)\Rightarrow-\partial u=cossec^2(x)$$

Exercício II

$$\int sec^3(x)tg(x)\partial x=\int sec^2(x)sec(x)tg(x)\partial x=\int u^2\partial u=\frac{u^3}{3}+c=\frac{sec^3(x)}{3}+c \tag{94}$$

$$u=sec(x)\Rightarrow\partial u=sec(x)tg(x)\partial x$$

Integral de Cosseno pelo Método da Substituição – <u>Aula 32</u>

Exercício I

$$\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\partial x = \int \cos\left(\pi x^{-1}\right)x^{-2}\partial x = \frac{-1}{\pi}\int \cos(u)\partial u = \frac{-1}{\pi}sen(u)+c = \frac{-sen\left(\frac{\pi}{x}\right)}{\pi}+c$$

$$u = \frac{\pi}{x} = \pi x^{-1} \rightarrow \partial u = -\pi x^{-2} \rightarrow \frac{-\partial u}{\pi} = x^{-2}\partial x$$

(95)

Integral de Potência de Tangente – <u>Aula 33</u>

Exercício I

$$\int tg^3(x)\partial x = \int tg^2(x)tg(x)\partial x = \int \left(sec^2(x)-1\right)tg(x)\partial x =$$

$$\int tg(x)sec^2(x)\partial x - \int tg(x)\partial x = \int tg(x)sec^2(x)\partial x - \int \frac{sen(x)}{\cos(x)}\partial x =$$

$$\int tg(x)sec^2(x)\partial x - \int \cos^{-1}(x)sen(x)\partial = \int u\,\partial u + \int v^{-1}\partial v = \frac{u^2}{2}+\ln|v|+c =$$

$$\frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$

$$u = tg(x) \rightarrow \partial u = sec^2(x)\partial x \,;\, v = \cos(x) \rightarrow -\partial v = sen(x)\partial x$$

(96)

$$\int tg^3(x)\partial x = \int tg^2(x)tg(x)\partial x = \int \left(sec^2(x)-1\right)tg(x)\partial x =$$

$$\int tg(x)sec^2(x)\partial x - \int tg(x)\partial x = \int u\,\partial u - \int tg(x)\partial x = \frac{u^2}{2}-\left(\ln|sec(x)|\right)+c =$$

$$\frac{u^2}{2}-\left(\ln|\cos^{-1}(x)|\right)+c = \frac{u^2}{2}-\left(-\ln|\cos(x)|\right)+c = \frac{u^2}{2}+\ln|\cos(x)|+c = \frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$

$$u = tg(x) \rightarrow \partial u = sec^2(x)\partial x$$

Integral de Potência de Cotangente – <u>Aula 34</u>

Exercício I

$$\int cotg^4(3x)\partial x = \int cotg^2(3x)cotg^2(3x)\partial x = \int \left(cossec^2(3x)-1\right)cotg^2(3x)\partial x =$$

$$\int cotg^2(3x)cossec^2(3x)\partial x - \int cotg^2(3x)\partial x =$$

$$\int cotg^2(3x)cossec^2(3x)\partial x - \int \left(cossec^2(3x)-1\right)\partial x =$$

$$\int cotg^2(3x)cossec^2(3x)\partial x - \int cossec^2(3x)\partial x + \int \partial x =$$

$$\frac{1}{3}\int cotg^2(u)cossec^2(u)\partial u - \frac{1}{3}\int cossec^2(u)\partial u + \int \partial x =$$

$$\frac{-1}{3}\int v^2\partial v + \frac{1}{3}\int \partial v + \int \partial x = \frac{-1}{3}\frac{v^3}{3}+\frac{1}{3}v+x+c = \frac{-cotg^3(u)}{9}+\frac{cotg(u)}{3}+x+c =$$

$$\frac{-cotg^3(3x)}{9}+\frac{cotg(3x)}{3}+x+c$$

$$u = 3x \rightarrow \frac{\partial u}{3} = \partial x \,;\, v = cotg(u) \rightarrow -\partial v = cossec^2(u)$$

(97)

Integral da Potência de Secante – Aula 35

Exercício I

$$\int cossec^6(x)\partial x = \int cossec^4(x)cossec^2(x)\partial x = \int \left(cossec^2(x)\right)^2 cossec^2(x)\partial x =$$
$$\int \left(cotg^2(x)+1\right)^2 cossec^2(x)\partial x = \int \left(cotg^4(x)+2\,cotg^2(x)+1\right)cossec^2(x)\partial x =$$
$$\int cotg^4(x)cossec^2(x)\partial x + 2\int cotg^2(x)cossec^2(x)\partial x + \int cossec^2(x)\partial x =$$
$$-\int u^4\partial u - 2u^2\partial u - \int \partial u = \frac{-u^5}{5} - 2\frac{u^3}{3} - u + c = \frac{-cotg^5(x)}{5} - \frac{2\,cotg^3(x)}{3} - cotg(x)+c$$
$$u=cotg(x) \rightarrow -\partial u = cossec^2(x)\partial x$$

(98)

Integral de Secante ao Cubo – Aula 36

$$\int u\,\partial v = uv - \int v\,\partial u$$

Exercício I

$$\int sec^3(x)\partial x = \int sec(x)sec^2(x)\partial x = sec(x)tg(x) - \int tg(x)sec(x)tg(x)\partial x =$$
$$sec(x)tg(x) - \int tg^2(x)sec(x)\partial x = sec(x)tg(x) - \int \left(sec^2(x)-1\right)sec(x)\partial x =$$
$$sec(x)tg(x) - \int sec^3(x)\partial x + \int sec(x)\partial x \rightarrow$$
$$\int sec^3(x)\partial x + \int sec^3(x)\partial x = sec(x)tg(x) + \int sec(x)\partial x \rightarrow$$
$$2\int sec^3(x)\partial x = sec(x)tg(x) + \ln|sec(x)+tg(x)| + c \rightarrow$$
$$\int sec^3(x)\partial x = \frac{sec(x)tg(x) + \ln|sec(x)+tg(x)| + c}{2} = \frac{sec(x)tg(x)}{2} + \frac{\ln|sec(x)+tg(x)|}{2} + \frac{c}{2} =$$
$$\frac{sec(x)tg(x)}{2} + \frac{\ln|sec(x)+tg(x)|}{2} + c$$
$$u=sec(x) \rightarrow \partial u = sec(x)tg(x)\partial x$$
$$v=tg(x) \rightarrow \partial v = sec^2(x)\partial x$$

(99)

Integral de Potência de Secante e Tangente – Aula 37

$$\int tg^m(x)sec^n(x)\partial x$$

n → par	Guarde $sec^2(x)$ Use $sec^2(x)=tg^2(x)+1$ $u=tg(x)$
m → impar	Guarde $sec(x)tg(x)$ Use $tg^2(x)=sec^2(x)-1$ $u=sec(x)$
m → par	
n → impar	Use $tg^2(x)=sec^2(x)-1$ para reduzir somente a $sec(x)$ Após aplique integral por partes

Exercício I

$$\int tg^5(x)\sec^4(x)\,\partial x = \int tg^5(x)\sec^2(x)\sec^2(x)\,\partial x = \int tg^5(x)\left(tg^2(x)+1\right)\sec^2(x)\,\partial x =$$
$$\int tg^5(x)\left(tg^2(x)\sec^2(x)+\sec^2(x)\right)\partial x = \int tg^7(x)\sec^2(x)\,\partial x + \int tg^5(x)\sec^2(x)\,\partial x =$$
$$\int u^7\,\partial u + \int u^5\,\partial u = \frac{u^8}{8}+\frac{u^6}{6}+c = \frac{tg^8(x)}{8}+\frac{tg^6(x)}{6}+c$$
$$u = tg(x) \rightarrow \partial u = \sec^2(x)\,\partial x$$

(100)

Integral de Potência de Secante e Tangente – Aula 38

Exercício I

$$\int tg^5(x)\sec^7(x)\,\partial x = \int tg^4(x)\sec^6(x)\sec(x)tg(x)\,\partial x =$$
$$\int \left(tg^2(x)\right)^2\sec^6(x)\sec(x)tg(x)\,\partial x = \int \left(\sec^2(x)-1\right)^2\sec^6(x)\sec(x)tg(x)\,\partial x =$$
$$\int \left(\sec^4(x)-2\sec^2(x)+1\right)\sec^6(x)\sec(x)tg(x)\,\partial x =$$
$$\int \sec^{10}(x)\sec(x)tg(x)\,\partial x - 2\int \sec^8(x)\sec(x)tg(x)\,\partial x + \int \sec^6(x)\sec(x)tg(x)\,\partial x =$$
$$\int u^{10}\,\partial u - 2\int u^8\,\partial u + \int u^6\,\partial u = \frac{u^{11}}{11}-2\frac{u^9}{9}+\frac{u^7}{7}+c = \frac{\sec^{11}(x)}{11}-\frac{2\sec^9(x)}{9}+\frac{\sec^7(x)}{7}+c$$
$$u = \sec(x) \rightarrow \partial u = \sec(x)tg(x)\,\partial x$$

(101)

Exercício II

$$\int tg^2(x)\sec(x)\,\partial x = \int \left(\sec^2(x)-1\right)\sec(x)\,\partial x = \int \sec(x)\sec^2(x)\,\partial x - \int \sec(x)\,\partial x \rightarrow$$
$$\int u\,\partial v = uv - \int v\,\partial u \rightarrow \int \sec(x)\sec^2(x)\,\partial x = \sec(x)tg(x) - \int tg(x)\sec(x)tg(x)\,\partial x =$$
$$\sec(x)tg(x) - \int tg^2(x)\sec(x)\,\partial x = \sec(x)tg(x) - \int \left(\sec^2(x)-1\right)\sec(x)\,\partial x =$$
$$\sec(x)tg(x) - \int \sec(x)\sec^2(x)\,\partial x + \int \sec(x)\,\partial x \rightarrow$$
$$\int \sec(x)\sec^2(x)\,\partial x + \int \sec(x)\sec^2(x)\,\partial x = \sec(x)tg(x) + \int \sec(x)\,\partial x \rightarrow$$
$$2\int \sec(x)\sec^2(x)\,\partial x = \sec(x)tg(x) + \ln|\sec(x)+tg(x)|+c \rightarrow$$
$$\int \sec(x)\sec^2(x)\,\partial x = \frac{\sec(x)tg(x)+\ln|\sec(x)+tg(x)|+c}{2} =$$
$$\int tg^2(x)\sec(x)\,\partial x = \left(\frac{\sec(x)tg(x)+\ln|\sec(x)+tg(x)|+c}{2}\right) - \int \sec(x)\,\partial x =$$
$$\frac{\sec(x)tg(x)+\ln|\sec(x)+tg(x)|+c}{2} - \ln|\sec(x)+tg(x)|+c =$$
$$\frac{\sec(x)tg(x)+\cancel{\ln|\sec(x)+tg(x)|}-2\ln|\sec(x)+tg(x)|+c}{2} =$$
$$\frac{\sec(x)tg(x)}{2} - \frac{\ln|\sec(x)+tg(x)|}{2}+c$$
$$u = \sec(x) \rightarrow \partial u = \sec(x)tg(x)\,\partial x$$
$$v = tg(x) \rightarrow \partial v = \sec^2(x)\,\partial x$$

(102)

Cálculo de área com integrais – <u>Aula 1</u>

$$\int_{c_1}^{c_2}[f(x)-g(x)]\partial x+\int_{c_2}^{c_3}[g(x)-f(x)]\partial x+\int_{c_3}^{c_4}[f(x)-g(x)]\partial x$$

Exercício I

$$f(x)=x^2\,;g(x)=x$$
$$f(x)=g(x)\rightarrow x^2=x\rightarrow x^2-x=0\rightarrow x(x-1)=0\rightarrow$$
$$x=0\rightarrow x_1=0\,;x-1=0\rightarrow x=1\rightarrow x_2=1$$
$$\int_0^1[g(x)-f(x)]\partial x=\int_0^1(x-x^2)\partial x=\int_0^1 x\partial x-\int_0^1 x^2\partial x=\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1=\left(\frac{1^2}{2}-\frac{1^3}{3}\right)-\left(\frac{0^2}{2}-\frac{0^3}{3}\right)=$$

(103)

$$\frac{1}{2}-\frac{1}{3}-0=\frac{3-2}{6}=\frac{1}{6}=0{,}166$$

Exercício II

$$f(x)=x^2\,;g(x)=1$$
$$\int_0^1[g(x)-f(x)]\partial x+\int_1^2[f(x)-g(x)]\partial x=\int_0^1(1-x^2)\partial x+\int_1^2(x^2-1)\partial x=$$
$$\int_0^1\partial x-\int_0^1 x^2\partial x+\int_1^2 x^2\partial x-\int_1^2\partial x=\left[x-\frac{x^3}{3}\right]_0^1+\left[\frac{x^3}{3}-x\right]_1^2=\left[\frac{3x-x^3}{3}\right]_0^1+\left[\frac{x^3-3x}{3}\right]_1^2=$$

(104)

$$\left[\frac{x(3-x^2)}{3}\right]_0^1+\left[\frac{x(x^2-3)}{3}\right]_1^2=\left[\frac{1(3-1^2)}{3}-\frac{0(3-0^2)}{3}\right]+\left[\frac{2(2^2-3)}{3}-\frac{1(1^2-3)}{3}\right]=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=$$

$$\frac{6}{3}=2$$

Exercício III

$$f(x)=2x-2; g(x)=0$$

$$\int_0^1 [g(x)-f(x)]\partial x+\int_1^3 [f(x)-g(x)]\partial x=\int_0^1 (0-(2x-2))\partial x+\int_1^3 (2x-2-0)\partial x=$$

$$-2\int_0^1 x\partial x+2\int_0^1 \partial x+2\int_1^3 x\partial x-2\int_1^3 \partial x=\left[-2\frac{x^2}{2}+2x\right]_0^1+\left[2\frac{x^2}{2}-2x\right]_1^3=$$

$$\left[-x^2+2x\right]_0^1+\left[x^2-2x\right]_1^3=\left[-x(x-2)\right]_0^1+\left[x(x-2)\right]_1^3=$$

$$[-1(1-2)-(\cancel{0(0-2)})]+[3(3-2)-1(1-2)]=1+3+1=5$$

(105)

Cálculo de área com integrais – <u>Aula 2</u>

Exercício I

$$y^2=4x\rightarrow y=\pm\sqrt{4x}=\pm2\sqrt{x}; x_1=1; x_2=4$$

$$y_1(1)=2\sqrt{1}=2; y_1(4)=2\sqrt{4}=2\cdot2=4$$

$$y_2(1)=-2\sqrt{1}=-2; y_2(4)=-2\sqrt{4}=-2\cdot2=-4$$

$$f(x)=2\sqrt{x}=2x^{\frac{1}{2}}; g(x)=0$$

$$\int_1^4 [f(x)-g(x)]\partial x=\int_1^4 [2\sqrt{x}-0]\partial x=2\int_1^4 x^{\frac{1}{2}}\partial x=\left[2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_1^4=\left[\frac{4\sqrt{x^3}}{3}\right]_1^4=\frac{4}{3}\left[\sqrt{x^3}\right]_1^4=$$

$$\frac{4}{3}\left[\sqrt{4^3}-\sqrt{1^3}\right]=\frac{4}{3}\left[\sqrt{(2^2)^3}-1\right]=\frac{4}{3}[2^3-1]=\frac{4}{3}[2^3-1]=\frac{4}{3}7=\frac{28}{3}=9,33$$

(106)

Exercício II

$$f(x)=x^2; g(x)=2x+3$$

$$f(x)=g(x)\rightarrow x^2=2x+3\rightarrow x^2-(2x+3)=0\rightarrow x^2-2x-3=0\rightarrow x^2-2x-3+1-1=0\rightarrow$$

$$x^2-2x+1-4=0\rightarrow(x-1)^2-4=0\rightarrow(x-1)^2=4\rightarrow x-1=\pm\sqrt{4}\rightarrow x=\pm2+1\rightarrow$$

$$x=2+1\rightarrow x_1=3; x=-2+1\rightarrow x_2=-1$$

$$f(0)=(0)^2=0; g(0)=2(0)+3=3$$

$$\int_{-1}^3 [g(x)-f(x)]\partial x=\int_{-1}^3 (2x+3-x^2)\partial x=2\int_{-1}^3 x\partial x+3\int_{-1}^3 \partial x-\int_{-1}^3 x^2\partial x=$$

$$\left[2\frac{x^2}{2}+3x-\frac{x^3}{3}\right]_{-1}^3=\left[x^2+3x-\frac{x^3}{3}\right]_{-1}^3=\left[\frac{3x^2+9x-x^3}{3}\right]_{-1}^3=\frac{1}{3}\left[x(3x+9-x^2)\right]_{-1}^3=$$

$$\frac{1}{3}\left[3(3\cdot3+9-3^2)-(-1)(3(-1)+9-(-1)^2)\right]=\frac{1}{3}[3\cdot9+5]=\frac{1}{3}32=\frac{32}{3}=10,66$$

(107)

Cálculo de área com integrais – Aula 3

Exercício I

$$f(x)=-x^2+4x-3\,;\,g(x)=0$$

$$f(x)=g(x)\rightarrow -x^2+4x-3=0\rightarrow x^2-4x+3+(2)^2-(2)^2=0\rightarrow x^2-4x+4-1=0\rightarrow$$

$$(x-2)^2-1=0\rightarrow x-2=\pm\sqrt{1}\rightarrow x=\pm 1+2\rightarrow$$

$$x=1+2\rightarrow x_1=3\,;\,x=-1+2\rightarrow x_2=1$$

$$f(2)=-(2)^2+4(2)-3=-4+8-3=1\,;\,g(2)=0$$

$$\int_1^3 [f(x)-g(x)]\partial x=\int_1^3 (-x^2+4x-3-0)\partial x=-\int_1^3 x^2\partial x+4\int_1^3 x\partial x-3\int_1^3 \partial x\rightarrow \tag{108}$$

$$\left[\frac{-x^3}{3}+4\frac{x^2}{2}-3x\right]_1^3=\left[\frac{-x^3}{3}+2x^2-3x\right]_1^3=\left[\frac{-x^3+6x^2-9x}{3}\right]_1^3=\frac{1}{3}\left[-x(x^2-6x+9)\right]_1^3=$$

$$\frac{1}{3}\left[-3(3^2-6\cdot 3+9)-(-1(1^2-6\cdot 1+9))\right]=\frac{1}{3}[-3\cdot 0+4]=\frac{1}{3}4=\frac{4}{3}=1,33$$

Exercício II

$$f(x)=-x^2+4\,;\,g(x)=-5$$

$$f(x)=g(x)\rightarrow -x^2+4=-5\rightarrow -x^2=-5-4\rightarrow x^2=9\rightarrow x=\pm\sqrt{9}\rightarrow x=\pm 3$$

$$x_1=-3\,;\,x_2=3$$

$$f(0)=-(0)^2+4=4\,;\,g(0)=-5$$

$$\int_{-3}^3 [f(x)-g(x)]\partial x=\int_{-3}^3 (-x^2+4+5)\partial x=-\int_{-3}^3 x^2\partial x+4\int_{-3}^3 \partial x+5\int_{-3}^3 \partial x= \tag{109}$$

$$\left[\frac{-x^3}{3}+4x+5x\right]_{-3}^3=\left[\frac{-x^3+12x+15x}{3}\right]_{-3}^3=\frac{1}{3}\left[-x(x^2-12-15)\right]_{-3}^3=\frac{1}{3}\left[-x(x^2-27)\right]_{-3}^3=$$

$$\frac{1}{3}\left[-3(3^2-27)+(-3)((-3)^2-27)\right]=\frac{1}{3}[-3(-18)-3(-18)]=\frac{1}{3}[54+54]=\frac{1}{3}108=36$$

Cálculo de áreas com integrais – Aula 4

Exercício I

$$g(x)=x+2\,;\,f(x)=x^2-x+2$$

$$f(x)=g(x)\rightarrow x^2-x+2=x+2\rightarrow x^2-x+2-x-2=0\rightarrow x^2-2x=0\rightarrow x(x-2)=0\rightarrow$$

$$x=0\rightarrow x_1=0\,;\,x-2=0\rightarrow x=2\rightarrow x_2=2$$

$$g(1)=1+2=3\,;\,f(1)=1^2-1+2=2$$

$$\int_0^2 [g(x)-f(x)]\partial x=\int_0^2 [x+2-(x^2-x+2)]\partial x=\int_0^2 (x+2-x^2+x-2)\partial x=\int_0^2 (-x^2+2x)\partial x= \tag{110}$$

$$-\int_0^2 x^2\partial x+2\int_0^2 x\partial x=\left[\frac{-x^3}{3}+2\frac{x^2}{2}\right]_0^2=\left[\frac{-x^3}{3}+x^2\right]_0^2=\left[\frac{-x^3+3x^2}{3}\right]_0^2=\frac{1}{3}\left[-x(x^2-3x)\right]_0^2=$$

$$\frac{1}{3}\left[-2(2^2-3\cdot 2)+0(0^2-3\cdot 0)\right]=\frac{1}{3}4=\frac{4}{3}=1,33$$

Cálculo de áreas com integrais – Aula 5

Exercício I

$$f(x)=4x\,;g(x)=x^3+3x^2$$

$$f(x)=g(x)\to 4x=x^3+3x^2\to x^3+3x^2-4x=0\to x(x^2+3x-4)=0\to$$

$$x=0\to x_1=0\,;x^2+3x-4=0\to x^2+3x-4+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=0\to x^2+3x-4+\frac{9}{4}-\frac{9}{4}=0\to$$

$$x^2+3x+\frac{9}{4}-\frac{25}{4}=0\to\left(x+\frac{3}{2}\right)^2=\frac{25}{4}\to x+\frac{3}{2}=\pm\sqrt{\frac{25}{4}}\to x=\frac{\pm 5}{2}-\frac{3}{2}\to$$

$$x=\frac{-5}{2}-\frac{3}{2}=\frac{-5-3}{2}=\frac{-8}{2}=-4\to x_2=-4\,;x=\frac{5}{2}-\frac{3}{2}=\frac{5-3}{2}=\frac{2}{2}=1\to x_3=1$$

$$f(-1)=4(-1)=-4\,;g(-1)=(-1)^3+3(-1)^2=-1+3=2$$

$$f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)=2\,;g\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2=\frac{1}{8}+\frac{3}{4}=\frac{1+6}{8}=\frac{7}{8}$$

$$\int_{-4}^{0}[g(x)-f(x)]\partial x+\int_{0}^{1}[f(x)-g(x)]\partial x=\int_{-4}^{0}\left(x^3+3x^2-4x\right)\partial x+\int_{0}^{1}\left(4x-x^3-3x^2\right)\partial x= \quad (111)$$

$$\int_{-4}^{0}x^3\partial x+3\int_{-4}^{0}x^2\partial x-4\int_{-4}^{0}x\partial x+4\int_{0}^{1}x\partial x-\int_{0}^{1}x^3\partial x-3\int_{0}^{1}x^2\partial x=$$

$$\left[\frac{x^4}{4}+3\frac{x^3}{3}-4\frac{x^2}{2}\right]_{-4}^{0}+\left[4\frac{x^2}{2}-\frac{x^4}{4}-3\frac{x^3}{3}\right]_{0}^{1}=\left[\frac{x^4}{4}+x^3-2x^2\right]_{-4}^{0}+\left[2x^2-\frac{x^4}{4}-x^3\right]_{0}^{1}=$$

$$\left[\frac{x^4+4x^3-8x^2}{4}\right]_{-4}^{0}+\left[\frac{8x^2-x^4-4x^3}{4}\right]_{0}^{1}=\frac{1}{4}\left[x^2\left(x^2+4x-8\right)\right]_{-4}^{0}+\frac{1}{4}\left[x^2\left(-x^2-4x+8\right)\right]_{0}^{1}=$$

$$\frac{1}{4}\left[\cancel{0^2\left(0^2+4\cdot 0-8\right)}-(-4)^2\left((-4)^2+4(-4)-8\right)\right]+\frac{1}{4}\left[1^2\left(-1^2-4\cdot 1+8\right)-\cancel{0^2\left(-0^2-4\cdot 0+8\right)}\right]=$$

$$\frac{1}{4}\left[-16\left(\cancel{16-16}-8\right)\right]+\frac{1}{4}\left[-1-4+8\right]=\frac{1}{4}\left(128+3\right)=\frac{131}{4}=32,75$$
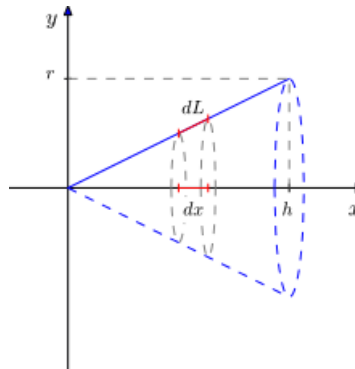
Cálculo de volume com integrais simples – Aula 1
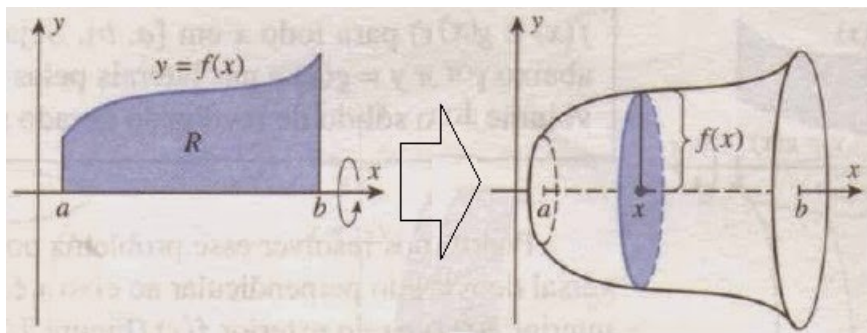
Volume do cilindro: $v=\pi r^2 h\,;r=y=f(x);h=\partial x;\partial v=\pi\left[f(x)\right]^2\partial x$

Volume do sólido: $v=\int_{a}^{b}\partial v=\int_{a}^{b}\pi\left[f(x)\right]^2\partial x\to v=\pi\int_{a}^{b}\left[f(x)\right]^2\partial x$

Exercício I

Determine o volume do sólido de revolução gerado pela rotação em torno do eixo dos x da região R delimitado pelo gráfico das equações: $y=x\,; y=0\,; x=0\,; x=2$

$$f(x)=x\,; x_1=0\,; x_2=2$$

$$v=\pi\int_0^2 x^2\,\partial x=\pi\left[\frac{x^3}{3}\right]_0^2=\frac{\pi}{3}\left[x^3\right]_0^2=\frac{\pi}{3}\left[2^3-0^3\right]=\frac{\pi}{3}8=\frac{8\pi}{3}=2{,}667\,\pi \tag{112}$$

Exercício II
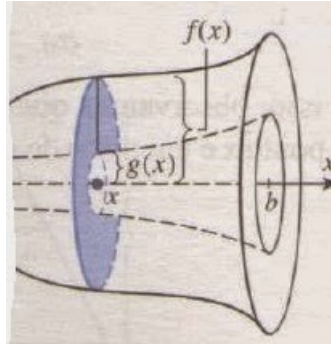
$$f(x)=\frac{x^2}{4}\,; x_1=1\,; x_2=4$$

$$v=\pi\int_1^4\left(\frac{x^2}{4}\right)^2\partial x=\pi\int_1^4\frac{x^4}{16}\partial x=\frac{\pi}{16}\int_1^4 x^4\,\partial x=\frac{\pi}{16}\left[\frac{x^5}{5}\right]_1^4=\frac{\pi}{80}\left[x^5\right]_1^4=\frac{\pi}{80}\left[4^5-1^5\right]=\frac{1023\pi}{80}= \tag{113}$$
$$12{,}7875\,\pi$$

Exercício III

$$f(x)=x^2\,; x_1=1\,; x_2=2$$

$$v=\pi\int_1^2 x^4\,\partial x=\pi\left[\frac{x^5}{5}\right]_1^2=\frac{\pi}{5}\left[x^5\right]_1^2=\frac{\pi}{5}\left[2^5-1^5\right]=\frac{31\pi}{5}=6{,}2\,\pi \tag{114}$$

Volume do cilindro: $v=\pi r_1^2 h - \pi r_2^2 h = \pi\left(r_1^2 - r_2^2\right)h\,; r_1 = f(x)\,; r_2 = g(x)\,; h = \partial x$

$$\partial v = \pi\left[(f(x))^2 - (g(x))^2\right]\partial x$$

Volume do sólido: $v = \int_a^b \partial v = \int_a^b \pi\left[(f(x))^2 - (g(x))^2\right]\partial x \rightarrow v = \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]\partial x$

Exercício IV

$$f(x)=x^2+1\,; g(x)=x+3$$

$$f(x)=g(x)\rightarrow x^2+1=x+3 \rightarrow x^2+1-x-3=0 \rightarrow x^2-x-2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0\rightarrow$$

$$x^2-x+\frac{1}{4}-\frac{9}{4}=0\rightarrow\left(x-\frac{1}{2}\right)^2=\frac{9}{4}\rightarrow x=\pm\sqrt{\frac{9}{4}}+\frac{1}{2}\rightarrow x=\frac{\pm3}{2}+\frac{1}{2}$$

$$x=\frac{-3}{2}+\frac{1}{2}=\frac{-2}{2}=-1\rightarrow x_1=-1\,; x=\frac{3}{2}+\frac{1}{2}=\frac{3+1}{2}=\frac{4}{2}=2\rightarrow x_2=2$$

$$f(0)=0^2+1=1\,; g(0)=0+3=3$$

$$v=\pi\int_{-1}^{2}\left[(g(x))^2-(f(x))^2\right]\partial x=\pi\int_{-1}^{2}\left[(x+3)^2-(x^2+1)^2\right]\partial x=$$

$$\pi\int_{-1}^{2}\left[(x^2+6x+9)-(x^4+2x^2+1)\right]\partial x=\pi\int_{-1}^{2}\left(x^2+6x+9-x^4-2x^2-1\right)\partial x=$$

$$\pi\int_{-1}^{2}\left(-x^4-x^2+6x+8\right)\partial x=-\pi\int_{-1}^{2}x^4\partial x-\pi\int_{-1}^{2}x^2\partial x+6\pi\int_{-1}^{2}x\partial x+8\pi\int_{-1}^{2}\partial x=$$

$$\left[-\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\right]_{-1}^{2}=\pi\left[\frac{-x^5}{5}-\frac{x^3}{3}+3x^2+8x\right]_{-1}^{2}=$$

$$\pi\left[\frac{-3x^5-5x^3+45x^2+120x}{15}\right]_{-1}^{2}=\frac{\pi}{15}\left[-x\left(3x^4+5x^2-45x-120\right)\right]_{-1}^{2}=$$

$$\frac{\pi}{15}\left[-2\left(3\cdot2^4+5\cdot2^2-45\cdot2-120\right)+(-1)\left(3(-1)^4+5(-1)^2-45(-1)-120\right)\right]=$$

$$\frac{\pi}{15}\left[-2(48+20-90-120)-(3+5+45-120)\right]=\frac{\pi}{15}\left[-2(-142)-(-67)\right]=\frac{\pi}{15}\left[284+67\right]=$$

$$\frac{\pi}{15}351=\frac{117\pi}{5}=23,4\pi$$

(115)

Cálculo de volume com integrais simples – <u>Aula 2</u>
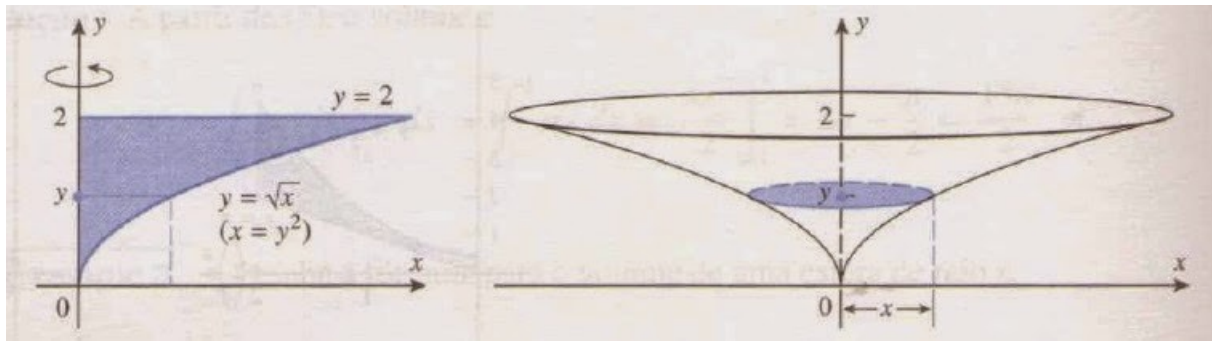
Volume do cilindro: $v=\pi r^2 h\,;r=x=f(y)\,;h=\partial y\,;\partial v=\pi\left[f(y)\right]^2\partial y$

Volume do sólido: $v=\int\limits_a^b\partial v=\int\limits_a^b\pi\left[f(y)\right]^2\partial y\rightarrow v=\pi\int\limits_a^b\left[f(y)\right]^2\partial y$

Exercício I

A região limitada pelas funções $y=x^3$ e $y=8$, gira em torno do eixo y. Determine o volume do sólido de revolução obtido.
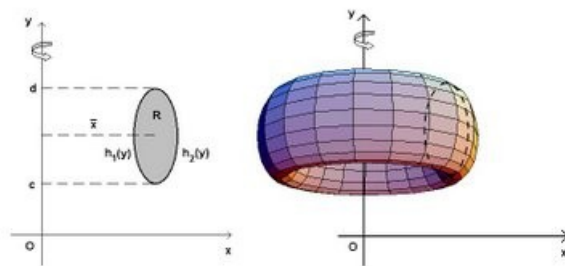
Volume do sólido em torno do eixo y:

$$y=f(x)=x^3\rightarrow x=f(y)=\sqrt[3]{y}$$

$$f(x)=0\rightarrow x^3=0\rightarrow x=\sqrt[3]{0}=0\rightarrow x_1=0\,;f(x)=8\rightarrow x^3=8\rightarrow x=\sqrt[3]{8}=2\rightarrow x_2=2$$

$$f(y)=0\rightarrow\sqrt[3]{y}=0\rightarrow y=0^3=0\rightarrow y_1=0\,;f(y)=2\rightarrow\sqrt[3]{y}=2\rightarrow y=2^3=8\rightarrow y_2=8$$

$$v=\pi\int\limits_0^8\left[f(y)\right]^2\partial y=\pi\int\limits_0^8\left(\sqrt[3]{y}\right)^2\partial y=\pi\int\limits_0^8 y^{\frac{2}{3}}\partial y=\pi\left[\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\right]_0^8=\pi\left[\frac{3\sqrt[3]{y^5}}{5}\right]_0^8=\frac{3\pi}{5}\left[\sqrt[3]{y^5}\right]_0^8=$$

$$\frac{3\pi}{5}\left[\sqrt[3]{8^5}-\sqrt[3]{0^5}\right]=\frac{3\sqrt[3]{\left(2^3\right)^5}\pi}{5}=\frac{3\cdot 2^5\pi}{5}=\frac{96\pi}{5}=19{,}2\,\pi$$

(116)

Volume do sólido em torno do eixo x:

$$v=\pi\int\limits_0^2\left[f(x)\right]^2\partial x=\pi\int\limits_0^2\left[x^3\right]^2\partial x=\pi\int\limits_0^2 x^6\partial x=\pi\left[\frac{x^7}{7}\right]_0^2=\frac{\pi}{7}\left[x^7\right]_0^2=\frac{\pi}{7}\left[2^7-0^7\right]=\frac{128\pi}{7}=$$

$$18{,}2857142\,\pi$$

$$\lim_{x\to\infty}f(x)\rightarrow\lim_{x\to\infty}\left[x^3\right]=\infty^3=\infty\,;\lim_{x\to-\infty}f(x)\rightarrow\lim_{x\to-\infty}\left[x^3\right]=(-\infty)^3=-\infty$$

Volume do cilindro: $v = \pi r_1^2 h - \pi r_2^2 h = \pi\left(r_1^2 - r_2^2\right)h ; r_1 = f(y) ; r_2 = g(y) ; h = \partial y$

$$\partial v = \pi\left[(f(y))^2 - (g(y))^2\right]\partial y$$

Volume do sólido: $v = \int\limits_a^b \partial v = \int\limits_a^b \pi\left[(f(y))^2 - (g(y))^2\right]\partial y \to v = \pi\int\limits_a^b \left[(f(y))^2 - (g(y))^2\right]\partial y$

Integral por Partes – Aula 1

$$\int u\,\partial v = uv - \int v\,\partial u$$

Regras: LIATE ou ILATE

L = Logaritmica: $\ln|x| ; \ln|x+1|$
I = Inversa da trigonométrica: $arc\,tg(x) ; arc\,sen(x)$
A = Aritmética: $x^2 ; x^{-3}$
T = Trigométrica: $sen(x) ; \cos(x)$
E = Exponencial: $e^x ; e^{x+1}$

Exercício I

$$\int x^2 sen(x)\partial x = x^2(-\cos(x)) - \int (-\cos(x))2x\,\partial x = -x^2\cos(x) + 2\int x\cos(x)\partial x =$$
$$-x^2\cos(x) + 2(x\,sen(x) + \cos(x) + c) = -x^2\cos(x) + 2x\,sen(x) + 2\cos(x) + c$$
$$u = x^2 \to \partial u = 2x\,\partial x$$
$$v = -\cos(x) \to \partial v = sen(x)\partial x$$
$$\int x\cos(x)\partial x = x\,sen(x) - \int sen(x)\partial = x\,sen(x) - (-\cos(x)) + c = x\,sen(x) + \cos(x) + c$$
$$u = x \to \partial u = \partial x$$
$$v = sen(x) \to \partial v = \cos(x)\partial x$$

(117)

$$\frac{\partial\left(-x^2\cos(x) + 2x\,sen(x) + 2\cos(x) + c\right)}{\partial x} =$$
$$\left[-\left(2x\cos(x) + x^2(-sen(x))\right)\right] + (2\,sen(x) + 2x\cos(x)) + 2(-sen(x)) + 0 =$$
$$-2x\cos(x) + x^2\,sen(x) + 2\,sen(x) + 2x\cos(x) - 2\,sen(x) = x^2\,sen(x)$$

Exercício II

$$\int \ln|x| \, tg(x) \, \partial x = \ln|x| \ln|sec(x)| - \int \ln|sec(x)| x^{-1} \partial x =$$

$$\ln|x| \ln|\cos^{-1}(x)| - \int \ln|\cos^{-1}(x)| x^{-1} \partial x = -\ln|x| \ln|\cos(x)| + \int \ln|\cos(x)| x^{-1} \partial x =$$

$$\cancel{-\ln|x| \ln|\cos(x)|} + \cancel{\ln|\cos(x)| \ln|x|} + \int \ln|x| \, tg(x) \, \partial x$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x$$

$$v = \ln|sec(x)| \rightarrow \partial v = tg(x) \partial x \tag{118}$$

$$\int \ln|\cos(x)| x^{-1} \partial x = \ln|\cos(x)| \ln|x| + \int \ln|x| \, tg(x) \, \partial x$$

$$u = \ln|\cos(x)| \rightarrow \partial u = \frac{1}{\cos(x)}(-sen(x)) \partial x = \frac{-sen(x)}{\cos(x)} \partial x = -tg(x) \partial x$$

$$v = \ln|x| \rightarrow \partial v = \frac{1}{x} \partial x$$

Exercício III

$$\int x \, sen(x) \partial x = -x\cos(x) + \int \cos(x) \partial x = -x\cos(x) + sen(x) + c$$

$$u = x \rightarrow \partial u = \partial x$$

$$v = -\cos(x) \rightarrow \partial v = sen(x) \partial x \tag{119}$$

$$\frac{\partial(-x\cos(x) + sen(x) + c)}{\partial x} = -[\cos(x) + x(-sen(x))] + \cos(x) + 0 =$$

$$\cancel{-\cos(x)} + x \, sen(x) + \cancel{\cos(x)} = x \, sen(x)$$

Exercício IV

$$\int x \, e^x \partial x = x \, e^x - \int e^x \partial x = x \, e^x - e^x + c$$

$$u = x \rightarrow \partial u = \partial x$$

$$\partial v = e^x \partial x \rightarrow v = \int \partial v = e^x \tag{120}$$

$$\frac{\partial(x \, e^x - e^x + c)}{\partial x} = \cancel{e^x} + x \, e^x - \cancel{e^x} + 0 = x \, e^x$$

Exercício V

$$\int \ln|x| \partial x = \ln|x| x - \int x \frac{1}{x} \partial x = x \ln|x| - \int \partial x = x \ln|x| - x + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x$$

$$\partial v = \partial x \rightarrow v = \int \partial v = x \tag{121}$$

$$\frac{\partial(x \ln|x| - x + c)}{\partial x} = \ln|x| + x\frac{1}{x} - 1 + 0 = \ln|x| + 1 - 1 = \ln|x|$$

Exercício VI

$$\int x^2 \cos(x)\,\partial x = x^2 \, sen(x) - 2\int sen(x)\, x\, \partial x = x^2\, sen(x) - 2\left(-x\cos(x) + sen(x)\right) + c =$$
$$x^2\, sen(x) + 2x\cos(x) - 2\, sen(x) + c$$
$$u = x^2 \rightarrow \partial u = 2x\, \partial x$$
$$\partial v = \cos(x)\, \partial x \rightarrow v = \int \partial v = sen(x)$$
$$\int sen(x)\, x\, \partial x = -x\cos(x) + \int \cos(x)\, \partial x = -x\cos(x) + sen(x) + c$$
$$u = x \rightarrow \partial u = \partial x$$
$$\partial v = sen(x)\, \partial x \rightarrow v = \int \partial v = -\cos(x)$$

(122)

$$\frac{\partial\left(x^2\, sen(x) + 2x\cos(x) - 2\, sen(x) + c\right)}{\partial x} =$$
$$2x\, sen(x) + x^2\cos(x) + \cancel{2\cos(x)} + 2x(-sen(x)) - \cancel{2\cos(x)} + 0 =$$
$$\cancel{2x\, sen(x)} + x^2\cos(x) - \cancel{2x\, sen(x)} = x^2\cos(x)$$

Exercício VII

$$\int \frac{x}{\sqrt{x+1}}\, \partial x = \int \frac{x}{(x+1)^{\frac{1}{2}}}\, \partial x = \int (x+1)^{\frac{-1}{2}}\, x\, \partial x = 2x\sqrt{x+1} - 2\int \sqrt{x+1}\, \partial x =$$
$$2x\sqrt{x+1} - 2\int (x+1)^{\frac{1}{2}}\, \partial x = 2x\sqrt{x+1} - 2\frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = 2x\sqrt{x+1} - \frac{4\sqrt{(x+1)^3}}{3} + c$$
$$u = x \rightarrow \partial u = \partial x$$
$$\partial v = (x+1)^{\frac{-1}{2}}\, \partial x \rightarrow v = \int \partial v = \frac{(x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = 2\sqrt{x+1}$$

(123)

$$\frac{\partial\left(2x\sqrt{x+1} - \frac{4\sqrt{(x+1)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(2x(x+1)^{\frac{1}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}} + c\right)}{\partial x} =$$
$$2(x+1)^{\frac{1}{2}} + 2x\frac{1}{2}(x+1)^{\frac{-1}{2}} - \frac{4}{3}\frac{3}{2}(x+1)^{\frac{1}{2}} + 0 = \cancel{2\sqrt{x+1}} + \frac{x}{\sqrt{x+1}} - \cancel{2\sqrt{x+1}} = \frac{x}{\sqrt{x+1}}$$

Exercício I

$$\int (2x+1)\,sen(x)\,\partial x = -(2x+1)\cos(x)+2\int \cos(x)\,\partial x = -(2x+1)\cos(x)+2\,sen(x)+c =$$
$$-2x\cos(x)-\cos(x)+2\,sen(x)+c$$
$$u=2x+1 \rightarrow \partial u = 2\,\partial x$$
$$\partial v = sen(x)\,\partial x \rightarrow v = \int \partial v = -\cos(x)$$

$$\tag{124}$$

$$\frac{\partial\left(-(2x+1)\cos(x)+2\,sen(x)+c\right)}{\partial x} = -\left[2\cos(x)+(2x+1)(-sen(x))\right]+2\cos(x)+0 =$$
$$\cancel{-2\cos(x)}+(2x+1)\,sen(x)+\cancel{2\cos(x)} = (2x+1)\,sen(x)$$

Exercício II

$$\int \ln|x|\frac{\partial x}{x^2} = \int \ln|x|\,x^{-2}\,\partial x = \ln|x|\left(\frac{-1}{x}\right)+\int \frac{1}{x}\frac{1}{x}\,\partial x = \frac{-\ln|x|}{x}+\int x^{-2}\,\partial x = \frac{-\ln|x|}{x}+\frac{x^{-1}}{(-1)}+c$$

$$\frac{-\ln|x|}{x}-\frac{1}{x}+c = \frac{-\ln|x|-1}{x}+c = \frac{\ln|x^{-1}|-1}{x}+c = \frac{\ln\left|\frac{1}{x}\right|-1}{x}+c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x}\,\partial x$$
$$\partial v = x^{-2}\,\partial x \rightarrow v = \int \partial v = \frac{x^{-1}}{(-1)} = \frac{-1}{x}$$

$$\tag{125}$$

$$\frac{\partial\left(\frac{\ln\left|\frac{1}{x}\right|-1}{x}+c\right)}{\partial x} = \frac{\partial\left(\frac{\ln|x^{-1}|-1}{x}+c\right)}{\partial x} = \frac{\frac{1}{x^{-1}}(-x^{-2})x-(\ln|x^{-1}|-1)}{x^2}+0 =$$

$$\frac{\frac{-1}{\left(\frac{1}{x}\right)}\left(\frac{1}{x^2}\right)x-\left(\ln\left|\frac{1}{x}\right|-1\right)}{x^2} = \frac{\cancel{-1}-\ln\left|\frac{1}{x}\right|\cancel{+1}}{x^2} = \frac{-\ln|x^{-1}|}{x^2} = \frac{\ln|x|}{x^2}$$

Exercício III

$$\int sen(x)\cos(x)\,\partial x = sen(x)\,sen(x)-\int sen(x)\cos(x)\,\partial x \rightarrow$$
$$\int sen(x)\cos(x)\,\partial x+\int sen(x)\cos(x)\,\partial x = sen^2(x) \rightarrow 2\int sen(x)\cos(x)\,\partial x = sen^2(x) \rightarrow$$
$$\int sen(x)\cos(x)\,\partial x = \frac{sen^2(x)}{2}+c$$
$$u = sen(x) \rightarrow \partial u = \cos(x)\,\partial x$$
$$\partial v = \cos(x)\,\partial x \rightarrow v = \int \partial v = sen(x)$$

$$\tag{126}$$

$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x} = \frac{\partial\left(\frac{1}{2}sen^2(x)+c\right)}{\partial x} = \frac{1}{\cancel{2}}\cancel{2}\,sen(x)\cos(x)+0 = sen(x)\cos(x)$$

Exercício IV

$$\int \sqrt{x}\ln|x|\partial x = \int \ln|x|x^{\frac{1}{2}}\partial x = \ln|x|\frac{2\sqrt{x^3}}{3} - \int \frac{2\sqrt{x^3}}{3}\frac{1}{x}\partial x = \frac{2\sqrt{x^3}\ln|x|}{3} - \frac{2}{3}\int x^{\frac{3}{2}}x^{-1}\partial x =$$

$$\frac{2\sqrt{x^3}\ln|x|}{3} - \frac{2}{3}\int x^{\frac{1}{2}}\partial x = \frac{2\sqrt{x^3}\ln|x|}{3} - \frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2\sqrt{x^3}\ln|x|}{3} - \frac{4\sqrt{x^3}}{9} + c =$$

$$\frac{2\sqrt{x^2 x}\ln|x|}{3} - \frac{4\sqrt{x^2 x}}{9} + c = \frac{2x\sqrt{x}\ln|x|}{3} - \frac{4x\sqrt{x}}{9} + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x}\partial x$$

(127)

$$\partial v = x^{\frac{1}{2}}\partial x \rightarrow v = \int \partial v = \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} = \frac{2\sqrt{x^3}}{3}$$

$$\frac{\partial\left(\frac{2x\sqrt{x}\ln|x|}{3} - \frac{4x\sqrt{x}}{9} + c\right)}{\partial x} = \frac{\partial\left(\frac{2}{3}x^{\frac{3}{2}}\ln|x| - \frac{4}{9}x^{\frac{3}{2}} + c\right)}{\partial x} = \frac{2}{3}\frac{3}{2}x^{\frac{1}{2}}\ln|x| + \frac{2}{3}x^{\frac{3}{2}}\frac{1}{x} - \frac{4}{9}\frac{3}{2}x^{\frac{1}{2}} + 0 =$$

$$x^{\frac{1}{2}}\ln|x| + \frac{2}{3}x^{\frac{3}{2}-1} - \frac{2}{3}x^{\frac{1}{2}} = x^{\frac{1}{2}}\ln|x|\frac{+2\sqrt{x}}{3}\frac{-2\sqrt{x}}{3} = \sqrt{x}\ln|x|$$

Integral por Partes – Aula 3

Exercício I

$$\int arc\,sen(x)\partial x = x\,arc\,sen(x) - \int x\frac{1}{\sqrt{1-x^2}}\partial x = x\,arc\,sen(x) - \int \left(1-x^2\right)^{\frac{-1}{2}}x\partial x =$$

$$x\,arc\,sen(x) + \sqrt{1-x^2} + c$$

$$u = arc\,sen(x) \rightarrow \partial u = \frac{1}{\sqrt{1-x^2}}\partial x$$

$$\partial v = \partial x \rightarrow v = \int \partial v = x$$

$$\int \left(1-x^2\right)^{\frac{-1}{2}}x\partial x = \int u^{\frac{-1}{2}}\left(\frac{-\partial u}{2}\right) = \frac{-1}{2}\int u^{\frac{-1}{2}}\partial u = \frac{-1}{2}\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c = -\sqrt{u} + c = -\sqrt{1-x^2} + c$$

(128)

$$u = 1-x^2 \rightarrow \partial u = -2x\partial x \rightarrow \frac{-\partial u}{2} = x\partial x$$

$$\frac{\partial\left(x\,arc\,sen(x) + \sqrt{1-x^2} + c\right)}{\partial x} = \frac{\partial\left(x\,arc\,sen(x) + \left(1-x^2\right)^{\frac{1}{2}} + c\right)}{\partial x} =$$

$$arc\,sen(x) + x\frac{1}{\sqrt{1-x^2}} + \frac{1}{2}\left(1-x^2\right)^{\frac{-1}{2}}(-2x) + 0 = arc\,sen(x) + \frac{x}{\sqrt{1-x^2}} - \frac{2x}{2\sqrt{1-x^2}} =$$

$$arc\,sen(x)\frac{+x}{\sqrt{1-x^2}}\frac{-x}{\sqrt{1-x^2}} = arc\,sen(x)$$

Exercício II

$$\int arc\,tg(x)\partial x = x\,arc\,tg(x) - \int x\frac{1}{1+x^2}\partial x = x\,arc\,tg(x) - \int \left(1+x^2\right)^{-1}x\partial x =$$

$$x\,arc\,tg(x) - \frac{\ln\left|1+x^2\right|}{2} + c$$

$$u = arc\,tg(x) \rightarrow \partial u = \frac{1}{1+x^2}\partial x$$

$$\partial v = \partial x \rightarrow v = \int \partial v = x$$

$$\int \left(1+x^2\right)^{-1}x\partial x = \int u^{-1}\frac{\partial u}{2} = \frac{1}{2}\int u^{-1}\partial u = \frac{1}{2}\ln|u| + c = \frac{\ln|u|}{2} + c = \frac{\ln\left|1+x^2\right|}{2} + c \qquad (129)$$

$$u = 1+x^2 \rightarrow \partial u = 2x\partial x \rightarrow \frac{\partial u}{2} = x\partial x$$

$$\frac{\partial\left(x\,arc\,tg(x) - \frac{\ln\left|1+x^2\right|}{2} + c\right)}{\partial x} = \frac{\partial\left(x\,arc\,tg(x) - \frac{1}{2}\ln\left|1+x^2\right| + c\right)}{\partial x} =$$

$$arc\,tg(x) + x\frac{1}{1+x^2} - \frac{1}{\cancel{2}}\frac{1}{1+x^2}\cancel{2}x + 0 = arc\,tg(x)\frac{+x}{1+x^2}\frac{-x}{1+x^2} = arc\,tg(x)$$

Exercício III

$$\int x\,arc\,tg(x)\partial x = \frac{x^2}{2}arc\,tg(x) - \int \frac{x^2}{2}\frac{1}{1+x^2}\partial x = \frac{x^2\,arc\,tg(x)}{2} - \frac{1}{2}\int \left(1+x^2\right)^{-1}x^2\partial x =$$

$$\frac{x^2\,arc\,tg(x)}{2} - \frac{1}{2}(x - arc\,tg(x)) + c = \frac{x^2\,arc\,tg(x)}{2} - \frac{x}{2} + \frac{arc\,tg(x)}{2} + c =$$

$$\frac{x^2\,arc\,tg(x) + arc\,tg(x) - x}{2} + c$$

$$u = arc\,tg(x) \rightarrow \partial u = \frac{1}{1+x^2}\partial x$$

$$\partial v = x\partial x \rightarrow v = \int \partial v = \frac{x^2}{2}$$

$$\int \left(1+x^2\right)^{-1}x^2\partial x = \int \frac{x^2}{1+x^2}\partial x = \int \left(1 - \frac{1}{1+x^2}\right)\partial x = \int \partial x - \int \frac{1}{1+x^2}\partial x = x - arc\,tg(x) + c \qquad (130)$$

$$\frac{\partial\left(\frac{x^2\,arc\,tg(x) + arc\,tg(x) - x}{2} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{2}x^2\,arc\,tg(x) + \frac{1}{2}arc\,tg(x) - \frac{1}{2}x + c\right)}{\partial x} =$$

$$\frac{1}{2}\left(2x\,arc\,tg(x) + x^2\frac{1}{1+x^2}\right) + \frac{1}{2}\frac{1}{1+x^2} - \frac{1}{2} + 0 = x\,arc\,tg(x) + \frac{x^2}{2\left(1+x^2\right)} + \frac{1}{2\left(1+x^2\right)} - \frac{1}{2} =$$

$$\frac{2\left(1+x^2\right)x\,arc\,tg(x) + x^2 + 1 - \left(1+x^2\right)}{2\left(1+x^2\right)} = \frac{2\cancel{\left(1+x^2\right)}x\,arc\,tg(x) + \cancel{x^2} + \cancel{1} - \cancel{1} - \cancel{x^2}}{2\cancel{\left(1+x^2\right)}} = x\,arc\,tg(x)$$

Integral por Partes – Aula 4

Exercício I

$$\int e^x sen(x) \partial x = sen(x) e^x - \int e^x \cos(x) \partial x = sen(x) e^x - \left(\cos(x) e^x + \int e^x sen(x) \partial x \right) =$$

$$sen(x) e^x - \cos(x) e^x - \int e^x sen(x) \partial x \rightarrow$$

$$\int e^x sen(x) \partial x + \int e^x sen(x) \partial x = sen(x) e^x - \cos(x) e^x \rightarrow$$

$$2 \int e^x sen(x) \partial x = sen(x) e^x - \cos(x) e^x \rightarrow \int e^x sen(x) \partial x = \frac{sen(x) e^x - \cos(x) e^x}{2} + c =$$

$$\frac{e^x \left(sen(x) - \cos(x) \right)}{2} + c$$

$$u = sen(x) \rightarrow \partial u = \cos(x) \partial x$$

$$\partial v = e^x \partial x \rightarrow v = \int \partial v = e^x$$

$$\int e^x \cos(x) \partial x = \cos(x) e^x + \int e^x sen(x) \partial x \tag{131}$$

$$u = \cos(x) \rightarrow \partial u = -sen(x) \partial x$$

$$\partial v = e^x \partial x \rightarrow v = \int \partial v = e^x$$

$$\frac{\partial \left(\frac{e^x \left(sen(x) - \cos(x) \right)}{2} + c \right)}{\partial x} = \frac{\partial \left(\frac{1}{2} e^x sen(x) - \frac{1}{2} e^x \cos(x) + c \right)}{\partial x} =$$

$$\frac{1}{2} \left(e^x sen(x) + e^x \cos(x) \right) - \frac{1}{2} \left[e^x \cos(x) + e^x \left(-sen(x) \right) \right] + 0 =$$

$$\frac{e^x sen(x)}{2} + \frac{e^x \cos(x)}{2} - \frac{e^x \cos(x)}{2} + \frac{e^x sen(x)}{2} = \frac{2 e^x sen(x)}{2} = e^x sen(x)$$

Exercício II

$$\int \ln|x| x^5 \partial x = \frac{x^6}{6} \ln|x| - \int \frac{x^6}{6} \frac{1}{x} \partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6} \int x^5 \partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6} \frac{x^6}{6} + c =$$

$$\frac{x^6 \ln|x|}{6} - \frac{x^6}{36} + c = \frac{6 x^6 \ln|x| - x^6}{36} + c = \frac{x^6 \left(6 \ln|x| - 1 \right)}{36} + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x$$

$$\partial v = x^5 \partial x \rightarrow v = \int \partial v = \frac{x^6}{6} \tag{132}$$

$$\frac{\partial \left(\frac{x^6 \left(6 \ln|x| - 1 \right)}{36} + c \right)}{\partial x} = \frac{\partial \left(\frac{1}{6} x^6 \ln|x| - \frac{1}{36} x^6 + c \right)}{\partial x} = \frac{1}{6} \left(6 x^5 \ln|x| + x^6 \frac{1}{x} \right) - \frac{1}{36} 6 x^5 + 0 =$$

$$x^5 \ln|x| + \frac{x^6}{6x} - \frac{x^5}{6} = \frac{6 x^6 5 \ln|x| + x^6 - x^6}{6x} = x^5 \ln|x|$$

Exercício III

$$\int sec^3(x)\partial x = \int sec(x)sec^2(x)\partial x = sec(x)tg(x) - \int tg(x)sec(x)tg(x)\partial x =$$
$$sec(x)tg(x) - \int tg^2(x)sec(x)\partial x = sec(x)tg(x) - \int \left(sec^2(x) - 1\right)sec(x)\partial x =$$
$$sec(x)tg(x) - \int sec^3(x)\partial x + \int sec(x)\partial x \rightarrow$$
$$\int sec^3(x)\partial x + \int sec^3(x)\partial x = sec(x)tg(x) + \int sec(x)\partial x \rightarrow$$
$$2\int sec^3(x)\partial x = sec(x)tg(x) + \ln|sec(x) + tg(x)| + c \rightarrow$$
$$\int sec^3(x)\partial x = \frac{sec(x)tg(x) + \ln|sec(x) + tg(x)|}{2} + c$$
$$u = sec(x) \rightarrow \partial u = sec(x)tg(x)\partial x$$
$$\partial v = sec^2(x)\partial x \rightarrow v = \int \partial v = tg(x)$$

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Integral por Partes – Aula 5

Exercício I

$$\int x\,sec^2(x)\partial x = x\,tg(x) - \int tg(x)\partial x = x\,tg(x) - \ln|sec(x)| + c$$
$$u = x \rightarrow \partial u = \partial x$$
$$\partial v = sec^2(x)\partial x \rightarrow v = \int \partial v = tg(x)$$

$$\frac{\partial\left(x\,tg(x) - \ln|sec(x)| + c\right)}{\partial x} = \frac{\partial\left(x\,tg(x) - \ln|\cos^{-1}(x)| + c\right)}{\partial x} = \frac{\partial\left(x\,tg(x) + \ln|\cos(x)| + c\right)}{\partial x} =$$
$$tg(x) + x\,sec^2(x) + \frac{1}{\cos(x)}(-sen(x)) + 0 = tg(x) + x\,sec^2(x) - \frac{sen(x)}{\cos(x)} =$$
$$\cancel{tg(x)} + x\,sec^2(x) \cancel{- tg(x)} = x\,sec^2(x)$$
$$\frac{\partial\left(x\,tg(x) - \ln|sec(x)| + c\right)}{\partial x} = tg(x) + x\,sec^2(x) - \frac{1}{\cancel{sec(x)}}\cancel{sec(x)}tg(x) + 0 =$$
$$\cancel{tg(x)} + x\,sec^2(x) \cancel{- tg(x)} = x\,sec^2(x)$$

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Integral por Frações Parciais – Aula 1

$$\int \frac{(x-2)\partial x}{(x-1)(x+1)(x-3)} = \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-3}\right)\partial x$$
$$\int \frac{(x-2)\partial x}{(x-1)^3(x+1)} = \int \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}\right)\partial x$$
$$\int \frac{(x-2)\partial x}{\left(x^2 - 2x + 1\right)(x+3)} = \int \frac{(Ax+B)\partial x}{x^2 - 2x + 1} + \int \frac{C\partial x}{x+3}$$

Exercício I

$$\int \frac{\partial x}{x^2-4}=\int \frac{\partial x}{(x-2)(x+2)}=\int \left(\frac{A}{x-2}+\frac{B}{x+2}\right)\partial x=\int \frac{\partial x}{x^2-4}=\int \left(\frac{\left(\frac{1}{4}\right)}{x-2}+\frac{\left(\frac{-1}{4}\right)}{x+2}\right)\partial x=$$

$$\int \left(\frac{1}{4(x-2)}-\frac{1}{4(x+2)}\right)\partial x=\frac{1}{4}\int (x-2)^{-1}\partial x-\frac{1}{4}\int (x+2)^{-1}\partial x=$$

$$\frac{1}{4}\ln|x-2|-\frac{1}{4}\ln|x+2|+c=\frac{1}{4}\left(\ln|x-2|-\ln|x+2|\right)+c$$

$$\frac{\frac{1}{(x-2)(x+2)}=\frac{A}{x-2}+\frac{B}{x+2}}{(x-2)(x+2)} \rightarrow$$

$$1=A(x+2)+B(x-2)$$

$$x=-2 \rightarrow 1=A(-2+2)+B(-2-2) \rightarrow 1=0-4B \rightarrow B=\frac{-1}{4}$$

$$x=2 \rightarrow 1=A(2+2)+B(2-2) \rightarrow 1=4A+0 \rightarrow A=\frac{1}{4}$$

(135)

$$\frac{\partial \left[\frac{1}{4}\left(\ln|x-2|-\ln|x+2|\right)+c\right]}{\partial x}=\frac{1}{4}\left(\frac{1}{x-2}-\frac{1}{x+2}\right)=\frac{1}{4}\left(\frac{x+2-(x-2)}{x^2-4}\right)=\frac{1}{4}\left(\frac{x+2-x+2}{x^2-4}\right)=$$

$$\frac{1}{4}\frac{4}{x^2-4}=\frac{1}{x^2-4}$$

Exercício II

$$\int \frac{(x-1)\partial x}{x^3-x^2-2x}=\int \frac{(x-1)\partial x}{x(x^2-x-2)}=\int \frac{(x-1)\partial x}{x(x-2)(x+1)}=\int\left(\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+1}\right)\partial x=$$

$$\int\left(\frac{\left(\frac{1}{2}\right)}{x}+\frac{\left(\frac{1}{6}\right)}{x-2}+\frac{\left(\frac{-2}{3}\right)}{x+1}\right)\partial x=\frac{1}{2}\int x^{-1}\partial x+\frac{1}{6}\int (x-2)^{-1}\partial x-\frac{2}{3}(x+1)^{-1}=$$

$$\frac{1}{2}\ln|x|+\frac{1}{6}\ln|x-2|-\frac{2}{3}\ln|x+1|+c$$

$$x^2-x-2=0\rightarrow x^2-x-2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2=0\rightarrow x^2-x+\frac{1}{4}-\frac{9}{4}=0\rightarrow\left(x-\frac{1}{2}\right)^2=\frac{9}{4}\rightarrow x=$$

$$\pm\sqrt{\frac{9}{4}}+\frac{1}{2}=\frac{\pm3}{2}+\frac{1}{2}$$

$$x_1=\frac{3+1}{2}=\frac{4}{2}=2\,;\,x_2=\frac{-3+1}{2}=\frac{-2}{2}=-1$$

$$ax^2+bx+c=a(x-x_1)(x-x_2)\rightarrow x^2-x-2=(x-2)(x-(-1))=(x-2)(x+1)$$

$$\frac{\frac{x-1}{x(x-2)(x+1)}=\frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+1}}{x(x-2)(x+1)}\rightarrow x-1=A(x-2)(x+1)+Bx(x+1)+Cx(x-2)$$

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$$x=-1\rightarrow -1-1=A(-1-2)(-1+1)+B(-1)(-1+1)+C(-1)(-1-2)\rightarrow -2=3C\rightarrow C=\frac{-2}{3}$$

$$x=2\rightarrow 2-1=A(2-2)(2+1)+B(2)(2+1)+C(2)(2-2)\rightarrow 1=6B\rightarrow B=\frac{1}{6}$$

$$x=0\rightarrow 0-1=A(0-2)(0+1)+B(0)(0+1)+C(0)(0-2)\rightarrow -1=-2A\rightarrow A=\frac{1}{2}$$

$$\frac{\partial\left(\frac{1}{2}\ln|x|+\frac{1}{6}\ln|x-2|-\frac{2}{3}\ln|x+1|+c\right)}{\partial x}=\frac{1}{2}\frac{1}{x}+\frac{1}{6}\frac{1}{x-2}-\frac{2}{3}\frac{1}{x+1}+0=$$

$$\frac{1}{2x}+\frac{1}{6(x-2)}-\frac{2}{3(x+1)}=\frac{18(x-2)(x+1)+6x(x+1)-24x(x-2)}{36x(x-2)(x+1)}=$$

$$\frac{6(3(x-2)(x+1)+x(x+1)-4x(x-2))}{36x(x-2)(x+1)}=\frac{3x^2-3x-6+x^2+x-4x^2+8x}{6x(x-2)(x+1)}=$$

$$\frac{6x-6}{6x(x-2)(x+1)}=\frac{6(x-1)}{6x(x-2)(x+1)}=\frac{x-1}{x(x^2-x-2)}=\frac{x-1}{x^3-x^2-2x}$$

Integral por Frações Parciais – <u>Aula 2</u>

Exercício I

$$\int \frac{(x-2)\,\partial x}{x^3-3x^2-x+3} = \int \frac{(x-2)\,\partial x}{x(x^2-1)-3(x^2-1)} = \int \frac{(x-2)\,\partial x}{(x^2-1)(x-3)} = \int \frac{(x-2)\,\partial x}{(x+1)(x-1)(x-3)} =$$

$$\int \left(\frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x-3} \right) \partial x = \int \left(\frac{\left(\frac{-3}{8}\right)}{x+1} + \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{\left(\frac{1}{8}\right)}{x-3} \right) \partial x =$$

$$\frac{-3}{8} \int (x+1)^{-1} \partial x + \frac{1}{4} \int (x-1)^{-1} \partial x + \frac{1}{8} \int (x-3)^{-1} \partial x =$$

$$\frac{-3}{8} \ln|x+1| + \frac{1}{4} \ln|x-1| + \frac{1}{8} \ln|x-3| + c = \frac{-3\ln|x+1|}{8} + \frac{\ln|x-1|}{4} + \frac{\ln|x-3|}{8} + c$$

$$\frac{\frac{x-2}{(x+1)(x-1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x-3}}{(x+1)(x-1)(x-3)} \rightarrow$$

$$x-2 = A(x-1)(x-3) + B(x+1)(x-3) + C(x+1)(x-1)$$

$$x=1 \rightarrow 1-2 = A(1-1)(1-3) + B(1+1)(1-3) + C(1+1)(1-1) \rightarrow -1 = -4B \rightarrow B = \frac{1}{4}$$

$$x=3 \rightarrow 3-2 = A(3-1)(3-3) + B(3+1)(3-3) + C(3+1)(3-1) \rightarrow 1 = 8C \rightarrow C = \frac{1}{8}$$

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$$x=-1 \rightarrow -1-2 = A(-1-1)(-1-3) + B(-1+1)(-1-3) + C(-1+1)(-1-1) \rightarrow -3 = 8A \rightarrow$$

$$A = \frac{-3}{8}$$

$$\frac{\partial \left(\frac{-3\ln|x+1|}{8} + \frac{\ln|x-1|}{4} + \frac{\ln|x-3|}{8} + c \right)}{\partial x} = \frac{\partial \left(\frac{-3}{8}\ln|x+1| + \frac{1}{4}\ln|x-1| + \frac{1}{8}\ln|x-3| + c \right)}{\partial x} =$$

$$\frac{-3}{8} \frac{1}{x+1} + \frac{1}{4} \frac{1}{x-1} + \frac{1}{8} \frac{1}{x-3} + 0 = \frac{-3}{8(x+1)} + \frac{1}{4(x-1)} + \frac{1}{8(x-3)} =$$

$$\frac{-96(x-1)(x-3) + 64(x+1)(x-3) + 32(x+1)(x-1)}{256(x+1)(x-1)(x-3)} =$$

$$\frac{-32\left[3(x^2-4x+3) - 2(x^2-2x-3) - (x^2-1) \right]}{256(x+1)(x-1)(x-3)} = \frac{-\left[3x^2-12x+9 -2x^2+4x+6 -x^2+1 \right]}{8(x+1)(x-1)(x-3)} =$$

$$\frac{-8\left[-x+2 \right]}{8(x+1)(x-1)(x-3)} = \frac{x-2}{(x^2-1)(x-3)} = \frac{x-2}{x^3-3x^2-x+3}$$

Exercício I

$$\int \frac{-4x^3 \partial x}{2x^3+x^2-2x-1} = \int \left(-2+\frac{2x^2-4x-2}{2x^3+x^2-2x-1}\right)\partial x = -2\int \partial x + \int \frac{2x^2-4x-2}{2x^3+x^2-2x-1}\partial x =$$

$$-2\int \partial x + \int \frac{2x^2-4x-2}{x^2(2x+1)-(2x+1)}\partial x = -2\int \partial x + \int \frac{2x^2-4x-2}{(2x+1)(x^2-1)}\partial x =$$

$$-2\int \partial x + \int \frac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}\partial x = -2x - \frac{\ln|2x+1|}{3} + 2\ln|x+1| - \frac{2\ln|x-1|}{3} + c$$

$$\int \frac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}\partial x = \int \left(\frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{x-1}\right)\partial x = \int \left(\frac{\left(\frac{-2}{3}\right)}{2x+1}+\frac{2}{x+1}+\frac{\left(\frac{-2}{3}\right)}{x-1}\right)\partial x =$$

$$\frac{-2}{3}\int (2x+1)^{-1}\partial x + 2\int (x+1)^{-1}\partial x + \frac{-2}{3}\int (x-1)^{-1}\partial x =$$

$$\frac{-2}{3}\int (u)^{-1}\frac{\partial u}{2} + 2\int (x+1)^{-1}\partial x + \frac{-2}{3}\int (x-1)^{-1}\partial x = \frac{-1}{3}\ln|u| + 2\ln|x+1| - \frac{2}{3}\ln|x-1| + c =$$

$$\frac{-\ln|2x+1|}{3} + 2\ln|x+1| - \frac{2\ln|x-1|}{3} + c$$

$$\frac{\frac{2x^2-4x-2}{(2x+1)(x+1)(x-1)}}{(2x+1)(x+1)(x-1)} = \frac{\frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{x-1}}{(2x+1)(x+1)(x-1)} \rightarrow$$

$$2x^2-4x-2 = A(x+1)(x-1)+B(2x+1)(x-1)+C(2x+1)(x+1)$$

$$x=-1 \rightarrow 2(-1)^2-4(-1)-2=$$

$$\cancel{A(-1+1)(-1-1)}+B(2(-1)+1)(-1-1)\cancel{+C(2(-1)+1)(-1+1)} \rightarrow$$

$$2+4-2=B(-2+1)(-2) \rightarrow 4=2B \rightarrow B=2$$

$$x=1 \rightarrow 2\cdot1^2-4\cdot1-2 = \cancel{A(1+1)(1-1)}+\cancel{B(2\cdot1+1)(1-1)}+C(2\cdot1+1)(1+1) \rightarrow$$

$$2-4-2=6C \rightarrow C=\frac{-4}{6} \rightarrow C=\frac{-2}{3}$$

$$x=\frac{-1}{2} \rightarrow 2\left(\frac{-1}{2}\right)^2-4\left(\frac{-1}{2}\right)-2=$$

$$A\left(\frac{-1}{2}+1\right)\left(\frac{-1}{2}-1\right)+B\left[\cancel{2\left(\frac{-1}{2}\right)+1}\right]\left(\frac{-1}{2}-1\right)+C\left[\cancel{2\left(\frac{-1}{2}\right)+1}\right]\left(\frac{-1}{2}+1\right) \rightarrow$$

$$2\left(\frac{1}{4}\right)+2-2=A\left(\frac{1}{2}\right)\left(\frac{-3}{2}\right) \rightarrow \frac{1}{2}=\frac{-3A}{4} \rightarrow A=\frac{1}{2}\left(\frac{4}{-3}\right) \rightarrow A=\frac{-2}{3}$$

$$\frac{\partial\left(-2x-\frac{1}{3}\ln|2x+1|+2\ln|x+1|-\frac{2}{3}\ln|x-1|+c\right)}{\partial x} = -2-\frac{1}{3}\frac{1}{2x+1}2+2\frac{1}{x+1}-\frac{2}{3}\frac{1}{x-1}+0=$$

$$-2-\frac{2}{3(2x+1)}+\frac{2}{x+1}-\frac{2}{3(x-1)}=$$

$$\frac{-18(2x+1)(x+1)(x-1)-6(x+1)(x-1)+18(2x+1)(x-1)-6(2x+1)(x+1)}{9(2x+1)(x+1)(x-1)}=$$

$$\frac{-6[3(2x+1)(x+1)(x-1)+(x+1)(x-1)-3(2x+1)(x-1)+(2x+1)(x+1)]}{9(2x+1)(x+1)(x-1)}=$$

$$\frac{-2[6 \; 2x^3+3x^2-6x-3+x^2-1-6x^2+3x+3+2x^2+3x+1]}{3(2x+1)(x+1)(x-1)} = \frac{-4x^3}{2x^3+x^2-2x-1}$$

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Exercício I

$$\int \frac{3x\,\partial x}{(2x+1)(x-1)^2}=\int\left(\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}\right)\partial x=\int\left(\frac{\left(\frac{-2}{3}\right)}{2x+1}+\frac{\left(\frac{1}{3}\right)}{x-1}+\frac{1}{(x-1)^2}\right)\partial x=$$

$$\frac{-2}{3}\int\frac{\partial x}{2x+1}+\frac{1}{3}\int\frac{\partial x}{x-1}+\int\frac{\partial x}{(x-1)^2}=\frac{-2}{3}\int u^{-1}\frac{\partial u}{2}+\frac{1}{3}\int (x-1)^{-1}\partial x+\int (x-1)^{-2}\partial x=$$

$$\frac{-1}{3}\ln|u|+\frac{1}{3}\ln|x-1|+\frac{(x-1)^{-1}}{(-1)}+c=\frac{-\ln|2x+1|}{3}+\frac{\ln|x-1|}{3}-\frac{1}{x-1}+c$$

$$u=2x+1\to\frac{\partial u}{2}=\partial x$$

$$\frac{\frac{3x}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}}{(2x+1)(x-1)^2}\to 3x=A(x-1)^2+B(2x+1)(x-1)+C(2x+1)$$

$$x=1\to 3\cdot 1=A(1-1)^2+B(2\cdot 1+1)(1-1)+C(2\cdot 1+1)\to 3=3C\to C=1$$

$$x=\frac{-1}{2}\to 3\left(\frac{-1}{2}\right)=A\left(\frac{-1}{2}-1\right)^2+B\left[2\left(\frac{-1}{2}\right)+1\right]\left[\frac{-1}{2}-1\right]+C\left[2\left(\frac{-1}{2}\right)+1\right]\to$$

$$\frac{-3}{2}=A\left(\frac{-3}{2}\right)^2\to\frac{-3}{2}=\frac{9A}{4}\to A=\frac{-3}{2}\frac{4}{9}\to A=\frac{-2}{3}$$

(139)

$$x=0\to 3\cdot 0=A(0-1)^2+B(2\cdot 0+1)(0-1)+C(2\cdot 0+1)\to 0=A-B+C\to B=A+C=\frac{1}{3}$$

$$\frac{\partial\left(\frac{-\ln|2x+1|}{3}+\frac{\ln|x-1|}{3}-\frac{1}{x-1}+c\right)}{\partial x}=\frac{\partial\left(\frac{-1}{3}\ln|2x+1|+\frac{1}{3}\ln|x-1|-(x-1)^{-1}+c\right)}{\partial x}=$$

$$\frac{-1}{3}\frac{1}{2x+1}2+\frac{1}{3}\frac{1}{x-1}+(x-1)^{-2}+0=\frac{-2}{3(2x+1)}+\frac{1}{3(x-1)}+\frac{1}{(x-1)^2}=$$

$$\frac{-6(x-1)^3+3(2x+1)(x-1)^2+9(2x+1)(x-1)}{9(2x+1)(x-1)^3}=$$

$$\frac{-3(x-1)\left[2(x-1)^2-(2x+1)(x-1)-3(2x+1)\right]}{93(2x+1)(x-1)^{32}}=$$

$$\frac{-\left[2(x^2-2x+1)-(2x^2-x-1)-3(2x+1)\right]}{3(2x+1)(x-1)^2}=\frac{-\left(2x^2-4x+2-2x^2+x+1-6x-3\right)}{3(2x+1)(x-1)^2}=$$

$$\frac{93x}{3(2x+1)(x-1)^2}=\frac{3x}{(2x+1)(x-1)^2}$$

Exercício II

$$\int \frac{(x+2)\partial x}{x^3(x-1)} = \int \frac{(x+2)\partial x}{(x+0)^3(x-1)} = \int \left(\frac{A}{x+0} + \frac{B}{(x+0)^2} + \frac{C}{(x+0)^3} + \frac{D}{x-1} \right) \partial x =$$

$$\int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} \right) \partial x = A \int x^{-1} \partial x + B \int x^{-2} \partial x + C \int x^{-3} \partial x + D \int (x-1)^{-1} \partial x =$$

$$A \ln|x| + B \frac{x^{-1}}{(-1)} + C \frac{x^{-2}}{(-2)} + D \ln|x-1| + c = A \ln|x| - B \frac{1}{x} - C \frac{1}{2x^2} + D \ln|x-1| + c =$$

$$-3 \ln|x| + 3 \frac{1}{x} + 2 \frac{1}{2 x^2} + 3 \ln|x-1| + c = -3\ln|x| + \frac{3}{x} + \frac{1}{x^2} + 3 \ln|x-1| + c$$

$$\frac{\frac{x+2}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}}{x^3(x-1)} \rightarrow x+2 = Ax^2(x-1) + Bx(x-1) + C(x-1) + Dx^3$$

$$x=1 \rightarrow 1+2 = A \cdot 1^2(1-1) + B \cdot 1(1-1) + C(1-1) + D \cdot 1^3 \rightarrow D=3$$

$$x=0 \rightarrow 0+2 = A \cdot 0^2(0-1) + B \cdot 0(0-1) + C(0-1) + D \cdot 0^3 \rightarrow 2 = -C \rightarrow C=-2$$

$$x+2 = Ax^2(x-1) + Bx(x-1) - 2(x-1) + 3x^3 \rightarrow x+2 = Ax^2(x-1) + Bx(x-1) - 2x + 2 + 3x^3 \rightarrow$$ (140)

$$x+2+2x-2-3x^3 = Ax^2(x-1) + Bx(x-1) \rightarrow -3x^3 + 3x = Ax^2(x-1) + Bx(x-1) \rightarrow$$

$$-3x(x^2-1) = Ax^2(x-1) + Bx(x-1) \rightarrow -3x(x+1)(x-1) = Ax^2(x-1) + Bx(x-1) \rightarrow$$

$$-3x(x+1) = x(Ax+B) \rightarrow \frac{-3x(x+1)}{x} = Ax+B \rightarrow -3(x+1) = Ax+B$$

$$x=0 \rightarrow -3(0+1) = A \cdot 0 + B \rightarrow B=-3$$

$$x=1 \rightarrow -3(1+1) = A \cdot 1 - 3 \rightarrow -6 = A-3 \rightarrow A = -6+3 \rightarrow A=-3$$

$$\frac{\partial \left(-3\ln|x| + \frac{3}{x} + \frac{1}{x^2} + 3\ln|x-1| + c \right)}{\partial x} = \frac{\partial \left(-3\ln|x| + 3x^{-1} + x^{-2} + 3\ln|x-1| + c \right)}{\partial x} =$$

$$-3 \frac{1}{x} - 3x^{-2} - 2x^{-3} + 3\frac{1}{x-1} + 0 = \frac{-3}{x} - \frac{3}{x^2} - \frac{2}{x^3} + \frac{3}{x-1} =$$

$$\frac{-3x^2(x-1) - 3x(x-1) - 2(x-1) + 3x^3}{x^3(x-1)} = \frac{-3x^3 + 3x^2 - 3x^2 + 3x - 2x + 2 + 3x^3}{x^3(x-1)} = \frac{x+2}{x^3(x-1)}$$

Exercício I

$$\int \frac{x^2\,\partial x}{(x-4)(x^2+1)}=\int\left(\frac{A}{x-4}+\frac{Bx+C}{x^2+1}\right)\partial x=\int\left(\frac{\left(\frac{16}{17}\right)}{x-4}+\frac{\left(\frac{1}{17}x+\frac{4}{17}\right)}{x^2+1}\right)\partial x=$$

$$\frac{16}{17}\int(x-4)^{-1}\partial x+\int\frac{\frac{1}{17}(x+4)}{x^2+1}\partial x=\frac{16}{17}\ln|x-4|+\frac{1}{17}\int\frac{x+4}{x^2+1}\partial x=$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\int\left(\frac{x}{x^2+1}+\frac{4}{x^2+1}\right)\partial x=\frac{16\ln|x-4|}{17}+\frac{1}{17}\int\frac{x}{x^2+1}\partial x+\frac{1}{17}\int\frac{4}{x^2+1}\partial x=$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\int(x^2+1)^{-1}x\,\partial x+\frac{4}{17}\int(x^2+1)^{-1}\partial x=$$

$$\frac{16\ln|x-4|}{17}+\frac{1}{17}\frac{\ln|x^2+1|}{2}+\frac{4}{17}arc\,tg(x)+c=\frac{16\ln|x-4|}{17}+\frac{\ln|x^2+1|}{34}+\frac{4\,arc\,tg(x)}{17}+c$$

$$\int(x^2+1)^{-1}x\,\partial x=\int u^{-1}\frac{\partial u}{2}=\frac{1}{2}\int u^{-1}\partial u=\frac{1}{2}\ln|u|+c=\frac{\ln|x^2+1|}{2}+c$$

$$u=x^2+1\rightarrow\frac{\partial u}{2}=x\,\partial x$$

$$\int(x^2+1)^{-1}\partial x=\int\frac{1}{1+x^2}\partial x=arc\,tg(x)+c$$

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$$\frac{\frac{x^2}{(x-4)(x^2+1)}=\frac{A}{x-4}+\frac{Bx+C}{x^2+1}}{(x-4)(x^2+1)}\rightarrow x^2=A(x^2+1)+(Bx+C)(x-4)$$

$$x=4\rightarrow 4^2=A(4^2+1)+\cancel{(4B+C)(4-4)}\rightarrow 16=17A\rightarrow A=\frac{16}{17}$$

$$x^2=\frac{16}{17}(x^2+1)+(Bx+C)(x-4)\rightarrow x^2-\frac{16}{17}(x^2+1)=(Bx+C)(x-4)\rightarrow$$

$$\frac{\left(\frac{17x^2-16(x^2+1)}{17}\right)}{x-4}=Bx+C\rightarrow\frac{17x^2-16(x^2+1)}{17(x-4)}=Bx+C$$

$$x=0\rightarrow\frac{\cancel{17\cdot 0^2}-16\cdot 4(0^2+1)}{17(0-4)}=B\cdot 0+C\rightarrow C=\frac{4}{17}$$

$$\frac{17x^2-16(x^2+1)}{17(x-4)}=Bx+\frac{4}{17}\rightarrow\frac{17x^2-16(x^2+1)}{17(x-4)}-\frac{4}{17}=Bx\rightarrow$$

$$\rightarrow\frac{17x^2-16(x^2+1)-4(x-4)}{17(x-4)}=Bx\rightarrow B=\frac{17x^2-16(x^2+1)-4(x-4)}{17x(x-4)}\rightarrow$$

$$\frac{17\cdot 1^2-16(1^2+1)-4(1-4)}{17\cdot 1(1-4)}=\frac{17-32+12}{(-51)}=\frac{-3}{(-51)}=\frac{1}{17}$$

Exercício II

$$\int \frac{3x^4+4x^3+16x^2+20x+9}{(x+2)(x^2+3)^2}\partial x = \int\left(\frac{A}{x+2}+\frac{Bx+C}{x^2+3}+\frac{Dx+E}{(x^2+3)^2}\right)\partial x =$$

$$\int\left(\frac{1}{x+2}+\frac{2x+0}{x^2+3}+\frac{4x+0}{(x^2+3)^2}\right)\partial x = \int(x+2)^{-1}\partial x + 2\int(x^2+3)^{-1}x\,\partial x + 4\int(x^2+3)^{-2}x\,\partial x =$$

$$\int(x+2)^{-1}\partial x + 2\!\!\!\!\not2\int u^{-1}\frac{\partial u}{\not2} + 4\cdot2\int u^{-2}\frac{\partial u}{\not2} = \ln|x+2|+\ln|u|+2\frac{u^{-1}}{(-1)}+c =$$

$$\ln|x+2|+\ln|u|-\frac{2}{u}+c = \ln|x+2|+\ln|x^2+3|-\frac{2}{x^2+3}+c$$

$$u=x^2+3 \rightarrow \frac{\partial u}{2}=x\,\partial x$$

$$\frac{\frac{3x^4+4x^3+16x^2+20x+9}{(x+2)(x^2+3)^2}=\frac{A}{x+2}+\frac{Bx+C}{x^2+3}+\frac{Dx+E}{(x^2+3)^2}}{(x+2)(x^2+3)^2} \rightarrow$$

$$3x^4+4x^3+16x^2+20x+9=A(x^2+3)^2+(Bx+C)(x+2)(x^2+3)+(Dx+E)(x+2)$$

$$x=-2 \rightarrow 3(-2)^4+4(-2)^3+16(-2)^2+20(-2)+9=$$

$$A\big((-2)^2+3\big)^2 + \cancel{(B(-2)+C)(-2+2)((-2)^2+3)} + \cancel{(D(-2)+E)(-2+2)} \rightarrow$$

$$48-32+64-40+9=A(4+3)^2 \rightarrow 49=49A \rightarrow A=\frac{49}{49}=1 \qquad\qquad (142)$$

$$\frac{-(x^2+3)^2+3x^4+4x^3+16x^2+20x+9}{x+2}=(Bx+C)(x^2+3)+Dx+E$$

$$x=0 \rightarrow \frac{-(0^2+3)^2+3\cdot0^4+4\cdot0^3+16\cdot0^2+20\cdot0+9}{0+2}=(B\cdot0+C)(0^2+3)+D\cdot0+E \rightarrow$$

$$\frac{-9+9}{2}=3C+E \rightarrow 3C+E=0 \rightarrow E=-3C=0$$

$$x=-1 \rightarrow \frac{-\big((-1)^2+3\big)^2+3(-1)^4+4(-1)^3+16(-1)^2+20(-1)+9}{-1+2}=$$

$$(B(-1)+C)\big((-1)^2+3\big)+D(-1)-3C \rightarrow -16+3-4+16-20+9=(-B+C)4-D-3C \rightarrow$$

$$-12=-4B+4C-D-3C \rightarrow -12=-4B+C-D \rightarrow D=-4B+C+12=4$$

$$x=1 \rightarrow \frac{-(1^2+3)^2+3\cdot1^4+4\cdot1^3+16\cdot1^2+20\cdot1+9}{1+2}=(B\cdot1+C)(1^2+3)-4B+C+12-3C \rightarrow$$

$$\frac{-16+3+4+16+20+9}{3}=4B+4C-4B+C+12-3C \rightarrow 12-12=2C \rightarrow C=0$$

$$x=2 \rightarrow \frac{-(2^2+3)^2+3\cdot2^4+4\cdot2^3+16\cdot2^2+20\cdot2+9}{2+2}=(B\cdot2+0)(2^2+3)+(-4B+0+12)2+0 \rightarrow$$

$$\frac{-49+48+32+64+40+9}{4}=14B-8B+24 \rightarrow 36-24=6B \rightarrow B=\frac{12}{6}=2$$