Introdução as Derivadas – Aula 1

$$\frac{\partial f(x)}{\partial x} = D_x f(x) = f'(x)$$

1) $y = c \rightarrow y' = 0$
2) $y = x \rightarrow y' = 1$
3) $y = x^c \rightarrow y' = c \cdot x^{c-1}$
4) $y = e^x \rightarrow y' = e^x$
5) $y = \ln(x) \rightarrow y' = \frac{1}{x}$
6) $y = f \cdot h \rightarrow y' = f' \cdot h + f \cdot h'$
7) $y = \frac{f}{h} \rightarrow y' = \frac{f' \cdot h - f \cdot h'}{h^2}$
8) $y = f^c \rightarrow y' = c \cdot f^{c-1} \cdot f'$
9) $y = e^f \rightarrow y' = e^f \cdot f'$
10) $y = c^f \rightarrow y' = c^f \cdot f' \cdot \ln(c)$
11) $y = \ln(f) \rightarrow y' = \frac{f'}{f}$
12) $y = \log_c(f) \rightarrow y' = \frac{f'}{f} \cdot \log_c(e)$

Exercício I

$$\begin{array}{l} \text{a) } y = 8 \rightarrow y' = 0 \\ \text{b) } y = \sqrt{3} \rightarrow y' = 0 \\ \text{c) } f(x) = \pi \rightarrow f'(x) = 0 \\ \text{d) } g(x) = (\pi - 1)^{\pi} \rightarrow g'(x) = 0 \end{array} \tag{1}$$

Exercício II

$$\begin{array}{l} \text{a) } y = x^5 \rightarrow y' = 5x^{5-1} = 5x^4 \\ \text{b) } h(x) = x^{-5} \rightarrow h'(x) = -5x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = -\frac{5}{x^6} \\ \text{c) } g(x) = 5x^3 \rightarrow g'(x) = 5 \cdot 3x^{3-1} = 15x^2 \end{array} \tag{2}$$

Exercício III

$$h(x) = 8x \rightarrow h'(x) = 8 \cdot 1 = 8 \tag{3}$$

Exercício IV

$$\begin{array}{c} f(x) = 7x^3 - 2x - 400 \\ f'(x) = 7 \cdot 3x^{3-1} - 2 \cdot 1 - 0 = 21x^2 - 2 \end{array} \tag{4}$$

Derivada com X no Denominador – Aula 2

Exercício I

$$g(x)=\frac{3}{x^5}=3x^{-5}$$
$$g'(x)=3(-5x^{-5-1})=-15x^{-6}=-\frac{15}{x^6} \tag{5}$$

Exercício II

$$h(x)=3x^5-\frac{2}{x^4}=3x^5-2x^{-4}$$
$$h'(x)=3(5x^{5-1})-2(-4x^{-4-1})=15x^4+8x^{-5}=15x^4+\frac{8}{x^5} \tag{6}$$

Derivada de Função Raiz – Aula 3

Exercício I

$$y=\sqrt[3]{x^4}=x^{\frac{4}{3}}$$
$$y'=\frac{4}{3}x^{\frac{4}{3}-1}=\frac{4}{3}x^{\frac{1}{3}}=\frac{4}{3}\sqrt[3]{x} \tag{7}$$

Exercício II

$$g(x)=7\sqrt[3]{x}=7x^{\frac{1}{3}}$$
$$g'(x)=7\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)=\frac{7}{3}x^{-\frac{2}{3}}=\frac{7}{3}\cdot\frac{1}{x^{\frac{2}{3}}}=\frac{7}{3\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right)=\frac{7\sqrt[3]{x}}{3x} \tag{8}$$

Derivada de uma Função Potência – Aula 4

Exercício I

$$y=x^3 \rightarrow y'=3x^2 \tag{9}$$

Exercício II

$$y=\frac{5x^4x^3}{x^2}=5x^{4+3-2}=5x^5$$
$$y'=5(5x^{5-1})=25x^4 \tag{10}$$

Derivada de uma Função Potência – Aula 5

Exercício I

$$\begin{gathered} y=\frac{x^2\sqrt{x}}{\sqrt[3]{x}}=x^2\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}=x^{2+\frac{1}{2}-\frac{1}{3}}=x^{\frac{12+3-2}{6}}=x^{\frac{13}{6}} \\ y'=\frac{13}{6}x^{\frac{13}{6}-1}=\frac{13}{6}x^{\frac{7}{6}}=\frac{13}{6}\sqrt[6]{x^7} \end{gathered} \tag{11}$$

Derivada de Função Exponencial e Logarítmica – Aula 6

Exercício I

$$\begin{gathered} f(x)=3e^x+10\cdot\ln(x) \\ f'(x)=3e^x+\frac{10}{x}=\frac{3xe^x+10}{x} \end{gathered} \tag{12}$$

Exercício II

$$\begin{gathered} g(x)=7e^x+9\cdot\ln(x)+3x^4-4x+100 \\ g'(x)=7e^x+\frac{9}{x}+12x^3-4=\frac{7xe^x+9+12x^4-4x}{x} \end{gathered} \tag{13}$$

Derivada de um Produto de Funções – Aula 7

Exercício I

$$\begin{gathered} f(x)=x\cdot\ln(x) \\ \begin{cases} g(x)=x & \rightarrow \quad g'(x)=1 \\ h(x)=\ln(x) & \rightarrow \quad h'(x)=\frac{1}{x} \end{cases} \end{gathered} \tag{14}$$

$$f(x)=g(x)\cdot h(x)\rightarrow \color{red}{f'(x)}=g'(x)\cdot h(x)+g(x)\cdot h'(x)=1\cdot\ln(x)+x\cdot\frac{1}{x}=\color{red}{\ln(x)+1}$$

Derivada de uma Divisão de Funções – Aula 8

Exercício I

$$f(x)=\frac{e^x}{3x}=e^x\cdot\frac{1}{3x}=e^x\cdot\frac{x^{-1}}{3}$$
$$\begin{cases} g(x)=e^x & \rightarrow \quad g'(x)=e^x \\ h(x)=\frac{x^{-1}}{3}=\frac{1}{3x} & \rightarrow \quad h'(x)=\frac{-x^{-2}}{3}=-\frac{1}{3x^2} \end{cases}$$
$$f(x)=g(x)\cdot h(x)\rightarrow f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)=$$
$$e^x\cdot\frac{1}{3x}+e^x\cdot\left(-\frac{1}{3x^2}\right)=\frac{e^x}{3x}-\frac{e^x}{3x^2}=\frac{x\cdot e^x-e^x}{3x^2}=\frac{e^x(x-1)}{3x^2}$$
$$f(x)=\frac{e^x}{3x}$$
$$\begin{cases} g(x)=e^x & \rightarrow \quad g'(x)=e^x \\ h(x)=3x & \rightarrow \quad h'(x)=3 \end{cases}$$
$$f(x)=\frac{g(x)}{h(x)}\rightarrow f'(x)=\frac{g'(x)\cdot h(x)-g(x)\cdot h'(x)}{h(x)^2}=$$
$$\frac{e^x\cdot 3x-e^x\cdot 3}{(3x)^2}=\frac{3e^x(x-1)}{9x^2}=\frac{e^x(x-1)}{3x^2} \tag{15}$$

Derivadas Básicas – Aula 9

Exercício I

$$y=\frac{2x^2+5x}{x}=\frac{2x^2}{x}+\frac{5x}{x}=2x+5$$
$$y'=2 \tag{16}$$

Exercício II

$$h(x)=\frac{3(x^2-1)}{x}=\frac{3x^2-3}{x}=\frac{3x^2}{x}-\frac{3}{x}=3x-3x^{-1}$$
$$h'(x)=3-(-3x^{-2})=3+\frac{3}{x^2}=\frac{3x^2+3}{x^2}=\frac{3(x^2+1)}{x^2} \tag{17}$$

Exercícios de Derivada – Aula 10

Exercício I

$$h(x)=3x^3(2+4x)=6x^3+12x^4$$
$$h'(x)=18x^2+48x^3=6x^2(8x+3) \tag{18}$$

Exercício II

$$g(x)=(x^2-1)(x^3+4)=x^5+4x^2-x^3-4$$
$$g'(x)=5x^4+8x-3x^2=x(5x^3-3x+8) \tag{19}$$

Curso básico de derivadas – Aula 12

Exercício I

$$g(x)=x^4+2e^x+e^2$$
$$g'(x)=4x^3+2e^x=2(2x^3+e^x) \tag{20}$$

Exercício II

$$g(x)=\sqrt[3]{x^7}+\frac{3}{x^2}+5=x^{\frac{7}{3}}+3x^{-2}+5$$
$$g'(x)=\frac{7x^{\frac{4}{3}}}{3}+(-6x^{-3})=\frac{7\sqrt[3]{x^4}}{3}-\frac{6}{x^3}=\frac{7x^3\sqrt[3]{x^4}-18}{3x^3}=\frac{7\sqrt[3]{x^{13}}-18}{3x^3} \tag{21}$$

Derivada de um Produto de Funções – Aula 13

Exercício I

$$y=8x\cdot\ln(x)$$
$$y'=8\cdot\ln(x)+8x\cdot\frac{1}{x}=8\cdot\ln(x)+8=8(\ln(x)+1) \tag{22}$$

Derivada de função composta, raiz, polinomial – Aula 14

Exercício I

$$y=x^3 \rightarrow y'=3x^2 \tag{23}$$

Exercício II

$$f(x)=(2x^2-1)^3$$
$$g(x)=2x^2-1 \rightarrow g'(x)=4x$$
$$f(x)=g(x)^p \rightarrow f'(x)=p\cdot g(x)^{p-1}\cdot g'(x)=3(2x^2-1)^2\cdot 4x=12x(2x^2-1)^2 \tag{24}$$

Exercício III

$$y=(3-x^2)^3$$
$$y'=3(3-x^2)^2(-2x)=-6x(3-x^2)^2 \tag{25}$$

Exercício IV

$$\begin{gathered} y=\frac{3}{(2x^2-1)^4}=3(2x^2-1)^{-4} \\ y'=3(-4)(2x^2-1)^{-5}\cdot 4x=-48x(2x^2-1)^{-5}=-\frac{48x}{(2x^2-1)^5} \end{gathered} \tag{26}$$

Exercício V

$$\begin{gathered} y=\sqrt{(2x^2-1)^3}=(2x^2-1)^{\frac{3}{2}} \\ y'=\frac{3}{2}(2x^2-1)^{\frac{1}{2}}\cdot 4x=6x(2x^2-1)^{\frac{1}{2}}=6x\sqrt{2x^2-1} \end{gathered} \tag{27}$$

Derivada de funções quociente, produto, polinomial – Aula 15

Exercício I

$$\begin{gathered} y=7x^4-2x^3+8x+5 \\ y'=28x^3-6x^2+8=2(14x^3-3x^2+4) \end{gathered} \tag{28}$$

Exercício II

$$\begin{gathered} y=(2x^3-4x^2)(3x^5+x^2)=6x^8+2x^5-12x^7-4x^4=6x^8-12x^7+2x^5-4x^4 \\ y'=48x^7-84x^6+10x^4-16x^3=2x^3(24x^4-42x^3+5x-8) \end{gathered} \tag{29}$$

Exercício III

$$\begin{gathered} h(x)=\frac{2x^3+4}{x^2-4x+1} \\ \begin{cases} f(x)=2x^3+4 & \rightarrow \quad f'(x)=6x^2 \\ g(x)=x^2-4x+1 & \rightarrow \quad g'(x)=2x-4 \end{cases} \\ h'(x)=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g(x)^2}=\frac{6x^2(x^2-4x+1)-(2x^3+4)(2x-4)}{(x^2-4x+1)^2}= \\ \frac{6x^4-24x^3+6x^2-(4x^4-8x^3+8x-16)}{(x^2-4x+1)^2}=\frac{6x^4-24x^3+6x^2-4x^4+8x^3-8x+16}{(x^2-4x+1)^2}= \\ \frac{2x^4-16x^3+6x^2-8x+16}{(x^2-4x+1)^2}=\frac{2(x^4-8x^3+3x^2-4x+8)}{(x^2-4x+1)^2} \end{gathered} \tag{30}$$

Exercício IV

$$y=\frac{3}{x^5}=3x^{-5}\rightarrow y'=-15x^{-6}=-\frac{15}{x^6} \tag{31}$$

Exercício V

$$v(r)=\frac{4}{3}\pi r^3 \rightarrow v'(r)=4\pi r^2 \tag{32}$$

Exercício VI

$$\begin{gathered} f(s)=\sqrt{3}(s^3-s^2)=\sqrt{3}s^3-\sqrt{3}s^2 \\ f'(s)=3\sqrt{3}s^2-2\sqrt{3}s=s\sqrt{3}(3s-2) \end{gathered} \tag{33}$$

Exercício VII

$$\begin{gathered} y=(4x^2+3)^2=16x^4+12x^2+12x^2+9=16x^4+24x^2+9 \\ y'=64x^3+48x=16x(4x^2+3) \end{gathered} \tag{34}$$

Derivadas de função quociente e produto – Aula 16

Exercício I

$$\begin{gathered} y=\frac{x^4-2x^2+5x+1}{x^4}=\frac{x^4}{x^4}-\frac{2x^2}{x^4}+\frac{5x}{x^4}+\frac{1}{x^4}=1-2x^{-2}+5x^{-3}+x^{-4} \\ y'=4x^{-3}-15x^{-4}-4x^{-5}=\frac{4}{x^3}-\frac{15}{x^4}-\frac{4}{x^5}=\frac{4x^2-15x-4}{x^5} \end{gathered} \tag{35}$$

Exercício II

$$\begin{gathered} y=\frac{x}{x-1} \\ y'=\frac{1(x-1)-x\cdot 1}{(x-1)^2}=\frac{x-1-x}{(x-1)^2}=-\frac{1}{(x-1)^2} \end{gathered} \tag{36}$$

Exercício III

$$\begin{gathered} y=\left(\frac{2x+1}{x+5}\right)(3x-1)=\frac{6x^2-2x+3x-1}{x+5}=\frac{6x^2+x-1}{x+5} \\ y'=\frac{(12x+1)(x+5)-(6x^2+x-1)\cdot 1}{(x+5)^2}=\frac{12x^2+60x+x+5-6x^2-x+1}{(x+5)^2}= \\ \frac{6x^2+60x+6}{(x+5)^2}=\frac{6(x^2+10x+1)}{(x+5)^2} \end{gathered} \tag{37}$$

Exercício IV

$$y=\frac{1}{8}x^8-4x^4 \rightarrow y'=x^7-16x^3=x^3(x^4-16) \tag{38}$$

Exercício V

$$\begin{gathered}y=x^2+3x+\frac{1}{x^2}=x^2+3x+x^{-2}\\ y'=2x+3-2x^{-3}=2x+3+\frac{2}{x^3}=\frac{2x^4+3x^3+2}{x^3}\end{gathered} \tag{39}$$

Exercício VI

$$y=\frac{3}{x^2}+\frac{5}{x^4}=3x^{-2}+5x^{-4}\rightarrow y'=-6x^{-3}-20x^{-5}=-\frac{6}{x^3}-\frac{20}{x^5}=\frac{-6x^2-20}{x^5} \tag{40}$$

Derivada de funções envolvendo raiz, quociente, produto – Aula 17

Exercício I

$$\begin{gathered}g(x)=\frac{3}{\sqrt{x}}=\frac{3}{x^{\frac{1}{2}}}=3x^{-\frac{1}{2}}\\ g'(x)=3\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}=\frac{-3}{2x^{\frac{3}{2}}}=-\frac{3}{2\sqrt{x^3}}\end{gathered} \tag{41}$$

Exercício II

$$\begin{gathered}g(x)=5\sqrt[3]{x^2}=5x^{\frac{2}{3}}\\ g'(x)=5\cdot\frac{2}{3}x^{-\frac{1}{3}}=\frac{10}{3x^{\frac{1}{3}}}=\frac{10}{3\sqrt[3]{x}}\end{gathered} \tag{42}$$

Exercício III

$$\begin{gathered}h(r)=6\sqrt[4]{r^3}+\frac{7}{r^3}=6r^{\frac{3}{4}}+7r^{-3}\\ h'(r)=6\cdot\frac{3}{4}r^{-\frac{1}{4}}+7(-3)r^{-4}=\frac{9}{2}r^{-\frac{1}{4}}-21r^{-4}=\frac{9}{2r^{\frac{1}{4}}}-\frac{21}{r^4}=\frac{9}{2\sqrt[4]{r}}-\frac{21}{r^4}=\\ \frac{9r^4-42\sqrt[4]{r}}{2r}=\frac{3}{2r}\left(3r^4-14\sqrt[4]{r}\right)\end{gathered} \tag{43}$$

Exercício IV

$$g(t)=\frac{5t^3}{6\sqrt{t}}=5t^3\cdot\frac{1}{6t^{\frac{1}{2}}}=5t^3\cdot\frac{t^{-\frac{1}{2}}}{6}$$

$$g'(t)=5\cdot 3t^{3-1}\cdot\frac{1}{6\sqrt{t}}+5t^3\left(\frac{1}{6}\left(-\frac{1}{2}\right)t^{-\frac{1}{2}-1}\right)=\frac{15t^2}{6\sqrt{t}}+5t^3\left(-\frac{t^{-\frac{3}{2}}}{12}\right)=\frac{5t^2}{2t^{\frac{1}{2}}}-\frac{5}{12}t^{3-\frac{3}{2}}=$$
$$\frac{5}{2}t^{2-\frac{1}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5}{2}t^{\frac{3}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5\sqrt{t^3}}{2}-\frac{5\sqrt{t^3}}{12}=\frac{30\sqrt{t^3}-5\sqrt{t^3}}{12}=\frac{5\sqrt{t^3}(6-1)}{12}=\frac{25\sqrt{t^3}}{12}$$

$$g(t)=\frac{5t^3}{6\sqrt{t}}=\frac{5t^3}{6t^{\frac{1}{2}}}$$
$$g'(t)=\frac{5\cdot 3t^{3-1}\cdot 6t^{\frac{1}{2}}-5t^3\cdot 6\left(\frac{1}{2}\right)t^{\frac{1}{2}-1}}{\left(6t^{\frac{1}{2}}\right)^2}=\frac{15t^2\cdot 6t^{\frac{1}{2}}-5t^3\cdot 3t^{-\frac{1}{2}}}{36t}=$$
$$\frac{90t^{2+\frac{1}{2}}-15t^{3-\frac{1}{2}}}{36t}=\frac{15\left(6t^{\frac{5}{2}}-t^{\frac{5}{2}}\right)}{36t}=\frac{5\sqrt{t^5}(6-1)}{12t}=\frac{30\sqrt{t^5}}{12t}-\frac{5\sqrt{t^5}}{12t}=$$
$$\frac{5}{2}t^{\frac{5}{2}-1}-\frac{5}{12}t^{\frac{5}{2}-1}=\frac{5}{2}t^{\frac{3}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5\sqrt{t^3}}{2}-\frac{5\sqrt{t^3}}{12}=\frac{5\sqrt{t^3}(6-1)}{12}=\frac{25\sqrt{t^3}}{12} \quad (44)$$

$$g(t)=\frac{5t^3}{6\sqrt{t}}=\frac{5t^3}{6t^{\frac{1}{2}}}=\frac{5}{6}t^{3-\frac{1}{2}}=\frac{5}{6}t^{\frac{5}{2}}$$
$$g'(t)=\frac{5}{6}\cdot\frac{5}{2}t^{\frac{5}{2}-1}=\frac{25t^{\frac{3}{2}}}{12}=\frac{25\sqrt{t^3}}{12}$$

Exercício V

$$h(x)=7x\sqrt[3]{x^2}=7x\cdot x^{\frac{2}{3}}=7x^{1+\frac{2}{3}}=7x^{\frac{5}{3}}$$
$$h'(x)=7\cdot\frac{5}{3}x^{\frac{5}{3}-1}=\frac{35x^{\frac{2}{3}}}{3}=\frac{35\sqrt[3]{x^2}}{3} \quad (45)$$

Exercício VI

$$g(t)=\frac{8t^3\sqrt{t}}{t}=\frac{8t^3t^{\frac{1}{2}}}{t}=8t^{3+\frac{1}{2}-1}=8t^{\frac{5}{2}}$$
$$g'(t)=8\cdot\frac{5}{2}t^{\frac{5}{2}-1}=20t^{\frac{3}{2}}=20\sqrt{t^3} \quad (46)$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 18

Exercício I

$$\begin{gathered} f(x)=\frac{3x^4-8x^2+4}{x^2}=\frac{3x^4}{x^2}-\frac{8x^2}{x^2}+\frac{4}{x^2}=3x^2-8+4x^{-2} \\ f'(x)=6x-8x^{-3}=6x-\frac{8}{x^3}=\frac{6x^4-8}{x^3} \end{gathered} \tag{47}$$

Exercício II

$$\begin{gathered} f(x)=\frac{\pi x^3}{\sqrt{x^3}}=\frac{\pi x^3}{x^{\frac{3}{2}}}=\pi x^{3-\frac{3}{2}}=\pi x^{\frac{3}{2}} \\ f'(x)=\pi\cdot\frac{3}{2}x^{\frac{3}{2}-1}=\frac{3\pi x^{\frac{1}{2}}}{2}=\frac{3\pi\sqrt{x}}{2} \end{gathered} \tag{48}$$

Exercício III

$$\begin{gathered} y=\frac{x^2}{e^x} \\ y'=\frac{2x\cdot e^x-x^2\cdot e^x}{(e^x)^2}=\frac{e^x\cdot x(2-x)}{e^{2x}}=\frac{x(2-x)}{e^x}=\frac{x}{e^x}(2-x) \end{gathered} \tag{49}$$

Exercício IV

$$\begin{gathered} y=x^5\cdot e^x \\ y'=5x^4\cdot e^x+x^5\cdot e^x=e^x\cdot x^4(5+x) \end{gathered} \tag{50}$$

Exercício V

$$\begin{gathered} y=\sqrt{x}\cdot\ln(x)=x^{\frac{1}{2}}\cdot\ln(x) \\ y'=\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)\ln(x)+x^{\frac{1}{2}}\cdot\frac{1}{x}=\frac{x^{-\frac{1}{2}}\cdot\ln(x)}{2}+x^{\frac{1}{2}}\cdot x^{-1}=\frac{\ln(x)}{2x^{\frac{1}{2}}}+x^{\frac{1}{2}-1}=\frac{\ln(x)}{2\sqrt{x}}+x^{-\frac{1}{2}}=\frac{\ln(x)}{2\sqrt{x}}+\frac{1}{x^{\frac{1}{2}}}= \\ \frac{\ln(x)}{2\sqrt{x}}+\frac{1}{\sqrt{x}}=\left(\frac{1}{\sqrt{x}}\left(\frac{\ln(x)}{2}+1\right)\right)\left(\frac{\sqrt{x}}{\sqrt{x}}\right)=\frac{\sqrt{x}}{x}\left(\frac{\ln(x)}{2}+1\right) \end{gathered} \tag{51}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 19

Exercício I

$$f(t)=e^t\cdot\ln(t)$$
$$f'(t)=e^t\cdot\ln(t)+e^t\cdot\frac{1}{t}=e^t\left(\ln(t)+\frac{1}{t}\right)=\frac{e^t(t\cdot\ln(t)+1)}{t}=\frac{e^t}{t}(t\cdot\ln(t)+1) \tag{52}$$

Exercício II

$$f(r)=\frac{e^r}{\sqrt{r}}=\frac{e^r}{r^{\frac{1}{2}}}$$
$$f'(r)=\frac{e^r\sqrt{r}-e^r\cdot\frac{1}{2}r^{\frac{1}{2}-1}}{(\sqrt{r})^2}=\frac{e^r\sqrt{r}-e^r\cdot\frac{r^{-\frac{1}{2}}}{2}}{r}=\frac{e^r\sqrt{r}-\frac{e^r}{2r^{\frac{1}{2}}}}{r}=\frac{e^r\sqrt{r}-\frac{e_r}{2\sqrt{r}}}{r}=\frac{\frac{2r\cdot e^r-e^r}{2\sqrt{r}}}{r}=$$
$$\frac{2r\cdot e^r-e^r}{2\sqrt{r}\cdot r}=\frac{e^r(2r-1)}{2r^{\frac{1}{2}+1}}=\frac{e^r(2r-1)}{2r^{\frac{3}{2}}}=\left(\frac{e^r(2r-1)}{2\sqrt{r^3}}\right)\left(\frac{\sqrt{r^3}}{\sqrt{r^3}}\right)=\frac{e^r\sqrt{r^3}(2r-1)}{2r^3}=\frac{e^r\sqrt{r^3}}{2r^3}(2r-1)$$
$$f(r)=\frac{e^r}{\sqrt{r}}=\frac{e^r}{r^{\frac{1}{2}}} \tag{53}$$
$$f'(r)=\frac{e^r\sqrt{r}-e^r\cdot\frac{1}{2}r^{\frac{1}{2}-1}}{(\sqrt{r})^2}=\frac{e^r\sqrt{r}-e^r\cdot\frac{r^{-\frac{1}{2}}}{2}}{r}=\frac{e^r\sqrt{r}-\frac{e^r}{2r^{\frac{1}{2}}}}{r}=\frac{e^r\sqrt{r}-\frac{e_r}{2\sqrt{r}}}{r}=\frac{e^r\left(\sqrt{r}-\frac{1}{2\sqrt{r}}\right)}{r}=$$
$$\frac{e^r}{r}\left(\sqrt{r}-\frac{1}{2\sqrt{r}}\right)=\frac{e^r}{r}\left(\frac{2r-1}{2\sqrt{r}}\right)\left(\frac{\sqrt{r}}{\sqrt{r}}\right)=\frac{e^r\sqrt{r}}{2r^2}(2r-1)$$

Exercício III

$$v(t)=\frac{t^3\cdot e^t}{t}=t^{3-1}\cdot e^t=t^2\cdot e^t$$
$$v'(t)=2t\cdot e^t+t^2\cdot e^t=e^t\cdot t(2+t) \tag{54}$$

Exercício IV

$$y=x^5\rightarrow y'=5x^4$$
$$y=(2x^2-4)^5\rightarrow y'=5(2x^2-4)^4\cdot 4x=20x(2x^2-4)^4 \tag{55}$$

Exercício V

$$\begin{gathered} y=\sqrt{2x^3-1}=(2x^3-1)^{\frac{1}{2}} \\ y'=\frac{1}{2}(2x^3-1)^{-\frac{1}{2}}\cdot 6x^2=3x^2(2x^3-1)^{-\frac{1}{2}}=\frac{3x^2}{(2x^3-1)^{\frac{1}{2}}}=\left(\frac{3x^2}{\sqrt{2x^3-1}}\right)\left(\frac{\sqrt{2x^3-1}}{\sqrt{2x^3-1}}\right)= \\ \frac{3x^2\sqrt{2x^3-1}}{2x^3-1} \end{gathered} \tag{56}$$

Exercício VI

$$\begin{gathered} y=\frac{5}{(3x^2+9)^4}=5(3x^2+9)^{-4} \\ y'=-20(3x^2+9)^{-5}\cdot 6x=-120x(3x^2+9)^{-5}=-\frac{120x}{(3x^2+9)^5} \end{gathered} \tag{57}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 20

Exercício I

$$\begin{gathered} y=\sqrt[3]{6x^2+7x+2}=(6x^2+7x+2)^{\frac{1}{3}} \\ y'=\frac{1}{3}(6x^2+7x+2)^{-\frac{2}{3}}\cdot(12x+7)=\frac{12x+7}{3(6x^2+7x+2)^{\frac{2}{3}}}=\left(\frac{12x+7}{3\sqrt[3]{(6x^2+7x+2)^2}}\right)\left(\frac{\sqrt[3]{6x^2+7x+2}}{\sqrt[3]{6x^2+7x+2}}\right) \\ \frac{(12x+7)\sqrt[3]{6x^2+7x+2}}{3(6x^2+7x+2)} \end{gathered} \tag{58}$$

Exercício II

$$\begin{gathered} y=e^{5x^2+4} \\ y'=e^{5x^2+4}\cdot 10x \end{gathered} \tag{59}$$

Exercício III

$$\begin{gathered} y=e^{\frac{1}{x^2}}=e^{x^{-2}} \\ y'=e^{\frac{1}{x^2}}(-2x^{-3})=e^{\frac{1}{x^2}}\left(-\frac{2}{x^3}\right)=-\frac{2e^{\frac{1}{x^2}}}{x^3} \end{gathered} \tag{60}$$

Exercício IV

$$\begin{gathered} y=3^{x^2} \\ y'=3^{x^2}\cdot\ln(3)\cdot 2x \end{gathered} \tag{61}$$

Exercício V

$$y=5^{2x^2+3x-1}$$
$$y'=5^{2x^2+3x-1}\cdot\ln(5)\cdot(4x+3) \tag{62}$$

Exercício VI

$$y=\left(\frac{1}{2}\right)^{\sqrt{x}}=\left(\frac{1}{2}\right)^{x^{\frac{1}{2}}}$$
$$y'=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2}x^{\frac{1}{2}-1}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2}x^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2x^{\frac{1}{2}}}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2\sqrt{x}} \tag{63}$$

Exercício VII

$$f=e^{\frac{x+1}{x-1}}\rightarrow g=\frac{x+1}{x-1}\rightarrow f=e^{g}$$
$$g'=\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}=\frac{x-1-x-1}{(x-1)^2}=\frac{-2}{(x-1)^2} \tag{64}$$
$$f'=e^{g}\cdot g'\rightarrow f'=e^{\frac{x+1}{x-1}}\cdot\frac{-2}{(x-1)^2}=\frac{-2e^{\frac{x+1}{x-1}}}{(x-1)^2}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 21

Exercício I

$$y=\ln(5x^2-4x)$$
$$y'=\frac{10x-4}{5x^2-4x}=\frac{2(5x-2)}{x(5x-4)} \tag{65}$$

Exercício II

$$y=\log_2(3x^2-7)$$
$$y'=\frac{6x}{3x^2-7}\cdot\log_2(e) \tag{66}$$

Exercício III

$$f=\log_{10}\left(\frac{x+1}{x^2+1}\right)\rightarrow g=\frac{x+1}{x^2+1}\rightarrow f=\log_{10}(g)$$

$$g'=\frac{1(x^2+1)-(x+1)2x}{(x^2+1)^2}=\frac{x^2+1-2x^2-2x}{(x^2+1)^2}=\frac{-x^2-2x+1}{(x^2+1)^2}=\frac{-(x^2+2x-1)}{(x^2+1)^2}$$

$$f'=\frac{g'}{g}\cdot\log_{10}(e)\rightarrow f'=\frac{\frac{-(x^2+2x-1)}{(x^2+1)^2}}{\frac{x+1}{x^2+1}}\cdot\log_{10}(e)=\frac{-(x^2+2x-1)(x^2+1)}{(x^2+1)^2(x+1)}\cdot\log_{10}(e)=$$

$$\frac{-(x^2+2x-1)}{(x^2+1)(x+1)}\cdot\log_{10}(e)=\frac{-x^2-2x+1}{x^3+x^2+x+1}\cdot\log_{10}(e) \tag{67}$$

$$f=\log_{10}\left(\frac{x+1}{x^2+1}\right)=\log_{10}(x+1)-\log_{10}(x^2+1)$$

$$f'=\frac{1}{x+1}\cdot\log_{10}(e)-\frac{2x}{x^2+1}\cdot\log_{10}(e)=\log_{10}(e)\left(\frac{1}{x+1}-\frac{2x}{x^2+1}\right)=$$

$$\log_{10}(e)\left(\frac{1(x^2+1)-2x(x+1)}{x^3+x+x^2+1}\right)=\log_{10}(e)\left(\frac{x^2+1-2x^2-2x}{x^3+x^2+x+1}\right)=$$

$$\log_{10}(e)\left(\frac{-x^2-2x+1}{x^3+x^2+x+1}\right)$$

Exercício IV

$$y=\ln\left(\frac{e^x}{x+1}\right)=\ln(e^x)-\ln(x+1)=x-\ln(x+1)$$

$$y'=1-\frac{1}{x+1}=\frac{x+1-1}{x+1}=\frac{x}{x+1} \tag{68}$$

Exercício V

$$y=\ln(2x-1)^3\rightarrow f=(2x-1)^3\rightarrow y=\ln(f)$$

$$f'=3(2x-1)^2\cdot 2=6(2x-1)^2$$

$$y'=\frac{f'}{f}=\frac{6(2x-1)^2}{(2x-1)^3}=\frac{6}{2x-1} \tag{69}$$

$$y=\ln(2x-1)^3=3\cdot\ln(2x-1)$$

$$y'=0\cdot\ln(2x-1)+3\cdot\frac{2}{2x-1}=\frac{6}{2x-1}$$

Exercício VI

$$\begin{gathered}y=\ln[(4x^2+3)(2x-1)]=\ln(4x^2+3)+\ln(2x-1)\\ y'=\frac{8x}{4x^2+3}+\frac{2}{2x-1}=\frac{8x(2x-1)+2(4x^2+3)}{8x^3-4x^2+6x-3}=\frac{16x^2-8x+8x^2+6}{8x^3-4x^2+6x-3}=\\ \frac{24x^2-8x+6}{8x^3-4x^2+6x-3}=\frac{2(12x^2-4x+3)}{8x^3-4x^2+6x-3}\end{gathered} \tag{70}$$

Exercício VII

$$\begin{gathered}f(x)=\left(\frac{2x+1}{3x-1}\right)^4\rightarrow g(x)=\frac{2x+1}{3x-1}\rightarrow f(x)=g(x)^4\\ g'(x)=\frac{2(3x-1)-(2x+1)3}{(3x-1)^2}=\frac{6x-2-6x-3}{(3x-1)^2}=\frac{-5}{(3x-1)^2}\\ f'(x)=4g(x)^{4-1}\cdot g'(x)=4\left(\frac{2x+1}{3x-1}\right)^3\cdot\frac{-5}{(3x-1)^2}=\frac{-20}{(3x-1)^2}\cdot\left(\frac{2x+1}{3x-1}\right)^3\end{gathered} \tag{71}$$

Derivação (ou diferenciação) Logarítmica – Aula 22

Exercício I

$$\begin{gathered}y=x^x\rightarrow\ln(y)=\ln(x^x)\rightarrow\ln(y)=x\cdot\ln(x)\\ \frac{y'}{y}=1\cdot\ln(x)+x\cdot\frac{1}{x}\rightarrow\frac{y'}{y}=\ln(x)+1\rightarrow y'=y(\ln(x)+1)=x^x(\ln(x)+1)\end{gathered} \tag{72}$$

Exercício II

$$\begin{gathered}y=c^f\rightarrow\ln(y)=\ln(c^f)\rightarrow\ln(y)=f\cdot\ln(c)\\ \frac{y'}{y}=f'\cdot\ln(c)+f\cdot\frac{0}{c}\rightarrow y'=y(f'\cdot\ln(c))\rightarrow y'=c^f\cdot f'\cdot\ln(c)\\ \\ y=e^f\rightarrow\ln(y)=\ln(e^f)\rightarrow\ln(y)=f\cdot\ln(e)\rightarrow\ln(y)=f\\ \frac{y'}{y}=f'\rightarrow y'=y\cdot f'\rightarrow y'=e^f\cdot f'\end{gathered} \tag{73}$$

Derivação (ou diferenciação) Logarítmica – Aula 23

Exercício I

$$\begin{gathered}y=x^{\sqrt{x}}\rightarrow\ln(y)=\ln(x^{\sqrt{x}})=\sqrt{x}\cdot\ln(x)=x^{\frac{1}{2}}\cdot\ln(x)\\ \frac{y'}{y}=\frac{1}{2}x^{\frac{1}{2}-1}\cdot\ln(x)+x^{\frac{1}{2}}\cdot\frac{1}{x}=\frac{x^{\frac{-1}{2}}}{2}\cdot\ln(x)+x^{\frac{1}{2}-1}=\frac{x^{\frac{-1}{2}}\cdot\ln(x)}{2}+x^{\frac{-1}{2}}=x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)\rightarrow\\ y'=y\cdot x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)=x^{\sqrt{x}}\cdot x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)=x^{\sqrt{x}-\frac{1}{x}}\left(\frac{\ln(x)}{2}+1\right)=x^{\frac{x\sqrt{x}-1}{x}}\left(\frac{\ln(x)+2}{2}\right)\end{gathered} \tag{74}$$

Derivada de função composta – Aula 24

Exercício I

$$y=\left(\frac{2x+1}{3x-1}\right)^4 \rightarrow f=\frac{2x+1}{3x-1} \rightarrow y=f^4$$

$$f'=\frac{2(3x-1)-(2x+1)3}{(3x-1)^2}=\frac{6x-2-6x-3}{(3x-1)^2}=\frac{-5}{(3x-1)^2}$$

$$y'=4f^{4-1}\cdot f'=4\left(\frac{2x+1}{3x-1}\right)^3\cdot\frac{-5}{(3x-1)^2}=\frac{-20}{(3x-1)^2}\cdot\left(\frac{2x+1}{3x-1}\right)^3=\frac{-20}{(3x-1)^2}\cdot\frac{(2x+1)^3}{(3x-1)^3}=$$

$$\frac{-20(2x+1)^3}{(3x-1)^5}$$

$$y=\left(\frac{2x+1}{3x-1}\right)^4 \rightarrow \ln(y)=\ln\left(\frac{2x+1}{3x-1}\right)^4=4\cdot\ln\left(\frac{2x+1}{3x-1}\right)=4(\ln(2x+1)-\ln(3x-1)) \tag{75}$$

$$\frac{y'}{y}=4\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right) \rightarrow y'=4y\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right)=$$

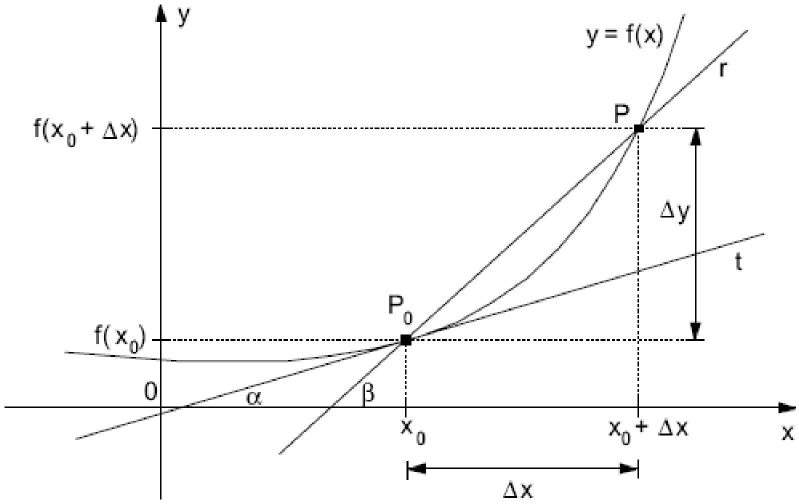
$$4\cdot\left(\frac{2x+1}{3x-1}\right)^4\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right)=4\cdot\left(\frac{2x+1}{3x-1}\right)^4\cdot\frac{2(3x-1)-3(2x+1)}{(2x+1)(3x-1)}=$$

$$\frac{4(2x+1)^4}{(3x-1)^4}\cdot\frac{2(3x-1)-3(2x+1)}{(2x+1)(3x-1)}=\frac{4(2x+1)^3(2(3x-1)-3(2x+1))}{(3x-1)^5}=$$

$$\frac{4(2x+1)^3(6x-2-6x-3)}{(3x-1)^5}=\frac{4(2x+1)^3(-5)}{(3x-1)^5}=\frac{-20(2x+1)^3}{(3x-1)^5}$$

O que é uma Derivada – Aula 25

Derivada é a principal ferramenta matemática utilizada para calcular e estudar taxas de variação.



$$tg\beta=\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

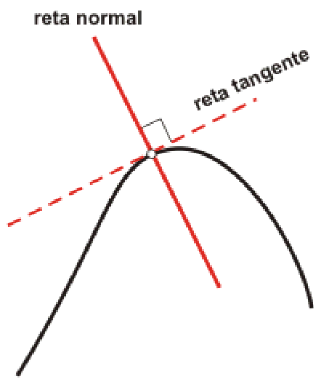
$$tg\alpha=\lim_{\Delta x\to 0}\left[\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\right]$$

$$f(x_0)=x_0^2$$

$$tg\alpha=f'(x_0)=2x_0$$

$$tg\alpha=\lim_{\Delta x\to 0}\left[\frac{(x_0+\Delta x)^2-(x_0)^2}{\Delta x}\right]=\frac{x_0^2+2x_0\Delta x+\Delta x^2-x_0^2}{\Delta x}=\frac{\Delta x(2x_0+\Delta x)}{\Delta x}=2x_0+\Delta x=2x_0+0=2x_0$$

Derivadas: Reta Tangente e Reta Normal – Aula 26



Reta tangente: $y-y_0=f'(x_0)(x-x_0)\to y=f'(x_0)(x-x_0)+y_0=f'(x_0)(x-x_0)+f(x_0)$

Reta normal: $y-y_0=\frac{-1}{f'(x_0)}(x-x_0)=\frac{-(x-x_0)}{f'(x_0)}\to y=\frac{-(x-x_0)}{f'(x_0)}+y_0=\frac{-(x-x_0)}{f'(x_0)}+f(x_0)$

Exercício I

$$f(x_0)=\frac{1}{x_0}=x_0^{-1}$$

$$f'(x_0)=-x_0^{-2}=\frac{-1}{x_0^2}$$

$$y_t=\frac{-1}{x_0^2}(x-x_0)+\frac{1}{x_0}=\frac{-x+x_0}{x_0^2}+\frac{1}{x_0}=\frac{-x+x_0+x_0}{x_0^2}=\frac{-x+2x_0}{x_0^2}$$

$$y_n=\frac{-(x-x_0)}{\frac{-1}{x_0^2}}+\frac{1}{x_0}=x_0^2(x-x_0)+\frac{1}{x_0}=x_0^2x-x_0^3+\frac{1}{x_0}=\frac{x_0^3x-x_0^4+1}{x_0}$$

$$
\begin{gathered}
x_0=1\\
y_0=f(x_0)=\frac{1}{1}=1\\
tg\,\alpha=f'(x_0)=\frac{-1}{1^2}=-1\\
y_t=\frac{-x+2\cdot 1}{1^2}=-x+2\\
y_n=\frac{1^3x-1^4+1}{1}=x
\end{gathered}
\tag{76}
$$

$$\lim_{x_0\to 0}f(x_0)=\lim_{x_0\to 0}\left[\frac{1}{x_0}\right]=\nexists$$

$$\lim_{x_0\to 0^+}\left[\frac{1}{x_0}\right]=\frac{1}{0^+}=+\infty$$

$$\lim_{x_0\to 0^+}\left[\frac{1}{x_0}\right]=\frac{1}{0^-}=-\infty$$

$$\lim_{x_0\to +\infty}f(x_0)=\lim_{x_0\to +\infty}\left[\frac{1}{x_0}\right]=\frac{1}{+\infty}=0$$

$$\lim_{x_0\to -\infty}f(x_0)=\lim_{x_0\to -\infty}\left[\frac{1}{x_0}\right]=\frac{1}{-\infty}=0$$

Exercício II

$$
\begin{gathered}
x_0=1 \\
f(x)=\sqrt{x}=x^{\frac{1}{2}} \\
f'(x)=\frac{1}{2}x^{\frac{-1}{2}}=\frac{1}{2\sqrt{x}} \\
y_0=f(x_0)=\sqrt{1}=1 \\
tg\,\alpha=f'(x_0)=\frac{1}{2\sqrt{1}}=\frac{1}{2} \\
y_t-y_0=f'(x_0)(x-x_0)\rightarrow y_t-1=\frac{1}{2}(x-1)\rightarrow y_t=\frac{x-1}{2}+1=\frac{x-1+2}{2}=\frac{x+1}{2} \\
y_n-y_0=\frac{-1}{f'(x_0)}(x-x_0)\rightarrow y_n-1=\frac{-1}{\frac{1}{2}}(x-1)\rightarrow y_n=-2(x-1)+1=-2x+2+1=-2x+3
\end{gathered}
\tag{77}
$$

Derivada pela Definição – Aula 27

Exercício I

$$
\begin{gathered}
f(x)=2x^2-3x+4 \\
f'(x)=4x-3 \\
\lim_{h\to 0}\left[\frac{(2(x+h)^2-3(x+h)+4)-(2x^2-3x+4)}{h}\right]= \\
\frac{2(x^2+2xh+h^2)-3x-3h+4-2x^2+3x-4}{h}= \\
\frac{2x^2+4xh+2h^2-3x-3h+4-2x^2+3x-4}{h}=\frac{2h^2+4xh-3h}{h}=\frac{h(2h+4x-3)}{h}= \\
2h+4x-3=2\cdot 0+4x-3=4x-3
\end{gathered}
\tag{78}
$$

Exercício II

$$
\begin{gathered}
f(x)=\frac{2}{x}=2x^{-1} \\
f'(x)=-2x^{-2}=\frac{-2}{x^2} \\
\lim_{h\to 0}\left[\frac{\left(\frac{2}{x+h}\right)-\left(\frac{2}{x}\right)}{h}\right]=\frac{\frac{2x-2(x+h)}{x(x+h)}}{h}=\frac{\frac{2x-2x-2h}{x(x+h)}}{h}=\frac{-2h}{xh(x+h)}=\frac{-2}{x(x+h)}= \\
\frac{-2}{x(x+0)}=\frac{-2}{x^2}
\end{gathered}
\tag{79}
$$

Derivada pela Regra da Cadeia – Aula 1

$y=f(g(x)) \quad \rightarrow \quad y'=f'(g(x)) \cdot g'(x)$

$y=(2x^2-1)^3=a^3 \rightarrow a=2x^2-1$
$y=e^{5x^2-1}=e^a \rightarrow a=5x^2-1$
$y=\ln[\cos(5x^2-1)]=\ln[a] \rightarrow a=\cos(5x^2-1)=\cos(b) \rightarrow b=5x^2-1$

Exercício I

$$\begin{gathered} y=(2x^2-1)^4=a^4 \rightarrow a=2x^2-1 \\ a'=4x \\ y'=4a^{4-1} \cdot a'=4(2x^2-1)^3 \cdot 4x=16x(2x^2-1)^3 \end{gathered} \tag{80}$$

Exercício II

$$\begin{gathered} y=e^{2x^2-1}=e^a \rightarrow a=2x^2-1 \\ a'=4x \\ y'=e^a \cdot a'=e^{2x^2-1} \cdot 4x \end{gathered} \tag{81}$$

Exercício III

$$\begin{gathered} y=\cos(2x^2-9)=\cos(a) \rightarrow a=2x^2-9 \\ a'=4x \\ y'=-sen(a) \cdot a'=-sen(2x^2-9) \cdot 4x=-4x \cdot sen(2x^2-9) \end{gathered} \tag{82}$$

Exercício IV

$$\begin{gathered} y=\ln[sen(x^3-1)]=\ln[a] \rightarrow a=sen(x^3-1)=sen(b) \rightarrow b=x^3-1 \\ b'=3x^2 \\ a'=\cos(b) \cdot b'=\cos(x^3-1) \cdot 3x^2 \\ y'=\frac{1}{a} \cdot a'=\frac{1}{sen(x^3-1)} \cdot \cos(x^3-1) \cdot 3x^2=3x^2 \cdot \frac{\cos(x^3-1)}{sen(x^3-1)}=3x^2 \cdot cotg(x^3-1) \end{gathered} \tag{83}$$

Exercício V

$$\begin{gathered} y=\ln[\cos(sen(2x^2-1))]=\ln[a] \\ a=\cos(sen(2x^2-1))=\cos(b) \\ b=sen(2x^2-1)=sen(c) \\ c=2x^2-1 \\ \\ c'=4x \\ b'=\cos(c) \cdot c'=\cos(2x^2-1) \cdot 4x \\ a'=-sen(b) \cdot b'=-sen(sen(2x^2-1)) \cdot \cos(2x^2-1) \cdot 4x \\ y'=\ln[a] \cdot a'=\frac{1}{\cos(sen(2x^2-1))}(-sen(sen(2x^2-1))) \cdot \cos(2x^2-1) \cdot 4x= \\ -4x \cdot \cos(2x^2-1) \cdot \frac{sen(sen(2x^2-1))}{\cos(sen(2x^2-1))}=-4x \cdot \cos(2x^2-1) \cdot tg(sen(2x^2-1)) \end{gathered} \tag{84}$$

Derivada pela Regra da Cadeia – Aula 2

Exercício I

$$y = x^4 \rightarrow \frac{\partial y}{\partial x} = 4x^3$$

$$y = (x^2+1)^4$$
$$\frac{\partial y}{\partial x} = 4(x^2+1)^3 \cdot 2x = 8x(x^2+1)^3 \tag{85}$$

Exercício II

$$y = \cos(x^2+1)^4 \rightarrow \frac{\partial y}{\partial x} = -sen(x^2+1)^4 \cdot 4(x^2+1)^3 \cdot 2x = -8x(x^2+1)^3 sen(x^2+1)^4 \tag{86}$$

Exercício III

$$y = \cos^2 x \rightarrow \frac{\partial y}{\partial x} = 2 \cdot \cos x(-sen\, x) = -2 \cdot sen\, x \cdot \cos x \tag{87}$$

Exercício IV

$$y = \cos^2(2x^3-1) \rightarrow \frac{\partial y}{\partial x} = 2\cos(2x^3-1)(-sen(2x^3-1))6x^2 = \\ -12x^2 sen(2x^3-1)\cos(2x^3-1) \tag{88}$$

Exercício V

$$y = (\ln x+1)^3 \rightarrow \frac{\partial y}{\partial x} = 3(\ln x+1)^2 \frac{1}{x} = \frac{3}{x}(\ln x+1)^2 \tag{89}$$

Exercício VI

$$y = \cos(\ln x+1)^3 \rightarrow \frac{\partial y}{\partial x} = -sen(\ln x+1)^3 3(\ln x+1)^2 \frac{1}{x} = \frac{-3}{x}(\ln x+1)^2 sen(\ln x+1)^3 \tag{90}$$

Exercício VII

$$y = \ln^3(2x^2-1) \rightarrow \frac{\partial y}{\partial x} = 3\ln^2(2x^2-1)\frac{1}{2x^2-1}4x = \frac{12x}{2x^2-1}\ln^2(2x^2-1) \tag{91}$$

Exercício VIII

$$y = e^{2x^2-1} \rightarrow \frac{\partial y}{\partial x} = e^{2x^2-1} 4x \tag{92}$$

Exercício IX

$$
\begin{gathered}
y=tg^2(x^2-1)=[tg(x^2-1)]^2=\left[\frac{sen(x^2-1)}{\cos(x^2-1)}\right]^2 \rightarrow \\
\frac{\partial y}{\partial x}=2\frac{sen(x^2-1)}{\cos(x^2-1)}\frac{\cos(x^2-1)\cos(x^2-1)-sen(x^2-1)(-sen(x^2-1))}{\cos^2(x^2-1)}2x= \\
4x\frac{sen(x^2-1)}{\cos(x^2-1)}\frac{\cos^2(x^2-1)+sen^2(x^2-1)}{\cos^2(x^2-1)}=4x\frac{sen(x^2-1)\cos^2(x^2-1)+sen^3(x^2-1)}{\cos^3(x^2-1)}= \\
4x\left[\frac{sen(x^2-1)\cos^2(x^2-1)}{\cos^3(x^2-1)}+\frac{sen^3(x^2-1)}{\cos^3(x^2-1)}\right]=4x\left[\frac{sen(x^2-1)}{\cos(x^2-1)}+\left(\frac{sen(x^2-1)}{\cos(x^2-1)}\right)^3\right]= \\
4x[tg(x^2-1)+tg^3(x^2-1)]=4x\cdot tg(x^2-1)[1+tg^2(x^2-1)]
\end{gathered}
\tag{93}
$$

$$
y=tg^2(x^2-1)\rightarrow\frac{\partial y}{\partial x}=2tg(x^2-1)sec^2(x^2-1)2x=4x\cdot tg(x^2-1)sec^2(x^2-1)
$$

Exercício X

$$
\begin{gathered}
y=\left(\frac{2x^2-1}{1-x^2}\right)^3\rightarrow\frac{\partial y}{\partial x}=3\left(\frac{2x^2-1}{1-x^2}\right)^2\frac{4x(1-x^2)-(2x^2-1)(-2x)}{(1-x^2)^2}= \\
3\left(\frac{2x^2-1}{1-x^2}\right)^2\frac{4x-4x^3+4x^3-2x}{(1-x^2)^2}=3\left(\frac{2x^2-1}{1-x^2}\right)^2\frac{2x}{(1-x^2)^2}=\frac{3(2x^2-1)^2}{(1-x^2)^2}\frac{2x}{(1-x^2)^2}= \\
\frac{6x(2x^2-1)^2}{(1-x^2)^4}=6x\left(\frac{2x^2-1}{(1-x^2)^2}\right)^2
\end{gathered}
\tag{94}
$$

Exercício XI

$$
\begin{gathered}
y=\ln[\cos(x^3-1)^4]\rightarrow\frac{\partial y}{\partial x}=\frac{1}{\cos(x^3-1)^4}(-sen(x^3-1)^4)4(x^3-1)^3 3x^2= \\
-12x^2(x^3-1)^3\frac{sen(x^3-1)^4}{\cos(x^3-1)^4}=-12x^2(x^3-1)^3 tg(x^3-1)^4
\end{gathered}
\tag{95}
$$

Derivada pela Regra da Cadeia – Aula 3

Exercício I

$$y=e^{x\sqrt[3]{2x-1}}=e^{a}$$
$$a=x\sqrt[3]{2x-1}=xb$$
$$b=\sqrt[3]{2x-1}=(2x-1)^{\frac{1}{3}}=c^{\frac{1}{3}}$$
$$c=2x-1$$

$$c'=2$$
$$b'=\frac{1}{3}c^{\frac{1}{3}-1}\cdot c'=\frac{1}{3}(2x-1)^{\frac{-2}{3}}2=\frac{2}{3(2x-1)^{\frac{2}{3}}}=\frac{2}{3\sqrt[3]{(2x-1)^{2}}} \tag{96}$$
$$a'=1\cdot b+x\cdot b'=1(\sqrt[3]{2x-1})+x\left(\frac{2}{3\sqrt[3]{(2x-1)^{2}}}\right)=\sqrt[3]{2x-1}+\frac{2x}{3\sqrt[3]{(2x-1)^{2}}}=$$
$$\frac{3\sqrt[3]{(2x-1)^{3}}+2x}{3\sqrt[3]{(2x-1)^{2}}}=\frac{3(2x-1)+2x}{3\sqrt[3]{(2x-1)^{2}}}=\frac{6x-3+2x}{3\sqrt[3]{(2x-1)^{2}}}=\frac{8x-3}{3\sqrt[3]{(2x-1)^{2}}}$$
$$y'=e^{a}\cdot a'=e^{x\sqrt[3]{2x-1}}\left(\frac{8x-3}{3\sqrt[3]{(2x-1)^{2}}}\right)=\frac{(8x-3)e^{x\sqrt[3]{2x-1}}}{3\sqrt[3]{(2x-1)^{2}}}$$

Derivada pela Regra da Cadeia – Aula 4

Exercício I

$$f(x)=sen^{3}(2^{5x^{4}})\rightarrow\frac{\partial f(x)}{\partial x}=3\,sen^{2}(2^{5x^{4}})\cos(2^{5x^{4}})2^{5x^{4}}\ln(2)20x^{3}=$$
$$\ln(2)60x^{3}2^{5x^{4}}sen^{2}(2^{5x^{4}})\cos(2^{5x^{4}}) \tag{97}$$

Derivada pela Regra da Cadeia – Aulа 5

Exercício I

$$
\begin{gathered}
y=2^{-3x^2}\ln x=a\cdot b\\
a=2^{-3x^2}=2^c\\
b=\ln x\\
c=-3x^2\\
\\
c'=-6x\\
b'=\frac{1}{x}\\
a'=2^c c'\ln(2)=2^{-3x^2}(-6x)\ln(2)=-\ln(2)6x\cdot 2^{-3x^2}\\
y'=a'b+ab'=(-\ln(2)6x\cdot 2^{-3x^2})\ln x+2^{-3x^2}\frac{1}{x}=-\ln(2)\ln(x)6x\cdot 2^{-3x^2}+\frac{2^{-3x^2}}{x}=\\
\frac{-\ln(2)\ln(x)6x^2 2^{-3x^2}+2^{-3x^2}}{x}=\frac{-2^{-3x^2}(\ln(2)\ln(x)6x^2-1)}{x}
\end{gathered}
\tag{98}
$$

Taxas Relacionadas – Aula 1

$$
\begin{gathered}
y=x^2\\
1\frac{\partial y}{\partial x}=2x\frac{\partial x}{\partial x}\rightarrow\frac{\partial y}{\partial x}=2x
\end{gathered}
$$

$$
\begin{gathered}
4y^3=x^2+1\\
12y^2\frac{\partial y}{\partial x}=2x\frac{\partial x}{\partial x}\rightarrow 12y^2\frac{\partial y}{\partial x}=2x\rightarrow\frac{\partial y}{\partial x}=\frac{2x}{12y^2}=\frac{x}{6y^2}
\end{gathered}
$$

$$
\begin{gathered}
y^3=x^2+y+2x^3+3=y^3-y=2x^3+x^2+3\\
(3y^2-1)\frac{\partial y}{\partial x}=(6x^2+2x)\frac{\partial x}{\partial x}\rightarrow\frac{\partial y}{\partial x}=\frac{6x^2+2x}{3y^2-1}=\frac{2x(3x+1)}{3y^2-1}
\end{gathered}
$$

$$
\begin{gathered}
y^3+1+z^2=x^2\\
3y^2\frac{\partial y}{\partial x}+2z\frac{\partial z}{\partial x}=2x\\
3y^2\frac{\partial y}{\partial z}+2z=2x\frac{\partial x}{\partial z}\\
3y^2+2z\frac{\partial z}{\partial y}=2x\frac{\partial x}{\partial y}\\
3y^2\frac{\partial y}{\partial t}+2z\frac{\partial z}{\partial t}=2x\frac{\partial x}{\partial t}
\end{gathered}
$$

1. Identificar as variáveis.
2. Achar uma relação entre as variáveis.
3. Derivar em relação a variável de referência.
4. Substituir os valores conhecidos.
5. Isolar o que se quer calcular.

Exercício I

O volume do balão esférico cresce a uma taxa de 100 centímetros cúbicos por segundo, qual é a taxa de crescimento do raio quando o mesmo mede 50 cm.

$$
\begin{aligned}
&1.\ \text{Volume (v): } cm^3\text{; Raio (r): } cm\text{; Tempo(t): } s\\
&2.\ v=\frac{4\pi r^3}{3}\\
&3.\ \frac{\partial v}{\partial t}=3\frac{4\pi r^{3-1}}{3}\frac{\partial r}{\partial t}=4\pi r^2\frac{\partial r}{\partial t}\\
&4.\ 100\frac{cm^3}{s}=4\pi(50\,cm)^2\frac{\partial r}{\partial t}=4\pi\,2500\,cm^2\frac{\partial r}{\partial t}=10000\pi\,cm^2\frac{\partial r}{\partial t}\\
&5.\ \frac{\partial r}{\partial t}=\frac{100\frac{cm^3}{s}}{10000\pi\,cm^2}=\frac{100\,cm^3}{10000\pi\,cm^2 s}=\frac{1}{100\pi}\frac{cm}{s}
\end{aligned}
\tag{99}
$$

Exercício II

Uma mancha de óleo expande-se em forma de círculo onde a área cresce a uma taxa constante de 26 quilômetros quadrados/h. Com que rapidez variará o raio da mancha quando a área for 9 quilômetros quadrados.

$$
\begin{aligned}
&1.\ \text{Área (a): } km^2\text{; Raio (r): } km\text{; Tempo (t): } h\\
&2.\ a=\pi r^2\rightarrow r=\sqrt{\frac{a}{\pi}}\rightarrow r=\sqrt{\frac{9\,km^2}{\pi}}=\frac{3\,km}{\sqrt{\pi}}=\frac{3\sqrt{\pi}km}{\pi}\\
&3.\ \frac{\partial a}{\partial t}=2\pi r\frac{\partial r}{\partial t}\\
&4.\ 26\frac{km^2}{h}=2\pi\frac{3\sqrt{\pi}km}{\pi}\frac{\partial r}{\partial t}=6\sqrt{\pi}km\frac{\partial r}{\partial t}\\
&5.\ \frac{\partial r}{\partial t}=\frac{26\frac{km^2}{h}}{6\sqrt{\pi}km}=\frac{26\,km^2}{6\sqrt{\pi}km\,h}=\frac{26\,km}{6\sqrt{\pi}h}=\frac{13}{3\sqrt{\pi}}\frac{km}{h}=\frac{13\sqrt{\pi}}{3\pi}\frac{km}{h}
\end{aligned}
\tag{100}
$$

Exercício III

Um foguete sobe verticalmente e é acompanhado por uma estação no solo a 5 km da base de lançamento. Com que rapidez o foguete subirá, quando sua altura for 4 km e a sua distância da estação estiver crescendo a 2000 km/h.

$$\begin{aligned}
&1.\ \text{Distância (d): } km\text{; Altura (a): } km\text{; Tempo (t): } h \\
&2.\ d=\sqrt{(5\,km)^2+a^2}=\sqrt{25\,km^2+a^2}=(25\,km^2+a^2)^{\frac{1}{2}} \\
&3.\ \frac{\partial d}{\partial t}=\frac{1}{2}(25\,km^2+a^2)^{\frac{-1}{2}}\cdot 2a\frac{\partial a}{\partial t}=\frac{a}{\sqrt{25\,km^2+a^2}}\frac{\partial a}{\partial t} \\
&4.\ 2000\frac{km}{h}=\frac{4\,km}{\sqrt{25\,km^2+(4\,km)^2}}\frac{\partial a}{\partial t}=\frac{4\,km}{\sqrt{25\,km^2+16\,km^2}}\frac{\partial a}{\partial t}=\frac{4\,km}{\sqrt{41}\,km}\frac{\partial a}{\partial t}=\frac{4\sqrt{41}}{41}\frac{\partial a}{\partial t} \\
&5.\ \frac{\partial a}{\partial t}=\frac{2000\frac{km}{h}}{\frac{4\sqrt{41}}{41}}=\frac{2000\,km}{\frac{4\sqrt{41}}{41}h}=\frac{41\cdot 500\,km}{\sqrt{41}\,h}=500\sqrt{41}\frac{km}{h}
\end{aligned} \tag{101}$$

Taxas Relacionadas – Aula 2

Exercício I

Um tanque de água tem a forma de um cone circular invertido com base de raio igual a 2 metros e altura igual a 4 metros. Se a água está sendo bombeada para dentro do tanque a uma taxa de 2 metros cúbicos por minuto, encontre a taxa na qual o nível de água estará elevado quando a água estiver a 3 metros de profundidade.

$$\begin{aligned}
&1.\ \text{Volume do tanque (v): } m^3\text{; Profundidade do tanque invertido (h): } m\text{; Tempo (t): } mi \\
&2.\ v=\frac{\pi r^2 h}{3}\rightarrow\frac{2m}{4m}=\frac{r}{h}\rightarrow r=\frac{2h}{4}=\frac{h}{2}\rightarrow v=\frac{\pi\left(\frac{h}{2}\right)^2 h}{3}=\frac{\pi\frac{h^2}{4}h}{3}=\frac{\pi\frac{h^3}{4}}{3}=\frac{\pi h^3}{12} \\
&3.\ \frac{\partial v}{\partial t}=\frac{3\pi h^2}{12}\frac{\partial h}{\partial t}=\frac{\pi h^2}{4}\frac{\partial h}{\partial t} \\
&4.\ 2\frac{m^3}{min}=\frac{\pi(3m)^2}{4}\frac{\partial h}{\partial t}=\frac{9\pi m^2}{4}\frac{\partial h}{\partial t} \\
&5.\ \frac{\partial h}{\partial t}=\frac{2\frac{m^3}{min}}{\frac{9\pi m^2}{4}}=\frac{8m^3}{9\pi m^2\,min}=\frac{8}{9\pi}\frac{m}{min}
\end{aligned} \tag{102}$$

Exercício II

Uma escada de 5 metros está apoiada em uma parede. A base é arrastada a 3 m/s. Qual a velocidade da escada a longo da parede, quando a base se encontra a 3 metros da parede.

$$\begin{array}{l} 1.\ \text{Base (b): } m\text{; Altura (h): } m\text{; Tempo (t): } s \\ 2.\ (5m)^2 = b^2 + h^2 \rightarrow b^2 + h^2 = 25m^2 \rightarrow \\ h = \sqrt{25m^2 - b^2} = \sqrt{25m^2 - (3m)^2} = \sqrt{25m^2 - 9m^2} = \sqrt{16m^2} = 4m \\ 3.\ 2b\dfrac{\partial b}{\partial t} + 2h\dfrac{\partial h}{\partial t} = 0 \\ 4.\ 2\cdot 3m\cdot 3\dfrac{m}{s} + 2\cdot 4m\cdot\dfrac{\partial h}{\partial t} = 0 \rightarrow 18\dfrac{m^2}{s} + 8m\dfrac{\partial h}{\partial t} = 0 \\ 5.\ \dfrac{\partial h}{\partial t} = \dfrac{-18\dfrac{m^2}{s}}{8m} = \dfrac{-18m^2}{8ms} = \dfrac{-9}{4}\dfrac{m}{s} \end{array} \tag{103}$$

Exercício III

Determine a taxa de variação instantânea da área em relação ao lado, quando esse mede 4 m.

$$\begin{array}{l} 1.\ \text{Área (a): } m^2\text{; Lado (l): } m \\ 2.\ a = l^2 \\ 3.\ \dfrac{\partial a}{\partial l} = 2l \\ 4.\ \dfrac{\partial a}{\partial l} = 2(4m) \\ 5.\ \dfrac{\partial a}{\partial l} = 8\dfrac{m^2}{m} \end{array} \tag{104}$$

Exercício IV

Uma pedra foi lançada no lago e com isso, gerou ondas circulares que se propagam com a velocidade constante de 3 m/s. Qual e taxa de crescimento da área após transcorrido 10 segundos.

$$\begin{array}{l} 1.\ \text{Área (a): } m^2\text{, Raio (r): } m\text{; Tempo (t): } s \\ 2.\ a = \pi r^2 \rightarrow r = 3\dfrac{m}{s}\cdot 10s = 30m \\ 3.\ \dfrac{\partial a}{\partial t} = 2\pi r\dfrac{\partial r}{\partial t} \\ 4.\ \dfrac{\partial a}{\partial t} = 2\pi(30m)3\dfrac{m}{s} \\ 5.\ \dfrac{\partial a}{\partial t} = 180\pi\dfrac{m^2}{s} \end{array} \tag{105}$$

Exercício V

Acumula-se areia em um monte com a forma de um cone. A altura do cone é igual ao raio da base. O volume de areia cresce a uma taxa de 10 metros cúbicos por hora. Com que razão aumenta a área da base quando a altura for 4 metros.

$$
\begin{aligned}
&1.\ \text{Volume (v): } m^3 \text{ Altura(r): } m\text{; Área (a): } m^2\text{; Tempo (t): } h \\
&2.\ a=\pi r^2 \rightarrow r=h \rightarrow v=\frac{\pi r^2 h}{3}=\frac{\pi r^3}{3} \\
&3.\ \frac{\partial a}{\partial t}=2\pi r\frac{\partial r}{\partial t} \\
&\frac{\partial v}{\partial t}=\frac{3\pi r^2}{3}\frac{\partial r}{\partial t}=\pi r^2\frac{\partial r}{\partial t} \\
&4.\ 10\frac{m^3}{h}=\pi(4m)^2\frac{\partial r}{\partial t}=16\pi m^2\frac{\partial r}{\partial t} \\
&\frac{\partial r}{\partial t}=\frac{10\frac{m^3}{h}}{16\pi m^2}=\frac{10m^3}{16\pi m^2 h}=\frac{5}{8\pi}\frac{m}{h} \\
&\frac{\partial a}{\partial t}=2\pi r\frac{5m}{8\pi h}=\frac{r\cdot 5m}{4h} \\
&5.\ \frac{\partial a}{\partial t}=\frac{(4m)5m}{4h}=\frac{20m^2}{4h}=5\frac{m^2}{h}
\end{aligned}
\tag{106}
$$

Diferenciação (ou Derivação) Implícita – Aula

Exercício I

$$2x+y=0 \rightarrow y=-2x \tag{107}$$

Exercício II

$$2x-xy+1=0 \rightarrow xy=2x+1 \rightarrow y=\frac{2x+1}{x}=2+\frac{1}{x} \tag{108}$$

Exercício III

$$x^2+y^2-4=0 \rightarrow y^2=-x^2+4 \rightarrow y=\pm\sqrt{-x^2+4} \tag{109}$$

Exercício IV

$$e^y-x=0 \rightarrow e^y=x \rightarrow y=\log_e x=\ln x \tag{110}$$

Exercício I

$$y^4-x^5=7 \rightarrow 4y^3\frac{\partial y}{\partial x}-5x^4 \rightarrow \frac{\partial y}{\partial x}=\frac{5x^4}{4y^3} \tag{111}$$

Exercício II

$$x^2+y^2=25 \rightarrow 2x+2y\frac{\partial y}{\partial x}=0 \rightarrow \frac{\partial y}{\partial x}=\frac{-2x}{2y}=\frac{-x}{y} \tag{112}$$

Exercício III

$$\begin{gathered}x^2 y+7x+8y=5 \rightarrow \left(2xy+x^2\frac{\partial y}{\partial x}\right)+7+8\frac{\partial y}{\partial x}=0 \rightarrow x^2\frac{\partial y}{\partial x}+8\frac{\partial y}{\partial x}=-2xy-7 \rightarrow \\ (x^2+8)\frac{\partial y}{\partial x}=-2xy-7 \rightarrow \frac{\partial y}{\partial x}=\frac{-2xy-7}{x^2+8}\end{gathered} \tag{113}$$

Exercício IV

$$\begin{gathered}x^2+2y^3=3xy \rightarrow 2x+6y^2\frac{\partial y}{\partial x}=3y+3x\frac{\partial y}{\partial x} \rightarrow 6y^2\frac{\partial y}{\partial x}-3x\frac{\partial y}{\partial x}=3y-2x \rightarrow \\ (6y^2-3x)\frac{\partial y}{\partial x}=3y-2x \rightarrow \frac{\partial y}{\partial x}=\frac{3y-2x}{6y^2-3x}=\frac{3y-2x}{3(2y^2-x)} \\ \frac{\partial y}{\partial x}(1,1)=\frac{3\cdot 1-2\cdot 1}{3(2(1)^2-1)}=\frac{1}{3}\end{gathered} \tag{114}$$

Exercício V

$$\begin{gathered}2xy-\ln(xy)+5=0 \rightarrow \left(2y+2x\frac{\partial y}{\partial x}\right)-\left[\frac{1}{xy}\left(y+x\frac{\partial y}{\partial x}\right)\right]+0=0 \rightarrow \\ 2y+2x\frac{\partial y}{\partial x}-\left(\frac{y}{xy}+\frac{x}{xy}\frac{\partial y}{\partial x}\right)=0 \rightarrow 2y+2x\frac{\partial y}{\partial x}-\frac{1}{x}-\frac{1}{y}\frac{\partial y}{\partial x}=0 \rightarrow \\ 2x\frac{\partial y}{\partial x}-\frac{1}{y}\frac{\partial y}{\partial x}=-2y+\frac{1}{x} \rightarrow \left(2x-\frac{1}{y}\right)\frac{\partial y}{\partial x}=-2y+\frac{1}{x} \rightarrow \\ \left(\frac{2xy-1}{y}\right)\frac{\partial y}{\partial x}=\frac{-2xy+1}{x} \rightarrow \frac{\partial y}{\partial x}=\frac{\frac{-(2xy-1)}{x}}{\frac{2xy-1}{y}}=\frac{-y(2xy-1)}{x(2xy-1)}=\frac{-y}{x}\end{gathered} \tag{115}$$

Exercício VI

$$\begin{gathered}\ln(xy)=2x-2y^2 \rightarrow \frac{1}{xy}\left(y+x\frac{\partial y}{\partial x}\right)=2-4y\frac{\partial y}{\partial x} \rightarrow \frac{y}{xy}+\frac{x}{xy}\frac{\partial y}{\partial x}=2-4y\frac{\partial y}{\partial x} \rightarrow \\ \frac{1}{x}+\frac{1}{y}\frac{\partial y}{\partial x}=2-4y\frac{\partial y}{\partial x} \rightarrow \frac{1}{y}\frac{\partial y}{\partial x}+4y\frac{\partial y}{\partial x}=2-\frac{1}{x} \rightarrow \left(\frac{1}{y}+4y\right)\frac{\partial y}{\partial x}=2-\frac{1}{x} \rightarrow \\ \left(\frac{1+4y^2}{y}\right)\frac{\partial y}{\partial x}=\frac{2x-1}{x} \rightarrow \frac{\partial y}{\partial x}=\frac{\frac{2x-1}{x}}{\frac{1+4y^2}{y}}=\frac{y(2x-1)}{x(1+4y^2)}\end{gathered} \tag{116}$$

Exercício VII

$$(2x-1)^4+10=y^2+20 \rightarrow [4(2x-1)^3 \cdot 2]+0=2y\frac{\partial y}{\partial x}+0 \rightarrow 8(2x-1)^3=2y\frac{\partial y}{\partial x} \rightarrow$$
$$\frac{\partial y}{\partial x}=\frac{8(2x-1)^3}{2y}=\frac{4(2x-1)^3}{y} \tag{117}$$

Exercício VIII

$$e^{xy}+3x=3y^3+4 \rightarrow \left[e^{xy}\left(y+x\frac{\partial y}{\partial x}\right)\right]+3=9y^2\frac{\partial y}{\partial x}+0 \rightarrow e^{xy}y+e^{xy}x\frac{\partial y}{\partial x}+3=9y^2\frac{\partial y}{\partial x} \rightarrow$$
$$e^{xy}x\frac{\partial y}{\partial x}-9y^2\frac{\partial y}{\partial x}=-e^{xy}y-3 \rightarrow (e^{xy}x-9y^2)\frac{\partial y}{\partial x}=-e^{xy}y-3 \rightarrow \frac{\partial y}{\partial x}=\frac{-e^{xy}y-3}{e^{xy}x-9y^2} \tag{118}$$
$$\frac{\partial y}{\partial x}(1{,}0)=\frac{-e^{1\cdot 0}0-3}{e^{1\cdot 0}1-9(0)^2}=-3$$

O Teorema de Rolle – Aula

1. A função deve ser contínua num intervalo [a, b]
2. A função deve ser diferenciável ou derivável em (a, b)
3. f(a) = f(b) = 0

Satisfeitas as 3 condições, conclui-se que existe pelo menos um x=c em (a, b), tal que, f'(c) = 0

Exercício I

$$f(x)=-x^2+4x \rightarrow [0;4]$$

1. A função polinomial é contínua para qualquer valor de x
2. Toda função polinomial é derivável
3. $f(0)=-(0)^2+4(0)=0 \rightarrow f(4)=-(4)^2+4(4)=-4^2+4^2=0 \rightarrow f(0)=f(4)=0$

$$f'(x)=-2x+4 \rightarrow -2x+4=0 \rightarrow -2x=-4 \rightarrow x=\frac{-4}{-2}=2 \rightarrow f'(2)=0 \tag{119}$$

Exercício II

$$f(x)=4x^3-9x \rightarrow \left[\frac{-3}{2};\frac{3}{2}\right]$$

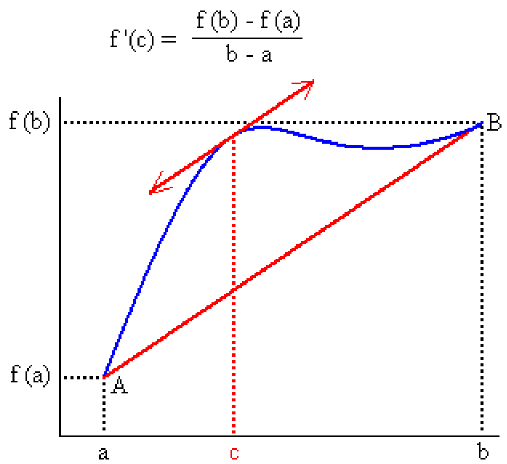
1. A função polinomial é contínua para qualquer valor de x
2. Toda função polinomial é derivável
3. $f\left(\frac{-3}{2}\right)=4\left(\frac{-3}{2}\right)^3-9\left(\frac{-3}{2}\right)=4\left(\frac{-27}{8}\right)+\frac{27}{2}=\frac{-27}{2}+\frac{27}{2}=0 \rightarrow$

$$f\left(\frac{3}{2}\right)=4\left(\frac{3}{2}\right)^3-9\left(\frac{3}{2}\right)=4\left(\frac{27}{8}\right)-\frac{27}{2}=\frac{27}{2}-\frac{27}{2}=0 \rightarrow f\left(\frac{-3}{2}\right)=f\left(\frac{3}{2}\right)=0 \tag{120}$$
$$f'(x)=12x^2-9 \rightarrow 12x^2-9=0 \rightarrow 12x^2=9 \rightarrow x=\pm\sqrt{\frac{9}{12}}=\frac{\pm 3}{2\sqrt{3}}=\frac{\pm\sqrt{3}}{2} \rightarrow$$
$$f'\left(\frac{-\sqrt{3}}{2}\right)=f'\left(\frac{\sqrt{3}}{2}\right)=0$$

Exercício III

$$
\begin{array}{l}
f(x)=sen\,x \rightarrow [0;2\pi] \\
\text{1. A função senoidal é contínua para qualquer valor de x} \\
\text{2. Toda função senoidal é derivável} \\
\text{3. } f(0)=sen\,0=0 \rightarrow f(2\pi)=sen\,2\pi=0 \rightarrow f(0)=f(2\pi)=0 \\
f'(x)=\cos x \rightarrow \cos x=0 \rightarrow x=\left[\frac{\pi}{2};\frac{3\pi}{2}\right] \rightarrow f'\left(\frac{\pi}{2}\right)=f'\left(\frac{3\pi}{2}\right)=0
\end{array}
\tag{121}
$$

Teorema do Valor Médio – Aula



1. A função deve ser contínua num intervalo [a, b]
2. A função deve ser diferenciável ou derivável em (a, b)

Satisfeitas as 2 condições, existe pelo menos um x=c em (a, b), tal que, $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Exercício I

$$
\begin{array}{l}
f(x)=x^2-2x \rightarrow [0;3] \\
\text{1. A função polinomial é contínua para qualquer valor de x} \\
\text{2. Toda função polinomial é derivável} \\
\text{3. } f'(x)=\dfrac{[(0)^2-2(0)]-[(3)^2-2(3)]}{0-3}=\dfrac{-(9-6)}{-3}=\dfrac{-3}{-3}=1 \rightarrow \\
f'(x)=2x-2 \rightarrow 2x-2=1 \rightarrow 2x=1+2 \rightarrow x=\dfrac{3}{2} \rightarrow f'\left(\dfrac{3}{2}\right)=1
\end{array}
\tag{122}
$$

Exercício II

$$
\begin{array}{l}
f(x)=\sqrt[3]{x^2}=x^{\frac{2}{3}} \rightarrow [-2;2] \\
\text{1. A função e contínua} \\
\text{2. A função não é derivável} \\
\text{3. } f'(x)=\dfrac{\left[\sqrt[3]{(2)^2}\right]-\left[\sqrt[3]{(-2)^2}\right]}{2-(-2)}=\dfrac{\sqrt[3]{4}-\sqrt[3]{4}}{4}=\dfrac{0}{4}=0 \rightarrow \\
f'(x)=\dfrac{2}{3}x^{\frac{-1}{3}}=\dfrac{2}{3\sqrt[3]{x}}=0 \rightarrow 2=0\cdot 3\sqrt[3]{x}
\end{array}
\tag{123}
$$

Regra de L'Hospital – Aula 1

Indeterminações:

1. $\frac{0}{0}$
2. $\frac{\infty}{\infty}$
3. $\infty \cdot 0$ (Indeterminação de produto)
4. $\infty - \infty$ (Indeterminação da diferença)
5. $0^0 \quad \infty^0 \quad 1^\infty$ (Indeterminação de potência)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Exercício I

$$\begin{gathered}
\lim_{x \to 1} \left[\frac{x^2 - 1}{x - 1} \right] = \frac{1^2 - 1}{1 - 1} = \frac{0}{0} \to (x = 1 \to x - 1 = 0) \\
\frac{(x-1)(x+1)}{x-1} = x + 1 \to \lim_{x \to 1} [x + 1] = 1 + 1 = 2 \\
\lim_{x \to 1} \left[\frac{2x}{1} \right] = \lim_{x \to 1} [2x] = 2 \cdot 1 = 2
\end{gathered} \tag{124}$$

Exercício II

$$\begin{gathered}
\lim_{x \to \infty} \left[\frac{e^x}{x^2} \right] = \frac{e^\infty}{\infty^2} = \frac{\infty}{\infty} \\
\lim_{x \to \infty} \left[\frac{e^x}{2x} \right] = \frac{e^\infty}{2\infty} = \frac{\infty}{\infty} \\
\lim_{x \to \infty} \left[\frac{e^x}{2} \right] = \frac{e^\infty}{2} = \frac{\infty}{2} = \infty
\end{gathered} \tag{125}$$

Exercício III

$$\begin{gathered}
\lim_{x \to 2} \left[\frac{x^2 + 2x - 8}{x^2 - x - 2} \right] = \frac{2^2 + 2 \cdot 2 - 8}{2^2 - 2 - 2} = \frac{4 + 4 - 8}{4 - 2 - 2} = \frac{0}{0} \to (x = 2 \to x - 2 = 0) \\
\frac{(x^2 + 2x - 8) \div (x - 2)}{(x^2 - x - 2) \div (x - 2)} = \frac{x + 4}{x + 1} \to \lim_{x \to 2} \left[\frac{x + 4}{x + 1} \right] = \frac{2 + 4}{2 + 1} = \frac{6}{3} = 2 \\
\lim_{x \to 2} \left[\frac{2x + 2}{2x - 1} \right] = \frac{2 \cdot 2 + 2}{2 \cdot 2 - 1} = \frac{6}{3} = 2
\end{gathered} \tag{126}$$

Exercício IV

$$\begin{gathered}\lim_{x\to 0}\left[\frac{e^x-x-1}{x^2}\right]=\frac{e^0-0-1}{0^2}=\frac{1-1}{0}=\frac{0}{0}\\ \lim_{x\to 0}\left[\frac{e^x-1}{2x}\right]=\frac{e^0-1}{2\cdot 0}=\frac{1-1}{0}=\frac{0}{0}\\ \lim_{x\to 0}\left[\frac{e^x}{2}\right]=\frac{e^0}{2}=\frac{1}{2}\end{gathered}\tag{127}$$

Exercício V

$$\begin{gathered}\lim_{x\to 1}\left[\frac{1-x+\ln x}{x^3-3x+2}\right]=\frac{1-1+\ln 1}{1^3-3\cdot 1+2}=\frac{1-1+0}{1-3+2}=\frac{0}{0}\\ \lim_{x\to 1}\left[\frac{-1+\frac{1}{x}}{3x^2-3}\right]=\lim_{x\to 1}\left[\frac{\frac{-x+1}{x}}{3x^2-3}\right]=\lim_{x\to 1}\left[\frac{-x+1}{x(3x^2-3)}\right]=\lim_{x\to 1}\left[\frac{-(x-1)}{3x(x^2-1)}\right]=\frac{-(1-1)}{3\cdot 1(1^2-1)}=\frac{0}{0}\\ \frac{-(x-1)}{3x(x^2-1)}=\frac{-(x-1)}{3x(x+1)(x-1)}=\frac{-1}{3x(x+1)}\rightarrow\lim_{x\to 1}\left[\frac{-1}{3x(x+1)}\right]=\frac{-1}{3\cdot 1(1+1)}=\frac{-1}{3\cdot 2}=\frac{-1}{6}\\ \lim_{x\to 1}\left[\frac{\frac{-1}{x^2}}{6x}\right]=\lim_{x\to 1}\left[\frac{-1}{6x^3}\right]=\frac{-1}{6\cdot 1^3}=\frac{-1}{6}\end{gathered}\tag{128}$$

Exercício VI

$$\begin{gathered}\lim_{x\to 0}[x\cdot\ln x]=0\cdot\ln 0=0(-\infty)\\ \lim_{x\to 0}[x\cdot\ln x]=\lim_{x\to 0}\left[\frac{x}{1}\cdot\ln x\right]=\lim_{x\to 0}\left[\frac{\ln x}{\frac{1}{x}}\right]=\frac{\ln 0}{\frac{1}{0}}=\frac{-\infty}{\infty}\\ \lim_{x\to 0}\left[\frac{\frac{1}{x}}{\frac{-1}{x^2}}\right]=\lim_{x\to 0}\left[\frac{x^2}{-x}\right]=\lim_{x\to 0}[-x]=-0=0\end{gathered}\tag{129}$$

Exercício VII

$$\begin{gathered}\lim_{x\to 1}\left[\frac{\ln x}{x-1}\right]=\frac{\ln 1}{1-1}=\frac{0}{0}\\ \lim_{x\to 1}\left[\frac{\frac{1}{x}}{1}\right]=\lim_{x\to 1}\left[\frac{1}{x}\right]=\frac{1}{1}=1\end{gathered}\tag{130}$$

Exercício VIII

$$\begin{aligned}\lim_{x\to\infty}\left[\frac{x^2}{e^x}\right]=\frac{\infty^2}{e^\infty}=\frac{\infty}{\infty}\\ \lim_{x\to\infty}\left[\frac{2x}{e^x}\right]=\frac{2\cdot\infty}{e^\infty}=\frac{\infty}{\infty}\\ \lim_{x\to\infty}\left[\frac{2}{e^x}\right]=\frac{2}{e^\infty}=\frac{2}{\infty}=0\end{aligned} \tag{131}$$

Exercício IX

$$\begin{gathered}\lim_{x\to0}[x^x]=0^0\\ y=\lim_{x\to0}[x^x]\to\ln y=\lim_{x\to0}[\ln x^x]=\lim_{x\to0}[x\cdot\ln x]=0\cdot\ln0=0(-\infty)\\ \ln y=\lim_{x\to0}[x\cdot\ln x]=\lim_{x\to0}\left[\frac{x}{1}\cdot\ln x\right]=\lim_{x\to0}\left[\frac{\ln x}{\frac{1}{x}}\right]=\frac{\ln0}{\frac{1}{0}}=\frac{-\infty}{\infty}\\ \ln y=\lim_{x\to0}\left[\frac{\ln x}{\frac{1}{x}}\right]=\lim_{x\to0}\left[\frac{\frac{1}{x}}{\frac{-1}{x^2}}\right]=\lim_{x\to0}\left[\frac{x^2}{-x}\right]=\lim_{x\to0}[-x]=-0=0\\ \ln y=0\to e^0=y\to y=1\end{gathered} \tag{132}$$

Exercício X

$$\begin{gathered}\lim_{x\to0}\left[(1+x)^{\frac{1}{x}}\right]=(1+0)^{\frac{1}{0}}=1^\infty\\ y=\lim_{x\to0}\left[(1+x)^{\frac{1}{x}}\right]\to\ln y=\lim_{x\to0}\left[\ln(1+x)^{\frac{1}{x}}\right]=\lim_{x\to0}\left[\frac{\ln(1+x)}{x}\right]=\frac{\ln(1+0)}{0}=\frac{\ln1}{0}=\frac{0}{0}\\ \ln y=\lim_{x\to0}\left[\frac{\frac{1}{1+x}\cdot1}{1}\right]=\lim_{x\to0}\left[\frac{1}{1+x}\right]=\frac{1}{1+0}=1\\ \ln y=1\to e^1=y\to y=e\end{gathered} \tag{133}$$