

## Introdução as Derivadas – Aula 1

$$\frac{\partial f(x)}{\partial x} = D_x f(x) = f'(x)$$

- 01.  $y = c \rightarrow y' = 0$
- 02.  $y = x \rightarrow y' = 1$
- 03.  $y = x^c \rightarrow y' = c \cdot x^{c-1}$
- 04.  $y = e^x \rightarrow y' = e^x$
- 05.  $y = \ln(x) \rightarrow y' = \frac{1}{x}$
- 06.  $y = f \cdot h \rightarrow y' = f' \cdot h + f \cdot h'$
- 07.  $y = \frac{f}{h} \rightarrow y' = \frac{f' \cdot h - f \cdot h'}{h^2}$
- 08.  $y = f^c \rightarrow y' = c \cdot f^{c-1} \cdot f'$
- 09.  $y = e^f \rightarrow y' = e^f \cdot f'$
- 10.  $y = c^f \rightarrow y' = c^f \cdot f' \cdot \ln(c)$
- 11.  $y = \ln(f) \rightarrow y' = \frac{f'}{f}$
- 12.  $y = \log_c(f) \rightarrow y' = \frac{f'}{f} \cdot \log_c(e)$

### Exercício I

- a)  $y = 8 \rightarrow y' = 0$
  - b)  $y = \sqrt{3} \rightarrow y' = 0$
  - c)  $f(x) = \pi \rightarrow f'(x) = 0$
  - d)  $g(x) = (\pi - 1)^\pi \rightarrow g'(x) = 0$
- (1)

### Exercício II

- a)  $y = x^5 \rightarrow y' = 5x^{5-1} = 5x^4$
  - b)  $h(x) = x^{-5} \rightarrow h'(x) = -5x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = -\frac{5}{x^6}$
  - c)  $g(x) = 5x^3 \rightarrow g'(x) = 5 \cdot 3x^{3-1} = 15x^2$
- (2)

### Exercício III

$$h(x) = 8x \rightarrow h'(x) = 8 \cdot 1 = 8$$

(3)

### Exercício IV

$$\begin{aligned} f(x) &= 7x^3 - 2x - 400 \\ f'(x) &= 7 \cdot 3x^{3-1} - 2 \cdot 1 - 0 = 21x^2 - 2 \end{aligned}$$

(4)

## Derivada com X no Denominador – [Aula 2](#)

### Exercício I

$$\begin{aligned}g(x) &= \frac{3}{x^5} = 3x^{-5} \\g'(x) &= 3(-5x^{-5-1}) = -15x^{-6} = -\frac{15}{x^6}\end{aligned}\tag{5}$$

### Exercício II

$$\begin{aligned}h(x) &= 3x^5 - \frac{2}{x^4} = 3x^5 - 2x^{-4} \\h'(x) &= 3(5x^{5-1}) - 2(-4x^{-4-1}) = 15x^4 + 8x^{-5} = 15x^4 + \frac{8}{x^5}\end{aligned}\tag{6}$$

## Derivada de Função Raiz – [Aula 3](#)

### Exercício I

$$\begin{aligned}y &= \sqrt[3]{x^4} = x^{\frac{4}{3}} \\y' &= \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}\end{aligned}\tag{7}$$

### Exercício II

$$\begin{aligned}g(x) &= 7\sqrt[3]{x} = 7x^{\frac{1}{3}} \\g'(x) &= 7\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) = \frac{7}{3}x^{-\frac{2}{3}} = \frac{7}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = \frac{7}{3\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right) = \frac{7\sqrt[3]{x}}{3x}\end{aligned}\tag{8}$$

## Derivada de uma Função Potência – [Aula 4](#)

### Exercício I

$$y = x^3 \rightarrow y' = 3x^2\tag{9}$$

### Exercício II

$$\begin{aligned}y &= \frac{5x^4x^3}{x^2} = 5x^{4+3-2} = 5x^5 \\y' &= 5(5x^{5-1}) = 25x^4\end{aligned}\tag{10}$$

## Derivada de uma Função Potência – [Aula 5](#)

### Exercício I

$$y = \frac{x^2 \sqrt[3]{x}}{\sqrt[3]{x}} = x^2 \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{2+\frac{1}{2}-\frac{1}{3}} = x^{\frac{12+3-2}{6}} = x^{\frac{13}{6}}$$
$$y' = \frac{13}{6} x^{\frac{13}{6}-1} = \frac{13}{6} x^{\frac{7}{6}} = \frac{13}{6} \sqrt[6]{x^7}$$
(11)

## Derivada de Função Exponencial e Logarítmica – [Aula 6](#)

### Exercício I

$$f(x) = 3e^x + 10 \cdot \ln(x)$$
$$f'(x) = 3e^x + \frac{10}{x} = \frac{3xe^x + 10}{x}$$
(12)

### Exercício II

$$g(x) = 7e^x + 9 \cdot \ln(x) + 3x^4 - 4x + 100$$
$$g'(x) = 7e^x + \frac{9}{x} + 12x^3 - 4 = \frac{7xe^x + 9 + 12x^4 - 4x}{x}$$
(13)

## Derivada de um Produto de Funções – [Aula 7](#)

### Exercício I

$$f(x) = x \cdot \ln(x)$$
$$\begin{cases} g(x) = x & \rightarrow g'(x) = 1 \\ h(x) = \ln(x) & \rightarrow h'(x) = \frac{1}{x} \end{cases}$$
(14)
$$f(x) = g(x) \cdot h(x) \rightarrow f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$$

## Derivada de uma Divisão de Funções – [Aula 8](#)

### Exercício I

$$f(x) = \frac{e^x}{3x} = e^x \cdot \frac{1}{3x} = e^x \cdot x^{-1}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = \frac{x^{-1}}{3} = \frac{1}{3x} & \rightarrow h'(x) = \frac{-x^{-2}}{3} = -\frac{1}{3x^2} \end{cases}$$

$$f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x) = e^x \cdot \frac{1}{3x} + e^x \cdot \left(-\frac{1}{3x^2}\right) = \frac{e^x}{3x} - \frac{e^x}{3x^2} = \frac{x \cdot e^x - e^x}{3x^2} = \frac{e^x(x-1)}{3x^2}$$

(15)

$$f(x) = \frac{e^x}{3x}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = 3x & \rightarrow h'(x) = 3 \end{cases}$$

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2} = \frac{e^x \cdot 3x - e^x \cdot 3}{(3x)^2} = \frac{3e^x(x-1)}{9x^2} = \frac{e^x(x-1)}{3x^2}$$

## Derivadas Básicas – [Aula 9](#)

### Exercício I

$$y = \frac{2x^2 + 5x}{x} = \frac{2x^2}{x} + \frac{5x}{x} = 2x + 5$$

$$y' = 2$$

(16)

### Exercício II

$$h(x) = \frac{3(x^2 - 1)}{x} = \frac{3x^2 - 3}{x} = \frac{3x^2}{x} - \frac{3}{x} = 3x - 3x^{-1}$$

$$h'(x) = 3 - (-3x^{-2}) = 3 + \frac{3}{x^2} = \frac{3x^2 + 3}{x^2} = \frac{3(x^2 + 1)}{x^2}$$

(17)

## Exercícios de Derivada – [Aula 10](#)

### Exercício I

$$h(x) = 3x^3(2 + 4x) = 6x^3 + 12x^4$$

$$h'(x) = 18x^2 + 48x^3 = 6x^2(8x + 3)$$

(18)

### Exercício II

$$\begin{aligned}g(x) &= (x^2 - 1)(x^3 + 4) = x^5 + 4x^2 - x^3 - 4 \\g'(x) &= 5x^4 + 8x - 3x^2 = x(5x^3 - 3x + 8)\end{aligned}\quad (19)$$

### Curso básico de derivadas – [Aula 12](#)

#### Exercício I

$$\begin{aligned}g(x) &= x^4 + 2e^x + e^2 \\g'(x) &= 4x^3 + 2e^x = 2(2x^3 + e^x)\end{aligned}\quad (20)$$

#### Exercício II

$$\begin{aligned}g(x) &= \sqrt[3]{x^7} + \frac{3}{x^2} + 5 = x^{\frac{7}{3}} + 3x^{-2} + 5 \\g'(x) &= \frac{7x^{\frac{4}{3}}}{3} + (-6x^{-3}) = \frac{7\sqrt[3]{x^4}}{3} - \frac{6}{x^3} = \frac{7x^3\sqrt[3]{x^4} - 18}{3x^3} = \frac{7\sqrt[3]{x^{13}} - 18}{3x^3}\end{aligned}\quad (21)$$

### Derivada de um Produto de Funções – [Aula 13](#)

#### Exercício I

$$\begin{aligned}y &= 8x \cdot \ln(x) \\y' &= 8 \cdot \ln(x) + 8x \cdot \frac{1}{x} = 8 \cdot \ln(x) + 8 = 8(\ln(x) + 1)\end{aligned}\quad (22)$$

### Derivada de função composta, raiz, polinomial – [Aula 14](#)

#### Exercício I

$$y = x^3 \rightarrow y' = 3x^2 \quad (23)$$

#### Exercício II

$$\begin{aligned}f(x) &= (2x^2 - 1)^3 \\g(x) &= 2x^2 - 1 \rightarrow g'(x) = 4x \\f'(x) &= g(x)^p \rightarrow f'(x) = p \cdot g(x)^{p-1} \cdot g'(x) = 3(2x^2 - 1)^2 \cdot 4x = 12x(2x^2 - 1)^2\end{aligned}\quad (24)$$

#### Exercício III

$$\begin{aligned}y &= (3 - x^2)^3 \\y' &= 3(3 - x^2)^2(-2x) = -6x(3 - x^2)^2\end{aligned}\quad (25)$$

#### Exercício IV

$$y = \frac{3}{(2x^2 - 1)^4} = 3(2x^2 - 1)^{-4}$$

$$y' = 3(-4)(2x^2 - 1)^{-5} \cdot 4x = -48x(2x^2 - 1)^{-5} = -\frac{48x}{(2x^2 - 1)^5} \quad (26)$$

#### Exercício V

$$y = \sqrt{(2x^2 - 1)^3} = (2x^2 - 1)^{\frac{3}{2}}$$

$$y' = \frac{3}{2}(2x^2 - 1)^{\frac{1}{2}} \cdot 4x = 6x(2x^2 - 1)^{\frac{1}{2}} = 6x\sqrt{2x^2 - 1} \quad (27)$$

### Derivada de funções quociente, produto, polinomial – [Aula 15](#)

#### Exercício I

$$y = 7x^4 - 2x^3 + 8x + 5$$

$$y' = 28x^3 - 6x^2 + 8 = 2(14x^3 - 3x^2 + 4) \quad (28)$$

#### Exercício II

$$y = (2x^3 - 4x^2)(3x^5 + x^2) = 6x^8 + 2x^5 - 12x^7 - 4x^4 = 6x^8 - 12x^7 + 2x^5 - 4x^4$$

$$y' = 48x^7 - 84x^6 + 10x^4 - 16x^3 = 2x^3(24x^4 - 42x^3 + 5x - 8) \quad (29)$$

#### Exercício III

$$h(x) = \frac{2x^3 + 4}{x^2 - 4x + 1}$$

$$\begin{cases} f(x) = 2x^3 + 4 & \rightarrow f'(x) = 6x^2 \\ g(x) = x^2 - 4x + 1 & \rightarrow g'(x) = 2x - 4 \end{cases}$$

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2} = \frac{6x^2(x^2 - 4x + 1) - (2x^3 + 4)(2x - 4)}{(x^2 - 4x + 1)^2} =$$

$$\frac{6x^4 - 24x^3 + 6x^2 - (4x^4 - 8x^3 + 8x - 16)}{(x^2 - 4x + 1)^2} = \frac{6x^4 - 24x^3 + 6x^2 - 4x^4 + 8x^3 - 8x + 16}{(x^2 - 4x + 1)^2} =$$

$$\frac{2x^4 - 16x^3 + 6x^2 - 8x + 16}{(x^2 - 4x + 1)^2} = \frac{2(x^4 - 8x^3 + 3x^2 - 4x + 8)}{(x^2 - 4x + 1)^2} \quad (30)$$

#### Exercício IV

$$y = \frac{3}{x^5} = 3x^{-5} \rightarrow y' = -15x^{-6} = -\frac{15}{x^6} \quad (31)$$

Exercício V

$$v(r) = \frac{4}{3} \pi r^3 \rightarrow v'(r) = 4 \pi r^2 \quad (32)$$

Exercício VI

$$\begin{aligned} f(s) &= \sqrt{3}(s^3 - s^2) = \sqrt{3}s^3 - \sqrt{3}s^2 \\ f'(s) &= 3\sqrt{3}s^2 - 2\sqrt{3}s = s\sqrt{3}(3s - 2) \end{aligned} \quad (33)$$

Exercício VII

$$\begin{aligned} y &= (4x^2 + 3)^2 = 16x^4 + 12x^2 + 12x^2 + 9 = 16x^4 + 24x^2 + 9 \\ y' &= 64x^3 + 48x = 16x(4x^2 + 3) \end{aligned} \quad (34)$$

Derivadas de função quociente e produto – [Aula 16](#)

Exercício I

$$\begin{aligned} y &= \frac{x^4 - 2x^2 + 5x + 1}{x^4} = \frac{x^4}{x^4} - \frac{2x^2}{x^4} + \frac{5x}{x^4} + \frac{1}{x^4} = 1 - 2x^{-2} + 5x^{-3} + x^{-4} \\ y' &= 4x^{-3} - 15x^{-4} - 4x^{-5} = \frac{4}{x^3} - \frac{15}{x^4} - \frac{4}{x^5} = \frac{4x^2 - 15x - 4}{x^5} \end{aligned} \quad (35)$$

Exercício II

$$\begin{aligned} y &= \frac{x}{x-1} \\ y' &= \frac{1(x-1) - x \cdot 1}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2} \end{aligned} \quad (36)$$

Exercício III

$$\begin{aligned} y &= \left( \frac{2x+1}{x+5} \right) (3x-1) = \frac{6x^2 - 2x + 3x - 1}{x+5} = \frac{6x^2 + x - 1}{x+5} \\ y' &= \frac{(12x+1)(x+5) - (6x^2 + x - 1) \cdot 1}{(x+5)^2} = \frac{12x^2 + 60x + x + 5 - 6x^2 - x + 1}{(x+5)^2} = \\ &= \frac{6x^2 + 60x + 6}{(x+5)^2} = \frac{6(x^2 + 10x + 1)}{(x+5)^2} \end{aligned} \quad (37)$$

Exercício IV

$$y = \frac{1}{8} x^8 - 4x^4 \rightarrow y' = x^7 - 16x^3 = x^3(x^4 - 16) \quad (38)$$

Exercício V

$$y = x^2 + 3x + \frac{1}{x^2} = x^2 + 3x + x^{-2}$$

$$y' = 2x + 3 - 2x^{-3} = 2x + 3 + \frac{2}{x^3} = \frac{2x^4 + 3x^3 + 2}{x^3} \quad (39)$$

Exercício VI

$$y = \frac{3}{x^2} + \frac{5}{x^4} = 3x^{-2} + 5x^{-4} \rightarrow y' = -6x^{-3} - 20x^{-5} = -\frac{6}{x^3} - \frac{20}{x^5} = \frac{-6x^2 - 20}{x^5} \quad (40)$$

Derivada de funções envolvendo raiz, quociente, produto – [Aula 17](#)

Exercício I

$$g(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{\frac{1}{2}}} = 3x^{-\frac{1}{2}}$$

$$g'(x) = 3 \left( -\frac{1}{2} \right) x^{-\frac{3}{2}} = \frac{-3}{2x^{\frac{3}{2}}} = -\frac{3}{2\sqrt{x^3}} \quad (41)$$

Exercício II

$$g(x) = 5\sqrt[3]{x^2} = 5x^{\frac{2}{3}}$$

$$g'(x) = 5 \cdot \frac{2}{3} x^{-\frac{1}{3}} = \frac{10}{3x^{\frac{1}{3}}} = \frac{10}{3\sqrt[3]{x}} \quad (42)$$

Exercício III

$$h(r) = 6\sqrt[4]{r^3} + \frac{7}{r^3} = 6r^{\frac{3}{4}} + 7r^{-3}$$

$$h'(r) = 6 \cdot \frac{3}{4} r^{-\frac{1}{4}} + 7(-3)r^{-4} = \frac{9}{2} r^{-\frac{1}{4}} - 21r^{-4} = \frac{9}{2r^{\frac{1}{4}}} - \frac{21}{r^4} = \frac{9}{2\sqrt[4]{r}} - \frac{21}{r^4} =$$

$$\frac{9r^4 - 42\sqrt[4]{r}}{2r} = \frac{3}{2r} (3r^4 - 14\sqrt[4]{r}) \quad (43)$$



#### Exercício IV

$$\begin{aligned}
 g(t) &= \frac{5t^3}{6\sqrt{t}} = 5t^3 \cdot \frac{1}{6t^{\frac{1}{2}}} = 5t^3 \cdot \frac{t^{-\frac{1}{2}}}{6} \\
 g'(t) &= 5 \cdot 3t^{3-1} \cdot \frac{1}{6\sqrt{t}} + 5t^3 \left( \frac{1}{6} \left( -\frac{1}{2} \right) t^{-\frac{1}{2}-1} \right) = \frac{15t^2}{6\sqrt{t}} + 5t^3 \left( -\frac{t^{-\frac{3}{2}}}{12} \right) = \frac{5t^2}{2\sqrt{t}} - \frac{5}{12} t^{3-\frac{3}{2}} = \\
 \frac{5}{2} t^{2-\frac{1}{2}} - \frac{5}{12} t^{\frac{3}{2}} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{30\sqrt{t^3} - 5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} \\
 g'(t) &= \frac{5 \cdot 3t^{3-1} \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 6 \left( \frac{1}{2} \right) t^{\frac{1}{2}-1}}{\left( 6t^{\frac{1}{2}} \right)^2} = \frac{15t^2 \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 3t^{-\frac{1}{2}}}{36t} = \\
 \frac{90t^{2+\frac{1}{2}} - 15t^{3-\frac{1}{2}}}{36t} &= \frac{15 \left( 6t^{\frac{5}{2}} - t^{\frac{5}{2}} \right)}{36t} = \frac{5\sqrt{t^5}(6-1)}{12t} = \frac{30\sqrt{t^5}}{12t} - \frac{5\sqrt{t^5}}{12t} = \\
 \frac{5}{2} t^{\frac{5}{2}-1} - \frac{5}{12} t^{\frac{5}{2}-1} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} = \frac{5}{6} t^{3-\frac{1}{2}} = \frac{5}{6} t^{\frac{5}{2}} \\
 g'(t) &= \frac{5}{6} \cdot \frac{5}{2} t^{\frac{5}{2}-1} = \frac{25t^{\frac{3}{2}}}{12} = \frac{25\sqrt{t^3}}{12}
 \end{aligned}
 \tag{44}$$

#### Exercício V

$$\begin{aligned}
 h(x) &= 7x\sqrt[3]{x^2} = 7x \cdot x^{\frac{2}{3}} = 7x^{1+\frac{2}{3}} = 7x^{\frac{5}{3}} \\
 h'(x) &= 7 \cdot \frac{5}{3} x^{\frac{5}{3}-1} = \frac{35x^{\frac{2}{3}}}{3} = \frac{35\sqrt[3]{x^2}}{3}
 \end{aligned}
 \tag{45}$$

#### Exercício VI

$$\begin{aligned}
 g(t) &= \frac{8t^3\sqrt{t}}{t} = \frac{8t^3t^{\frac{1}{2}}}{t} = 8t^{3+\frac{1}{2}-1} = 8t^{\frac{5}{2}} \\
 g'(t) &= 8 \cdot \frac{5}{2} t^{\frac{5}{2}-1} = 20t^{\frac{3}{2}} = 20\sqrt{t^3}
 \end{aligned}
 \tag{46}$$

## Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 18](#)

### Exercício I

$$f(x) = \frac{3x^4 - 8x^2 + 4}{x^2} = \frac{3x^4}{x^2} - \frac{8x^2}{x^2} + \frac{4}{x^2} = 3x^2 - 8 + 4x^{-2}$$

$$f'(x) = 6x - 8x^{-3} = 6x - \frac{8}{x^3} = \frac{6x^4 - 8}{x^3} \quad (47)$$

### Exercício II

$$f(x) = \frac{\pi x^3}{\sqrt{x^3}} = \frac{\pi x^3}{x^{\frac{3}{2}}} = \pi x^{3 - \frac{3}{2}} = \pi x^{\frac{3}{2}}$$

$$f'(x) = \pi \cdot \frac{3}{2} x^{\frac{3}{2} - 1} = \frac{3\pi x^{\frac{1}{2}}}{2} = \frac{3\pi\sqrt{x}}{2} \quad (48)$$

### Exercício III

$$y = \frac{x^2}{e^x}$$

$$y' = \frac{2x \cdot e^x - x^2 \cdot e^x}{(e^x)^2} = \frac{e^x \cdot x(2 - x)}{e^{2x}} = \frac{x(2 - x)}{e^x} = \frac{x}{e^x}(2 - x) \quad (49)$$

### Exercício IV

$$y = x^5 \cdot e^x$$

$$y' = 5x^4 \cdot e^x + x^5 \cdot e^x = e^x \cdot x^4(5 + x) \quad (50)$$

### Exercício V

$$y = \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x)$$

$$y' = \left( \frac{1}{2} x^{\frac{1}{2} - 1} \right) \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{-\frac{1}{2}} \cdot \ln(x)}{2} + x^{\frac{1}{2}} \cdot x^{-1} = \frac{\ln(x)}{2x^{\frac{1}{2}}} + x^{\frac{1}{2} - 1} = \frac{\ln(x)}{2\sqrt{x}} + x^{-\frac{1}{2}} = \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{x^{\frac{1}{2}}} =$$

$$\frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} = \left( \frac{1}{\sqrt{x}} \left( \frac{\ln(x)}{2} + 1 \right) \right) \left( \frac{\sqrt{x}}{\sqrt{x}} \right) = \frac{\sqrt{x}}{x} \left( \frac{\ln(x)}{2} + 1 \right) \quad (51)$$

## Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 19](#)

### Exercício I

$$f(t) = e^t \cdot \ln(t)$$

$$f'(t) = e^t \cdot \ln(t) + e^t \cdot \frac{1}{t} = e^t \left( \ln(t) + \frac{1}{t} \right) = \frac{e^t(t \cdot \ln(t) + 1)}{t} = \frac{e^t}{t} (t \cdot \ln(t) + 1) \quad (52)$$

### Exercício II

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} =$$

$$\frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} = \frac{e^r(2r-1)}{2r^{\frac{1}{2}+1}} = \frac{e^r(2r-1)}{2r^{\frac{3}{2}}} = \left( \frac{e^r(2r-1)}{2\sqrt{r^3}} \right) \left( \frac{\sqrt{r^3}}{\sqrt{r^3}} \right) = \frac{e^r \sqrt{r^3}(2r-1)}{2r^3} = \frac{e^r \sqrt{r^3}}{2r^3} (2r-1) \quad (53)$$

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{e^r \left( \sqrt{r} - \frac{1}{2\sqrt{r}} \right)}{r} =$$

$$\frac{e^r}{r} \left( \sqrt{r} - \frac{1}{2\sqrt{r}} \right) = \frac{e^r}{r} \left( \frac{2r-1}{2\sqrt{r}} \right) \left( \frac{\sqrt{r}}{\sqrt{r}} \right) = \frac{e^r \sqrt{r}}{2r^2} (2r-1)$$

### Exercício III

$$v(t) = \frac{t^3 \cdot e^t}{t} = t^{3-1} \cdot e^t = t^2 \cdot e^t$$

$$v'(t) = 2t \cdot e^t + t^2 \cdot e^t = e^t \cdot t(2+t) \quad (54)$$

### Exercício IV

$$y = x^5 \rightarrow y' = 5x^4$$

$$y = (2x^2 - 4)^5 \rightarrow y' = 5(2x^2 - 4)^4 \cdot 4x = 20x(2x^2 - 4)^4 \quad (55)$$

### Exercício V

$$y = \sqrt{2x^3 - 1} = (2x^3 - 1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(2x^3 - 1)^{-\frac{1}{2}} \cdot 6x^2 = 3x^2(2x^3 - 1)^{-\frac{1}{2}} = \frac{3x^2}{(2x^3 - 1)^{\frac{1}{2}}} = \left( \frac{3x^2}{\sqrt{2x^3 - 1}} \right) \left( \frac{\sqrt{2x^3 - 1}}{\sqrt{2x^3 - 1}} \right) =$$

$$\frac{3x^2 \sqrt{2x^3 - 1}}{2x^3 - 1} \quad (56)$$

### Exercício VI

$$y = \frac{5}{(3x^2 + 9)^4} = 5(3x^2 + 9)^{-4}$$

$$y' = -20(3x^2 + 9)^{-5} \cdot 6x = -120x(3x^2 + 9)^{-5} = -\frac{120x}{(3x^2 + 9)^5} \quad (57)$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 20](#)

### Exercício I

$$y = \sqrt[3]{6x^2 + 7x + 2} = (6x^2 + 7x + 2)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(6x^2 + 7x + 2)^{-\frac{2}{3}} \cdot (12x + 7) = \frac{12x + 7}{3(6x^2 + 7x + 2)^{\frac{2}{3}}} = \left( \frac{12x + 7}{3\sqrt[3]{(6x^2 + 7x + 2)^2}} \right) \left( \frac{\sqrt[3]{6x^2 + 7x + 2}}{\sqrt[3]{6x^2 + 7x + 2}} \right)$$

$$\frac{(12x + 7)\sqrt[3]{6x^2 + 7x + 2}}{3(6x^2 + 7x + 2)} \quad (58)$$

### Exercício II

$$y = e^{5x^2 + 4}$$

$$y' = e^{5x^2 + 4} \cdot 10x \quad (59)$$

### Exercício III

$$y = e^{\frac{1}{x^2}} = e^{x^{-2}}$$

$$y' = e^{\frac{1}{x^2}} (-2x^{-3}) = e^{\frac{1}{x^2}} \left( -\frac{2}{x^3} \right) = -\frac{2e^{\frac{1}{x^2}}}{x^3} \quad (60)$$

### Exercício IV

$$y = 3^{x^2}$$

$$y' = 3^{x^2} \cdot \ln(3) \cdot 2x \quad (61)$$

### Exercício V

$$y = 5^{2x^2+3x-1}$$

$$y' = 5^{2x^2+3x-1} \cdot \ln(5) \cdot (4x+3) \quad (62)$$

### Exercício VI

$$y = \left(\frac{1}{2}\right)^{\sqrt{x}} = \left(\frac{1}{2}\right)^{x^{\frac{1}{2}}}$$

$$y' = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{\frac{1}{2}-1} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{-\frac{1}{2}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2x^{\frac{1}{2}}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2\sqrt{x}} \quad (63)$$

### Exercício VII

$$f = e^{\frac{x+1}{x-1}} \rightarrow g = \frac{x+1}{x-1} \rightarrow f = e^g$$

$$g' = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2} \quad (64)$$

$$f' = e^g \cdot g' \rightarrow f' = e^{\frac{x+1}{x-1}} \cdot \frac{-2}{(x-1)^2} = \frac{-2e^{\frac{x+1}{x-1}}}{(x-1)^2}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 21](#)

### Exercício I

$$y = \ln(5x^2 - 4x)$$

$$y' = \frac{10x-4}{5x^2-4x} = \frac{2(5x-2)}{x(5x-4)} \quad (65)$$

### Exercício II

$$y = \log_2(3x^2 - 7)$$

$$y' = \frac{6x}{3x^2-7} \cdot \log_2(e) \quad (66)$$

### Exercício III

$$\begin{aligned}
 f &= \log_{10} \left( \frac{x+1}{x^2+1} \right) \rightarrow g = \frac{x+1}{x^2+1} \rightarrow f = \log_{10}(g) \\
 g' &= \frac{1(x^2+1) - (x+1)2x}{(x^2+1)^2} = \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2} = \frac{-(x^2+2x-1)}{(x^2+1)^2} \\
 f' &= \frac{g'}{g} \cdot \log_{10}(e) \rightarrow f' = \frac{\frac{-(x^2+2x-1)}{(x^2+1)^2}}{\frac{x+1}{x^2+1}} \cdot \log_{10}(e) = \frac{-(x^2+2x-1)(x^2+1)}{(x^2+1)^2(x+1)} \cdot \log_{10}(e) = \\
 &= \frac{-(x^2+2x-1)}{(x^2+1)(x+1)} \cdot \log_{10}(e) = \frac{-x^2-2x+1}{x^3+x^2+x+1} \cdot \log_{10}(e)
 \end{aligned} \tag{67}$$

$$\begin{aligned}
 f &= \log_{10} \left( \frac{x+1}{x^2+1} \right) = \log_{10}(x+1) - \log_{10}(x^2+1) \\
 f' &= \frac{1}{x+1} \cdot \log_{10}(e) - \frac{2x}{x^2+1} \cdot \log_{10}(e) = \log_{10}(e) \left( \frac{1}{x+1} - \frac{2x}{x^2+1} \right) = \\
 &= \log_{10}(e) \left( \frac{1(x^2+1) - 2x(x+1)}{x^3+x+x^2+1} \right) = \log_{10}(e) \left( \frac{x^2+1-2x^2-2x}{x^3+x^2+x+1} \right) = \\
 &= \log_{10}(e) \left( \frac{-x^2-2x+1}{x^3+x^2+x+1} \right)
 \end{aligned}$$

### Exercício IV

$$\begin{aligned}
 y &= \ln \left( \frac{e^x}{x+1} \right) = \ln(e^x) - \ln(x+1) = x - \ln(x+1) \\
 y' &= 1 - \frac{1}{x+1} = \frac{x+1-1}{x+1} = \frac{x}{x+1}
 \end{aligned} \tag{68}$$

### Exercício V

$$\begin{aligned}
 y &= \ln(2x-1)^3 \rightarrow f = (2x-1)^3 \rightarrow y = \ln(f) \\
 f' &= 3(2x-1)^2 \cdot 2 = 6(2x-1)^2 \\
 y' &= \frac{f'}{f} = \frac{6(2x-1)^2}{(2x-1)^3} = \frac{6}{2x-1} \\
 y &= \ln(2x-1)^3 = 3 \cdot \ln(2x-1) \\
 y' &= 0 \cdot \ln(2x-1) + 3 \cdot \frac{2}{2x-1} = \frac{6}{2x-1}
 \end{aligned} \tag{69}$$

Exercício VI

$$\begin{aligned}
 y &= \ln[(4x^2+3)(2x-1)] = \ln(4x^2+3) + \ln(2x-1) \\
 y' &= \frac{8x}{4x^2+3} + \frac{2}{2x-1} = \frac{8x(2x-1) + 2(4x^2+3)}{8x^3-4x^2+6x-3} = \frac{16x^2-8x+8x^2+6}{8x^3-4x^2+6x-3} = \\
 &= \frac{24x^2-8x+6}{8x^3-4x^2+6x-3} = \frac{2(12x^2-4x+3)}{8x^3-4x^2+6x-3}
 \end{aligned} \tag{70}$$

Exercício VII

$$\begin{aligned}
 f(x) &= \left(\frac{2x+1}{3x-1}\right)^4 \rightarrow g(x) = \frac{2x+1}{3x-1} \rightarrow f(x) = g(x)^4 \\
 g'(x) &= \frac{2(3x-1) - (2x+1)3}{(3x-1)^2} = \frac{6x-2-6x-3}{(3x-1)^2} = \frac{-5}{(3x-1)^2} \\
 f'(x) &= 4g(x)^{4-1} \cdot g'(x) = 4\left(\frac{2x+1}{3x-1}\right)^3 \cdot \frac{-5}{(3x-1)^2} = \frac{-20}{(3x-1)^2} \cdot \left(\frac{2x+1}{3x-1}\right)^3
 \end{aligned} \tag{71}$$

## Derivação (ou diferenciação) Logarítmica – [Aula 22](#)

Exercício I

$$\begin{aligned}
 y &= x^x \rightarrow \ln(y) = \ln(x^x) \rightarrow \ln(y) = x \cdot \ln(x) \\
 \frac{y'}{y} &= 1 \cdot \ln(x) + x \cdot \frac{1}{x} \rightarrow \frac{y'}{y} = \ln(x) + 1 \rightarrow y' = y(\ln(x) + 1) = x^x(\ln(x) + 1)
 \end{aligned} \tag{72}$$

Exercício II

$$\begin{aligned}
 y &= c^f \rightarrow \ln(y) = \ln(c^f) \rightarrow \ln(y) = f \cdot \ln(c) \\
 \frac{y'}{y} &= f' \cdot \ln(c) + f \cdot \frac{0}{c} \rightarrow y' = y(f' \cdot \ln(c)) \rightarrow y' = c^f \cdot f' \cdot \ln(c) \\
 y &= e^f \rightarrow \ln(y) = \ln(e^f) \rightarrow \ln(y) = f \cdot \ln(e) \rightarrow \ln(y) = f \\
 \frac{y'}{y} &= f' \rightarrow y' = y \cdot f' \rightarrow y' = e^f \cdot f'
 \end{aligned} \tag{73}$$

## Derivação (ou diferenciação) Logarítmica – [Aula 23](#)

Exercício I

$$\begin{aligned}
 y &= x^{\sqrt{x}} \rightarrow \ln(y) = \ln(x^{\sqrt{x}}) = \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x) \\
 \frac{y'}{y} &= \frac{1}{2} x^{\frac{1}{2}-1} \cdot \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{\frac{-1}{2}}}{2} \cdot \ln(x) + x^{\frac{1}{2}-1} = \frac{x^{\frac{-1}{2}} \cdot \ln(x)}{2} + x^{\frac{-1}{2}} = x^{\frac{-1}{2}} \left( \frac{\ln(x)}{2} + 1 \right) \rightarrow \\
 y' &= y \cdot x^{\frac{-1}{2}} \left( \frac{\ln(x)}{2} + 1 \right) = x^{\sqrt{x}} \cdot x^{\frac{-1}{2}} \left( \frac{\ln(x)}{2} + 1 \right) = x^{\sqrt{x}-\frac{1}{2}} \left( \frac{\ln(x)}{2} + 1 \right) = x^{\frac{x\sqrt{x}-1}{x}} \left( \frac{\ln(x)+2}{2} \right)
 \end{aligned} \tag{74}$$

## Derivada de função composta – [Aula 24](#)

### Exercício I

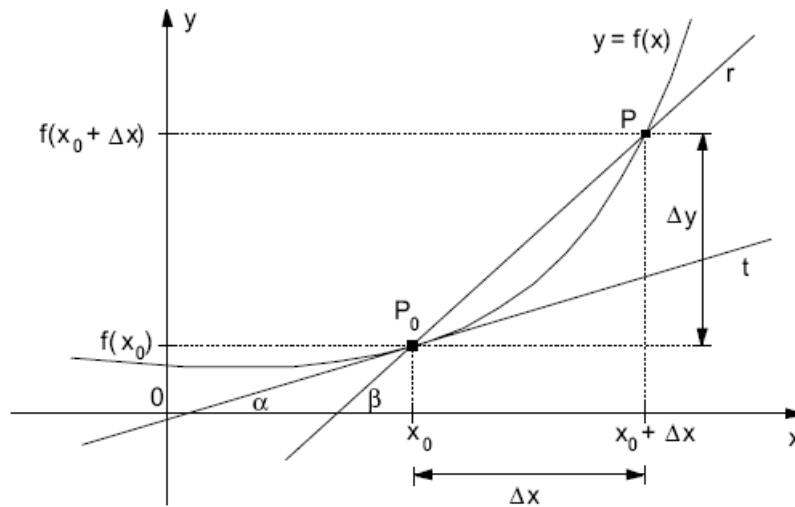
$$\begin{aligned}
 y &= \left( \frac{2x+1}{3x-1} \right)^4 \rightarrow f = \frac{2x+1}{3x-1} \rightarrow y = f^4 \\
 f' &= \frac{2(3x-1) - (2x+1)3}{(3x-1)^2} = \frac{6x-2-6x-3}{(3x-1)^2} = \frac{-5}{(3x-1)^2} \\
 y' &= 4f^{4-1} \cdot f' = 4 \left( \frac{2x+1}{3x-1} \right)^3 \cdot \frac{-5}{(3x-1)^2} = \frac{-20}{(3x-1)^2} \cdot \left( \frac{2x+1}{3x-1} \right)^3 = \frac{-20}{(3x-1)^2} \cdot \frac{(2x+1)^3}{(3x-1)^3} = \\
 &\quad \frac{-20(2x+1)^3}{(3x-1)^5}
 \end{aligned}$$

$$\begin{aligned}
 y &= \left( \frac{2x+1}{3x-1} \right)^4 \rightarrow \ln(y) = \ln \left( \frac{2x+1}{3x-1} \right)^4 = 4 \cdot \ln \left( \frac{2x+1}{3x-1} \right) = 4(\ln(2x+1) - \ln(3x-1)) \quad (75) \\
 \frac{y'}{y} &= 4 \cdot \left( \frac{2}{2x+1} - \frac{3}{3x-1} \right) \rightarrow y' = 4y \cdot \left( \frac{2}{2x+1} - \frac{3}{3x-1} \right) = \\
 &= 4 \cdot \left( \frac{2x+1}{3x-1} \right)^4 \cdot \left( \frac{2}{2x+1} - \frac{3}{3x-1} \right) = 4 \cdot \left( \frac{2x+1}{3x-1} \right)^4 \cdot \frac{2(3x-1) - 3(2x+1)}{(2x+1)(3x-1)} = \\
 &= \frac{4(2x+1)^4 \cdot 2(3x-1) - 3(2x+1)}{(3x-1)^4 \cdot (2x+1)(3x-1)} = \frac{4(2x+1)^3(2(3x-1) - 3(2x+1))}{(3x-1)^5} = \\
 &= \frac{4(2x+1)^3(6x-2-6x-3)}{(3x-1)^5} = \frac{4(2x+1)^3(-5)}{(3x-1)^5} = \frac{-20(2x+1)^3}{(3x-1)^5}
 \end{aligned}$$



## O que é uma Derivada – [Aula 25](#)

Derivada é a principal ferramenta matemática utilizada para calcular e estudar taxas de variação.



$$\operatorname{tg} \beta = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

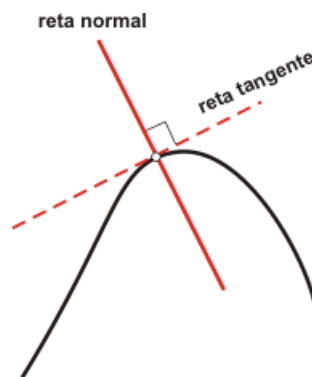
$$\operatorname{tg} \alpha = \lim_{\Delta x \rightarrow 0} \left[ \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \right]$$

$$f(x_0) = x_0^2$$

$$\operatorname{tg} \alpha = f'(x_0) = 2x_0$$

$$\operatorname{tg} \alpha = \lim_{\Delta x \rightarrow 0} \left[ \frac{(x_0 + \Delta x)^2 - (x_0)^2}{\Delta x} \right] = \frac{x_0^2 + 2x_0\Delta x + \Delta x^2 - x_0^2}{\Delta x} = \frac{\Delta x(2x_0 + \Delta x)}{\Delta x} = 2x_0 + \Delta x = 2x_0 + 0 = 2x_0$$

## Derivadas: Reta Tangente e Reta Normal – [Aula 26](#)



Reta tangente:  $y - y_0 = f'(x_0)(x - x_0) \rightarrow y = f'(x_0)(x - x_0) + y_0 = f'(x_0)(x - x_0) + f(x_0)$

Reta normal:  $y - y_0 = \frac{-1}{f'(x_0)}(x - x_0) = \frac{-(x - x_0)}{f'(x_0)} \rightarrow y = \frac{-(x - x_0)}{f'(x_0)} + y_0 = \frac{-(x - x_0)}{f'(x_0)} + f(x_0)$

# Exercício I

$$f(x_0) = \frac{1}{x_0} = x_0^{-1}$$

$$f'(x_0) = -x_0^{-2} = \frac{-1}{x_0^2}$$

$$y_t = \frac{-1}{x_0^2}(x - x_0) + \frac{1}{x_0} = \frac{-x + x_0}{x_0^2} + \frac{1}{x_0} = \frac{-x + x_0 + x_0}{x_0^2} = \frac{-x + 2x_0}{x_0^2}$$

$$y_n = \frac{-(x - x_0)}{\frac{-1}{x_0^2}} + \frac{1}{x_0} = x_0^2(x - x_0) + \frac{1}{x_0} = x_0^2 x - x_0^3 + \frac{1}{x_0} = \frac{x_0^3 x - x_0^4 + 1}{x_0}$$

$$x_0 = 1$$

$$y_0 = f(x_0) = \frac{1}{1} = 1$$

$$\tan \alpha = f'(x_0) = \frac{-1}{1^2} = -1$$

$$y_t = \frac{-x + 2 \cdot 1}{1^2} = -x + 2$$

$$y_n = \frac{1^3 x - 1^4 + 1}{1} = x$$

(76)

$$\lim_{x_0 \rightarrow 0} f(x_0) = \lim_{x_0 \rightarrow 0} \left[ \frac{1}{x_0} \right] = \nexists$$

$$\lim_{x_0 \rightarrow 0^+} \left[ \frac{1}{x_0} \right] = \frac{1}{0^+} = +\infty$$

$$\lim_{x_0 \rightarrow 0^+} \left[ \frac{1}{x_0} \right] = \frac{1}{0^-} = -\infty$$

$$\lim_{x_0 \rightarrow +\infty} f(x_0) = \lim_{x_0 \rightarrow +\infty} \left[ \frac{1}{x_0} \right] = \frac{1}{+\infty} = 0$$

$$\lim_{x_0 \rightarrow -\infty} f(x_0) = \lim_{x_0 \rightarrow -\infty} \left[ \frac{1}{x_0} \right] = \frac{1}{-\infty} = 0$$

## Exercício II

$$\begin{aligned}
 x_0 &= 1 \\
 f(x) &= \sqrt{x} = x^{\frac{1}{2}} \\
 f'(x) &= \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}} \\
 y_0 &= f(x_0) = \sqrt{1} = 1 \\
 \tan \alpha &= f'(x_0) = \frac{1}{2\sqrt{1}} = \frac{1}{2} \\
 y_t - y_0 &= f'(x_0)(x - x_0) \rightarrow y_t - 1 = \frac{1}{2}(x - 1) \rightarrow y_t = \frac{x-1}{2} + 1 = \frac{x-1+2}{2} = \frac{x+1}{2} \\
 y_n - y_0 &= \frac{-1}{f'(x_0)}(x - x_0) \rightarrow y_n - 1 = \frac{-1}{\frac{1}{2}}(x - 1) \rightarrow y_n = -2(x - 1) + 1 = -2x + 2 + 1 = -2x + 3
 \end{aligned} \tag{77}$$

## Derivada pela Definição – [Aula 27](#)

### Exercício I

$$\begin{aligned}
 f(x) &= 2x^2 - 3x + 4 \\
 f'(x) &= 4x - 3 \\
 \lim_{h \rightarrow 0} \left[ \frac{(2(x+h)^2 - 3(x+h) + 4) - (2x^2 - 3x + 4)}{h} \right] &= \\
 \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 4 - 2x^2 + 3x - 4}{h} &= \\
 \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 4 - 2x^2 + 3x - 4}{h} &= \frac{2h^2 + 4xh - 3h}{h} = \frac{h(2h + 4x - 3)}{h} = \\
 2h + 4x - 3 &= 2 \cdot 0 + 4x - 3 = 4x - 3
 \end{aligned} \tag{78}$$

### Exercício II

$$\begin{aligned}
 f(x) &= \frac{2}{x} = 2x^{-1} \\
 f'(x) &= -2x^{-2} = \frac{-2}{x^2} \\
 \lim_{h \rightarrow 0} \left[ \frac{\left( \frac{2}{x+h} \right) - \left( \frac{2}{x} \right)}{h} \right] &= \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h} = \frac{\frac{2x - 2x - 2h}{x(x+h)}}{h} = \frac{-2h}{xh(x+h)} = \frac{-2}{x(x+h)} = \\
 \frac{-2}{x(x+0)} &= \frac{-2}{x^2}
 \end{aligned} \tag{79}$$

## Derivada pela Regra da Cadeia – [Aula 1](#)

$$y=f(g(x)) \rightarrow y'=f'(g(x)) \cdot g'(x)$$

$$y=(2x^2-1)^3=a^3 \rightarrow a=2x^2-1$$

$$y=e^{5x^2-1}=e^a \rightarrow a=5x^2-1$$

$$y=\ln[\cos(5x^2-1)]=\ln[a] \rightarrow a=\cos(5x^2-1)=\cos(b) \rightarrow b=5x^2-1$$

Exercício I

$$\begin{aligned} y &= (2x^2-1)^4 = a^4 \rightarrow a = 2x^2-1 \\ a' &= 4x \\ y' &= 4a^{4-1} \cdot a' = 4(2x^2-1)^3 \cdot 4x = 16x(2x^2-1)^3 \end{aligned} \quad (80)$$

Exercício II

$$\begin{aligned} y &= e^{2x^2-1} = e^a \rightarrow a = 2x^2-1 \\ a' &= 4x \\ y' &= e^a \cdot a' = e^{2x^2-1} \cdot 4x \end{aligned} \quad (81)$$

Exercício III

$$\begin{aligned} y &= \cos(2x^2-9) = \cos(a) \rightarrow a = 2x^2-9 \\ a' &= 4x \\ y' &= -\sin(a) \cdot a' = -\sin(2x^2-9) \cdot 4x = -4x \cdot \sin(2x^2-9) \end{aligned} \quad (82)$$

Exercício IV

$$\begin{aligned} y &= \ln[\sin(x^3-1)] = \ln[a] \rightarrow a = \sin(x^3-1) = \sin(b) \rightarrow b = x^3-1 \\ b' &= 3x^2 \\ a' &= \cos(b) \cdot b' = \cos(x^3-1) \cdot 3x^2 \\ y' &= \frac{1}{a} \cdot a' = \frac{1}{\sin(x^3-1)} \cdot \cos(x^3-1) \cdot 3x^2 = 3x^2 \cdot \frac{\cos(x^3-1)}{\sin(x^3-1)} = 3x^2 \cdot \cotg(x^3-1) \end{aligned} \quad (83)$$

Exercício V

$$\begin{aligned} y &= \ln[\cos(\sin(2x^2-1))] = \ln[a] \\ a &= \cos(\sin(2x^2-1)) = \cos(b) \\ b &= \sin(2x^2-1) = \sin(c) \\ c &= 2x^2-1 \\ c' &= 4x \\ b' &= \cos(c) \cdot c' = \cos(2x^2-1) \cdot 4x \\ a' &= -\sin(b) \cdot b' = -\sin(\sin(2x^2-1)) \cdot \cos(2x^2-1) \cdot 4x \\ y' &= \ln[a] \cdot a' = \frac{1}{\cos(\sin(2x^2-1))} (-\sin(\sin(2x^2-1))) \cdot \cos(2x^2-1) \cdot 4x = \\ &= -4x \cdot \cos(2x^2-1) \cdot \frac{\sin(\sin(2x^2-1))}{\cos(\sin(2x^2-1))} = -4x \cdot \cos(2x^2-1) \cdot \tg(\sin(2x^2-1)) \end{aligned} \quad (84)$$

## Derivada pela Regra da Cadeia – [Aula 2](#)

### Exercício I

$$y = x^4 \rightarrow \frac{\partial y}{\partial x} = 4x^3$$

$$y = (x^2 + 1)^4$$
$$\frac{\partial y}{\partial x} = 4(x^2 + 1)^3 \cdot 2x = 8x(x^2 + 1)^3 \quad (85)$$

### Exercício II

$$y = \cos(x^2 + 1)^4 \rightarrow \frac{\partial y}{\partial x} = -\operatorname{sen}(x^2 + 1)^4 \cdot 4(x^2 + 1)^3 \cdot 2x = -8x(x^2 + 1)^3 \operatorname{sen}(x^2 + 1)^4 \quad (86)$$

### Exercício III

$$y = \cos^2 x \rightarrow \frac{\partial y}{\partial x} = 2 \cdot \cos x (-\operatorname{sen} x) = -2 \cdot \operatorname{sen} x \cdot \cos x \quad (87)$$

### Exercício IV

$$y = \cos^2(2x^3 - 1) \rightarrow \frac{\partial y}{\partial x} = 2 \cos(2x^3 - 1) (-\operatorname{sen}(2x^3 - 1)) 6x^2 =$$
$$-12x^2 \operatorname{sen}(2x^3 - 1) \cos(2x^3 - 1) \quad (88)$$

### Exercício V

$$y = (\ln x + 1)^3 \rightarrow \frac{\partial y}{\partial x} = 3(\ln x + 1)^2 \frac{1}{x} = \frac{3}{x} (\ln x + 1)^2 \quad (89)$$

### Exercício VI

$$y = \cos(\ln x + 1)^3 \rightarrow \frac{\partial y}{\partial x} = -\operatorname{sen}(\ln x + 1)^3 3(\ln x + 1)^2 \frac{1}{x} = -\frac{3}{x} (\ln x + 1)^2 \operatorname{sen}(\ln x + 1)^3 \quad (90)$$

### Exercício VII

$$y = \ln^3(2x^2 - 1) \rightarrow \frac{\partial y}{\partial x} = 3 \ln^2(2x^2 - 1) \frac{1}{2x^2 - 1} 4x = \frac{12x}{2x^2 - 1} \ln^2(2x^2 - 1) \quad (91)$$

### Exercício VIII

$$y = e^{2x^2 - 1} \rightarrow \frac{\partial y}{\partial x} = e^{2x^2 - 1} 4x \quad (92)$$

Exercício IX

$$\begin{aligned}
 y &= \text{tg}^2(x^2-1) = [\text{tg}(x^2-1)]^2 = \left[ \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \right]^2 \rightarrow \\
 \frac{\partial y}{\partial x} &= 2 \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \frac{\cos(x^2-1) \cos(x^2-1) - \text{sen}(x^2-1)(-\text{sen}(x^2-1))}{\cos^2(x^2-1)} 2x = \\
 4x \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \frac{\cos^2(x^2-1) + \text{sen}^2(x^2-1)}{\cos^2(x^2-1)} &= 4x \frac{\text{sen}(x^2-1) \cos^2(x^2-1) + \text{sen}^3(x^2-1)}{\cos^3(x^2-1)} = \\
 4x \left[ \frac{\text{sen}(x^2-1) \cos^2(x^2-1)}{\cos^3(x^2-1)} + \frac{\text{sen}^3(x^2-1)}{\cos^3(x^2-1)} \right] &= 4x \left[ \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} + \left( \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \right)^3 \right] = \\
 4x [\text{tg}(x^2-1) + \text{tg}^3(x^2-1)] &= 4x \cdot \text{tg}(x^2-1) [1 + \text{tg}^2(x^2-1)]
 \end{aligned} \tag{93}$$

$$y = \text{tg}^2(x^2-1) \rightarrow \frac{\partial y}{\partial x} = 2 \text{tg}(x^2-1) \sec^2(x^2-1) 2x = 4x \cdot \text{tg}(x^2-1) \sec^2(x^2-1)$$

Exercício X

$$\begin{aligned}
 y &= \left( \frac{2x^2-1}{1-x^2} \right)^3 \rightarrow \frac{\partial y}{\partial x} = 3 \left( \frac{2x^2-1}{1-x^2} \right)^2 \frac{4x(1-x^2) - (2x^2-1)(-2x)}{(1-x^2)^2} = \\
 3 \left( \frac{2x^2-1}{1-x^2} \right)^2 \frac{4x-4x^3+4x^3-2x}{(1-x^2)^2} &= 3 \left( \frac{2x^2-1}{1-x^2} \right)^2 \frac{2x}{(1-x^2)^2} = \frac{3(2x^2-1)^2}{(1-x^2)^2} \frac{2x}{(1-x^2)^2} = \\
 \frac{6x(2x^2-1)^2}{(1-x^2)^4} &= 6x \left( \frac{2x^2-1}{(1-x^2)^2} \right)^2
 \end{aligned} \tag{94}$$

Exercício XI

$$\begin{aligned}
 y &= \ln[\cos(x^3-1)^4] \rightarrow \frac{\partial y}{\partial x} = \frac{1}{\cos(x^3-1)^4} (-\text{sen}(x^3-1)^4) 4(x^3-1)^3 3x^2 = \\
 -12x^2(x^3-1)^3 \frac{\text{sen}(x^3-1)^4}{\cos(x^3-1)^4} &= -12x^2(x^3-1)^3 \text{tg}(x^3-1)^4
 \end{aligned} \tag{95}$$

## Derivada pela Regra da Cadeia – [Aula 3](#)

### Exercício I

$$\begin{aligned}
 y &= e^{x\sqrt[3]{2x-1}} = e^a \\
 a &= x\sqrt[3]{2x-1} = xb \\
 b &= \sqrt[3]{2x-1} = (2x-1)^{\frac{1}{3}} = c^{\frac{1}{3}} \\
 c &= 2x-1 \\
 c' &= 2 \\
 b' &= \frac{1}{3}c^{\frac{1}{3}-1} \cdot c' = \frac{1}{3}(2x-1)^{-\frac{2}{3}} \cdot 2 = \frac{2}{3(2x-1)^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{(2x-1)^2}} \\
 a' &= 1 \cdot b + x \cdot b' = 1(\sqrt[3]{2x-1}) + x \left( \frac{2}{3\sqrt[3]{(2x-1)^2}} \right) = \sqrt[3]{2x-1} + \frac{2x}{3\sqrt[3]{(2x-1)^2}} = \\
 &= \frac{3\sqrt[3]{(2x-1)^3} + 2x}{3\sqrt[3]{(2x-1)^2}} = \frac{3(2x-1) + 2x}{3\sqrt[3]{(2x-1)^2}} = \frac{6x-3+2x}{3\sqrt[3]{(2x-1)^2}} = \frac{8x-3}{3\sqrt[3]{(2x-1)^2}} \\
 y' &= e^a \cdot a' = e^{x\sqrt[3]{2x-1}} \left( \frac{8x-3}{3\sqrt[3]{(2x-1)^2}} \right) = \frac{(8x-3)e^{x\sqrt[3]{2x-1}}}{3\sqrt[3]{(2x-1)^2}}
 \end{aligned} \tag{96}$$

## Derivada pela Regra da Cadeia – [Aula 4](#)

### Exercício I

$$\begin{aligned}
 f(x) &= \sin^3(2^{5x^4}) \rightarrow \frac{\partial f(x)}{\partial x} = 3\sin^2(2^{5x^4})\cos(2^{5x^4})2^{5x^4}\ln(2)20x^3 = \\
 &= \ln(2)60x^32^{5x^4}\sin^2(2^{5x^4})\cos(2^{5x^4})
 \end{aligned} \tag{97}$$

## Derivada pela Regra da Cadeia – [Aula 5](#)

### Exercício I

$$\begin{aligned}
 y &= 2^{-3x^2} \ln x = a \cdot b \\
 a &= 2^{-3x^2} = 2^c \\
 b &= \ln x \\
 c &= -3x^2 \\
 c' &= -6x \\
 b' &= \frac{1}{x} \\
 a' &= 2^c c' \ln(2) = 2^{-3x^2} (-6x) \ln(2) = -\ln(2) 6x \cdot 2^{-3x^2} \\
 y' &= a' b + a b' = (-\ln(2) 6x \cdot 2^{-3x^2}) \ln x + 2^{-3x^2} \frac{1}{x} = -\ln(2) \ln(x) 6x \cdot 2^{-3x^2} + \frac{2^{-3x^2}}{x} = \\
 &= \frac{-\ln(2) \ln(x) 6x^2 2^{-3x^2} + 2^{-3x^2}}{x} = \frac{-2^{-3x^2} (\ln(2) \ln(x) 6x^2 - 1)}{x}
 \end{aligned} \tag{98}$$

## Taxas Relacionadas – [Aula 1](#)

$$\begin{aligned}
 y &= x^2 \\
 1 \frac{\partial y}{\partial x} &= 2x \frac{\partial x}{\partial x} \rightarrow \frac{\partial y}{\partial x} = 2x \\
 4y^3 &= x^2 + 1 \\
 12y^2 \frac{\partial y}{\partial x} &= 2x \frac{\partial x}{\partial x} \rightarrow 12y^2 \frac{\partial y}{\partial x} = 2x \rightarrow \frac{\partial y}{\partial x} = \frac{2x}{12y^2} = \frac{x}{6y^2} \\
 y^3 &= x^2 + y + 2x^3 + 3 = y^3 - y = 2x^3 + x^2 + 3 \\
 (3y^2 - 1) \frac{\partial y}{\partial x} &= (6x^2 + 2x) \frac{\partial x}{\partial x} \rightarrow \frac{\partial y}{\partial x} = \frac{6x^2 + 2x}{3y^2 - 1} = \frac{2x(3x + 1)}{3y^2 - 1} \\
 y^3 + 1 + z^2 &= x^2 \\
 3y^2 \frac{\partial y}{\partial x} + 2z \frac{\partial z}{\partial x} &= 2x \\
 3y^2 \frac{\partial y}{\partial z} + 2z &= 2x \frac{\partial x}{\partial z} \\
 3y^2 + 2z \frac{\partial z}{\partial y} &= 2x \frac{\partial x}{\partial y} \\
 3y^2 \frac{\partial y}{\partial t} + 2z \frac{\partial z}{\partial t} &= 2x \frac{\partial x}{\partial t}
 \end{aligned}$$

1. Identificar as variáveis.
2. Achar uma relação entre as variáveis.
3. Derivar em relação a variável de referência.
4. Substituir os valores conhecidos.
5. Isolar o que se quer calcular.



### Exercício I

O **volume** do balão esférico cresce a uma taxa de **100 centímetros cúbicos por segundo**, qual é a taxa de crescimento do **raio** quando o mesmo mede **50 cm**.

1. Volume (v):  $cm^3$ ; Raio (r):  $cm$ ; Tempo(t):  $s$
2.  $v = \frac{4\pi r^3}{3}$
3.  $\frac{\partial v}{\partial t} = 3 \frac{4\pi r^{3-1}}{3} \frac{\partial r}{\partial t} = 4\pi r^2 \frac{\partial r}{\partial t}$
4.  $100 \frac{cm^3}{s} = 4\pi (50cm)^2 \frac{\partial r}{\partial t} = 4\pi 2500 cm^2 \frac{\partial r}{\partial t} = 10000\pi cm^2 \frac{\partial r}{\partial t}$
5.  $\frac{\partial r}{\partial t} = \frac{100 \frac{cm^3}{s}}{10000\pi cm^2} = \frac{100 cm^3}{10000\pi cm^2 s} = \frac{1}{100\pi} \frac{cm}{s}$

(99)

### Exercício II

Uma mancha de óleo expande-se em forma de círculo onde a **área** cresce a uma taxa constante de **26 quilômetros quadrados/h**. Com que rapidez variará o raio da mancha quando a **área** for **9 quilômetros quadrados**.

1. Área (a):  $km^2$ ; Raio (r):  $km$ ; Tempo (t):  $h$
2.  $a = \pi r^2 \rightarrow r = \sqrt{\frac{a}{\pi}} \rightarrow r = \sqrt{\frac{9km^2}{\pi}} = \frac{3km}{\sqrt{\pi}} = \frac{3\sqrt{\pi}km}{\pi}$
3.  $\frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t}$
4.  $26 \frac{km^2}{h} = 2\pi \frac{3\sqrt{\pi}km}{\pi} \frac{\partial r}{\partial t} = 6\sqrt{\pi}km \frac{\partial r}{\partial t}$
5.  $\frac{\partial r}{\partial t} = \frac{26 \frac{km^2}{h}}{6\sqrt{\pi}km} = \frac{26 km^2}{6\sqrt{\pi}km h} = \frac{26 km}{6\sqrt{\pi}h} = \frac{13}{3\sqrt{\pi}} \frac{km}{h} = \frac{13\sqrt{\pi} km}{3\pi h}$

(100)

### Exercício III

Um foguete sobe verticalmente e é acompanhado por uma estação no solo a 5 km da base de lançamento. Com que rapidez o foguete subirá, quando sua altura for 4 km e a sua distância da estação estiver crescendo a 2000 km/h.

1. Distância (d): km; Altura (a): km; Tempo (t): h

$$2. d = \sqrt{(5 \text{ km})^2 + a^2} = \sqrt{25 \text{ km}^2 + a^2} = (25 \text{ km}^2 + a^2)^{\frac{1}{2}}$$

$$3. \frac{\partial d}{\partial t} = \frac{1}{2} (25 \text{ km}^2 + a^2)^{-\frac{1}{2}} \cdot 2a \frac{\partial a}{\partial t} = \frac{a}{\sqrt{25 \text{ km}^2 + a^2}} \frac{\partial a}{\partial t}$$

$$4. 2000 \frac{\text{km}}{\text{h}} = \frac{4 \text{ km}}{\sqrt{25 \text{ km}^2 + (4 \text{ km})^2}} \frac{\partial a}{\partial t} = \frac{4 \text{ km}}{\sqrt{25 \text{ km}^2 + 16 \text{ km}^2}} \frac{\partial a}{\partial t} = \frac{4 \text{ km}}{\sqrt{41 \text{ km}^2}} \frac{\partial a}{\partial t} = \frac{4\sqrt{41}}{41} \frac{\partial a}{\partial t} \quad (101)$$

$$5. \frac{\partial a}{\partial t} = \frac{2000 \frac{\text{km}}{\text{h}}}{\frac{4\sqrt{41}}{41}} = \frac{2000 \text{ km}}{\frac{4\sqrt{41}}{41} h} = \frac{41 \cdot 500 \text{ km}}{\sqrt{41} h} = 500 \sqrt{41} \frac{\text{km}}{h}$$

## Taxas Relacionadas – [Aula 2](#)

### Exercício I

Um tanque de água tem a forma de um cone circular invertido com base de raio igual a 2 metros e altura igual a 4 metros. Se a água está sendo bombeada para dentro do tanque a uma taxa de 2 metros cúbicos por minuto, encontre a taxa na qual o nível de água estará elevado quando a água estiver a 3 metros de profundidade.

1. Volume do tanque (v): m<sup>3</sup>; Profundidade do tanque invertido (h): m; Tempo (t): min

$$2. v = \frac{\pi r^2 h}{3} \rightarrow \frac{2 \text{ m}}{4 \text{ m}} = \frac{r}{h} \rightarrow r = \frac{2h}{4} = \frac{h}{2} \rightarrow v = \frac{\pi \left(\frac{h}{2}\right)^2 h}{3} = \frac{\pi \frac{h^2}{4} h}{3} = \frac{\pi \frac{h^3}{4}}{3} = \frac{\pi h^3}{12}$$

$$3. \frac{\partial v}{\partial t} = \frac{3\pi h^2}{12} \frac{\partial h}{\partial t} = \frac{\pi h^2}{4} \frac{\partial h}{\partial t}$$

$$4. 2 \frac{\text{m}^3}{\text{min}} = \frac{\pi (3 \text{ m})^2}{4} \frac{\partial h}{\partial t} = \frac{9\pi \text{ m}^2}{4} \frac{\partial h}{\partial t}$$

$$5. \frac{\partial h}{\partial t} = \frac{2 \frac{\text{m}^3}{\text{min}}}{\frac{9\pi \text{ m}^2}{4}} = \frac{8 \text{ m}^3}{9\pi \text{ m}^2 \text{ min}} = \frac{8}{9\pi} \frac{\text{m}}{\text{min}}$$

(102)

### Exercício II

Uma escada de 5 metros está apoiada em uma parede. A base é arrastada a 3 m/s. Qual a velocidade da escada a longo da parede, quando a base se encontra a 3 metros da parede.

1. Base (b): m; Altura (h): m; Tempo (t): s
2.  $(5m)^2 = b^2 + h^2 \rightarrow b^2 + h^2 = 25m^2 \rightarrow$   
 $h = \sqrt{25m^2 - b^2} = \sqrt{25m^2 - (3m)^2} = \sqrt{25m^2 - 9m^2} = \sqrt{16m^2} = 4m$
3.  $2b \frac{\partial b}{\partial t} + 2h \frac{\partial h}{\partial t} = 0$
4.  $2 \cdot 3m \cdot 3 \frac{m}{s} + 2 \cdot 4m \cdot \frac{\partial h}{\partial t} = 0 \rightarrow 18 \frac{m^2}{s} + 8m \frac{\partial h}{\partial t} = 0$
5.  $\frac{\partial h}{\partial t} = \frac{-18 \frac{m^2}{s}}{8m} = \frac{-18m^2}{8ms} = \frac{-9}{4} \frac{m}{s}$

(103)

### Exercício III

Determine a taxa de variação instantânea da área em relação ao lado, quando esse mede 4 m.

1. Área (a):  $m^2$ ; Lado (l): m
2.  $a = l^2$
3.  $\frac{\partial a}{\partial l} = 2l$
4.  $\frac{\partial a}{\partial l} = 2(4m)$
5.  $\frac{\partial a}{\partial l} = 8 \frac{m^2}{m}$

(104)

### Exercício IV

Uma pedra foi lançada no lago e com isso, gerou ondas circulares que se propagam com a velocidade constante de 3 m/s. Qual a taxa de crescimento da área após transcorrido 10 segundos.

1. Área (a):  $m^2$ , Raio (r): m; Tempo (t): s
2.  $a = \pi r^2 \rightarrow r = 3 \frac{m}{s} \cdot 10s = 30m$
3.  $\frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t}$
4.  $\frac{\partial a}{\partial t} = 2\pi(30m)3 \frac{m}{s}$
5.  $\frac{\partial a}{\partial t} = 180\pi \frac{m^2}{s}$

(105)

## Exercício V

Acumula-se areia em um monte com a forma de um cone. A altura do cone é igual ao raio da base. O **volume** de areia cresce a uma taxa de **10 metros cúbicos por hora**. Com que razão aumenta a **área** da base quando a **altura** for **4 metros**.

$$\begin{aligned}
 &1. \text{ Volume (v): } m^3 \text{ Altura(r): } m; \text{ Área (a): } m^2; \text{ Tempo (t): } h \\
 &2. \text{ } a = \pi r^2 \rightarrow r = h \rightarrow v = \frac{\pi r^2 h}{3} = \frac{\pi r^3}{3} \\
 &3. \frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t} \\
 &\frac{\partial v}{\partial t} = \frac{3\pi r^2}{3} \frac{\partial r}{\partial t} = \pi r^2 \frac{\partial r}{\partial t} \\
 &4. 10 \frac{m^3}{h} = \pi (4m)^2 \frac{\partial r}{\partial t} = 16\pi m^2 \frac{\partial r}{\partial t} \quad (106) \\
 &\frac{\partial r}{\partial t} = \frac{10 \frac{m^3}{h}}{16\pi m^2} = \frac{10m^3}{16\pi m^2 h} = \frac{5}{8\pi} \frac{m}{h} \\
 &\frac{\partial a}{\partial t} = 2\pi r \frac{5m}{8\pi h} = \frac{r \cdot 5m}{4h} \\
 &5. \frac{\partial a}{\partial t} = \frac{(4m)5m}{4h} = \frac{20m^2}{4h} = 5 \frac{m^2}{h}
 \end{aligned}$$

## Diferenciação (ou Derivação) Implícita – [Aula](#)

### Exercício I

$$2x + y = 0 \rightarrow y = -2x \quad (107)$$

### Exercício II

$$2x - xy + 1 = 0 \rightarrow xy = 2x + 1 \rightarrow y = \frac{2x+1}{x} = 2 + \frac{1}{x} \quad (108)$$

### Exercício III

$$x^2 + y^2 - 4 = 0 \rightarrow y^2 = -x^2 + 4 \rightarrow y = \pm \sqrt{-x^2 + 4} \quad (109)$$

### Exercício IV

$$e^y - x = 0 \rightarrow e^y = x \rightarrow y = \log_e x = \ln x \quad (110)$$

### Exercício I

$$y^4 - x^5 = 7 \rightarrow 4y^3 \frac{\partial y}{\partial x} - 5x^4 \rightarrow \frac{\partial y}{\partial x} = \frac{5x^4}{4y^3} \quad (111)$$

Exercício II

$$x^2 + y^2 = 25 \rightarrow 2x + 2y \frac{\partial y}{\partial x} = 0 \rightarrow \frac{\partial y}{\partial x} = \frac{-2x}{2y} = \frac{-x}{y} \quad (112)$$

Exercício III

$$\begin{aligned} x^2 y + 7x + 8y = 5 &\rightarrow \left( 2xy + x^2 \frac{\partial y}{\partial x} \right) + 7 + 8 \frac{\partial y}{\partial x} = 0 \rightarrow x^2 \frac{\partial y}{\partial x} + 8 \frac{\partial y}{\partial x} = -2xy - 7 \rightarrow \\ (x^2 + 8) \frac{\partial y}{\partial x} &= -2xy - 7 \rightarrow \frac{\partial y}{\partial x} = \frac{-2xy - 7}{x^2 + 8} \end{aligned} \quad (113)$$

Exercício IV

$$\begin{aligned} x^2 + 2y^3 = 3xy &\rightarrow 2x + 6y^2 \frac{\partial y}{\partial x} = 3y + 3x \frac{\partial y}{\partial x} \rightarrow 6y^2 \frac{\partial y}{\partial x} - 3x \frac{\partial y}{\partial x} = 3y - 2x \rightarrow \\ (6y^2 - 3x) \frac{\partial y}{\partial x} &= 3y - 2x \rightarrow \frac{\partial y}{\partial x} = \frac{3y - 2x}{6y^2 - 3x} = \frac{3y - 2x}{3(2y^2 - x)} \\ \frac{\partial y}{\partial x}(1, 1) &= \frac{3 \cdot 1 - 2 \cdot 1}{3(2(1)^2 - 1)} = \frac{1}{3} \end{aligned} \quad (114)$$

Exercício V

$$\begin{aligned} 2xy - \ln(xy) + 5 = 0 &\rightarrow \left( 2y + 2x \frac{\partial y}{\partial x} \right) - \left[ \frac{1}{xy} \left( y + x \frac{\partial y}{\partial x} \right) \right] + 0 = 0 \rightarrow \\ 2y + 2x \frac{\partial y}{\partial x} - \left( \frac{y}{xy} + \frac{x}{xy} \frac{\partial y}{\partial x} \right) &= 0 \rightarrow 2y + 2x \frac{\partial y}{\partial x} - \frac{1}{x} - \frac{1}{y} \frac{\partial y}{\partial x} = 0 \rightarrow \\ 2x \frac{\partial y}{\partial x} - \frac{1}{y} \frac{\partial y}{\partial x} &= -2y + \frac{1}{x} \rightarrow \left( 2x - \frac{1}{y} \right) \frac{\partial y}{\partial x} = -2y + \frac{1}{x} \rightarrow \\ \left( \frac{2xy - 1}{y} \right) \frac{\partial y}{\partial x} &= \frac{-2xy + 1}{x} \rightarrow \frac{\partial y}{\partial x} = \frac{\frac{-(2xy - 1)}{y}}{\frac{2xy - 1}{x}} = \frac{-y(2xy - 1)}{x(2xy - 1)} = \frac{-y}{x} \end{aligned} \quad (115)$$

Exercício VI

$$\begin{aligned} \ln(xy) = 2x - 2y^2 &\rightarrow \frac{1}{xy} \left( y + x \frac{\partial y}{\partial x} \right) = 2 - 4y \frac{\partial y}{\partial x} \rightarrow \frac{y}{xy} + \frac{x}{xy} \frac{\partial y}{\partial x} = 2 - 4y \frac{\partial y}{\partial x} \rightarrow \\ \frac{1}{x} + \frac{1}{y} \frac{\partial y}{\partial x} &= 2 - 4y \frac{\partial y}{\partial x} \rightarrow \frac{1}{y} \frac{\partial y}{\partial x} + 4y \frac{\partial y}{\partial x} = 2 - \frac{1}{x} \rightarrow \left( \frac{1}{y} + 4y \right) \frac{\partial y}{\partial x} = 2 - \frac{1}{x} \rightarrow \\ \left( \frac{1 + 4y^2}{y} \right) \frac{\partial y}{\partial x} &= \frac{2x - 1}{x} \rightarrow \frac{\partial y}{\partial x} = \frac{\frac{2x - 1}{x}}{\frac{1 + 4y^2}{y}} = \frac{y(2x - 1)}{x(1 + 4y^2)} \end{aligned} \quad (116)$$

### Exercício VII

$$(2x-1)^4 + 10 = y^2 + 20 \rightarrow [4(2x-1)^3 \cdot 2] + 0 = 2y \frac{\partial y}{\partial x} + 0 \rightarrow 8(2x-1)^3 = 2y \frac{\partial y}{\partial x} \rightarrow$$

$$\frac{\partial y}{\partial x} = \frac{8(2x-1)^3}{2y} = \frac{4(2x-1)^3}{y} \quad (117)$$

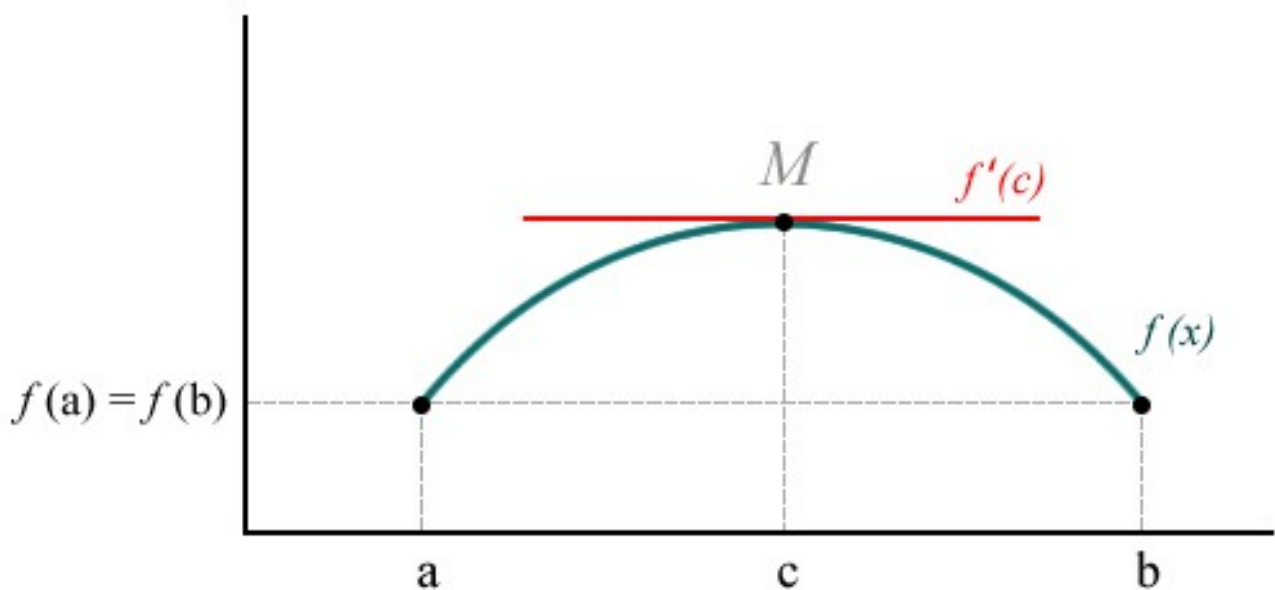
### Exercício VIII

$$e^{xy} + 3x = 3y^3 + 4 \rightarrow \left[ e^{xy} \left( y + x \frac{\partial y}{\partial x} \right) \right] + 3 = 9y^2 \frac{\partial y}{\partial x} + 0 \rightarrow e^{xy} y + e^{xy} x \frac{\partial y}{\partial x} + 3 = 9y^2 \frac{\partial y}{\partial x} \rightarrow$$

$$e^{xy} x \frac{\partial y}{\partial x} - 9y^2 \frac{\partial y}{\partial x} = -e^{xy} y - 3 \rightarrow (e^{xy} x - 9y^2) \frac{\partial y}{\partial x} = -e^{xy} y - 3 \rightarrow \frac{\partial y}{\partial x} = \frac{-e^{xy} y - 3}{e^{xy} x - 9y^2} \quad (118)$$

$$\frac{\partial y}{\partial x}(1,0) = \frac{-e^{1 \cdot 0} 0 - 3}{e^{1 \cdot 0} 1 - 9(0)^2} = -3$$

### O Teorema de Rolle – [Aula](#)



1. A função deve ser contínua num intervalo  $[a, b]$
2. A função deve ser diferenciável ou derivável em  $(a, b)$
3.  $f(a) = f(b) = 0$

Satisfeitas as 3 condições, conclui-se que existe pelo menos um  $x=c$  em  $(a, b)$ , tal que,  $f'(c) = 0$

### Exercício I

$$f(x) = -x^2 + 4x \rightarrow [0; 4]$$

1. A função polinomial é contínua para qualquer valor de x

2. Toda função polinomial é derivável

$$3. f(0) = -(0)^2 + 4(0) = 0 \rightarrow f(4) = -(4)^2 + 4(4) = -4^2 + 4^2 = 0 \rightarrow f(0) = f(4) = 0$$

(119)

$$f'(x) = -2x + 4 \rightarrow -2x + 4 = 0 \rightarrow -2x = -4 \rightarrow x = \frac{-4}{-2} = 2 \rightarrow f'(2) = 0$$

### Exercício II

$$f(x) = 4x^3 - 9x \rightarrow \left[ \frac{-3}{2}; \frac{3}{2} \right]$$

1. A função polinomial é contínua para qualquer valor de x

2. Toda função polinomial é derivável

$$3. f\left(\frac{-3}{2}\right) = 4\left(\frac{-3}{2}\right)^3 - 9\left(\frac{-3}{2}\right) = 4\left(\frac{-27}{8}\right) + \frac{27}{2} = \frac{-27}{2} + \frac{27}{2} = 0 \rightarrow$$

$$f\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right) = 4\left(\frac{27}{8}\right) - \frac{27}{2} = \frac{27}{2} - \frac{27}{2} = 0 \rightarrow f\left(\frac{-3}{2}\right) = f\left(\frac{3}{2}\right) = 0$$

(120)

$$f'(x) = 12x^2 - 9 \rightarrow 12x^2 - 9 = 0 \rightarrow 12x^2 = 9 \rightarrow x = \pm \sqrt{\frac{9}{12}} = \frac{\pm 3}{2\sqrt{3}} = \frac{\pm \sqrt{3}}{2} \rightarrow$$

$$f'\left(\frac{-\sqrt{3}}{2}\right) = f'\left(\frac{\sqrt{3}}{2}\right) = 0$$

### Exercício III

$$f(x) = \sin x \rightarrow [0; 2\pi]$$

1. A função senoidal é contínua para qualquer valor de x

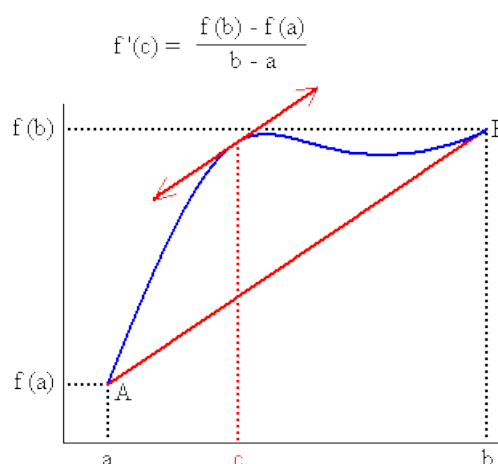
2. Toda função senoidal é derivável

$$3. f(0) = \sin 0 = 0 \rightarrow f(2\pi) = \sin 2\pi = 0 \rightarrow f(0) = f(2\pi) = 0$$

(121)

$$f'(x) = \cos x \rightarrow \cos x = 0 \rightarrow x = \left[ \frac{\pi}{2}; \frac{3\pi}{2} \right] \rightarrow f'\left(\frac{\pi}{2}\right) = f'\left(\frac{3\pi}{2}\right) = 0$$

## Teorema do Valor Médio – [Aula](#)



1. A função deve ser contínua num intervalo  $[a, b]$
2. A função deve ser diferenciável ou derivável em  $(a, b)$

Satisfeitas as 2 condições, existe pelo menos um  $x=c$  em  $(a, b)$ , tal que,  $f'(c) = \frac{f(b) - f(a)}{b - a}$

Exercício I

$$f(x) = x^2 - 2x \rightarrow [0; 3]$$

1. A função polinomial é contínua para qualquer valor de  $x$
2. Toda função polinomial é derivável

$$3. f'(x) = \frac{[(0)^2 - 2(0)] - [(3)^2 - 2(3)]}{0 - 3} = \frac{-(9 - 6)}{-3} = \frac{-3}{-3} = 1 \rightarrow \quad (122)$$

$$f'(x) = 2x - 2 \rightarrow 2x - 2 = 1 \rightarrow 2x = 1 + 2 \rightarrow x = \frac{3}{2} \rightarrow f'\left(\frac{3}{2}\right) = 1$$

Exercício II

$$f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}} \rightarrow [-2; 2]$$

1. A função é contínua
2. A função não é derivável

$$3. f'(x) = \frac{[\sqrt[3]{(2)^2}] - [\sqrt[3]{(-2)^2}]}{2 - (-2)} = \frac{\sqrt[3]{4} - \sqrt[3]{4}}{4} = \frac{0}{4} = 0 \rightarrow \quad (123)$$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}} = 0 \rightarrow 2 = 0 \cdot 3\sqrt[3]{x}$$



## Regra de L'Hospital – [Aula 1](#)

Indeterminações:

1.  $\frac{0}{0}$
2.  $\frac{\infty}{\infty}$
3.  $\infty \cdot 0$  (Indeterminação de produto)
4.  $\infty - \infty$  (Indeterminação da diferença)
5.  $0^0 \quad \infty^0 \quad 1^\infty$  (Indeterminação de potência)

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Exercício I

$$\begin{aligned} \lim_{x \rightarrow 1} \left[ \frac{x^2 - 1}{x - 1} \right] &= \frac{1^2 - 1}{1 - 1} = \frac{0}{0} \rightarrow (x = 1 \rightarrow x - 1 = 0) \\ \frac{(x - 1)(x + 1)}{x - 1} &= x + 1 \rightarrow \lim_{x \rightarrow 1} [x + 1] = 1 + 1 = 2 \\ \lim_{x \rightarrow 1} \left[ \frac{2x}{1} \right] &= \lim_{x \rightarrow 1} [2x] = 2 \cdot 1 = 2 \end{aligned} \quad (124)$$

Exercício II

$$\begin{aligned} \lim_{x \rightarrow \infty} \left[ \frac{e^x}{x^2} \right] &= \frac{e^\infty}{\infty^2} = \frac{\infty}{\infty} \\ \lim_{x \rightarrow \infty} \left[ \frac{e^x}{2x} \right] &= \frac{e^\infty}{2\infty} = \frac{\infty}{\infty} \\ \lim_{x \rightarrow \infty} \left[ \frac{e^x}{2} \right] &= \frac{e^\infty}{2} = \frac{\infty}{2} = \infty \end{aligned} \quad (125)$$

Exercício III

$$\begin{aligned} \lim_{x \rightarrow 2} \left[ \frac{x^2 + 2x - 8}{x^2 - x - 2} \right] &= \frac{2^2 + 2 \cdot 2 - 8}{2^2 - 2 - 2} = \frac{4 + 4 - 8}{4 - 2 - 2} = \frac{0}{0} \rightarrow (x = 2 \rightarrow x - 2 = 0) \\ \frac{(x^2 + 2x - 8) \div (x - 2)}{(x^2 - x - 2) \div (x - 2)} &= \frac{x + 4}{x + 1} \rightarrow \lim_{x \rightarrow 2} \left[ \frac{x + 4}{x + 1} \right] = \frac{2 + 4}{2 + 1} = \frac{6}{3} = 2 \\ \lim_{x \rightarrow 2} \left[ \frac{2x + 2}{2x - 1} \right] &= \frac{2 \cdot 2 + 2}{2 \cdot 2 - 1} = \frac{6}{3} = 2 \end{aligned} \quad (126)$$

### Exercício IV

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ \frac{e^x - x - 1}{x^2} \right] &= \frac{e^0 - 0 - 1}{0^2} = \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{2x} \right] &= \frac{e^0 - 1}{2 \cdot 0} = \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{e^x}{2} \right] &= \frac{e^0}{2} = \frac{1}{2}\end{aligned}\tag{127}$$

### Exercício V

$$\begin{aligned}\lim_{x \rightarrow 1} \left[ \frac{1 - x + \ln x}{x^3 - 3x + 2} \right] &= \frac{1 - 1 + \ln 1}{1^3 - 3 \cdot 1 + 2} = \frac{1 - 1 + 0}{1 - 3 + 2} = \frac{0}{0} \\ \lim_{x \rightarrow 1} \left[ \frac{-1 + \frac{1}{x}}{3x^2 - 3} \right] &= \lim_{x \rightarrow 1} \left[ \frac{\frac{-x + 1}{x}}{3x^2 - 3} \right] = \lim_{x \rightarrow 1} \left[ \frac{-x + 1}{x(3x^2 - 3)} \right] = \lim_{x \rightarrow 1} \left[ \frac{-(x - 1)}{3x(x^2 - 1)} \right] = \frac{-(1 - 1)}{3 \cdot 1(1^2 - 1)} = \frac{0}{0} \\ \frac{-(x - 1)}{3x(x^2 - 1)} &= \frac{-(x - 1)}{3x(x + 1)(x - 1)} = \frac{-1}{3x(x + 1)} \rightarrow \lim_{x \rightarrow 1} \left[ \frac{-1}{3x(x + 1)} \right] = \frac{-1}{3 \cdot 1(1 + 1)} = \frac{-1}{3 \cdot 2} = \frac{-1}{6} \\ \lim_{x \rightarrow 1} \left[ \frac{\frac{-1}{x^2}}{6x} \right] &= \lim_{x \rightarrow 1} \left[ \frac{-1}{6x^3} \right] = \frac{-1}{6 \cdot 1^3} = \frac{-1}{6}\end{aligned}\tag{128}$$

### Exercício VI

$$\begin{aligned}\lim_{x \rightarrow 0} [x \cdot \ln x] &= 0 \cdot \ln 0 = 0(-\infty) \\ \lim_{x \rightarrow 0} [x \cdot \ln x] &= \lim_{x \rightarrow 0} \left[ \frac{x}{1} \cdot \ln x \right] = \lim_{x \rightarrow 0} \left[ \frac{\ln x}{\frac{1}{x}} \right] = \frac{\ln 0}{\frac{1}{0}} = \frac{-\infty}{\infty} \\ \lim_{x \rightarrow 0} \left[ \frac{\frac{1}{x}}{\frac{-1}{x^2}} \right] &= \lim_{x \rightarrow 0} \left[ \frac{x^2}{-x} \right] = \lim_{x \rightarrow 0} [-x] = -0 = 0\end{aligned}\tag{129}$$

### Exercício VII

$$\begin{aligned}\lim_{x \rightarrow 1} \left[ \frac{\ln x}{x - 1} \right] &= \frac{\ln 1}{1 - 1} = \frac{0}{0} \\ \lim_{x \rightarrow 1} \left[ \frac{\frac{1}{x}}{\frac{1}{1}} \right] &= \lim_{x \rightarrow 1} \left[ \frac{1}{x} \right] = \frac{1}{1} = 1\end{aligned}\tag{130}$$

### Exercício VIII

$$\begin{aligned}\lim_{x \rightarrow \infty} \left[ \frac{x^2}{e^x} \right] &= \frac{\infty^2}{e^\infty} = \frac{\infty}{\infty} \\ \lim_{x \rightarrow \infty} \left[ \frac{2x}{e^x} \right] &= \frac{2 \cdot \infty}{e^\infty} = \frac{\infty}{\infty} \\ \lim_{x \rightarrow \infty} \left[ \frac{2}{e^x} \right] &= \frac{2}{e^\infty} = \frac{2}{\infty} = 0\end{aligned}\tag{131}$$

### Exercício IX

$$\begin{aligned}\lim_{x \rightarrow 0} [x^x] &= 0^0 \\ y = \lim_{x \rightarrow 0} [x^x] &\rightarrow \ln y = \lim_{x \rightarrow 0} [\ln x^x] = \lim_{x \rightarrow 0} [x \cdot \ln x] = 0 \cdot \ln 0 = 0(-\infty) \\ \ln y = \lim_{x \rightarrow 0} [x \cdot \ln x] &= \lim_{x \rightarrow 0} \left[ \frac{x}{1} \cdot \ln x \right] = \lim_{x \rightarrow 0} \left[ \frac{\ln x}{\frac{1}{x}} \right] = \frac{\ln 0}{\frac{1}{\infty}} = \frac{-\infty}{\infty} \\ \ln y = \lim_{x \rightarrow 0} \left[ \frac{\ln x}{\frac{1}{x}} \right] &= \lim_{x \rightarrow 0} \left[ \frac{\frac{1}{x}}{-\frac{1}{x^2}} \right] = \lim_{x \rightarrow 0} \left[ \frac{x^2}{-x} \right] = \lim_{x \rightarrow 0} [-x] = -0 = 0 \\ \ln y = 0 &\rightarrow e^0 = y \rightarrow y = 1\end{aligned}\tag{132}$$

### Exercício X

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ (1+x)^{\frac{1}{x}} \right] &= (1+0)^{\frac{1}{0}} = 1^\infty \\ y = \lim_{x \rightarrow 0} \left[ (1+x)^{\frac{1}{x}} \right] &\rightarrow \ln y = \lim_{x \rightarrow 0} \left[ \ln (1+x)^{\frac{1}{x}} \right] = \lim_{x \rightarrow 0} \left[ \frac{\ln(1+x)}{x} \right] = \frac{\ln(1+0)}{0} = \frac{\ln 1}{0} = \frac{0}{0} \\ \ln y = \lim_{x \rightarrow 0} \left[ \frac{\frac{1}{1+x} \cdot 1}{1} \right] &= \lim_{x \rightarrow 0} \left[ \frac{1}{1+x} \right] = \frac{1}{1+0} = 1 \\ \ln y = 1 &\rightarrow e^1 = y \rightarrow y = e\end{aligned}\tag{133}$$

## Regra de L'Hospital – [Aula 2](#)

### Exercício I

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ \frac{\sin(x)}{x} \right] &= \frac{\sin(0)}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{\cos(x)}{1} \right] &= \lim_{x \rightarrow 0} [\cos(x)] = \cos(0) = 1\end{aligned}\tag{134}$$

### Exercício II

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ \frac{\text{sen}(x)}{x^2 + x} \right] &= \frac{\text{sen}(0)}{0^2 + 0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{\cos(x)}{2x + 1} \right] &= \frac{\cos(0)}{2 \cdot 0 + 1} = \frac{1}{1} = 1\end{aligned}\quad (135)$$

### Exercício III

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ \frac{\text{sen}(2x)}{x} \right] &= \frac{\text{sen}(2 \cdot 0)}{0} = \frac{\text{sen}(0)}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{\cos(2x) - 2}{1} \right] &= \lim_{x \rightarrow 0} [2 \cos(2x)] = 2 \cos(2 \cdot 0) = 2 \cos(0) = 2 \cdot 1 = 2\end{aligned}\quad (136)$$

### Exercício IV

$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{2}} \left[ \frac{1 - \text{sen}(x)}{\cos(x)} \right] &= \frac{1 - \text{sen}\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)} = \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow \frac{\pi}{2}} \left[ \frac{-(\cos(x))}{-\text{sen}(x)} \right] &= \lim_{x \rightarrow \frac{\pi}{2}} \left[ \frac{\cos(x)}{\text{sen}(x)} \right] = \frac{\cos\left(\frac{\pi}{2}\right)}{\text{sen}\left(\frac{\pi}{2}\right)} = \frac{0}{1} = 0\end{aligned}\quad (137)$$

### Exercício V

$$\begin{aligned}\lim_{x \rightarrow 0} \left[ \frac{1 - \cos(x)}{x^2} \right] &= \frac{1 - \cos(0)}{0^2} = \frac{1 - 1}{0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{-(-\text{sen}(x))}{2x} \right] &= \lim_{x \rightarrow 0} \left[ \frac{\text{sen}(x)}{2x} \right] = \frac{\text{sen}(0)}{2 \cdot 0} = \frac{0}{0} \\ \lim_{x \rightarrow 0} \left[ \frac{\cos(x)}{2} \right] &= \frac{\cos(0)}{2} = \frac{1}{2}\end{aligned}\quad (138)$$

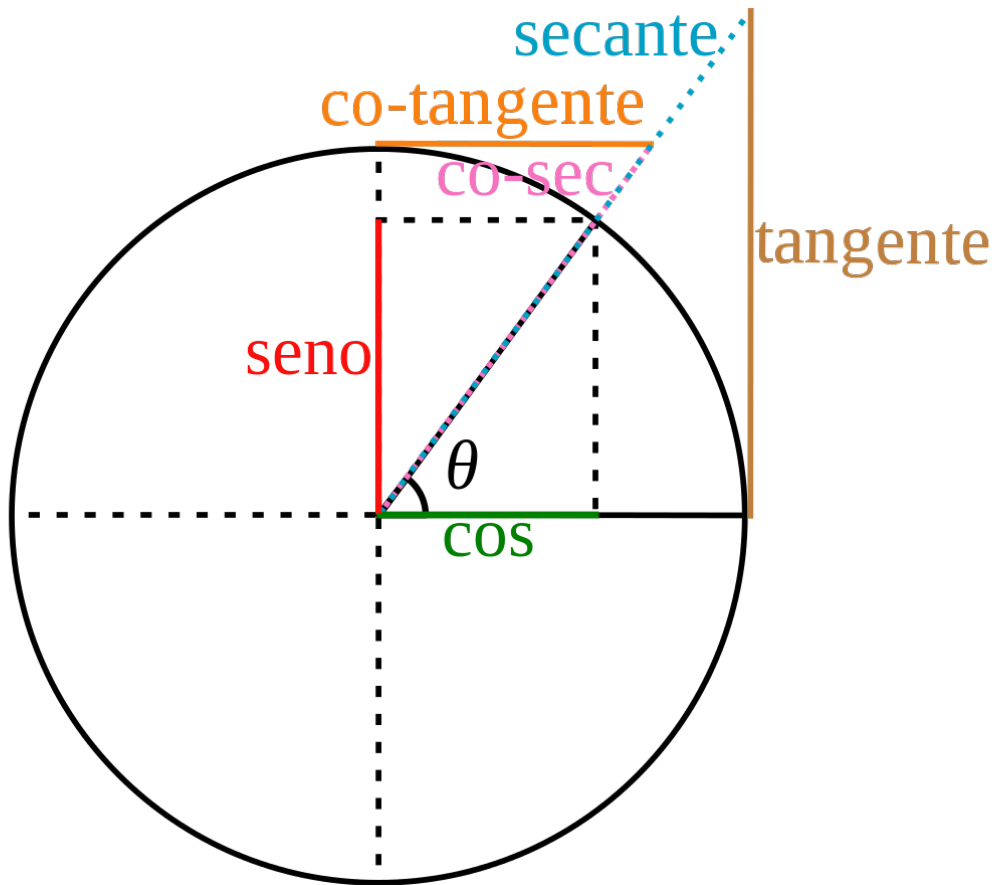
### Exercício VI

$$\begin{aligned}\lim_{x \rightarrow 0^+} \left[ \frac{1}{x} - \frac{1}{\text{sen}(x)} \right] &= \frac{1}{0^+} - \frac{1}{\text{sen}(0^+)} = \infty - \infty \\ \lim_{x \rightarrow 0^+} \left[ \frac{\text{sen}(x) - x}{x \text{sen}(x)} \right] &= \frac{\text{sen}(0) - 0}{0 \text{sen}(0)} = \frac{0 - 0}{0 \cdot 0} = \frac{0}{0} \\ \lim_{x \rightarrow 0^+} \left[ \frac{\cos(x) - 1}{\text{sen}(x) + x \cos(x)} \right] &= \frac{\cos(0) - 1}{\text{sen}(0) + 0 \cos(0)} = \frac{1 - 1}{0 + 0} = \frac{0}{0} \\ \lim_{x \rightarrow 0^+} \left[ \frac{-\text{sen}(x)}{\cos(x) + (\cos(x) + x(-\text{sen}(x)))} \right] &= \lim_{x \rightarrow 0^+} \left[ \frac{-\text{sen}(x)}{\cos(x) + \cos(x) - x \text{sen}(x)} \right] = \\ \lim_{x \rightarrow 0^+} \left[ \frac{-\text{sen}(x)}{2 \cos(x) - x \text{sen}(x)} \right] &= \frac{-\text{sen}(0)}{2 \cos(0) - 0 \text{sen}(0)} = \frac{-0}{2 \cdot 1 - 0 \cdot 0} = \frac{0}{2} = 0\end{aligned}\quad (139)$$

Exercício VII

$$\lim_{x \rightarrow \pi} \left[ \frac{\text{sen}(x)}{1 - \cos(x)} \right] = \frac{\text{sen}(\pi)}{1 - \cos(\pi)} = \frac{0}{1 - (-1)} = \frac{0}{2} = 0 \quad (140)$$

## FUNÇÕES TRIGONOMÉTRICAS



	0°	30°	45°	60°	90°
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	#

$$\text{tg } x = \frac{\text{sen } x}{\cos x} \rightarrow \text{cotg } x = \frac{\cos x}{\text{sen } x}$$

$$\text{sec } x = \frac{1}{\cos x} \rightarrow \text{cossec } x = \frac{1}{\text{sen } x}$$

$$\text{sen}^2 x + \cos^2 x = 1$$

$$\text{sec}^2 x - \text{tg}^2 x = 1 \rightarrow \frac{1}{\cos^2 x} - \frac{\text{sen}^2 x}{\cos^2 x} = \frac{1 - \text{sen}^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$$

$$\text{cossec}^2 x - \text{cotg}^2 x = 1 \rightarrow \frac{1}{\text{sen}^2 x} - \frac{\cos^2 x}{\text{sen}^2 x} = \frac{1 - \cos^2 x}{\text{sen}^2 x} = \frac{\text{sen}^2 x}{\text{sen}^2 x} = 1$$

## Derivadas Trigonométricas – $\sin(x)$ e $\cos(x)$ – [Aula 1](#)

01.  $y = \sin x \rightarrow y' = \cos x$
02.  $y = \cos x \rightarrow y' = -\sin x$
03.  $y = \tan x \rightarrow y' = \frac{1}{\cos^2 x} = \sec^2 x$
04.  $y = \cotg x \rightarrow y' = \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$
05.  $y = \sec x \rightarrow y' = \frac{\sin x}{\cos^2 x} = \sec x \cdot \tan x$
06.  $y = \operatorname{cosec} x \rightarrow y' = \frac{-\cos x}{\sin^2 x} = -\operatorname{cosec} x \cdot \cotg x$
07.  $y = \operatorname{arc} \sin x \rightarrow y' = \frac{1}{\sqrt{1-x^2}}$
08.  $y = \operatorname{arc} \cos x \rightarrow y' = \frac{-1}{\sqrt{1-x^2}}$
09.  $y = \operatorname{arc} \tan x \rightarrow y' = \frac{1}{1+x^2}$
10.  $y = \operatorname{arc} \cotg x \rightarrow y' = \frac{-1}{1+x^2}$
11.  $y = \operatorname{arc} \sec x \rightarrow y' = \frac{1}{|x| \cdot \sqrt{x^2-1}}$
12.  $y = \operatorname{arc} \operatorname{cosec} x \rightarrow y' = \frac{-1}{|x| \cdot \sqrt{x^2-1}}$

### Exercício I

$$\begin{aligned}
 y &= 3 \cos(x) + 7 \sin(x) + x^3 + 7 \\
 y' &= 3(-\sin(x)) + 7 \cos(x) + 3x^2 = -3 \sin(x) + 7 \cos(x) + 3x^2
 \end{aligned}
 \tag{141}$$

### Exercício II

$$y = \sin^2 x \rightarrow y' = 2 \sin x \cdot \cos x \tag{142}$$

### Exercício III

$$y = \sin^2 x \rightarrow y' = 2 \sin x \cdot \cos x \tag{143}$$

### Exercício IV

$$\begin{aligned}
 y &= 3 \sin x \cdot \cos x \rightarrow y' = 3 \cos x \cdot \cos x + 3 \sin x (-\sin x) = 3 \cos^2 x - 3 \sin^2 x = \\
 &= 3(\cos^2 x - \sin^2 x) = 3(\cos x - \sin x)(\cos x + \sin x)
 \end{aligned}
 \tag{144}$$

## Derivadas Trigonométricas – tg(x) e cotg(x) – [Aula 2](#)

### Exercício I

$$y = \text{tg } x = \frac{\text{sen } x}{\cos x} \rightarrow y' = \frac{\cos x \cdot \cos x - \text{sen } x(-\text{sen } x)}{\cos^2 x} = \frac{\cos^2 x + \text{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \left(\frac{1}{\cos x}\right)^2 = \text{sec}^2 x \quad (145)$$

### Exercício II

$$y = \text{cotg } x = \frac{1}{\text{tg } x} = \text{tg}^{-1} x \rightarrow y' = (-1) \text{tg}^{-2} x \cdot \text{sec}^2 x = \frac{-\text{sec}^2 x}{\text{tg}^2 x} = \frac{-\left(\frac{1}{\cos^2 x}\right)}{\frac{\text{sen}^2 x}{\cos^2 x}} = \frac{-\cos^2 x}{\cos^2 x \cdot \text{sen}^2 x} = \frac{-1}{\text{sen}^2 x} = -\left(\frac{1}{\text{sen } x}\right)^2 = -\text{cosec}^2 x \quad (146)$$

### Exercício III

$$y = 3 \text{tg } x + 4 \text{cotg } x \rightarrow y' = 3 \text{sec}^2 x + 4(-\text{cosec}^2 x) = 3 \text{sec}^2 x - 4 \text{cosec}^2 x = 3 \frac{1}{\cos^2 x} - 4 \frac{1}{\text{sen}^2 x} = \frac{3 \text{sen}^2 x - 4 \cos^2 x}{\cos^2 x \text{sen}^2 x} \quad (147)$$

### Exercício IV

$$y = \text{tg}(4x^2) \rightarrow y' = \text{sec}^2(4x^2) 8x = \frac{8x}{\cos^2(4x^2)} = \frac{8x}{\cos^2(2^2 x^2)} = \frac{8x}{\cos^4(2x)} \quad (148)$$

## Derivadas Trigonométricas, sen(x), cos(x), log(x) – [Aula 3](#)

### Exercício I

$$y = \ln(2x^2 - 1) \rightarrow y' = \frac{1}{2x^2 - 1} 4x = \frac{4x}{2x^2 - 1} = \frac{4x}{(x\sqrt{2} - 1)(x\sqrt{2} + 1)} \quad (149)$$

### Exercício II

$$y = e^{2x^2 - 4} \rightarrow \ln y = \ln(e^{2x^2 - 4}) = (2x^2 - 4) \ln(e) = (2x^2 - 4) 1 = 2x^2 - 4$$

$$\frac{1}{y} \frac{\partial y}{\partial x} = 4x \rightarrow \frac{\partial y}{\partial x} = 4xy = 4x \cdot e^{2x^2 - 4} \quad (150)$$

### Exercício III

$$y = 2 \text{sen } x + 3 \cos x + x^3 - 10 \rightarrow y' = 2 \cos x + 3(-\text{sen } x) + 3x^2 = 2 \cos x - 3 \text{sen } x + 3x^2 \quad (151)$$

#### Exercício IV

$$y = \operatorname{sen} x \cdot \cos x \rightarrow y' = \cos x \cdot \cos x + \operatorname{sen} x (-\operatorname{sen} x) = \cos^2 x - \operatorname{sen}^2 x = (\cos x - \operatorname{sen} x)(\cos x + \operatorname{sen} x) \quad (152)$$

### Derivada Trigonométrica de função composta de $\operatorname{sen}(x)$ – [Aula 4](#)

#### Exercício I

$$y = e^{\operatorname{sen} x} \rightarrow y' = e^{\operatorname{sen} x} \cdot \cos x \quad (153)$$

#### Exercício II

$$y = \frac{\operatorname{sen} x}{x} \rightarrow y' = \frac{\cos x \cdot x - \operatorname{sen} x \cdot 1}{x^2} = \frac{x \cdot \cos x - \operatorname{sen} x}{x^2} \quad (154)$$

#### Exercício III

$$y = \operatorname{sen} x \cdot e^x \rightarrow y' = \cos x \cdot e^x + \operatorname{sen} x \cdot e^x = e^x (\operatorname{sen} x + \cos x) \quad (155)$$

#### Exercício IV

$$y = \ln x \cdot e^x \rightarrow y' = \frac{1}{x} e^x + \ln x \cdot e^x = e^x \left( \frac{1}{x} + \ln x \right) = e^x \left( \frac{1 + x \cdot \ln x}{x} \right) \quad (156)$$

#### Exercício V

$$y = \operatorname{sen}(2x^2 - 1) \rightarrow y' = \cos(2x^2 - 1) 4x \quad (157)$$

### Derivada Trigonométrica de função composta $\operatorname{sen}(x)$ e $\cos(x)$ – [Aula 5](#)

#### Exercício I

$$y = \cos(5x^2 + 5) \rightarrow y' = -\operatorname{sen}(5x^2 + 5) 10x = -10x \cdot \operatorname{sen}(5x^2 + 5) \quad (158)$$

#### Exercício II

$$y = \operatorname{sen}(\sqrt{x}) = \operatorname{sen}\left(x^{\frac{1}{2}}\right) \rightarrow y' = \cos(\sqrt{x}) \frac{1}{2} x^{-\frac{1}{2}} = \frac{\cos(\sqrt{x})}{2\sqrt{x}} = \frac{\sqrt{x} \cdot \cos(\sqrt{x})}{2x} \quad (159)$$

### Derivada de produto, $\operatorname{tg}(x)$ , $\sec(x)$ , $\operatorname{cotg}(x)$ , $\operatorname{cossec}(x)$ – [Aula 6](#)

#### Exercício I

$$y = e^{2x} \cdot \cos(3x) \rightarrow y' = 2e^{2x} \cdot \cos(3x) + e^{2x} (-3 \operatorname{sen}(3x)) = 2e^{2x} \cdot \cos(3x) - 3e^{2x} \cdot \operatorname{sen}(3x) = e^{2x} (2 \cos(3x) - 3 \operatorname{sen}(3x)) \quad (160)$$



Exercício II

$$y = \operatorname{tg} x \rightarrow y' = \sec^2 x \quad (161)$$

Exercício III

$$y = \operatorname{cotg} x \rightarrow y' = -\operatorname{cosec}^2 x \quad (162)$$

Exercício IV

$$y = \sec x = \frac{1}{\cos x} = \cos^{-1} x \rightarrow y' = -\cos^{-2} x (-\sin x) = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \frac{\sin x}{\cos x} = \sec x \cdot \operatorname{tg} x \quad (163)$$

Exercício V

$$\begin{aligned} y = \operatorname{cosec} x &= \frac{1}{\sin x} = \sin^{-1} x \rightarrow y' = -\sin^{-2} x \cdot \cos x = \frac{-\cos x}{\sin^2 x} = \\ &= -\frac{1}{\sin x} \frac{\cos x}{\sin x} = -\operatorname{cosec} x \cdot \operatorname{cotg} x \end{aligned} \quad (164)$$

## Derivada de funções trigonométricas – [Aula 7](#)

Exercício I

$$y = 7 \operatorname{tg}(2x^2 - 1) \rightarrow y' = 7 \sec^2(2x^2 - 1) 4x = 28x \cdot \sec^2(2x^2 - 1) \quad (165)$$

Exercício II

$$y = \sin x^4 \rightarrow y' = 4x^3 \cdot \cos x^4 \quad (166)$$

Exercício III

$$y = \sin^4 x \rightarrow y' = 4 \sin^3 x \cdot \cos x \quad (167)$$

Exercício IV

$$y = \ln(\cos x) \rightarrow y' = \frac{1}{\cos x} (-\sin x) = \frac{-\sin x}{\cos x} = -\operatorname{tg} x \quad (168)$$

Exercício V

$$\begin{aligned} y = 5 \sec(2x^2 - 1) \rightarrow y' &= 5 \sec(2x^2 - 1) \cdot \operatorname{tg}(2x^2 - 1) 4x = 20x \cdot \sec(2x^2 - 1) \cdot \operatorname{tg}(2x^2 - 1) = \\ &= 20x \frac{1}{\cos(2x^2 - 1)} \frac{\sin(2x^2 - 1)}{\cos(2x^2 - 1)} = \frac{20x \cdot \sin(2x^2 - 1)}{\cos^2(2x^2 - 1)} \end{aligned} \quad (169)$$

## Exercício VI

$$\begin{aligned}
 y = \sec x \cdot \operatorname{tg} x \rightarrow y' &= \sec x \cdot \operatorname{tg} x \cdot \operatorname{tg} x + \sec x \cdot \sec^2 x = \sec x \cdot \operatorname{tg}^2 x + \sec^3 x = \\
 \sec x (\operatorname{tg}^2 x + \sec^2 x) &= \sec x \left( \frac{\operatorname{sen}^2 x}{\cos^2 x} + \frac{1}{\cos^2 x} \right) = \sec x \left( \frac{\operatorname{sen}^2 x + 1}{\cos^2 x} \right) = \frac{1}{\cos x} \frac{\operatorname{sen}^2 x + 1}{\cos^2 x} = \\
 &= \frac{\operatorname{sen}^2 x + 1}{\cos^3 x}
 \end{aligned} \tag{170}$$

## Exercício V

$$\begin{aligned}
 y = \operatorname{cosec} \left( \frac{x+1}{x-1} \right) \rightarrow y' &= -\operatorname{cosec} \left( \frac{x+1}{x-1} \right) \cdot \cotg \left( \frac{x+1}{x-1} \right) \left( \frac{(x-1) - (x+1)}{(x-1)^2} \right) = \\
 -\operatorname{cosec} \left( \frac{x+1}{x-1} \right) \cdot \cotg \left( \frac{x+1}{x-1} \right) &\left( \frac{x-1-x-1}{(x-1)^2} \right) = -\operatorname{cosec} \left( \frac{x+1}{x-1} \right) \cdot \cotg \left( \frac{x+1}{x-1} \right) \left( \frac{-2}{(x-1)^2} \right) = \\
 \frac{2}{(x-1)^2} \operatorname{cosec} \left( \frac{x+1}{x-1} \right) \cdot \cotg \left( \frac{x+1}{x-1} \right) &= \frac{2}{(x-1)^2} \frac{1}{\operatorname{sen} \left( \frac{x+1}{x-1} \right)} \frac{\cos \left( \frac{x+1}{x-1} \right)}{\operatorname{sen} \left( \frac{x+1}{x-1} \right)} = \\
 \frac{2}{(x-1)^2} \frac{\cos \left( \frac{x+1}{x-1} \right)}{\operatorname{sen}^2 \left( \frac{x+1}{x-1} \right)}
 \end{aligned} \tag{171}$$

## Derivada de Funções Trigonométricas Inversas – [Aula 8](#)

### Exercício I

$$\begin{aligned}
 y = \operatorname{arc} \operatorname{sen}(x+1) \rightarrow y' &= \frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{1-(x^2+2x+1)}} = \frac{1}{\sqrt{1-x^2-2x-1}} = \\
 &= \frac{1}{\sqrt{-x(x+2)}}
 \end{aligned} \tag{172}$$

### Exercício II

$$\begin{aligned}
 y = \text{arc tg} \left( \frac{1-x^2}{1+x^2} \right) \rightarrow y' &= \frac{1}{1 + \left( \frac{1-x^2}{1+x^2} \right)^2} \left( \frac{(-2x)(1+x^2) - (1-x^2)2x}{(1+x^2)^2} \right) = \\
 &= \frac{1}{1 + \left( \frac{1-x^2}{1+x^2} \right)^2} \left( \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} \right) = \frac{-4x}{1 + \left( \frac{1-x^2}{1+x^2} \right)^2} = \frac{-4x}{(x^2+1)^2 \cdot \left[ 1 + \left( \frac{-x^2+1}{x^2+1} \right)^2 \right]} = \\
 &= \frac{-4x}{(x^2+1)^2 \cdot \left( 1 + \frac{x^4-2x^2+1}{x^4+2x^2+1} \right)} = \frac{-4x}{(x^2+1)^2 \cdot \left( \frac{x^4+2x^2+1+x^4-2x^2+1}{(x^2+1)^2} \right)} = \frac{-4x}{(x^2+1)^2 \cdot \frac{2x^4+2}{(x^2+1)^2}} = \\
 &= \frac{-4x}{2x^4+2} = \frac{-4x}{2(x^4+1)} = \frac{-2x}{x^4+1}
 \end{aligned} \tag{173}$$

### Exercício III

$$y = (\text{arc sen } x)^2 \rightarrow y' = 2(\text{arc sen } x) \left( \frac{1}{\sqrt{1-x^2}} \right) = \frac{2 \text{arc sen } x}{\sqrt{1-x^2}} \tag{174}$$

### Exercício IV

$$\begin{aligned}
 y = \text{arc cos}(\text{sen}(t)) \rightarrow y' &= \frac{-1}{\sqrt{1-\text{sen}^2(t)}} \cos(t) = \frac{-\cos(t)}{\sqrt{1-\text{sen}^2(t)}} \\
 \text{sen}^2(t) + \cos^2(t) &= 1 \rightarrow \cos^2(t) = 1 - \text{sen}^2(t) \rightarrow \cos(t) = \pm \sqrt{1-\text{sen}^2(t)} = \sqrt{1-\text{sen}^2(t)} \\
 y' &= \frac{-\cos(t)}{\sqrt{1-\text{sen}^2(t)}} = \frac{-\cos(t)}{\cos(t)} = -1
 \end{aligned} \tag{175}$$

### Exercício V

$$y = \text{arc cos} \left( \frac{2x}{3} \right) \rightarrow y' = \frac{-1}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} \cdot \frac{2}{3} = \frac{-2}{3\sqrt{1-\frac{4x^2}{9}}} = \frac{-2}{3\sqrt{\frac{9-4x^2}{9}}} = \frac{-2}{\sqrt{9-4x^2}} \tag{176}$$

## Derivada de funções trigonométricas inversas – [Aula 9](#)

### Exercício I

$$\begin{aligned}
 f(t) = t \cdot \text{arc cos}(3t) \rightarrow f'(t) &= \text{arc cos}(3t) + t \left( \frac{-1}{\sqrt{1-(3t)^2}} \right) 3 = \text{arc cos}(3t) - \frac{3t}{\sqrt{1-9t^2}} = \\
 &= \frac{\sqrt{1-9t^2} \cdot \text{arc cos}(3t) - 3t}{\sqrt{1-9t^2}} = \frac{(1-9t^2) \text{arc cos}(3t) - 3t \sqrt{1-9t^2}}{1-9t^2}
 \end{aligned} \tag{177}$$

### Exercício II

$$y = \operatorname{arc\,sec}(\sqrt{x}) = \operatorname{arc\,sec}\left(x^{\frac{1}{2}}\right) \rightarrow y' = \frac{1}{|\sqrt{x}| \cdot \sqrt{(\sqrt{x})^2 - 1}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{x} \cdot \sqrt{x-1}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x\sqrt{x-1}} = \frac{\sqrt{x-1}}{2x(x-1)}$$
(178)

### Exercício III

$$y = \operatorname{arc\,cossec}(e^x + 1) \rightarrow y' = \frac{-1}{|e^x + 1| \cdot \sqrt{(e^x + 1)^2 - 1}} e^x = \frac{-e^x}{(e^x + 1) \sqrt{e^{2x} + 2e^x + 1 - 1}} = \frac{-e^x}{(e^x + 1) \sqrt{e^x(e^x + 2)}}$$
(179)

### Exercício IV

$$f(t) = t^2 \operatorname{arc\,cossec}(2t+3) \rightarrow f'(t) = 2t \cdot \operatorname{arc\,cossec}(2t+3) + t^2 \left( \frac{-1}{|2t+3| \cdot \sqrt{(2t+3)^2 - 1}} \right) 2 = 2t \cdot \operatorname{arc\,cossec}(2t+3) - \frac{2t^2}{|2t+3| \cdot \sqrt{4t^2 + 12t + 9 - 1}} = 2t \left( \operatorname{arc\,cossec}(2t+3) - \frac{t}{|2t+3| \cdot \sqrt{4(t^2 + 3t + 2)}} \right) = 2t \left( \operatorname{arc\,cossec}(2t+3) - \frac{t}{2|2t+3| \cdot \sqrt{t^2 + 3t + 2}} \right)$$
(180)

### Exercício V

$$f(\theta) = 2 \cos^2(\theta) \cdot \operatorname{sen}(2\theta) \rightarrow f'(\theta) = 2 \cdot 2 \cos(\theta) (-\operatorname{sen}(\theta)) \operatorname{sen}(2\theta) + 2 \cos^2(\theta) \cdot \cos(2\theta) 2 = -4 \cos(\theta) \cdot \operatorname{sen}(\theta) \cdot \operatorname{sen}(2\theta) + 4 \cos^2(\theta) \cdot \cos(2\theta) = -4 \cos(\theta) [\operatorname{sen}(\theta) \cdot \operatorname{sen}(2\theta) - \cos(\theta) \cdot \cos(2\theta)]$$
(181)

## Derivada de funções trigonométricas – [Aula 10](#)

### Exercício I

$$y = \frac{\operatorname{sen}(x+1)}{e^x} \rightarrow y' = \frac{\cos(x+1) \cdot e^x - \operatorname{sen}(x+1) \cdot e^x}{e^{2x}} = \frac{e^x (\cos(x+1) - \operatorname{sen}(x+1))}{e^{2x}} = \frac{\cos(x+1) - \operatorname{sen}(x+1)}{e^x}$$
(182)

### Exercício II

$$y = \arctg \left( \frac{1}{1-x^2} \right) = \arctg (1-x^2)^{-1} \rightarrow y' = \frac{1}{1 + \left( \frac{1}{1-x^2} \right)^2} \left( -(1-x^2)^{-2} (-2x) \right) =$$

$$\frac{1}{1 + \frac{1}{x^2 - 2x^2 + 1}} \frac{2x}{(1-x^2)^2} = \frac{1}{\frac{x^2 - 2x^2 + 1 + 1}{(1-x^2)^2}} \frac{2x}{(1-x^2)^2} = \frac{2x}{x^2 - 2x^2 + 2} \quad (183)$$

### Exercício III

$$y = (x \cdot \operatorname{tg} x)^2 \rightarrow y' = 2(x \cdot \operatorname{tg} x)(\operatorname{tg} x + x \cdot \sec^2 x) = 2x \cdot \operatorname{tg} x (\operatorname{tg} x + x \cdot \sec^2 x) =$$

$$2x \frac{\operatorname{sen} x}{\cos x} \left( \frac{\operatorname{sen} x}{\cos x} + \frac{x}{\cos^2 x} \right) = 2x \frac{\operatorname{sen} x}{\cos x} \left( \frac{\operatorname{sen} x \cos x + x}{\cos^2 x} \right) = 2x \frac{\operatorname{sen}^2 x \cos x + x \cdot \operatorname{sen} x}{\cos^3 x} =$$

$$2x \left( \frac{\operatorname{sen}^2 x}{\cos^2 x} + \frac{x \cdot \operatorname{sen} x}{\cos^3 x} \right) = 2x \left( \operatorname{tg}^2 x + \frac{x}{\cos^2 x} \frac{\operatorname{sen} x}{\cos x} \right) = 2x \left( \operatorname{tg}^2 x + \operatorname{tg} x \frac{x}{\cos^2 x} \right) \quad (184)$$

### Exercício IV

$$y = \log_2(3x - \cos(2x)) \rightarrow y' = \frac{1}{3x - \cos(2x)} \log_2(e) (3 - (-\operatorname{sen}(2x) \cdot 2)) =$$

$$\log_2(e) \frac{3 + 2 \operatorname{sen}(2x)}{3x - \cos(2x)} \quad (185)$$

### Exercício V

$$f(t) = \ln(\cos^2(t)) \rightarrow f'(t) = \frac{1}{\cos^2(t)} 2 \cos(t) (-\operatorname{sen}(t)) = -2 \frac{\operatorname{sen}(t)}{\cos(t)} = -2 \operatorname{tg}(t)$$

$$f(t) = \ln(\cos^2(t)) = 2 \cdot \ln(\cos(t)) \rightarrow f'(t) = 2 \frac{1}{\cos(t)} (-\operatorname{sen}(t)) = -2 \frac{\operatorname{sen}(t)}{\cos(t)} = -2 \operatorname{tg}(t) \quad (186)$$

### Exercício VI

$$f(t) = e^{2 \cos(2t)} \rightarrow \ln(f(t)) = \ln(e^{2 \cos(2t)}) = 2 \cos(2t) \cdot \ln(e) = 2 \cos(2t)$$

$$\frac{1}{f(t)} \frac{\partial f(t)}{\partial t} = 2(-\operatorname{sen}(2t)) = -4 \operatorname{sen}(2t) \rightarrow$$

$$\frac{\partial f(t)}{\partial t} = f(t)(-4 \operatorname{sen}(2t)) = e^{2 \cos(2t)} (-4 \operatorname{sen}(2t)) = -4 e^{2 \cos(2t)} \cdot \operatorname{sen}(2t) \quad (187)$$

## Derivadas trigonométricas – [Aula 11](#)

### Exercício I

$$\begin{aligned}
 f(x) &= \frac{3 \sec^2 x}{x} = 3 \sec^2 x \cdot x^{-1} \rightarrow f'(x) = 3 \cdot 2 \sec x \cdot \sec x \cdot \operatorname{tg} x \cdot \frac{1}{x} + 3 \sec^2 x (-x^{-2}) = \\
 &= \frac{6 \sec^2 x \cdot \operatorname{tg} x}{x} + 3 \sec^2 x \left( \frac{-1}{x^2} \right) = \frac{6 \sec^2 x \cdot \operatorname{tg} x}{x} - \frac{3 \sec^2 x}{x^2} = \frac{6x \cdot \sec^2 x \cdot \operatorname{tg} x - 3 \sec^2 x}{x^2} = \\
 &= \frac{3 \sec^2 x (2x \cdot \operatorname{tg} x - 1)}{x^2} = \frac{3 \frac{1}{\cos^2 x} \left( 2x \frac{\operatorname{sen} x}{\cos x} - 1 \right)}{x^2} = \frac{3}{\cos^2 x} \left( \frac{2x \cdot \operatorname{sen} x - \cos x}{\cos x} \right) = \\
 &= \frac{3(2x \cdot \operatorname{sen} x - \cos x)}{\cos^3 x} = \frac{3(2x \cdot \operatorname{sen} x - \cos x)}{x^2 \cdot \cos^3 x} = \frac{3}{x^2} \frac{2x \cdot \operatorname{sen} x - \cos x}{\cos^3 x}
 \end{aligned} \tag{188}$$

### Exercício II

$$\begin{aligned}
 f(x) &= \left( \frac{1}{\operatorname{sen} x} \right)^2 = (\operatorname{sen}^{-1} x)^2 = \operatorname{sen}^{-2} x \rightarrow f'(x) = -2 \operatorname{sen}^{-3} x \cdot \cos x = \frac{-2}{\operatorname{sen}^3 x} \cos x = \\
 &= \frac{-2 \cos x}{\operatorname{sen}^3 x} = -2 \frac{1}{\operatorname{sen}^2 x} \frac{\cos x}{\operatorname{sen} x} = -2 \operatorname{cosec}^2 x \cdot \cotg x \\
 f(x) &= \left( \frac{1}{\operatorname{sen} x} \right)^2 = \operatorname{cosec}^2 x \rightarrow f'(x) = 2 \operatorname{cosec} x (-\operatorname{cosec} x \cdot \cotg x) = -2 \operatorname{cosec}^2 x \cdot \cotg x
 \end{aligned} \tag{189}$$

## Derivada de Função Trigonométrica Inversa – [Aula 12](#)

### Exercício I

$$y = \operatorname{arc} \operatorname{sen}(x^3) \rightarrow y' = \frac{1}{\sqrt{1-(x^3)^2}} 3x^2 = \frac{3x^2}{\sqrt{1-x^6}} = \frac{3x^2 \sqrt{1-x^6}}{1-x^6} \tag{190}$$

### Exercício II

$$y = \operatorname{arc} \operatorname{tg}(x^3) \rightarrow y' = \frac{1}{1+(x^3)^2} 3x^2 = \frac{3x^2}{1+x^6} \tag{191}$$

### Exercício III

$$y = e^x \cdot \operatorname{arc} \sec x \rightarrow y' = e^x \cdot \operatorname{arc} \sec x + e^x \frac{1}{|x| \cdot \sqrt{x^2+1}} = e^x \left( \operatorname{arc} \sec x + \frac{1}{|x| \cdot \sqrt{x^2+1}} \right) \tag{192}$$

## Derivada de Função Trigonométrica – [Aula 13](#)

Exercício I

$$y = (1 + \cos^2 x)^6 \rightarrow y' = 6(1 + \cos^2 x)^5 \cdot 2 \cos x (-\sin x) = -12(1 + \cos^2 x)^5 \cdot \cos x \cdot \sin x \quad (193)$$

## Derivada de Função com tg(x) e sec(x) – [Aula 14](#)

Exercício I

$$f(x) = \sec^2 x - \tan^2 x \rightarrow f'(x) = 2 \sec x \cdot \sec x \cdot \tan x - 2 \tan x \cdot \sec^2 x = \\ 2 \sec^2 x \cdot \tan x - 2 \sec^2 x \cdot \tan x = 0$$

$$f(x) = \sec^2 x - \tan^2 x = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1 \rightarrow f'(x) = 0 \quad (194)$$
$$\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$$

$$f(x) = \sec^2 x - \tan^2 x = 1 + \tan^2 x - \tan^2 x = 1 \rightarrow f'(x) = 0$$
$$\sec^2 x = 1 + \tan^2 x$$

## Derivada de função com seno, secante e tangente – [Aula 15](#)

### Exercício I

$$\begin{aligned}
 f(x) &= \frac{\text{sen } x \cdot \sec x}{1+x \cdot \text{tg } x} = \frac{\text{sen } x \cdot \frac{1}{\cos x}}{1+x \cdot \text{tg } x} = \frac{\text{tg } x}{1+x \cdot \text{tg } x} \rightarrow \\
 f'(x) &= \frac{\sec^2 x (1+x \cdot \text{tg } x) - \text{tg } x (\text{tg } x + x \cdot \sec^2 x)}{(1+x \cdot \text{tg } x)^2} = \\
 &= \frac{\sec^2 x + x \cdot \text{tg } x \cdot \sec^2 x - (\text{tg}^2 x + x \cdot \sec^2 x \cdot \text{tg } x)}{(1+x \cdot \text{tg } x)^2} = \frac{\sec^2 x + x \cdot \text{tg } x \cdot \sec^2 x - \text{tg}^2 x - x \cdot \text{tg } x \cdot \sec^2 x}{(1+x \cdot \text{tg } x)^2} = \\
 &= \frac{\sec^2 x - \text{tg}^2 x}{(1+x \cdot \text{tg } x)^2} = \frac{\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}}{(1+x \cdot \text{tg } x)^2} = \frac{1 - \sin^2 x}{\cos^2 x (1+x \cdot \text{tg } x)^2} = \frac{\cos^2 x}{\cos^2 x (1+x \cdot \text{tg } x)^2} = \frac{1}{(1+x \cdot \text{tg } x)^2} = \\
 &= \frac{1}{\left(1+x \cdot \frac{\text{sen } x}{\cos x}\right)^2} = \frac{1}{\left(\frac{\cos x + x \cdot \text{sen } x}{\cos x}\right)^2} = \frac{\cos^2 x}{(\cos x + x \cdot \text{sen } x)^2} = \\
 &= \frac{\cos^2 x}{\cos^2 x + 2x \cdot \text{sen } x \cdot \cos x + x^2 \cdot \text{sen}^2 x} = \frac{\cos^2 x}{x \cdot \text{sen } x (x \cdot \text{sen } x + 2 \cos x) + \cos^2 x}
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \frac{\text{sen } x \cdot \sec x}{1+x \cdot \text{tg } x} \rightarrow \\
 f'(x) &= \frac{(\cos x \cdot \sec x + \text{sen } x \cdot \sec x \cdot \text{tg } x)(1+x \cdot \text{tg } x) - (\text{sen } x \cdot \sec x)(\text{tg } x + x \cdot \sec^2 x)}{(1+x \cdot \text{tg } x)^2} = \\
 &= \frac{[\sec x (\cos x + \text{sen } x \cdot \text{tg } x)](1+x \cdot \text{tg } x) - (\text{sen } x \cdot \sec x)(\text{tg } x + x \cdot \sec^2 x)}{(1+x \cdot \text{tg } x)^2} = \\
 &= \frac{x^2 \cdot \text{tg}^2 x + 2x \cdot \text{tg } x + 1}{\left[\frac{1}{\cos x} \left(\cos x + \text{sen } x \cdot \frac{\text{sen } x}{\cos x}\right)\right] \left(1+x \cdot \frac{\text{sen } x}{\cos x}\right) - \left(\text{sen } x \cdot \frac{1}{\cos x}\right) \left(\frac{\text{sen } x}{\cos x} + x \cdot \frac{1}{\cos^2 x}\right)} = \\
 &= \frac{x^2 \cdot \frac{\text{sen}^2 x}{\cos^2 x} + 2x \cdot \frac{\text{sen } x}{\cos x} + 1}{\frac{1}{\cos x} \cdot \frac{\cos^2 x + \text{sen}^2 x}{\cos x} \cdot \frac{\cos x + x \cdot \text{sen } x}{\cos x} - \frac{\text{sen } x}{\cos x} \cdot \frac{\text{sen } x \cdot \cos x + x}{\cos^2 x}} = \\
 &= \frac{x^2 \cdot \text{sen}^2 x + 2x \cdot \text{sen } x \cdot \cos x + \cos^2 x}{\cos^2 x} = \\
 &= \frac{\frac{\cos x + x \cdot \text{sen } x}{\cos^3 x} - \frac{\text{sen}^2 x \cdot \cos x + x \cdot \text{sen } x}{\cos^3 x}}{\frac{x^2 \cdot \text{sen}^2 x + 2x \cdot \text{sen } x \cdot \cos x + \cos^2 x}{\cos^2 x}} = \frac{\cos x + x \cdot \text{sen } x - (\text{sen}^2 x \cdot \cos x + x \cdot \text{sen } x)}{\cos^3 x} = \\
 &= \frac{\cos x + x \cdot \text{sen } x - \text{sen}^2 x \cdot \cos x - x \cdot \text{sen } x}{\cos x (x^2 \cdot \text{sen}^2 x + 2x \cdot \text{sen } x \cdot \cos x + \cos^2 x)} = \frac{\cos x (1 - \text{sen}^2 x)}{\cos x (x^2 \cdot \text{sen}^2 x + 2x \cdot \text{sen } x \cdot \cos x + \cos^2 x)} = \\
 &= \frac{\cos^2 x}{x \cdot \text{sen } x (x \cdot \text{sen } x + 2 \cdot \cos x) + \cos^2 x}
 \end{aligned}$$

(195)



## Derivada de função com cossecante e cotangente – [Aula 16](#)

Exercício I

$$\begin{aligned}
 f(x) &= \frac{\cotg x}{1 + \operatorname{cossec} x} \rightarrow f'(x) = \frac{-\operatorname{cossec}^2 x (1 + \operatorname{cossec} x) - \cotg x (-\operatorname{cossec} x \cdot \cotg x)}{(1 + \operatorname{cossec} x)^2} = \\
 &= \frac{-\operatorname{cossec}^2 x - \operatorname{cossec}^3 x + \operatorname{cossec} x \cdot \cotg^2 x}{(1 + \operatorname{cossec} x)^2} = \frac{-\operatorname{cossec} x (\operatorname{cossec} x + (\operatorname{cossec}^2 x - \cotg^2 x))}{(1 + \operatorname{cossec} x)^2} = \\
 &= \frac{-\operatorname{cossec} x (\operatorname{cossec} x + 1)}{(\operatorname{cossec} x + 1)^2} = \frac{-\operatorname{cossec} x}{\operatorname{cossec} x + 1} = \frac{\frac{-1}{\operatorname{sen} x}}{\frac{1}{\operatorname{sen} x} + 1} = \frac{\frac{-1}{\operatorname{sen} x}}{\frac{1 + \operatorname{sen} x}{\operatorname{sen} x}} = \frac{-1}{1 + \operatorname{sen} x}
 \end{aligned} \tag{196}$$

## Derivada de função com seno e cosseno – [Aula 17](#)

Exercício I

$$f(x) = \operatorname{sen}^2 x + \cos^2 x = 1 \rightarrow f'(x) = 0 \tag{197}$$

## Derivada de função com secante e tangente – [Aula 18](#)

Exercício I

$$\begin{aligned}
 f(x) &= \sec x \cdot \tg x \rightarrow f'(x) = \sec x \cdot \tg x \cdot \tg x + \sec x \cdot \sec^2 x = \sec x \cdot \tg^2 x + \sec^3 x = \\
 &= \sec x (\tg^2 x + \sec^2 x) = \sec x ((-1 + \sec^2 x) + \sec^2 x) = -\sec x (1 - 2 \sec^2 x) = \\
 &= \frac{-1}{\cos x} \left( 1 - 2 \frac{1}{\cos^2 x} \right) = \frac{-1}{\cos x} \left( \frac{\cos^2 x - 2}{\cos^2 x} \right) = \frac{2 - \cos^2 x}{\cos^3 x} = \frac{1 + 1 - \cos^2 x}{\cos^3 x} = \frac{1 + \operatorname{sen}^2 x}{\cos^3 x}
 \end{aligned} \tag{198}$$

## Derivada de função com cosseno e tangente – [Aula 19](#)

Exercício I

$$\begin{aligned}
 y &= \tg(\cos x) \rightarrow y' = \sec^2(\cos x) (-\operatorname{sen} x) = -\operatorname{sen} x \cdot \sec^2(\cos x) = -\operatorname{sen} x \frac{1}{\cos^2(\cos x)} = \\
 &= \frac{-\operatorname{sen} x}{\cos^2(\cos x)}
 \end{aligned} \tag{199}$$

## Derivada de função com tangente – [Aula 20](#)

Exercício I

$$\begin{aligned}
 y = \sqrt[3]{1+tg\,x} &= (1+tg\,x)^{\frac{1}{3}} \rightarrow y' = \frac{1}{3}(1+tg\,x)^{-\frac{2}{3}} \cdot sec^2\,x = \frac{sec^2\,x}{3\sqrt[3]{(tg\,x+1)^2}} = \frac{sec^2\,x}{3\sqrt[3]{tg^2\,x+2tg\,x+1}} = \\
 \frac{sec^2\,x}{3\sqrt[3]{-1+sec^2\,x+2tg\,x+1}} &= \frac{sec^2\,x}{3\sqrt[3]{sec^2\,x+2tg\,x}} = \frac{\frac{1}{\cos^2\,x}}{3\sqrt[3]{\frac{1}{\cos^2\,x}+2\frac{\sin\,x}{\cos\,x}}} = \frac{\frac{1}{\cos^2\,x}}{3\sqrt[3]{\frac{1+2\sin\,x\cdot\cos\,x}{\cos^2\,x}}} = \\
 \frac{\frac{1}{\cos^2\,x}}{\frac{3\sqrt[3]{1+2\sin\,x\cdot\cos\,x}}{\cos\,x}} &= \frac{1}{3\cos\,x\sqrt[3]{1+2\sin\,x\cdot\cos\,x}}
 \end{aligned} \tag{200}$$

## Derivada de função com cossecante – [Aula 21](#)

Exercício I

$$\begin{aligned}
 y = \cos\,x - x \cdot \cossec\,x &\rightarrow y' = -\sin\,x - (\cossec\,x + x(-\cossec\,x \cdot \cotg\,x)) = \\
 -\sin\,x - (\cossec\,x - x \cdot \cossec\,x \cdot \cotg\,x) &= -\sin\,x - \cossec\,x + x \cdot \cossec\,x \cdot \cotg\,x = \\
 -\sin\,x - \cossec\,x(1 - x \cdot \cotg\,x) &= -\sin\,x - \frac{1}{\sin\,x} \left(1 - x \frac{\cos\,x}{\sin\,x}\right) = \\
 -\sin\,x - \frac{1}{\sin\,x} \left(\frac{\sin\,x - x \cdot \cos\,x}{\sin\,x}\right) &= -\sin\,x - \frac{\sin\,x - x \cdot \cos\,x}{\sin^2\,x} = \frac{-\sin^3\,x - \sin\,x - x \cdot \cos\,x}{\sin^2\,x} = \\
 \frac{-\sin\,x(\sin^2\,x + 1) - x \cdot \cos\,x}{\sin^2\,x}
 \end{aligned} \tag{201}$$

## Derivada segunda de função cosseno – [Aula 22](#)

Exercício I

$$\begin{aligned}
 f(x) = x \cdot \cos\,x &\rightarrow f'(x) = \cos\,x + x(-\sin\,x) = \cos\,x - x \cdot \sin\,x \rightarrow \\
 f''(x) &= -\sin\,x - (\sin\,x + x \cdot \cos\,x) = -\sin\,x - \sin\,x - x \cdot \cos\,x = -2\sin\,x - x \cdot \cos\,x
 \end{aligned} \tag{202}$$

## Derivada segunda de função secante – [Aula 23](#)

Exercício I

$$\begin{aligned}
 f(x) = \cossec\,x &\rightarrow f'(x) = -\cossec\,x \cdot \cotg\,x \rightarrow \\
 f''(x) &= -[-\cossec\,x \cdot \cotg\,x \cdot \cotg\,x + \cossec\,x(-\cossec^2\,x)] = \\
 -(-\cossec\,x \cdot \cotg^2\,x - \cossec^3\,x) &= -(-\cossec\,x(\cotg^2\,x + \cossec^2\,x)) = \\
 \cossec\,x(\cotg^2\,x + \cossec^2\,x) &= \cossec\,x(\cotg^2\,x + (1 + \cotg^2\,x)) = \\
 \cossec\,x(1 + 2\cotg^2\,x)
 \end{aligned} \tag{203}$$

## Derivada da cossecante – [Aula 24](#)

Exercício I

$$f(x) = \operatorname{cossec} x \rightarrow f'(x) = -\operatorname{cossec} x \cdot \cotg x \quad (204)$$

## Derivada de função com sen(x) e cos(x) – [Aula 25](#)

Exercício I

$$\begin{aligned} f(x) &= \sqrt{\operatorname{sen} x \cdot \cos x} = (\operatorname{sen} x \cdot \cos x)^{\frac{1}{2}} \rightarrow \\ f'(x) &= \frac{1}{2} (\operatorname{sen} x \cdot \cos x)^{-\frac{1}{2}} (\cos x \cdot \cos x + \operatorname{sen} x (-\operatorname{sen} x)) = \frac{\cos^2 x - \operatorname{sen}^2 x}{2\sqrt{\operatorname{sen} x \cdot \cos x}} = \\ &= \frac{(1 - \operatorname{sen}^2 x) - \operatorname{sen}^2 x}{2\sqrt{\operatorname{sen} x \cdot \cos x}} = \frac{1 - 2\operatorname{sen}^2 x}{2\sqrt{\operatorname{sen} x \cdot \cos x}} = \frac{1 - 2\operatorname{sen}^2 x}{2\sqrt{\operatorname{sen} x} \frac{\operatorname{sen} x}{\operatorname{tg} x}} = \frac{1 - 2\operatorname{sen}^2 x}{2\sqrt{\frac{\operatorname{sen}^2 x}{\operatorname{tg} x}}} = \frac{1 - 2\operatorname{sen}^2 x}{\frac{2\operatorname{sen} x}{\sqrt{\operatorname{tg} x}}} = \\ &= \sqrt{\operatorname{tg} x} \frac{1 - 2\operatorname{sen}^2 x}{2\operatorname{sen} x} \end{aligned} \quad (205)$$

## Derivada segunda de função tg (x) – [Aula 26](#)

Exercício I

$$\begin{aligned} f(x) = \operatorname{tg} x \rightarrow f'(x) = \sec^2 x \rightarrow f''(x) &= 2 \sec x \cdot \sec x \cdot \operatorname{tg} x = 2 \sec^2 x \cdot \operatorname{tg} x = 2 \frac{1}{\cos^2 x} \frac{\operatorname{sen} x}{\cos x} = \\ &= 2 \frac{\operatorname{sen} x}{\cos^3 x} \end{aligned} \quad (206)$$

## Derivada de função com seno – [Aula 27](#)

Exercício I

$$\begin{aligned} f(x) &= \sqrt{\cos(6x) + \operatorname{sen}(x^2)} = (\cos(6x) + \operatorname{sen}(x^2))^{\frac{1}{2}} \rightarrow \\ f'(x) &= \frac{1}{2} (\cos(6x) + \operatorname{sen}(x^2))^{-\frac{1}{2}} [(-\operatorname{sen}(6x) \cdot 6) + \cos(x^2) \cdot 2x] = \\ &= \frac{-6\operatorname{sen}(6x) + 2x \cdot \cos(x^2)}{2\sqrt{\cos(6x) + \operatorname{sen}(x^2)}} = \frac{-2(3\operatorname{sen}(6x) - x \cdot \cos(x^2))}{2\sqrt{\cos(6x) + \operatorname{sen}(x^2)}} = \frac{-3\operatorname{sen}(6x) + x \cdot \cos(x^2)}{\sqrt{\cos(6x) + \operatorname{sen}(x^2)}} \end{aligned} \quad (207)$$

## Derivada de função com cossecante – [Aula 28](#)

Exercício I

$$\begin{aligned}
 f(x) &= \cotg^7(x^5) \rightarrow f'(x) = 7 \cotg^6(x^5) (-\operatorname{cossec}^2(x^5) \cdot 5x^4) = \\
 &= -35x^4 \cdot \cotg^6(x^5) \cdot \operatorname{cossec}^2(x^5) = -35x^4 \frac{\cos^6(x^5)}{\sin^6(x^5)} \frac{1}{\sin^2(x^5)} = -35x^4 \frac{\cos^6(x^5)}{\sin^8(x^5)} = \\
 &= -35x^4 \left( \frac{\cos^3(x^5)}{\sin^4(x^5)} \right)^2
 \end{aligned} \tag{208}$$

## Derivada de Função com Tangente – [Aula 29](#)

Exercício I

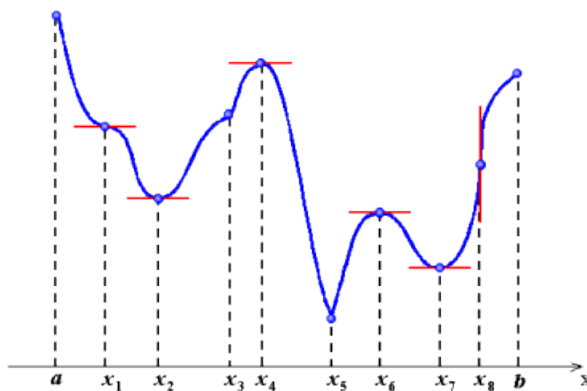
$$y = \operatorname{tg}^3(x) + \operatorname{tg}(x^3) \rightarrow y' = 3\operatorname{tg}^2(x) \cdot \sec^2(x) + \sec^2(x^3) \cdot 3x^2 = 3(\operatorname{tg}^2(x) \cdot \sec^2(x) + x^2 \cdot \sec^2(x^3)) \tag{209}$$

## Derivada de função com cos (x) – [Aula 30](#)

Exercício I

$$\begin{aligned}
 y &= \frac{1}{\sqrt{\cos(x^2)+1}} = \frac{1}{(\cos(x^2)+1)^{\frac{1}{2}}} = (\cos(x^2)+1)^{-\frac{1}{2}} \rightarrow \\
 y' &= \frac{-1}{2} (\cos(x^2)+1)^{-\frac{3}{2}} (-\sin(x^2) \cdot 2x) = \frac{2x \cdot \sin(x^2)}{2\sqrt{(\cos(x^2)+1)^3}} = \frac{x \cdot \sin(x^2)}{\sqrt{(\cos(x^2)+1)^3}}
 \end{aligned} \tag{210}$$

## Máximos e Mínimos de uma função – [Aula 1](#)



- Pontos onde podem existir extremos relativos:

$$\text{Números Críticos: } \begin{cases} f' = 0: x_1, x_2, x_4, x_6, x_7 \\ \nexists f': x_3, x_5 \text{ e } x_8 \end{cases}$$

- Extremos Relativos:

Valores Máximos Relativos:  $f(x_4)$  e  $f(x_6)$

Valores Mínimos Relativos:  $f(x_2)$ ,  $f(x_5)$  e  $f(x_7)$

- Extremos Absolutos:

Valor Máximo Absoluto:  $f(a)$

Valor Mínimo Absoluto:  $f(x_5)$

## Exercício I

$$f(x) = x^3 - 3x^2 - 9x + 7$$

$$1. f'(x) = 3x^2 - 6x - 9$$

$$2. f'(x) = 0 \rightarrow 3x^2 - 6x - 9 = 0$$

$$\Delta = b^2 - 4ac = (-6)^2 - 4 \cdot 3 \cdot (-9) = 36 + 108 = 144$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(-6) \pm \sqrt{144}}{2 \cdot 3} = \frac{6 \pm 12}{6} = \frac{6(1 \pm 2)}{6} = 1 \pm 2 \quad (211)$$

$$x_1 = 1 + 2 = 3 \rightarrow x_2 = 1 - 2 = -1$$

$$f'(3) = f'(-1) = 0$$

5.

$$f(x_1) = (3)^3 - 3(3)^2 - 9(3) + 7 = 27 - 27 - 27 + 7 = -20$$

$$f(x_2) = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = -1 - 3 + 9 + 7 = 12$$

## Máximos, Mínimos, Modelagem e Otimização – [Aula 2](#)

### Exercício I

Quero construir um **jardim retangular**, usando a **casa como um dos lados**. Encontre as **dimensões do maior jardim** que se pode ser cercado com **40 metros de cerca**.

$$\text{Área do jardim: } a = xy = x(40 - 2x) = 40x - 2x^2$$

$$\text{Comprimento da cerca: } 2x + y = 40 \rightarrow y = 40 - 2x$$

$$a(x) = 40x - 2x^2 \rightarrow \frac{\partial a(x)}{\partial x} = 40 - 4x$$

$$\frac{\partial a(x)}{\partial x} = 0 \rightarrow 40 - 4x = 0 \rightarrow -4x = -40 = x = \frac{40}{4} = 10 \quad (212)$$

$$x = 10 \text{ m}$$

$$y = 40 - 2(10) = 40 - 20 = 20 \text{ m}$$

$$a = 40(10) - 2(10)^2 = 400 - 200 = 200 \text{ m}^2$$

## Exercício II

Uma loja quer construir um **cercado retangular** com **600 metros quadrados**. **Três lados** serão construídos de madeira a um custo de **R\$14,00 o metro**. No **quarto lado** será usado bloco de cimento e o custo será de **R\$28,00 o metro**. Quais as dimensões que **minimizará o custo** do cercado.

$$\text{Área do cercado: } xy = 600 \rightarrow y = \frac{600}{x}$$

$$\text{Custo do cercado: } c = 14 \cdot 2x + 14y + 28y = 28x + 42y = 28x + 42 \frac{600}{x} = 28x + \frac{25200}{x}$$

$$c(x) = 28x + 25200x^{-1} \rightarrow \frac{\partial c(x)}{\partial x} = 28 - 25200x^{-2} = 28 - \frac{25200}{x^2}$$

$$\frac{\partial c(x)}{\partial x} = 0 \rightarrow 28 - \frac{25200}{x^2} = 0 \rightarrow -\frac{25200}{x^2} = -28 \rightarrow x^2 = \frac{25200}{28} \rightarrow x = \pm \sqrt{\frac{25200}{28}} = \frac{\pm \sqrt{2^2 \cdot 2^2 \cdot 3^2 \cdot 5^2 \cdot 7}}{\sqrt{2^2 \cdot 7}} = \frac{\pm 60\sqrt{7}}{2\sqrt{7}} = \pm 30 \quad (213)$$

$$x = 30 \text{ m}$$

$$y = \frac{600}{30} = 20 \text{ m}$$

$$c = 28(30) + 42(20) = 840 + 840 = \text{R\$ } 1680,00$$

## Exercício III

Encontre as dimensões de uma **caixa retangular** de **base quadrada** e **volume** igual a **8000 centímetros cúbicos**, que possa ser feita com o **mínimo material** possível.

$$\text{Volume da caixa: } yx^2 = 8000 \rightarrow y = \frac{8000}{x^2}$$

$$\text{Área do material: } a = 2x^2 + 4xy = 2x^2 + 4x \frac{8000}{x^2} = 2x^2 + 4 \frac{8000}{x} = 2x^2 + \frac{32000}{x}$$

$$a(x) = 2x^2 + 32000x^{-1} \rightarrow \frac{\partial a(x)}{\partial x} = 4x - 32000x^{-2} = 4x - \frac{32000}{x^2}$$

$$\frac{\partial a(x)}{\partial x} = 0 \rightarrow 4x - \frac{32000}{x^2} = 0 \rightarrow -\frac{32000}{x^2} = -4x \rightarrow 4x^3 = 32000 \rightarrow x^3 = \frac{32000}{4} \rightarrow x^3 = \frac{32000}{4} \quad (214)$$

$$x = \sqrt[3]{\frac{32000}{4}} = \frac{\sqrt[3]{2^3 \cdot 2^3 \cdot 5^3 \cdot 4}}{\sqrt[3]{4}} = \frac{20\sqrt[3]{4}}{\sqrt[3]{4}} = 20$$

$$x = 20 \text{ cm}$$

$$y = \frac{8000}{20^2} = \frac{8000}{400} = 20 \text{ cm}$$

$$a = 2(20)^2 + 4(20)(20) = 800 + 1600 = 2400 \text{ cm}^2$$

## Máximos, Mínimos, Modelagem e Otimização – [Aula 3](#)

### Exercício I

Encontre o maior produto possível de dois números positivos  $x$  e  $y$  cuja soma é 100.

$$\text{Soma: } x + y = 100 \rightarrow y = 100 - x$$

$$\text{Produto: } p = xy = x(100 - x) = 100x - x^2$$

$$p(x) = -x^2 + 100x \rightarrow \frac{\partial p(x)}{\partial x} = -2x + 100$$

$$\frac{\partial p(x)}{\partial x} = 0 \rightarrow -2x + 100 = 0 \rightarrow -2x = -100 \rightarrow x = \frac{100}{2} = 50 \quad (215)$$

$$x = 50$$

$$y = 100 - 50 = 50$$

$$p = 50 \cdot 50 = 2500$$

### Exercício II

Deseja-se construir uma caixa retangular aberta, cortando-se os cantos de um pedaço de papelão quadrado de lado 16 cm e dobrando-se as abas.

$$\text{Lado da caixa: } y = 16 - 2x$$

$$\text{Volume da caixa: } v = xy^2 = x(16 - 2x)^2$$

$$v(x) = x(16 - 2x)^2 \rightarrow \frac{\partial v(x)}{\partial x} = (16 - 2x)^2 + x[2(16 - 2x)(-2)] =$$

$$(16 - 2x)^2 - 4x(16 - 2x) = (16 - 2x)[(16 - 2x) - 4x] = (16 - 2x)(16 - 6x) =$$

$$2(8 - x)6\left(\frac{8}{3} - x\right) = 12(8 - x)\left(\frac{8}{3} - x\right)$$

$$\frac{\partial v(x)}{\partial x} = 0 \rightarrow 12(8 - x)\left(\frac{8}{3} - x\right) = 0$$

$$8 - x = 0 \rightarrow -x = -8 \rightarrow x_1 = 8$$

$$\frac{8}{3} - x = 0 \rightarrow -x = -\frac{8}{3} \rightarrow x_2 = \frac{8}{3}$$

(216)

$$v_1 = (8)(16 - 2(8))^2 = 8 \cdot 0^2 = 0$$

$$v_2 = \left(\frac{8}{3}\right)\left(16 - 2\left(\frac{8}{3}\right)\right)^2 = \frac{8}{3}\left(\frac{48 - 16}{3}\right)^2 = \frac{8}{3}\frac{1024}{9} = \frac{8192}{27} = 303,4074 \text{ cm}^3$$

$$x = \frac{8}{3} = 2,6 \text{ cm}$$

$$y = 16 - 2\frac{8}{3} = \frac{48 - 16}{3} = \frac{32}{3} = 10,6 \text{ cm}$$

$$v = \frac{8192}{27} = 303,4074 \text{ cm}^3$$

### Exercício III

Uma companhia aérea transporta 8000 passageiros, cada um paga R\$50,00. O departamento de pesquisa de mercado estima que cada R\$1,00 de aumento, a companhia irá perder 100 passageiros. Determine o preço que maximiza o faturamento da companhia aérea.

Faturamento da companhia aérea:  $f = (8000 - 100x)(50 + x)$

$$\begin{aligned} f(x) &= (8000 - 100x)(50 + x) \rightarrow \frac{\partial f(x)}{\partial x} = -100(50 + x) + (8000 - 100x) = \\ &= -5000 - 100x + 8000 - 100x = 3000 - 200x \\ \frac{\partial f(x)}{\partial x} &= 0 \rightarrow 3000 - 200x = 0 \rightarrow -200x = -3000 \rightarrow x = \frac{3000}{200} = 15 \end{aligned} \quad (217)$$

$$\text{preço da passagem} = 15 + 50 = \text{R\$ } 65,00$$

$$\text{faturamento da companhia} = (8000 - 100(15))(50 + 15) = 6500 \cdot 65 = \text{R\$ } 422500,00$$

### Ponto de Inflexão e Concavidade da função – [Aula 4](#)

1. Ache a  $f''(x)$
2.  $f''(x) = 0$

### Exercício I

$$\begin{aligned} f(x) &= x^3 - 3x^2 - 9x + 7 \rightarrow f'(x) = 3x^2 - 6x - 9 \rightarrow f''(x) = 6x - 6 \\ f''(x) &= 0 \rightarrow 6x - 6 = 0 \rightarrow 6x = 6 \rightarrow x = \frac{6}{6} = 1 \\ f(1) &= (1)^3 - 3(1)^2 - 9(1) + 7 = 1 - 3 - 9 + 7 = 8 - 12 = -4 \end{aligned} \quad (218)$$