Integral Indefinida – Aula 1

01. $\int \partial x \quad = \quad x+c$

02. $\int x^p \partial x \quad = \quad \frac{x^{p+1}}{p+1}+c \quad \rightarrow \quad p \neq -1$

03. $\int e^x \partial x \quad = \quad e^x+c$

04. $\int \frac{\partial x}{x} \quad = \quad \ln|x|+c$

05. $\int u^p \partial u \quad = \quad \frac{u^{p+1}}{p+1}+c \quad \rightarrow \quad p \neq -1$

06. $\int e^u \partial u \quad = \quad e^u+c$

07. $\int \frac{\partial u}{u} \quad = \quad \ln|u|+c$

08. $\int a^u \partial u \quad = \quad \frac{a^u}{\ln|a|}+c$

Exercício I

$$\int \partial x = x+c \qquad (1)$$
$$\int x^3 \partial x = \frac{x^4}{4}+c$$

Exercício II

$$\int 3x^4 \partial x = 3\int x^4 \partial x = 3\frac{x^5}{5}+c = \frac{3x^5}{5}+c \qquad (2)$$

Exercício III

$$\int (4x^5+7)\partial x = \int 4x^5 \partial x + \int 7\partial x = 4\int x^5 \partial x + 7\int \partial x = 4\frac{x^6}{6}+7x+c = \frac{2x^6}{3}+7x+c \qquad (3)$$

Exercício IV

$$\int 3\partial x = 3\int \partial x = 3x+c \qquad (4)$$

Exercício V

$$\int 5x^7 \partial x = 5\int x^7 \partial x = \frac{5x^8}{8}+c \qquad (5)$$

Exercício VI

$$\int \left(5+3x^2-7x^3\right)\partial x=5\int \partial x+3\int x^2\partial x-7\int x^3\partial x=5x+\frac{3x^3}{3}-\frac{7x^4}{4}+c=\frac{-7x^4}{4}+x^3+5x+c \tag{6}$$

Integral Indefinida – Aula 2

Exercício I

$$\int \frac{\partial x}{x^4}=\int x^{-4}\partial x=\frac{x^{-3}}{-3}+c=\frac{-1}{3x^3}+c \tag{7}$$

Exercício II

$$\int \sqrt{x^3}\partial x=\int x^{\frac{3}{2}}\partial x=\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+c=\sqrt{x^5}\cdot\frac{2}{5}=\frac{2\sqrt{x^5}}{5}+c \tag{8}$$

Exercício II

$$\int \left(7\sqrt[5]{x^2}+\frac{3}{x^3}\right)\partial x=\int \left(7x^{\frac{2}{5}}+3x^{-3}\right)\partial x=7\int x^{\frac{2}{5}}\partial x+3\int x^{-3}\partial x=7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)}+3\frac{x^{-2}}{-2}+c=5\sqrt[5]{x^7}-\frac{3}{2x^2}+c \tag{9}$$

Integral indefinida – Aula 3

Exercício I

$$\int \left(3x^{-4}-3x+4\right)\partial x=3\int x^{-4}\partial x-3\int x\partial x+4\int \partial x=3\frac{x^{-3}}{(-3)}-3\frac{x^2}{2}+4x+c=\frac{-1}{x^3}-\frac{3x^2}{2}+4x+c \tag{10}$$

Exercício II

$$\int \frac{2}{3}\sqrt{x}\partial x=\frac{2}{3}\int x^{\frac{1}{2}}\partial x=\frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{4\sqrt{x^3}}{9}+c \tag{11}$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}x^3}{\sqrt[3]{x^2}}\partial x=\int \frac{x^{\frac{1}{2}}x^3}{x^{\frac{2}{3}}}\partial x=\int x^{\frac{1}{2}+3-\frac{2}{3}}\partial x=\int x^{\frac{3+18-4}{6}}\partial x=\int x^{\frac{17}{6}}\partial x=\frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)}+c=\frac{6\sqrt[6]{x^{23}}}{23}+c \tag{12}$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3}=\int x^{-3}\partial x=\frac{x^{-2}}{-2}+c=\frac{-1}{2x^2}+c \tag{13}$$

Exercício II

$$\int \frac{\partial x}{x}=\ln x+c \tag{14}$$

Exercício III

$$\begin{gathered}\int \sqrt{2x+1}\partial x=\int \sqrt{u}\frac{\partial u}{2}=\frac{1}{2}\int u^{\frac{1}{2}}\partial u=\frac{1}{2}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{1}{2}\frac{2}{3}\sqrt{u^3}+c=\frac{\sqrt{u^3}}{3}+c=\frac{\sqrt{(2x+1)^3}}{3}+c\\ u=2x+1,\frac{\partial u}{\partial x}=2\rightarrow\partial x=\frac{\partial u}{2}\\ \frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+c\right)}{\partial x}=\frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1}\cdot 2+0=(2x+1)^{\frac{1}{2}}=\sqrt{2x+1}\end{gathered} \tag{15}$$

Exercício IV

$$\begin{gathered}\int \frac{5x^3+2x+3}{x}\partial x=\int\left(\frac{5x^3}{x}+\frac{2x}{x}+\frac{3}{x}\right)\partial x=\int\left(5x^2+2+\frac{3}{x}\right)\partial x=\\ 5\int x^2\partial x+2\int\partial x+3\int\frac{\partial x}{x}=5\frac{x^3}{3}+2x+3\ln|x|+c=\frac{5x^3}{3}+2x+3\ln|x|+c\\ \frac{\partial\left(\frac{5x^3}{3}+2x+3\ln|x|+c\right)}{\partial x}=\frac{5}{3}3x^2+2+3\frac{1}{x}+0=5x^2+2+\frac{3}{x}=\frac{5x^3+2x+3}{x}\end{gathered} \tag{16}$$

Exercício V

$$\int\left(2x^4+3+5e^x+\frac{7}{x}\right)\partial x=2\int x^4\partial x+3\int\partial x+5\int e^x\partial x+7\int\frac{\partial x}{x}=$$
$$2\frac{x^5}{5}+3x+5e^x+7\ln|x|+c=\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c \tag{17}$$
$$\frac{\partial\left(\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c\right)}{\partial x}=\frac{2}{5}5x^4+3+5e^x+7\frac{1}{x}+0=2x^4+3+5e^x+\frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int\frac{5t^2+7}{\sqrt[3]{t^4}}\partial t=\int\frac{5t^2+7}{t^{\frac{4}{3}}}\partial t=\int t^{\frac{-4}{3}}(5t^2+7)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\partial t=$$
$$5\int t^{\frac{2}{3}}\partial t+7\int t^{\frac{-4}{3}}\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot 3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c \tag{18}$$
$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$
$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int\frac{\partial x}{\sqrt{x}}=\int\frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c \tag{19}$$

Exercício II

$$\int\left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int\frac{\partial x}{x}=3e^x+2\ln|x|+c \tag{20}$$

Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \int u^3 \cancel{x} \frac{\partial u}{4\cancel{x}} = \frac{1}{4}\int u^3 \partial u = \frac{1}{4}\frac{u^4}{4} + c = \frac{u^4}{16} + c = \frac{(2x^2+1)^4}{16} + c =$$
$$\frac{(2x^2+1)^4}{2^4} + c = \left(\frac{2x^2+1}{2}\right)^4 + c = \left(x^2 + \frac{1}{2}\right)^4 + c$$
$$u = 2x^2+1, \frac{\partial u}{\partial x} = 4x \rightarrow \partial x = \frac{\partial u}{4x} \tag{21}$$
$$\frac{\partial\left[\left(x^2+\frac{1}{2}\right)^4 + c\right]}{\partial x} = 4\left(x^2+\frac{1}{2}\right)^3 \cdot 2x + 0 = 8x\left(x^2+\frac{1}{2}\right)^3 = 8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2 =$$
$$(8x^3+4x)\left(x^4+x^2+\frac{1}{4}\right) = 8x^7+8x^5+2x^3+4x^5+4x^3+x = 8x^7+12x^5+6x^3+x$$
$$(2x^2+1)^3 x = (2x^2+1)^2(2x^2+1)x = (4x^4+4x^2+1)(2x^3+x) = 8x^7+4x^5+8x^5+4x^3+2x^3+x =$$
$$8x^7+12x^5+6x^3+x$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x \partial x = 3\int e^x \partial x = 3e^x + c$$
$$\int e^{x^2+1} x \partial x = \int e^u \cancel{x} \frac{\partial u}{2\cancel{x}} = \frac{1}{2}\int e^u \partial u = \frac{1}{2}e^u + c = \frac{e^{x^2+1}}{2} + c$$
$$u = x^2+1, \frac{\partial u}{\partial x} = 2x \rightarrow \partial x = \frac{\partial u}{2x} \tag{22}$$
$$\frac{\partial\left(\frac{e^{x^2+1}}{2} + c\right)}{\partial x} = \frac{1}{2}e^{x^2+1} 2x + 0 = e^{x^2+1} x$$

Exercício II

$$\int e^{x^4+1} x^3 \partial x = \int e^u \cancel{x^3} \frac{\partial u}{4\cancel{x^3}} = \frac{1}{4}\int e^u \partial u = \frac{1}{4}e^u + c = \frac{e^{x^4+1}}{4} + c$$
$$u = x^4+1, \frac{\partial u}{\partial x} = 4x^3 \rightarrow \partial x = \frac{\partial u}{4x^3} \tag{23}$$
$$\frac{\partial\left(\frac{e^{x^4+1}}{4} + c\right)}{\partial x} = \frac{1}{4}e^{x^4+1} 4x^3 + 0 = e^{x^4+1} x^3$$

Exercício III

$$\int \frac{x}{\left(2x^2-1\right)^3}\partial x=\int \left(2x^2-1\right)^{-3} x\,\partial x=\int u^{-3}\cancel{x}\frac{\partial u}{4\cancel{x}}=\frac{1}{4}\int u^{-3}\partial u=\frac{1}{4}\frac{u^{-2}}{(-2)}+c=\frac{-1}{8u^2}+c=$$
$$\frac{-1}{8\left(2x^2-1\right)^2}+c$$
$$u=2x^2-1,\frac{\partial u}{\partial x}=4x\rightarrow\partial x=\frac{\partial u}{4x}$$
$$\frac{\partial\left(\frac{-1}{8\left(2x^2-1\right)^2}+c\right)}{\partial x}=\frac{\partial\left(\frac{-\left(2x^2-1\right)^{-2}}{8}+c\right)}{\partial x}=\frac{-1}{8}(-2)\left(2x^2-1\right)^{-3}4x+0=\left(2x^2-1\right)^{-3}x=$$
$$\frac{x}{\left(2x^2-1\right)^3} \tag{24}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\partial x=\int \left(2x^2-1\right)^{-1} x\,\partial x=\int u^{-1}\cancel{x}\frac{\partial u}{4\cancel{x}}=\frac{1}{4}\int u^{-1}\partial u=\frac{1}{4}\ln|u|+c=\frac{\ln\left|2x^2-1\right|}{4}+c$$
$$u=2x^2-1,\frac{\partial u}{\partial x}=4x\rightarrow\partial x=\frac{\partial u}{4x}$$
$$\frac{\partial\left(\frac{\ln\left|2x^2-1\right|}{4}+c\right)}{\partial x}=\frac{1}{4}\frac{1}{2x^2-1}4x+0=\frac{x}{2x^2-1} \tag{25}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u-1)^2\sqrt{u}\,\partial u = \int (u-1)^2 u^{\frac{1}{2}}\,\partial u = \int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u =$$
$$\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u = \int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u = \int u^{\frac{5}{2}}\,\partial u - 2\int u^{\frac{3}{2}}\,\partial u + \int u^{\frac{1}{2}}\,\partial u =$$
$$\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - 2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2\sqrt{u^7}}{7} - \frac{4\sqrt{u^5}}{5} + \frac{2\sqrt{u^3}}{3} + c = \frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c$$
$$u=1+x \rightarrow x=u-1,\ \frac{\partial x}{\partial u}=1 \rightarrow \partial x=\partial u$$

(26)

$$\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7} - \frac{4(1+x)^{\frac{5}{2}}}{5} + \frac{2(1+x)^{\frac{3}{2}}}{3} + c\right)}{\partial x} =$$
$$\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}} - \frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}} + \frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}} + 0 = (1+x)^{\frac{5}{2}} - 2(1+x)^{\frac{3}{2}} + (1+x)^{\frac{1}{2}} =$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}} - 2(1+x)^{\frac{3}{2}-\frac{1}{2}} + 1\right) = (1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}} - 2(1+x)^{\frac{2}{2}} + 1\right) =$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^2 - 2(1+x) + 1\right) = (1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1) = (1+x)^{\frac{1}{2}}x^2 = x^2\sqrt{1+x}$$

Exercício II

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u^2-1)^2 u\,2u\,\partial u = 2\int (u^2-1)^2u^2\,\partial u = 2\int (u^4-2u^2+1)u^2\,\partial u =$$
$$2\int (u^6-2u^4+u^2)\,\partial u = 2\int u^6\,\partial u - 4\int u^4\,\partial u + 2\int u^2\,\partial u = 2\frac{u^7}{7} - 4\frac{u^5}{5} + 2\frac{u^3}{3} + c =$$
$$\frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c$$
$$u=\sqrt{1+x} \rightarrow u^2=1+x \rightarrow x=u^2-1,\ \frac{\partial x}{\partial u}=2u \rightarrow \partial x=2u\,\partial u$$

(27)

O que é uma Integral Definida – Aula 10

Exercício I

$$\int_1^2 x^3\,\partial x = \left.\frac{x^4}{4}\right]_1^2 = \frac{(2)^4}{4} - \frac{(1)^4}{4} = \frac{16}{4} - \frac{1}{4} = \frac{16-1}{4} = \frac{15}{4} = 3{,}75 \qquad (28)$$

Integral Definida – Aula 10a

Exercício I

$$\int_0^2 (6x^2-4x+5)\partial x=6\int_0^2 x^2\partial x-4\int_0^2 x\partial x+5\int_0^2\partial x=6\frac{x^3}{3}-4\frac{x^2}{2}+5x\Big]_0^2=2x^3-2x^2+5x\Big]_0^2= \\ x(2x^2-2x+5)\Big]_0^2=[(2)(2(2)^2-2(2)+5)]-[(0)(2(0)^2-2(0)+5)]=2(8-4+5)=2\cdot9=18 \tag{29}$$

Exercício II

$$\int_{-1}^0 (2x-e^x)\partial x=2\int_{-1}^0 x\partial x-\int_{-1}^0 e^x\partial x=2\frac{x^2}{2}-e^x\Big]_{-1}^0=x^2-e^x\Big]_{-1}^0=((0)^2-e^{(0)})-((-1)^2-e^{(-1)})= \\ -1-\left(1-\frac{1}{e}\right)=-1-1+\frac{1}{e}=-2+\frac{1}{e} \tag{30}$$

Integral definida – Aula 11

Exercício I

$$\frac{5\pi}{4}\int_0^2\frac{r\partial r}{1+r^2}=\frac{5\pi}{4}\int_0^2(1+r^2)^{-1}r\partial r=\frac{5\pi}{4}\int_0^2 u^{-1}\cancel{r}\frac{\partial u}{2\cancel{r}}=\frac{5\pi}{4}\frac{1}{2}\int_0^2 u^{-1}\partial u=\frac{5\pi}{8}\int_0^2 u^{-1}\partial u= \\ \frac{5\pi}{8}\ln|u|\Big]_0^2=\frac{5\pi}{8}\ln|1+r^2|\Big]_0^2=\left[\frac{5\pi}{8}\ln|1+(2)^2|\right]-\left[\frac{5\pi}{8}\ln|1+(0)^2|\right]=\frac{5\pi}{8}\ln|5|-\frac{5\pi}{8}\ln|1|= \\ \frac{5\pi}{8}(\ln|5|-\ln|1|)=\frac{5\pi}{8}(\ln|5|-0)=\frac{5\pi}{8}\ln|5| \\ u=1+r^2,\frac{\partial u}{\partial r}=2r\to\partial x=\frac{\partial u}{2r} \\ \ln|1|=x\to e^x=1=e^0\to x=0 \\ \ln|5|=x\to e^x=5 \tag{31}$$

Exercício II

$$2\pi\int_0^2 r^2\partial r=2\pi\frac{r^3}{3}\Big]_0^2=\frac{2\pi r^3}{3}\Big]_0^2=\left(\frac{2\pi(2)^3}{3}\right)-\left(\frac{2\pi(0)^3}{3}\right)=\frac{16\pi}{3} \tag{32}$$

Exercício II

$$2\pi\int_0^{\sqrt2}(4r-2r^3)\partial r=8\pi\int_0^{\sqrt2}r\partial r-4\pi\int_0^{\sqrt2}r^3\partial r=8\pi\frac{r^2}{2}-4\pi\frac{r^4}{4}\Big]_0^{\sqrt2}=4\pi r^2-\pi r^4\Big]_0^{\sqrt2}= \\ \pi r^2(4-r^2)\Big]_0^{\sqrt2}=[\pi(\sqrt2)^2(4-(\sqrt2)^2)]-[\pi(0)^2(4-(0)^2)]=2\pi(4-2)=4\pi \tag{33}$$

Exercício III

$$\pi\int_0^2 x^2\,\partial x=\pi\frac{x^3}{3}\Big]_0^2=\frac{\pi x^3}{3}\Big]_0^2=\left(\frac{\pi(2)^3}{3}\right)-\left(\frac{\pi(0)^3}{3}\right)=\frac{8\pi}{3}\tag{34}$$

Exercício IV

$$\frac{\pi}{16}\int_1^4 x^4\,\partial x=\frac{\pi}{16}\,\frac{x^5}{5}\Big]_1^4=\frac{\pi x^5}{80}\Big]_1^4=\left(\frac{\pi(4)^5}{80}\right)-\left(\frac{\pi(1)^5}{80}\right)=\frac{4^5\pi}{80}-\frac{\pi}{80}=\frac{1024\pi-\pi}{80}=\frac{1023\pi}{80}\tag{35}$$

Exercício V

$$\pi\int_1^2 (x^2)^2\,\partial x=\pi\int_1^2 x^4\,\partial x=\pi\frac{x^5}{5}\Big]_1^2=\frac{\pi x^5}{5}\Big]_1^2=\left(\frac{\pi(2)^5}{5}\right)-\left(\frac{\pi(1)^5}{5}\right)=\frac{32\pi}{5}-\frac{\pi}{5}=\frac{32\pi-\pi}{5}=\frac{31\pi}{5}\tag{36}$$

Exercício VI

$$\begin{aligned}&\pi\int_{-1}^2\left(-x^4-x^2+6x+8\right)\partial x=-\pi\int_{-1}^2 x^4\,\partial x-\pi\int_{-1}^2 x^2\partial x+6\pi\int_{-1}^2 x\,\partial x+8\pi\int_{-1}^2\partial x=\\&-\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\Big]_{-1}^2=\frac{-\pi x^5}{5}-\frac{\pi x^3}{3}+3\pi x^2+8\pi x\Big]_{-1}^2=-\pi x\left(\frac{x^4}{5}+\frac{x^2}{3}-3x-8\right)\Bigg]_{-1}^2\\&\left[-\pi(2)\left(\frac{(2)^4}{5}+\frac{(2)^2}{3}-3(2)-8\right)\right]-\left[-\pi(-1)\left(\frac{(-1)^4}{5}+\frac{(-1)^2}{3}-3(-1)-8\right)\right]=\\&-2\pi\left(\frac{16}{5}+\frac{4}{3}-6-8\right)-\pi\left(\frac{1}{5}+\frac{1}{3}+3-8\right)=-2\pi\left(\frac{48+20-210}{15}\right)-\pi\left(\frac{3+5-75}{15}\right)=\\&2\pi\left(\frac{142}{15}\right)+\pi\left(\frac{67}{15}\right)=\pi\left(\frac{284}{15}+\frac{67}{15}\right)=\frac{351\pi}{15}=\frac{3^3\cdot 13\pi}{3\cdot 5}=\frac{3^2\cdot 13\pi}{5}=\frac{117\pi}{5}\end{aligned}\tag{37}$$

Exercício VII

$$\begin{aligned}&\pi\int_0^8\left(\sqrt[3]{y}\right)^2\partial y=\pi\int_0^8\left(y^{\frac{1}{3}}\right)^2\partial y=\pi\int_0^8 y^{\frac{2}{3}}\partial y=\pi\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\Bigg]_0^8=\frac{3\pi\sqrt[3]{y^5}}{5}\Big]_0^8=\\&\left(\frac{3\pi\sqrt[3]{(8)^5}}{5}\right)-\left(\frac{3\pi\sqrt[3]{(0)^5}}{5}\right)=\frac{3\pi\sqrt[3]{8^3\cdot 8^2}}{5}=\frac{3\pi 2^3\sqrt[3]{(2^3)^2}}{5}=\frac{3\pi 2^3 2^2}{5}=\frac{3\pi 2^5}{5}=\frac{96\pi}{5}\end{aligned}\tag{38}$$

Integral definida – Aula 12

Exercício I

$$\int_1^2 2x\,\partial x=2\int_1^2 x\,\partial x=2\frac{x^2}{2}\Big]_1^2=x^2\Big]_1^2=\left((2)^2\right)-\left((1)^2\right)=4-1=4-1=3\tag{39}$$

Exercício II

$$\int_1^4 2\sqrt{x}\,\partial x=\int_1^4 2x^{\frac{1}{2}}\partial x=2\int_1^4 x^{\frac{1}{2}}\partial x=2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_1^4=\frac{4\sqrt{x^3}}{3}\Bigg]_1^4=\left(\frac{4\sqrt{(4)^3}}{3}\right)-\left(\frac{4\sqrt{(1)^3}}{3}\right)=$$
$$\frac{4\sqrt{4^2 2^2}}{3}-\frac{4}{3}=\frac{32}{3}-\frac{4}{3}=\frac{32-4}{3}=\frac{28}{3} \tag{40}$$

Exercício III

$$\int_1^2 4x^2\partial x=4\int_1^2 x^2\partial x=4\frac{x^3}{3}\Bigg]_1^2=\frac{4}{3}x^3\Bigg]_1^2=\frac{4}{3}\left(2^3-1^3\right)=\frac{4}{3}7=\frac{28}{3} \tag{41}$$

Integrais definidas e indefinidas – Aula 13

Exercício I

$$\int\left(\frac{3}{x^4}+\frac{2x^2}{3}-2x+7\right)\partial x=\int\left(3x^{-4}+\frac{2}{3}x^2-2x+7\right)\partial x=$$
$$3\int x^{-4}\partial x+\frac{2}{3}\int x^2\partial x-2\int x\,\partial x+7\int\partial x=3\frac{x^{-3}}{-3}+\frac{2}{3}\frac{x^3}{3}-2\frac{x^2}{2}+7x+c=$$
$$\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c$$
$$\frac{\partial\left(\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c\right)}{\partial x}=\frac{\partial\left(-x^{-3}+\frac{2}{9}x^3-x^2+7x+c\right)}{\partial x}=3x^{-4}+\frac{2}{9}3x^2-2x+7+0=$$
$$\frac{3}{x^4}+\frac{2x^2}{3}-2x+7 \tag{42}$$

Exercício II

$$\int 5\sqrt[3]{x^2}\,\partial x=\int 5x^{\frac{2}{3}}\partial x=5\int x^{\frac{2}{3}}\partial x=5\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+c=3\sqrt[3]{x^5}+c$$
$$\frac{\partial\left(3\sqrt[3]{x^5}+c\right)}{\partial x}=\frac{\partial\left(3x^{\frac{5}{3}}+c\right)}{\partial x}=3\frac{5}{3}x^{\frac{2}{3}}+0=5\sqrt[3]{x^2} \tag{43}$$

Exercício III

$$\int_2^4 2x^3 \partial x = 2\int_2^4 x^3 \partial x = 2\frac{x^4}{4}\Big]_2^4 = \frac{1}{2}x^4\Big]_2^4 = \frac{1}{2}(4^4-2^4) = \frac{1}{2}((2\cdot 2)^4-2^4) = \frac{1}{2}(2^4\cdot 2^4-2^4) = \frac{2^4}{2}(2^4-1) = 2^3(16-1) = 8\cdot 15 = 120 \quad (44)$$

Exercício IV

$$\int_1^2 (3x^2-2x)\partial x = 3\int_1^2 x^2 \partial x - 2\int_1^2 x \partial x = 3\frac{x^3}{3} - 2\frac{x^2}{2}\Big]_1^2 = x^3 - x^2\big]_1^2 = x^2(x-1)\big]_1^2 = [2^2(2-1)]-[1^2(1-1)] = 4 \quad (45)$$

Integral definida pelo método da substituição – U du – Aula 14

Exercício I

$$\int_0^2 \sqrt{2x^2+1}\, x\, \partial x = \int_0^2 \sqrt{u}\, \cancel{x} \frac{\partial u}{4\cancel{x}} = \frac{1}{4}\int_0^2 u^{\frac{1}{2}} \partial u = \frac{1}{4}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_0^2 = \frac{1}{4}\frac{2}{3}\sqrt{u^3}\Big]_0^2 = \frac{1}{6}\sqrt{u^3}\Big]_0^2 = \frac{1}{6}\sqrt{(2x^2+1)^3}\Big]_0^2$$
$$\frac{1}{6}\left[\sqrt{(2\cdot 2^2+1)^3} - \sqrt{(2\cdot 0^2+1)^3}\right] = \frac{1}{6}\left(\sqrt{9^3}-\sqrt{1^3}\right) = \frac{1}{6}\left(\sqrt{9^2 3^2}-1\right) = \frac{1}{6}(27-1) = \frac{1}{6}26 = \frac{13}{3} \quad (46)$$
$$u = 2x^2+1, \frac{\partial u}{\partial x} = 4x \rightarrow \partial x = \frac{\partial u}{4x}$$

Integral Método da Substituição – Aula 15

Exercício I

$$\int (x^3-1)^4 x^2 \partial x = \int u^4 \cancel{x^2} \frac{\partial u}{3\cancel{x^2}} = \frac{1}{3}\int u^4 \partial u = \frac{1}{3}\frac{u^5}{5} + c = \frac{(x^3-1)^5}{15} + c \quad (47)$$
$$u = x^3-1, \frac{\partial u}{\partial x} = 3x^2 \rightarrow \partial x = \frac{\partial u}{3x^2}$$

Exercício II

$$\int \frac{x}{(x^2-1)^3}\partial x = \int (x^2-1)^{-3} x \partial x = \int u^{-3}\frac{\partial u}{2} = \frac{1}{2}\int u^{-3}\partial u = \frac{1}{2}\frac{u^{-2}}{(-2)} + c = \frac{-1}{4u^2} + c = \frac{-1}{4(x^2-1)^2} + c \quad (48)$$
$$u = x^2-1, \partial u = 2x\partial x \rightarrow \frac{\partial u}{2} = x\partial x$$

Exercício III

$$\int \frac{x}{(x^2-1)}\partial x=\int (x^2-1)^{-1}x\partial x=\int u^{-1}\frac{\partial u}{2}=\frac{1}{2}\int u^{-1}\partial u=\frac{1}{2}\ln|u|+c=\frac{\ln|x^2-1|}{2}+c$$
$$u=x^2-1,\partial u=2x\partial x\rightarrow\frac{\partial u}{2}=x\partial x \quad (49)$$

Exercício IV

$$\int e^{x^2-1}x\partial x=\int e^u\frac{\partial u}{2}=\frac{1}{2}e^u\partial u=\frac{1}{2}e^u+c=\frac{e^{x^2-1}}{2}+c$$
$$u=x^2-1,\partial u=2x\partial x\rightarrow\frac{\partial u}{2}=x\partial x \quad (50)$$

Exercício V

$$\int \sqrt{x^3-4}\,x^2\partial x=\int u^{\frac{1}{2}}\frac{\partial u}{3}=\frac{1}{3}\int u^{\frac{1}{2}}\partial u=\frac{1}{3}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{1}{3}\frac{2}{3}\sqrt{u^3}+c=\frac{2\sqrt{u^3}}{9}+c=$$
$$\frac{2\sqrt{(x^3-4)^3}}{9}+c$$
$$u=x^3-4,\partial u=3x^2\partial x\rightarrow\frac{\partial u}{3}=x^2\partial x \quad (51)$$

Exercício VI

$$\int e^{\sqrt{x}}\frac{\partial x}{\sqrt{x}}=\int e^{x^{\frac{1}{2}}}\frac{\partial x}{x^{\frac{1}{2}}}=\int e^{x^{\frac{1}{2}}}x^{\frac{-1}{2}}\partial x=\int e^u 2\partial u=2\int e^u\partial u=2e^u+c=2e^{\sqrt{x}}+c$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x=2\partial u=x^{\frac{-1}{2}}\partial x \quad (52)$$

Exercício VII

$$\int \frac{x\partial x}{\sqrt[5]{x^2-1}}=\int \frac{x}{(x^2-1)^{\frac{1}{5}}}\partial x=\int (x^2-1)^{\frac{-1}{5}}x\partial x=\int u^{\frac{-1}{5}}\frac{\partial u}{2}=\frac{1}{2}\int u^{\frac{-1}{5}}\partial u=\frac{1}{2}\frac{u^{\frac{4}{5}}}{\left(\frac{4}{5}\right)}+c=$$
$$\frac{1}{2}\frac{5}{4}\sqrt[5]{u^4}+c=\frac{5\sqrt[5]{u^4}}{8}+c=\frac{5\sqrt[5]{(x^2-1)^4}}{8}+c$$
$$u=x^2-1,\partial u=2x\partial x\rightarrow\frac{\partial u}{2}=x\partial x \quad (53)$$

Exercício VIII

$$\int \frac{e^t \partial t}{e^t+4} = \int \left(e^t+4\right)^{-1} e^t \partial t = \int u^{-1} \partial u = \ln|u| + c = \ln\left|e^t+4\right| + c \quad (54)$$

$$u = e^t+4, \partial u = e^t \partial t$$

Integral de uma Função Exponencial Qualquer – Aula 16

Exercício I

$$\int \sqrt{10^{3x}} \partial x = \int 10^{\frac{3x}{2}} \partial x = \int 10^u \frac{2}{3} \partial u = \frac{2}{3} \int 10^u \partial u = \frac{2}{3} \frac{10^u}{\ln|10|} + c = \frac{2 \cdot 10^{\frac{3x}{2}}}{3 \ln|10|} + c = \frac{2\sqrt{10^{3x}}}{3 \ln|10|} + c$$

$$u = \frac{3x}{2}, \partial u = \frac{3}{2} \partial x = \frac{2}{3} \partial u = \partial x$$

$$\frac{\partial\left(\frac{2\sqrt{10^{3x}}}{3\ln|10|} + c\right)}{\partial x} = \frac{\partial\left(\frac{2}{3\ln|10|} 10^{\frac{3x}{2}} + c\right)}{\partial x} = \frac{\cancel{2}}{\cancel{3\ln|10|}} \frac{\cancel{3\ln|10|}\sqrt{10^{3x}}}{\cancel{2}} + 0 = \sqrt{10^{3x}} \quad (55)$$

$$y = 10^{\frac{3x}{2}} \rightarrow \ln|y| = \ln\left|10^{\frac{3x}{2}}\right| = \frac{3x}{2} \ln|10| = \frac{3\ln|10|}{2} x$$

$$\frac{\partial(\ln|y|)}{\partial y} = \frac{\partial\left(\frac{3\ln|10|}{2} x\right)}{\partial x} \rightarrow \frac{1}{y} \partial y = \frac{3\ln|10|}{2} \partial x \rightarrow \partial y = y \frac{3\ln|10|}{2} \partial x \rightarrow \frac{\partial y}{\partial x} =$$

$$10^{\frac{3x}{2}} \frac{3\ln|10|}{2} = \frac{3\ln|10|\sqrt{10^{3x}}}{2}$$

Integral de função marginal – Aula 17

Exercício I

O custo marginal por unidade x é dado pela expressão $\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5}$. Determine a função custo total c(x) da produção, sabendo-se que o custo fixo para x = 0 é de R$2000,00.

$$\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5} \rightarrow \cancel{\int \partial c(x)} = \int \left(20 - \frac{4x^3}{5}\right) \partial x \rightarrow \int \left(20 - \frac{4x^3}{5}\right) \partial x = 20 \int \partial x - \frac{4}{5} \int x^3 \partial x =$$

$$20x - \frac{4}{5} \frac{x^4}{4} + c = 20x + \frac{x^4}{5} + c \quad (56)$$

$$20 \cdot 0 + \frac{0^4}{5} + c = 2000 \rightarrow c = 2000$$

$$c(x) = 20x + \frac{x^4}{5} + 2000$$

Exercício II

O rendimento marginal de um bem em quantidade (x) é dado pela expressão $Rm=800-2x^2$. Ache o rendimento total para x = 6, sabendo que quando x = 0, R = 0.

$$\frac{\partial r(x)}{\partial x}=800-2x^2\rightarrow\cancel{\int\partial r(x)}=\int(800-2x^2)\partial x\rightarrow\int(800-2x^2)\partial x=800\int\partial x-2\int x^2\partial x$$
$$800x-2\frac{x^3}{3}+c=800x-\frac{2x^3}{3}+c$$
$$800\cdot 0-\frac{2\cdot 0^2}{3}+c=0\rightarrow c=0 \tag{57}$$
$$r(x)=800x-\frac{2x^3}{3}$$
$$r(6)=800\cdot 6-\frac{2\cdot 6^3}{3}=4800-\frac{2\cdot 2^3\cdot 3^3}{3}=4800-2^4\cdot 3^2=4800-16\cdot 9=4800-144=4656$$

Método da substituição com sen(x) e cos(x) – Aula 18

01.	$\int sen(u)\partial u$	$=$	$-\cos(u)+c$
02.	$\int \cos(u)\partial u$	$=$	$sen(u)+c$
03.	$\int tg(u)\partial u$	$=$	$\ln\lvert sec(u)\rvert+c$
04.	$\int cotg(u)\partial u$	$=$	$\ln\lvert sen(u)\rvert+c$
05.	$\int sec(u)\partial u$	$=$	$\ln\lvert sec(u)+tg(u)\rvert+c$
06.	$\int cossec(u)\partial u$	$=$	$\ln\lvert cossec(u)-cotg(u)\rvert+c$

Exercício I

$$\int sen(2x^2-1)x\partial x=\int sen(u)\frac{\partial u}{4}=\frac{1}{4}sen(u)\partial u=\frac{1}{4}(-\cos(u))+c=\frac{-\cos(u)}{4}+c=$$
$$\frac{-\cos(2x^2-1)}{4}+c$$
$$u=2x^2-1,\partial u=4x\partial x\rightarrow\frac{\partial u}{4}=x\partial x \tag{58}$$
$$\frac{\partial\left(\frac{-\cos(2x^2-1)}{4}+c\right)}{\partial x}=\frac{-1}{4}[-sen(2x^2-1)]4x+0=sen(2x^2-1)x$$

Exercício II

$$\int \cos(3x^3+4)x^2\partial x=\int \cos(u)\frac{\partial u}{9}=\frac{1}{9}\int \cos(u)\partial u=\frac{1}{9}sen(u)+c=\frac{sen(3x^3+4)}{9}+c$$
$$u=3x^3+4,\partial u=9x^2\partial x\rightarrow\frac{\partial u}{9}=x^2\partial x \quad (59)$$
$$\frac{\partial\left(\frac{sen(3x^3+4)}{9}+c\right)}{\partial x}=\frac{1}{9}\cos(3x^3+4)9x^2+0=\cos(3x^3+4)x^2$$

Exercício III

$$\int sen(\sqrt{x})\frac{\partial x}{\sqrt{x}}=\int sen\left(x^{\frac{1}{2}}\right)\frac{\partial x}{x^{\frac{1}{2}}}=\int sen\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int sen(u)2\partial u=2\int sen(u)\partial u=$$
$$2(-\cos(u))+c=-2\cos\sqrt{x}+c \quad (60)$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

Exercício IV

$$\int sen(x)\cos(x)\partial x=\int u\partial u=\frac{u^2}{2}+c=\frac{sen^2(x)}{2}+c$$
$$u=sen(x),\partial u=\cos(x)\partial x \quad (61)$$
$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x}=\frac{1}{2}2sen(x)\cos(x)+0=sen(x)\cos(x)$$

Exercício V

$$\int sen(\cos(x))sen(x)\partial x=\int sen(u)(-\partial u)=-\int sen(u)\partial u=-(-\cos(u))+c=$$
$$\cos(\cos(x))+c$$
$$u=\cos(x),\partial u=-sen(x)\partial x\rightarrow-\partial u=sen(x)\partial x \quad (62)$$
$$\frac{\partial[\cos(\cos(x))+c]}{\partial x}=-sen(\cos(x))(-sen(x))+0=sen(\cos(x))sen(x)$$

Exercício VI

$$\int\sqrt{sen(\theta)}\cos(\theta)\partial\theta=\int u^{\frac{1}{2}}\partial u=\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{sen^3(\theta)}}{3}+c \quad (63)$$
$$u=sen(\theta),\partial u=\cos(\theta)\partial\theta$$

Exercício VI

$$\int \ln|x| \frac{\partial x}{x} = \int \ln|x| x^{-1} \partial x = \int u \partial u = \frac{u^2}{2} + c = \frac{\ln^2|x|}{2} + c$$
$$u = \ln|x|, \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \tag{64}$$
$$\frac{\partial\left(\frac{\ln^2|x|}{2} + c\right)}{\partial x} = \frac{1}{2} 2 \ln|x| \frac{1}{x} + 0 = \frac{\ln|x|}{x}$$

Exercício VII

$$\int \frac{\partial x}{x \ln^2|x|} = \int \left(x \ln^2|x|\right)^{-1} \partial x = \int \ln^{-2}|x| x^{-1} \partial x = \int u^{-2} \partial u = \frac{u^{-1}}{(-1)} = \frac{-1}{u} + c = \frac{-1}{\ln|x|} + c$$
$$u = \ln|x|, \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \tag{65}$$
$$\frac{\partial\left(\frac{-1}{\ln|x|} + c\right)}{\partial x} = \frac{\partial\left(-\ln^{-1}|x| + c\right)}{\partial x} = \ln^{-2} \frac{1}{x} + 0 = \frac{1}{x \ln^2|x|}$$

Exercício VIII

$$\int \frac{sen(\theta) \partial \theta}{(5 - \cos(\theta))^3} = \int (5 - \cos(\theta))^{-3} sen(\theta) \partial \theta = \int u^{-3} \partial u = \frac{u^{-2}}{(-2)} + c = \frac{-1}{2u^2} + c =$$
$$\frac{-1}{2(5 - \cos(\theta))^2} + c$$
$$u = 5 - \cos(\theta), \partial u = -(-sen(\theta)) \partial x \rightarrow \partial u = sen(\theta) \partial \theta \tag{66}$$
$$\frac{\partial\left(\frac{-1}{2(5 - \cos(\theta))^2} + c\right)}{\partial \theta} = \frac{\partial\left(\frac{-1}{2}(5 - \cos(\theta))^{-2} + c\right)}{\partial \theta} = \frac{-1}{2}(-2)(5 - \cos(\theta))^{-3}(-(-sen(\theta))) =$$
$$(5 - \cos(\theta))^{-3} sen(\theta) = \frac{sen(\theta)}{(5 - \cos(\theta))^3}$$

Integração de funções trigonométricas – Aula 19

Exercício I

$$\int tg(x)\partial x=\int \frac{sen(x)}{\cos(x)}\partial x=\int \cos^{-1}(x)sen(x)\partial x=\int u^{-1}(-\partial u)=-\int u^{-1}\partial u=$$
$$-\ln|u|+c=\ln|u^{-1}|+c=\ln\left|\frac{1}{u}\right|+c=\ln\left|\frac{1}{\cos(x)}\right|+c=\ln|sec(x)|+c$$
$$u=\cos(x),\partial u=-sen(x)\partial x\rightarrow -\partial u=sen(x)\partial x$$
$$\frac{\partial(\ln|sec(x)|+c)}{\partial x}=\frac{\partial\left(\ln\left|\frac{1}{\cos(x)}\right|+c\right)}{\partial x}=\frac{\partial(\ln|\cos^{-1}(x)|+c)}{\partial x}=\frac{\partial(-\ln|\cos(x)|+c)}{\partial x}=$$
$$\frac{-1}{\cos(x)}(-sen(x))+0=\frac{sen(x)}{\cos(x)}=tg(x) \tag{67}$$

Exercício II

$$\int cotg(x)\partial x=\int \frac{\cos(x)}{sen(x)}\partial x=\int sen^{-1}(x)\cos(x)\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sen(x)|+c$$
$$u=sen(x),\partial u=\cos(x)\partial x$$
$$\frac{\partial(\ln|sen(x)|+c)}{\partial x}=\frac{1}{sen(x)}\cos(x)+0=\frac{\cos(x)}{sen(x)}=cotg(x) \tag{68}$$

Exercício III

$$\int sec(x)\partial x=\int sec(x)\left(\frac{sec(x)+tg(x)}{sec(x)+tg(x)}\right)\partial x=\int \frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}\partial x=$$
$$\int [sec(x)+tg(x)]^{-1}[sec(x)tg(x)+sec^2(x)]\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sec(x)+tg(x)|+c$$
$$u=sec(x)+tg(x),\partial u=[sec(x)tg(x)+sec^2(x)]\partial x$$
$$\frac{\partial(\ln|sec(x)+tg(x)|+c)}{\partial x}=\frac{1}{sec(x)+tg(x)}[sec(x)tg(x)+sec^2(x)]+0=$$
$$\frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}=\frac{sec(x)\cancel{[sec(x)+tg(x)]}}{\cancel{sec(x)+tg(x)}}=sec(x) \tag{69}$$

Exercício IV

$$\int cossec(x)\partial x \int cossec(x)\left(\frac{cossec(x)-cotg(x)}{cossec(x)-cotg(x)}\right)\partial x=$$
$$\int \frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}\partial x=$$
$$\int [cossec(x)-cotg(x)]^{-1}\left[-cossec(x)cotg(x)+cossec^2(x)\right]\partial x=\int u^{-1}\partial u=$$
$$\ln|u|+c=\ln|cossec(x)-cotg(x)|+c$$
$$u=cossec(x)-cotg(x)=\partial u=\left[-cossec(x)cotg(x)-(-cossec^2(x))\right]\partial x\rightarrow$$
$$\partial u=\left[-cossec(x)cotg(x)+cossec^2(x)\right]\partial x \quad (70)$$

$$\frac{\partial(\ln|cossec(x)-cotg(x)|+c)}{\partial x}=$$
$$\frac{1}{cossec(x)-cotg(x)}\left[-cossec(x)cotg(x)-(-cossec^2(x))\right]+0=$$
$$\frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}=\frac{cossec(x)\cancel{[cossec(x)-cotg(x)]}}{\cancel{cossec(x)-cotg(x)}}=cossec(x)$$

Exercício V

$$\int tg(3x)\partial x=\int tg(u)\frac{\partial u}{3}=\frac{1}{3}tg(u)\partial u=\frac{1}{3}\ln|sec(u)|+c=\frac{\ln|sec(u)|}{3}+c=\frac{\ln|sec(3x)|}{3}+c$$
$$u=3x,\partial u=3\partial x\rightarrow\frac{\partial u}{3}=\partial x \quad (71)$$

Exercício VI

$$\int\frac{\partial x}{sen(2x)}=\int sen^{-1}(2x)\partial x=\int cossec(2x)\partial x=\int cossec(u)\frac{\partial u}{2}=\frac{1}{2}\int cossec(u)\partial u=$$
$$\frac{1}{2}\ln|cossec(u)-cotg(u)|+c=\frac{\ln|cossec(u)-cotg(u)|}{2}+c=\frac{\ln|cossec(2x)-cotg(2x)|}{2}+c \quad (72)$$
$$u=2x,\partial u=2\partial x\rightarrow\frac{\partial u}{2}=\partial x$$

Integração de funções trigonométricas – Aula 20

Exercício I

$$\int\frac{tg(\sqrt{x})\partial x}{\sqrt{x}}=\int\frac{tg\left(x^{\frac{1}{2}}\right)\partial x}{x^{\frac{1}{2}}}=\int tg\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int tg(u)2\partial u=2\int tg(u)\partial u=$$
$$2\ln|sec(u)|+c=2\ln|sec(\sqrt{x})|+c \quad (73)$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

Exercício II

$$\int \frac{cotg(\ln|x|)\partial x}{x} = \int cotg(\ln|x|)x^{-1}\partial x = \int cotg(u)\partial u = \ln|sen(u)| + c = \ln|sen(\ln|x|)| + c$$
$$u = \ln|x|, \partial u = \frac{1}{x}\partial x \rightarrow \partial u = x^{-1}\partial x \quad (74)$$

Exercício III

$$\int sec(5x-\pi)\partial x = \int sec(u)\frac{\partial u}{5} = \frac{1}{5}\int sec(u)\partial u = \frac{1}{5}\ln|sec(u)+tg(u)|+c =$$
$$\frac{\ln|sec(5x-\pi)+tg(5x-\pi)|}{5} + c \quad (75)$$
$$u = 5x-\pi, \partial u = 5\partial x \rightarrow \frac{\partial u}{5} = \partial x$$

Integral de potência sen(x) ou cos(x) – Aula 21

$$\int \cos^n(x)\partial x$$
$$\int sen^n(x)\partial x$$

$n \rightarrow$ ímpar $\quad sen^2(x)+\cos^2(x)=1$

$n \rightarrow$ par $\quad sen^2(x) = \frac{1-\cos(2x)}{2}$, $\quad \cos^2(x) = \frac{1+\cos(2x)}{2}$

Exercício I

$$\int \cos^5(x)\partial x=\int \cos^4(x)\cos(x)\partial x=\int \left(\cos^2(x)\right)^2\cos(x)\partial x=\int \left(1-sen^2(x)\right)^2\cos(x)\partial x=$$
$$\int \left[1-2sen^2(x)+sen^4(x)\right]\cos(x)\partial x=\int \left[\cos(x)-2sen^2(x)\cos(x)+sen^4(x)\cos(x)\right]\partial x=$$
$$\int \cos(x)\partial x-2\int sen^2(x)\cos(x)\partial x+\int sen^4(x)\cos(x)\partial x=\int \partial u-2\int u^2\partial u+\int u^4\partial u=$$
$$u-2\frac{u^3}{3}+\frac{u^5}{5}+c=sen(x)-\frac{2sen^3(x)}{3}+\frac{sen^5(x)}{5}+c$$
$$u=sen(x),\partial u=\cos(x)\partial x$$

$$\frac{\partial\left(sen(x)-\frac{2sen^3(x)}{3}+\frac{sen^5(x)}{5}+c\right)}{\partial x}=\frac{\partial\left(sen(x)-\frac{2}{3}sen^3(x)+\frac{1}{5}sen^5(x)+c\right)}{\partial x}= \tag{76}$$
$$\cos(x)-\frac{2}{\cancel{3}}\cancel{3}sen^2(x)\cos(x)+\frac{1}{\cancel{5}}\cancel{5}sen^4(x)\cos(x)+0=\cos(x)\left[1-2sen^2(x)+sen^4(x)\right]=$$
$$\cos(x)\left[1-\left(sen^2(x)+sen^2(x)\right)+\left(sen^2(x)\right)^2\right]=$$
$$\cos(x)\left[\left(1-sen^2(x)\right)-sen^2(x)+\left(1-\cos^2(x)\right)^2\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+\left(1-2\cos^2(x)+\cos^4(x)\right)\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+1-\left(\cos^2(x)+\cos^2(x)\right)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}+\left(1-sen^2(x)\right)-\cancel{\cos^2(x)}-\cos^2(x)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}-\cancel{\cos^2(x)}+\cos^4(x)\right]=\cos(x)\cos^4(x)=\cos^5(x)$$

Exercício II

$$\begin{gathered}
\int sen^4(x)\partial x=\int\left(sen^2(x)\right)^2\partial x=\int\left(\frac{1-\cos(2x)}{2}\right)^2\partial x=\int\frac{1-2\cos(2x)+\cos^2(2x)}{4}\partial x=\\
\frac{1}{4}\int\left[1-2\cos(2x)+\frac{1+\cos(2\cdot 2x)}{2}\right]\partial x=\frac{1}{4}\int\partial x-\frac{1}{2}\int\cos(2x)\partial x+\frac{1}{8}\int[1+\cos(4x)]\partial x=\\
\frac{1}{4}\int\partial x-\frac{1}{2}\int\cos(2x)\partial x+\frac{1}{8}\int\partial x+\frac{1}{8}\int\cos(4x)\partial x=\\
\frac{1}{4}\int\frac{\partial u}{2}-\frac{1}{2}\int\cos(u)\frac{\partial u}{2}+\frac{1}{8}\int\frac{\partial u}{2}+\frac{1}{8}\int\cos(2u)\frac{\partial u}{2}=\\
\frac{1}{8}\int\partial u-\frac{1}{4}\int\cos(u)\partial u+\frac{1}{16}\int\partial u+\frac{1}{16}\int\cos(2u)\partial u=\\
\frac{1}{8}\int\partial u-\frac{1}{4}\int\cos(u)\partial u+\frac{1}{16}\int\partial u+\frac{1}{16}\int\cos(v)\frac{\partial v}{2}=\\
\frac{1}{8}\int\partial u-\frac{1}{4}\int\cos(u)\partial u+\frac{1}{16}\int\partial u+\frac{1}{32}\int\cos(v)\partial v=\\
\frac{1}{8}u-\frac{1}{4}sen(u)+\frac{1}{16}u+\frac{1}{32}sen(v)+c=\frac{2u+u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\\
\frac{3u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\frac{3\cdot 2x}{16}-\frac{sen(2x)}{4}+\frac{sen(2\cdot 2x)}{32}+c=\\
\frac{3x}{8}-\frac{sen(2x)}{4}+\frac{sen(4x)}{32}+c\\
u=2x,\frac{\partial u}{2}=\partial x\\
v=2u,\frac{\partial v}{2}=\partial u
\end{gathered}\tag{77}$$