Introdução aos limites – Aula 1

Exercício I

$$\lim_{x \to 1} [2x+1] = 3 \tag{1}$$

Exercício II

$$\lim_{x \to 3} \left[\frac{2x+2}{x+1} \right] = 2 \tag{2}$$

Exercício III

$$f(x) = \begin{cases} 2x+1 & x \geq 1 \\ x^2+2 & x < 1 \end{cases}$$
$$\lim_{x \to 1} f(x) = 3 \tag{3}$$

Exercício IV

$$f(x) = \begin{cases} x^2+3x & x \geq 2 \\ 3x+1 & x < 2 \end{cases}$$
$$\lim_{x \to 2^+} f(x) = 10 \tag{4}$$
$$\lim_{x \to 2^-} f(x) = 7$$

Indeterminação de limites – Aula 2

Exercício I

$$\lim_{x \to 0} \left[\frac{x^2+2x}{x} \right] = \frac{0^2+2\cdot 0}{0} = \frac{0}{0} \rightarrow (x=0)$$

$$\frac{(x^2+2x) \div x}{x \div x} = x+2 \tag{5}$$

$$\lim_{x \to 0} \left[\frac{x^2+2x}{x} \right] = 0+2 = 2$$

Exercício II

$$\lim_{x \to 2}\left[\frac{x^2-4}{x-2}\right]=\frac{2^2-4}{2-2}=\frac{0}{0}\to(x=2\to x-2=0)$$

$$\frac{(x^2-4)\div(x-2)}{(x-2)\div(x-2)}=x+2 \tag{6}$$

$$\lim_{x \to 2}\left[\frac{x^2-4}{x-2}\right]=2+2=4$$

Exercício III

$$\lim_{x \to 2}\left[\frac{2x^2-2x-4}{x-2}\right]=\frac{2\cdot 2^2-2\cdot 2-4}{2-2}=\frac{8-4-4}{0}=\frac{0}{0}\to(x=2\to x-2=0)$$

$$\frac{(2x^2-2x-4)\div(x-2)}{(x-2)\div(x-2)}=2x+2 \tag{7}$$

$$\lim_{x \to 2}\left[\frac{2x^2-2x-4}{x-2}\right]=2\cdot 2+2=6$$

Indeterminação de limites – Aula 3

Exercício I

$$\lim_{x \to 3}\left[\frac{x^2-9}{x-3}\right]=\frac{3^2-9}{3-3}=\frac{0}{0}\to(x=3\to x-3=0)$$

$$\frac{(x^2-9)\div(x-3)}{(x-3)\div(x-3)}=x+3 \tag{8}$$

$$\lim_{x \to 3}\left[\frac{x^2-9}{x-3}\right]=3+3=6$$

Exercício II

$$\lim_{x \to -2}\left[\frac{x+2}{x^2-4}\right]=\frac{-2+2}{(-2)^2-4}=\frac{0}{0}\to(x=-2\to x+2=0)$$

$$\frac{(x+2)\div(x+2)}{(x^2-4)\div(x+2)}=\frac{1}{x-2} \tag{9}$$

$$\lim_{x \to -2}\left[\frac{x+2}{x^2-4}\right]=\frac{1}{-2-2}=-\frac{1}{4}$$

Exercício III

$$\lim_{x \to 3}\left[\frac{2x^3-6x^2+x-3}{x-3}\right]=\frac{54-54+3-3}{3-3}=\frac{0}{0} \to (x=3 \to x-3=0)$$

$$\frac{(2x^3-6x^2+x-3)\div(x-3)}{(x-3)\div(x-3)}=2x^2+1 \tag{10}$$

$$\lim_{x \to 3}\left[\frac{2x^3-6x^2+x-3}{x-3}\right]=2\cdot 3^2+1=19$$

Indeterminação de limites 0/0 – Aula 3a

Exercício I

$$\lim_{x \to 1}\left[\frac{x^2-x}{2x^2+5x-7}\right]=\frac{1^2-1}{2\cdot 1^2+5\cdot 1-7}=\frac{0}{0} \to (x=1 \to x-1=0)$$

$$\frac{(x^2-x)\div(x-1)}{(2x^2+5x-7)\div(x-1)}=\frac{x}{2x+7} \tag{11}$$

$$\lim_{x \to 1}\left[\frac{x^2-x}{2x^2+5x-7}\right]=\frac{1}{2\cdot 1+7}=\frac{1}{9}$$

Exercício II

$$\lim_{x \to 2}\left[\frac{x^3-8}{x^2-4}\right]=\frac{2^3-8}{2^2-4}=\frac{0}{0} \to (x=2 \to x-2=0)$$

$$\frac{(x^3-8)\div(x-2)}{(x^2-4)\div(x-2)}=\frac{x^2+2x+4}{x+2} \tag{12}$$

$$\lim_{x \to 2}\left[\frac{x^3-8}{x^2-4}\right]=\frac{2^2+2\cdot 2+4}{2+2}=\frac{12}{4}=3$$

Indeterminação polinomial de limites – Aula 4

Exercício I

$$\lim_{h\to 0}\left[\frac{(x+h)^3-x^3}{h}\right]=\frac{(x+0)^3-x^3}{0}=\frac{x^3-x^3}{0}=\frac{0}{0}$$

$$\frac{(x+h)^3-x^3}{h}=\frac{(x+h)^2(x+h)-x^3}{h}=\frac{(x^2+2xh+h^2)(x+h)-x^3}{h}=$$
$$\frac{x^3+2x^2h+xh^2+x^2h+2xh^2+h^3-x^3}{h}=\frac{3x^2h+3xh^2+h^3}{h}=\frac{h(3x^2+3xh+h^2)}{h}=3x^2+3xh+h^2 \tag{13}$$
$$\lim_{h\to 0}\left[\frac{(x+h)^3-x^3}{h}\right]=3x^2+3x\cdot 0+0^2=3x^2$$

Exercício II

$$\lim_{x\to -1}\left[\frac{x^3+1}{x^2-1}\right]=\frac{(-1)^3+1}{(-1)^2-1}=\frac{-1+1}{1-1}=\frac{0}{0}\to(x=-1\to x+1=0)$$

$$\frac{(x^3+1)\div(x+1)}{(x^2-1)\div(x+1)}=\frac{x^2-x+1}{x-1} \tag{14}$$
$$\lim_{x\to -1}\left[\frac{x^3+1}{x^2-1}\right]=\frac{(-1)^2-(-1)+1}{-1-1}=\frac{1+1+1}{-2}=-\frac{3}{2}$$

Indeterminação polinomial de limites – Aula 5

Exercício I

$$\lim_{t\to -2}\left[\frac{t^3+4t^2+4t}{(t+2)(t-3)}\right]=\frac{(-2)^3+4\cdot(-2)^2+4\cdot(-2)}{(-2+2)(-2-3)}=\frac{-8+16-8}{0\cdot(-5)}=\frac{0}{0}\to(x=-2\to x+2=0)$$

$$\frac{(t^3+4t^2+4t)\div(x+2)}{[(t+2)(t-3)]\div(x+2)}=\frac{t^2+2t}{t-3} \tag{15}$$
$$\lim_{t\to -2}\left[\frac{t^3+4t^2+4t}{(t+2)(t-3)}\right]=\frac{(-2)^2+2\cdot(-2)}{-2-3}=\frac{4-4}{-5}=\frac{0}{-5}=0$$

Exercício II

$$\lim_{t\to 0}\left[\frac{(4+t)^2-16}{t}\right]=\frac{(4+0)^2-16}{0}=\frac{16-16}{0}=\frac{0}{0}\to(t=0)$$

$$\frac{(4+t)^2-16}{t}=\frac{16+8t+t^2-16}{t}=\frac{t(8+t)}{t}=8+t \tag{16}$$
$$\lim_{t\to 0}\left[\frac{(4+t)^2-16}{t}\right]=8+0=8$$

Exercício III

$$\lim_{x\to a}\left[\frac{x^2+(1-a)x-a}{x-a}\right]=\frac{a^2+(1-a)a-a}{a-a}=\frac{a^2+a-a^2-a}{0}=\frac{0}{0}\to(x=a\to x-a=0)$$

$$\frac{[x^2+(1-a)x-a]\div(x-a)}{(x-a)\div(x-a)}=x+1 \qquad (17)$$

$$\lim_{x\to a}\left[\frac{x^2+(1-a)x-a}{x-a}\right]=a+1$$

Indeterminação de limites com raiz – Aula 6

Exercício I

$$\lim_{x\to 1}\left[\frac{x-1}{\sqrt{x}-1}\right]=\frac{1-1}{\sqrt{1}-1}=\frac{0}{0}\to(x=1\to x-1=0)$$

$$\left(\frac{x-1}{\sqrt{x}-1}\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)=\frac{(x-1)(\sqrt{x}+1)}{x-1}=\sqrt{x}+1 \qquad (18)$$

$$\lim_{x\to 1}\left[\frac{x-1}{\sqrt{x}-1}\right]=\sqrt{1}+1=2$$

Exercício II

$$\lim_{x\to 0}\left[\frac{\sqrt{x+2}-\sqrt{2}}{x}\right]=\frac{\sqrt{0+2}-\sqrt{2}}{0}=\frac{\sqrt{2}-\sqrt{2}}{0}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{x+2}-\sqrt{2}}{x}\right)\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right)=\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}=\frac{x}{x(\sqrt{x+2}+\sqrt{2})}=\frac{1}{\sqrt{x+2}+\sqrt{2}} \qquad (19)$$

$$\lim_{x\to 0}\left[\frac{\sqrt{x+2}-\sqrt{2}}{x}\right]=\frac{1}{\sqrt{0+2}+\sqrt{2}}=\frac{1}{\sqrt{2}+\sqrt{2}}=\left(\frac{1}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{4}$$

Exercício III

$$\lim_{x\to 4}\left[\frac{x^2-16}{\sqrt{x}-2}\right]=\frac{4^2-16}{\sqrt{4}-2}=\frac{16-16}{2-2}=\frac{0}{0}\to(x=4\to x-4=0)$$

$$\left(\frac{x^2-16}{\sqrt{x}-2}\right)\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right)=\frac{(x-4)(x+4)(\sqrt{x}+2)}{x-4}=(x+4)(\sqrt{x}+2) \qquad (20)$$

$$\lim_{x\to 4}\left[\frac{x^2-16}{\sqrt{x}-2}\right]=(4+4)(\sqrt{4}+2)=8\cdot 4=32$$

Exercício IV

$$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x}\right]=\frac{\sqrt{4+0}-2}{0}=\frac{2-2}{0}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{4+x}-2}{x}\right)\left(\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)=\frac{4+x-4}{x(\sqrt{4+x}+2)}=\frac{x}{x(\sqrt{4+x}+2)}=\frac{1}{\sqrt{4+x}+2} \qquad (21)$$

$$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x}\right]=\frac{1}{\sqrt{4+0}+2}=\frac{1}{2+2}=\frac{1}{4}$$

Indeterminação de limites com raiz – Aula 7

Exercício I

$$\lim_{x\to 7}\left[\frac{2-\sqrt{x-3}}{x^2-49}\right]=\frac{2-\sqrt{7-3}}{7^2-49}=\frac{2-\sqrt{4}}{49-49}=\frac{0}{0}\to(x=7\to x-7=0)$$

$$\left(\frac{2-\sqrt{x-3}}{x^2-49}\right)\left(\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}\right)=\frac{4-(x-3)}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{4-x+3}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{}{(x+7)(x} \qquad (22)$$

$$\frac{-(x-7)}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{-1}{(x+7)(2+\sqrt{x-3})}$$

$$\lim_{x\to 7}\left[\frac{2-\sqrt{x-3}}{x^2-49}\right]=\frac{-1}{(7+7)(2+\sqrt{7-3})}=\frac{-1}{14\cdot(2+\sqrt{4})}=\frac{-1}{14\cdot 4}=-\frac{1}{56}$$

Exercício II

$$\lim_{x\to 0}\left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right]=\frac{\sqrt{0^2+a^2}-a}{\sqrt{0^2+b^2}-b}=\frac{\sqrt{a^2}-a}{\sqrt{b^2}-b}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right)\left(\frac{\sqrt{x^2+b^2}+b}{\sqrt{x^2+b^2}+b}\right)=\left(\frac{(\sqrt{x^2+a^2}-a)(\sqrt{x^2+b^2}+b)}{x^2}\right)\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}+a}\right)=\frac{x^2(\sqrt{x^2+b^2}+b)}{x^2(\sqrt{x^2+a^2}+a)}= \qquad (23)$$

$$\lim_{x\to 0}\left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right]=\frac{\sqrt{0^2+b^2}+b}{\sqrt{0^2+a^2}+a}=\frac{\sqrt{b^2}+b}{\sqrt{a^2}+a}=\frac{b+b}{a+a}=\frac{2b}{2a}=\frac{b}{a}$$

Indeterminação de limites com raiz – Aula 8

Exercício I

$$\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]=\frac{3-\sqrt{5+4}}{1-\sqrt{5-4}}=\frac{3-\sqrt{9}}{1-\sqrt{1}}=\frac{0}{0}\rightarrow(x=4\rightarrow x-4=0)$$

$$\begin{gathered}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)\left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right)=\frac{(3-\sqrt{5+x})(1+\sqrt{5-x})}{1-(5-x)}=\\ \left(\frac{(3-\sqrt{5+x})(1+\sqrt{5-x})}{-4+x}\right)\left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)=\frac{(9-(5+x))(1+\sqrt{5-x})}{(-4+x)(3+\sqrt{5+x})}=\\ \frac{(4-x)(1+\sqrt{5-x})}{(-4+x)(3+\sqrt{5+x})}=\frac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}=\frac{-(1+\sqrt{5-x})}{3+\sqrt{5+x}}=\frac{-1-\sqrt{5-x}}{3+\sqrt{5+x}}\\ \lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]=\frac{-1-\sqrt{5-4}}{3+\sqrt{5+4}}=\frac{-1-\sqrt{1}}{3+\sqrt{9}}=\frac{-2}{6}=-\frac{1}{3}\end{gathered}\tag{24}$$

Indeterminação de limites com raiz – Aula 9

Exercício I

$$\lim_{x\to 0}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\right]=\frac{\sqrt{1+0}-\sqrt{1-0}}{0}=\frac{\sqrt{1}-\sqrt{1}}{0}=\frac{0}{0}\rightarrow(x=0)$$

$$\begin{gathered}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\right)\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{1+x-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}=\frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})}=\\ \frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}=\frac{2}{\sqrt{1+x}+\sqrt{1-x}}\\ \lim_{x\to 0}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\right]=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}=\frac{2}{\sqrt{1}+\sqrt{1}}=\frac{2}{2}=1\end{gathered}\tag{25}$$

Limites com módulo – Aula 10

Exercício I

$$\lim_{x\to 4}\left[\frac{|x-4|}{x-4}\right]=\nexists$$

$$\begin{gathered}|x-4|=\begin{cases}(x-4) & \text{para} \quad (x-4)\geq 0\\ -(x-4) & \text{para} \quad (x-4)<0\end{cases}\\ |x-4|=\begin{cases}(x-4) & \text{para} \quad x\geq 4\\ -(x-4) & \text{para} \quad x<4\end{cases}\\ \lim_{x\to 4^+}\left[\frac{x-4}{x-4}\right]=1\\ \lim_{x\to 4^-}\left[\frac{-(x-4)}{x-4}\right]=-1\end{gathered} \tag{26}$$

Exercício II

$$\lim_{x\to 0}\left[\frac{|x|}{x}\right]=\nexists$$

$$\begin{gathered}|x|=\begin{cases}x & \text{para} \quad x\geq 0\\ -x & \text{para} \quad x<0\end{cases}\\ \lim_{x\to 0^+}\left[\frac{x}{x}\right]=1\\ \lim_{x\to 0^-}\left[\frac{-x}{x}\right]=-1\end{gathered} \tag{27}$$

Exercício III

$$\lim_{x\to 0}\left[\frac{|x|}{x^2}\right]=+\infty$$

$$\begin{gathered}|x|=\begin{cases}x & \text{para} \quad x\geq 0\\ -x & \text{para} \quad x<0\end{cases}\\ \lim_{x\to 0^+}\left[\frac{x}{x^2}\right]=\frac{1}{x}=\frac{1}{0^+}=+\infty\\ \lim_{x\to 0^-}\left[\frac{-x}{x^2}\right]=\frac{-1}{x}=\frac{-1}{0^-}=+\infty\end{gathered} \tag{28}$$

Limites no infinito – Aula 11

Exercício I

$$\lim_{x\to+\infty}\left[\frac{2x-5}{x+8}\right]=2$$

$$\frac{x\left(\frac{2x}{x}-\frac{5}{x}\right)}{x\left(\frac{x}{x}+\frac{8}{x}\right)}=\frac{2-\frac{5}{x}}{1+\frac{8}{x}}=\frac{2-0}{1+0}=2 \tag{29}$$

Exercício II

$$\lim_{x\to-\infty}\left[\frac{2x^3-3x+5}{4x^5-2}\right]=0$$

$$\frac{x^3\left(\frac{2x^3}{x^3}-\frac{3x}{x^3}+\frac{5}{x^3}\right)}{x^3\left(\frac{4x^5}{x^3}-\frac{2}{x^3}\right)}=\frac{2-\frac{3}{x^2}+\frac{5}{x^3}}{4x^2-\frac{2}{x^3}}=\frac{2-0+0}{4x^2-0}=\frac{1}{2x^2}=0 \tag{30}$$

Exercício III

$$\lim_{x\to+\infty}\left[\frac{5-x^3}{8x+2}\right]=\frac{-(+\infty)^2}{8}=\frac{-\infty}{8}=-\infty$$

$$\frac{x\left(\frac{5}{x}-\frac{x^3}{x}\right)}{x\left(\frac{8x}{x}+\frac{2}{x}\right)}=\frac{\frac{5}{x}-x^2}{8+\frac{2}{x}}=\frac{0-x^2}{8+0}=\frac{-x^2}{8} \tag{31}$$

Limites com x tendendo ao infinito – Aula 11a

Exercício I

$$\lim_{x\to+\infty}\left[\frac{2x+5}{\sqrt{2x^2-5}}\right]=\sqrt{2}$$

$$\frac{x\left(\frac{2x}{x}+\frac{5}{x}\right)}{x\left(\frac{\sqrt{2x^2-5}}{x}\right)}=\frac{2+\frac{5}{x}}{\frac{\sqrt{2x^2-5}}{\sqrt{x^2}}}=\frac{2+\frac{5}{x}}{\sqrt{\frac{2x^2}{x^2}-\frac{5}{x^2}}}=\frac{2+\frac{5}{x}}{\sqrt{2-\frac{5}{x^2}}}=\frac{2+0}{\sqrt{2-0}}=\left(\frac{2}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{2\sqrt{2}}{2}=\sqrt{2} \tag{32}$$

Exercício II

$$\lim_{x\to-\infty}\left[\frac{2x+5}{\sqrt{2x^2-5}}\right]=-\sqrt{2}$$

$$\frac{x\left(\frac{2x}{x}+\frac{5}{x}\right)}{x\left(\frac{\sqrt{2x^2-5}}{x}\right)}=\frac{2+\frac{5}{x}}{\frac{\sqrt{2x^2-5}}{-\sqrt{x^2}}}=\frac{2+\frac{5}{x}}{-\sqrt{\frac{2x^2}{x^2}-\frac{5}{x^2}}}=\frac{2+\frac{5}{x}}{-\sqrt{2-\frac{5}{x^2}}}=\frac{2+0}{-\sqrt{2-0}}=\left(\frac{2}{-\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=-\sqrt{2} \tag{33}$$

Limites no infinito – Aula 12

Exercício I

$$\lim_{x\to+\infty}\left[\frac{2x^2-3x}{x+1}\right]=2\cdot(+\infty)=+\infty$$

$$\frac{x\left(\frac{2x^2}{x}-\frac{3x}{x}\right)}{x\left(\frac{x}{x}+\frac{1}{x}\right)}=\frac{2x-3}{1+\frac{1}{x}}=\frac{2x-3}{1+0}=2x \tag{34}$$

Exercício II

$$\lim_{x\to+\infty}\left[\frac{2x^4+3x^2+2x+1}{4-x^4}\right]=-2$$

$$\frac{x^4\left(\frac{2x^4}{x^4}+\frac{3x^2}{x^4}+\frac{2x}{x^4}+\frac{1}{x^4}\right)}{x^4\left(\frac{4}{x^4}-\frac{x^4}{x^4}\right)}=\frac{2+\frac{3}{x^2}+\frac{2}{x^3}+\frac{1}{x^4}}{\frac{4}{x^4}-1}=\frac{2+0+0+0}{0-1}=\frac{2}{-1}=-2 \tag{35}$$

Exercício III

$$\lim_{x\to+\infty}\left[\frac{x^2+3x-1}{x^3-2}\right]=0$$

$$\frac{x^2\left(\frac{x^2}{x^2}+\frac{3x}{x^2}-\frac{1}{x^2}\right)}{x^2\left(\frac{x^3}{x^2}-\frac{2}{x^2}\right)}=\frac{1+\frac{3}{x}-\frac{1}{x^2}}{x-\frac{2}{x^2}}=\frac{1+0-0}{x-0}=\frac{1}{x}=0 \tag{36}$$

Limites infinitos – Aula 13

Exercício I

$$\lim_{x \to 3}\left[\frac{1}{x-3}\right]=\frac{1}{3-3}=\frac{1}{0}$$

$$\begin{aligned}\lim_{x \to 3^+}\left[\frac{1}{x-3}\right]&=\frac{1}{3^+-3}=\frac{1}{0^+}=+\infty\\ \lim_{x \to 3^-}\left[\frac{1}{x-3}\right]&=\frac{1}{3^--3}=\frac{1}{0^-}=-\infty\\ \lim_{x \to 3}\left[\frac{1}{x-3}\right]&=\nexists\end{aligned} \tag{37}$$

Exercício II

$$\lim_{x \to 1}\left[\frac{1}{(x-1)^2}\right]=\frac{1}{(1-1)^2}=\frac{1}{0^2}=\frac{1}{0}$$

$$\begin{aligned}\lim_{x \to 1^+}\left[\frac{1}{(x-1)^2}\right]&=\frac{1}{(1^+-1)^2}=\frac{1}{(0^+)^2}=\frac{1}{0^+}=+\infty\\ \lim_{x \to 1^-}\left[\frac{1}{(x-1)^2}\right]&=\frac{1}{(1^--1)^2}=\frac{1}{(0^-)^2}=\frac{1}{0^+}=+\infty\\ \lim_{x \to 1}\left[\frac{1}{(x-1)^2}\right]&=+\infty\end{aligned} \tag{38}$$

Exercício III

$$\lim_{x \to 3}\left[\frac{5-6x}{3x-9}\right]=\frac{5-6\cdot 3}{3\cdot 3-9}=\frac{5-18}{9-9}=\frac{-13}{0}$$

$$\begin{aligned}\lim_{x \to 3^+}\left[\frac{5-6x}{3x-9}\right]&=\frac{5-6(3^+)}{3(3^+)-9}=\frac{5-18^+}{9^+-9}=\frac{-13^+}{0^+}=-\infty\\ \lim_{x \to 3^-}\left[\frac{5-6x}{3x-9}\right]&=\frac{5-6(3^-)}{3(3^-)-9}=\frac{5-18^-}{9^--9}=\frac{-13^-}{0^-}=+\infty\\ \lim_{x \to 3}\left[\frac{5-6x}{3x-9}\right]&=\nexists\end{aligned} \tag{39}$$

Limites – aplicações das propriedades – Aula 14

Exercício I

$$\begin{gathered}\lim_{x\to+\infty} f(x)=3\\ \lim_{x\to+\infty} g(x)=5\\ \lim_{x\to+\infty} h(x)=0\\ \\ \lim_{x\to+\infty}[f(x)+3\cdot g(x)]=\\ \lim_{x\to+\infty} f(x)+\lim_{x\to+\infty} 3\cdot g(x)=\\ \lim_{x\to+\infty} f(x)+\lim_{x\to+\infty}[3]\cdot\lim_{x\to+\infty} g(x)=3+3\cdot 5=3+15=18\end{gathered} \tag{40}$$

Exercício II

$$\begin{gathered}\lim_{x\to+\infty} f(x)=3\\ \lim_{x\to+\infty} g(x)=5\\ \lim_{x\to+\infty} h(x)=0\\ \\ \lim_{x\to+\infty}[f(x)\cdot g(x)]=\\ \lim_{x\to+\infty} f(x)\cdot\lim_{x\to+\infty} g(x)=3\cdot 5=15\end{gathered} \tag{41}$$

Exercício III

$$\begin{gathered}\lim_{x\to+\infty} f(x)=3\\ \lim_{x\to+\infty} g(x)=5\\ \lim_{x\to+\infty} h(x)=0\\ \\ \lim_{x\to+\infty}\left[\frac{3\cdot h(x)+4}{x}\right]=\\ \frac{\lim_{x\to+\infty}[3\cdot h(x)+4]}{\lim_{x\to+\infty}[x]}=\\ \frac{\lim_{x\to+\infty}[3\cdot h(x)]+\lim_{x\to+\infty}[4]}{\lim_{x\to+\infty}[x]}=\\ \frac{\lim_{x\to+\infty}[3]\cdot\lim_{x\to+\infty} h(x)+\lim_{x\to+\infty}[4]}{\lim_{x\to+\infty}[x]}=\frac{3\cdot 0+4}{+\infty}=\frac{0+4}{+\infty}=\frac{4}{+\infty}=0\end{gathered} \tag{42}$$

Exercício IV

$$
\begin{gathered}
f(x)=\begin{cases} k\cdot x-1 & x\geq 3 \\ 3x-7 & x<3 \end{cases} \\
\lim_{x\to 3^+}[k\cdot x-1]=3k-1 \\
\lim_{x\to 3^-}[3x-7]=3\cdot 3-7=9-7=2 \\
3k-1=2\to 3k=2+1\to 3k=3\to k=\frac{3}{3}\to k=1
\end{gathered} \tag{43}
$$

Exercício V

$$
\begin{gathered}
f(x)=\begin{cases} \dfrac{x^2-9}{x+3} & x\geq -3 \\ k+1 & x<-3 \end{cases} \\
\lim_{x\to -3^+}\left[\frac{x^2-9}{x+3}\right]=\frac{(-3)^2-9}{-3+3}=\frac{9-9}{0}=\frac{0}{0}\to(x=-3\to x+3=0) \\
\frac{x^2-9}{x+3}=\frac{(x-3)(x+3)}{x+3}=x-3 \\
\lim_{x\to -3^+}[x-3]=-3-3=-6 \\
\lim_{x\to -3^-}[k+1]=k+1 \\
k+1=-6\to k=-6-1\to k=-7
\end{gathered} \tag{44}
$$

Limites no infinito – Aula 15

Exercício I

$$
\lim_{x\to +\infty}[3x^3+4x^2-1]=x^3\left(\frac{3x^3}{x^3}+\frac{4x^2}{x^3}-\frac{1}{x^3}\right)=x^3\left(3+\frac{4}{x}-\frac{1}{x^3}\right)=x^3(3+0-0)=3(+\infty)^3=+\infty \tag{45}
$$

Exercício II

$$
\lim_{x\to +\infty}[3x^5-4x^3+1]=x^5\left(\frac{3x^5}{x^5}-\frac{4x^3}{x^5}+\frac{1}{x^5}\right)=x^5\left(3-\frac{4}{x^2}+\frac{1}{x^5}\right)=x^5(3-0+0)=3(+\infty)^3=+\infty \tag{46}
$$

Continuidade de uma função – Aula 17

Continuidade da *f(x)* em *x=a*

a) $f(a)$ está definida.
b) $\lim_{x\to a} f(x)$ deve existir.
c) $\lim_{x\to a} f(x)=f(a)$

Exercício I

$$a=1$$
$$f(x)=\begin{cases} \dfrac{3}{x-1} & x\neq 1 \\ 3 & x=1 \end{cases}$$

a) $f(a)=f(1)=3$

$$\text{b) } \lim_{x\to a} f(x)=\lim_{x\to 1} f(x)=\lim_{x\to 1}\left[\frac{3}{x-1}\right]=\frac{3}{1-1}=\frac{1}{0} \quad (47)$$
$$\lim_{x\to 1^+} f(x)=\frac{3}{1^+-1}=\frac{3}{0^+}=+\infty$$
$$\lim_{x\to 1^-} f(x)=\frac{3}{1^--1}=\frac{3}{0^-}=-\infty$$
$$\lim_{x\to 1} f(x)=\nexists$$

$f(x)$ não é contínua

Exercício II

$$a=5$$
$$f(x)=\begin{cases} \dfrac{x^2-25}{x-5} & x\neq 5 \\ x+5 & x=5 \end{cases}$$

a) $f(a)=f(5)=5+5=10$

$$\text{b) } \lim_{x\to a} f(x)=\lim_{x\to 5} f(x)=\lim_{x\to 5}\left[\frac{x^2-25}{x-5}\right]=\frac{5^2-25}{5-5}=\frac{0}{0}\to(x=5\to x-5=0) \quad (48)$$
$$\frac{x^2-25}{x-5}=\frac{(x-5)(x+5)}{(x-5)}=x+5$$
$$\lim_{x\to 5} f(x)=5+5=10$$

c) $f(5)=\lim_{x\to 5} f(x)$

$f(x)$ é contínua

Exercício III

$$a = 4$$
$$f(x) = \begin{cases} 2x+3 & x \leq 4 \\ 7 + \dfrac{16}{x} & x > 4 \end{cases}$$

a) $f(a) = f(4) = 2 \cdot 4 + 3 = 11$
b) $\lim_{x \to a} f(x) = \lim_{x \to 4} f(x) = 11$
$$\lim_{x \to 4^+} \left[7 + \frac{16}{x}\right] = 7 + \frac{16}{4} = 7 + 4 = 11 \qquad (49)$$
$$\lim_{x \to 4^-} [2x+3] = 2 \cdot 4 + 3 = 11$$
c) $f(4) = \lim_{x \to 4} f(x)$

$f(x)$ é contínua

Exercício IV

$$a = 1$$
$$f(x) = \begin{cases} 7x - 2 & x \leq 1 \\ k \cdot x^2 & x > 1 \end{cases}$$

a) $f(a) = f(1) = 7 \cdot 1 - 2 = 5$
b) $\lim_{x \to a} f(x) = \lim_{x \to 1} f(x) = 5$
$$\lim_{x \to 1^+} [k \cdot x^2] = k(1)^2 = k \qquad (50)$$
$$\lim_{x \to 1^-} [7x - 2] = 7 \cdot 1 - 2 = 5$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) \rightarrow k = 5$$
c) $f(1) = \lim_{x \to 1} f(x)$

Para que $f(x)$ seja contínua, o valor de k é 5

Exercício V

$$a=-3$$
$$f(x)=\begin{cases} k\cdot x^2 & x\geq -3 \\ 2x+k & x<-3 \end{cases}$$

$$\text{a) } f(a)=f(-3)=k(-3)^2=9k=9\left(-\frac{3}{4}\right)=-\frac{27}{4}$$
$$\text{b) } \lim_{x\to a} f(x)=\lim_{x\to -3} f(x)=-\frac{27}{4}$$
$$\lim_{x\to -3^+}[k\cdot x^2]=k(-3)^2=9k \tag{51}$$
$$\lim_{x\to -3^-}[2x+k]=2(-3)+k=-6+k$$
$$\lim_{x\to -3^+} f(x)=\lim_{x\to -3^-} f(x)\to 9k=-6+k\to 9k-k=-6\to 8k=-6\to k=\frac{-6}{8}=-\frac{3}{4}$$
$$\text{c) } f(-3)=\lim_{x\to -3} f(x)$$

Para que $f(x)$ seja contínua, o valor de k é $\left(-\frac{27}{8}\right)$

Exercício VI

$$a=3$$
$$f(x)=\begin{cases} x^2+k\cdot x+2 & x\neq 3 \\ 3 & x=3 \end{cases}$$

$$\text{a) } f(a)=f(3)=3$$
$$\text{b) } \lim_{x\to a} f(x)=\lim_{x\to 3} f(x)=\lim_{x\to 3}[x^2+k\cdot x+2]=3^2+3k+2=3k+11 \tag{52}$$
$$\text{c) } f(3)=\lim_{x\to 3} f(x)\to 3=3k+11\to 3k=3-11\to k=-\frac{8}{3}$$

Para que $f(x)$ seja contínua, o valor de k é $\left(-\frac{8}{3}\right)$

Exercício VII

$$a=0$$
$$f(x)=\begin{cases} e^{2x} & x\neq 0 \\ k^3-7 & x=0 \end{cases}$$

$$\text{a) } f(a)=f(0)=k^3-7$$
$$\text{b) } \lim_{x\to a} f(x)=\lim_{x\to 0} f(x)=\lim_{x\to 0}[e^{2x}]=e^{2(0)}=e^0=1 \tag{53}$$
$$\text{c) } f(0)=\lim_{x\to 0} f(x)\to k^3-7=1\to k^3=1+7\to k=\sqrt[3]{8}=2$$

Para que $f(x)$ seja contínua, o valor de k é 2

Limites fundamentais – Aula 19

$$\lim_{x \to 0}\left[\frac{sen(x)}{x}\right]=1$$

Exercício I

$$\lim_{x \to 0}\left[\frac{sen(2x)}{x}\right]=\lim_{x \to 0}\left[2\left(\frac{sen(2x)}{2x}\right)\right]=2\cdot\lim_{x \to 0}\left[\frac{sen(2x)}{2x}\right]=2\cdot 1=2 \tag{54}$$

Exercício II

$$\lim_{x \to 0}\left[\frac{sen(3x)}{sen(4x)}\right]=\lim_{x \to 0}\left[\frac{\frac{sen(3x)}{x}}{\frac{sen(4x)}{x}}\right]=\lim_{x \to 0}\left[\frac{3\left(\frac{sen(3x)}{3x}\right)}{4\left(\frac{sen(4x)}{4x}\right)}\right]=\frac{3\cdot\lim\limits_{x \to 0}\left[\frac{sen(3x)}{3x}\right]}{4\cdot\lim\limits_{x \to 0}\left[\frac{sen(4x)}{4x}\right]}=\frac{3\cdot 1}{4\cdot 1}=\frac{3}{4} \tag{55}$$

Exercício III

$$\begin{gathered}\lim_{x \to 0}\left[\frac{tg(x)}{x}\right]=\lim_{x \to 0}\left[tg(x)\cdot\frac{1}{x}\right]=\lim_{x \to 0}\left[\frac{sen(x)}{\cos(x)}\cdot\frac{1}{x}\right]=\lim_{x \to 0}\left[\frac{1}{\cos(x)}\cdot\frac{sen(x)}{x}\right]=\\ \lim_{x \to 0}\left[\frac{1}{\cos(x)}\right]\cdot\lim_{x \to 0}\left[\frac{sen(x)}{x}\right]=\frac{1}{\cos(0)}\cdot 1=\frac{1}{1}=1\end{gathered} \tag{56}$$

Exercício IV

$$\lim_{x \to 0}\left[\frac{sen(9x)}{x}\right]=\lim_{x \to 0}\left[9\cdot\left(\frac{sen(9x)}{9x}\right)\right]=\lim_{x \to 0}[9\cdot 1]=9 \tag{57}$$

Exercício V

$$\begin{gathered}\lim_{x \to 0}\left[\frac{6x-sen(2x)}{2x+3\cdot sen(4x)}\right]=\lim_{x \to 0}\left[\frac{\frac{6x-sen(2x)}{x}}{\frac{2x+3\cdot sen(4x)}{x}}\right]=\lim_{x \to 0}\left[\frac{\frac{6x}{x}-\frac{sen(2x)}{x}}{\frac{2x}{x}+\frac{3\cdot sen(4x)}{x}}\right]=\\ \lim_{x \to 0}\left[\frac{6-2\cdot\left(\frac{sen(2x)}{2x}\right)}{2+4\cdot 3\cdot\left(\frac{sen(4x)}{4x}\right)}\right]=\frac{6-2\cdot\lim\limits_{x \to 0}\left[\frac{sen(2x)}{2x}\right]}{2+12\cdot\lim\limits_{x \to 0}\left[\frac{sen(4x)}{4x}\right]}=\frac{6-2\cdot 1}{2+12\cdot 1}=\frac{4}{14}=\frac{2}{7}\end{gathered} \tag{58}$$