Integral Indefinida – Aula 1

01. $\int \partial x = x + c$

02. $\int x^p \partial x = \frac{x^{p+1}}{p+1} + c \rightarrow p \neq -1$

03. $\int e^x \partial x = e^x + c$

04. $\int \frac{\partial x}{x} = \ln x + c$

05. $\int f^p \partial f = \frac{f^{p+1}}{p+1} + c \rightarrow p \neq -1$

06. $\int e^f \partial f = e^f + c$

07. $\int \frac{\partial f}{f} = \ln f + c$

Exercício I

$$\int \partial x = x + c$$
$$\int x^3 \partial x = \frac{x^4}{4} + c \quad (1)$$

Exercício II

$$\int 3x^4 \partial x = 3\int x^4 \partial x = 3\frac{x^5}{5} + c = \frac{3x^5}{5} + c \quad (2)$$

Exercício III

$$\int (4x^5 + 7)\partial x = \int 4x^5 \partial x + \int 7 \partial x = 4\int x^5 \partial x + 7\int \partial x = 4\frac{x^6}{6} + 7x + c = \frac{2x^6}{3} + 7x + c \quad (3)$$

Exercício IV

$$\int 3\partial x = 3\int \partial x = 3x + c \quad (4)$$

Exercício V

$$\int 5x^7 \partial x = 5\int x^7 \partial x = \frac{5x^8}{8} + c \quad (5)$$

Exercício VI

$$\int (5 + 3x^2 - 7x^3)\partial x = 5\int \partial x + 3\int x^2 \partial x - 7\int x^3 \partial x = 5x + \frac{3x^3}{3} - \frac{7x^4}{4} + c =$$
$$\frac{-7x^4}{4} + x^3 + 5x + c \quad (6)$$

Integral Indefinida – Aula 2

Exercício I

$$\int \frac{\partial x}{x^4} = \int x^{-4} \partial x = \frac{x^{-3}}{-3} + c = \frac{-1}{3x^3} + c \tag{7}$$

Exercício II

$$\int \sqrt{x^3} \partial x = \int x^{\frac{3}{2}} \partial x = \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + c = \sqrt{x^5} \cdot \frac{2}{5} = \frac{2\sqrt{x^5}}{5} + c \tag{8}$$

Exercício II

$$\int \left(7\sqrt[5]{x^2} + \frac{3}{x^3}\right) \partial x = \int \left(7x^{\frac{2}{5}} + 3x^{-3}\right) \partial x = 7\int x^{\frac{2}{5}} \partial x + 3\int x^{-3} \partial x = 7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)} + 3\frac{x^{-2}}{-2} + c = \\ 5\sqrt[5]{x^7} - \frac{3}{2x^2} + c \tag{9}$$

Integral indefinida – Aula 3

Exercício I

$$\int \left(3x^{-4} - 3x + 4\right) \partial x = 3\int x^{-4} \partial x - 3\int x \partial x + 4\int \partial x = 3\frac{x^{-3}}{-3} - 3\frac{x^2}{2} + 4x + c = \\ \frac{-1}{x^3} - \frac{3x^2}{2} + 4x + c \tag{10}$$

Exercício II

$$\int \frac{2}{3}\sqrt{x} \partial x = \frac{2}{3}\int x^{\frac{1}{2}} \partial x = \frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{4\sqrt{x^3}}{9} + c \tag{11}$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}\, x^3}{\sqrt[3]{x^2}} \partial x = \int \frac{x^{\frac{1}{2}} x^3}{x^{\frac{2}{3}}} \partial x = \int x^{\frac{1}{2}+3-\frac{2}{3}} \partial x = \int x^{\frac{3+18-4}{6}} \partial x = \int x^{\frac{17}{6}} \partial x = \frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)} + c = \frac{6\sqrt[6]{x^{23}}}{23} + c \tag{12}$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3} = \int x^{-3} \partial x = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + c \tag{13}$$

Exercício II

$$\int \frac{\partial x}{x} = \ln x + c \tag{14}$$

Exercício III

$$\int \sqrt{2x+1}\,\partial x = \frac{1}{2}\int (2x+1)^{\frac{1}{2}} \cdot 2\,\partial x = \frac{1}{2}\int (2x+1)^{\frac{1}{2}}\partial x = \frac{1}{2}\frac{(2x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{1}{2}\frac{2}{3}\sqrt{(2x+1)^3} + c =$$
$$\frac{\sqrt{(2x+1)^3}}{3} + c \tag{15}$$
$$\frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}} + c\right)}{\partial x} = \frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1} \cdot 2 + 0 = (2x+1)^{\frac{1}{2}} = \sqrt{2x+1}$$

Exercício IV

$$\int \frac{5x^3+2x+3}{x}\partial x = \int \frac{5x^3}{x} + \frac{2x}{x} + \frac{3}{x}\partial x = \int 5x^2 + 2 + \frac{3}{x}\partial x =$$
$$5\int x^2\partial x + 2\int \partial x + 3\int \frac{\partial x}{x} = 5\frac{x^3}{3} + 2x + 3\ln x + c = \frac{5x^3}{3} + 2x + 3\ln x + c \tag{16}$$
$$\frac{\partial\left(\frac{5x^3}{3} + 2x + 3\ln x + c\right)}{\partial x} = \frac{5}{3}3x^2 + 2 + 3\frac{1}{x} + 0 = 5x^2 + 2 + \frac{3}{x} = \frac{5x^3+2x+3}{x}$$

Exercício V

$$\int \left(2x^4 + 3 + 5e^x + \frac{7}{x}\right)\partial x = 2\int x^4\partial x + 3\int \partial x + 5\int e^x\partial x + 7\int \frac{\partial x}{x} =$$
$$2\frac{x^5}{5} + 3x + 5e^x + 7\ln x + c = \frac{2x^5}{5} + 3x + 5e^x + 7\ln x + c \tag{17}$$
$$\frac{\partial\left(\frac{2x^5}{5} + 3x + 5e^x + 7\ln x + c\right)}{\partial x} = \frac{2}{5}5x^4 + 3 + 5e^x + 7\frac{1}{x} + 0 = 2x^4 + 3 + 5e^x + \frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int \frac{5t^2+7}{\sqrt[3]{t^4}}\partial t=\int \frac{5t^2+7}{t^{\frac{4}{3}}}\partial t=\int t^{\frac{-4}{3}}\left(5t^2+7\right)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\partial t=$$
$$5\int t^{\frac{2}{3}}\partial t+7\int t^{\frac{-4}{3}}\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot 3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c$$
$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$
$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}} \tag{18}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int \frac{\partial x}{\sqrt{x}}=\int \frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c \tag{19}$$

Exercício II

$$\int\left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int \frac{\partial x}{x}=3e^x+2\ln x+c \tag{20}$$

Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \frac{1}{4}\int (2x^2+1)^3 4x \partial x = \frac{1}{4}\int (2x^2+1)^3 \partial x = \frac{1}{4}\frac{(2x^2+1)^4}{4} + c = \frac{(2x^2+1)^4}{16} + c$$

$$\frac{(2x^2+1)^4}{2^4} + c = \left(\frac{2x^2+1}{2}\right)^4 + c = \left(x^2+\frac{1}{2}\right)^4 + c$$

$$\frac{\partial\left[\left(x^2+\frac{1}{2}\right)^4+c\right]}{\partial x} = 4\left(x^2+\frac{1}{2}\right)^3 \cdot 2x + 0 = 8x\left(x^2+\frac{1}{2}\right)^3 = 8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2 = \tag{21}$$

$$(8x^3+4x)\left(x^4+x^2+\frac{1}{4}\right) = 8x^7+8x^5+2x^3+4x^5+4x^3+x = 8x^7+12x^5+6x^3+x$$

$$(2x^2+1)^3 x = (2x^2+1)^2(2x^2+1)x = (4x^4+4x^2+1)(2x^3+x) = 8x^7+4x^5+8x^5+4x^3+2x^3+x =$$

$$8x^7+12x^5+6x^3+x$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x \partial x = 3\int e^x \partial x = 3e^x + c$$

$$\int e^{x^2+1} x \partial x = \frac{1}{2}\int e^{x^2+1} 2x \partial x = \frac{1}{2}\int e^{x^2+1} \partial x = \frac{1}{2}e^{x^2+1} + c = \frac{e^{x^2+1}}{2} + c \tag{22}$$

$$\frac{\partial\left(\frac{e^{x^2+1}}{2}+c\right)}{\partial x} = \frac{1}{2}e^{x^2+1}2x + 0 = e^{x^2+1}x$$

Exercício II

$$\int e^{x^4+1} x^3 \partial x = \frac{1}{4}\int e^{x^4+1} 4x^3 \partial x = \frac{1}{4}\int e^{x^4+1} \partial x = \frac{1}{4}e^{x^4+1} + c = \frac{e^{x^4+1}}{4} + c \tag{23}$$

$$\frac{\partial\left(\frac{e^{x^4+1}}{4}+c\right)}{\partial x} = \frac{1}{4}e^{x^4+1}4x^3 + 0 = e^{x^4+1}x^3$$

Exercício III

$$\int \frac{x}{(2x^2-1)^3}\partial x=\int (2x^2-1)^{-3}x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-3}4x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-3}\partial x=$$
$$\frac{1}{4}\frac{(2x^2-1)^{-2}}{-2}+c=\frac{-1}{8(2x^2-1)^2}+c$$
$$\frac{\partial\left(\frac{-1}{8(2x^2-1)^2}+c\right)}{\partial x}=\frac{\partial\left(\frac{-(2x^2-1)^{-2}}{8}+c\right)}{\partial x}=\frac{-1}{8}(-2)(2x^2-1)^{-3}4x+0=(2x^2-1)^{-3}x=$$
$$\frac{x}{(2x^2-1)^3} \tag{24}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\partial x=\int (2x^2-1)^{-1}x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-1}4x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-1}\partial x=$$
$$\frac{1}{4}\ln(2x^2-1)+c=\frac{\ln(2x^2-1)}{4}+c$$
$$\frac{\partial\left(\frac{\ln(2x^2-1)}{4}+c\right)}{\partial x}=\frac{1}{4}\frac{1}{2x^2-1}4x+0=\frac{x}{2x^2-1} \tag{25}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$
\begin{gathered}
\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u-1)^2\sqrt{u}\,\partial u=\int (u-1)^2 u^{\frac{1}{2}}\,\partial u=\int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u=\\
\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u=\int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u=\int u^{\frac{5}{2}}\,\partial u-2\int u^{\frac{3}{2}}\,\partial u+\int u^{\frac{1}{2}}\,\partial u=\\
\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)}-2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^7}}{7}-\frac{4\sqrt{u^5}}{5}+\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\\
u=1+x\rightarrow x=u-1\rightarrow \frac{\partial x}{\partial u}=1\rightarrow \partial x=\partial u\\
\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{3}{2}}}{3}+c\right)}{\partial x}=\\
\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}}-\frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}}+\frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}}+0=(1+x)^{\frac{5}{2}}-2(1+x)^{\frac{3}{2}}+(1+x)^{\frac{1}{2}}=\\
(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}}-2(1+x)^{\frac{3}{2}-\frac{1}{2}}+1\right)=(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}}-2(1+x)^{\frac{2}{2}}+1\right)=\\
(1+x)^{\frac{1}{2}}\left((1+x)^2-2(1+x)+1\right)=(1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1)=(1+x)^{\frac{1}{2}}x^2=x^2\sqrt{1+x}
\end{gathered}
\tag{26}
$$

Exercício II

$$
\begin{gathered}
\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u^2-1)^2 u\,2u\,\partial u=2\int (u^2-1)^2 u^2\,\partial u=2\int (u^4-2u^2+1)u^2\,\partial u=\\
2\int (u^6-2u^4+u^2)\partial u=2\int u^6\,\partial u-4\int u^4\,\partial u+2\int u^2\,\partial u=2\frac{u^7}{7}-4\frac{u^5}{5}+2\frac{u^3}{3}+c=\\
\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\\
u=\sqrt{1+x}\rightarrow u^2=1+x\rightarrow x=u^2-1\rightarrow \frac{\partial x}{\partial u}=2u\rightarrow \partial x=2u\,\partial u
\end{gathered}
\tag{27}
$$