Limites de Funções de duas Variáveis – Aula 1

Exercício I

$$\lim_{(x,y)\to(-1,2)}\left[\frac{xy}{x^2+y^2}\right]=\lim_{\substack{x\to-1\\y\to2}}\left[\frac{xy}{x^2+y^2}\right]=\frac{(-1)2}{(-1)^2+2^2}=\frac{-2}{1+4}=-\frac{2}{5} \tag{1}$$

Exercício II

$$\lim_{(x,y)\to(0,0)}\left[\frac{5}{x^2+3y^2}\right]=\frac{5}{(0)^2+3(0)^2}=\frac{5}{0^+}=+\infty \tag{2}$$

Exercício III

$$\lim_{(x,y)\to(1,-2)}\left[\frac{4x^2-y^2}{2x+y}\right]=\frac{4(1)^2-(-2)^2}{2(1)+(-2)}=\frac{4-4}{2-2}=\frac{0}{0}$$
$$\frac{(2x)^2-y^2}{2x+y}=\frac{(2x-y)(2x+y)}{2x+y}=2x-y \tag{3}$$
$$\lim_{(x,y)\to(1,-2)}[2x-y]=2\cdot1-(-2)=2+2=4$$

Exercício IV

$$\lim_{\substack{x\to0\\y\to0}}\left[\frac{2xy}{x^2+y^2}\right]=\frac{2(0)(0)}{(0)^2+(0)^2}=\frac{0}{0}$$
$$\lim_{\substack{x\to0\\y=0}}\left[\frac{2x(0)}{x^2+(0)^2}\right]=\lim_{x\to0}\left[\frac{0}{x^2}\right]=\lim_{x\to0}[0]=0$$
$$\lim_{\substack{x=0\\y\to0}}\left[\frac{2(0)y}{(0)^2+y^2}\right]=\lim_{y\to0}\left[\frac{0}{y^2}\right]=\lim_{y\to0}[0]=0$$
$$\lim_{\substack{x\to0\\y=x}}\left[\frac{2x(x)}{x^2+(x)^2}\right]=\lim_{x\to0}\left[\frac{2x^2}{2x^2}\right]=\lim_{x\to0}[1]=1 \tag{4}$$
$$\lim_{\substack{x\to0\\y=x^2}}\left[\frac{2xy}{x^2+y^2}\right]=\lim_{x\to0}\left[\frac{2x(x^2)}{x^2+(x^2)^2}\right]=\lim_{x\to0}\left[\frac{2x^3}{x^2+x^4}\right]=\lim_{x\to0}\left[\frac{2x^3}{x^2(1+x^2)}\right]=\lim_{x\to0}\left[\frac{2x}{1+x^2}\right]=\frac{2(0)}{1+(0)^2}=0$$
$$\lim_{\substack{x\to0\\y\to0}}\left[\frac{2xy}{x^2+y^2}\right]=\nexists$$