Integral Indefinida – Aula 1

01. $\int \partial x = x + c$

02. $\int x^p \partial x = \frac{x^{p+1}}{p+1} + c \rightarrow p \neq -1$

03. $\int e^x \partial x = e^x + c$

04. $\int \frac{\partial x}{x} = \ln x + c$

05. $\int u^p \partial u = \frac{u^{p+1}}{p+1} + c \rightarrow p \neq -1$

06. $\int e^u \partial u = e^u + c$

07. $\int \frac{\partial u}{u} = \ln u + c$

Exercício I

$$\int \partial x = x + c$$
$$\int x^3 \partial x = \frac{x^4}{4} + c \tag{1}$$

Exercício II

$$\int 3x^4 \partial x = 3\int x^4 \partial x = 3\frac{x^5}{5} + c = \frac{3x^5}{5} + c \tag{2}$$

Exercício III

$$\int (4x^5 + 7)\partial x = \int 4x^5 \partial x + \int 7 \partial x = 4\int x^5 \partial x + 7\int \partial x = 4\frac{x^6}{6} + 7x + c = \frac{2x^6}{3} + 7x + c \tag{3}$$

Exercício IV

$$\int 3 \partial x = 3\int \partial x = 3x + c \tag{4}$$

Exercício V

$$\int 5x^7 \partial x = 5\int x^7 \partial x = \frac{5x^8}{8} + c \tag{5}$$

Exercício VI

$$\int (5 + 3x^2 - 7x^3)\partial x = 5\int \partial x + 3\int x^2 \partial x - 7\int x^3 \partial x = 5x + \frac{3x^3}{3} - \frac{7x^4}{4} + c =$$
$$\frac{-7x^4}{4} + x^3 + 5x + c \tag{6}$$

Integral Indefinida – Aula 2

Exercício I

$$\int \frac{\partial x}{x^4} = \int x^{-4}\partial x = \frac{x^{-3}}{-3} + c = \frac{-1}{3x^3} + c \tag{7}$$

Exercício II

$$\int \sqrt{x^3}\partial x = \int x^{\frac{3}{2}}\partial x = \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + c = \sqrt{x^5}\cdot\frac{2}{5} = \frac{2\sqrt{x^5}}{5} + c \tag{8}$$

Exercício II

$$\int \left(7\sqrt[5]{x^2} + \frac{3}{x^3}\right)\partial x = \int \left(7x^{\frac{2}{5}} + 3x^{-3}\right)\partial x = 7\int x^{\frac{2}{5}}\partial x + 3\int x^{-3}\partial x = 7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)} + 3\frac{x^{-2}}{-2} + c = \\ 5\sqrt[5]{x^7} - \frac{3}{2x^2} + c \tag{9}$$

Integral indefinida – Aula 3

Exercício I

$$\int \left(3x^{-4} - 3x + 4\right)\partial x = 3\int x^{-4}\partial x - 3\int x\partial x + 4\int \partial x = 3\frac{x^{-3}}{-3} - 3\frac{x^2}{2} + 4x + c = \\ \frac{-1}{x^3} - \frac{3x^2}{2} + 4x + c \tag{10}$$

Exercício II

$$\int \frac{2}{3}\sqrt{x}\partial x = \frac{2}{3}\int x^{\frac{1}{2}}\partial x = \frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{4\sqrt{x^3}}{9} + c \tag{11}$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}x^3}{\sqrt[3]{x^2}}\partial x = \int \frac{x^{\frac{1}{2}}x^3}{x^{\frac{2}{3}}}\partial x = \int x^{\frac{1}{2}+3-\frac{2}{3}}\partial x = \int x^{\frac{3+18-4}{6}}\partial x = \int x^{\frac{17}{6}}\partial x = \frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)} + c = \frac{6\sqrt[6]{x^{23}}}{23} + c \tag{12}$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3} = \int x^{-3}\partial x = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + c \tag{13}$$

Exercício II

$$\int \frac{\partial x}{x} = \ln x + c \tag{14}$$

Exercício III

$$\int \sqrt{2x+1}\,\partial x = \frac{1}{2}\int (2x+1)^{\frac{1}{2}} \cdot 2\,\partial x = \frac{1}{2}\int (2x+1)^{\frac{1}{2}}\partial x = \frac{1}{2}\frac{(2x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{1}{2}\frac{2}{3}\sqrt{(2x+1)^3} + c = \frac{\sqrt{(2x+1)^3}}{3} + c \tag{15}$$

$$\frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}} + c\right)}{\partial x} = \frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1} \cdot 2 + 0 = (2x+1)^{\frac{1}{2}} = \sqrt{2x+1}$$

Exercício IV

$$\int \frac{5x^3+2x+3}{x}\partial x = \int \frac{5x^3}{x} + \frac{2x}{x} + \frac{3}{x}\partial x = \int 5x^2 + 2 + \frac{3}{x}\partial x =$$
$$5\int x^2\partial x + 2\int \partial x + 3\int \frac{\partial x}{x} = 5\frac{x^3}{3} + 2x + 3\ln x + c = \frac{5x^3}{3} + 2x + 3\ln x + c \tag{16}$$

$$\frac{\partial\left(\frac{5x^3}{3} + 2x + 3\ln x + c\right)}{\partial x} = \frac{5}{3}3x^2 + 2 + 3\frac{1}{x} + 0 = 5x^2 + 2 + \frac{3}{x} = \frac{5x^3+2x+3}{x}$$

Exercício V

$$\int \left(2x^4 + 3 + 5e^x + \frac{7}{x}\right)\partial x = 2\int x^4\partial x + 3\int \partial x + 5\int e^x\partial x + 7\int \frac{\partial x}{x} =$$
$$2\frac{x^5}{5} + 3x + 5e^x + 7\ln x + c = \frac{2x^5}{5} + 3x + 5e^x + 7\ln x + c \tag{17}$$

$$\frac{\partial\left(\frac{2x^5}{5} + 3x + 5e^x + 7\ln x + c\right)}{\partial x} = \frac{2}{5}5x^4 + 3 + 5e^x + 7\frac{1}{x} + 0 = 2x^4 + 3 + 5e^x + \frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int \frac{5t^2+7}{\sqrt[3]{t^4}}\partial t=\int \frac{5t^2+7}{t^{\frac{4}{3}}}\partial t=\int t^{\frac{-4}{3}}\left(5t^2+7\right)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\partial t=$$
$$5\int t^{\frac{2}{3}}\partial t+7\int t^{\frac{-4}{3}}\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot 3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c \tag{18}$$
$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$
$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int \frac{\partial x}{\sqrt{x}}=\int \frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c \tag{19}$$

Exercício II

$$\int \left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int \frac{\partial x}{x}=3e^x+2\ln x+c \tag{20}$$

Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \frac{1}{4}\int (2x^2+1)^3 4x \partial x = \frac{1}{4}\int (2x^2+1)^3 \partial x = \frac{1}{4}\frac{(2x^2+1)^4}{4} + c = \frac{(2x^2+1)^4}{16} + c$$

$$\frac{(2x^2+1)^4}{2^4} + c = \left(\frac{2x^2+1}{2}\right)^4 + c = \left(x^2+\frac{1}{2}\right)^4 + c$$

$$\frac{\partial\left[\left(x^2+\frac{1}{2}\right)^4 + c\right]}{\partial x} = 4\left(x^2+\frac{1}{2}\right)^3 \cdot 2x + 0 = 8x\left(x^2+\frac{1}{2}\right)^3 = 8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2 =$$

$$(8x^3+4x)\left(x^4+x^2+\frac{1}{4}\right) = 8x^7+8x^5+2x^3+4x^5+4x^3+x = 8x^7+12x^5+6x^3+x$$

$$(2x^2+1)^3 x = (2x^2+1)^2(2x^2+1)x = (4x^4+4x^2+1)(2x^3+x) = 8x^7+4x^5+8x^5+4x^3+2x^3+x =$$

$$8x^7+12x^5+6x^3+x \tag{21}$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x \partial x = 3\int e^x \partial x = 3e^x + c$$

$$\int e^{x^2+1} x \partial x = \frac{1}{2}\int e^{x^2+1} 2x \partial x = \frac{1}{2}\int e^{x^2+1} \partial x = \frac{1}{2}e^{x^2+1} + c = \frac{e^{x^2+1}}{2} + c \tag{22}$$

$$\frac{\partial\left(\frac{e^{x^2+1}}{2} + c\right)}{\partial x} = \frac{1}{2}e^{x^2+1} 2x + 0 = e^{x^2+1} x$$

Exercício II

$$\int e^{x^4+1} x^3 \partial x = \frac{1}{4}\int e^{x^4+1} 4x^3 \partial x = \frac{1}{4}\int e^{x^4+1} \partial x = \frac{1}{4}e^{x^4+1} + c = \frac{e^{x^4+1}}{4} + c \tag{23}$$

$$\frac{\partial\left(\frac{e^{x^4+1}}{4} + c\right)}{\partial x} = \frac{1}{4}e^{x^4+1} 4x^3 + 0 = e^{x^4+1} x^3$$

Exercício III

$$\int \frac{x}{\left(2x^2-1\right)^3}\partial x=\int \left(2x^2-1\right)^{-3} x\,\partial x=\frac{1}{4}\int \left(2x^2-1\right)^{-3} 4x\,\partial x=\frac{1}{4}\int \left(2x^2-1\right)^{-3}\partial x=$$
$$\frac{1}{4}\frac{\left(2x^2-1\right)^{-2}}{-2}+c=\frac{-1}{8\left(2x^2-1\right)^2}+c$$
$$\frac{\partial\left(\frac{-1}{8\left(2x^2-1\right)^2}+c\right)}{\partial x}=\frac{\partial\left(\frac{-\left(2x^2-1\right)^{-2}}{8}+c\right)}{\partial x}=\frac{-1}{8}(-2)\left(2x^2-1\right)^{-3}4x+0=\left(2x^2-1\right)^{-3}x=$$
$$\frac{x}{\left(2x^2-1\right)^3} \tag{24}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\partial x=\int (2x^2-1)^{-1}x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-1}4x\,\partial x=\frac{1}{4}\int (2x^2-1)^{-1}\partial x=$$
$$\frac{1}{4}\ln(2x^2-1)+c=\frac{\ln(2x^2-1)}{4}+c$$
$$\frac{\partial\left(\frac{\ln(2x^2-1)}{4}+c\right)}{\partial x}=\frac{1}{4}\frac{1}{2x^2-1}4x+0=\frac{x}{2x^2-1} \tag{25}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$\int x^2\sqrt{1+x}\,\partial x \to \int (u-1)^2\sqrt{u}\,\partial u = \int (u-1)^2 u^{\frac{1}{2}}\,\partial u = \int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u =$$
$$\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u = \int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u = \int u^{\frac{5}{2}}\,\partial u - 2\int u^{\frac{3}{2}}\,\partial u + \int u^{\frac{1}{2}}\,\partial u =$$
$$\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - 2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2\sqrt{u^7}}{7} - \frac{4\sqrt{u^5}}{5} + \frac{2\sqrt{u^3}}{3} + c = \frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c$$
$$u=1+x \to x=u-1 \to \frac{\partial x}{\partial u}=1 \to \partial x=\partial u \tag{26}$$
$$\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c\right)}{\partial x} = \frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7} - \frac{4(1+x)^{\frac{5}{2}}}{5} + \frac{2(1+x)^{\frac{3}{2}}}{3} + c\right)}{\partial x} =$$
$$\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}} - \frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}} + \frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}} + 0 = (1+x)^{\frac{5}{2}} - 2(1+x)^{\frac{3}{2}} + (1+x)^{\frac{1}{2}} =$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}} - 2(1+x)^{\frac{3}{2}-\frac{1}{2}} + 1\right) = (1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}} - 2(1+x)^{\frac{2}{2}} + 1\right) =$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^2 - 2(1+x) + 1\right) = (1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1) = (1+x)^{\frac{1}{2}}x^2 = x^2\sqrt{1+x}$$

Exercício II

$$\int x^2\sqrt{1+x}\,\partial x \to \int (u^2-1)^2 u\, 2u\,\partial u = 2\int (u^2-1)^2 u^2\,\partial u = 2\int (u^4-2u^2+1)u^2\,\partial u =$$
$$2\int (u^6-2u^4+u^2)\,\partial u = 2\int u^6\,\partial u - 4\int u^4\,\partial u + 2\int u^2\,\partial u = 2\frac{u^7}{7} - 4\frac{u^5}{5} + 2\frac{u^3}{3} + c =$$
$$\frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c \tag{27}$$
$$u=\sqrt{1+x} \to u^2=1+x \to x=u^2-1 \to \frac{\partial x}{\partial u}=2u \to \partial x=2u\,\partial u$$

O que é uma Integral Definida – Aula 10

Exercício I

$$\int_1^2 x^3\,\partial x = \left.\frac{x^4}{4}\right]_1^2 = \frac{(2)^4}{4} - \frac{(1)^4}{4} = \frac{16}{4} - \frac{1}{4} = \frac{16-1}{4} = \frac{15}{4} = 3{,}75 \tag{28}$$

Integral Definida – Aula 10a

Exercício I

$$\int_0^2 (6x^2-4x+5)\partial x = 6\int_0^2 x^2\partial x - 4\int_0^2 x\partial x + 5\int_0^2 \partial x = 6\frac{x^3}{3} - 4\frac{x^2}{2} + 5x\Big]_0^2 = 2x^3-2x^2+5x\Big]_0^2 = \\ x(2x^2-2x+5)\Big]_0^2 = [(2)(2(2)^2-2(2)+5)] - [(0)(2(0)^2-2(0)+5)] = 2(8-4+5) = 2\cdot 9 = 18 \quad (29)$$

Exercício II

$$\int_{-1}^0 (2x-e^x)\partial x = 2\int_{-1}^0 x\partial x - \int_{-1}^0 e^x\partial x = 2\frac{x^2}{2} - e^x\Big]_{-1}^0 = x^2-e^x\Big]_{-1}^0 = ((0)^2-e^{(0)}) - ((-1)^2-e^{(-1)}) = \\ -1-\left(1-\frac{1}{e}\right) = -1-1+\frac{1}{e} = -2+\frac{1}{e} \quad (30)$$

Integral definida – Aula 11

Exercício I

$$\frac{5\pi}{4}\int_0^2 \frac{r\partial r}{1+r^2} = \frac{5\pi}{4}\int_0^2 (1+r^2)^{-1} r\partial r = \frac{5\pi}{4}\frac{1}{2}\int_0^2 (1+r^2)^{-1} 2r\partial r = \frac{5\pi}{8}\int_0^2 (1+r^2)^{-1}\partial r = \\ \frac{5\pi}{8}\ln(1+r^2)\Big]_0^2 = \left[\frac{5\pi}{8}\ln(1+(2)^2)\right] - \left[\frac{5\pi}{8}\ln(1+(0)^2)\right] = \frac{5\pi}{8}\ln(5) - \frac{5\pi}{8}\ln(1) = \\ \frac{5\pi}{8}(\ln(5)-\ln(1)) = \frac{5\pi}{8}(\ln(5)-0) = \frac{5\pi}{8}\ln(5) \\ \ln(1) = x \rightarrow e^x = 1 = e^0 \rightarrow x = 0 \\ \ln(5) = x \rightarrow e^x = 5 \quad (31)$$

Exercício II

$$2\pi\int_0^2 r^2\partial r = 2\pi\frac{r^3}{3}\Big]_0^2 = \frac{2\pi r^3}{3}\Big]_0^2 = \left(\frac{2\pi(2)^3}{3}\right) - \left(\frac{2\pi(0)^3}{3}\right) = \frac{16\pi}{3} \quad (32)$$

Exercício II

$$2\pi\int_0^{\sqrt{2}} (4r-2r^3)\partial r = 8\pi\int_0^{\sqrt{2}} r\partial r - 4\pi\int_0^{\sqrt{2}} r^3\partial r = 8\pi\frac{r^2}{2} - 4\pi\frac{r^4}{4}\Big]_0^{\sqrt{2}} = 4\pi r^2 - \pi r^4\Big]_0^{\sqrt{2}} = \\ \pi r^2(4-r^2)\Big]_0^{\sqrt{2}} = [\pi(\sqrt{2})^2(4-(\sqrt{2})^2)] - [\pi(0)^2(4-(0)^2)] = 2\pi(4-2) = 4\pi \quad (33)$$

Exercício III

$$\pi\int_0^2 x^2\partial x = \pi\frac{x^3}{3}\Big]_0^2 = \frac{\pi x^3}{3}\Big]_0^2 = \left(\frac{\pi(2)^3}{3}\right) - \left(\frac{\pi(0)^3}{3}\right) = \frac{8\pi}{3} \quad (34)$$

Exercício IV

$$\frac{\pi}{16}\int_{1}^{4} x^4 \partial x=\frac{\pi}{16}\frac{x^5}{5}\Big]_1^4=\frac{\pi x^5}{80}\Big]_1^4=\left(\frac{\pi(4)^5}{80}\right)-\left(\frac{\pi(1)^5}{80}\right)=\frac{4^5\pi}{80}-\frac{\pi}{80}=\frac{1024\pi-\pi}{80}=\frac{1023\pi}{80} \quad (35)$$

Exercício V

$$\pi\int_{1}^{2}(x^2)^2\partial x=\pi\int_{1}^{2}x^4\partial x=\pi\frac{x^5}{5}\Big]_1^2=\frac{\pi x^5}{5}\Big]_1^2=\left(\frac{\pi(2)^5}{5}\right)-\left(\frac{\pi(1)^5}{5}\right)=\frac{32\pi}{5}-\frac{\pi}{5}=\frac{32\pi-\pi}{5}=\frac{31\pi}{5} \quad (36)$$

Exercício VI

$$\begin{gathered}\pi\int_{-1}^{2}(-x^4-x^2+6x+8)\partial x=-\pi\int_{-1}^{2}x^4\partial x-\pi\int_{-1}^{2}x^2\partial x+6\pi\int_{-1}^{2}x\partial x+8\pi\int_{-1}^{2}\partial x=\\ -\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\Big]_{-1}^2=\frac{-\pi x^5}{5}-\frac{\pi x^3}{3}+3\pi x^2+8\pi x\Big]_{-1}^2=-\pi x\left(\frac{x^4}{5}+\frac{x^2}{3}-3x-8\right)\Big]_{-1}^2\\ \left[-\pi(2)\left(\frac{(2)^4}{5}+\frac{(2)^2}{3}-3(2)-8\right)\right]-\left[-\pi(-1)\left(\frac{(-1)^4}{5}+\frac{(-1)^2}{3}-3(-1)-8\right)\right]=\\ -2\pi\left(\frac{16}{5}+\frac{4}{3}-6-8\right)-\pi\left(\frac{1}{5}+\frac{1}{3}+3-8\right)=-2\pi\left(\frac{48+20-210}{15}\right)-\pi\left(\frac{3+5-75}{15}\right)=\\ 2\pi\left(\frac{142}{15}\right)+\pi\left(\frac{67}{15}\right)=\pi\left(\frac{284}{15}+\frac{67}{15}\right)=\frac{351\pi}{15}=\frac{3^3\cdot 13\pi}{3\cdot 5}=\frac{3^2\cdot 13\pi}{5}=\frac{117\pi}{5}\end{gathered} \quad (37)$$

Exercício VII

$$\begin{gathered}\pi\int_{0}^{8}(\sqrt[3]{y})^2\partial y=\pi\int_{0}^{8}\left(y^{\frac{1}{3}}\right)^2\partial y=\pi\int_{0}^{8}y^{\frac{2}{3}}\partial y=\pi\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\Bigg]_0^8=\frac{3\pi\sqrt[3]{y^5}}{5}\Big]_0^8=\\ \left(\frac{3\pi\sqrt[3]{(8)^5}}{5}\right)-\left(\frac{3\pi\sqrt[3]{(0)^5}}{5}\right)=\frac{3\pi\sqrt[3]{8^3\cdot 8^2}}{5}=\frac{3\pi 2^3\sqrt[3]{(2^3)^2}}{5}=\frac{3\pi 2^3 2^2}{5}=\frac{3\pi 2^5}{5}=\frac{96\pi}{5}\end{gathered} \quad (38)$$

Integral definida – Aula 12

Exercício I

$$\int_{1}^{2}2x\partial x=2\int_{1}^{2}x\partial x=2\frac{x^2}{2}\Big]_1^2=x^2\Big]_1^2=((2)^2)-((1)^2)=4-1=4-1=3 \quad (39)$$

Exercício II

$$\int_1^4 2\sqrt{x}\,\partial x=\int_1^4 2x^{\frac{1}{2}}\partial x=2\int_1^4 x^{\frac{1}{2}}\partial x=2\left.\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_1^4=\left.\frac{4\sqrt{x^3}}{3}\right]_1^4=\left(\frac{4\sqrt{(4)^3}}{3}\right)-\left(\frac{4\sqrt{(1)^3}}{3}\right)=$$
$$\frac{4\sqrt{4^2 2^2}}{3}-\frac{4}{3}=\frac{32}{3}-\frac{4}{3}=\frac{32-4}{3}=\frac{28}{3} \tag{40}$$

Exercício III

$$\int_1^2 4x^2\partial x=4\int_1^2 x^2\partial x=4\left.\frac{x^3}{3}\right]_1^2=\left.\frac{4}{3}x^3\right]_1^2=\frac{4}{3}\left(2^3-1^3\right)=\frac{4}{3}7=\frac{28}{3} \tag{41}$$

Integrais definidas e indefinidas – Aula 13

Exercício I

$$\int\left(\frac{3}{x^4}+\frac{2x^2}{3}-2x+7\right)\partial x=\int\left(3x^{-4}+\frac{2}{3}x^2-2x+7\right)\partial x=$$
$$3\int x^{-4}\partial x+\frac{2}{3}\int x^2\partial x-2\int x\,\partial x+7\int\partial x=3\frac{x^{-3}}{-3}+\frac{2}{3}\frac{x^3}{3}-2\frac{x^2}{2}+7x+c=$$
$$\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c$$
$$\frac{\partial\left(\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c\right)}{\partial x}=\frac{\partial\left(-x^{-3}+\frac{2}{9}x^3-x^2+7x+c\right)}{\partial x}=3x^{-4}+\frac{2}{9}3x^2-2x+7+0=$$
$$\frac{3}{x^4}+\frac{2x^2}{3}-2x+7 \tag{42}$$

Exercício II

$$\int 5\sqrt[3]{x^2}\,\partial x=\int 5x^{\frac{2}{3}}\partial x=5\int x^{\frac{2}{3}}\partial x=5\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+c=3\sqrt[3]{x^5}+c$$
$$\frac{\partial\left(3\sqrt[3]{x^5}+c\right)}{\partial x}=\frac{\partial\left(3x^{\frac{5}{3}}+c\right)}{\partial x}=3\frac{5}{3}x^{\frac{2}{3}}+0=5\sqrt[3]{x^2} \tag{43}$$

Exercício III

$$\int_2^4 2x^3\partial x=2\int_2^4 x^3\partial x=2\frac{x^4}{4}\Big]_2^4=\frac{1}{2}x^4\Big]_2^4=\frac{1}{2}\left(4^4-2^4\right)=\frac{1}{2}\left((2\cdot 2)^4-2^4\right)=\frac{1}{2}\left(2^4\cdot 2^4-2^4\right)=$$
$$\frac{2^4}{2}\left(2^4-1\right)=2^3(16-1)=8\cdot 15=120 \tag{44}$$

Exercício IV

$$\int_1^2\left(3x^2-2x\right)\partial x=3\int_1^2 x^2\partial x-2\int_1^2 x\,\partial x=3\frac{x^3}{3}-2\frac{x^2}{2}\Big]_1^2=x^3-x^2\Big]_1^2=x^2(x-1)\Big]_1^2=$$
$$\left[2^2(2-1)\right]-\left[1^2(1-1)\right]=4 \tag{45}$$

Integral definida pelo método da substituição – U du – Aula 14

Exercício I

$$\int_0^2\sqrt{2x^2+1}\,x\,\partial x=\frac{1}{4}\int_0^2\left(2x^2+1\right)^{\frac{1}{2}}4x\,\partial x=\frac{1}{4}\int_0^2\left(2x^2+1\right)^{\frac{1}{2}}\partial x=\frac{1}{4}\frac{\left(2x^2+1\right)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_0^2=$$
$$\frac{1}{4}\frac{2}{3}\sqrt{\left(2x^2+1\right)^3}\Big]_0^2=\frac{1}{6}\sqrt{\left(2x^2+1\right)^3}\Big]_0^2=\frac{1}{6}\left[\sqrt{\left(2\cdot 2^2+1\right)^3}-\sqrt{\left(2\cdot 0^2+1\right)^3}\right]=\frac{1}{6}\left(\sqrt{9^3}-\sqrt{1^3}\right)=$$
$$\frac{1}{6}\left(\sqrt{9^2 3^2}-1\right)=\frac{1}{6}(27-1)=\frac{1}{6}26=\frac{13}{3} \tag{46}$$
$$u=2x^2+1\rightarrow\frac{\partial u}{\partial x}=4x\rightarrow\partial u=4x\,\partial x$$