

## Integral Indefinida – [Aula 1](#)

- 01.  $\int \partial x = x + c$
- 02.  $\int x^p \partial x = \frac{x^{p+1}}{p+1} + c \rightarrow p \neq -1$
- 03.  $\int e^x \partial x = e^x + c$
- 04.  $\int \frac{\partial x}{x} = \ln|x| + c$
- 05.  $\int u^p \partial u = \frac{u^{p+1}}{p+1} + c \rightarrow p \neq -1$
- 06.  $\int e^u \partial u = e^u + c$
- 07.  $\int \frac{\partial u}{u} = \ln|u| + c$
- 08.  $\int a^u \partial u = \frac{a^u}{\ln|a|} + c$

Exercício I

$$\begin{aligned}\int \partial x &= x + c \\ \int x^3 \partial x &= \frac{x^4}{4} + c\end{aligned}\tag{1}$$

Exercício II

$$\int 3x^4 \partial x = 3 \int x^4 \partial x = 3 \frac{x^5}{5} + c = \frac{3x^5}{5} + c\tag{2}$$

Exercício III

$$\int (4x^5 + 7) \partial x = \int 4x^5 \partial x + \int 7 \partial x = 4 \int x^5 \partial x + 7 \int \partial x = 4 \frac{x^6}{6} + 7x + c = \frac{2x^6}{3} + 7x + c\tag{3}$$

Exercício IV

$$\int 3 \partial x = 3 \int \partial x = 3x + c\tag{4}$$

Exercício V

$$\int 5x^7 \partial x = 5 \int x^7 \partial x = \frac{5x^8}{8} + c\tag{5}$$

## Exercício VI

$$\int (5+3x^2-7x^3) \partial x = 5 \int \partial x + 3 \int x^2 \partial x - 7 \int x^3 \partial x = 5x + \frac{3x^3}{3} - \frac{7x^4}{4} + c = \frac{-7x^4}{4} + x^3 + 5x + c \quad (6)$$

## Integral Indefinida – [Aula 2](#)

### Exercício I

$$\int \frac{\partial x}{x^4} = \int x^{-4} \partial x = \frac{x^{-3}}{-3} + c = \frac{-1}{3x^3} + c \quad (7)$$

### Exercício II

$$\int \sqrt{x^3} \partial x = \int x^{\frac{3}{2}} \partial x = \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + c = \sqrt{x^5} \cdot \frac{2}{5} = \frac{2\sqrt{x^5}}{5} + c \quad (8)$$

### Exercício II

$$\int \left( 7\sqrt[5]{x^2} + \frac{3}{x^3} \right) \partial x = \int \left( 7x^{\frac{2}{5}} + 3x^{-3} \right) \partial x = 7 \int x^{\frac{2}{5}} \partial x + 3 \int x^{-3} \partial x = 7 \frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)} + 3 \frac{x^{-2}}{-2} + c = 5\sqrt[5]{x^7} - \frac{3}{2x^2} + c \quad (9)$$

## Integral indefinida – [Aula 3](#)

### Exercício I

$$\int (3x^{-4} - 3x + 4) \partial x = 3 \int x^{-4} \partial x - 3 \int x \partial x + 4 \int \partial x = 3 \frac{x^{-3}}{(-3)} - 3 \frac{x^2}{2} + 4x + c = \frac{-1}{x^3} - \frac{3x^2}{2} + 4x + c \quad (10)$$

### Exercício II

$$\int \frac{2}{3} \sqrt{x} \partial x = \frac{2}{3} \int x^{\frac{1}{2}} \partial x = \frac{2}{3} \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{4\sqrt{x^3}}{9} + c \quad (11)$$

## Integral de uma função Potência – [Aula 4](#)

### Exercício I

$$\int \frac{\sqrt{x} x^3}{\sqrt[3]{x^2}} \partial x = \int \frac{x^{\frac{1}{2}} x^3}{x^{\frac{2}{3}}} \partial x = \int x^{\frac{1}{2}+3-\frac{2}{3}} \partial x = \int x^{\frac{3+18-4}{6}} \partial x = \int x^{\frac{17}{6}} \partial x = \frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)} + c = \frac{6\sqrt[6]{x^{23}}}{23} + c \quad (12)$$

## Integral Indefinida – [Aula 5](#)

### Exercício I

$$\int \frac{\partial x}{x^3} = \int x^{-3} \partial x = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + c \quad (13)$$

### Exercício II

$$\int \frac{\partial x}{x} = \ln x + c \quad (14)$$

### Exercício III

$$\int \sqrt{2x+1} \partial x = \int \sqrt{u} \frac{\partial u}{2} = \frac{1}{2} \int u^{\frac{1}{2}} \partial u = \frac{1}{2} \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{1}{2} \frac{2}{3} \sqrt{u^3} + c = \frac{\sqrt{u^3}}{3} + c = \frac{\sqrt{(2x+1)^3}}{3} + c$$

$$u = 2x+1 \rightarrow \frac{\partial u}{2} = \partial x \quad (15)$$

$$\frac{\partial \left( \frac{\sqrt{(2x+1)^3}}{3} + c \right)}{\partial x} = \frac{\partial \left( \frac{1}{3} (2x+1)^{\frac{3}{2}} + c \right)}{\partial x} = \frac{1}{3} \frac{3}{2} (2x+1)^{\frac{3}{2}-1} \cdot 2 + 0 = (2x+1)^{\frac{1}{2}} = \sqrt{2x+1}$$

### Exercício IV

$$\int \frac{5x^3+2x+3}{x} \partial x = \int \left( \frac{5x^3}{x} + \frac{2x}{x} + \frac{3}{x} \right) \partial x = \int \left( 5x^2 + 2 + \frac{3}{x} \right) \partial x =$$

$$5 \int x^2 \partial x + 2 \int \partial x + 3 \int \frac{\partial x}{x} = 5 \frac{x^3}{3} + 2x + 3 \ln|x| + c = \frac{5x^3}{3} + 2x + 3 \ln|x| + c \quad (16)$$

$$\frac{\partial \left( \frac{5x^3}{3} + 2x + 3 \ln|x| + c \right)}{\partial x} = \frac{5}{3} 3x^2 + 2 + 3 \frac{1}{x} + 0 = 5x^2 + 2 + \frac{3}{x} = \frac{5x^3+2x+3}{x}$$

## Exercício V

$$\int \left( 2x^4 + 3 + 5e^x + \frac{7}{x} \right) \partial x = 2 \int x^4 \partial x + 3 \int \partial x + 5 \int e^x \partial x + 7 \int \frac{\partial x}{x} =$$

$$2 \frac{x^5}{5} + 3x + 5e^x + 7 \ln|x| + c = \frac{2x^5}{5} + 3x + 5e^x + 7 \ln|x| + c$$
(17)

$$\frac{\partial \left( \frac{2x^5}{5} + 3x + 5e^x + 7 \ln|x| + c \right)}{\partial x} = \frac{2}{5} 5x^4 + 3 + 5e^x + 7 \frac{1}{x} + 0 = 2x^4 + 3 + 5e^x + \frac{7}{x}$$

## Integral Indefinida – [Aula 6](#)

### Exercício I

$$\int \frac{5t^2 + 7}{\sqrt[3]{t^4}} \partial t = \int \frac{5t^2 + 7}{t^{\frac{4}{3}}} \partial t = \int t^{\frac{-4}{3}} (5t^2 + 7) \partial t = \int 5t^{2 - \frac{4}{3}} + 7t^{\frac{-4}{3}} \partial t = \int 5t^{\frac{2}{3}} + 7t^{\frac{-4}{3}} \partial t =$$

$$5 \int t^{\frac{2}{3}} \partial t + 7 \int t^{\frac{-4}{3}} \partial t = 5 \frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} + 7 \frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + c = 5 \frac{3}{5} \sqrt[3]{t^5} - 7 \cdot 3 \frac{1}{\sqrt[3]{t}} + c = 3\sqrt[3]{t^5} - \frac{21}{\sqrt[3]{t}} + c$$
(18)

$$\frac{\partial \left( 3\sqrt[3]{t^5} - \frac{21}{\sqrt[3]{t}} + c \right)}{\partial t} = \frac{\partial \left( 3t^{\frac{5}{3}} - 21t^{\frac{-1}{3}} + c \right)}{\partial t} = 3 \frac{5}{3} t^{\frac{2}{3}} - 21 \left( \frac{-1}{3} \right) t^{\frac{-4}{3}} + 0 = 5\sqrt[3]{t^2} + \frac{7}{\sqrt[3]{t^4}} =$$

$$\frac{5t^{\frac{2}{3}} t^{\frac{4}{3}} + 7}{\sqrt[3]{t^4}} = \frac{5t^{\frac{2}{3} + \frac{4}{3}} + 7}{\sqrt[3]{t^4}} = \frac{5t^{\frac{6}{3}} + 7}{\sqrt[3]{t^4}} = \frac{5t^2 + 7}{\sqrt[3]{t^4}}$$

## Integral Indefinida e Composta – [Aula 7](#)

### Exercício I

$$\int \frac{\partial x}{\sqrt{x}} = \int \frac{\partial x}{x^{\frac{1}{2}}} = \int x^{\frac{-1}{2}} \partial x = \frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c = 2\sqrt{x} + c$$
(19)

### Exercício II

$$\int \left( 3e^x + \frac{2}{x} \right) \partial x = 3 \int e^x \partial x + 2 \int \frac{\partial x}{x} = 3e^x + 2 \ln|x| + c$$
(20)

### Exercício III

$$\int x^3 \partial x = \frac{x^4}{4} + c$$

$$\int (2x^2+1)^3 x \partial x = \int u^3 \frac{\partial u}{4} = \frac{1}{4} \int u^3 \partial u = \frac{1}{4} \frac{u^4}{4} + c = \frac{u^4}{16} + c = \frac{(2x^2+1)^4}{16} + c =$$

$$\frac{(2x^2+1)^4}{2^4} + c = \left( \frac{2x^2+1}{2} \right)^4 + c = \left( x^2 + \frac{1}{2} \right)^4 + c$$

$$u = 2x^2+1 \rightarrow \frac{\partial u}{4} = x \partial x$$

(21)

$$\frac{\partial \left[ \left( x^2 + \frac{1}{2} \right)^4 + c \right]}{\partial x} = 4 \left( x^2 + \frac{1}{2} \right)^3 \cdot 2x + 0 = 8x \left( x^2 + \frac{1}{2} \right)^3 = 8x \left( x^2 + \frac{1}{2} \right) \left( x^2 + \frac{1}{2} \right)^2 =$$

$$(8x^3 + 4x) \left( x^4 + x^2 + \frac{1}{4} \right) = 8x^7 + 8x^5 + 2x^3 + 4x^5 + 4x^3 + x = 8x^7 + 12x^5 + 6x^3 + x$$

$$(2x^2+1)^3 x = (2x^2+1)^2 (2x^2+1) x = (4x^4 + 4x^2 + 1) (2x^3 + x) = 8x^7 + 4x^5 + 8x^5 + 4x^3 + 2x^3 + x =$$

$$8x^7 + 12x^5 + 6x^3 + x$$

### Integral indefinida e composta – [Aula 8](#)

#### Exercício I

$$\int 3e^x \partial x = 3 \int e^x \partial x = 3e^x + c$$

$$\int e^{x^2+1} x \partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2} \int e^u \partial u = \frac{1}{2} e^u + c = \frac{e^{x^2+1}}{2} + c$$

$$u = x^2+1 \rightarrow \frac{\partial u}{2} = x \partial x$$

(22)

$$\frac{\partial \left( \frac{e^{x^2+1}}{2} + c \right)}{\partial x} = \frac{1}{2} e^{x^2+1} 2x + 0 = e^{x^2+1} x$$

#### Exercício II

$$\int e^{x^4+1} x^3 \partial x = \int e^u \frac{\partial u}{4} = \frac{1}{4} \int e^u \partial u = \frac{1}{4} e^u + c = \frac{e^{x^4+1}}{4} + c$$

$$u = x^4+1 \rightarrow \frac{\partial u}{4} = x^3 \partial x$$

(23)

$$\frac{\partial \left( \frac{e^{x^4+1}}{4} + c \right)}{\partial x} = \frac{1}{4} e^{x^4+1} 4x^3 + 0 = e^{x^4+1} x^3$$

### Exercício III

$$\int \frac{x}{(2x^2-1)^3} \partial x = \int (2x^2-1)^{-3} x \partial x = \int u^{-3} \frac{\partial u}{4} = \frac{1}{4} \int u^{-3} \partial u = \frac{1}{4} \frac{u^{-2}}{(-2)} + c = \frac{-1}{8u^2} + c =$$

$$\frac{-1}{8(2x^2-1)^2} + c$$

$$u = 2x^2 - 1 \rightarrow \frac{\partial u}{4} = x \partial x$$

(24)

$$\frac{\partial \left( \frac{-1}{8(2x^2-1)^2} + c \right)}{\partial x} = \frac{\partial \left( \frac{-(2x^2-1)^{-2}}{8} + c \right)}{\partial x} = \frac{-1}{8} (-2) (2x^2-1)^{-3} 4x + 0 = (2x^2-1)^{-3} x = \frac{x}{(2x^2-1)^3}$$

### Exercício IV

$$\int \frac{x}{2x^2-1} \partial x = \int (2x^2-1)^{-1} x \partial x = \int u^{-1} \frac{\partial u}{4} = \frac{1}{4} \int u^{-1} \partial u = \frac{1}{4} \ln|u| + c = \frac{\ln|2x^2-1|}{4} + c$$

$$u = 2x^2 - 1 \rightarrow \frac{\partial u}{4} = x \partial x$$

(25)

$$\frac{\partial \left( \frac{\ln|2x^2-1|}{4} + c \right)}{\partial x} = \frac{1}{4} \frac{1}{2x^2-1} 4x + 0 = \frac{x}{2x^2-1}$$

## Integral pelo Método da Substituição não tão evidente – [Aula 9](#)

### Exercício I

$$\begin{aligned}
 \int x^2 \sqrt{1+x} \partial x &\rightarrow \int (u-1)^2 \sqrt{u} \partial u = \int (u-1)^2 u^{\frac{1}{2}} \partial u = \int (u^2 - 2u + 1) u^{\frac{1}{2}} \partial u = \\
 &\int \left( u^{2+\frac{1}{2}} - 2u^{1+\frac{1}{2}} + u^{\frac{1}{2}} \right) \partial u = \int \left( u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}} \right) \partial u = \int u^{\frac{5}{2}} \partial u - 2 \int u^{\frac{3}{2}} \partial u + \int u^{\frac{1}{2}} \partial u = \\
 &\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - 2 \frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2\sqrt{u^7}}{7} - \frac{4\sqrt{u^5}}{5} + \frac{2\sqrt{u^3}}{3} + c = \frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c \\
 &u = 1+x \rightarrow x = u-1 \rightarrow \partial u = \partial x \\
 &\frac{\partial \left( \frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c \right)}{\partial x} = \frac{\partial \left( \frac{2(1+x)^{\frac{7}{2}}}{7} - \frac{4(1+x)^{\frac{5}{2}}}{5} + \frac{2(1+x)^{\frac{3}{2}}}{3} + c \right)}{\partial x} = \\
 &\frac{2}{7} \frac{7}{2} (1+x)^{\frac{5}{2}} - \frac{4}{5} \frac{5}{2} (1+x)^{\frac{3}{2}} + \frac{2}{3} \frac{3}{2} (1+x)^{\frac{1}{2}} + 0 = (1+x)^{\frac{5}{2}} - 2(1+x)^{\frac{3}{2}} + (1+x)^{\frac{1}{2}} = \\
 &(1+x)^{\frac{1}{2}} \left( (1+x)^{\frac{5}{2}-\frac{1}{2}} - 2(1+x)^{\frac{3}{2}-\frac{1}{2}} + 1 \right) = (1+x)^{\frac{1}{2}} \left( (1+x)^{\frac{4}{2}} - 2(1+x)^{\frac{2}{2}} + 1 \right) = \\
 &(1+x)^{\frac{1}{2}} \left( (1+x)^2 - 2(1+x) + 1 \right) = (1+x)^{\frac{1}{2}} (1+2x+x^2-2-2x+1) = (1+x)^{\frac{1}{2}} x^2 = x^2 \sqrt{1+x}
 \end{aligned} \tag{26}$$

### Exercício II

$$\begin{aligned}
 \int x^2 \sqrt{1+x} \partial x &\rightarrow \int (u^2-1)^2 u \partial u = 2 \int (u^2-1)^2 u^2 \partial u = 2 \int (u^4 - 2u^2 + 1) u^2 \partial u = \\
 &2 \int (u^6 - 2u^4 + u^2) \partial u = 2 \int u^6 \partial u - 4 \int u^4 \partial u + 2 \int u^2 \partial u = 2 \frac{u^7}{7} - 4 \frac{u^5}{5} + 2 \frac{u^3}{3} + c = \\
 &\frac{2\sqrt{(1+x)^7}}{7} - \frac{4\sqrt{(1+x)^5}}{5} + \frac{2\sqrt{(1+x)^3}}{3} + c \\
 &u = \sqrt{1+x} \rightarrow u^2 = 1+x \rightarrow x = u^2-1 \rightarrow \partial x = 2u \partial u
 \end{aligned} \tag{27}$$

## O que é uma Integral Definida – [Aula 10](#)

### Exercício I

$$\int_1^2 x^3 \partial x = \frac{x^4}{4} \Big|_1^2 = \frac{(2)^4}{4} - \frac{(1)^4}{4} = \frac{16}{4} - \frac{1}{4} = \frac{16-1}{4} = \frac{15}{4} = 3,75 \tag{28}$$

## Integral Definida – [Aula 10a](#)

### Exercício I

$$\int_0^2 (6x^2 - 4x + 5) \partial x = 6 \int_0^2 x^2 \partial x - 4 \int_0^2 x \partial x + 5 \int_0^2 \partial x = 6 \left[ \frac{x^3}{3} - 4 \frac{x^2}{2} + 5x \right]_0^2 = 2x^3 - 2x^2 + 5x \Big|_0^2 = x(2x^2 - 2x + 5) \Big|_0^2 = [(2)(2(2)^2 - 2(2) + 5)] - [(0)(2(0)^2 - 2(0) + 5)] = 2(8 - 4 + 5) = 2 \cdot 9 = 18 \quad (29)$$

### Exercício II

$$\int_{-1}^0 (2x - e^x) \partial x = 2 \int_{-1}^0 x \partial x - \int_{-1}^0 e^x \partial x = 2 \left[ \frac{x^2}{2} - e^x \right]_{-1}^0 = x^2 - e^x \Big|_{-1}^0 = ((0)^2 - e^{(0)}) - ((-1)^2 - e^{(-1)}) = -1 - \left(1 - \frac{1}{e}\right) = -1 - 1 + \frac{1}{e} = -2 + \frac{1}{e} \quad (30)$$

## Integral definida – [Aula 11](#)

### Exercício I

$$\begin{aligned} \frac{5\pi}{4} \int_0^2 \frac{r \partial r}{1+r^2} &= \frac{5\pi}{4} \int_0^2 (1+r^2)^{-1} r \partial r = \frac{5\pi}{4} \int_0^2 u^{-1} \frac{\partial u}{2} = \frac{5\pi}{4} \frac{1}{2} \int_0^2 u^{-1} \partial u = \frac{5\pi}{8} \int_0^2 u^{-1} \partial u = \\ \frac{5\pi}{8} \ln|u| \Big|_0^2 &= \frac{5\pi}{8} \ln|1+r^2| \Big|_0^2 = \left[ \frac{5\pi}{8} \ln|1+(2)^2| \right] - \left[ \frac{5\pi}{8} \ln|1+(0)^2| \right] = \frac{5\pi}{8} \ln|5| - \frac{5\pi}{8} \ln|1| = \\ \frac{5\pi}{8} (\ln|5| - \ln|1|) &= \frac{5\pi}{8} (\ln|5| - 0) = \frac{5\pi}{8} \ln|5| \quad (31) \\ u = 1+r^2 &\rightarrow \frac{\partial u}{2} = r \partial r \\ \ln|1| = x &\rightarrow e^x = 1 = e^0 \rightarrow x = 0 \\ \ln|5| = x &\rightarrow e^x = 5 \end{aligned}$$

### Exercício II

$$2\pi \int_0^2 r^2 \partial r = 2\pi \left[ \frac{r^3}{3} \right]_0^2 = \frac{2\pi r^3}{3} \Big|_0^2 = \left( \frac{2\pi(2)^3}{3} \right) - \left( \frac{2\pi(0)^3}{3} \right) = \frac{16\pi}{3} \quad (32)$$

### Exercício II

$$\begin{aligned} 2\pi \int_0^{\sqrt{2}} (4r - 2r^3) \partial r &= 8\pi \int_0^{\sqrt{2}} r \partial r - 4\pi \int_0^{\sqrt{2}} r^3 \partial r = 8\pi \left[ \frac{r^2}{2} - 4\pi \frac{r^4}{4} \right]_0^{\sqrt{2}} = 4\pi r^2 - \pi r^4 \Big|_0^{\sqrt{2}} = \\ \pi r^2 (4 - r^2) \Big|_0^{\sqrt{2}} &= \left[ \pi (\sqrt{2})^2 (4 - (\sqrt{2})^2) \right] - \left[ \pi (0)^2 (4 - (0)^2) \right] = 2\pi (4 - 2) = 4\pi \quad (33) \end{aligned}$$



### Exercício III

$$\pi \int_0^2 x^2 \partial x = \pi \left[ \frac{x^3}{3} \right]_0^2 = \frac{\pi x^3}{3} \Big|_0^2 = \left( \frac{\pi(2)^3}{3} \right) - \left( \frac{\pi(0)^3}{3} \right) = \frac{8\pi}{3} \quad (34)$$

### Exercício IV

$$\frac{\pi}{16} \int_1^4 x^4 \partial x = \frac{\pi}{16} \left[ \frac{x^5}{5} \right]_1^4 = \frac{\pi x^5}{80} \Big|_1^4 = \left( \frac{\pi(4)^5}{80} \right) - \left( \frac{\pi(1)^5}{80} \right) = \frac{4^5 \pi}{80} - \frac{\pi}{80} = \frac{1024\pi - \pi}{80} = \frac{1023\pi}{80} \quad (35)$$

### Exercício V

$$\pi \int_1^2 (x^2)^2 \partial x = \pi \int_1^2 x^4 \partial x = \pi \left[ \frac{x^5}{5} \right]_1^2 = \frac{\pi x^5}{5} \Big|_1^2 = \left( \frac{\pi(2)^5}{5} \right) - \left( \frac{\pi(1)^5}{5} \right) = \frac{32\pi}{5} - \frac{\pi}{5} = \frac{32\pi - \pi}{5} = \frac{31\pi}{5} \quad (36)$$

### Exercício VI

$$\begin{aligned} \pi \int_{-1}^2 (-x^4 - x^2 + 6x + 8) \partial x &= -\pi \int_{-1}^2 x^4 \partial x - \pi \int_{-1}^2 x^2 \partial x + 6\pi \int_{-1}^2 x \partial x + 8\pi \int_{-1}^2 \partial x = \\ &= -\pi \left[ \frac{x^5}{5} - \pi \frac{x^3}{3} + 6\pi \frac{x^2}{2} + 8\pi x \right]_{-1}^2 = \left[ \frac{-\pi x^5}{5} - \frac{\pi x^3}{3} + 3\pi x^2 + 8\pi x \right]_{-1}^2 = -\pi x \left( \frac{x^4}{5} + \frac{x^2}{3} - 3x - 8 \right) \Big|_{-1}^2 \\ &= \left[ -\pi(2) \left( \frac{(2)^4}{5} + \frac{(2)^2}{3} - 3(2) - 8 \right) \right] - \left[ -\pi(-1) \left( \frac{(-1)^4}{5} + \frac{(-1)^2}{3} - 3(-1) - 8 \right) \right] = \\ &= -2\pi \left( \frac{16}{5} + \frac{4}{3} - 6 - 8 \right) - \pi \left( \frac{1}{5} + \frac{1}{3} + 3 - 8 \right) = -2\pi \left( \frac{48 + 20 - 210}{15} \right) - \pi \left( \frac{3 + 5 - 75}{15} \right) = \\ &= 2\pi \left( \frac{142}{15} \right) + \pi \left( \frac{67}{15} \right) = \pi \left( \frac{284}{15} + \frac{67}{15} \right) = \frac{351\pi}{15} = \frac{3^3 \cdot 13\pi}{3 \cdot 5} = \frac{3^2 \cdot 13\pi}{5} = \frac{117\pi}{5} \end{aligned} \quad (37)$$

### Exercício VII

$$\begin{aligned} \pi \int_0^8 (\sqrt[3]{y})^2 \partial y &= \pi \int_0^8 \left( y^{\frac{1}{3}} \right)^2 \partial y = \pi \int_0^8 y^{\frac{2}{3}} \partial y = \pi \left[ \frac{y^{\frac{5}{3}}}{\frac{5}{3}} \right]_0^8 = \frac{3\pi \sqrt[3]{y^5}}{5} \Big|_0^8 = \\ &= \left( \frac{3\pi \sqrt[3]{(8)^5}}{5} \right) - \left( \frac{3\pi \sqrt[3]{(0)^5}}{5} \right) = \frac{3\pi \sqrt[3]{8^3 \cdot 8^2}}{5} = \frac{3\pi 2^3 \sqrt[3]{(2^3)^2}}{5} = \frac{3\pi 2^3 2^2}{5} = \frac{3\pi 2^5}{5} = \frac{96\pi}{5} \end{aligned} \quad (38)$$

## Integral definida – [Aula 12](#)

### Exercício I

$$\int_1^2 2x \partial x = 2 \int_1^2 x \partial x = 2 \left[ \frac{x^2}{2} \right]_1^2 = x^2 \Big|_1^2 = ((2)^2) - ((1)^2) = 4 - 1 = 4 - 1 = 3 \quad (39)$$

## Exercício II

$$\int_1^4 2\sqrt{x} \partial x = \int_1^4 2x^{\frac{1}{2}} \partial x = 2 \int_1^4 x^{\frac{1}{2}} \partial x = 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \frac{4\sqrt{x^3}}{3} \Big|_1^4 = \left( \frac{4\sqrt{(4)^3}}{3} \right) - \left( \frac{4\sqrt{(1)^3}}{3} \right) =$$

$$\frac{4\sqrt{4^2 \cdot 2^2}}{3} - \frac{4}{3} = \frac{32}{3} - \frac{4}{3} = \frac{32-4}{3} = \frac{28}{3}$$
(40)

## Exercício III

$$\int_1^2 4x^2 \partial x = 4 \int_1^2 x^2 \partial x = 4 \left[ \frac{x^3}{3} \right]_1^2 = \frac{4}{3} x^3 \Big|_1^2 = \frac{4}{3} (2^3 - 1^3) = \frac{4}{3} 7 = \frac{28}{3}$$
(41)

## Integrais definidas e indefinidas – [Aula 13](#)

### Exercício I

$$\int \left( \frac{3}{x^4} + \frac{2x^2}{3} - 2x + 7 \right) \partial x = \int \left( 3x^{-4} + \frac{2}{3}x^2 - 2x + 7 \right) \partial x =$$

$$3 \int x^{-4} \partial x + \frac{2}{3} \int x^2 \partial x - 2 \int x \partial x + 7 \int \partial x = 3 \frac{x^{-3}}{-3} + \frac{2}{3} \frac{x^3}{3} - 2 \frac{x^2}{2} + 7x + c =$$

$$\frac{-1}{x^3} + \frac{2x^3}{9} - x^2 + 7x + c$$
(42)

$$\frac{\partial \left( \frac{-1}{x^3} + \frac{2x^3}{9} - x^2 + 7x + c \right)}{\partial x} = \frac{\partial \left( -x^{-3} + \frac{2}{9}x^3 - x^2 + 7x + c \right)}{\partial x} = 3x^{-4} + \frac{2}{9}3x^2 - 2x + 7 + 0 =$$

$$\frac{3}{x^4} + \frac{2x^2}{3} - 2x + 7$$

### Exercício II

$$\int 5\sqrt[3]{x^2} \partial x = \int 5x^{\frac{2}{3}} \partial x = 5 \int x^{\frac{2}{3}} \partial x = 5 \left[ \frac{x^{\frac{5}{3}}}{\frac{5}{3}} \right] + c = 3\sqrt[3]{x^5} + c$$
(43)

$$\frac{\partial (3\sqrt[3]{x^5} + c)}{\partial x} = \frac{\partial (3x^{\frac{5}{3}} + c)}{\partial x} = 3 \frac{5}{3} x^{\frac{2}{3}} + 0 = 5\sqrt[3]{x^2}$$

### Exercício III

$$\int_2^4 2x^3 \partial x = 2 \int_2^4 x^3 \partial x = 2 \left[ \frac{x^4}{4} \right]_2^4 = \frac{1}{2} x^4 \Big|_2^4 = \frac{1}{2} (4^4 - 2^4) = \frac{1}{2} ((2 \cdot 2)^4 - 2^4) = \frac{1}{2} (2^4 \cdot 2^4 - 2^4) = \frac{2^4}{2} (2^4 - 1) = 2^3 (16 - 1) = 8 \cdot 15 = 120 \quad (44)$$

### Exercício IV

$$\int_1^2 (3x^2 - 2x) \partial x = 3 \int_1^2 x^2 \partial x - 2 \int_1^2 x \partial x = 3 \left[ \frac{x^3}{3} \right]_1^2 - 2 \left[ \frac{x^2}{2} \right]_1^2 = x^3 - x^2 \Big|_1^2 = x^2 (x - 1) \Big|_1^2 = [2^2(2 - 1)] - [1^2(1 - 1)] = 4 \quad (45)$$

## Integral definida pelo método da substituição – U du – [Aula 14](#)

### Exercício I

$$\int_0^2 \sqrt{2x^2 + 1} x \partial x = \int_0^2 \sqrt{u} \frac{\partial u}{4} = \frac{1}{4} \int_0^2 u^{\frac{1}{2}} \partial u = \frac{1}{4} \left[ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 = \frac{1}{4} \frac{2}{3} \sqrt{u^3} \Big|_0^2 = \frac{1}{6} \sqrt{u^3} \Big|_0^2 = \frac{1}{6} \sqrt{(2x^2 + 1)^3} \Big|_0^2 = \frac{1}{6} [\sqrt{(2 \cdot 2^2 + 1)^3} - \sqrt{(2 \cdot 0^2 + 1)^3}] = \frac{1}{6} (\sqrt{9^3} - \sqrt{1^3}) = \frac{1}{6} (\sqrt{9^2 \cdot 3} - 1) = \frac{1}{6} (27 - 1) = \frac{1}{6} 26 = \frac{13}{3}$$

$$u = 2x^2 + 1 \rightarrow \frac{\partial u}{4} = x \partial x \quad (46)$$

## Integral Método da Substituição – [Aula 15](#)

### Exercício I

$$\int (x^3 - 1)^4 x^2 \partial x = \int u^4 \frac{\partial u}{3} = \frac{1}{3} \int u^4 \partial u = \frac{1}{3} \frac{u^5}{5} + c = \frac{(x^3 - 1)^5}{15} + c$$

$$u = x^3 - 1 \rightarrow \frac{\partial u}{3} = x^2 \partial x \quad (47)$$

### Exercício II

$$\int \frac{x}{(x^2 - 1)^3} \partial x = \int (x^2 - 1)^{-3} x \partial x = \int u^{-3} \frac{\partial u}{2} = \frac{1}{2} \int u^{-3} \partial u = \frac{1}{2} \frac{u^{-2}}{(-2)} + c = \frac{-1}{4u^2} + c = \frac{-1}{4(x^2 - 1)^2} + c$$

$$u = x^2 - 1 \rightarrow \frac{\partial u}{2} = x \partial x \quad (48)$$

### Exercício III

$$\int \frac{x}{(x^2-1)} \partial x = \int (x^2-1)^{-1} x \partial x = \int u^{-1} \frac{\partial u}{2} = \frac{1}{2} \int u^{-1} \partial u = \frac{1}{2} \ln|u| + c = \frac{\ln|x^2-1|}{2} + c$$

$$u = x^2 - 1 \rightarrow \frac{\partial u}{2} = x \partial x$$
(49)

### Exercício IV

$$\int e^{x^2-1} x \partial x = \int e^u \frac{\partial u}{2} = \frac{1}{2} e^u \partial u = \frac{1}{2} e^u + c = \frac{e^{x^2-1}}{2} + c$$

$$u = x^2 - 1 \rightarrow \frac{\partial u}{2} = x \partial x$$
(50)

### Exercício V

$$\int \sqrt{x^3-4} x^2 \partial x = \int u^{\frac{1}{2}} \frac{\partial u}{3} = \frac{1}{3} \int u^{\frac{1}{2}} \partial u = \frac{1}{3} \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{1}{3} \frac{2}{3} \sqrt{u^3} + c = \frac{2\sqrt{u^3}}{9} + c =$$

$$\frac{2\sqrt{(x^3-4)^3}}{9} + c$$

$$u = x^3 - 4 \rightarrow \frac{\partial u}{3} = x^2 \partial x$$
(51)

### Exercício VI

$$\int e^{\sqrt{x}} \frac{\partial x}{\sqrt{x}} = \int e^{x^{\frac{1}{2}}} \frac{\partial x}{x^{\frac{1}{2}}} = \int e^{x^{\frac{1}{2}}} x^{-\frac{1}{2}} \partial x = \int e^u 2 \partial u = 2 \int e^u \partial u = 2e^u + c = 2e^{\sqrt{x}} + c$$

$$u = \sqrt{x} = x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2} x^{-\frac{1}{2}} \partial x \rightarrow 2 \partial u = x^{-\frac{1}{2}} \partial x$$
(52)

### Exercício VII

$$\int \frac{x \partial x}{\sqrt[5]{x^2-1}} = \int \frac{x}{(x^2-1)^{\frac{1}{5}}} \partial x = \int (x^2-1)^{-\frac{1}{5}} x \partial x = \int u^{-\frac{1}{5}} \frac{\partial u}{2} = \frac{1}{2} \int u^{-\frac{1}{5}} \partial u = \frac{1}{2} \frac{u^{\frac{4}{5}}}{\left(\frac{4}{5}\right)} + c =$$

$$\frac{1}{2} \frac{5}{4} \sqrt[5]{u^4} + c = \frac{5\sqrt[5]{u^4}}{8} + c = \frac{5\sqrt[5]{(x^2-1)^4}}{8} + c$$

$$u = x^2 - 1 \rightarrow \frac{\partial u}{2} = x \partial x$$
(53)

## Exercício VIII

$$\int \frac{e^t \partial t}{e^t + 4} = \int (e^t + 4)^{-1} e^t \partial t = \int u^{-1} \partial u = \ln|u| + c = \ln|e^t + 4| + c$$

$$u = e^t + 4 \rightarrow \partial u = e^t \partial t$$
(54)

## Integral de uma Função Exponencial Qualquer – [Aula 16](#)

### Exercício I

$$\int \sqrt{10^{3x}} \partial x = \int 10^{\frac{3x}{2}} \partial x = \int 10^u \frac{2}{3} \partial u = \frac{2}{3} \int 10^u \partial u = \frac{2}{3} \frac{10^u}{\ln|10|} + c = \frac{2 \cdot 10^{\frac{3x}{2}}}{3 \ln|10|} + c = \frac{2\sqrt{10^{3x}}}{3 \ln|10|} + c$$

$$u = \frac{3x}{2} \rightarrow \partial u = \frac{3}{2} \partial x = \frac{2}{3} \partial u = \partial x$$

$$\frac{\partial \left( \frac{2\sqrt{10^{3x}}}{3 \ln|10|} + c \right)}{\partial x} = \frac{\partial \left( \frac{2}{3 \ln|10|} 10^{\frac{3x}{2}} + c \right)}{\partial x} = \frac{2}{3 \ln|10|} \frac{3 \ln|10| \sqrt{10^{3x}}}{2} + 0 = \sqrt{10^{3x}}$$

$$y = 10^{\frac{3x}{2}} \rightarrow \ln|y| = \ln|10^{\frac{3x}{2}}| = \frac{3x}{2} \ln|10| = \frac{3 \ln|10|}{2} x$$

$$\frac{\partial(\ln|y|)}{\partial y} = \frac{\partial \left( \frac{3 \ln|10|}{2} x \right)}{\partial x} \rightarrow \frac{1}{y} \partial y = \frac{3 \ln|10|}{2} \partial x \rightarrow \partial y = y \frac{3 \ln|10|}{2} \partial x \rightarrow \frac{\partial y}{\partial x} =$$

$$10^{\frac{3x}{2}} \frac{3 \ln|10|}{2} = \frac{3 \ln|10| \sqrt{10^{3x}}}{2}$$
(55)

## Integral de função marginal – [Aula 17](#)

### Exercício I

O custo marginal por unidade  $x$  é dado pela expressão  $\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5}$ . Determine a função custo total  $c(x)$  da produção, sabendo-se que o custo fixo para  $x = 0$  é de R\$2000,00.

$$\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5} \rightarrow \int \partial c(x) = \int \left( 20 - \frac{4x^3}{5} \right) \partial x \rightarrow \int \left( 20 - \frac{4x^3}{5} \right) \partial x = 20 \int \partial x - \frac{4}{5} \int x^3 \partial x =$$

$$20x - \frac{4}{5} \frac{x^4}{4} + c = 20x + \frac{x^4}{5} + c$$

$$20 \cdot 0 + \frac{0^4}{5} + c = 2000 \rightarrow c = 2000$$

$$c(x) = 20x + \frac{x^4}{5} + 2000$$
(56)

## Exercício II

O rendimento marginal de um bem em quantidade (x) é dado pela expressão  $Rm = 800 - 2x^2$ . Ache o rendimento total para  $x = 6$ , sabendo que quando  $x = 0$ ,  $R = 0$ .

$$\begin{aligned}
 \frac{\partial r(x)}{\partial x} &= 800 - 2x^2 \rightarrow \int \partial r(x) = \int (800 - 2x^2) \partial x \rightarrow \int (800 - 2x^2) \partial x = 800 \int \partial x - 2 \int x^2 \partial x \\
 800x - 2 \frac{x^3}{3} + c &= 800x - \frac{2x^3}{3} + c \\
 800 \cdot 0 - \frac{2 \cdot 0^3}{3} + c &= 0 \rightarrow c = 0 \\
 r(x) &= 800x - \frac{2x^3}{3} \\
 r(6) &= 800 \cdot 6 - \frac{2 \cdot 6^3}{3} = 4800 - \frac{2 \cdot 2^3 \cdot 3^3}{3} = 4800 - 2^4 \cdot 3^2 = 4800 - 16 \cdot 9 = 4800 - 144 = 4656
 \end{aligned}
 \tag{57}$$

## Método da substituição com $\sin(x)$ e $\cos(x)$ – [Aula 18](#)

01.  $\int \sin(u) \partial u = -\cos(u) + c$
02.  $\int \cos(u) \partial u = \sin(u) + c$
03.  $\int \operatorname{tg}(u) \partial u = \ln|\sec(u)| + c$
04.  $\int \operatorname{cotg}(u) \partial u = \ln|\sin(u)| + c$
05.  $\int \sec(u) \partial u = \ln|\sec(u) + \operatorname{tg}(u)| + c$
06.  $\int \operatorname{cosec}(u) \partial u = \ln|\operatorname{cosec}(u) - \operatorname{cotg}(u)| + c$
07.  $\int \sec^2(u) \partial u = \operatorname{tg}(u) + c$
08.  $\int \operatorname{cosec}^2(u) \partial u = -\operatorname{cotg}(u) + c$
09.  $\int \sec(u) \operatorname{tg}(u) \partial u = \sec(u) + c$
10.  $\int \operatorname{cosec}(u) \operatorname{cotg}(u) \partial u = -\operatorname{cosec}(u) + c$

## Exercício I

$$\begin{aligned}
 \int \sin(2x^2 - 1) x \partial x &= \int \sin(u) \frac{\partial u}{4} = \frac{1}{4} \sin(u) \partial u = \frac{1}{4} (-\cos(u)) + c = \frac{-\cos(u)}{4} + c = \\
 &\quad \frac{-\cos(2x^2 - 1)}{4} + c \\
 u &= 2x^2 - 1 \rightarrow \frac{\partial u}{4} = x \partial x
 \end{aligned}
 \tag{58}$$

$$\frac{\partial \left( \frac{-\cos(2x^2 - 1)}{4} + c \right)}{\partial x} = \frac{-1}{4} [-\sin(2x^2 - 1)] 4x + 0 = \sin(2x^2 - 1) x$$

Exercício II

$$\begin{aligned}
 \int \cos(3x^3+4) x^2 \partial x &= \int \cos(u) \frac{\partial u}{9} = \frac{1}{9} \int \cos(u) \partial u = \frac{1}{9} \operatorname{sen}(u) + c = \frac{\operatorname{sen}(3x^3+4)}{9} + c \\
 u &= 3x^3+4 \rightarrow \frac{\partial u}{9} = x^2 \partial x \\
 \frac{\partial \left( \frac{\operatorname{sen}(3x^3+4)}{9} + c \right)}{\partial x} &= \frac{1}{9} \cos(3x^3+4) 9x^2 + 0 = \cos(3x^3+4) x^2
 \end{aligned}
 \tag{59}$$

Exercício III

$$\begin{aligned}
 \int \operatorname{sen}(\sqrt{x}) \frac{\partial x}{\sqrt{x}} &= \int \operatorname{sen}\left(x^{\frac{1}{2}}\right) \frac{\partial x}{x^{\frac{1}{2}}} = \int \operatorname{sen}\left(x^{\frac{1}{2}}\right) x^{\frac{-1}{2}} \partial x = \int \operatorname{sen}(u) 2 \partial u = 2 \int \operatorname{sen}(u) \partial u = \\
 &2(-\cos(u)) + c = -2 \cos \sqrt{x} + c \\
 u &= \sqrt{x} = x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2} x^{\frac{-1}{2}} \partial x \rightarrow 2 \partial u = x^{\frac{-1}{2}} \partial x
 \end{aligned}
 \tag{60}$$

Exercício IV

$$\begin{aligned}
 \int \operatorname{sen}(x) \cos(x) \partial x &= \int u \partial u = \frac{u^2}{2} + c = \frac{\operatorname{sen}^2(x)}{2} + c \\
 u &= \operatorname{sen}(x) \rightarrow \partial u = \cos(x) \partial x \\
 \frac{\partial \left( \frac{\operatorname{sen}^2(x)}{2} + c \right)}{\partial x} &= \frac{1}{2} 2 \operatorname{sen}(x) \cos(x) + 0 = \operatorname{sen}(x) \cos(x)
 \end{aligned}
 \tag{61}$$

Exercício V

$$\begin{aligned}
 \int \operatorname{sen}(\cos(x)) \operatorname{sen}(x) \partial x &= \int \operatorname{sen}(u) (-\partial u) = - \int \operatorname{sen}(u) \partial u = -(-\cos(u)) + c = \\
 &\cos(\cos(x)) + c \\
 u &= \cos(x) \rightarrow -\partial u = \operatorname{sen}(x) \partial x \\
 \frac{\partial [\cos(\cos(x)) + c]}{\partial x} &= -\operatorname{sen}(\cos(x)) (-\operatorname{sen}(x)) + 0 = \operatorname{sen}(\cos(x)) \operatorname{sen}(x)
 \end{aligned}
 \tag{62}$$

Exercício VI

$$\begin{aligned}
 \int \sqrt{\operatorname{sen}(\theta)} \cos(\theta) \partial \theta &= \int u^{\frac{1}{2}} \partial u = \frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{2\sqrt{u^3}}{3} + c = \frac{2\sqrt{\operatorname{sen}^3(\theta)}}{3} + c \\
 u &= \operatorname{sen}(\theta) \rightarrow \partial u = \cos(\theta) \partial \theta
 \end{aligned}
 \tag{63}$$

Exercício VI

$$\begin{aligned}
 \int \ln|x| \frac{\partial x}{x} &= \int \ln|x| x^{-1} \partial x = \int u \partial u = \frac{u^2}{2} + c = \frac{\ln^2|x|}{2} + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \\
 \frac{\partial \left( \frac{\ln^2|x|}{2} + c \right)}{\partial x} &= \frac{1}{2} 2 \ln|x| \frac{1}{x} + 0 = \frac{\ln|x|}{x}
 \end{aligned}
 \tag{64}$$

Exercício VII

$$\begin{aligned}
 \int \frac{\partial x}{x \ln^2|x|} &= \int (x \ln^2|x|)^{-1} \partial x = \int \ln^{-2}|x| x^{-1} \partial x = \int u^{-2} \partial u = \frac{u^{-1}}{(-1)} = \frac{-1}{u} + c = \frac{-1}{\ln|x|} + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \\
 \frac{\partial \left( \frac{-1}{\ln|x|} + c \right)}{\partial x} &= \frac{\partial (-\ln^{-1}|x| + c)}{\partial x} = \ln^{-2} \frac{1}{x} + 0 = \frac{1}{x \ln^2|x|}
 \end{aligned}
 \tag{65}$$

Exercício VIII

$$\begin{aligned}
 \int \frac{\sin(\theta) \partial \theta}{(5 - \cos(\theta))^3} &= \int (5 - \cos(\theta))^{-3} \sin(\theta) \partial \theta = \int u^{-3} \partial u = \frac{u^{-2}}{(-2)} + c = \frac{-1}{2u^2} + c = \\
 &\quad \frac{-1}{2(5 - \cos(\theta))^2} + c \\
 u &= 5 - \cos(\theta) \rightarrow \partial u = -(-\sin(\theta)) \partial \theta \rightarrow \partial u = \sin(\theta) \partial \theta \\
 \frac{\partial \left( \frac{-1}{2(5 - \cos(\theta))^2} + c \right)}{\partial \theta} &= \frac{\partial \left( \frac{-1}{2} (5 - \cos(\theta))^{-2} + c \right)}{\partial \theta} = \frac{-1}{2} (-2) (5 - \cos(\theta))^{-3} (-(-\sin(\theta))) = \\
 &\quad (5 - \cos(\theta))^{-3} \sin(\theta) = \frac{\sin(\theta)}{(5 - \cos(\theta))^3}
 \end{aligned}
 \tag{66}$$



## Integração de funções trigonométricas – Aula 19

### Exercício I

$$\begin{aligned}
 \int \textcolor{red}{tg}(x) \partial x &= \int \frac{\text{sen}(x)}{\cos(x)} \partial x = \int \cos^{-1}(x) \text{sen}(x) \partial x = \int u^{-1} (-\partial u) = - \int u^{-1} \partial u = \\
 &= -\ln|u| + c = \ln|u^{-1}| + c = \ln\left|\frac{1}{u}\right| + c = \ln\left|\frac{1}{\cos(x)}\right| + c = \textcolor{red}{\ln|\sec(x)|} + c \\
 &\quad u = \cos(x) \rightarrow -\partial u = \textcolor{green}{\text{sen}(x) \partial x}
 \end{aligned}$$

(67)

$$\begin{aligned}
 \frac{\partial(\textcolor{blue}{\ln|\sec(x)|+c})}{\partial x} &= \frac{\partial\left(\ln\left|\frac{1}{\cos(x)}\right|+c\right)}{\partial x} = \frac{\partial(\ln|\cos^{-1}(x)|+c)}{\partial x} = \frac{\partial(-\ln|\cos(x)|+c)}{\partial x} = \\
 &= \frac{-\frac{1}{\cos(x)}(-\text{sen}(x))+0}{\cos(x)} = \frac{\text{sen}(x)}{\cos(x)} = \textcolor{blue}{tg}(x)
 \end{aligned}$$

### Exercício II

$$\begin{aligned}
 \int \textcolor{red}{cotg}(x) \partial x &= \int \frac{\cos(x)}{\text{sen}(x)} \partial x = \int \text{sen}^{-1}(x) \cos(x) \partial x = \int u^{-1} \partial u = \ln|u| + c = \textcolor{red}{\ln|\text{sen}(x)|} + c \\
 &\quad u = \text{sen}(x) \rightarrow \partial u = \textcolor{green}{\cos(x) \partial x}
 \end{aligned}$$

(68)

$$\frac{\partial(\textcolor{blue}{\ln|\text{sen}(x)|+c})}{\partial x} = \frac{1}{\text{sen}(x)} \cos(x) + 0 = \frac{\cos(x)}{\text{sen}(x)} = \textcolor{blue}{cotg}(x)$$

### Exercício III

$$\begin{aligned}
 \int \textcolor{red}{\sec}(x) \partial x &= \int \sec(x) \left( \frac{\sec(x) + tg(x)}{\sec(x) + tg(x)} \right) \partial x = \int \frac{\sec^2(x) + \sec(x)tg(x)}{\sec(x) + tg(x)} \partial x = \\
 &= \int [\sec(x) + tg(x)]^{-1} [\sec(x)tg(x) + \sec^2(x)] \partial x = \int u^{-1} \partial u = \ln|u| + c = \textcolor{red}{\ln|\sec(x) + tg(x)|} + c \\
 &\quad u = \sec(x) + tg(x) \rightarrow \partial u = \textcolor{green}{[\sec(x)tg(x) + \sec^2(x)] \partial x}
 \end{aligned}$$

(69)

$$\begin{aligned}
 \frac{\partial(\textcolor{blue}{\ln|\sec(x) + tg(x)|+c})}{\partial x} &= \frac{1}{\sec(x) + tg(x)} [\sec(x)tg(x) + \sec^2(x)] + 0 = \\
 &= \frac{\sec^2(x) + \sec(x)tg(x)}{\sec(x) + tg(x)} = \frac{\sec(x)[\sec(x) + tg(x)]}{\sec(x) + tg(x)} = \textcolor{blue}{\sec}(x)
 \end{aligned}$$

#### Exercício IV

$$\begin{aligned}
 \int \operatorname{cosec}(x) \partial x &= \int \operatorname{cosec}(x) \left( \frac{\operatorname{cosec}(x) - \cotg(x)}{\operatorname{cosec}(x) - \cotg(x)} \right) \partial x = \\
 &= \int \frac{\operatorname{cosec}^2(x) - \operatorname{cosec}(x) \cotg(x)}{\operatorname{cosec}(x) - \cotg(x)} \partial x = \\
 &= \int [\operatorname{cosec}(x) - \cotg(x)]^{-1} [-\operatorname{cosec}(x) \cotg(x) + \operatorname{cosec}^2(x)] \partial x = \int u^{-1} \partial u = \\
 &= \ln|u| + c = \ln|\operatorname{cosec}(x) - \cotg(x)| + c \\
 u &= \operatorname{cosec}(x) - \cotg(x) \rightarrow \partial u = [-\operatorname{cosec}(x) \cotg(x) - (-\operatorname{cosec}^2(x))] \partial x \rightarrow \\
 \partial u &= [-\operatorname{cosec}(x) \cotg(x) + \operatorname{cosec}^2(x)] \partial x \\
 \frac{\partial (\ln|\operatorname{cosec}(x) - \cotg(x)| + c)}{\partial x} &= \\
 \frac{1}{\operatorname{cosec}(x) - \cotg(x)} [-\operatorname{cosec}(x) \cotg(x) - (-\operatorname{cosec}^2(x))] + 0 &= \\
 \frac{\operatorname{cosec}^2(x) - \operatorname{cosec}(x) \cotg(x)}{\operatorname{cosec}(x) - \cotg(x)} &= \frac{\operatorname{cosec}(x) [\operatorname{cosec}(x) - \cotg(x)]}{\operatorname{cosec}(x) - \cotg(x)} = \operatorname{cosec}(x)
 \end{aligned} \tag{70}$$

#### Exercício V

$$\begin{aligned}
 \int \operatorname{tg}(3x) \partial x &= \int \operatorname{tg}(u) \frac{\partial u}{3} = \frac{1}{3} \operatorname{tg}(u) \partial u = \frac{1}{3} \ln|\sec(u)| + c = \frac{\ln|\sec(u)|}{3} + c = \frac{\ln|\sec(3x)|}{3} + c \\
 u &= 3x \rightarrow \frac{\partial u}{3} = \partial x
 \end{aligned} \tag{71}$$

#### Exercício VI

$$\begin{aligned}
 \int \frac{\partial x}{\operatorname{sen}(2x)} &= \int \operatorname{sen}^{-1}(2x) \partial x = \int \operatorname{cosec}(2x) \partial x = \int \operatorname{cosec}(u) \frac{\partial u}{2} = \frac{1}{2} \int \operatorname{cosec}(u) \partial u = \\
 \frac{1}{2} \ln|\operatorname{cosec}(u) - \cotg(u)| + c &= \frac{\ln|\operatorname{cosec}(u) - \cotg(u)|}{2} + c = \frac{\ln|\operatorname{cosec}(2x) - \cotg(2x)|}{2} + c \\
 u &= 2x \rightarrow \frac{\partial u}{2} = \partial x
 \end{aligned} \tag{72}$$

### Integração de funções trigonométricas – [Aula 20](#)

#### Exercício I

$$\begin{aligned}
 \int \frac{\operatorname{tg}(\sqrt{x}) \partial x}{\sqrt{x}} &= \int \frac{\operatorname{tg}\left(x^{\frac{1}{2}}\right) \partial x}{x^{\frac{1}{2}}} = \int \operatorname{tg}\left(x^{\frac{1}{2}}\right) x^{-\frac{1}{2}} \partial x = \int \operatorname{tg}(u) 2 \partial u = 2 \int \operatorname{tg}(u) \partial u = \\
 2 \ln|\sec(u)| + c &= 2 \ln|\sec(\sqrt{x})| + c \\
 u &= \sqrt{x} = x^{\frac{1}{2}} \rightarrow \partial u = \frac{1}{2} x^{-\frac{1}{2}} \partial x \rightarrow 2 \partial u = x^{-\frac{1}{2}} \partial x
 \end{aligned} \tag{73}$$

## Exercício II

$$\int \frac{\cotg(\ln|x|) \partial x}{x} = \int \cotg(\ln|x|) x^{-1} \partial x = \int \cotg(u) \partial u = \ln|\operatorname{sen}(u)| + c = \ln|\operatorname{sen}(\ln|x|)| + c$$

$$u = \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x$$
(74)

## Exercício III

$$\int \sec(5x - \pi) \partial x = \int \sec(u) \frac{\partial u}{5} = \frac{1}{5} \int \sec(u) \partial u = \frac{1}{5} \ln|\sec(u) + \operatorname{tg}(u)| + c =$$

$$\frac{\ln|\sec(5x - \pi) + \operatorname{tg}(5x - \pi)|}{5} + c$$

$$u = 5x - \pi \rightarrow \frac{\partial u}{5} = \partial x$$
(75)

## Integral de potência $\operatorname{sen}(x)$ ou $\cos(x)$ – [Aula 21](#)

$$\int \cos^n(x) \partial x$$

$$\int \operatorname{sen}^n(x) \partial x$$

$$n \rightarrow \text{ímpar} \quad \operatorname{sen}^2(x) + \cos^2(x) = 1$$

$$\operatorname{sen}^2(x) = \frac{1 - \cos(2x)}{2}$$

$$n \rightarrow \text{par} \quad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\int \operatorname{sen}^m(x) \cos^n(x) \partial x$$

$$n \vee m \rightarrow \text{ímpar} \quad \text{Separa o } \partial u \begin{cases} \operatorname{sen}(x) \partial x \\ \cos(x) \partial x \end{cases}$$

Transforma em  $\operatorname{sen}^p(x)$  ou  $\cos^p(x)$  através de  $\operatorname{sen}^2(x) + \cos^2(x) = 1$

$$\operatorname{sen}^2(x) = \frac{1 - \cos(2x)}{2}$$

$$n \wedge m \rightarrow \text{par} \quad \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

# Exercício I

$$\begin{aligned}
 \int \cos^5(x) \partial x &= \int \cos^4(x) \cos(x) \partial x = \int (\cos^2(x))^2 \cos(x) \partial x = \int (1 - \sin^2(x))^2 \cos(x) \partial x = \\
 &= \int [1 - 2\sin^2(x) + \sin^4(x)] \cos(x) \partial x = \int [\cos(x) - 2\sin^2(x) \cos(x) + \sin^4(x) \cos(x)] \partial x = \\
 &= \int \cos(x) \partial x - 2 \int \sin^2(x) \cos(x) \partial x + \int \sin^4(x) \cos(x) \partial x = \int \partial u - 2 \int u^2 \partial u + \int u^4 \partial u = \\
 &= u - 2 \frac{u^3}{3} + \frac{u^5}{5} + c = \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5} + c \\
 &\quad u = \sin(x) \rightarrow \partial u = \cos(x) \partial x
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \left( \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5} + c \right)}{\partial x} &= \frac{\partial \left( \sin(x) - \frac{2}{3} \sin^3(x) + \frac{1}{5} \sin^5(x) + c \right)}{\partial x} = \\
 \cos(x) - \frac{2}{3} 3 \sin^2(x) \cos(x) + \frac{1}{5} 5 \sin^4(x) \cos(x) + 0 &= \cos(x) [1 - 2\sin^2(x) + \sin^4(x)] = \\
 \cos(x) [1 - (\sin^2(x) + \sin^2(x)) + (\sin^2(x))^2] &= \\
 \cos(x) [(1 - \sin^2(x)) - \sin^2(x) + (1 - \cos^2(x))^2] &= \\
 \cos(x) [\cos^2(x) - \sin^2(x) + (1 - 2\cos^2(x) + \cos^4(x))] &= \\
 \cos(x) [\cos^2(x) - \sin^2(x) + 1 - (\cos^2(x) + \cos^2(x)) + \cos^4(x)] &= \\
 \cos(x) [\cos^2(x) + (1 - \sin^2(x)) - \cos^2(x) - \cos^2(x) + \cos^4(x)] &= \\
 \cos(x) [\cos^2(x) - \cos^2(x) + \cos^4(x)] &= \cos(x) \cos^4(x) = \cos^5(x)
 \end{aligned} \tag{76}$$

Exercício II

$$\begin{aligned}
 \int \text{sen}^4(x) \partial x &= \int (\text{sen}^2(x))^2 \partial x = \int \left( \frac{1 - \cos(2x)}{2} \right)^2 \partial x = \int \frac{1 - 2\cos(2x) + \cos^2(2x)}{4} \partial x = \\
 \frac{1}{4} \int \left[ 1 - 2\cos(2x) + \frac{1 + \cos(2 \cdot 2x)}{2} \right] \partial x &= \frac{1}{4} \int \partial x - \frac{1}{2} \int \cos(2x) \partial x + \frac{1}{8} \int [1 + \cos(4x)] \partial x = \\
 \frac{1}{4} \int \partial x - \frac{1}{2} \int \cos(2x) \partial x + \frac{1}{8} \int \partial x + \frac{1}{8} \int \cos(4x) \partial x &= \\
 \frac{1}{4} \int \frac{\partial u}{2} - \frac{1}{2} \int \cos(u) \frac{\partial u}{2} + \frac{1}{8} \int \frac{\partial u}{2} + \frac{1}{8} \int \cos(2u) \frac{\partial u}{2} &= \\
 \frac{1}{8} \int \partial u - \frac{1}{4} \int \cos(u) \partial u + \frac{1}{16} \int \partial u + \frac{1}{16} \int \cos(2u) \partial u &= \\
 \frac{1}{8} \int \partial u - \frac{1}{4} \int \cos(u) \partial u + \frac{1}{16} \int \partial u + \frac{1}{16} \int \cos(v) \frac{\partial v}{2} &= \\
 \frac{1}{8} \int \partial u - \frac{1}{4} \int \cos(u) \partial u + \frac{1}{16} \int \partial u + \frac{1}{32} \int \cos(v) \partial v &= \\
 \frac{1}{8} u - \frac{1}{4} \text{sen}(u) + \frac{1}{16} u + \frac{1}{32} \text{sen}(v) + c &= \frac{2u+u}{16} - \frac{\text{sen}(u)}{4} + \frac{\text{sen}(2u)}{32} + c = \\
 \frac{3u}{16} - \frac{\text{sen}(u)}{4} + \frac{\text{sen}(2u)}{32} + c &= \frac{3 \cdot 2x}{16} - \frac{\text{sen}(2x)}{4} + \frac{\text{sen}(2 \cdot 2x)}{32} + c = \\
 \frac{3x}{8} - \frac{\text{sen}(2x)}{4} + \frac{\text{sen}(4x)}{32} + c &= \\
 u = 2x \rightarrow \frac{\partial u}{2} = \partial x &= \\
 v = 2u \rightarrow \frac{\partial v}{2} = \partial u &=
 \end{aligned}
 \tag{77}$$

## Integral do produto de potência entre $\sin(x)$ e $\cos(x)$ – [Aula 22](#)

### Exercício I

$$\begin{aligned}
 \int \sin^5(x) \cos^2(x) \partial x &= \int \sin^4(x) \cos^2(x) \sin(x) \partial x = \int (\sin^2(x))^2 \cos^2(x) \sin(x) \partial x = \\
 &= \int (1 - \cos^2(x))^2 \cos^2(x) \sin(x) \partial x = \int [1 - 2\cos^2(x) + \cos^4(x)] \cos^2(x) \sin(x) \partial x = \\
 &= \int [\cos^2(x) - 2\cos^4(x) + \cos^6(x)] \sin(x) \partial x = \\
 &= \int \cos^2(x) \sin(x) \partial x - 2 \int \cos^4(x) \sin(x) \partial x + \int \cos^6(x) \sin(x) \partial x = \\
 &= \int u^2 (-\partial u) - 2 \int u^4 (-\partial u) + \int u^6 (-\partial u) = - \int u^2 \partial u + 2 \int u^4 \partial u - \int u^6 \partial u = \\
 &= \frac{-u^3}{3} + 2 \frac{u^5}{5} - \frac{u^7}{7} + c = \frac{-\cos^3(x)}{3} + \frac{2\cos^5(x)}{5} - \frac{\cos^7(x)}{7} + c \\
 &\quad u = \cos(x) \rightarrow -\partial u = \sin(x) \partial x
 \end{aligned}$$

(78)

$$\begin{aligned}
 \frac{\partial \left( \frac{-\cos^3(x)}{3} + \frac{2\cos^5(x)}{5} - \frac{\cos^7(x)}{7} + c \right)}{\partial x} &= \frac{\partial \left( -\frac{1}{3} \cos^3(x) + \frac{2}{5} \cos^5(x) - \frac{1}{7} \cos^7(x) + c \right)}{\partial x} = \\
 &= -\frac{1}{3} 3 \cos^2(x) (-\sin(x)) + \frac{2}{5} 5 \cos^4(x) (-\sin(x)) - \frac{1}{7} 7 \cos^6(x) (-\sin(x)) + 0 = \\
 \cos^2(x) \sin(x) - 2 \cos^4(x) \sin(x) + \cos^6(x) \sin(x) &= \sin(x) [\cos^2(x) - 2 \cos^4(x) + \cos^6(x)] = \\
 \sin(x) \cos^2(x) [1 - 2 \cos^2(x) + \cos^4(x)] &= (1 - \cos^2(x))^2 \cos^2(x) \sin(x) = \\
 (\sin^2(x))^2 \cos^2(x) \sin(x) &= \sin^4(x) \cos^2(x) \sin(x) = \sin^5(x) \cos^2(x)
 \end{aligned}$$

## Exercício II

$$\begin{aligned}
 \int \text{sen}^2(x) \cos^4(x) \partial x &= \int \frac{1 - \cos(2x)}{2} (\cos^2(x))^2 \partial x = \frac{1}{2} \int (1 - \cos(2x)) \left( \frac{1 + \cos(2x)}{2} \right)^2 \partial x \\
 &= \frac{1}{2} \int (1 - \cos(2x)) \left( \frac{1 + 2\cos(2x) + \cos^2(2x)}{4} \right) \partial x = \\
 &= \frac{1}{8} \int (1 - \cos(2x)) (1 + 2\cos(2x) + \cos^2(2x)) \partial x = \\
 &= \frac{1}{8} \int (1 + 2\cos(2x) + \cos^2(2x) - \cos(2x) - 2\cos^2(2x) - \cos^3(2x)) \partial x = \\
 &= \frac{1}{8} \int (\text{sen}^2(2x) + \cos(2x) - \cos^2(2x) \cos(2x)) \partial x = \\
 &= \frac{1}{8} \int \text{sen}^2(2x) \partial x + \frac{1}{8} \int \cos(2x) \partial x - \frac{1}{8} \int \cos^2(2x) \cos(2x) \partial x = \\
 &= \frac{1}{8} \int \frac{1 - \cos(4x)}{2} \partial x + \frac{1}{8} \int \cos(2x) \partial x - \frac{1}{8} \int (1 - \text{sen}^2(2x)) \cos(2x) \partial x = \quad (79) \\
 &= \frac{1}{16} \int (1 - \cos(4x)) \partial x + \frac{1}{8} \int \cos(2x) \partial x - \frac{1}{8} \int (\cos(2x) - \text{sen}^2(2x) \cos(2x)) \partial x = \\
 &= \frac{1}{16} \int \partial x - \frac{1}{16} \int \cos(4x) \partial x + \frac{1}{8} \int \cos(2x) \partial x - \frac{1}{8} \int \cos(2x) \partial x + \frac{1}{8} \int \text{sen}^2(2x) \cos(2x) \partial x \\
 &= \frac{1}{16} \int \partial x - \frac{1}{32} \int \cos(2y) \partial y + \frac{1}{16} \int \text{sen}^2(y) \cos(y) \partial y = \\
 &= \frac{1}{16} \int \partial x - \frac{1}{64} \int \cos(z) \partial z + \frac{1}{16} \int u^2 \partial u = \frac{1}{16} x - \frac{1}{64} \text{sen}(z) + \frac{1}{16} \frac{u^3}{3} + c = \\
 &= \frac{x}{16} - \frac{\text{sen}(2y)}{64} + \frac{\text{sen}^3(y)}{48} + c = \frac{x}{16} - \frac{\text{sen}(4x)}{64} + \frac{\text{sen}^3(2x)}{48} + c \\
 &= y = 2x \rightarrow \frac{\partial y}{2} = \partial x; z = 2y \rightarrow \frac{\partial z}{2} = \partial y; u = \text{sen}(y) \rightarrow \partial u = \cos(y) \partial y
 \end{aligned}$$

## Integral de Funções Trigonométricas – [Aula 23](#)

$$\begin{aligned}
 \text{tg}(x) &= \frac{\text{sen}(x)}{\cos(x)}, \text{cotg}(x) = \frac{\cos(x)}{\text{sen}(x)} \\
 \text{sec}(x) &= \frac{1}{\cos(x)}, \text{cossec}(x) = \frac{1}{\text{sen}(x)} \\
 \text{sen}^2(x) + \cos^2(x) &= 1
 \end{aligned}$$

$$\frac{\text{sen}^2(x)}{\text{sen}^2(x)} + \frac{\cos^2(x)}{\text{sen}^2(x)} = \frac{1}{\text{sen}^2(x)} \rightarrow 1 + \text{cotg}^2(x) = \text{cossec}^2(x) \rightarrow \text{cossec}^2(x) - \text{cotg}^2(x) = 1$$

$$\frac{\text{sen}^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} \rightarrow \text{tg}^2(x) + 1 = \text{sec}^2(x) \rightarrow \text{sec}^2(x) - \text{tg}^2(x) = 1$$

### Exercício I

$$\begin{aligned}
 \int [tg(2x) + cotg(2x)]^2 dx &= \int [tg^2(2x) + 2tg(2x)cotg(2x) + cotg^2(2x)] dx = \\
 &= \int tg^2(2x) dx + 2 \int tg(2x)cotg(2x) dx + \int cotg^2(2x) dx = \\
 &= \int (sec^2(2x) - 1) dx + 2 \int \frac{\sin(2x)}{\cos(2x)} \frac{\cos(2x)}{\sin(2x)} dx + \int (cosec^2(2x) - 1) dx = \\
 &= \int sec^2(2x) dx - \int dx + 2 \int dx + \int cosec^2(2x) dx - \int dx = \\
 &= \frac{1}{2} \int sec^2(u) du - \int dx + 2 \int dx + \frac{1}{2} \int cosec^2(u) du - \int dx = \\
 &= \frac{1}{2} tg(u) - x + 2x + \frac{1}{2} (-cotg(u)) - x + c = \frac{tg(2x)}{2} - \frac{cotg(2x)}{2} + c
 \end{aligned} \tag{80}$$

$$u = 2x \rightarrow \frac{\partial u}{2} = dx$$

$$\begin{aligned}
 \frac{\partial \left( \frac{tg(2x)}{2} - \frac{cotg(2x)}{2} + c \right)}{\partial x} &= \frac{1}{2} sec^2(2x) \cdot 2 - \frac{1}{2} (-cosec^2(2x)) \cdot 2 + 0 = \\
 sec^2(2x) + cosec^2(2x) &= (1 + tg^2(2x)) + (1 + cosec^2(2x)) = 2 + tg^2(2x) + cotg^2(2x) = \\
 tg^2(2x) + 2tg(2x)cotg(2x) + cotg^2(2x) &= [tg(2x) + cotg(2x)]^2
 \end{aligned}$$

### Integral Definida com Seno e Cosseno – [Aula 24](#)

	$0(0^\circ)$	$\frac{\pi}{6}(30^\circ)$	$\frac{\pi}{4}(45^\circ)$	$\frac{\pi}{3}(60^\circ)$	$\frac{\pi}{2}(90^\circ)$
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\infty$

### Exercício I

$$\begin{aligned}
 \int_0^{\frac{\pi}{4}} \left( \frac{1 + \cos^2(\theta)}{\cos^2(\theta)} \right) d\theta &= \int_0^{\frac{\pi}{4}} \left( \frac{1}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} \right) d\theta = \int_0^{\frac{\pi}{4}} (sec^2(\theta) + 1) d\theta = \\
 \int_0^{\frac{\pi}{4}} sec^2(\theta) d\theta + \int_0^{\frac{\pi}{4}} d\theta &= tg(\theta) \Big|_0^{\frac{\pi}{4}} + \theta \Big|_0^{\frac{\pi}{4}} = \left( tg\left(\frac{\pi}{4}\right) - tg(0) \right) + \left( \frac{\pi}{4} - 0 \right) = (1 - 0) + \left( \frac{\pi}{4} - 0 \right) = 1 + \frac{\pi}{4} = \\
 &= \frac{4 + \pi}{4}
 \end{aligned} \tag{81}$$

### Exercício II

$$\begin{aligned}
 \int_0^{\pi} (4 \sin(\theta) - 3 \cos(\theta)) d\theta &= 4 \int_0^{\pi} \sin(\theta) d\theta - 3 \int_0^{\pi} \cos(\theta) d\theta = 4(-\cos(\theta)) \Big|_0^{\pi} - 3 \sin(\theta) \Big|_0^{\pi} = \\
 -4 \cos(\theta) \Big|_0^{\pi} - 3 \sin(\theta) \Big|_0^{\pi} &= -4(\cos(\pi) - \cos(0)) - 3(\sin(\pi) - \sin(0)) = \\
 -4(-1 - 1) - 3(0 - 0) &= -4(-2) = 8
 \end{aligned} \tag{82}$$



## Integral Definida para funções trigonométricas – [Aula 25](#)

### Exercício I

$$\begin{aligned}
 \int_0^{\frac{\pi}{3}} \left( \frac{\text{sen}(\theta) + \text{sen}(\theta) \text{tg}^2(\theta)}{\sec^2(\theta)} \right) \partial \theta &= \int_0^{\frac{\pi}{3}} \left( \frac{\text{sen}(\theta)}{\sec^2(\theta)} + \frac{\text{sen}(\theta) \text{tg}^2(\theta)}{\sec^2(\theta)} \right) \partial \theta = \\
 \int_0^{\frac{\pi}{3}} \left( \text{sen}(\theta) \left( \frac{1}{\cos^2(\theta)} \right) + \text{sen}(\theta) \frac{\text{sen}^2(\theta)}{\cos^2(\theta)} \left( \frac{1}{\cos^2(\theta)} \right) \right) \partial \theta &= \int_0^{\frac{\pi}{3}} (\cos^2(\theta) \text{sen}(\theta) + \text{sen}^3(\theta)) \partial \theta = \\
 \int_0^{\frac{\pi}{3}} \cos^2(\theta) \text{sen}(\theta) \partial \theta + \int_0^{\frac{\pi}{3}} \text{sen}^2(\theta) \text{sen}(\theta) \partial \theta &= \\
 \int_0^{\frac{\pi}{3}} \cos^2(\theta) \text{sen}(\theta) \partial \theta + \int_0^{\frac{\pi}{3}} (1 - \cos^2(\theta)) \text{sen}(\theta) \partial \theta &= \\
 \int_0^{\frac{\pi}{3}} \cos^2(\theta) \text{sen}(\theta) \partial \theta + \int_0^{\frac{\pi}{3}} (\text{sen}(\theta) - \cos^2(\theta) \text{sen}(\theta)) \partial \theta &= \\
 \int_0^{\frac{\pi}{3}} \cos^2(\theta) \text{sen}(\theta) \partial \theta + \int_0^{\frac{\pi}{3}} \text{sen}(\theta) \partial \theta - \int_0^{\frac{\pi}{3}} \cos^2(\theta) \text{sen}(\theta) \partial \theta &= \\
 -\cos(\theta) \Big|_0^{\frac{\pi}{3}} = -\left( \cos\left(\frac{\pi}{3}\right) - \cos(0) \right) = -\left( \frac{1}{2} - 1 \right) = -\left( \frac{-1}{2} \right) = \frac{1}{2} &
 \end{aligned} \tag{83}$$

$$\begin{aligned}
 \int_0^{\frac{\pi}{3}} \left( \frac{\text{sen}(\theta) + \text{sen}(\theta) \text{tg}^2(\theta)}{\sec^2(\theta)} \right) \partial \theta &= \int_0^{\frac{\pi}{3}} \frac{\text{sen}(\theta)(1 + \text{tg}^2(\theta))}{\sec^2(\theta)} \partial \theta = \int_0^{\frac{\pi}{3}} \frac{\text{sen}(\theta) \sec^2(\theta)}{\sec^2(\theta)} \partial \theta = \\
 \int_0^{\frac{\pi}{3}} \text{sen}(\theta) \partial \theta &= -\cos(\theta) \Big|_0^{\frac{\pi}{3}} = -\left( \cos\left(\frac{\pi}{3}\right) - \cos(0) \right) = -\left( \frac{1}{2} - 1 \right) = -\left( \frac{-1}{2} \right) = \frac{1}{2}
 \end{aligned}$$

### Exercício II

$$\begin{aligned}
 \int_0^{\pi} \sec^2\left(\frac{t}{4}\right) \partial t &= 4 \int_0^{\pi} \sec^2(u) \partial u = 4 \text{tg}(u) \Big|_0^{\pi} = 4 \text{tg}\left(\frac{t}{4}\right) \Big|_0^{\pi} = 4 \left( \text{tg}\left(\frac{\pi}{4}\right) - \text{tg}(0) \right) = 4(1 - 0) = 4 \\
 u = \frac{t}{4} \rightarrow 4 \partial u &= \partial t
 \end{aligned} \tag{84}$$

## Integral envolvendo Funções Seno e Cosseno – [Aula 26](#)

$$\text{sen}(2x) = 2 \text{sen}(x) \cos(x)$$

### Exercício I

$$\int \frac{\text{sen}(x)}{1-\text{sen}^2(x)} \partial x = \int \frac{\text{sen}(x)}{\cos^2(x)} \partial x = \int \cos^{-2}(x) \text{sen}(x) \partial x = - \int u^{-2} \partial u = \frac{-u^{-1}}{(-1)} + c =$$

$$u^{-1} + c = \frac{1}{u} + c = \frac{1}{\cos(x)} + c = \text{sec}(x) + c$$

$$u = \cos(x) \rightarrow -\partial u = \text{sen}(x) \partial x$$
(85)

### Exercício II

$$\int \frac{\text{sen}(2x)}{\cos(x)} \partial x = \int \frac{2\text{sen}(x)\cos(x)}{\cos(x)} \partial x = 2 \int \text{sen}(x) \partial x = 2(-\cos(x)) + c = -2\cos(x) + c$$
(86)

## Integral de uma função exponencial de seno – [Aula 27](#)

### Exercício I

$$\int e^{\text{sen}(\theta)} \cos(\theta) \partial \theta = \int e^u \partial u = e^u + c = e^{\text{sen}(\theta)} + c$$

$$u = \text{sen}(\theta) \rightarrow \partial u = \cos(\theta) \partial \theta$$
(87)

### Exercício II

$$\int \text{sen}(\pi t) \partial t = \frac{1}{\pi} \int \text{sen}(u) \partial u = \frac{1}{\pi} (-\cos(u)) + c = \frac{-\cos(u)}{\pi} + c = \frac{-\cos(\pi t)}{\pi} + c$$

$$u = \pi t \rightarrow \frac{\partial u}{\pi} = \partial t$$
(88)

## Integral Definida do Produto de secante e tangente – [Aula 28](#)

### Exercício I

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec(\theta) \text{tg}(\theta) \partial \theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos(\theta)} \frac{\text{sen}(\theta)}{\cos(\theta)} \partial \theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\text{sen}(\theta)}{\cos^2(\theta)} \partial \theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^{-2}(\theta) \text{sen}(\theta) \partial \theta =$$

$$- \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} u^{-2} \partial u = \left[ \frac{-u^{-1}}{(-1)} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[ u^{-1} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[ \frac{1}{u} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[ \frac{1}{\cos(\theta)} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} - \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} =$$

$$2 - \frac{2}{\sqrt{2}} = 2 - \frac{2\sqrt{2}}{2} = 2 - \sqrt{2}$$

$$u = \cos(\theta) \rightarrow -\partial u = \text{sen}(\theta) \partial \theta$$
(89)

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec(\theta) \text{tg}(\theta) \partial \theta = \left[ \sec(\theta) \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \left[ \frac{1}{\cos(\theta)} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} - \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{2}\right)} - \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 2 - \frac{2}{\sqrt{2}} =$$

$$2 - \frac{2\sqrt{2}}{2} = 2 - \sqrt{2}$$

## Integral de seno pelo Método da Substituição – [Aula 29](#)

Exercício I

$$\int \sqrt{x} \operatorname{sen}(1+\sqrt{x^3}) \partial x = \int \operatorname{sen}\left(1+x^{\frac{3}{2}}\right) x^{\frac{1}{2}} \partial x = \frac{2}{3} \operatorname{sen}(u) \partial u = \frac{2}{3} (-\cos(u)) + c =$$

$$\frac{-2 \cos(1+\sqrt{x^3})}{3} + c \quad (90)$$

$$u = 1 + \sqrt{x^3} = 1 + x^{\frac{3}{2}} \rightarrow \partial u = \frac{3}{2} x^{\frac{1}{2}} \partial x = \frac{2 \partial u}{3} = x^{\frac{1}{2}} \partial x$$

## Integral Trigonométrica – Método da Substituição – [Aula 30](#)

Exercício I

$$\int (1+\operatorname{tg}(\theta))^5 \sec^2(\theta) \partial \theta = \int u^5 \partial u = \frac{u^6}{6} + c = \frac{(1+\operatorname{tg}(\theta))^6}{6} + c \quad (91)$$

$$u = 1 + \operatorname{tg}(\theta) \rightarrow \partial u = \sec^2(\theta) \partial \theta$$

Exercício II

$$\int \sec(2\theta) \operatorname{tg}(2\theta) \partial \theta = \frac{1}{2} \int \sec(u) \operatorname{tg}(u) \partial u = \frac{1}{2} \sec(u) + c = \frac{\sec(2\theta)}{2} + c \quad (92)$$

$$u = 2\theta \rightarrow \frac{\partial u}{2} = \partial \theta$$

## Integral Trigonométrica – Método da Substituição – [Aula 31](#)

Exercício I

$$\int \sqrt{\cot g(x)} \operatorname{cosec}^2(x) \partial x = \int \cot g^{\frac{1}{2}}(x) \operatorname{cosec}^2(x) \partial x = - \int u^{\frac{1}{2}} \partial u = \frac{-u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = \frac{-2\sqrt{u^3}}{3} + c =$$

$$\frac{-2\sqrt{\cot g^3(x)}}{3} + c \quad (93)$$

$$u = \cot g(x) \rightarrow -\partial u = \operatorname{cosec}^2(x)$$

Exercício II

$$\int \sec^3(x) \operatorname{tg}(x) \partial x = \int \sec^2(x) \sec(x) \operatorname{tg}(x) \partial x = \int u^2 \partial u = \frac{u^3}{3} + c = \frac{\sec^3(x)}{3} + c \quad (94)$$

$$u = \sec(x) \rightarrow \partial u = \sec(x) \operatorname{tg}(x) \partial x$$

## Integral de Cosseno pelo Método da Substituição – [Aula 32](#)

Exercício I

$$\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2} \partial x = \int \cos(\pi x^{-1}) x^{-2} \partial x = \frac{-1}{\pi} \int \cos(u) \partial u = \frac{-1}{\pi} \operatorname{sen}(u) + c = \frac{-\operatorname{sen}\left(\frac{\pi}{x}\right)}{\pi} + c \quad (95)$$

$$u = \frac{\pi}{x} = \pi x^{-1} \rightarrow \partial u = -\pi x^{-2} \rightarrow \frac{-\partial u}{\pi} = x^{-2} \partial x$$

## Integral de Potência de Tangente – [Aula 33](#)

Exercício I

$$\begin{aligned} \int \operatorname{tg}^3(x) \partial x &= \int \operatorname{tg}^2(x) \operatorname{tg}(x) \partial x = \int (\sec^2(x) - 1) \operatorname{tg}(x) \partial x = \\ &= \int \operatorname{tg}(x) \sec^2(x) \partial x - \int \operatorname{tg}(x) \partial x = \int \operatorname{tg}(x) \sec^2(x) \partial x - \int \frac{\operatorname{sen}(x)}{\cos(x)} \partial x = \\ &= \int \operatorname{tg}(x) \sec^2(x) \partial x - \int \cos^{-1}(x) \operatorname{sen}(x) \partial x = \int u \partial u + \int v^{-1} \partial v = \frac{u^2}{2} + \ln|v| + c = \\ &= \frac{\operatorname{tg}^2(x)}{2} + \ln|\cos(x)| + c \end{aligned} \quad (96)$$

$$u = \operatorname{tg}(x) \rightarrow \partial u = \sec^2(x) \partial x; v = \cos(x) \rightarrow -\partial v = \operatorname{sen}(x) \partial x$$

$$\begin{aligned} \int \operatorname{tg}^3(x) \partial x &= \int \operatorname{tg}^2(x) \operatorname{tg}(x) \partial x = \int (\sec^2(x) - 1) \operatorname{tg}(x) \partial x = \\ &= \int \operatorname{tg}(x) \sec^2(x) \partial x - \int \operatorname{tg}(x) \partial x = \int u \partial u - \int \operatorname{tg}(x) \partial x = \frac{u^2}{2} - (\ln|\sec(x)|) + c = \\ &= \frac{u^2}{2} - (\ln|\cos^{-1}(x)|) + c = \frac{u^2}{2} - (-\ln|\cos(x)|) + c = \frac{u^2}{2} + \ln|\cos(x)| + c = \frac{\operatorname{tg}^2(x)}{2} + \ln|\cos(x)| + c \end{aligned}$$

$$u = \operatorname{tg}(x) \rightarrow \partial u = \sec^2(x) \partial x$$

## Integral de Potência de Cotangente – [Aula 34](#)

Exercício I

$$\begin{aligned} \int \operatorname{cotg}^4(3x) \partial x &= \int \operatorname{cotg}^2(3x) \operatorname{cotg}^2(3x) \partial x = \int (\operatorname{cosec}^2(3x) - 1) \operatorname{cotg}^2(3x) \partial x = \\ &= \int \operatorname{cotg}^2(3x) \operatorname{cosec}^2(3x) \partial x - \int \operatorname{cotg}^2(3x) \partial x = \\ &= \int \operatorname{cotg}^2(3x) \operatorname{cosec}^2(3x) \partial x - \int (\operatorname{cosec}^2(3x) - 1) \partial x = \\ &= \int \operatorname{cotg}^2(3x) \operatorname{cosec}^2(3x) \partial x - \int \operatorname{cosec}^2(3x) \partial x + \int \partial x = \\ &= \frac{1}{3} \int \operatorname{cotg}^2(u) \operatorname{cosec}^2(u) \partial u - \frac{1}{3} \int \operatorname{cosec}^2(u) \partial u + \int \partial x = \\ &= \frac{-1}{3} \int v^2 \partial v + \frac{1}{3} \int \partial v + \int \partial x = \frac{-1}{3} \frac{v^3}{3} + \frac{1}{3} v + x + c = \frac{-\operatorname{cotg}^3(u)}{9} + \frac{\operatorname{cotg}(u)}{3} + x + c = \\ &= \frac{-\operatorname{cotg}^3(3x)}{9} + \frac{\operatorname{cotg}(3x)}{3} + x + c \end{aligned} \quad (97)$$

$$u = 3x \rightarrow \frac{\partial u}{3} = \partial x; v = \operatorname{cotg}(u) \rightarrow -\partial v = \operatorname{cosec}^2(u)$$

## Integral da Potência de Secante – [Aula 35](#)

Exercício I

$$\begin{aligned}
 \int \operatorname{cosec}^6(x) \partial x &= \int \operatorname{cosec}^4(x) \operatorname{cosec}^2(x) \partial x = \int (\operatorname{cosec}^2(x))^2 \operatorname{cosec}^2(x) \partial x = \\
 &= \int (\cotg^2(x)+1)^2 \operatorname{cosec}^2(x) \partial x = \int (\cotg^4(x)+2\cotg^2(x)+1) \operatorname{cosec}^2(x) \partial x = \\
 &= \int \cotg^4(x) \operatorname{cosec}^2(x) \partial x + 2 \int \cotg^2(x) \operatorname{cosec}^2(x) \partial x + \int \operatorname{cosec}^2(x) \partial x = \\
 &= -\int u^4 \partial u - 2u^2 \partial u - \int \partial u = \frac{-u^5}{5} - 2\frac{u^3}{3} - u + c = \frac{-\cotg^5(x)}{5} - \frac{2\cotg^3(x)}{3} - \cotg(x) + c \\
 &\quad u = \cotg(x) \rightarrow -\partial u = \operatorname{cosec}^2(x) \partial x
 \end{aligned} \tag{98}$$

## Integral de Secante ao Cubo – [Aula 36](#)

$$\int u \partial v = uv - \int v \partial u$$

Exercício I

$$\begin{aligned}
 \int \sec^3(x) \partial x &= \int \sec(x) \sec^2(x) \partial x = \sec(x) \tg(x) - \int \tg(x) \sec(x) \tg(x) \partial x = \\
 &= \sec(x) \tg(x) - \int \tg^2(x) \sec(x) \partial x = \sec(x) \tg(x) - \int (\sec^2(x) - 1) \sec(x) \partial x = \\
 &= \sec(x) \tg(x) - \int \sec^3(x) \partial x + \int \sec(x) \partial x \rightarrow \\
 &\quad \int \sec^3(x) \partial x + \int \sec^3(x) \partial x = \sec(x) \tg(x) + \int \sec(x) \partial x \rightarrow \\
 &\quad 2 \int \sec^3(x) \partial x = \sec(x) \tg(x) + \ln|\sec(x) + \tg(x)| + c \rightarrow \\
 \int \sec^3(x) \partial x &= \frac{\sec(x) \tg(x) + \ln|\sec(x) + \tg(x)| + c}{2} = \frac{\sec(x) \tg(x)}{2} + \frac{\ln|\sec(x) + \tg(x)|}{2} + \frac{c}{2} = \\
 &\quad \frac{\sec(x) \tg(x)}{2} + \frac{\ln|\sec(x) + \tg(x)|}{2} + c \\
 &\quad u = \sec(x) \rightarrow \partial u = \sec(x) \tg(x) \partial x \\
 &\quad v = \tg(x) \rightarrow \partial v = \sec^2(x) \partial x
 \end{aligned} \tag{99}$$

## Integral de Potência de Secante e Tangente – [Aula 37](#)

$$\int \tg^m(x) \sec^n(x) \partial x$$

- $n \rightarrow$  par      Guarde  $\sec^2(x)$   
                     Use  $\sec^2(x) = \tg^2(x) + 1$   
                      $u = \tg(x)$   
 $m \rightarrow$  ímpar      Guarde  $\sec(x) \tg(x)$   
                     Use  $\tg^2(x) = \sec^2(x) - 1$   
                      $u = \sec(x)$   
 $m \rightarrow$  par  
 $n \rightarrow$  ímpar      Use  $\tg^2(x) = \sec^2(x) - 1$  para reduzir somente a  $\sec(x)$   
                     Após aplique integral por partes

Exercício I

$$\begin{aligned}
 \int tg^5(x) sec^4(x) \partial x &= \int tg^5(x) sec^2(x) sec^2(x) \partial x = \int tg^5(x) (tg^2(x) + 1) sec^2(x) \partial x = \\
 &= \int tg^5(x) (tg^2(x) sec^2(x) + sec^2(x)) \partial x = \int tg^7(x) sec^2(x) \partial x + \int tg^5(x) sec^2(x) \partial x = \\
 &= \int u^7 \partial u + \int u^5 \partial u = \frac{u^8}{8} + \frac{u^6}{6} + c = \frac{tg^8(x)}{8} + \frac{tg^6(x)}{6} + c \\
 u &= tg(x) \rightarrow \partial u = sec^2(x) \partial x
 \end{aligned}
 \tag{100}$$

## Integral de Potência de Secante e Tangente – [Aula 38](#)

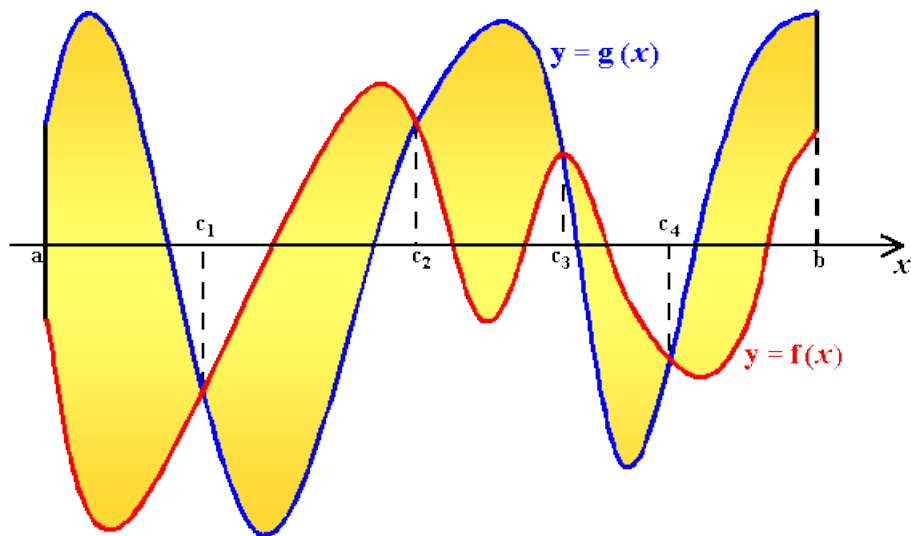
Exercício I

$$\begin{aligned}
 \int tg^5(x) sec^7(x) \partial x &= \int tg^4(x) sec^6(x) sec(x) tg(x) \partial x = \\
 &= \int (tg^2(x))^2 sec^6(x) sec(x) tg(x) \partial x = \int (sec^2(x) - 1)^2 sec^6(x) sec(x) tg(x) \partial x = \\
 &= \int (sec^4(x) - 2 sec^2(x) + 1) sec^6(x) sec(x) tg(x) \partial x = \\
 &= \int sec^{10}(x) sec(x) tg(x) \partial x - 2 \int sec^8(x) sec(x) tg(x) \partial x + \int sec^6(x) sec(x) tg(x) \partial x = \\
 &= \int u^{10} \partial u - 2 \int u^8 \partial u + \int u^6 \partial u = \frac{u^{11}}{11} - 2 \frac{u^9}{9} + \frac{u^7}{7} + c = \frac{sec^{11}(x)}{11} - \frac{2 sec^9(x)}{9} + \frac{sec^7(x)}{7} + c \\
 u &= sec(x) \rightarrow \partial u = sec(x) tg(x) \partial x
 \end{aligned}
 \tag{101}$$

Exercício II

$$\begin{aligned}
 \int tg^2(x) sec(x) \partial x &= \int (sec^2(x) - 1) sec(x) \partial x = \int sec(x) sec^2(x) \partial x - \int sec(x) \partial x \rightarrow \\
 \int u \partial v &= uv - \int v \partial u \rightarrow \int sec(x) sec^2(x) \partial x = sec(x) tg(x) - \int tg(x) sec(x) tg(x) \partial x = \\
 &= sec(x) tg(x) - \int tg^2(x) sec(x) \partial x = sec(x) tg(x) - \int (sec^2(x) - 1) sec(x) \partial x = \\
 &= sec(x) tg(x) - \int sec(x) sec^2(x) \partial x + \int sec(x) \partial x \rightarrow \\
 &= \int sec(x) sec^2(x) \partial x + \int sec(x) sec^2(x) \partial x = sec(x) tg(x) + \int sec(x) \partial x \rightarrow \\
 &= 2 \int sec(x) sec^2(x) \partial x = sec(x) tg(x) + \ln|sec(x) + tg(x)| + c \rightarrow \\
 \int sec(x) sec^2(x) \partial x &= \frac{sec(x) tg(x) + \ln|sec(x) + tg(x)| + c}{2} = \\
 \int tg^2(x) sec(x) \partial x &= \left( \frac{sec(x) tg(x) + \ln|sec(x) + tg(x)| + c}{2} \right) - \int sec(x) \partial x = \\
 &= \frac{sec(x) tg(x) + \ln|sec(x) + tg(x)| + c}{2} - \ln|sec(x) + tg(x)| + c = \\
 &= \frac{sec(x) tg(x) + \ln|sec(x) + tg(x)| - 2 \ln|sec(x) + tg(x)| + c}{2} = \\
 &= \frac{sec(x) tg(x)}{2} - \frac{\ln|sec(x) + tg(x)|}{2} + c \\
 u &= sec(x) \rightarrow \partial u = sec(x) tg(x) \partial x \\
 v &= tg(x) \rightarrow \partial v = sec^2(x) \partial x
 \end{aligned}
 \tag{102}$$

## Cálculo de área com integrais – Aula 1



$$\int_{c_1}^{c_2} [f(x) - g(x)] dx + \int_{c_2}^{c_3} [g(x) - f(x)] dx + \int_{c_3}^{c_4} [f(x) - g(x)] dx$$

Exercício I

$$\begin{aligned}
 & f(x) = x^2; g(x) = x \\
 & f(x) = g(x) \rightarrow x^2 = x \rightarrow x^2 - x = 0 \rightarrow x(x-1) = 0 \rightarrow \\
 & \quad x = 0 \rightarrow x_1 = 0; x-1 = 0 \rightarrow x = 1 \rightarrow x_2 = 1 \\
 & \int_0^1 [g(x) - f(x)] dx = \int_0^1 (x - x^2) dx = \int_0^1 x dx - \int_0^1 x^2 dx = \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left( \frac{1^2}{2} - \frac{1^3}{3} \right) - \left( \frac{0^2}{2} - \frac{0^3}{3} \right) = \\
 & \quad \frac{1}{2} - \frac{1}{3} - 0 = \frac{3-2}{6} = \frac{1}{6} = 0,166
 \end{aligned} \tag{103}$$

Exercício II

$$\begin{aligned}
 & f(x) = x^2; g(x) = 1 \\
 & \int_0^1 [g(x) - f(x)] dx + \int_1^2 [f(x) - g(x)] dx = \int_0^1 (1 - x^2) dx + \int_1^2 (x^2 - 1) dx = \\
 & \int_0^1 dx - \int_0^1 x^2 dx + \int_1^2 x^2 dx - \int_1^2 dx = \left[ x - \frac{x^3}{3} \right]_0^1 + \left[ \frac{x^3}{3} - x \right]_1^2 = \left[ \frac{3x - x^3}{3} \right]_0^1 + \left[ \frac{x^3 - 3x}{3} \right]_1^2 = \\
 & \left[ \frac{x(3 - x^2)}{3} \right]_0^1 + \left[ \frac{x(x^2 - 3)}{3} \right]_1^2 = \left[ \frac{1(3 - 1^2)}{3} - \frac{0(3 - 0^2)}{3} \right] + \left[ \frac{2(2^2 - 3)}{3} - \frac{1(1^2 - 3)}{3} \right] = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \\
 & \quad \frac{6}{3} = 2
 \end{aligned} \tag{104}$$

### Exercício III

$$\begin{aligned}
 f(x) &= 2x - 2; g(x) = 0 \\
 \int_0^1 [g(x) - f(x)] dx + \int_1^3 [f(x) - g(x)] dx &= \int_0^1 (0 - (2x - 2)) dx + \int_1^3 (2x - 2 - 0) dx = \\
 -2 \int_0^1 x dx + 2 \int_0^1 dx + 2 \int_1^3 x dx - 2 \int_1^3 dx &= \left[ -2 \frac{x^2}{2} + 2x \right]_0^1 + \left[ 2 \frac{x^2}{2} - 2x \right]_1^3 = \\
 [-x^2 + 2x]_0^1 + [x^2 - 2x]_1^3 &= [-x(x-2)]_0^1 + [x(x-2)]_1^3 = \\
 [-1(1-2) - (0(0-2))] + [3(3-2) - 1(1-2)] &= 1 + 3 + 1 = 5
 \end{aligned} \tag{105}$$

### Cálculo de área com integrais – [Aula 2](#)

#### Exercício I

$$\begin{aligned}
 y^2 &= 4x \rightarrow y = \pm \sqrt{4x} = \pm 2\sqrt{x}; x_1 = 1; x_2 = 4 \\
 y_1(1) &= 2\sqrt{1} = 2; y_1(4) = 2\sqrt{4} = 2 \cdot 2 = 4 \\
 y_2(1) &= -2\sqrt{1} = -2; y_2(4) = -2\sqrt{4} = -2 \cdot 2 = -4 \\
 f(x) &= 2\sqrt{x} = 2x^{\frac{1}{2}}; g(x) = 0 \\
 \int_1^4 [f(x) - g(x)] dx &= \int_1^4 [2\sqrt{x} - 0] dx = 2 \int_1^4 x^{\frac{1}{2}} dx = 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 = \left[ \frac{4\sqrt{x^3}}{3} \right]_1^4 = \frac{4}{3} [\sqrt{x^3}]_1^4 = \\
 \frac{4}{3} [\sqrt{4^3} - \sqrt{1^3}] &= \frac{4}{3} [\sqrt{(2^2)^3} - 1] = \frac{4}{3} [2^3 - 1] = \frac{4}{3} [8 - 1] = \frac{4}{3} \cdot 7 = \frac{28}{3} = 9,33
 \end{aligned} \tag{106}$$

#### Exercício II

$$\begin{aligned}
 f(x) &= x^2; g(x) = 2x + 3 \\
 f(x) &= g(x) \rightarrow x^2 = 2x + 3 \rightarrow x^2 - (2x + 3) = 0 \rightarrow x^2 - 2x - 3 = 0 \rightarrow x^2 - 2x - 3 + 1 - 1 = 0 \rightarrow \\
 x^2 - 2x + 1 - 4 &= 0 \rightarrow (x-1)^2 - 4 = 0 \rightarrow (x-1)^2 = 4 \rightarrow x-1 = \pm \sqrt{4} \rightarrow x = \pm 2 + 1 \rightarrow \\
 x &= 2 + 1 \rightarrow x_1 = 3; x = -2 + 1 \rightarrow x_2 = -1 \\
 f(0) &= (0)^2 = 0; g(0) = 2(0) + 3 = 3 \\
 \int_{-1}^3 [g(x) - f(x)] dx &= \int_{-1}^3 (2x + 3 - x^2) dx = 2 \int_{-1}^3 x dx + 3 \int_{-1}^3 dx - \int_{-1}^3 x^2 dx = \\
 \left[ 2 \frac{x^2}{2} + 3x - \frac{x^3}{3} \right]_{-1}^3 &= \left[ x^2 + 3x - \frac{x^3}{3} \right]_{-1}^3 = \left[ \frac{3x^2 + 9x - x^3}{3} \right]_{-1}^3 = \frac{1}{3} [x(3x + 9 - x^2)]_{-1}^3 = \\
 \frac{1}{3} [3(3 \cdot 3 + 9 - 3^2) - (-1)(3(-1) + 9 - (-1)^2)] &= \frac{1}{3} [3 \cdot 9 + 5] = \frac{1}{3} 32 = \frac{32}{3} = 10,66
 \end{aligned} \tag{107}$$



## Cálculo de área com integrais – [Aula 3](#)

### Exercício I

$$\begin{aligned}
 f(x) &= -x^2 + 4x - 3; g(x) = 0 \\
 f(x) = g(x) &\rightarrow -x^2 + 4x - 3 = 0 \rightarrow x^2 - 4x + 3 + (2)^2 - (2)^2 = 0 \rightarrow x^2 - 4x + 4 - 1 = 0 \rightarrow \\
 (x-2)^2 - 1 &= 0 \rightarrow x-2 = \pm\sqrt{1} \rightarrow x = \pm 1 + 2 \rightarrow \\
 x = 1+2 &\rightarrow x_1 = 3; x = -1+2 \rightarrow x_2 = 1 \\
 f(2) &= -(2)^2 + 4(2) - 3 = -4 + 8 - 3 = 1; g(2) = 0 \\
 \int_1^3 [f(x) - g(x)] dx &= \int_1^3 (-x^2 + 4x - 3 - 0) dx = -\int_1^3 x^2 dx + 4 \int_1^3 x dx - 3 \int_1^3 dx \rightarrow \\
 \left[ \frac{-x^3}{3} + 4 \frac{x^2}{2} - 3x \right]_1^3 &= \left[ \frac{-x^3}{3} + 2x^2 - 3x \right]_1^3 = \left[ \frac{-x^3 + 6x^2 - 9x}{3} \right]_1^3 = \frac{1}{3} [-x(x^2 - 6x + 9)]_1^3 = \\
 \frac{1}{3} [-3(3^2 - 6 \cdot 3 + 9) - (-1(1^2 - 6 \cdot 1 + 9))] &= \frac{1}{3} [-3 \cdot 0 + 4] = \frac{1}{3} 4 = \frac{4}{3} = 1,33
 \end{aligned} \tag{108}$$

### Exercício II

$$\begin{aligned}
 f(x) &= -x^2 + 4; g(x) = -5 \\
 f(x) = g(x) &\rightarrow -x^2 + 4 = -5 \rightarrow -x^2 = -5 - 4 \rightarrow x^2 = 9 \rightarrow x = \pm\sqrt{9} \rightarrow x = \pm 3 \\
 x_1 &= -3; x_2 = 3 \\
 f(0) &= -(0)^2 + 4 = 4; g(0) = -5 \\
 \int_{-3}^3 [f(x) - g(x)] dx &= \int_{-3}^3 (-x^2 + 4 + 5) dx = -\int_{-3}^3 x^2 dx + 4 \int_{-3}^3 dx + 5 \int_{-3}^3 dx = \\
 \left[ \frac{-x^3}{3} + 4x + 5x \right]_{-3}^3 &= \left[ \frac{-x^3 + 12x + 15x}{3} \right]_{-3}^3 = \frac{1}{3} [-x(x^2 - 12 - 15)]_{-3}^3 = \frac{1}{3} [-x(x^2 - 27)]_{-3}^3 = \\
 \frac{1}{3} [-3(3^2 - 27) + (-3)((-3)^2 - 27)] &= \frac{1}{3} [-3(-18) - 3(-18)] = \frac{1}{3} [54 + 54] = \frac{1}{3} 108 = 36
 \end{aligned} \tag{109}$$

## Cálculo de áreas com integrais – [Aula 4](#)

### Exercício I

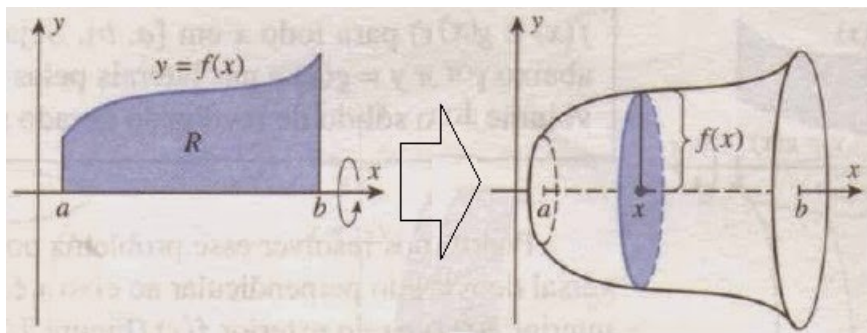
$$\begin{aligned}
 g(x) &= x + 2; f(x) = x^2 - x + 2 \\
 f(x) = g(x) &\rightarrow x^2 - x + 2 = x + 2 \rightarrow x^2 - x + 2 - x - 2 = 0 \rightarrow x^2 - 2x = 0 \rightarrow x(x-2) = 0 \rightarrow \\
 x = 0 &\rightarrow x_1 = 0; x - 2 = 0 \rightarrow x = 2 \rightarrow x_2 = 2 \\
 g(1) &= 1 + 2 = 3; f(1) = 1^2 - 1 + 2 = 2 \\
 \int_0^2 [g(x) - f(x)] dx &= \int_0^2 [x + 2 - (x^2 - x + 2)] dx = \int_0^2 (x + 2 - x^2 + x - 2) dx = \int_0^2 (-x^2 + 2x) dx = \\
 -\int_0^2 x^2 dx + 2 \int_0^2 x dx &= \left[ \frac{-x^3}{3} + 2 \frac{x^2}{2} \right]_0^2 = \left[ \frac{-x^3}{3} + x^2 \right]_0^2 = \left[ \frac{-x^3 + 3x^2}{3} \right]_0^2 = \frac{1}{3} [-x(x^2 - 3x)]_0^2 = \\
 \frac{1}{3} [-2(2^2 - 3 \cdot 2) + 0(0^2 - 3 \cdot 0)] &= \frac{1}{3} 4 = \frac{4}{3} = 1,33
 \end{aligned} \tag{110}$$

## Cálculo de áreas com integrais – [Aula 5](#)

### Exercício I

$$\begin{aligned}
 & f(x) = 4x; g(x) = x^3 + 3x^2 \\
 & f(x) = g(x) \rightarrow 4x = x^3 + 3x^2 \rightarrow x^3 + 3x^2 - 4x = 0 \rightarrow x(x^2 + 3x - 4) = 0 \rightarrow \\
 & x = 0 \rightarrow x_1 = 0; x^2 + 3x - 4 = 0 \rightarrow x^2 + 3x - 4 + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 = 0 \rightarrow x^2 + 3x - 4 + \frac{9}{4} - \frac{9}{4} = 0 \rightarrow \\
 & x^2 + 3x + \frac{9}{4} - \frac{25}{4} = 0 \rightarrow \left(x + \frac{3}{2}\right)^2 = \frac{25}{4} \rightarrow x + \frac{3}{2} = \pm \sqrt{\frac{25}{4}} \rightarrow x = \frac{\pm 5}{2} - \frac{3}{2} \rightarrow \\
 & x = \frac{-5}{2} - \frac{3}{2} = \frac{-5-3}{2} = \frac{-8}{2} = -4 \rightarrow x_2 = -4; x = \frac{5}{2} - \frac{3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1 \rightarrow x_3 = 1 \\
 & f(-1) = 4(-1) = -4; g(-1) = (-1)^3 + 3(-1)^2 = -1 + 3 = 2 \\
 & f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right) = 2; g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 = \frac{1}{8} + \frac{3}{4} = \frac{1+6}{8} = \frac{7}{8} \\
 & \int_{-4}^0 [g(x) - f(x)] \partial x + \int_0^1 [f(x) - g(x)] \partial x = \int_{-4}^0 (x^3 + 3x^2 - 4x) \partial x + \int_0^1 (4x - x^3 - 3x^2) \partial x = \quad (111) \\
 & \int_{-4}^0 x^3 \partial x + 3 \int_{-4}^0 x^2 \partial x - 4 \int_{-4}^0 x \partial x + 4 \int_0^1 x \partial x - \int_0^1 x^3 \partial x - 3 \int_0^1 x^2 \partial x = \\
 & \left[ \frac{x^4}{4} + 3 \frac{x^3}{3} - 4 \frac{x^2}{2} \right]_{-4}^0 + \left[ 4 \frac{x^2}{2} - \frac{x^4}{4} - 3 \frac{x^3}{3} \right]_0^1 = \left[ \frac{x^4}{4} + x^3 - 2x^2 \right]_{-4}^0 + \left[ 2x^2 - \frac{x^4}{4} - x^3 \right]_0^1 = \\
 & \left[ \frac{x^4 + 4x^3 - 8x^2}{4} \right]_{-4}^0 + \left[ \frac{8x^2 - x^4 - 4x^3}{4} \right]_0^1 = \frac{1}{4} [x^2(x^2 + 4x - 8)]_{-4}^0 + \frac{1}{4} [x^2(-x^2 - 4x + 8)]_0^1 = \\
 & \frac{1}{4} [0^2(0^2 + 4 \cdot 0 - 8) - (-4)^2((-4)^2 + 4(-4) - 8)] + \frac{1}{4} [1^2(-1^2 - 4 \cdot 1 + 8) - 0^2(-0^2 - 4 \cdot 0 + 8)] = \\
 & \frac{1}{4} [-16(16 - 16 - 8)] + \frac{1}{4} [-1 - 4 + 8] = \frac{1}{4} (128 + 3) = \frac{131}{4} = 32,75
 \end{aligned}$$

## Cálculo de volume com integrais simples – [Aula 1](#)

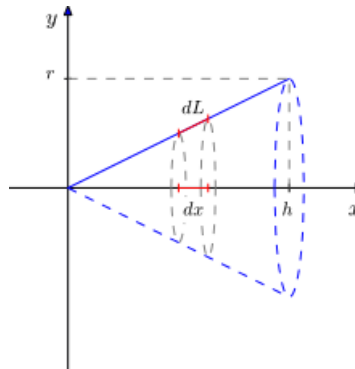


Volume do cilindro:  $v = \pi r^2 h$ ;  $r = y = f(x)$ ;  $h = \partial x$ ;  $\partial v = \pi [f(x)]^2 \partial x$

Volume do sólido:  $v = \int_a^b \partial v = \int_a^b \pi [f(x)]^2 \partial x \rightarrow v = \pi \int_a^b [f(x)]^2 \partial x$

### Exercício I

Determine o volume do sólido de revolução gerado pela rotação em torno do eixo dos x da região R delimitado pelo gráfico das equações:  $y=x$ ;  $y=0$ ;  $x=0$ ;  $x=2$



$$f(x)=x; x_1=0; x_2=2$$

$$V=\pi \int_0^2 x^2 dx = \pi \left[ \frac{x^3}{3} \right]_0^2 = \frac{\pi}{3} [x^3]_0^2 = \frac{\pi}{3} [2^3 - 0^3] = \frac{\pi}{3} 8 = \frac{8\pi}{3} = 2,667 \pi \quad (112)$$

### Exercício II

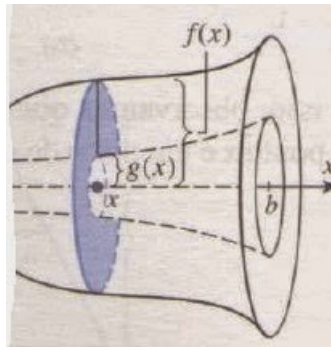
$$f(x)=\frac{x^2}{4}; x_1=1; x_2=4$$

$$V=\pi \int_1^4 \left( \frac{x^2}{4} \right)^2 dx = \pi \int_1^4 \frac{x^4}{16} dx = \frac{\pi}{16} \int_1^4 x^4 dx = \frac{\pi}{16} \left[ \frac{x^5}{5} \right]_1^4 = \frac{\pi}{80} [x^5]_1^4 = \frac{\pi}{80} [4^5 - 1^5] = \frac{1023\pi}{80} = 12,7875 \pi \quad (113)$$

### Exercício III

$$f(x)=x^2; x_1=1; x_2=2$$

$$V=\pi \int_1^2 x^4 dx = \pi \left[ \frac{x^5}{5} \right]_1^2 = \frac{\pi}{5} [x^5]_1^2 = \frac{\pi}{5} [2^5 - 1^5] = \frac{31\pi}{5} = 6,2 \pi \quad (114)$$



Volume do cilindro:  $v = \pi r_1^2 h - \pi r_2^2 h = \pi (r_1^2 - r_2^2) h$ ;  $r_1 = f(x)$ ;  $r_2 = g(x)$ ;  $h = \Delta x$

$$\partial v = \pi [(f(x))^2 - (g(x))^2] \partial x$$

Volume do sólido:  $v = \int_a^b \partial v = \int_a^b \pi [(f(x))^2 - (g(x))^2] \partial x \rightarrow v = \pi \int_a^b [(f(x))^2 - (g(x))^2] \partial x$

#### Exercício IV

$$f(x) = x^2 + 1; g(x) = x + 3$$

$$f(x) = g(x) \rightarrow x^2 + 1 = x + 3 \rightarrow x^2 + 1 - x - 3 = 0 \rightarrow x^2 - x - 2 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 0 \rightarrow$$

$$x^2 - x + \frac{1}{4} - \frac{9}{4} = 0 \rightarrow \left(x - \frac{1}{2}\right)^2 = \frac{9}{4} \rightarrow x = \pm \sqrt{\frac{9}{4}} + \frac{1}{2} \rightarrow x = \frac{\pm 3}{2} + \frac{1}{2}$$

$$x = \frac{-3}{2} + \frac{1}{2} = \frac{-2}{2} = -1 \rightarrow x_1 = -1; x = \frac{3}{2} + \frac{1}{2} = \frac{3+1}{2} = \frac{4}{2} = 2 \rightarrow x_2 = 2$$

$$f(0) = 0^2 + 1 = 1; g(0) = 0 + 3 = 3$$

$$v = \pi \int_{-1}^2 [(g(x))^2 - (f(x))^2] \partial x = \pi \int_{-1}^2 [(x+3)^2 - (x^2+1)^2] \partial x =$$

$$\pi \int_{-1}^2 [(x^2 + 6x + 9) - (x^4 + 2x^2 + 1)] \partial x = \pi \int_{-1}^2 (x^2 + 6x + 9 - x^4 - 2x^2 - 1) \partial x =$$

$$\pi \int_{-1}^2 (-x^4 - x^2 + 6x + 8) \partial x = -\pi \int_{-1}^2 x^4 \partial x - \pi \int_{-1}^2 x^2 \partial x + 6\pi \int_{-1}^2 x \partial x + 8\pi \int_{-1}^2 \partial x =$$

$$\left[ -\pi \frac{x^5}{5} - \pi \frac{x^3}{3} + 6\pi \frac{x^2}{2} + 8\pi x \right]_{-1}^2 = \pi \left[ \frac{-x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x \right]_{-1}^2 =$$

$$\pi \left[ \frac{-3x^5 - 5x^3 + 45x^2 + 120x}{15} \right]_{-1}^2 = \frac{\pi}{15} [-x(3x^4 + 5x^2 - 45x - 120)]_{-1}^2 =$$

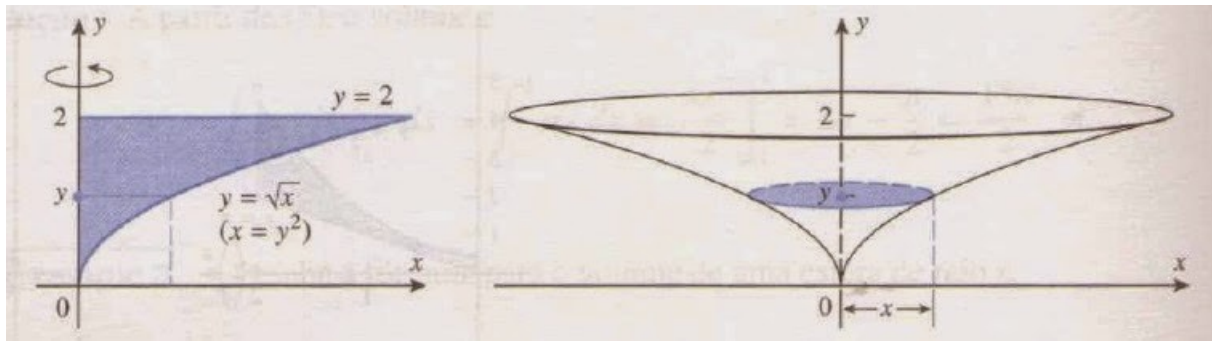
$$\frac{\pi}{15} [-2(3 \cdot 2^4 + 5 \cdot 2^2 - 45 \cdot 2 - 120) + (-1)(3(-1)^4 + 5(-1)^2 - 45(-1) - 120)] =$$

$$\frac{\pi}{15} [-2(48 + 20 - 90 - 120) - (3 + 5 + 45 - 120)] = \frac{\pi}{15} [-2(-142) - (-67)] = \frac{\pi}{15} [284 + 67] =$$

$$\frac{\pi}{15} 351 = \frac{117\pi}{5} = 23,4\pi$$

(115)

## Cálculo de volume com integrais simples – Aula 2



Volume do cilindro:  $v = \pi r^2 h$ ;  $r = x = f(y)$ ;  $h = \partial y$ ;  $\partial v = \pi [f(y)]^2 \partial y$

Volume do sólido:  $v = \int_a^b \partial v = \int_a^b \pi [f(y)]^2 \partial y \rightarrow v = \pi \int_a^b [f(y)]^2 \partial y$

### Exercício I

A região limitada pelas funções  $y = x^3$  e  $y = 8$ , gira em torno do eixo  $y$ . Determine o volume do sólido de revolução obtido.

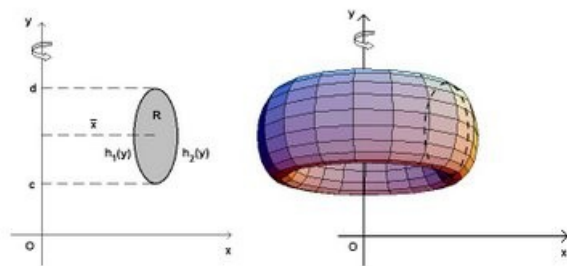
Volume do sólido em torno do eixo  $y$ :

$$\begin{aligned}
 y &= f(x) = x^3 \rightarrow x = f(y) = \sqrt[3]{y} \\
 f(x) = 0 &\rightarrow x^3 = 0 \rightarrow x = \sqrt[3]{0} = 0 \rightarrow x_1 = 0; f(x) = 8 \rightarrow x^3 = 8 \rightarrow x = \sqrt[3]{8} = 2 \rightarrow x_2 = 2 \\
 f(y) = 0 &\rightarrow \sqrt[3]{y} = 0 \rightarrow y = 0^3 = 0 \rightarrow y_1 = 0; f(y) = 2 \rightarrow \sqrt[3]{y} = 2 \rightarrow y = 2^3 = 8 \rightarrow y_2 = 8 \\
 v &= \pi \int_0^8 [f(y)]^2 \partial y = \pi \int_0^8 (\sqrt[3]{y})^2 \partial y = \pi \int_0^8 y^{\frac{2}{3}} \partial y = \pi \left[ \frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} \right]_0^8 = \pi \left[ \frac{3 \sqrt[3]{y^5}}{5} \right]_0^8 = \frac{3\pi}{5} [\sqrt[3]{y^5}]_0^8 = \\
 &= \frac{3\pi}{5} [\sqrt[3]{8^5} - \sqrt[3]{0^5}] = \frac{3 \sqrt[3]{(2^3)^5} \pi}{5} = \frac{3 \cdot 2^5 \pi}{5} = \frac{96\pi}{5} = 19,2\pi
 \end{aligned} \tag{116}$$

Volume do sólido em torno do eixo  $x$ :

$$v = \pi \int_0^2 [f(x)]^2 \partial x = \pi \int_0^2 [x^3]^2 \partial x = \pi \int_0^2 x^6 \partial x = \pi \left[ \frac{x^7}{7} \right]_0^2 = \frac{\pi}{7} [x^7]_0^2 = \frac{\pi}{7} [2^7 - 0^7] = \frac{128\pi}{7} = 18,2857142\pi$$

$$\lim_{x \rightarrow \infty} f(x) \rightarrow \lim_{x \rightarrow \infty} [x^3] = \infty^3 = \infty; \lim_{x \rightarrow -\infty} f(x) \rightarrow \lim_{x \rightarrow -\infty} [x^3] = (-\infty)^3 = -\infty$$



Volume do cilindro:  $v = \pi r_1^2 h - \pi r_2^2 h = \pi (r_1^2 - r_2^2) h$ ;  $r_1 = f(y)$ ;  $r_2 = g(y)$ ;  $h = dy$

$$\partial v = \pi [(f(y))^2 - (g(y))^2] \partial y$$

Volume do sólido:  $v = \int_a^b \partial v = \int_a^b \pi [(f(y))^2 - (g(y))^2] \partial y \rightarrow v = \pi \int_a^b [(f(y))^2 - (g(y))^2] \partial y$

## Integral por Partes – Aula 1

$$\int u \partial v = uv - \int v \partial u$$

Regras: LIATE ou ILATE

L = Logaritmica:  $\ln|x|$ ;  $\ln|x+1|$

I = Inversa da trigonometria:  $\arctg(x)$ ;  $\arcsen(x)$

A = Aritmética:  $x^2$ ;  $x^{-3}$

T = Trigonétrica:  $\sen(x)$ ;  $\cos(x)$

E = Exponencial:  $e^x$ ;  $e^{x+1}$

Exercício I

$$\int x^2 \sen(x) \partial x = x^2(-\cos(x)) - \int (-\cos(x)) 2x \partial x = -x^2 \cos(x) + 2 \int x \cos(x) \partial x =$$

$$-x^2 \cos(x) + 2(x \sen(x) + \cos(x) + c) = -x^2 \cos(x) + 2x \sen(x) + 2 \cos(x) + c$$

$$u = x^2 \rightarrow \partial u = 2x \partial x$$

$$v = -\cos(x) \rightarrow \partial v = \sen(x) \partial x$$

$$\int x \cos(x) \partial x = x \sen(x) - \int \sen(x) \partial = x \sen(x) - (-\cos(x)) + c = x \sen(x) + \cos(x) + c$$

$$u = x \rightarrow \partial u = \partial x$$

$$v = \sen(x) \rightarrow \partial v = \cos(x) \partial x$$

(117)

$$\frac{\partial (-x^2 \cos(x) + 2x \sen(x) + 2 \cos(x) + c)}{\partial x} =$$

$$\left[ -(2x \cos(x) + x^2(-\sen(x))) \right] + (2 \sen(x) + 2x \cos(x)) + 2(-\sen(x)) + 0 =$$

$$-2x \cos(x) + x^2 \sen(x) + 2 \sen(x) + 2x \cos(x) - 2 \sen(x) = x^2 \sen(x)$$

### Exercício II

$$\begin{aligned}
 \int \ln|x| \operatorname{tg}(x) \partial x &= \ln|x| \ln|\sec(x)| - \int \ln|\sec(x)| x^{-1} \partial x = \\
 \ln|x| \ln|\cos^{-1}(x)| - \int \ln|\cos^{-1}(x)| x^{-1} \partial x &= -\ln|x| \ln|\cos(x)| + \int \ln|\cos(x)| x^{-1} \partial x = \\
 -\ln|x| \ln|\cos(x)| + \ln|\cos(x)| \ln|x| + \int \ln|x| \operatorname{tg}(x) \partial x \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \\
 v &= \ln|\sec(x)| \rightarrow \partial v = \operatorname{tg}(x) \partial x \\
 \int \ln|\cos(x)| x^{-1} \partial x &= \ln|\cos(x)| \ln|x| + \int \ln|x| \operatorname{tg}(x) \partial x \\
 u &= \ln|\cos(x)| \rightarrow \partial u = \frac{1}{\cos(x)} (-\operatorname{sen}(x)) \partial x = \frac{-\operatorname{sen}(x)}{\cos(x)} \partial x = -\operatorname{tg}(x) \partial x \\
 v &= \ln|x| \rightarrow \partial v = \frac{1}{x} \partial x
 \end{aligned} \tag{118}$$

### Exercício III

$$\begin{aligned}
 \int x \operatorname{sen}(x) \partial x &= -x \cos(x) + \int \cos(x) \partial x = -x \cos(x) + \operatorname{sen}(x) + c \\
 u &= x \rightarrow \partial u = \partial x \\
 v &= -\cos(x) \rightarrow \partial v = \operatorname{sen}(x) \partial x \\
 \frac{\partial(-x \cos(x) + \operatorname{sen}(x) + c)}{\partial x} &= -[\cos(x) + x(-\operatorname{sen}(x))] + \cos(x) + 0 = \\
 -\cos(x) + x \operatorname{sen}(x) + \cos(x) &= x \operatorname{sen}(x)
 \end{aligned} \tag{119}$$

### Exercício IV

$$\begin{aligned}
 \int x e^x \partial x &= x e^x - \int e^x \partial x = x e^x - e^x + c \\
 u &= x \rightarrow \partial u = \partial x \\
 \partial v &= e^x \partial x \rightarrow v = \int \partial v = e^x \\
 \frac{\partial(x e^x - e^x + c)}{\partial x} &= e^x + x e^x - e^x + 0 = x e^x
 \end{aligned} \tag{120}$$

### Exercício V

$$\begin{aligned}
 \int \ln|x| \partial x &= \ln|x| x - \int x \frac{1}{x} \partial x = x \ln|x| - \int \partial x = x \ln|x| - x + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \\
 \partial v &= \partial x \rightarrow v = \int \partial v = x \\
 \frac{\partial(x \ln|x| - x + c)}{\partial x} &= \ln|x| + x \frac{1}{x} - 1 + 0 = \ln|x| + 1 - 1 = \ln|x|
 \end{aligned} \tag{121}$$

Exercício VI

$$\begin{aligned}
 \int x^2 \cos(x) \partial x &= x^2 \operatorname{sen}(x) - 2 \int \operatorname{sen}(x) x \partial x = x^2 \operatorname{sen}(x) - 2(-x \cos(x) + \operatorname{sen}(x)) + c = \\
 &\quad x^2 \operatorname{sen}(x) + 2x \cos(x) - 2 \operatorname{sen}(x) + c \\
 u &= x^2 \rightarrow \partial u = 2x \partial x \\
 \partial v &= \cos(x) \partial x \rightarrow v = \int \partial v = \operatorname{sen}(x) \\
 \int \operatorname{sen}(x) x \partial x &= -x \cos(x) + \int \cos(x) \partial x = -x \cos(x) + \operatorname{sen}(x) + c \\
 u &= x \rightarrow \partial u = \partial x \\
 \partial v &= \operatorname{sen}(x) \partial x \rightarrow v = \int \partial v = -\cos(x) \\
 &\quad \frac{\partial (x^2 \operatorname{sen}(x) + 2x \cos(x) - 2 \operatorname{sen}(x) + c)}{\partial x} = \\
 &\quad 2x \operatorname{sen}(x) + x^2 \cos(x) + 2 \cos(x) + 2x(-\operatorname{sen}(x)) - 2 \cos(x) + 0 = \\
 &\quad 2x \operatorname{sen}(x) + x^2 \cos(x) - 2x \operatorname{sen}(x) = x^2 \cos(x)
 \end{aligned} \tag{122}$$

Exercício VII

$$\begin{aligned}
 \int \frac{x}{\sqrt{x+1}} \partial x &= \int \frac{x}{(x+1)^{\frac{1}{2}}} \partial x = \int (x+1)^{-\frac{1}{2}} x \partial x = 2x \sqrt{x+1} - 2 \int \sqrt{x+1} \partial x = \\
 2x \sqrt{x+1} - 2 \int (x+1)^{\frac{1}{2}} \partial x &= 2x \sqrt{x+1} - 2 \frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c = 2x \sqrt{x+1} - \frac{4 \sqrt{(x+1)^3}}{3} + c \\
 u &= x \rightarrow \partial u = \partial x \\
 \partial v &= (x+1)^{-\frac{1}{2}} \partial x \rightarrow v = \int \partial v = \frac{(x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = 2 \sqrt{x+1} \\
 &\quad \frac{\partial \left( 2x \sqrt{x+1} - \frac{4 \sqrt{(x+1)^3}}{3} + c \right)}{\partial x} = \frac{\partial \left( 2x(x+1)^{\frac{1}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}} + c \right)}{\partial x} = \\
 2(x+1)^{\frac{1}{2}} + 2x \frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{4}{3} \frac{3}{2}(x+1)^{\frac{1}{2}} + 0 &= 2 \sqrt{x+1} + \frac{x}{\sqrt{x+1}} - 2 \sqrt{x+1} = \frac{x}{\sqrt{x+1}}
 \end{aligned} \tag{123}$$



## Integral por Partes – Aula 2

### Exercício I

$$\begin{aligned}
 \int (2x+1) \text{sen}(x) \partial x &= -(2x+1) \cos(x) + 2 \int \cos(x) \partial x = -(2x+1) \cos(x) + 2 \text{sen}(x) + c = \\
 &\quad -2x \cos(x) - \cos(x) + 2 \text{sen}(x) + c \\
 u &= 2x+1 \rightarrow \partial u = 2 \partial x \\
 \partial v &= \text{sen}(x) \partial x \rightarrow v = \int \partial v = -\cos(x)
 \end{aligned}
 \tag{124}$$

$$\begin{aligned}
 \frac{\partial \left( -(2x+1) \cos(x) + 2 \text{sen}(x) + c \right)}{\partial x} &= -[2 \cos(x) + (2x+1)(-\text{sen}(x))] + 2 \cos(x) + 0 = \\
 &\quad -2 \cos(x) + (2x+1) \text{sen}(x) + 2 \cos(x) = (2x+1) \text{sen}(x)
 \end{aligned}$$

### Exercício II

$$\begin{aligned}
 \int \ln|x| \frac{\partial x}{x^2} &= \int \ln|x| x^{-2} \partial x = \ln|x| \left( \frac{-1}{x} \right) + \int \frac{1}{x} \frac{1}{x} \partial x = \frac{-\ln|x|}{x} + \int x^{-2} \partial x = \frac{-\ln|x|}{x} + \frac{x^{-1}}{(-1)} + c \\
 \frac{-\ln|x|}{x} - \frac{1}{x} + c &= \frac{-\ln|x|-1}{x} + c = \frac{\ln|x^{-1}|-1}{x} + c = \frac{\ln\left|\frac{1}{x}\right|-1}{x} + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \\
 \partial v &= x^{-2} \partial x \rightarrow v = \int \partial v = \frac{x^{-1}}{(-1)} = \frac{-1}{x}
 \end{aligned}
 \tag{125}$$

$$\begin{aligned}
 \frac{\partial \left( \frac{\ln\left|\frac{1}{x}\right|-1}{x} + c \right)}{\partial x} &= \frac{\partial \left( \frac{\ln|x^{-1}|-1}{x} + c \right)}{\partial x} = \frac{\frac{1}{x^{-1}}(-x^{-2})x - (\ln|x^{-1}|-1)}{x^2} + 0 = \\
 \frac{-\frac{1}{x} \left( \frac{1}{x^2} \right) x - \left( \ln\left|\frac{1}{x}\right|-1 \right)}{x^2} &= \frac{-1 - \ln\left|\frac{1}{x}\right| + 1}{x^2} = \frac{-\ln|x^{-1}|}{x^2} = \frac{\ln|x|}{x^2}
 \end{aligned}$$

### Exercício III

$$\begin{aligned}
 \int \text{sen}(x) \cos(x) \partial x &= \text{sen}(x) \text{sen}(x) - \int \text{sen}(x) \cos(x) \partial x \rightarrow \\
 \int \text{sen}(x) \cos(x) \partial x + \int \text{sen}(x) \cos(x) \partial x &= \text{sen}^2(x) \rightarrow 2 \int \text{sen}(x) \cos(x) \partial x = \text{sen}^2(x) \rightarrow \\
 \int \text{sen}(x) \cos(x) \partial x &= \frac{\text{sen}^2(x)}{2} + c \\
 u &= \text{sen}(x) \rightarrow \partial u = \cos(x) \partial x \\
 \partial v &= \cos(x) \partial x \rightarrow v = \int \partial v = \text{sen}(x)
 \end{aligned}
 \tag{126}$$

$$\frac{\partial \left( \frac{\text{sen}^2(x)}{2} + c \right)}{\partial x} = \frac{\partial \left( \frac{1}{2} \text{sen}^2(x) + c \right)}{\partial x} = \frac{1}{2} 2 \text{sen}(x) \cos(x) + 0 = \text{sen}(x) \cos(x)$$

## Exercício IV

$$\begin{aligned}
 \int \sqrt{x} \ln|x| \partial x &= \int \ln|x| x^{\frac{1}{2}} \partial x = \ln|x| \frac{2\sqrt{x^3}}{3} - \int \frac{2\sqrt{x^3}}{3} \frac{1}{x} \partial x = \frac{2\sqrt{x^3} \ln|x|}{3} - \frac{2}{3} \int x^{\frac{3}{2}} x^{-1} \partial x = \\
 &= \frac{2\sqrt{x^3} \ln|x|}{3} - \frac{2}{3} \int x^{\frac{1}{2}} \partial x = \frac{2\sqrt{x^3} \ln|x|}{3} - \frac{2}{3} \left( \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + c = \frac{2\sqrt{x^3} \ln|x|}{3} - \frac{4\sqrt{x^3}}{9} + c = \\
 &= \frac{2\sqrt{x^2 x} \ln|x|}{3} - \frac{4\sqrt{x^2 x}}{9} + c = \frac{2x\sqrt{x} \ln|x|}{3} - \frac{4x\sqrt{x}}{9} + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \\
 \partial v &= x^{\frac{1}{2}} \partial x \rightarrow v = \int \partial v = \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} = \frac{2\sqrt{x^3}}{3}
 \end{aligned} \tag{127}$$

$$\begin{aligned}
 \frac{\partial \left( \frac{2x\sqrt{x} \ln|x|}{3} - \frac{4x\sqrt{x}}{9} + c \right)}{\partial x} &= \frac{\partial \left( \frac{2}{3} x^{\frac{3}{2}} \ln|x| - \frac{4}{9} x^{\frac{3}{2}} + c \right)}{\partial x} = \frac{2}{3} \frac{3}{2} x^{\frac{1}{2}} \ln|x| + \frac{2}{3} x^{\frac{3}{2}} \frac{1}{x} - \frac{4}{9} \frac{3}{2} x^{\frac{1}{2}} + 0 = \\
 &= x^{\frac{1}{2}} \ln|x| + \frac{2}{3} x^{\frac{3}{2}-1} - \frac{2}{3} x^{\frac{1}{2}} = x^{\frac{1}{2}} \ln|x| + \frac{2\sqrt{x}}{3} - \frac{2\sqrt{x}}{3} = \sqrt{x} \ln|x|
 \end{aligned}$$

## Integral por Partes – [Aula 3](#)

### Exercício I

$$\begin{aligned}
 \int \arcsen(x) \partial x &= x \arcsen(x) - \int x \frac{1}{\sqrt{1-x^2}} \partial x = x \arcsen(x) - \int (1-x^2)^{-\frac{1}{2}} x \partial x = \\
 &= x \arcsen(x) + \sqrt{1-x^2} + c \\
 u &= \arcsen(x) \rightarrow \partial u = \frac{1}{\sqrt{1-x^2}} \partial x \\
 \partial v &= \partial x \rightarrow v = \int \partial v = x \\
 \int (1-x^2)^{-\frac{1}{2}} x \partial x &= \int u^{-\frac{1}{2}} \left( \frac{-\partial u}{2} \right) = \frac{-1}{2} \int u^{-\frac{1}{2}} \partial u = \frac{-1}{2} \frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c = -\sqrt{u} + c = -\sqrt{1-x^2} + c \\
 u &= 1-x^2 \rightarrow \partial u = -2x \partial x \rightarrow \frac{-\partial u}{2} = x \partial x
 \end{aligned} \tag{128}$$

$$\begin{aligned}
 \frac{\partial \left( x \arcsen(x) + \sqrt{1-x^2} + c \right)}{\partial x} &= \frac{\partial \left( x \arcsen(x) + (1-x^2)^{\frac{1}{2}} + c \right)}{\partial x} = \\
 &= \arcsen(x) + x \frac{1}{\sqrt{1-x^2}} + \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x) + 0 = \arcsen(x) + \frac{x}{\sqrt{1-x^2}} - \frac{2x}{2\sqrt{1-x^2}} = \\
 &= \arcsen(x) + \frac{+x}{\sqrt{1-x^2}} - \frac{-x}{\sqrt{1-x^2}} = \arcsen(x)
 \end{aligned}$$

## Exercício II

$$\begin{aligned}
 \int \text{arctg}(x) \partial x &= x \text{arctg}(x) - \int x \frac{1}{1+x^2} \partial x = x \text{arctg}(x) - \int (1+x^2)^{-1} x \partial x = \\
 &= x \text{arctg}(x) - \frac{\ln|1+x^2|}{2} + c \\
 u &= \text{arctg}(x) \rightarrow \partial u = \frac{1}{1+x^2} \partial x \\
 \partial v &= \partial x \rightarrow v = \int \partial v = x \\
 \int (1+x^2)^{-1} x \partial x &= \int u^{-1} \frac{\partial u}{2} = \frac{1}{2} \int u^{-1} \partial u = \frac{1}{2} \ln|u| + c = \frac{\ln|u|}{2} + c = \frac{\ln|1+x^2|}{2} + c \\
 u &= 1+x^2 \rightarrow \partial u = 2x \partial x \rightarrow \frac{\partial u}{2} = x \partial x
 \end{aligned}
 \tag{129}$$

$$\begin{aligned}
 \frac{\partial \left( x \text{arctg}(x) - \frac{\ln|1+x^2|}{2} + c \right)}{\partial x} &= \frac{\partial \left( x \text{arctg}(x) - \frac{1}{2} \ln|1+x^2| + c \right)}{\partial x} = \\
 \text{arctg}(x) + x \frac{1}{1+x^2} - \frac{1}{2} \frac{1}{1+x^2} 2x + 0 &= \text{arctg}(x) + \frac{x}{1+x^2} - \frac{x}{1+x^2} = \text{arctg}(x)
 \end{aligned}$$

## Exercício III

$$\begin{aligned}
 \int x \text{arctg}(x) \partial x &= \frac{x^2}{2} \text{arctg}(x) - \int \frac{x^2}{2} \frac{1}{1+x^2} \partial x = \frac{x^2 \text{arctg}(x)}{2} - \frac{1}{2} \int (1+x^2)^{-1} x^2 \partial x = \\
 \frac{x^2 \text{arctg}(x)}{2} - \frac{1}{2} (x - \text{arctg}(x)) + c &= \frac{x^2 \text{arctg}(x)}{2} - \frac{x}{2} + \frac{\text{arctg}(x)}{2} + c = \\
 &= \frac{x^2 \text{arctg}(x) + \text{arctg}(x) - x}{2} + c \\
 u &= \text{arctg}(x) \rightarrow \partial u = \frac{1}{1+x^2} \partial x \\
 \partial v &= x \partial x \rightarrow v = \int \partial v = \frac{x^2}{2} \\
 \int (1+x^2)^{-1} x^2 \partial x &= \int \frac{x^2}{1+x^2} \partial x = \int \left( 1 - \frac{1}{1+x^2} \right) \partial x = \int \partial x - \int \frac{1}{1+x^2} \partial x = x - \text{arctg}(x) + c \\
 \frac{\partial \left( \frac{x^2 \text{arctg}(x) + \text{arctg}(x) - x}{2} + c \right)}{\partial x} &= \frac{\partial \left( \frac{1}{2} x^2 \text{arctg}(x) + \frac{1}{2} \text{arctg}(x) - \frac{1}{2} x + c \right)}{\partial x} = \\
 \frac{1}{2} \left( 2x \text{arctg}(x) + x^2 \frac{1}{1+x^2} \right) + \frac{1}{2} \frac{1}{1+x^2} - \frac{1}{2} + 0 &= x \text{arctg}(x) + \frac{x^2}{2(1+x^2)} + \frac{1}{2(1+x^2)} - \frac{1}{2} = \\
 \frac{2(1+x^2)x \text{arctg}(x) + x^2 + 1 - (1+x^2)}{2(1+x^2)} &= \frac{2(1+x^2)x \text{arctg}(x) + x^2 + 1 - 1 - x^2}{2(1+x^2)} = x \text{arctg}(x)
 \end{aligned}
 \tag{130}$$

## Integral por Partes – Aula 4

### Exercício I

$$\begin{aligned}
 \int e^x \text{sen}(x) \partial x &= \text{sen}(x) e^x - \int e^x \cos(x) \partial x = \text{sen}(x) e^x - \left( \cos(x) e^x + \int e^x \text{sen}(x) \partial x \right) = \\
 &\quad \text{sen}(x) e^x - \cos(x) e^x - \int e^x \text{sen}(x) \partial x \rightarrow \\
 \int e^x \text{sen}(x) \partial x + \int e^x \text{sen}(x) \partial x &= \text{sen}(x) e^x - \cos(x) e^x \rightarrow \\
 2 \int e^x \text{sen}(x) \partial x &= \text{sen}(x) e^x - \cos(x) e^x \rightarrow \int e^x \text{sen}(x) \partial x = \frac{\text{sen}(x) e^x - \cos(x) e^x}{2} + c = \\
 &\quad \frac{e^x (\text{sen}(x) - \cos(x))}{2} + c \\
 u &= \text{sen}(x) \rightarrow \partial u = \cos(x) \partial x \\
 \partial v &= e^x \partial x \rightarrow v = \int \partial v = e^x \\
 \int e^x \cos(x) \partial x &= \cos(x) e^x + \int e^x \text{sen}(x) \partial x \\
 u &= \cos(x) \rightarrow \partial u = -\text{sen}(x) \partial x \\
 \partial v &= e^x \partial x \rightarrow v = \int \partial v = e^x
 \end{aligned} \tag{131}$$

$$\begin{aligned}
 \frac{\partial \left( \frac{e^x (\text{sen}(x) - \cos(x))}{2} + c \right)}{\partial x} &= \frac{\partial \left( \frac{1}{2} e^x \text{sen}(x) - \frac{1}{2} e^x \cos(x) + c \right)}{\partial x} = \\
 \frac{1}{2} (e^x \text{sen}(x) + e^x \cos(x)) - \frac{1}{2} [e^x \cos(x) + e^x (-\text{sen}(x))] + 0 &= \\
 \frac{e^x \text{sen}(x)}{2} + \frac{e^x \cos(x)}{2} - \frac{e^x \cos(x)}{2} + \frac{e^x \text{sen}(x)}{2} &= \frac{2 e^x \text{sen}(x)}{2} = e^x \text{sen}(x)
 \end{aligned}$$

### Exercício II

$$\begin{aligned}
 \int \ln|x| x^5 \partial x &= \frac{x^6}{6} \ln|x| - \int \frac{x^6}{6} \frac{1}{x} \partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6} \int x^5 \partial x = \frac{x^6 \ln|x|}{6} - \frac{1}{6} \frac{x^6}{6} + c = \\
 \frac{x^6 \ln|x|}{6} - \frac{x^6}{36} + c &= \frac{6 x^6 \ln|x| - x^6}{36} + c = \frac{x^6 (6 \ln|x| - 1)}{36} + c \\
 u &= \ln|x| \rightarrow \partial u = \frac{1}{x} \partial x \\
 \partial v &= x^5 \partial x \rightarrow v = \int \partial v = \frac{x^6}{6}
 \end{aligned} \tag{132}$$

$$\begin{aligned}
 \frac{\partial \left( \frac{x^6 (6 \ln|x| - 1)}{36} + c \right)}{\partial x} &= \frac{\partial \left( \frac{1}{6} x^6 \ln|x| - \frac{1}{36} x^6 + c \right)}{\partial x} = \frac{1}{6} \left( 6 x^5 \ln|x| + x^6 \frac{1}{x} \right) - \frac{1}{36} 6 x^5 + 0 = \\
 x^5 \ln|x| + \frac{x^6}{6x} - \frac{x^5}{6} &= \frac{6 x^5 \ln|x| + x^6 - x^6}{6x} = x^5 \ln|x|
 \end{aligned}$$

### Exercício III

$$\begin{aligned}
 \int \sec^3(x) \partial x &= \int \sec(x) \sec^2(x) \partial x = \sec(x) \operatorname{tg}(x) - \int \operatorname{tg}(x) \sec(x) \operatorname{tg}(x) \partial x = \\
 &= \sec(x) \operatorname{tg}(x) - \int \operatorname{tg}^2(x) \sec(x) \partial x = \sec(x) \operatorname{tg}(x) - \int (\sec^2(x) - 1) \sec(x) \partial x = \\
 &= \sec(x) \operatorname{tg}(x) - \int \sec^3(x) \partial x + \int \sec(x) \partial x \rightarrow \\
 \int \sec^3(x) \partial x + \int \sec^3(x) \partial x &= \sec(x) \operatorname{tg}(x) + \int \sec(x) \partial x \rightarrow \\
 2 \int \sec^3(x) \partial x &= \sec(x) \operatorname{tg}(x) + \ln |\sec(x) + \operatorname{tg}(x)| + c \rightarrow \\
 \int \sec^3(x) \partial x &= \frac{\sec(x) \operatorname{tg}(x) + \ln |\sec(x) + \operatorname{tg}(x)|}{2} + c \\
 u &= \sec(x) \rightarrow \partial u = \sec(x) \operatorname{tg}(x) \partial x \\
 \partial v &= \sec^2(x) \partial x \rightarrow v = \int \partial v = \operatorname{tg}(x)
 \end{aligned}
 \tag{133}$$