Introdução às Derivadas Parciais de 1ª ordem – Aula 1

Exercício I

$$
\begin{aligned}
f(x,y) &= 4\frac{x^3}{y^2} - 2xy - 3x - 4y - 7 = 4x^3y^{-2} - 2xy - 3x - 4y - 7 \\
\frac{\partial f(x,y)}{\partial x} &= 4y^{-2}\frac{\partial(x^3)}{\partial x} - 2y\frac{\partial(x)}{\partial x} - 3\frac{\partial(x)}{\partial x} - 0 - 0 = 4y^{-2}3x^2 - 2y - 3 = \frac{12x^2}{y^2} - 2y - 3 \\
\frac{\partial f(x,y)}{\partial y} &= 4x^3\frac{\partial(y^{-2})}{\partial y} - 2x\frac{\partial(y)}{\partial y} - 0 - 4\frac{\partial(y)}{\partial y} - 0 = 4x^3(-2y^{-3}) - 2x - 4 = \frac{-8x^3}{y^3} - 2x - 4
\end{aligned} \tag{1}
$$

Derivadas Parciais: Interpretação Geométrica – Aula 2

Exercício I

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano x = -1, no ponto P(-1, 1, -2).

$$
\begin{aligned}
f(x,y) &= x^2 + y^2 - 2x^3y + 5xy^4 - 1 \\
z = f(-1,1) &= (-1)^2 + (1)^2 - 2(-1)^3(1) + 5(-1)(1)^4 - 1 = 1 + 1 + 2 - 5 - 1 = 4 - 6 = -2 \\
\frac{\partial f(x,y)}{\partial y} &= 0 + \frac{\partial(y^2)}{\partial y} - 2x^3\frac{\partial(y)}{\partial y} + 5x\frac{\partial(y^4)}{\partial y} - 0 = 2y - 2x^3 + 5x4y^3 = 2y + 20xy^3 - 2x^3 \\
\frac{\partial f(-1,1)}{\partial y} &= 2(1) + 20(-1)(1)^3 - 2(-1)^3 = 2 - 20 + 2 = -16
\end{aligned} \tag{2}
$$

Exercício II

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano y = 2, no ponto P(2, 2, 8).

$$
\begin{aligned}
f(x,y) &= x^2 + y^2 \\
z = f(2,2) &= (2)^2 + (2)^2 = 4 + 4 = 8 \\
\frac{\partial f(x,y)}{\partial x} &= \frac{\partial(x^2)}{\partial x} + 0 = 2x \\
\frac{\partial f(2,2)}{\partial x} &= 2(2) = 4
\end{aligned} \tag{3}
$$

Derivadas Parciais de 2ª ordem – Aula 3

Exercício I

$$
\begin{aligned}
f(x,y) &= x^2+y^2-2x^3y+5xy^4-1\\
\frac{\partial f(x,y)}{\partial x} &= 2x+0-2y3x^2+5y^4-0=2x-6x^2y+5y^4\\
\frac{\partial^2 f(x,y)}{\partial x^2} &= 2-6y2x=-12xy+2\\
\frac{\partial^2 f(x,y)}{\partial y\partial x} &= 0-6x^2+5\cdot 4y^3=-6x^2+20y^3\\
\frac{\partial f(x,y)}{\partial y} &= 0+2y-2x^3+5x4y^3-0=-2x^3+20xy^3+2y\\
\frac{\partial^2 f(x,y)}{\partial y^2} &= -0+20x3y^2+2=60xy^2+2\\
\frac{\partial^2 f(x,y)}{\partial x\partial y} &= -2\cdot 3x^2+20y^3+0=-6x^2+20y^3
\end{aligned}
\tag{4}
$$

Exercício II

$$
\begin{aligned}
z &= x^2y-xy^2+2x-y\\
\frac{\partial z}{\partial x} &= y\cdot 2x-y^2+2-0=2xy-y^2+2\\
\frac{\partial^2 z}{\partial x^2} &= 2y-0+0=2y\\
\frac{\partial^2 z}{\partial y\partial x} &= 2x-2y+0=2x-2y\\
\frac{\partial z}{\partial y} &= x^2-x2y+0-1=x^2-2xy-1\\
\frac{\partial^2 z}{\partial y^2} &= 0-2x-0=-2x\\
\frac{\partial^2 z}{\partial x\partial y} &= 2x-2y-0=2x-2y
\end{aligned}
\tag{5}
$$

Exercício III

$$z = xy$$

$$\frac{\partial z}{\partial x} = y$$
$$\frac{\partial^2 z}{\partial x^2} = 0$$
$$\frac{\partial^2 z}{\partial y \partial x} = 1 \tag{6}$$

$$\frac{\partial z}{\partial y} = x$$
$$\frac{\partial^2 z}{\partial y^2} = 0$$
$$\frac{\partial^2 z}{\partial x \partial y} = 1$$

Exercício IV

$$z = \ln(xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy} y = \frac{1}{x} = x^{-1}$$
$$\frac{\partial^2 z}{\partial x^2} = -x^{-2} = \frac{-1}{x^2}$$
$$\frac{\partial^2 z}{\partial y \partial x} = 0 \tag{7}$$

$$\frac{\partial z}{\partial y} = \frac{1}{xy} x = \frac{1}{y} = y^{-1}$$
$$\frac{\partial^2 z}{\partial y^2} = -y^{-2} = \frac{-1}{y^2}$$
$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

Derivadas Parciais de 2ª ordem – Aula 4

Exercício I

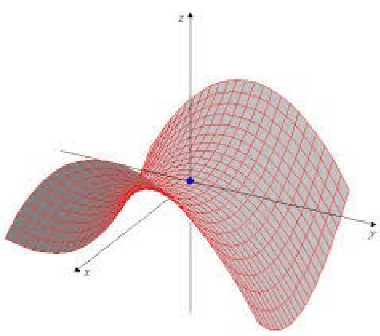
$$z=e^{-xy^2}$$

$$\frac{\partial z}{\partial x}=e^{-xy^2}(-y^2)=-y^2e^{-xy^2}$$
$$\frac{\partial^2 z}{\partial x^2}=-y^2e^{-xy^2}(-y^2)=y^4e^{-xy^2}$$
$$\frac{\partial^2 z}{\partial y\partial x}=-\left[2ye^{-xy^2}+y^2e^{-xy^2}(-x2y)\right]=-\left(2ye^{-xy^2}-2xy^3e^{-xy^2}\right)=2ye^{-xy^2}\left(xy^2-1\right) \tag{8}$$

$$\frac{\partial z}{\partial y}=e^{-xy^2}(-x2y)=-2xye^{-xy^2}$$
$$\frac{\partial^2 z}{\partial y^2}=-\left[2xe^{-xy^2}+2xye^{-xy^2}(-x2y)\right]=-\left(2xe^{-xy^2}-4x^2y^2e^{-xy^2}\right)=2xe^{-xy^2}\left(2xy^2-1\right)$$
$$\frac{\partial^2 z}{\partial x\partial y}=-\left[2ye^{-xy^2}+2xye^{-xy^2}(-y^2)\right]=-\left(2ye^{-xy^2}-2xy^3e^{-xy^2}\right)=2ye^{-xy^2}\left(xy^2-1\right)$$

Máximos, Mínimos e Sela através do Hessiano – Aula 5

1. Ache o **x** e o **y** crítico, igualando a **0** a derivada de **z** em relação a x e a y:
   $$\frac{\partial f(x,y)}{\partial x}=\frac{\partial z}{\partial x}=0\rightarrow x_c$$
   $$\frac{\partial f(x,y)}{\partial y}=\frac{\partial z}{\partial y}=0\rightarrow y_c$$
2. Calcule o determinante de **x** e **y** crítico: $H(x_c,y_c)=\begin{vmatrix}\frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y\partial x}\\ \frac{\partial^2 z}{\partial x\partial y} & \frac{\partial^2 z}{\partial y^2}\end{vmatrix}$
3. $H<0\rightarrow$ ponto de sela
   $H>0\rightarrow\frac{\partial^2 z}{\partial x^2}>0\rightarrow$ Mínimo, $\frac{\partial^2 z}{\partial x^2}<0\rightarrow$ Máximo
   $H=0\rightarrow$ NPA = Nada podemos afirmar



Exercício I

$$z=3x^4+8x^3-18x^2+6y^2+12y-4$$

1.

$$\frac{\partial z}{\partial x}=12x^3+24x^2-36x=12x(x^2+2x-3)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 12x(x^2+2x-3)=0$$

$$12x=0\rightarrow x=\frac{0}{12}=0\rightarrow x_1=0$$

$$x^2+2x-3=0\rightarrow x^2+2x-3+1-1=0\rightarrow (x^2+2x+1)-4=0\rightarrow (x+1)^2-4=0\rightarrow$$
$$(x+1)^2=4\rightarrow x+1=\pm\sqrt{4}\rightarrow x=\pm 2-1\rightarrow x_2=1,x_3=-3$$

$$\frac{\partial z}{\partial y}=12y+12=12(y+1)$$

$$\frac{\partial z}{\partial y}=0\rightarrow 12(y+1)=0$$

$$y+1=0\rightarrow y=-1$$

$P_1(-3,-1),P_2(0,-1),P_1(1,-1)$

2.

$$\frac{\partial^2 z}{\partial x^2}=36x^2+48x-36=12(3x^2+4x-3)$$

$$\frac{\partial^2 f(-3,-1)}{\partial x^2}=12(3(-3)^2+4(-3)-3)=12(27-12-3)=12\cdot 12=144$$

$$\frac{\partial^2 f(0,-1)}{\partial x^2}=12(3(0)^2+4(0)-3)=12(-3)=-36 \tag{9}$$

$$\frac{\partial^2 f(1,-1)}{\partial x^2}=12(3(1)^2+4(1)-3)=12(3+4-3)=12\cdot 4=48$$

$$\frac{\partial^2 z}{\partial y\,\partial x}=0$$

$$\frac{\partial^2 z}{\partial x\,\partial y}=0$$

$$\frac{\partial^2 z}{\partial y^2}=12$$

$$H(-3,-1)=\begin{vmatrix}144 & 0\\ 0 & 12\end{vmatrix}=(144\cdot 12)-(0\cdot 0)=1728$$

$$H(0,-1)=\begin{vmatrix}-36 & 0\\ 0 & 12\end{vmatrix}=-36\cdot 12=-432$$

$$H(1,-1)=\begin{vmatrix}48 & 0\\ 0 & 12\end{vmatrix}=48\cdot 12=576$$

3.

$$P_1(-3,-1)\rightarrow H(-3,-1)>0\rightarrow \frac{\partial^2 f(-3,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

$$P_2(0,-1)\rightarrow H(0,-1)<0\rightarrow \text{é pto de sela}$$

$$P_3(1,-1)\rightarrow H(1,-1)>0\rightarrow \frac{\partial^2 f(1,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

Exercício II

$$z=x^3+3xy+y^2-2$$

1.

$$\frac{\partial z}{\partial x}=3x^2+3y=3(x^2+y)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 3(x^2+y)=0$$

$$x^2+y=0\rightarrow y=-x^2\rightarrow 3x+2(-x^2)=0\rightarrow 3x-2x^2=0\rightarrow x(3-2x)=0$$

$$x_1=0$$

$$3-2x=0\rightarrow 3=2x\rightarrow x_2=\frac{3}{2}$$

$$\frac{\partial z}{\partial y}=3x+2y$$

$$\frac{\partial z}{\partial y}=0\rightarrow 3x+2y=0\rightarrow 2y=-3x\rightarrow y=\frac{-3x}{2}$$

$$y=\frac{-3(0)}{2}\rightarrow y_1=0$$

$$y=\frac{-3\left(\frac{3}{2}\right)}{2}\rightarrow y_2=\frac{-9}{4}$$

$$P_1(0,0),P_2\left(\frac{3}{2},\frac{-9}{4}\right)$$

2.

$$\frac{\partial^2 z}{\partial x^2}=6x \tag{10}$$

$$\frac{\partial^2 f(0,0)}{\partial x^2}=6(0)=0$$

$$\frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}=6\left(\frac{3}{2}\right)=9$$

$$\frac{\partial^2 z}{\partial y\partial x}=3$$

$$\frac{\partial^2 z}{\partial x\partial y}=3$$

$$\frac{\partial^2 z}{\partial y^2}=2$$

$$H(0,0)=\begin{vmatrix}0 & 3\\ 3 & 2\end{vmatrix}=(0\cdot 2)-(3\cdot 3)=0-9=-9$$

$$H\left(\frac{3}{2},\frac{-9}{4}\right)=\begin{vmatrix}9 & 3\\ 3 & 2\end{vmatrix}=(9\cdot 2)-(3\cdot 3)=18-9=9$$

3.

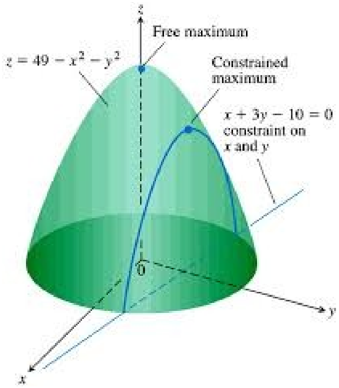
$$P_1(0,0)\rightarrow H(0,0)<0\rightarrow \text{é pto de sela}$$

$$P_2\left(\frac{3}{2},\frac{-9}{4}\right)\rightarrow H\left(\frac{3}{2},\frac{-9}{4}\right)>0\rightarrow \frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

Máximos e Mínimos Condicionados com Multiplicadores de Lagrange – Aula 6

Função: $f(x,y)$
Restrição: $r(x,y)=0$

Função de Lagrange: $L(x,y,\lambda)=f(x,y)-\lambda r(x,y)$
$$\frac{\partial L}{\partial x}=0, \frac{\partial L}{\partial y}=0, r(x,y)=0$$



Exercício I

Ache o ponto de máximo ou mínimo da função a seguir:
$f(x,y)=x^2+y^2$, sujeito a restrição $x+y=4$

$$
\begin{gathered}
r(x,y)=0 \rightarrow x+y-4=0 \rightarrow r(x,y)=x+y-4 \\
L(x,y,\lambda)=f(x,y)-\lambda r(x,y)=(x^2+y^2)-\lambda(x+y-4)=x^2+y^2-\lambda x-\lambda y+4\lambda \\
\frac{\partial L}{\partial x}=2x+0-\lambda-0+0=2x-\lambda \\
\frac{\partial L}{\partial x}=0 \rightarrow 2x-\lambda=0 \rightarrow 2x=\lambda \rightarrow x=\frac{\lambda}{2} \\
\frac{\partial L}{\partial y}=0+2y-0-\lambda+0=2y-\lambda \\
\frac{\partial L}{\partial y}=0 \rightarrow 2y-\lambda=0 \rightarrow 2y=\lambda \rightarrow y=\frac{\lambda}{2} \\
x+y-4=0 \rightarrow \frac{\lambda}{2}+\frac{\lambda}{2}-4=0 \rightarrow \frac{2\lambda}{2}=4 \rightarrow \lambda=4 \\
x=\frac{4}{2}=2, y=\frac{4}{2}=2 \rightarrow P(2,2) \\
f(2,2)=(2)^2+(2)^2=4+4=8 \\
x=0 \rightarrow x+y=4 \rightarrow y=4-x=4-(0)=4 \rightarrow f(0,4)=(0)^2+(4)^2=16 \\
P(2,2) \rightarrow f(0,4)>f(2,2) \rightarrow \text{pto de mínimo}
\end{gathered}
\tag{11}
$$

Exercício II

$$\begin{gathered}
\text{Função: } f(x,y)=9-x^2-y^2 \\
\text{Restrição: } x+y=2 \rightarrow x+y-2=0 \rightarrow r(x,y)=x+y-2 \\
L(x,y,\lambda)=(9-x^2-y^2)-\lambda(x+y-2)=9-x^2-y^2-\lambda x-\lambda y+2\lambda \\
\frac{\partial L}{\partial x}=0-2x-0-\lambda-0+0=-2x-\lambda \\
\frac{\partial L}{\partial x}=0 \rightarrow -2x-\lambda=0 \rightarrow -\lambda=2x \rightarrow x=\frac{-\lambda}{2} \\
\frac{\partial L}{\partial y}=0-0-2y-0-\lambda+0=-2y-\lambda \\
\frac{\partial L}{\partial y}=0 \rightarrow -2y-\lambda=0 \rightarrow -\lambda=2y \rightarrow y=\frac{-\lambda}{2} \\
x+y=2 \rightarrow -\frac{\lambda}{2}+\left(\frac{-\lambda}{2}\right)=2 \rightarrow \frac{-2\lambda}{2}=2 \rightarrow \lambda=-2 \\
x=\frac{-\lambda}{2}=\frac{-(-2)}{2}=1,\ y=\frac{-\lambda}{2}=\frac{-(-2)}{2}=1 \rightarrow P(1,1) \\
f(1,1)=9-(1)^2-(1)^2=9-1-1=7 \\
x=0 \rightarrow x+y=2 \rightarrow y=2-x=2-(0)=2 \rightarrow f(0,2)=9-(0)^2-(2)^2=9-4=5 \\
P(1,1) \rightarrow f(1,1)>f(0,2) \rightarrow \text{pto de máximo}
\end{gathered} \tag{12}$$

Exercício III

Seja a função lucro de uma indústria, $f(x,y)=-2x^2-y^2+32x+20y$ que produz e comercializa dois produtos em quantidades x e y. Calcule o lucro máximo, sabendo que a produção da indústria é limitada em 24 unidades.

Função: $f(x,y)=-2x^2-y^2+32x+20y$

Restrição: $x+y=24 \rightarrow x+y-24=0 \rightarrow r(x,y)=x+y-24$

$$
\begin{gathered}
L(x,y,\lambda)=(-2x^2-y^2+32x+20y)-\lambda(x+y-24)= \\
-2x^2-y^2+32x+20y-\lambda x-\lambda y+24\lambda \\
\frac{\partial L}{\partial x}=-4x-0+32+0-\lambda-0+0=-4x+32-\lambda \\
\frac{\partial L}{\partial x}=0 \rightarrow -4x+32-\lambda=0 \rightarrow 32-\lambda=4x \rightarrow x=\frac{32-\lambda}{4} \\
\frac{\partial L}{\partial y}=-0-2y+0+20-0-\lambda+0=-2y+20-\lambda \\
\frac{\partial L}{\partial y}=0 \rightarrow -2y+20-\lambda=0 \rightarrow 20-\lambda=2y \rightarrow y=\frac{20-\lambda}{2} \\
x+y=24 \rightarrow \frac{32-\lambda}{4}+\frac{20-\lambda}{2}=24 \rightarrow 32-\lambda+2(20-\lambda)=96 \rightarrow 32-\lambda+40-2\lambda=96 \rightarrow \\
72-3\lambda=96 \rightarrow -3\lambda=96-72 \rightarrow 3\lambda=-24 \rightarrow \lambda=\frac{-24}{3}=-8 \\
x=\frac{32-\lambda}{4}=\frac{32-(-8)}{4}=\frac{40}{4}=10,\ y=\frac{20-\lambda}{2}=\frac{20-(-8)}{2}=\frac{28}{2}=14 \rightarrow P(10,14) \\
f(10,14)=-2(10)^2-(14)^2+32(10)+20(14)= \\
-200-196+320+280=204 \\
x=0 \rightarrow x+y=24 \rightarrow y=24-x=24-(0)=24 \rightarrow f(0,24)=-2(0)^2-(24)^2+32(0)+20(24)= \\
-576+480=-96 \\
P(10,14) \rightarrow f(10,14)>f(0,24) \rightarrow \text{pto de máximo}
\end{gathered}
\tag{13}
$$

Derivada Direcional – Aula 7

$$
\begin{gathered}
D_u f(x,y)=\frac{\partial f(x,y)}{\partial x}U_1+\frac{\partial f(x,y)}{\partial y}U_2 \\
D_u f(x,y)=U\cdot\nabla f(x,y) \\
D_u f(x,y,z)=\frac{\partial f(x,y,z)}{\partial x}U_1+\frac{\partial f(x,y,z)}{\partial y}U_2+\frac{\partial f(x,y,z)}{\partial z}U_3
\end{gathered}
$$

$$
\begin{gathered}
\vec{U}=U_1\vec{i}+U_2\vec{j} \\
|\vec{U}|=\sqrt{x^2+y^2}=1 \\
|\vec{U}|\neq 1 \rightarrow \frac{\vec{U}}{|\vec{U}|}
\end{gathered}
$$

Exercício I

Qual o valor da derivada direcional da função f(x, y), no ponto P(1, 1) e na direção do vetor U.

$$\begin{gathered} f(x,y)=x^2+y^2, P(1,1), \vec{U}=\left(\frac{3}{5},\frac{4}{5}\right) \\ \frac{\partial f(x,y)}{\partial x}=2x, \frac{\partial f(x,y)}{\partial y}=2y \\ D_u f(x,y)=2x\frac{3}{5}+2y\frac{4}{5}=\frac{6x}{5}+\frac{8y}{5}=\frac{6x+8y}{5}=\frac{2}{5}(3x+4y) \\ D_u f(1,1)=\frac{2}{5}[3(1)+4(1)]=\frac{2}{5}7=\frac{14}{5}=2{,}8 \\ |\vec{U}|=1 \rightarrow \sqrt{x^2+y^2}=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2}=\sqrt{\frac{9}{25}+\frac{16}{25}}=\sqrt{\frac{9+16}{25}}=\sqrt{\frac{25}{25}}=\sqrt{1}=1 \end{gathered} \tag{14}$$

Exercício II

$$\begin{gathered} f(x,y)=x^2+y^2, P(1,1), \vec{V}=\left(\frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right) \\ D_u f(x,y)=2x\left(\frac{-\sqrt{2}}{2}\right)+2y\frac{\sqrt{2}}{2}=-x\sqrt{2}+y\sqrt{2}=-\sqrt{2}(x-y) \\ D_u f(1,1)=-\sqrt{2}[(1)-(1)]=-\sqrt{2}\cdot 0=0 \\ |\vec{V}|=\sqrt{\left(\frac{-\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2}=\sqrt{\frac{2}{4}+\frac{2}{4}}=\sqrt{\frac{2+2}{4}}=\sqrt{\frac{4}{4}}=\sqrt{1}=1 \end{gathered} \tag{15}$$

Exercício III

$$\begin{gathered} f(x,y)=x^2+y^2, P(1,1), \vec{W}=(0,-1) \\ D_u f(x,y)=2x\cdot 0+2y(-1)=-2y \\ D_u f(1,1)=-2(1)=-2 \\ |\vec{W}|=\sqrt{0^2+(-1)^2}=\sqrt{1}=1 \end{gathered} \tag{16}$$

Exercício IV

$$\begin{gathered} f(x,y)=x^5+\operatorname{sen} y, P(1,2), \vec{U}=\frac{3}{5}\vec{i}-\frac{4}{5}\vec{j} \\ |\vec{U}|=\sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{-4}{5}\right)^2}=\frac{9}{25}+\frac{16}{25}=\sqrt{\frac{9+16}{25}}=\sqrt{\frac{25}{25}}=\sqrt{1}=1 \\ D_u f(x,y)=5x^4\frac{3}{5}+\cos y\left(\frac{-4}{5}\right)=3x^4-\frac{4}{5}\cos y \\ D_u f(1,2)=3(1)^4-\frac{4}{5}\cos 2=3-\frac{4}{5}\cos 2=\frac{15-4}{5}\cos 2=\frac{11}{5}\cos 2 \end{gathered} \tag{17}$$

Exercício V

Qual o valor da derivada direcional da função f(x, y), quando um ponto se desloca a partir do ponto P(3, 1) e na direção do ponto P(4, -3).

$$\begin{gathered}
f(x,y)=x^2y^5,\ P_1(3,1),\ P_2(4,-3)\\
\vec{U}=\overrightarrow{P_1P_2}=P_2-P_1=(4,-3)-(3,1)=(1,-4)\\
|\vec{U}|=\sqrt{(1)^2+(-4)^2}=\sqrt{1+16}=\sqrt{17}\rightarrow|\vec{U}|\neq 1\\
\frac{\vec{U}}{|\vec{U}|}=\left(\frac{1}{\sqrt{17}},\frac{-4}{\sqrt{17}}\right)=\left(\frac{\sqrt{17}}{17},\frac{-4\sqrt{17}}{17}\right)\\
|VE\vec{R}S\,U|=\sqrt{\left(\frac{1}{\sqrt{17}}\right)^2+\left(\frac{-4}{\sqrt{17}}\right)^2}=\sqrt{\frac{1}{17}+\frac{16}{17}}=\sqrt{\frac{17}{17}}=1\\
D_uf(x,y)=y^5\,2x\frac{\sqrt{17}}{17}+x^2\,5y^4\left(\frac{-4\sqrt{17}}{17}\right)=2xy^5\frac{\sqrt{17}}{17}+5x^2y^4\left(\frac{-4\sqrt{17}}{17}\right)=\\
\frac{2\sqrt{17}xy^5-20\sqrt{17}x^2y^4}{17}=\frac{2\sqrt{17}xy^4}{17}(y-10x)\\
D_uf(3,1)=\frac{2\sqrt{17}(3)(1)^4}{17}((1)-10(3))=\frac{6\sqrt{17}}{17}(1-30)=\frac{6\sqrt{17}}{17}(-29)=\frac{-174\sqrt{17}}{17}
\end{gathered} \tag{18}$$

Exercício VI

Se $f(x,y)=3x^2-y^2+4x$ e $\vec{U}$ é o vetor unitário na direção $\frac{\pi}{6}$, calcule $D_uf(1,0)$

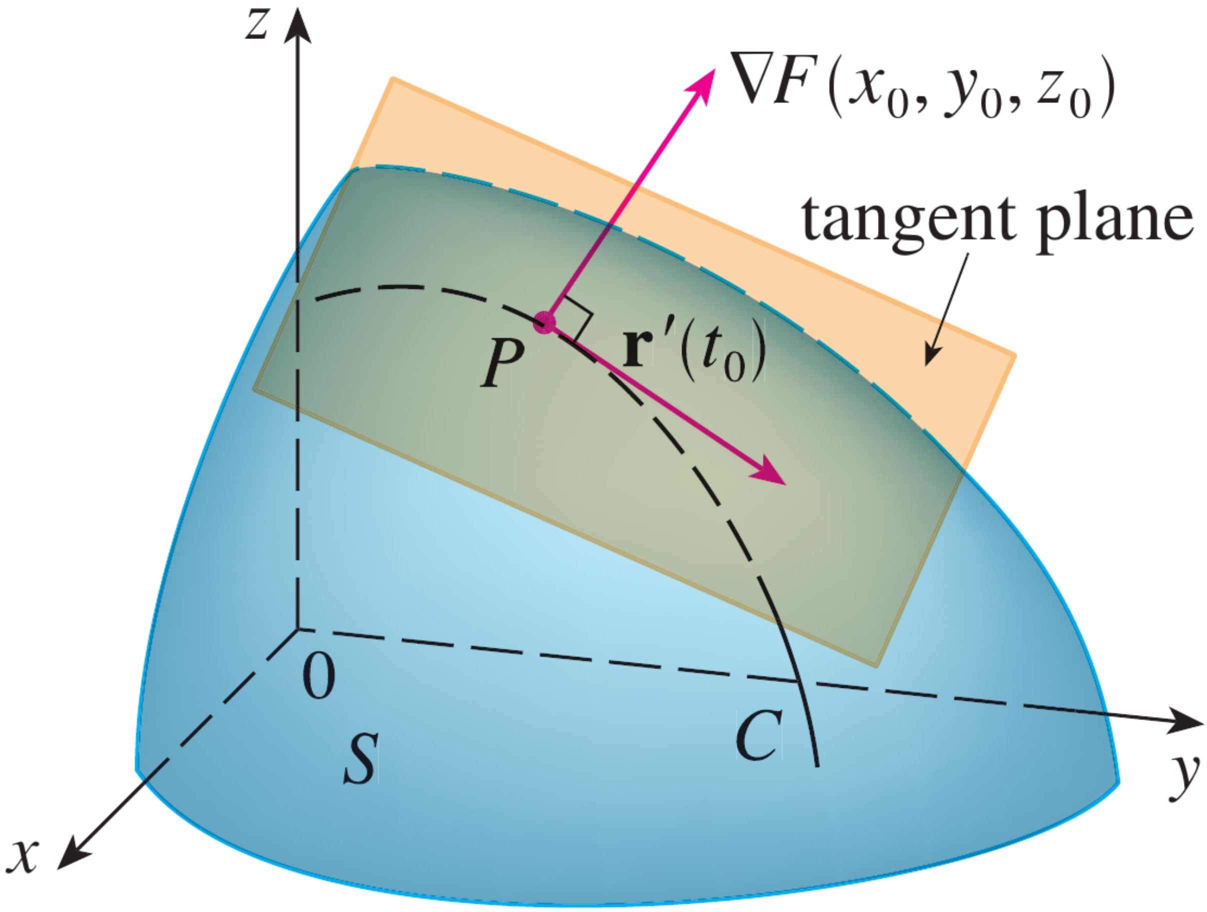
$$\begin{gathered}
f(x,y)=3x^2-y^2+4x,\ P(1,0),\ \vec{U}=\left(\cos\frac{\pi}{6},sen\frac{\pi}{6}\right)=(\cos 30^o,sen\,30^o)=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)\\
|\vec{U}|=\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{3}{4}+\frac{1}{4}}=\sqrt{\frac{3+1}{4}}=\sqrt{\frac{4}{4}}=\sqrt{1}=1\\
D_uf(x,y)=(6x+4)\frac{\sqrt{3}}{2}+(-2y)\frac{1}{2}=\frac{\sqrt{3}(6x+4)}{2}-y=\frac{6\sqrt{3}x+4\sqrt{3}}{2}-y=3\sqrt{3}x+2\sqrt{3}-y\\
D_uf(1,0)=3\sqrt{3}(1)+2\sqrt{3}-(0)=5\sqrt{3}
\end{gathered} \tag{19}$$

Vetor Gradiente – Aula 8

Gradiente ou Vetor Gradiente é um vetor que indica o sentido e a direção de maior alteração no valor de uma quantidade por unidade de espaço.

$$\nabla f(x,y)=\frac{\partial f(x,y)}{\partial x}\vec{i}+\frac{\partial f(x,y)}{\partial y}\vec{j}$$

$$\nabla f(x,y,z)=\frac{\partial f(x,y,z)}{\partial x}\vec{i}+\frac{\partial f(x,y,z)}{\partial y}\vec{j}+\frac{\partial f(x,y,z)}{\partial z}\vec{k}$$



Exercício I

$$\begin{gathered}
f(x,y)=x^2e^y, P(-2,0)\\
\frac{\partial f(x,y)}{\partial x}=e^y 2x=2xe^y \rightarrow \frac{\partial f(-2,0)}{\partial x}=2(-2)e^0=-4\\
\frac{\partial f(x,y)}{\partial y}=x^2e^y \rightarrow \frac{\partial f(-2,0)}{\partial y}=(-2)^2e^0=4\\
\nabla f(-2,0)=\frac{\partial f(-2,0)}{\partial x}\vec{i}+\frac{\partial f(-2,0)}{\partial y}\vec{j}=-4\,\vec{i}+4\,\vec{j}\\
|\nabla f(-2,0)|=\sqrt{(-4)^2+(4)^2}=\sqrt{16-16}=\sqrt{32}=\sqrt{2^2\cdot 2^2\cdot 2}=4\sqrt{2}
\end{gathered} \tag{20}$$

Exercício II

$$\begin{gathered}
f(x,y,z)=xy^2z^3, P(1,1,1)\\
\frac{\partial f(x,y,z)}{\partial x}=y^2z^3 \rightarrow \frac{\partial f(1,1,1)}{\partial x}=1^2 1^3=1\\
\frac{\partial f(x,y,z)}{\partial y}=xz^3 2y=2xyz^3 \rightarrow \frac{\partial f(1,1,1)}{\partial y}=2\cdot 1\cdot 1\cdot 1^3=2\\
\frac{\partial f(x,y,z)}{\partial z}=xy^2 3z^2=3xy^2z^2 \rightarrow \frac{\partial f(1,1,1)}{\partial z}=3\cdot 1\cdot 1^2\cdot 1^2=3\\
\nabla f(1,1,1)=\vec{i}+2\vec{j}+3\vec{k}\\
|\nabla f(1,1,1)|=\sqrt{1^2+2^2+3^2}=\sqrt{1+4+9}=\sqrt{14}
\end{gathered} \tag{21}$$

Exercício III

$$\begin{gathered}
f(x,y)=x^2+y^2, P(3,4)\\
\nabla f(x,y)=2x\vec{i}+2y\vec{j} \rightarrow \nabla f(3,4)=2(3)\vec{i}+2(4)\vec{j}=6\vec{i}+8\vec{j}\\
|\nabla f(3,4)|=\sqrt{6^2+8^2}=\sqrt{36+64}=\sqrt{100}=10
\end{gathered} \tag{22}$$

Exercício IV

$$\begin{gathered}
f(x,y)=2x^2-y^2+3x-y, P(1,-2)\\
\nabla f(x,y)=(4x+3)\vec{i}+(-2y-1)\vec{j} \rightarrow \nabla f(1,-2)=(4(1)+3)\vec{i}+(-2(-2)-1)\vec{j}=7\vec{i}+3\vec{j}\\
|\nabla f(1,-2)|=\sqrt{7^2+3^2}=\sqrt{49+9}=\sqrt{58}
\end{gathered} \tag{23}$$

Exercício V

A temperatura em qualquer ponto de uma lâmina plana, é determinada pela função $f(x,y)=x^2+y^2$

a) Encontre a taxa de variação da temperatura no ponto P(3, 4) e na direção pi/3 com o sentido positivo do eixo dos x.
b) Encontre a taxa de variação máxima no ponto P(3, 4).

$$\begin{gathered}
f(x,y)=x^2+y^2, P(3,4), \vec{U}=(\cos\frac{\pi}{3}, sen\frac{\pi}{3})=(\cos 60^o, sen 60^o)=\frac{1}{2}\vec{i}+\frac{\sqrt{3}}{2}\vec{j}\\
\nabla f(x,y)=2x\vec{i}+2y\vec{j} \rightarrow \nabla f(3,4)=2(3)\vec{i}+2(4)\vec{j}=6\vec{i}+8\vec{j}\\
|\vec{U}|=1\\
D_u f(x,y)=U\cdot\nabla f(x,y) \rightarrow D_u f(3,4)=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)(6,8)=\frac{1}{2}6+\frac{\sqrt{3}}{2}8=3+4\sqrt{3}\\
|\nabla f(x,y)|=\sqrt{6^2+8^2}=\sqrt{36+64}=\sqrt{100}=10
\end{gathered} \tag{24}$$