Integral Indefinida – Aula 1

01. $\int \partial x \quad = \quad x+c$

02. $\int x^p \partial x \quad = \quad \dfrac{x^{p+1}}{p+1}+c \rightarrow p \neq -1$

03. $\int e^x \partial x \quad = \quad e^x+c$

04. $\int \dfrac{\partial x}{x} \quad = \quad \ln|x|+c$

05. $\int u^p \partial u \quad = \quad \dfrac{u^{p+1}}{p+1}+c \rightarrow p \neq -1$

06. $\int e^u \partial u \quad = \quad e^u+c$

07. $\int \dfrac{\partial u}{u} \quad = \quad \ln|u|+c$

08. $\int a^u \partial u \quad = \quad \dfrac{a^u}{\ln|a|}+c$

Exercício I

$$\int \partial x = x+c$$
$$\int x^3 \partial x = \frac{x^4}{4}+c \tag{1}$$

Exercício II

$$\int 3x^4 \partial x = 3\int x^4 \partial x = 3\frac{x^5}{5}+c = \frac{3x^5}{5}+c \tag{2}$$

Exercício III

$$\int (4x^5+7)\partial x = \int 4x^5 \partial x + \int 7 \partial x = 4\int x^5 \partial x + 7\int \partial x = 4\frac{x^6}{6}+7x+c = \frac{2x^6}{3}+7x+c \tag{3}$$

Exercício IV

$$\int 3\partial x = 3\int \partial x = 3x+c \tag{4}$$

Exercício V

$$\int 5x^7 \partial x = 5\int x^7 \partial x = \frac{5x^8}{8}+c \tag{5}$$

Exercício VI

$$\int \left(5+3x^2-7x^3\right)\partial x=5\int \partial x+3\int x^2\partial x-7\int x^3\partial x=5x+\frac{3x^3}{3}-\frac{7x^4}{4}+c=\frac{-7x^4}{4}+x^3+5x+c \qquad (6)$$

Integral Indefinida – Aula 2

Exercício I

$$\int \frac{\partial x}{x^4}=\int x^{-4}\partial x=\frac{x^{-3}}{-3}+c=\frac{-1}{3x^3}+c \qquad (7)$$

Exercício II

$$\int \sqrt{x^3}\partial x=\int x^{\frac{3}{2}}\partial x=\frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+c=\sqrt{x^5}\cdot\frac{2}{5}=\frac{2\sqrt{x^5}}{5}+c \qquad (8)$$

Exercício II

$$\int \left(7\sqrt[5]{x^2}+\frac{3}{x^3}\right)\partial x=\int \left(7x^{\frac{2}{5}}+3x^{-3}\right)\partial x=7\int x^{\frac{2}{5}}\partial x+3\int x^{-3}\partial x=7\frac{x^{\frac{7}{5}}}{\left(\frac{7}{5}\right)}+3\frac{x^{-2}}{-2}+c=5\sqrt[5]{x^7}-\frac{3}{2x^2}+c \qquad (9)$$

Integral indefinida – Aula 3

Exercício I

$$\int \left(3x^{-4}-3x+4\right)\partial x=3\int x^{-4}\partial x-3\int x\partial x+4\int \partial x=3\frac{x^{-3}}{(-3)}-3\frac{x^2}{2}+4x+c=\frac{-1}{x^3}-\frac{3x^2}{2}+4x+c \qquad (10)$$

Exercício II

$$\int \frac{2}{3}\sqrt{x}\partial x=\frac{2}{3}\int x^{\frac{1}{2}}\partial x=\frac{2}{3}\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{4\sqrt{x^3}}{9}+c \qquad (11)$$

Integral de uma função Potência – Aula 4

Exercício I

$$\int \frac{\sqrt{x}x^3}{\sqrt[3]{x^2}}\partial x=\int \frac{x^{\frac{1}{2}}x^3}{x^{\frac{2}{3}}}\partial x=\int x^{\frac{1}{2}+3-\frac{2}{3}}\partial x=\int x^{\frac{3+18-4}{6}}\partial x=\int x^{\frac{17}{6}}\partial x=\frac{x^{\frac{23}{6}}}{\left(\frac{23}{6}\right)}+c=\frac{6\sqrt[6]{x^{23}}}{23}+c \tag{12}$$

Integral Indefinida – Aula 5

Exercício I

$$\int \frac{\partial x}{x^3}=\int x^{-3}\partial x=\frac{x^{-2}}{-2}+c=\frac{-1}{2x^2}+c \tag{13}$$

Exercício II

$$\int \frac{\partial x}{x}=\ln x+c \tag{14}$$

Exercício III

$$\begin{gathered}\int \sqrt{2x+1}\partial x=\int \sqrt{u}\frac{\partial u}{2}=\frac{1}{2}\int u^{\frac{1}{2}}\partial u=\frac{1}{2}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{1}{2}\frac{2}{3}\sqrt{u^3}+c=\frac{\sqrt{u^3}}{3}+c=\frac{\sqrt{(2x+1)^3}}{3}+c\\ u=2x+1,\frac{\partial u}{\partial x}=2\rightarrow\partial x=\frac{\partial u}{2}\\ \frac{\partial\left(\frac{\sqrt{(2x+1)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+c\right)}{\partial x}=\frac{1}{3}\frac{3}{2}(2x+1)^{\frac{3}{2}-1}\cdot 2+0=(2x+1)^{\frac{1}{2}}=\sqrt{2x+1}\end{gathered} \tag{15}$$

Exercício IV

$$\begin{gathered}\int \frac{5x^3+2x+3}{x}\partial x=\int\left(\frac{5x^3}{x}+\frac{2x}{x}+\frac{3}{x}\right)\partial x=\int\left(5x^2+2+\frac{3}{x}\right)\partial x=\\ 5\int x^2\partial x+2\int\partial x+3\int\frac{\partial x}{x}=5\frac{x^3}{3}+2x+3\ln|x|+c=\frac{5x^3}{3}+2x+3\ln|x|+c\\ \frac{\partial\left(\frac{5x^3}{3}+2x+3\ln|x|+c\right)}{\partial x}=\frac{5}{3}3x^2+2+3\frac{1}{x}+0=5x^2+2+\frac{3}{x}=\frac{5x^3+2x+3}{x}\end{gathered} \tag{16}$$

Exercício V

$$\int\left(2x^4+3+5e^x+\frac{7}{x}\right)\partial x=2\int x^4\partial x+3\int\partial x+5\int e^x\partial x+7\int\frac{\partial x}{x}=$$
$$2\frac{x^5}{5}+3x+5e^x+7\ln|x|+c=\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c \tag{17}$$
$$\frac{\partial\left(\frac{2x^5}{5}+3x+5e^x+7\ln|x|+c\right)}{\partial x}=\frac{2}{5}5x^4+3+5e^x+7\frac{1}{x}+0=2x^4+3+5e^x+\frac{7}{x}$$

Integral Indefinida – Aula 6

Exercício I

$$\int\frac{5t^2+7}{\sqrt[3]{t^4}}\partial t=\int\frac{5t^2+7}{t^{\frac{4}{3}}}\partial t=\int t^{\frac{-4}{3}}\left(5t^2+7\right)\partial t=\int 5t^{2-\frac{4}{3}}+7t^{\frac{-4}{3}}\partial t=\int 5t^{\frac{2}{3}}+7t^{\frac{-4}{3}}\partial t=$$
$$5\int t^{\frac{2}{3}}\partial t+7\int t^{\frac{-4}{3}}\partial t=5\frac{t^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+7\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+c=5\frac{3}{5}\sqrt[3]{t^5}-7\cdot3\frac{1}{\sqrt[3]{t}}+c=3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c \tag{18}$$
$$\frac{\partial\left(3\sqrt[3]{t^5}-\frac{21}{\sqrt[3]{t}}+c\right)}{\partial t}=\frac{\partial\left(3t^{\frac{5}{3}}-21t^{\frac{-1}{3}}+c\right)}{\partial t}=3\frac{5}{3}t^{\frac{2}{3}}-21\left(\frac{-1}{3}\right)t^{\frac{-4}{3}}+0=5\sqrt[3]{t^2}+\frac{7}{\sqrt[3]{t^4}}=$$
$$\frac{5t^{\frac{2}{3}}t^{\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{2}{3}+\frac{4}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^{\frac{6}{3}}+7}{\sqrt[3]{t^4}}=\frac{5t^2+7}{\sqrt[3]{t^4}}$$

Integral Indefinida e Composta – Aula 7

Exercício I

$$\int\frac{\partial x}{\sqrt{x}}=\int\frac{\partial x}{x^{\frac{1}{2}}}=\int x^{\frac{-1}{2}}\partial x=\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+c=2\sqrt{x}+c \tag{19}$$

Exercício II

$$\int\left(3e^x+\frac{2}{x}\right)\partial x=3\int e^x+2\int\frac{\partial x}{x}=3e^x+2\ln|x|+c \tag{20}$$

Exercício III

$$\int x^3 \partial x=\frac{x^4}{4}+c$$

$$\int (2x^2+1)^3 x\,\partial x=\int u^3 \cancel{x}\frac{\partial u}{4\cancel{x}}=\frac{1}{4}\int u^3\partial u=\frac{1}{4}\frac{u^4}{4}+c=\frac{u^4}{16}+c=\frac{(2x^2+1)^4}{16}+c=$$
$$\frac{(2x^2+1)^4}{2^4}+c=\left(\frac{2x^2+1}{2}\right)^4+c=\left(x^2+\frac{1}{2}\right)^4+c$$
$$u=2x^2+1,\frac{\partial u}{\partial x}=4x\rightarrow\partial x=\frac{\partial u}{4x} \tag{21}$$
$$\frac{\partial\left[\left(x^2+\frac{1}{2}\right)^4+c\right]}{\partial x}=4\left(x^2+\frac{1}{2}\right)^3\cdot 2x+0=8x\left(x^2+\frac{1}{2}\right)^3=8x\left(x^2+\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)^2=$$
$$(8x^3+4x)\left(x^4+x^2+\frac{1}{4}\right)=8x^7+8x^5+2x^3+4x^5+4x^3+x=8x^7+12x^5+6x^3+x$$
$$(2x^2+1)^3x=(2x^2+1)^2(2x^2+1)x=(4x^4+4x^2+1)(2x^3+x)=8x^7+4x^5+8x^5+4x^3+2x^3+x=$$
$$8x^7+12x^5+6x^3+x$$

Integral indefinida e composta – Aula 8

Exercício I

$$\int 3e^x\partial x=3\int e^x\partial x=3e^x+c$$
$$\int e^{x^2+1}x\,\partial x=\int e^u\cancel{x}\frac{\partial u}{2\cancel{x}}=\frac{1}{2}\int e^u\partial u=\frac{1}{2}e^u+c=\frac{e^{x^2+1}}{2}+c$$
$$u=x^2+1,\frac{\partial u}{\partial x}=2x\rightarrow\partial x=\frac{\partial u}{2x} \tag{22}$$
$$\frac{\partial\left(\frac{e^{x^2+1}}{2}+c\right)}{\partial x}=\frac{1}{2}e^{x^2+1}2x+0=e^{x^2+1}x$$

Exercício II

$$\int e^{x^4+1}x^3\partial x=\int e^u\cancel{x^3}\frac{\partial u}{4\cancel{x^3}}=\frac{1}{4}\int e^u\partial u=\frac{1}{4}e^u+c=\frac{e^{x^4+1}}{4}+c$$
$$u=x^4+1,\frac{\partial u}{\partial x}=4x^3\rightarrow\partial x=\frac{\partial u}{4x^3} \tag{23}$$
$$\frac{\partial\left(\frac{e^{x^4+1}}{4}+c\right)}{\partial x}=\frac{1}{4}e^{x^4+1}4x^3+0=e^{x^4+1}x^3$$

Exercício III

$$\int \frac{x}{(2x^2-1)^3}\partial x=\int (2x^2-1)^{-3} x\,\partial x=\int u^{-3}\cancel{x}\frac{\partial u}{4\cancel{x}}=\frac{1}{4}\int u^{-3}\partial u=\frac{1}{4}\frac{u^{-2}}{(-2)}+c=\frac{-1}{8u^2}+c=\frac{-1}{8(2x^2-1)^2}+c$$

$$u=2x^2-1,\frac{\partial u}{\partial x}=4x\rightarrow \partial x=\frac{\partial u}{4x} \tag{24}$$

$$\frac{\partial\left(\frac{-1}{8(2x^2-1)^2}+c\right)}{\partial x}=\frac{\partial\left(\frac{-(2x^2-1)^{-2}}{8}+c\right)}{\partial x}=\frac{-1}{8}(-2)(2x^2-1)^{-3}4x+0=(2x^2-1)^{-3}x=\frac{x}{(2x^2-1)^3}$$

Exercício IV

$$\int \frac{x}{2x^2-1}\partial x=\int (2x^2-1)^{-1} x\,\partial x=\int u^{-1}\cancel{x}\frac{\partial u}{4\cancel{x}}=\frac{1}{4}\int u^{-1}\partial u=\frac{1}{4}\ln|u|+c=\frac{\ln|2x^2-1|}{4}+c$$

$$u=2x^2-1,\frac{\partial u}{\partial x}=4x\rightarrow \partial x=\frac{\partial u}{4x} \tag{25}$$

$$\frac{\partial\left(\frac{\ln|2x^2-1|}{4}+c\right)}{\partial x}=\frac{1}{4}\frac{1}{2x^2-1}4x+0=\frac{x}{2x^2-1}$$

Integral pelo Método da Substituição não tão evidente – Aula 9

Exercício I

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u-1)^2\sqrt{u}\,\partial u=\int (u-1)^2 u^{\frac{1}{2}}\,\partial u=\int (u^2-2u+1)u^{\frac{1}{2}}\,\partial u=$$
$$\int \left(u^{2+\frac{1}{2}}-2u^{1+\frac{1}{2}}+u^{\frac{1}{2}}\right)\partial u=\int \left(u^{\frac{5}{2}}-2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)\partial u=\int u^{\frac{5}{2}}\,\partial u-2\int u^{\frac{3}{2}}\,\partial u+\int u^{\frac{1}{2}}\,\partial u=$$
$$\frac{u^{\frac{7}{2}}}{\left(\frac{7}{2}\right)}-2\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}+\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^7}}{7}-\frac{4\sqrt{u^5}}{5}+\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c$$
$$u=1+x\rightarrow x=u-1,\frac{\partial x}{\partial u}=1\rightarrow \partial x=\partial u$$
$$\frac{\partial\left(\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c\right)}{\partial x}=\frac{\partial\left(\frac{2(1+x)^{\frac{7}{2}}}{7}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{3}{2}}}{3}+c\right)}{\partial x}= \tag{26}$$
$$\frac{2}{7}\frac{7}{2}(1+x)^{\frac{5}{2}}-\frac{4}{5}\frac{5}{2}(1+x)^{\frac{3}{2}}+\frac{2}{3}\frac{3}{2}(1+x)^{\frac{1}{2}}+0=(1+x)^{\frac{5}{2}}-2(1+x)^{\frac{3}{2}}+(1+x)^{\frac{1}{2}}=$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{5}{2}-\frac{1}{2}}-2(1+x)^{\frac{3}{2}-\frac{1}{2}}+1\right)=(1+x)^{\frac{1}{2}}\left((1+x)^{\frac{4}{2}}-2(1+x)^{\frac{2}{2}}+1\right)=$$
$$(1+x)^{\frac{1}{2}}\left((1+x)^2-2(1+x)+1\right)=(1+x)^{\frac{1}{2}}(1+2x+x^2-2-2x+1)=(1+x)^{\frac{1}{2}}x^2=x^2\sqrt{1+x}$$

Exercício II

$$\int x^2\sqrt{1+x}\,\partial x \rightarrow \int (u^2-1)^2 u\,2u\,\partial u=2\int (u^2-1)^2u^2\,\partial u=2\int (u^4-2u^2+1)u^2\,\partial u=$$
$$2\int (u^6-2u^4+u^2)\partial u=2\int u^6\,\partial u-4\int u^4\,\partial u+2\int u^2\,\partial u=2\frac{u^7}{7}-4\frac{u^5}{5}+2\frac{u^3}{3}+c=$$
$$\frac{2\sqrt{(1+x)^7}}{7}-\frac{4\sqrt{(1+x)^5}}{5}+\frac{2\sqrt{(1+x)^3}}{3}+c \tag{27}$$
$$u=\sqrt{1+x}\rightarrow u^2=1+x\rightarrow x=u^2-1,\frac{\partial x}{\partial u}=2u\rightarrow \partial x=2u\,\partial u$$

O que é uma Integral Definida – Aula 10

Exercício I

$$\int_1^2 x^3\,\partial x=\left.\frac{x^4}{4}\right]_1^2=\frac{(2)^4}{4}-\frac{(1)^4}{4}=\frac{16}{4}-\frac{1}{4}=\frac{16-1}{4}=\frac{15}{4}=3{,}75 \tag{28}$$

Integral Definida – Aula 10a

Exercício I

$$\int_0^2 (6x^2-4x+5)\partial x = 6\int_0^2 x^2\partial x - 4\int_0^2 x\partial x + 5\int_0^2 \partial x = 6\frac{x^3}{3} - 4\frac{x^2}{2} + 5x\Big]_0^2 = 2x^3-2x^2+5x\Big]_0^2 =$$
$$x(2x^2-2x+5)\Big]_0^2 = \left[(2)(2(2)^2-2(2)+5)\right] - \left[(0)(2(0)^2-2(0)+5)\right] = 2(8-4+5) = 2\cdot 9 = 18 \tag{29}$$

Exercício II

$$\int_{-1}^0 (2x-e^x)\partial x = 2\int_{-1}^0 x\partial x - \int_{-1}^0 e^x\partial x = 2\frac{x^2}{2} - e^x\Big]_{-1}^0 = x^2-e^x\Big]_{-1}^0 = \left((0)^2-e^{(0)}\right) - \left((-1)^2-e^{(-1)}\right) =$$
$$-1-\left(1-\frac{1}{e}\right) = -1-1+\frac{1}{e} = -2+\frac{1}{e} \tag{30}$$

Integral definida – Aula 11

Exercício I

$$\frac{5\pi}{4}\int_0^2 \frac{r\partial r}{1+r^2} = \frac{5\pi}{4}\int_0^2 (1+r^2)^{-1} r\partial r = \frac{5\pi}{4}\int_0^2 u^{-1}\cancel{r}\frac{\partial u}{2\cancel{r}} = \frac{5\pi}{4}\frac{1}{2}\int_0^2 u^{-1}\partial u = \frac{5\pi}{8}\int_0^2 u^{-1}\partial u =$$
$$\frac{5\pi}{8}\ln|u|\Big]_0^2 = \frac{5\pi}{8}\ln|1+r^2|\Big]_0^2 = \left[\frac{5\pi}{8}\ln|1+(2)^2|\right] - \left[\frac{5\pi}{8}\ln|1+(0)^2|\right] = \frac{5\pi}{8}\ln|5| - \frac{5\pi}{8}\ln|1| =$$
$$\frac{5\pi}{8}(\ln|5|-\ln|1|) = \frac{5\pi}{8}(\ln|5|-0) = \frac{5\pi}{8}\ln|5| \tag{31}$$
$$u=1+r^2, \frac{\partial u}{\partial r} = 2r \rightarrow \partial x = \frac{\partial u}{2r}$$
$$\ln|1| = x \rightarrow e^x = 1 = e^0 \rightarrow x = 0$$
$$\ln|5| = x \rightarrow e^x = 5$$

Exercício II

$$2\pi\int_0^2 r^2\partial r = 2\pi\frac{r^3}{3}\Big]_0^2 = \frac{2\pi r^3}{3}\Big]_0^2 = \left(\frac{2\pi(2)^3}{3}\right) - \left(\frac{2\pi(0)^3}{3}\right) = \frac{16\pi}{3} \tag{32}$$

Exercício II

$$2\pi\int_0^{\sqrt{2}} (4r-2r^3)\partial r = 8\pi\int_0^{\sqrt{2}} r\partial r - 4\pi\int_0^{\sqrt{2}} r^3\partial r = 8\pi\frac{r^2}{2} - 4\pi\frac{r^4}{4}\Big]_0^{\sqrt{2}} = 4\pi r^2 - \pi r^4\Big]_0^{\sqrt{2}} =$$
$$\pi r^2(4-r^2)\Big]_0^{\sqrt{2}} = \left[\pi(\sqrt{2})^2(4-(\sqrt{2})^2)\right] - \left[\pi(0)^2(4-(0)^2)\right] = 2\pi(4-2) = 4\pi \tag{33}$$

Exercício III

$$\pi\int_0^2 x^2\,\partial x=\pi\frac{x^3}{3}\Bigg]_0^2=\frac{\pi x^3}{3}\Bigg]_0^2=\left(\frac{\pi(2)^3}{3}\right)-\left(\frac{\pi(0)^3}{3}\right)=\frac{8\pi}{3} \tag{34}$$

Exercício IV

$$\frac{\pi}{16}\int_1^4 x^4\,\partial x=\frac{\pi}{16}\frac{x^5}{5}\Bigg]_1^4=\frac{\pi x^5}{80}\Bigg]_1^4=\left(\frac{\pi(4)^5}{80}\right)-\left(\frac{\pi(1)^5}{80}\right)=\frac{4^5\pi}{80}-\frac{\pi}{80}=\frac{1024\pi-\pi}{80}=\frac{1023\pi}{80} \tag{35}$$

Exercício V

$$\pi\int_1^2 (x^2)^2\,\partial x=\pi\int_1^2 x^4\,\partial x=\pi\frac{x^5}{5}\Bigg]_1^2=\frac{\pi x^5}{5}\Bigg]_1^2=\left(\frac{\pi(2)^5}{5}\right)-\left(\frac{\pi(1)^5}{5}\right)=\frac{32\pi}{5}-\frac{\pi}{5}=\frac{32\pi-\pi}{5}=\frac{31\pi}{5} \tag{36}$$

Exercício VI

$$\begin{gathered}
\pi\int_{-1}^2 (-x^4-x^2+6x+8)\,\partial x=-\pi\int_{-1}^2 x^4\,\partial x-\pi\int_{-1}^2 x^2\partial x+6\pi\int_{-1}^2 x\,\partial x+8\pi\int_{-1}^2 \partial x=\\
-\pi\frac{x^5}{5}-\pi\frac{x^3}{3}+6\pi\frac{x^2}{2}+8\pi x\Bigg]_{-1}^2=\frac{-\pi x^5}{5}-\frac{\pi x^3}{3}+3\pi x^2+8\pi x\Bigg]_{-1}^2=-\pi x\left(\frac{x^4}{5}+\frac{x^2}{3}-3x-8\right)\Bigg]_{-1}^2\\
\left[-\pi(2)\left(\frac{(2)^4}{5}+\frac{(2)^2}{3}-3(2)-8\right)\right]-\left[-\pi(-1)\left(\frac{(-1)^4}{5}+\frac{(-1)^2}{3}-3(-1)-8\right)\right]=\\
-2\pi\left(\frac{16}{5}+\frac{4}{3}-6-8\right)-\pi\left(\frac{1}{5}+\frac{1}{3}+3-8\right)=-2\pi\left(\frac{48+20-210}{15}\right)-\pi\left(\frac{3+5-75}{15}\right)=\\
2\pi\left(\frac{142}{15}\right)+\pi\left(\frac{67}{15}\right)=\pi\left(\frac{284}{15}+\frac{67}{15}\right)=\frac{351\pi}{15}=\frac{3^3\cdot 13\pi}{3\cdot 5}=\frac{3^2\cdot 13\pi}{5}=\frac{117\pi}{5}
\end{gathered} \tag{37}$$

Exercício VII

$$\begin{gathered}
\pi\int_0^8 (\sqrt[3]{y})^2\,\partial y=\pi\int_0^8 \left(y^{\frac{1}{3}}\right)^2\partial y=\pi\int_0^8 y^{\frac{2}{3}}\partial y=\pi\frac{y^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}\Bigg]_0^8=\frac{3\pi\sqrt[3]{y^5}}{5}\Bigg]_0^8=\\
\left(\frac{3\pi\sqrt[3]{(8)^5}}{5}\right)-\left(\frac{3\pi\sqrt[3]{(0)^5}}{5}\right)=\frac{3\pi\sqrt[3]{8^3\cdot 8^2}}{5}=\frac{3\pi 2^3\sqrt[3]{(2^3)^2}}{5}=\frac{3\pi 2^3 2^2}{5}=\frac{3\pi 2^5}{5}=\frac{96\pi}{5}
\end{gathered} \tag{38}$$

Integral definida – Aula 12

Exercício I

$$\int_1^2 2x\,\partial x=2\int_1^2 x\,\partial x=2\frac{x^2}{2}\Bigg]_1^2=x^2\Big]_1^2=\left((2)^2\right)-\left((1)^2\right)=4-1=4-1=3 \tag{39}$$

Exercício II

$$\int_1^4 2\sqrt{x}\,\partial x=\int_1^4 2x^{\frac{1}{2}}\,\partial x=2\int_1^4 x^{\frac{1}{2}}\,\partial x=2\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_1^4=\frac{4\sqrt{x^3}}{3}\Bigg]_1^4=\left(\frac{4\sqrt{(4)^3}}{3}\right)-\left(\frac{4\sqrt{(1)^3}}{3}\right)= \\ \frac{4\sqrt{4^2 2^2}}{3}-\frac{4}{3}=\frac{32}{3}-\frac{4}{3}=\frac{32-4}{3}=\frac{28}{3} \quad (40)$$

Exercício III

$$\int_1^2 4x^2\,\partial x=4\int_1^2 x^2\,\partial x=4\frac{x^3}{3}\Bigg]_1^2=\frac{4}{3}x^3\Bigg]_1^2=\frac{4}{3}(2^3-1^3)=\frac{4}{3}7=\frac{28}{3} \quad (41)$$

Integrais definidas e indefinidas – Aula 13

Exercício I

$$\int\left(\frac{3}{x^4}+\frac{2x^2}{3}-2x+7\right)\partial x=\int\left(3x^{-4}+\frac{2}{3}x^2-2x+7\right)\partial x= \\ 3\int x^{-4}\,\partial x+\frac{2}{3}\int x^2\,\partial x-2\int x\,\partial x+7\int \partial x=3\frac{x^{-3}}{-3}+\frac{2}{3}\frac{x^3}{3}-2\frac{x^2}{2}+7x+c= \\ \frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c \\ \frac{\partial\left(\frac{-1}{x^3}+\frac{2x^3}{9}-x^2+7x+c\right)}{\partial x}=\frac{\partial\left(-x^{-3}+\frac{2}{9}x^3-x^2+7x+c\right)}{\partial x}=3x^{-4}+\frac{2}{9}3x^2-2x+7+0= \\ \frac{3}{x^4}+\frac{2x^2}{3}-2x+7 \quad (42)$$

Exercício II

$$\int 5\sqrt[3]{x^2}\,\partial x=\int 5x^{\frac{2}{3}}\,\partial x=5\int x^{\frac{2}{3}}\,\partial x=5\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)}+c=3\sqrt[3]{x^5}+c \\ \frac{\partial\left(3\sqrt[3]{x^5}+c\right)}{\partial x}=\frac{\partial\left(3x^{\frac{5}{3}}+c\right)}{\partial x}=3\frac{5}{3}x^{\frac{2}{3}}+0=5\sqrt[3]{x^2} \quad (43)$$

Exercício III

$$\int_2^4 2x^3 \partial x = 2\int_2^4 x^3 \partial x = 2\frac{x^4}{4}\Big]_2^4 = \frac{1}{2}x^4\Big]_2^4 = \frac{1}{2}(4^4-2^4) = \frac{1}{2}((2\cdot 2)^4-2^4) = \frac{1}{2}(2^4\cdot 2^4-2^4) = \frac{2^4}{2}(2^4-1) = 2^3(16-1) = 8\cdot 15 = 120 \quad (44)$$

Exercício IV

$$\int_1^2 (3x^2-2x)\partial x = 3\int_1^2 x^2 \partial x - 2\int_1^2 x \partial x = 3\frac{x^3}{3} - 2\frac{x^2}{2}\Big]_1^2 = x^3-x^2\Big]_1^2 = x^2(x-1)\Big]_1^2 = [2^2(2-1)]-[1^2(1-1)] = 4 \quad (45)$$

Integral definida pelo método da substituição – U du – Aula 14

Exercício I

$$\int_0^2 \sqrt{2x^2+1}\, x\, \partial x = \int_0^2 \sqrt{u}\, \cancel{x} \frac{\partial u}{4\cancel{x}} = \frac{1}{4}\int_0^2 u^{\frac{1}{2}} \partial u = \frac{1}{4}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\Bigg]_0^2 = \frac{1}{4}\frac{2}{3}\sqrt{u^3}\Big]_0^2 = \frac{1}{6}\sqrt{u^3}\Big]_0^2 = \frac{1}{6}\sqrt{(2x^2+1)^3}\Big]_0^2$$

$$\frac{1}{6}\left[\sqrt{(2\cdot 2^2+1)^3}-\sqrt{(2\cdot 0^2+1)^3}\right] = \frac{1}{6}\left(\sqrt{9^3}-\sqrt{1^3}\right) = \frac{1}{6}\left(\sqrt{9^2 3^2}-1\right) = \frac{1}{6}(27-1) = \frac{1}{6}26 = \frac{13}{3} \quad (46)$$

$$u = 2x^2+1, \frac{\partial u}{\partial x} = 4x \rightarrow \partial x = \frac{\partial u}{4x}$$

Integral Método da Substituição – Aula 15

Exercício I

$$\int (x^3-1)^4 x^2 \partial x = \int u^4 \cancel{x^2} \frac{\partial u}{3\cancel{x^2}} = \frac{1}{3}\int u^4 \partial u = \frac{1}{3}\frac{u^5}{5}+c = \frac{(x^3-1)^5}{15}+c \quad (47)$$

$$u = x^3-1, \frac{\partial u}{\partial x} = 3x^2 \rightarrow \partial x = \frac{\partial u}{3x^2}$$

Exercício II

$$\int \frac{x}{(x^2-1)^3}\partial x = \int (x^2-1)^{-3} x \partial x = \int u^{-3}\frac{\partial u}{2} = \frac{1}{2}\int u^{-3}\partial u = \frac{1}{2}\frac{u^{-2}}{(-2)}+c = \frac{-1}{4u^2}+c = \frac{-1}{4(x^2-1)^2}+c \quad (48)$$

$$u = x^2-1, \partial u = 2x\partial x \rightarrow \frac{\partial u}{2} = x\partial x$$

Exercício III

$$\int \frac{x}{(x^2-1)}\partial x=\int (x^2-1)^{-1}x\,\partial x=\int u^{-1}\frac{\partial u}{2}=\frac{1}{2}\int u^{-1}\partial u=\frac{1}{2}\ln|u|+c=\frac{\ln|x^2-1|}{2}+c$$
$$u=x^2-1,\partial u=2x\,\partial x\rightarrow\frac{\partial u}{2}=x\,\partial x \tag{49}$$

Exercício IV

$$\int e^{x^2-1}x\,\partial x=\int e^u\frac{\partial u}{2}=\frac{1}{2}e^u\partial u=\frac{1}{2}e^u+c=\frac{e^{x^2-1}}{2}+c$$
$$u=x^2-1,\partial u=2x\,\partial x\rightarrow\frac{\partial u}{2}=x\,\partial x \tag{50}$$

Exercício V

$$\int\sqrt{x^3-4}\,x^2\partial x=\int u^{\frac{1}{2}}\frac{\partial u}{3}=\frac{1}{3}\int u^{\frac{1}{2}}\partial u=\frac{1}{3}\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{1}{3}\frac{2}{3}\sqrt{u^3}+c=\frac{2\sqrt{u^3}}{9}+c=$$
$$\frac{2\sqrt{(x^3-4)^3}}{9}+c \tag{51}$$
$$u=x^3-4,\partial u=3x^2\partial x\rightarrow\frac{\partial u}{3}=x^2\partial x$$

Exercício VI

$$\int e^{\sqrt{x}}\frac{\partial x}{\sqrt{x}}=\int e^{x^{\frac{1}{2}}}\frac{\partial x}{x^{\frac{1}{2}}}=\int e^{x^{\frac{1}{2}}}x^{\frac{-1}{2}}\partial x=\int e^u 2\partial u=2\int e^u\partial u=2e^u+c=2e^{\sqrt{x}}+c$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x=2\partial u=x^{\frac{-1}{2}}\partial x \tag{52}$$

Exercício VII

$$\int\frac{x\,\partial x}{\sqrt[5]{x^2-1}}=\int\frac{x}{(x^2-1)^{\frac{1}{5}}}\partial x=\int (x^2-1)^{\frac{-1}{5}}x\,\partial x=\int u^{\frac{-1}{5}}\frac{\partial u}{2}=\frac{1}{2}\int u^{\frac{-1}{5}}\partial u=\frac{1}{2}\frac{u^{\frac{4}{5}}}{\left(\frac{4}{5}\right)}+c=$$
$$\frac{1}{2}\frac{5}{4}\sqrt[5]{u^4}+c=\frac{5\sqrt[5]{u^4}}{8}+c=\frac{5\sqrt[5]{(x^2-1)^4}}{8}+c \tag{53}$$
$$u=x^2-1,\partial u=2x\,\partial x\rightarrow\frac{\partial u}{2}=x\,\partial x$$

Exercício VIII

$$\int \frac{e^t \partial t}{e^t+4} = \int (e^t+4)^{-1} e^t \partial t = \int u^{-1} \partial u = \ln|u| + c = \ln|e^t+4| + c \tag{54}$$
$$u = e^t+4, \partial u = e^t \partial t$$

Integral de uma Função Exponencial Qualquer – Aula 16

Exercício I

$$\int \sqrt{10^{3x}} \partial x = \int 10^{\frac{3x}{2}} \partial x = \int 10^u \frac{2}{3} \partial u = \frac{2}{3} \int 10^u \partial u = \frac{2}{3} \frac{10^u}{\ln|10|} + c = \frac{2 \cdot 10^{\frac{3x}{2}}}{3\ln|10|} + c = \frac{2\sqrt{10^{3x}}}{3\ln|10|} + c$$
$$u = \frac{3x}{2}, \partial u = \frac{3}{2} \partial x = \frac{2}{3} \partial u = \partial x$$

$$\frac{\partial\left(\frac{2\sqrt{10^{3x}}}{3\ln|10|}+c\right)}{\partial x} = \frac{\partial\left(\frac{2}{3\ln|10|}10^{\frac{3x}{2}}+c\right)}{\partial x} = \frac{\cancel{2}}{\cancel{3\ln|10|}} \frac{\cancel{3\ln|10|}\sqrt{10^{3x}}}{\cancel{2}} + 0 = \sqrt{10^{3x}} \tag{55}$$
$$y = 10^{\frac{3x}{2}} \rightarrow \ln|y| = \ln\left|10^{\frac{3x}{2}}\right| = \frac{3x}{2}\ln|10| = \frac{3\ln|10|}{2} x$$
$$\frac{\partial(\ln|y|)}{\partial y} = \frac{\partial\left(\frac{3\ln|10|}{2}x\right)}{\partial x} \rightarrow \frac{1}{y} \partial y = \frac{3\ln|10|}{2} \partial x \rightarrow \partial y = y \frac{3\ln|10|}{2} \partial x \rightarrow \frac{\partial y}{\partial x} =$$
$$10^{\frac{3x}{2}} \frac{3\ln|10|}{2} = \frac{3\ln|10|\sqrt{10^{3x}}}{2}$$

Integral de função marginal – Aula 17

Exercício I

O custo marginal por unidade x é dado pela expressão $\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5}$. Determine a função custo total c(x) da produção, sabendo-se que o custo fixo para x = 0 é de R$2000,00.

$$\frac{\partial c(x)}{\partial x} = 20 - \frac{4x^3}{5} \rightarrow \cancel{\int \partial c(x)} = \int \left(20 - \frac{4x^3}{5}\right) \partial x \rightarrow \int \left(20 - \frac{4x^3}{5}\right) \partial x = 20 \int \partial x - \frac{4}{5} \int x^3 \partial x =$$
$$20x - \frac{4}{5} \frac{x^4}{4} + c = 20x + \frac{x^4}{5} + c \tag{56}$$
$$20 \cdot 0 + \frac{0^4}{5} + c = 2000 \rightarrow c = 2000$$
$$c(x) = 20x + \frac{x^4}{5} + 2000$$

Exercício II

O rendimento marginal de um bem em quantidade (x) é dado pela expressão $Rm=800-2x^2$. Ache o rendimento total para x = 6, sabendo que quando x = 0, R = 0.

$$\frac{\partial r(x)}{\partial x}=800-2x^2 \to \cancel{\int \partial r(x)}=\int (800-2x^2)\partial x \to \int (800-2x^2)\partial x=800\int \partial x-2\int x^2 \partial x$$

$$800x-2\frac{x^3}{3}+c=800x-\frac{2x^3}{3}+c$$

$$800\cdot 0-\frac{2\cdot 0^2}{3}+c=0 \to c=0 \tag{57}$$

$$r(x)=800x-\frac{2x^3}{3}$$

$$r(6)=800\cdot 6-\frac{2\cdot 6^3}{3}=4800-\frac{2\cdot 2^3\cdot 3^3}{3}=4800-2^4\cdot 3^2=4800-16\cdot 9=4800-144=4656$$

Método da substituição com sen(x) e cos(x) – Aula 18

01.	$\int sen(u)\partial u$	$=-\cos(u)+c$
02.	$\int \cos(u)\partial u$	$= sen(u)+c$
03.	$\int tg(u)\partial u$	$= \ln\lvert sec(u)\rvert+c$
04.	$\int cotg(u)\partial u$	$= \ln\lvert sen(u)\rvert+c$
05.	$\int sec(u)\partial u$	$= \ln\lvert sec(u)+tg(u)\rvert+c$
06.	$\int cossec(u)\partial u$	$= \ln\lvert cossec(u)-cotg(u)\rvert+c$
07.	$\int sec^2(u)\partial u$	$= tg(u)+c$
08.	$\int cossec^2(u)\partial u$	$=-cotg(u)+c$
09.	$\int sec(u)tg(u)\partial u$	$= sec(u)+c$
10.	$\int cossec(u)cotg(u)\partial u$	$=-cossec(u)+c$

Exercício I

$$\int sen(2x^2-1)x\partial x=\int sen(u)\frac{\partial u}{4}=\frac{1}{4}sen(u)\partial u=\frac{1}{4}(-\cos(u))+c=\frac{-\cos(u)}{4}+c=$$

$$\frac{-\cos(2x^2-1)}{4}+c$$

$$u=2x^2-1, \partial u=4x\partial x \to \frac{\partial u}{4}=x\partial x \tag{58}$$

$$\frac{\partial\left(\frac{-\cos(2x^2-1)}{4}+c\right)}{\partial x}=\frac{-1}{4}[-sen(2x^2-1)]4x+0=sen(2x^2-1)x$$

Exercício II

$$\int \cos\left(3x^3+4\right)x^2\partial x=\int \cos(u)\frac{\partial u}{9}=\frac{1}{9}\int \cos(u)\partial u=\frac{1}{9}sen(u)+c=\frac{sen\left(3x^3+4\right)}{9}+c$$
$$u=3x^3+4,\partial u=9x^2\partial x\rightarrow\frac{\partial u}{9}=x^2\partial x \tag{59}$$
$$\frac{\partial\left(\frac{sen\left(3x^3+4\right)}{9}+c\right)}{\partial x}=\frac{1}{9}\cos\left(3x^3+4\right)9x^2+0=\cos\left(3x^3+4\right)x^2$$

Exercício III

$$\int sen(\sqrt{x})\frac{\partial x}{\sqrt{x}}=\int sen\left(x^{\frac{1}{2}}\right)\frac{\partial x}{x^{\frac{1}{2}}}=\int sen\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int sen(u)2\partial u=2\int sen(u)\partial u=$$
$$2(-\cos(u))+c=-2\cos\sqrt{x}+c \tag{60}$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

Exercício IV

$$\int sen(x)\cos(x)\partial x=\int u\partial u=\frac{u^2}{2}+c=\frac{sen^2(x)}{2}+c$$
$$u=sen(x),\partial u=\cos(x)\partial x \tag{61}$$
$$\frac{\partial\left(\frac{sen^2(x)}{2}+c\right)}{\partial x}=\frac{1}{2}2sen(x)\cos(x)+0=sen(x)\cos(x)$$

Exercício V

$$\int sen(\cos(x))sen(x)\partial x=\int sen(u)(-\partial u)=-\int sen(u)\partial u=-(-\cos(u))+c=$$
$$\cos(\cos(x))+c$$
$$u=\cos(x),\partial u=-sen(x)\partial x\rightarrow-\partial u=sen(x)\partial x \tag{62}$$
$$\frac{\partial[\cos(\cos(x))+c]}{\partial x}=-sen(\cos(x))(-sen(x))+0=sen(\cos(x))sen(x)$$

Exercício VI

$$\int\sqrt{sen(\theta)}\cos(\theta)\partial\theta=\int u^{\frac{1}{2}}\partial u=\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{2\sqrt{u^3}}{3}+c=\frac{2\sqrt{sen^3(\theta)}}{3}+c \tag{63}$$
$$u=sen(\theta),\partial u=\cos(\theta)\partial\theta$$

Exercício VI

$$\int \ln|x| \frac{\partial x}{x} = \int \ln|x| x^{-1} \partial x = \int u \partial u = \frac{u^2}{2} + c = \frac{\ln^2|x|}{2} + c$$
$$u = \ln|x|, \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \tag{64}$$
$$\frac{\partial\left(\frac{\ln^2|x|}{2} + c\right)}{\partial x} = \frac{1}{2} 2 \ln|x| \frac{1}{x} + 0 = \frac{\ln|x|}{x}$$

Exercício VII

$$\int \frac{\partial x}{x \ln^2|x|} = \int \left(x \ln^2|x|\right)^{-1} \partial x = \int \ln^{-2}|x| x^{-1} \partial x = \int u^{-2} \partial u = \frac{u^{-1}}{(-1)} = \frac{-1}{u} + c = \frac{-1}{\ln|x|} + c$$
$$u = \ln|x|, \partial u = \frac{1}{x} \partial x \rightarrow \partial u = x^{-1} \partial x \tag{65}$$
$$\frac{\partial\left(\frac{-1}{\ln|x|} + c\right)}{\partial x} = \frac{\partial\left(-\ln^{-1}|x| + c\right)}{\partial x} = \ln^{-2} \frac{1}{x} + 0 = \frac{1}{x \ln^2|x|}$$

Exercício VIII

$$\int \frac{sen(\theta) \partial \theta}{(5 - \cos(\theta))^3} = \int (5 - \cos(\theta))^{-3} sen(\theta) \partial \theta = \int u^{-3} \partial u = \frac{u^{-2}}{(-2)} + c = \frac{-1}{2u^2} + c =$$
$$\frac{-1}{2(5 - \cos(\theta))^2} + c$$
$$u = 5 - \cos(\theta), \partial u = -(-sen(\theta)) \partial x \rightarrow \partial u = sen(\theta) \partial \theta \tag{66}$$
$$\frac{\partial\left(\frac{-1}{2(5 - \cos(\theta))^2} + c\right)}{\partial \theta} = \frac{\partial\left(\frac{-1}{2}(5 - \cos(\theta))^{-2} + c\right)}{\partial \theta} = \frac{-1}{2}(-2)(5 - \cos(\theta))^{-3}(-(-sen(\theta))) =$$
$$(5 - \cos(\theta))^{-3} sen(\theta) = \frac{sen(\theta)}{(5 - \cos(\theta))^3}$$

Integração de funções trigonométricas – Aula 19

Exercício I

$$\int tg(x)\partial x=\int \frac{sen(x)}{\cos(x)}\partial x=\int \cos^{-1}(x)sen(x)\partial x=\int u^{-1}(-\partial u)=-\int u^{-1}\partial u=$$
$$-\ln|u|+c=\ln|u^{-1}|+c=\ln\left|\frac{1}{u}\right|+c=\ln\left|\frac{1}{\cos(x)}\right|+c=\ln|sec(x)|+c$$
$$u=\cos(x),\partial u=-sen(x)\partial x\rightarrow -\partial u=sen(x)\partial x \tag{67}$$
$$\frac{\partial(\ln|sec(x)|+c)}{\partial x}=\frac{\partial\left(\ln\left|\frac{1}{\cos(x)}\right|+c\right)}{\partial x}=\frac{\partial(\ln|\cos^{-1}(x)|+c)}{\partial x}=\frac{\partial(-\ln|\cos(x)|+c)}{\partial x}=$$
$$\frac{-1}{\cos(x)}(-sen(x))+0=\frac{sen(x)}{\cos(x)}=tg(x)$$

Exercício II

$$\int cotg(x)\partial x=\int \frac{\cos(x)}{sen(x)}\partial x=\int sen^{-1}(x)\cos(x)\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sen(x)|+c$$
$$u=sen(x),\partial u=\cos(x)\partial x \tag{68}$$
$$\frac{\partial(\ln|sen(x)|+c)}{\partial x}=\frac{1}{sen(x)}\cos(x)+0=\frac{\cos(x)}{sen(x)}=cotg(x)$$

Exercício III

$$\int sec(x)\partial x=\int sec(x)\left(\frac{sec(x)+tg(x)}{sec(x)+tg(x)}\right)\partial x=\int \frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}\partial x=$$
$$\int [sec(x)+tg(x)]^{-1}[sec(x)tg(x)+sec^2(x)]\partial x=\int u^{-1}\partial u=\ln|u|+c=\ln|sec(x)+tg(x)|+c$$
$$u=sec(x)+tg(x),\partial u=[sec(x)tg(x)+sec^2(x)]\partial x \tag{69}$$
$$\frac{\partial(\ln|sec(x)+tg(x)|+c)}{\partial x}=\frac{1}{sec(x)+tg(x)}[sec(x)tg(x)+sec^2(x)]+0=$$
$$\frac{sec^2(x)+sec(x)tg(x)}{sec(x)+tg(x)}=\frac{sec(x)\cancel{[sec(x)+tg(x)]}}{\cancel{sec(x)+tg(x)}}=sec(x)$$

Exercício IV

$$\int cossec(x)\partial x \int cossec(x)\left(\frac{cossec(x)-cotg(x)}{cossec(x)-cotg(x)}\right)\partial x=$$
$$\int \frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}\partial x=$$
$$\int [cossec(x)-cotg(x)]^{-1}[-cossec(x)cotg(x)+cossec^2(x)]\partial x=\int u^{-1}\partial u=$$
$$\ln|u|+c=\ln|cossec(x)-cotg(x)|+c$$
$$u=cossec(x)-cotg(x)=\partial u=[-cossec(x)cotg(x)-(-cossec^2(x))]\partial x\rightarrow$$
$$\partial u=[-cossec(x)cotg(x)+cossec^2(x)]\partial x \tag{70}$$

$$\frac{\partial(\ln|cossec(x)-cotg(x)|+c)}{\partial x}=$$
$$\frac{1}{cossec(x)-cotg(x)}[-cossec(x)cotg(x)-(-cossec^2(x))]+0=$$
$$\frac{cossec^2(x)-cossec(x)cotg(x)}{cossec(x)-cotg(x)}=\frac{cossec(x)\cancel{[cossec(x)-cotg(x)]}}{\cancel{cossec(x)-cotg(x)}}=cossec(x)$$

Exercício V

$$\int tg(3x)\partial x=\int tg(u)\frac{\partial u}{3}=\frac{1}{3}tg(u)\partial u=\frac{1}{3}\ln|sec(u)|+c=\frac{\ln|sec(u)|}{3}+c=\frac{\ln|sec(3x)|}{3}+c$$
$$u=3x,\partial u=3\partial x\rightarrow\frac{\partial u}{3}=\partial x \tag{71}$$

Exercício VI

$$\int\frac{\partial x}{sen(2x)}=\int sen^{-1}(2x)\partial x=\int cossec(2x)\partial x=\int cossec(u)\frac{\partial u}{2}=\frac{1}{2}\int cossec(u)\partial u=$$
$$\frac{1}{2}\ln|cossec(u)-cotg(u)|+c=\frac{\ln|cossec(u)-cotg(u)|}{2}+c=\frac{\ln|cossec(2x)-cotg(2x)|}{2}+c \tag{72}$$
$$u=2x,\partial u=2\partial x\rightarrow\frac{\partial u}{2}=\partial x$$

Integração de funções trigonométricas – Aula 20

Exercício I

$$\int\frac{tg(\sqrt{x})\partial x}{\sqrt{x}}=\int\frac{tg\left(x^{\frac{1}{2}}\right)\partial x}{x^{\frac{1}{2}}}=\int tg\left(x^{\frac{1}{2}}\right)x^{\frac{-1}{2}}\partial x=\int tg(u)2\partial u=2\int tg(u)\partial u=$$
$$2\ln|sec(u)|+c=2\ln|sec(\sqrt{x})|+c \tag{73}$$
$$u=\sqrt{x}=x^{\frac{1}{2}},\partial u=\frac{1}{2}x^{\frac{-1}{2}}\partial x\rightarrow 2\partial u=x^{\frac{-1}{2}}\partial x$$

Exercício II

$$\int \frac{cotg(\ln|x|)\partial x}{x} = \int cotg(\ln|x|)x^{-1}\partial x = \int cotg(u)\partial u = \ln|sen(u)|+c = \ln|sen(\ln|x|)|+c$$
$$u=\ln|x|, \partial u = \frac{1}{x}\partial x \rightarrow \partial u = x^{-1}\partial x \quad (74)$$

Exercício III

$$\int sec(5x-\pi)\partial x = \int sec(u)\frac{\partial u}{5} = \frac{1}{5}\int sec(u)\partial u = \frac{1}{5}\ln|sec(u)+tg(u)|+c =$$
$$\frac{\ln|sec(5x-\pi)+tg(5x-\pi)|}{5}+c \quad (75)$$
$$u=5x-\pi, \partial u = 5\partial x \rightarrow \frac{\partial u}{5} = \partial x$$

Integral de potência sen(x) ou cos(x) – Aula 21

$\int \cos^n(x)\partial x$

$\int sen^n(x)\partial x$

$n \rightarrow$ ímpar $\quad sen^2(x)+\cos^2(x)=1$

$n \rightarrow$ par $\quad sen^2(x)=\frac{1-\cos(2x)}{2}$, $\quad \cos^2(x)=\frac{1+\cos(2x)}{2}$

$\int sen^m(x)\cos^n(x)\partial x$

$n \vee m \rightarrow$ ímpar $\quad$ Separa o $\partial u \begin{cases} sen(x)\partial x \\ \cos(x)\partial x \end{cases}$

Transfoma em $sen^p(x)$ ou $\cos^p(x)$ através de $sen^2(x)+\cos^2(x)=1$

$n \wedge m \rightarrow$ par $\quad sen^2(x)=\frac{1-\cos(2x)}{2}$, $\quad \cos^2(x)=\frac{1+\cos(2x)}{2}$

Exercício I

$$\int \cos^5(x)\partial x=\int \cos^4(x)\cos(x)\partial x=\int \left(\cos^2(x)\right)^2\cos(x)\partial x=\int \left(1-sen^2(x)\right)^2\cos(x)\partial x=$$
$$\int \left[1-2\,sen^2(x)+sen^4(x)\right]\cos(x)\partial x=\int \left[\cos(x)-2\,sen^2(x)\cos(x)+sen^4(x)\cos(x)\right]\partial x=$$
$$\int \cos(x)\partial x-2\int sen^2(x)\cos(x)\partial x+\int sen^4(x)\cos(x)\partial x=\int \partial u-2\int u^2\partial u+\int u^4\partial u=$$
$$u-2\frac{u^3}{3}+\frac{u^5}{5}+c=sen(x)-\frac{2\,sen^3(x)}{3}+\frac{sen^5(x)}{5}+c$$
$$u=sen(x),\partial u=\cos(x)\partial x$$

$$\frac{\partial\left(sen(x)-\frac{2\,sen^3(x)}{3}+\frac{sen^5(x)}{5}+c\right)}{\partial x}=\frac{\partial\left(sen(x)-\frac{2}{3}sen^3(x)+\frac{1}{5}sen^5(x)+c\right)}{\partial x}= \tag{76}$$
$$\cos(x)-\frac{2}{\cancel{3}}\cancel{3}\,sen^2(x)\cos(x)+\frac{1}{\cancel{5}}\cancel{5}\,sen^4(x)\cos(x)+0=\cos(x)\left[1-2\,sen^2(x)+sen^4(x)\right]=$$
$$\cos(x)\left[1-\left(sen^2(x)+sen^2(x)\right)+\left(sen^2(x)\right)^2\right]=$$
$$\cos(x)\left[\left(1-sen^2(x)\right)-sen^2(x)+\left(1-\cos^2(x)\right)^2\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+\left(1-2\cos^2(x)+\cos^4(x)\right)\right]=$$
$$\cos(x)\left[\cos^2(x)-sen^2(x)+1-\left(\cos^2(x)+\cos^2(x)\right)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}+\left(1-sen^2(x)\right)-\cancel{\cos^2(x)}-\cos^2(x)+\cos^4(x)\right]=$$
$$\cos(x)\left[\cancel{\cos^2(x)}-\cancel{\cos^2(x)}+\cos^4(x)\right]=\cos(x)\cos^4(x)=\cos^5(x)$$

Exercício II

$$
\begin{gathered}
\int sen^4(x)\partial x=\int \left(sen^2(x)\right)^2\partial x=\int \left(\frac{1-\cos(2x)}{2}\right)^2\partial x=\int \frac{1-2\cos(2x)+\cos^2(2x)}{4}\partial x=\\
\frac{1}{4}\int \left[1-2\cos(2x)+\frac{1+\cos(2\cdot 2x)}{2}\right]\partial x=\frac{1}{4}\int \partial x-\frac{1}{2}\int \cos(2x)\partial x+\frac{1}{8}\int [1+\cos(4x)]\partial x=\\
\frac{1}{4}\int \partial x-\frac{1}{2}\int \cos(2x)\partial x+\frac{1}{8}\int \partial x+\frac{1}{8}\int \cos(4x)\partial x=\\
\frac{1}{4}\int \frac{\partial u}{2}-\frac{1}{2}\int \cos(u)\frac{\partial u}{2}+\frac{1}{8}\int \frac{\partial u}{2}+\frac{1}{8}\int \cos(2u)\frac{\partial u}{2}=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{16}\int \cos(2u)\partial u=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{16}\int \cos(v)\frac{\partial v}{2}=\\
\frac{1}{8}\int \partial u-\frac{1}{4}\int \cos(u)\partial u+\frac{1}{16}\int \partial u+\frac{1}{32}\int \cos(v)\partial v=\\
\frac{1}{8}u-\frac{1}{4}sen(u)+\frac{1}{16}u+\frac{1}{32}sen(v)+c=\frac{2u+u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\\
\frac{3u}{16}-\frac{sen(u)}{4}+\frac{sen(2u)}{32}+c=\frac{3\cdot 2x}{16}-\frac{sen(2x)}{4}+\frac{sen(2\cdot 2x)}{32}+c=\\
\frac{3x}{8}-\frac{sen(2x)}{4}+\frac{sen(4x)}{32}+c\\
u=2x,\frac{\partial u}{2}=\partial x\\
v=2u,\frac{\partial v}{2}=\partial u
\end{gathered}
\tag{77}
$$

Integral do produto de potência entre sen(x) e cos(x) – Aula 22

Exercício I

$$\int sen^5(x)\cos^2(x)\partial x=\int sen^4(x)\cos^2(x)sen(x)\partial x=\int \left(sen^2(x)\right)^2\cos^2(x)sen(x)\partial x=$$
$$\int \left(1-\cos^2(x)\right)^2\cos^2(x)sen(x)\partial x=\int \left[1-2\cos^2(x)+\cos^4(x)\right]\cos^2(x)sen(x)\partial x=$$
$$\int \left[\cos^2(x)-2\cos^4(x)+\cos^6(x)\right]sen(x)\partial x=$$
$$\int \cos^2(x)sen(x)\partial x-2\int \cos^4(x)sen(x)\partial x+\int \cos^6(x)sen(x)\partial x=$$
$$\int u^2(-\partial u)-2\int u^4(-\partial u)+\int u^6(-\partial u)=-\int u^2\partial u+2\int u^4\partial u-\int u^6\partial u=$$
$$\frac{-u^3}{3}+2\frac{u^5}{5}-\frac{u^7}{7}+c=\frac{-\cos^3(x)}{3}+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}+c$$
$$u=\cos(x),-\partial u=sen(x)\partial x \tag{78}$$

$$\frac{\partial\left(\frac{-\cos^3(x)}{3}+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}+c\right)}{\partial x}=\frac{\partial\left(-\frac{1}{3}\cos^3(x)+\frac{2}{5}\cos^5(x)-\frac{1}{7}\cos^7(x)+c\right)}{\partial x}=$$
$$-\frac{1}{3}3\cos^2(x)(-sen(x))+\frac{2}{5}5\cos^4(x)(-sen(x))-\frac{1}{7}7\cos^6(x)(-sen(x))+0=$$
$$\cos^2(x)sen(x)-2\cos^4(x)sen(x)+\cos^6(x)sen(x)=sen(x)\left[\cos^2(x)-2\cos^4(x)+\cos^6(x)\right]=$$
$$sen(x)\cos^2(x)\left[1-2\cos^2(x)+\cos^4(x)\right]=\left(1-\cos^2(x)\right)^2\cos^2(x)sen(x)=$$
$$\left(sen^2(x)\right)^2\cos^2(x)sen(x)=sen^4(x)\cos^2(x)sen(x)=sen^5(x)\cos^2(x)$$

Exercício II

$$\int sen^2(x)\cos^4(x)\partial x=\int \frac{1-\cos(2x)}{2}\left(\cos^2(x)\right)^2\partial x=\frac{1}{2}\int \left(1-\cos(2x)\right)\left(\frac{1+\cos(2x)}{2}\right)^2\partial x$$

$$\frac{1}{2}\int \left(1-\cos(2x)\right)\left(\frac{1+2\cos(2x)+\cos^2(2x)}{4}\right)\partial x=$$

$$\frac{1}{8}\int \left(1-\cos(2x)\right)\left(1+2\cos(2x)+\cos^2(2x)\right)\partial x=$$

$$\frac{1}{8}\int \left(1+\cancel{2}\cos(2x)+\cancel{\cos^2(2x)}-\cancel{\cos(2x)}-\cancel{2}\cos^2(2x)-\cos^3(2x)\right)\partial x=$$

$$\frac{1}{8}\int \left(sen^2(2x)+\cos(2x)-\cos^2(2x)\cos(2x)\right)\partial x=$$

$$\frac{1}{8}\int sen^2(2x)\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \cos^2(2x)\cos(2x)\partial x=$$

$$\frac{1}{8}\int \frac{1-\cos(4x)}{2}\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \left(1-sen^2(2x)\right)\cos(2x)\partial x= \tag{79}$$

$$\frac{1}{16}\int \left(1-\cos(4x)\right)\partial x+\frac{1}{8}\int \cos(2x)\partial x-\frac{1}{8}\int \left(\cos(2x)-sen^2(2x)\cos(2x)\right)\partial x=$$

$$\frac{1}{16}\int \partial x-\frac{1}{16}\int \cos(4x)\partial x+\cancel{\frac{1}{8}\int \cos(2x)\partial x}-\cancel{\frac{1}{8}\int \cos(2x)\partial x}+\frac{1}{8}\int sen^2(2x)\cos(2x)\partial$$

$$\frac{1}{16}\int \partial x-\frac{1}{32}\int \cos(2y)\partial y+\frac{1}{16}\int sen^2(y)\cos(y)\partial y=$$

$$\frac{1}{16}\int \partial x-\frac{1}{64}\int \cos(z)\partial z+\frac{1}{16}\int u^2\partial u=\frac{1}{16}x-\frac{1}{64}sen(z)+\frac{1}{16}\frac{u^3}{3}+c=$$

$$\frac{x}{16}-\frac{sen(2y)}{64}+\frac{sen^3(y)}{48}+c=\frac{x}{16}-\frac{sen(4x)}{64}+\frac{sen^3(2x)}{48}+c$$

$$y=2x,\frac{\partial y}{2}=\partial x;z=2y,\frac{\partial z}{2}=\partial y;u=sen(y),\partial u=\cos(y)\partial y$$

Integral de Funções Trigonométricas – Aula 23

$$tg(x)=\frac{sen(x)}{\cos(x)},cotg(x)=\frac{\cos(s)}{sen(x)}$$

$$sec(x)=\frac{1}{\cos(x)},cossec(x)=\frac{1}{sen(x)}$$

$$sen^2(x)+\cos^2(x)=1$$

$$\frac{sen^2(x)}{sen^2(x)}+\frac{\cos^2(x)}{sen^2(x)}=\frac{1}{sen^2(x)}\rightarrow 1+cotg^2(x)=cossec^2(x)\rightarrow cossec^2(x)-cotg^2(x)=1$$

$$\frac{sen^2(x)}{\cos^2(x)}+\frac{\cos^2(x)}{\cos^2(x)}=\frac{1}{\cos^2(x)}\rightarrow tg^2(x)+1=sec^2(x)\rightarrow sec^2(x)-tg^2(x)=1$$

Exercício I

$$\begin{gathered}\int [tg(2x)+cotg(2x)]^2 \partial x = \int \left[tg^2(2x)+2\,tg(2x)cotg(2x)+cotg^2(2x)\right]\partial x = \\ \int tg^2(2x)\partial x + 2\int tg(2x)cotg(2x)\partial x + \int cotg^2(2x)\partial x = \\ \int \left(sec^2(2x)-1\right)\partial x + 2\int \frac{\cancel{sen(2x)}}{\cancel{\cos(2x)}}\frac{\cancel{\cos(2x)}}{\cancel{sen(2x)}}\partial x + \int \left(cossec^2(2x)-1\right)\partial x = \\ \int sec^2(2x)\partial x - \int \partial x + 2\int \partial x + \int cossec^2(2x)\partial x - \int \partial x = \\ \frac{1}{2}\int sec^2(u)\partial u - \int \partial x + 2\int \partial x + \frac{1}{2}\int cossec^2(u)\partial u - \int \partial x = \\ \frac{1}{2}tg(u) - \cancel{x} + \cancel{2x} + \frac{1}{2}(-cotg(u)) - \cancel{x} + c = \frac{tg(2x)}{2} - \frac{cotg(2x)}{2} + c \\ u = 2x;\frac{\partial u}{2} = \partial x\end{gathered} \tag{80}$$

$$\begin{gathered}\frac{\partial\left(\frac{tg(2x)}{2} - \frac{cotg(2x)}{2} + c\right)}{\partial x} = \frac{1}{2}sec^2(2x)2 - \frac{1}{2}\left(-cossec^2(2x)\right)2 + 0 = \\ sec^2(2x)+cossec^2(2x) = \left(1+tg^2(2x)\right)+\left(1+cossec^2(2x)\right) = 2+tg^2(2x)+cotg^2(2x) = \\ tg^2(2x)+2\,\cancel{tg(2x)cotg(2x)}+cotg^2(2x) = [tg(2x)+cotg(2x)]^2\end{gathered}$$

Integral Definida com Seno e Cosseno – Aula 24

	$0(0^o)$	$\frac{\pi}{6}(30^o)$	$\frac{\pi}{4}(45^o)$	$\frac{\pi}{3}(60^o)$	$\frac{\pi}{2}(90^o)$
sen	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

Exercício I

$$\begin{gathered}\int_0^{\frac{\pi}{4}}\left(\frac{1+\cos^2(\theta)}{\cos^2(\theta)}\right)\partial\theta = \int_0^{\frac{\pi}{4}}\left(\frac{1}{\cos^2(\theta)}+\frac{\cos^2(\theta)}{\cos^2(\theta)}\right)\partial\theta = \int_0^{\frac{\pi}{4}}\left(sec^2(\theta)+1\right)\partial\theta = \\ \int_0^{\frac{\pi}{4}} sec^2(\theta)\partial\theta + \int_0^{\frac{\pi}{4}}\partial\theta = tg(\theta)]_0^{\frac{\pi}{4}} + \theta]_0^{\frac{\pi}{4}} = \left(tg\left(\frac{\pi}{4}\right)-tg(0)\right)+\left(\frac{\pi}{4}-0\right) = (1-0)+\left(\frac{\pi}{4}-0\right) = 1+\frac{\pi}{4} = \\ \frac{4+\pi}{4}\end{gathered} \tag{81}$$

Exercício II

$$\begin{gathered}\int_0^{\pi}(4\,sen(\theta)-3\cos(\theta))\partial\theta = 4\int_0^{\pi} sen(\theta)\partial\theta - 3\int_0^{\pi}\cos(\theta)\partial\theta = 4(-\cos(\theta))]_0^{\pi} - 3\,sen(\theta)]_0^{\pi} = \\ -4\cos(\theta)]_0^{\pi} - 3\,sen(\theta)]_0^{\pi} = -4(\cos(\pi)-\cos(0)) - 3(sen(\pi)-sen(0)) = \\ -4(-1-1)-3(0-0) = -4(-2) = 8\end{gathered} \tag{82}$$

Integral Definida para funções trigonométricas – Aula 25

Exercício I

$$\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)+sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)}{sec^2(\theta)}+\frac{sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\left(sen(\theta)\frac{1}{\left(\frac{1}{\cos^2(\theta)}\right)}+sen(\theta)\frac{sen^2(\theta)}{\cancel{\cos^2(\theta)}}\frac{1}{\left(\frac{1}{\cancel{\cos^2(\theta)}}\right)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\left(\cos^2(\theta)sen(\theta)+sen^3(\theta)\right)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}sen^2(\theta)sen(\theta)\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}\left(1-\cos^2(\theta)\right)sen(\theta)\partial\theta= \tag{83}$$
$$\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta+\int_0^{\frac{\pi}{3}}\left(sen(\theta)-\cos^2(\theta)sen(\theta)\right)\partial\theta=$$
$$\cancel{\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta}+\int_0^{\frac{\pi}{3}}sen(\theta)\partial\theta-\cancel{\int_0^{\frac{\pi}{3}}\cos^2(\theta)sen(\theta)\partial\theta}=$$
$$-\cos(\theta)\Big]_0^{\frac{\pi}{3}}=-\left(\cos\left(\frac{\pi}{3}\right)-\cos(0)\right)=-\left(\frac{1}{2}-1\right)=-\left(\frac{-1}{2}\right)=\frac{1}{2}$$

$$\int_0^{\frac{\pi}{3}}\left(\frac{sen(\theta)+sen(\theta)tg^2(\theta)}{sec^2(\theta)}\right)\partial\theta=\int_0^{\frac{\pi}{3}}\frac{sen(\theta)\left(1+tg^2(\theta)\right)}{sec^2(\theta)}\partial\theta=\int_0^{\frac{\pi}{3}}\frac{sen(\theta)\cancel{sec^2(\theta)}}{\cancel{sec^2(\theta)}}\partial\theta=$$
$$\int_0^{\frac{\pi}{3}}sen(\theta)\partial\theta=-\cos(\theta)\Big]_0^{\frac{\pi}{3}}=-\left(\cos\left(\frac{\pi}{3}\right)-\cos(0)\right)=-\left(\frac{1}{2}-1\right)=-\left(\frac{-1}{2}\right)=\frac{1}{2}$$

Exercício II

$$\int_0^{\pi}sec^2\left(\frac{t}{4}\right)\partial t=4\int_0^{\pi}sec^2(u)\partial u=4\,tg(u)\Big]_0^{\pi}=4\,tg\left(\frac{t}{4}\right)\Big]_0^{\pi}=4\left(tg\left(\frac{\pi}{4}\right)-tg(0)\right)=4(1-0)=4 \tag{84}$$
$$u=\frac{t}{4},4\partial u=\partial t$$

Integral envolvendo Funções Seno e Cosseno – Aula 26

$sen(2x)=2sen(x)\cos(x)$

Exercício I

$$\int \frac{sen(x)}{1-sen^2(x)}\partial x=\int \frac{sen(x)}{\cos^2(x)}\partial x=\int \cos^{-2}(x)sen(x)\partial x=-\int u^{-2}\partial u=\frac{-u^{-1}}{(-1)}+c=$$
$$u^{-1}+c=\frac{1}{u}+c=\frac{1}{\cos(x)}+c=sec(x)+c$$
$$u=\cos(x),-\partial u=sen(x)\partial x \tag{85}$$

Exercício II

$$\int \frac{sen(2x)}{\cos(x)}\partial x=\int \frac{2sen(x)\cancel{\cos(x)}}{\cancel{\cos(x)}}\partial x=2\int sen(x)\partial x=2(-\cos(x))+c=-2\cos(x)+c \tag{86}$$

Integral de uma função exponencial de seno – Aula 27

Exercício I

$$\int e^{sen(\theta)}\cos(\theta)\partial\theta=\int e^u\partial u=e^u+c=e^{sen(\theta)}+c$$
$$u=sen(\theta),\partial u=\cos(\theta)\partial\theta \tag{87}$$

Exercício II

$$\int sen(\pi t)\partial t=\frac{1}{\pi}\int sen(u)\partial u=\frac{1}{\pi}(-\cos(u))+c=\frac{-\cos(u)}{\pi}+c=\frac{-\cos(\pi t)}{\pi}+c$$
$$u=\pi t,\frac{\partial u}{\pi}=\partial t \tag{88}$$

Integral Definida do Produto de secante e tangente – Aula 28

Exercício I

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{\cos(\theta)}\frac{sen(\theta)}{\cos(\theta)}\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{sen(\theta)}{\cos^2(\theta)}\partial\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\cos^{-2}(\theta)sen(\theta)\partial\theta=$$
$$-\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} u^{-2}\partial u=\left[\frac{-u^{-1}}{(-1)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=[u^{-1}]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{u}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{\cos\left(\frac{\pi}{3}\right)}-\frac{1}{\cos\left(\frac{\pi}{4}\right)}=\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{\left(\frac{\sqrt{2}}{2}\right)}=$$
$$2-\frac{2}{\sqrt{2}}=2-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}=2-\sqrt{2}$$
$$u=\cos(\theta),-\partial u=sen(\theta)\partial\theta \tag{89}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec(\theta)tg(\theta)\partial\theta=[sec(\theta)]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\left[\frac{1}{\cos(\theta)}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{1}{\cos\left(\frac{\pi}{3}\right)}-\frac{1}{\cos\left(\frac{\pi}{4}\right)}=\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{\left(\frac{\sqrt{2}}{2}\right)}=2-\frac{2}{\sqrt{2}}=$$
$$2-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}=2-\sqrt{2}$$

Integral de seno pelo Método da Substituição – Aula 29

Exercício I

$$\int \sqrt{x}\,sen\left(1+\sqrt{x^3}\right)\partial x=\int sen\left(1+x^{\frac{3}{2}}\right)x^{\frac{1}{2}}\partial x=\frac{2}{3}sen(u)\partial u=\frac{2}{3}(-\cos(u))+c= \frac{-2\cos\left(1+\sqrt{x^3}\right)}{3}+c \quad (90)$$
$$u=1+\sqrt{x^3}=1+x^{\frac{3}{2}},\partial u=\frac{3}{2}x^{\frac{1}{2}}\partial x=\frac{2\partial u}{3}=x^{\frac{1}{2}}\partial x$$

Integral Trigonométrica – Método da Substituição – Aula 30

Exercício I

$$\int (1+tg(\theta))^5 sec^2(\theta)\partial\theta=\int u^5\partial u=\frac{u^6}{6}+c=\frac{(1+tg(\theta))^6}{6}+c \quad (91)$$
$$u=1+tg(\theta),\partial u=sec^2(\theta)\partial\theta$$

Exercício II

$$\int sec(2\theta)tg(2\theta)\partial\theta=\frac{1}{2}\int sec(u)tg(u)\partial u=\frac{1}{2}sec(u)+c=\frac{sec(2\theta)}{2}+c \quad (92)$$
$$u=2\theta,\frac{\partial u}{2}=\partial\theta$$

Integral Trigonométrica – Método da Substituição – Aula 31

Exercício I

$$\int \sqrt{cotg(x)}cossec^2(x)\partial x=\int cotg^{\frac{1}{2}}(x)cossec^2(x)\partial x=-\int u^{\frac{1}{2}}\partial u=\frac{-u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+c=\frac{-2\sqrt{u^3}}{3}+c= \frac{-2\sqrt{cotg^3(x)}}{3}+c \quad (93)$$
$$u=cotg(x),-\partial u=cossec^2(x)$$

Exercício II

$$\int sec^3(x)tg(x)\partial x=\int sec^2(x)sec(x)tg(x)\partial x=\int u^2\partial u=\frac{u^3}{3}+c=\frac{sec^3(x)}{3}+c \quad (94)$$
$$u=sec(x),\partial u=sec(x)tg(x)\partial x$$

Integral de Cosseno pelo Método da Substituição – Aula 32

Exercício I

$$\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\partial x=\int \cos\left(\pi x^{-1}\right)x^{-2}\partial x=\frac{-1}{\pi}\int \cos(u)\partial u=\frac{-1}{\pi}sen(u)+c=\frac{-sen\left(\frac{\pi}{x}\right)}{\pi}+c$$
$$u=\frac{\pi}{x}=\pi x^{-1},\partial u=-\pi x^{-2}\rightarrow\frac{-\partial u}{\pi}=x^{-2}\partial x \tag{95}$$

Integral de Potência de Tangente – Aula 33

Exercício I

$$\int tg^3(x)\partial x=\int tg^2(x)tg(x)\partial x=\int \left(sec^2(x)-1\right)tg(x)\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int tg(x)\partial x=\int tg(x)sec^2(x)\partial x-\int\frac{sen(x)}{\cos(x)}\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int \cos^{-1}(x)sen(x)\partial=\int u\partial u+\int v^{-1}\partial v=\frac{u^2}{2}+\ln|v|+c=$$
$$\frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$
$$u=tg(x),\partial u=sec^2(x)\partial x;v=\cos(x),-\partial v=sen(x)\partial x \tag{96}$$
$$\int tg^3(x)\partial x=\int tg^2(x)tg(x)\partial x=\int \left(sec^2(x)-1\right)tg(x)\partial x=$$
$$\int tg(x)sec^2(x)\partial x-\int tg(x)\partial x=\int u\partial u-\int tg(x)\partial x=\frac{u^2}{2}-\left(\ln|sec(x)|\right)+c=$$
$$\frac{u^2}{2}-\left(\ln\left|\cos^{-1}(x)\right|\right)+c=\frac{u^2}{2}-\left(-\ln|\cos(x)|\right)+c=\frac{u^2}{2}+\ln|\cos(x)|+c=\frac{tg^2(x)}{2}+\ln|\cos(x)|+c$$
$$u=tg(x),\partial u=sec^2(x)\partial x$$

Integral de Potência de Cotangente – Aula 34

Exercício I

$$\int cotg^4(3x)\partial x=\int cotg^2(3x)cotg^2(3x)\partial x=\int \left(cossec^2(3x)-1\right)cotg^2(3x)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int cotg^2(3x)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int \left(cossec^2(3x)-1\right)\partial x=$$
$$\int cotg^2(3x)cossec^2(3x)\partial x-\int cossec^2(3x)\partial x+\int\partial x=$$
$$\frac{1}{3}\int cotg^2(u)cossec^2(u)\partial u-\frac{1}{3}\int cossec^2(u)\partial u+\int\partial x=$$
$$\frac{-1}{3}\int v^2\partial v+\frac{1}{3}\int\partial v+\int\partial x=\frac{-1}{3}\frac{v^3}{3}+\frac{1}{3}v+x+c=\frac{-cotg^3(u)}{9}+\frac{cotg(u)}{3}+x+c= \tag{97}$$
$$\frac{-cotg^3(3x)}{9}+\frac{cotg(3x)}{3}+x+c$$
$$u=3x,\frac{\partial u}{3}=\partial x;v=cotg(u),-\partial v=cossec^2(u)$$