

Introdução as Derivadas – Aula 1

$$\frac{\partial f(x)}{\partial x} = D_x f(x) = f'(x)$$

- 1) $y = c \rightarrow y' = 0$
- 2) $y = x \rightarrow y' = 1$
- 3) $y = x^c \rightarrow y' = c \cdot x^{c-1}$
- 4) $y = e^x \rightarrow y' = e^x$
- 5) $y = \ln(x) \rightarrow y' = \frac{1}{x}$
- 6) $y = f \cdot h \rightarrow y' = f' \cdot h + f \cdot h'$
- 7) $y = \frac{f}{h} \rightarrow y' = \frac{f' \cdot h - f \cdot h'}{h^2}$
- 8) $y = f^c \rightarrow y' = c \cdot f^{c-1} \cdot f'$
- 9) $y = e^f \rightarrow y' = e^f \cdot f'$
- 10) $y = c^f \rightarrow y' = c^f \cdot f' \cdot \ln(c)$
- 11) $y = \ln(f) \rightarrow y' = \frac{f'}{f}$
- 12) $y = \log_c(f) \rightarrow y' = \frac{f'}{f} \cdot \log_c(e)$

Exercício I

- a) $y = 8 \rightarrow y' = 0$
 - b) $y = \sqrt{3} \rightarrow y' = 0$
 - c) $f(x) = \pi \rightarrow f'(x) = 0$
 - d) $g(x) = (\pi - 1)^\pi \rightarrow g'(x) = 0$
- (1)

Exercício II

- a) $y = x^5 \rightarrow y' = 5x^{5-1} = 5x^4$
 - b) $h(x) = x^{-5} \rightarrow h'(x) = -5x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = -\frac{5}{x^6}$
 - c) $g(x) = 5x^3 \rightarrow g'(x) = 5 \cdot 3x^{3-1} = 15x^2$
- (2)

Exercício III

$$h(x) = 8x \rightarrow h'(x) = 8 \cdot 1 = 8$$

(3)

Exercício IV

$$\begin{aligned} f(x) &= 7x^3 - 2x - 400 \\ f'(x) &= 7 \cdot 3x^{3-1} - 2 \cdot 1 - 0 = 21x^2 - 2 \end{aligned}$$

(4)

Derivada com X no Denominador – [Aula 2](#)

Exercício I

$$\begin{aligned}g(x) &= \frac{3}{x^5} = 3x^{-5} \\g'(x) &= 3(-5x^{-5-1}) = -15x^{-6} = -\frac{15}{x^6}\end{aligned}\tag{5}$$

Exercício II

$$\begin{aligned}h(x) &= 3x^5 - \frac{2}{x^4} = 3x^5 - 2x^{-4} \\h'(x) &= 3(5x^{5-1}) - 2(-4x^{-4-1}) = 15x^4 + 8x^{-5} = 15x^4 + \frac{8}{x^5}\end{aligned}\tag{6}$$

Derivada de Função Raiz – [Aula 3](#)

Exercício I

$$\begin{aligned}y &= \sqrt[3]{x^4} = x^{\frac{4}{3}} \\y' &= \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}\end{aligned}\tag{7}$$

Exercício II

$$\begin{aligned}g(x) &= 7\sqrt[3]{x} = 7x^{\frac{1}{3}} \\g'(x) &= 7\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) = \frac{7}{3}x^{-\frac{2}{3}} = \frac{7}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = \frac{7}{3\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right) = \frac{7\sqrt[3]{x}}{3x}\end{aligned}\tag{8}$$

Derivada de uma Função Potência – [Aula 4](#)

Exercício I

$$y = x^3 \rightarrow y' = 3x^2\tag{9}$$

Exercício II

$$\begin{aligned}y &= \frac{5x^4x^3}{x^2} = 5x^{4+3-2} = 5x^5 \\y' &= 5(5x^{5-1}) = 25x^4\end{aligned}\tag{10}$$

Derivada de uma Função Potência – [Aula 5](#)

Exercício I

$$y = \frac{x^2 \sqrt[3]{x}}{\sqrt[3]{x}} = x^2 \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{2+\frac{1}{2}-\frac{1}{3}} = x^{\frac{12+3-2}{6}} = x^{\frac{13}{6}}$$
$$y' = \frac{13}{6} x^{\frac{13}{6}-1} = \frac{13}{6} x^{\frac{7}{6}} = \frac{13}{6} \sqrt[6]{x^7}$$
(11)

Derivada de Função Exponencial e Logarítmica – [Aula 6](#)

Exercício I

$$f(x) = 3e^x + 10 \cdot \ln(x)$$
$$f'(x) = 3e^x + \frac{10}{x} = \frac{3xe^x + 10}{x}$$
(12)

Exercício II

$$g(x) = 7e^x + 9 \cdot \ln(x) + 3x^4 - 4x + 100$$
$$g'(x) = 7e^x + \frac{9}{x} + 12x^3 - 4 = \frac{7xe^x + 9 + 12x^4 - 4x}{x}$$
(13)

Derivada de um Produto de Funções – [Aula 7](#)

Exercício I

$$f(x) = x \cdot \ln(x)$$
$$\begin{cases} g(x) = x & \rightarrow g'(x) = 1 \\ h(x) = \ln(x) & \rightarrow h'(x) = \frac{1}{x} \end{cases}$$
(14)
$$f(x) = g(x) \cdot h(x) \rightarrow f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$$

Derivada de uma Divisão de Funções – [Aula 8](#)

Exercício I

$$f(x) = \frac{e^x}{3x} = e^x \cdot \frac{1}{3x} = e^x \cdot x^{-1}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = \frac{x^{-1}}{3} = \frac{1}{3x} & \rightarrow h'(x) = \frac{-x^{-2}}{3} = -\frac{1}{3x^2} \end{cases}$$

$$f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x) = e^x \cdot \frac{1}{3x} + e^x \cdot \left(-\frac{1}{3x^2}\right) = \frac{e^x}{3x} - \frac{e^x}{3x^2} = \frac{x \cdot e^x - e^x}{3x^2} = \frac{e^x(x-1)}{3x^2}$$

(15)

$$f(x) = \frac{e^x}{3x}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = 3x & \rightarrow h'(x) = 3 \end{cases}$$

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2} = \frac{e^x \cdot 3x - e^x \cdot 3}{(3x)^2} = \frac{3e^x(x-1)}{9x^2} = \frac{e^x(x-1)}{3x^2}$$

Derivadas Básicas – [Aula 9](#)

Exercício I

$$y = \frac{2x^2 + 5x}{x} = \frac{2x^2}{x} + \frac{5x}{x} = 2x + 5$$

$$y' = 2$$

(16)

Exercício II

$$h(x) = \frac{3(x^2 - 1)}{x} = \frac{3x^2 - 3}{x} = \frac{3x^2}{x} - \frac{3}{x} = 3x - 3x^{-1}$$

$$h'(x) = 3 - (-3x^{-2}) = 3 + \frac{3}{x^2} = \frac{3x^2 + 3}{x^2} = \frac{3(x^2 + 1)}{x^2}$$

(17)

Exercícios de Derivada – [Aula 10](#)

Exercício I

$$h(x) = 3x^3(2 + 4x) = 6x^3 + 12x^4$$

$$h'(x) = 18x^2 + 48x^3 = 6x^2(8x + 3)$$

(18)

Exercício II

$$\begin{aligned}g(x) &= (x^2 - 1)(x^3 + 4) = x^5 + 4x^2 - x^3 - 4 \\g'(x) &= 5x^4 + 8x - 3x^2 = x(5x^3 - 3x + 8)\end{aligned}\quad (19)$$

Curso básico de derivadas – [Aula 12](#)

Exercício I

$$\begin{aligned}g(x) &= x^4 + 2e^x + e^2 \\g'(x) &= 4x^3 + 2e^x = 2(2x^3 + e^x)\end{aligned}\quad (20)$$

Exercício II

$$\begin{aligned}g(x) &= \sqrt[3]{x^7} + \frac{3}{x^2} + 5 = x^{\frac{7}{3}} + 3x^{-2} + 5 \\g'(x) &= \frac{7x^{\frac{4}{3}}}{3} + (-6x^{-3}) = \frac{7\sqrt[3]{x^4}}{3} - \frac{6}{x^3} = \frac{7x^3\sqrt[3]{x^4} - 18}{3x^3} = \frac{7\sqrt[3]{x^{13}} - 18}{3x^3}\end{aligned}\quad (21)$$

Derivada de um Produto de Funções – [Aula 13](#)

Exercício I

$$\begin{aligned}y &= 8x \cdot \ln(x) \\y' &= 8 \cdot \ln(x) + 8x \cdot \frac{1}{x} = 8 \cdot \ln(x) + 8 = 8(\ln(x) + 1)\end{aligned}\quad (22)$$

Derivada de função composta, raiz, polinomial – [Aula 14](#)

Exercício I

$$y = x^3 \rightarrow y' = 3x^2 \quad (23)$$

Exercício II

$$\begin{aligned}f(x) &= (2x^2 - 1)^3 \\g(x) &= 2x^2 - 1 \rightarrow g'(x) = 4x \\f'(x) &= g(x)^p \rightarrow f'(x) = p \cdot g(x)^{p-1} \cdot g'(x) = 3(2x^2 - 1)^2 \cdot 4x = 12x(2x^2 - 1)^2\end{aligned}\quad (24)$$

Exercício III

$$\begin{aligned}y &= (3 - x^2)^3 \\y' &= 3(3 - x^2)^2(-2x) = -6x(3 - x^2)^2\end{aligned}\quad (25)$$

Exercício IV

$$y = \frac{3}{(2x^2 - 1)^4} = 3(2x^2 - 1)^{-4}$$

$$y' = 3(-4)(2x^2 - 1)^{-5} \cdot 4x = -48x(2x^2 - 1)^{-5} = -\frac{48x}{(2x^2 - 1)^5} \quad (26)$$

Exercício V

$$y = \sqrt{(2x^2 - 1)^3} = (2x^2 - 1)^{\frac{3}{2}}$$

$$y' = \frac{3}{2}(2x^2 - 1)^{\frac{1}{2}} \cdot 4x = 6x(2x^2 - 1)^{\frac{1}{2}} = 6x\sqrt{2x^2 - 1} \quad (27)$$

Derivada de funções quociente, produto, polinomial – [Aula 15](#)

Exercício I

$$y = 7x^4 - 2x^3 + 8x + 5$$

$$y' = 28x^3 - 6x^2 + 8 = 2(14x^3 - 3x^2 + 4) \quad (28)$$

Exercício II

$$y = (2x^3 - 4x^2)(3x^5 + x^2) = 6x^8 + 2x^5 - 12x^7 - 4x^4 = 6x^8 - 12x^7 + 2x^5 - 4x^4$$

$$y' = 48x^7 - 84x^6 + 10x^4 - 16x^3 = 2x^3(24x^4 - 42x^3 + 5x - 8) \quad (29)$$

Exercício III

$$h(x) = \frac{2x^3 + 4}{x^2 - 4x + 1}$$

$$\begin{cases} f(x) = 2x^3 + 4 & \rightarrow f'(x) = 6x^2 \\ g(x) = x^2 - 4x + 1 & \rightarrow g'(x) = 2x - 4 \end{cases}$$

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2} = \frac{6x^2(x^2 - 4x + 1) - (2x^3 + 4)(2x - 4)}{(x^2 - 4x + 1)^2} =$$

$$\frac{6x^4 - 24x^3 + 6x^2 - (4x^4 - 8x^3 + 8x - 16)}{(x^2 - 4x + 1)^2} = \frac{6x^4 - 24x^3 + 6x^2 - 4x^4 + 8x^3 - 8x + 16}{(x^2 - 4x + 1)^2} =$$

$$\frac{2x^4 - 16x^3 + 6x^2 - 8x + 16}{(x^2 - 4x + 1)^2} = \frac{2(x^4 - 8x^3 + 3x^2 - 4x + 8)}{(x^2 - 4x + 1)^2} \quad (30)$$

Exercício IV

$$y = \frac{3}{x^5} = 3x^{-5} \rightarrow y' = -15x^{-6} = -\frac{15}{x^6} \quad (31)$$

Exercício V

$$v(r) = \frac{4}{3} \pi r^3 \rightarrow v'(r) = 4 \pi r^2 \quad (32)$$

Exercício VI

$$\begin{aligned} f(s) &= \sqrt{3}(s^3 - s^2) = \sqrt{3}s^3 - \sqrt{3}s^2 \\ f'(s) &= 3\sqrt{3}s^2 - 2\sqrt{3}s = s\sqrt{3}(3s - 2) \end{aligned} \quad (33)$$

Exercício VII

$$\begin{aligned} y &= (4x^2 + 3)^2 = 16x^4 + 12x^2 + 12x^2 + 9 = 16x^4 + 24x^2 + 9 \\ y' &= 64x^3 + 48x = 16x(4x^2 + 3) \end{aligned} \quad (34)$$

Derivadas de função quociente e produto – [Aula 16](#)

Exercício I

$$\begin{aligned} y &= \frac{x^4 - 2x^2 + 5x + 1}{x^4} = \frac{x^4}{x^4} - \frac{2x^2}{x^4} + \frac{5x}{x^4} + \frac{1}{x^4} = 1 - 2x^{-2} + 5x^{-3} + x^{-4} \\ y' &= 4x^{-3} - 15x^{-4} - 4x^{-5} = \frac{4}{x^3} - \frac{15}{x^4} - \frac{4}{x^5} = \frac{4x^2 - 15x - 4}{x^5} \end{aligned} \quad (35)$$

Exercício II

$$\begin{aligned} y &= \frac{x}{x-1} \\ y' &= \frac{1(x-1) - x \cdot 1}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2} \end{aligned} \quad (36)$$

Exercício III

$$\begin{aligned} y &= \left(\frac{2x+1}{x+5} \right) (3x-1) = \frac{6x^2 - 2x + 3x - 1}{x+5} = \frac{6x^2 + x - 1}{x+5} \\ y' &= \frac{(12x+1)(x+5) - (6x^2 + x - 1) \cdot 1}{(x+5)^2} = \frac{12x^2 + 60x + x + 5 - 6x^2 - x + 1}{(x+5)^2} = \\ &= \frac{6x^2 + 60x + 6}{(x+5)^2} = \frac{6(x^2 + 10x + 1)}{(x+5)^2} \end{aligned} \quad (37)$$

Exercício IV

$$y = \frac{1}{8} x^8 - 4x^4 \rightarrow y' = x^7 - 16x^3 = x^3(x^4 - 16) \quad (38)$$

Exercício V

$$y = x^2 + 3x + \frac{1}{x^2} = x^2 + 3x + x^{-2}$$

$$y' = 2x + 3 - 2x^{-3} = 2x + 3 + \frac{2}{x^3} = \frac{2x^4 + 3x^3 + 2}{x^3} \quad (39)$$

Exercício VI

$$y = \frac{3}{x^2} + \frac{5}{x^4} = 3x^{-2} + 5x^{-4} \rightarrow y' = -6x^{-3} - 20x^{-5} = -\frac{6}{x^3} - \frac{20}{x^5} = \frac{-6x^2 - 20}{x^5} \quad (40)$$

Derivada de funções envolvendo raiz, quociente, produto – [Aula 17](#)

Exercício I

$$g(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{\frac{1}{2}}} = 3x^{-\frac{1}{2}}$$

$$g'(x) = 3 \left(-\frac{1}{2} \right) x^{-\frac{3}{2}} = \frac{-3}{2x^{\frac{3}{2}}} = -\frac{3}{2\sqrt{x^3}} \quad (41)$$

Exercício II

$$g(x) = 5\sqrt[3]{x^2} = 5x^{\frac{2}{3}}$$

$$g'(x) = 5 \cdot \frac{2}{3} x^{-\frac{1}{3}} = \frac{10}{3x^{\frac{1}{3}}} = \frac{10}{3\sqrt[3]{x}} \quad (42)$$

Exercício III

$$h(r) = 6\sqrt[4]{r^3} + \frac{7}{r^3} = 6r^{\frac{3}{4}} + 7r^{-3}$$

$$h'(r) = 6 \cdot \frac{3}{4} r^{-\frac{1}{4}} + 7(-3)r^{-4} = \frac{9}{2} r^{-\frac{1}{4}} - 21r^{-4} = \frac{9}{2r^{\frac{1}{4}}} - \frac{21}{r^4} = \frac{9}{2\sqrt[4]{r}} - \frac{21}{r^4} =$$

$$\frac{9r^4 - 42\sqrt[4]{r}}{2r} = \frac{3}{2r} (3r^4 - 14\sqrt[4]{r}) \quad (43)$$

Exercício IV

$$\begin{aligned}
 g(t) &= \frac{5t^3}{6\sqrt{t}} = 5t^3 \cdot \frac{1}{6t^{\frac{1}{2}}} = 5t^3 \cdot \frac{t^{-\frac{1}{2}}}{6} \\
 g'(t) &= 5 \cdot 3t^{3-1} \cdot \frac{1}{6\sqrt{t}} + 5t^3 \left(\frac{1}{6} \left(-\frac{1}{2} \right) t^{-\frac{1}{2}-1} \right) = \frac{15t^2}{6\sqrt{t}} + 5t^3 \left(-\frac{t^{-\frac{3}{2}}}{12} \right) = \frac{5t^2}{2\sqrt{t}} - \frac{5}{12} t^{3-\frac{3}{2}} = \\
 \frac{5}{2} t^{2-\frac{1}{2}} - \frac{5}{12} t^{\frac{3}{2}} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{30\sqrt{t^3} - 5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} \\
 g'(t) &= \frac{5 \cdot 3t^{3-1} \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 6 \left(\frac{1}{2} \right) t^{\frac{1}{2}-1}}{\left(6t^{\frac{1}{2}} \right)^2} = \frac{15t^2 \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 3t^{-\frac{1}{2}}}{36t} = \\
 \frac{90t^{2+\frac{1}{2}} - 15t^{3-\frac{1}{2}}}{36t} &= \frac{15 \left(6t^{\frac{5}{2}} - t^{\frac{5}{2}} \right)}{36t} = \frac{5\sqrt{t^5}(6-1)}{12t} = \frac{30\sqrt{t^5}}{12t} - \frac{5\sqrt{t^5}}{12t} = \\
 \frac{5}{2} t^{\frac{5}{2}-1} - \frac{5}{12} t^{\frac{5}{2}-1} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} = \frac{5}{6} t^{3-\frac{1}{2}} = \frac{5}{6} t^{\frac{5}{2}} \\
 g'(t) &= \frac{5}{6} \cdot \frac{5}{2} t^{\frac{5}{2}-1} = \frac{25t^{\frac{3}{2}}}{12} = \frac{25\sqrt{t^3}}{12}
 \end{aligned}
 \tag{44}$$

Exercício V

$$\begin{aligned}
 h(x) &= 7x\sqrt[3]{x^2} = 7x \cdot x^{\frac{2}{3}} = 7x^{1+\frac{2}{3}} = 7x^{\frac{5}{3}} \\
 h'(x) &= 7 \cdot \frac{5}{3} x^{\frac{5}{3}-1} = \frac{35x^{\frac{2}{3}}}{3} = \frac{35\sqrt[3]{x^2}}{3}
 \end{aligned}
 \tag{45}$$

Exercício VI

$$\begin{aligned}
 g(t) &= \frac{8t^3\sqrt{t}}{t} = \frac{8t^3t^{\frac{1}{2}}}{t} = 8t^{3+\frac{1}{2}-1} = 8t^{\frac{5}{2}} \\
 g'(t) &= 8 \cdot \frac{5}{2} t^{\frac{5}{2}-1} = 20t^{\frac{3}{2}} = 20\sqrt{t^3}
 \end{aligned}
 \tag{46}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 18](#)

Exercício I

$$f(x) = \frac{3x^4 - 8x^2 + 4}{x^2} = \frac{3x^4}{x^2} - \frac{8x^2}{x^2} + \frac{4}{x^2} = 3x^2 - 8 + 4x^{-2}$$

$$f'(x) = 6x - 8x^{-3} = 6x - \frac{8}{x^3} = \frac{6x^4 - 8}{x^3} \quad (47)$$

Exercício II

$$f(x) = \frac{\pi x^3}{\sqrt{x^3}} = \frac{\pi x^3}{x^{\frac{3}{2}}} = \pi x^{3 - \frac{3}{2}} = \pi x^{\frac{3}{2}}$$

$$f'(x) = \pi \cdot \frac{3}{2} x^{\frac{3}{2} - 1} = \frac{3\pi x^{\frac{1}{2}}}{2} = \frac{3\pi \sqrt{x}}{2} \quad (48)$$

Exercício III

$$y = \frac{x^2}{e^x}$$

$$y' = \frac{2x \cdot e^x - x^2 \cdot e^x}{(e^x)^2} = \frac{e^x \cdot x(2 - x)}{e^{2x}} = \frac{x(2 - x)}{e^x} = \frac{x}{e^x} (2 - x) \quad (49)$$

Exercício IV

$$y = x^5 \cdot e^x$$

$$y' = 5x^4 \cdot e^x + x^5 \cdot e^x = e^x \cdot x^4 (5 + x) \quad (50)$$

Exercício V

$$y = \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x)$$

$$y' = \left(\frac{1}{2} x^{\frac{1}{2} - 1} \right) \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{-\frac{1}{2}} \cdot \ln(x)}{2} + x^{\frac{1}{2}} \cdot x^{-1} = \frac{\ln(x)}{2x^{\frac{1}{2}}} + x^{\frac{1}{2} - 1} = \frac{\ln(x)}{2\sqrt{x}} + x^{-\frac{1}{2}} = \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{x^{\frac{1}{2}}} =$$

$$\frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} = \left(\frac{1}{\sqrt{x}} \left(\frac{\ln(x)}{2} + 1 \right) \right) \left(\frac{\sqrt{x}}{\sqrt{x}} \right) = \frac{\sqrt{x}}{x} \left(\frac{\ln(x)}{2} + 1 \right) \quad (51)$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 19](#)

Exercício I

$$f(t) = e^t \cdot \ln(t)$$

$$f'(t) = e^t \cdot \ln(t) + e^t \cdot \frac{1}{t} = e^t \left(\ln(t) + \frac{1}{t} \right) = \frac{e^t(t \cdot \ln(t) + 1)}{t} = \frac{e^t}{t} (t \cdot \ln(t) + 1) \quad (52)$$

Exercício II

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} =$$

$$\frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} = \frac{e^r(2r-1)}{2r^{\frac{1}{2}+1}} = \frac{e^r(2r-1)}{2r^{\frac{3}{2}}} = \left(\frac{e^r(2r-1)}{2\sqrt{r^3}} \right) \left(\frac{\sqrt{r^3}}{\sqrt{r^3}} \right) = \frac{e^r \sqrt{r^3}(2r-1)}{2r^3} = \frac{e^r \sqrt{r^3}}{2r^3} (2r-1) \quad (53)$$

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{e^r \left(\sqrt{r} - \frac{1}{2\sqrt{r}} \right)}{r} =$$

$$\frac{e^r}{r} \left(\sqrt{r} - \frac{1}{2\sqrt{r}} \right) = \frac{e^r}{r} \left(\frac{2r-1}{2\sqrt{r}} \right) \left(\frac{\sqrt{r}}{\sqrt{r}} \right) = \frac{e^r \sqrt{r}}{2r^2} (2r-1)$$

Exercício III

$$v(t) = \frac{t^3 \cdot e^t}{t} = t^{3-1} \cdot e^t = t^2 \cdot e^t$$

$$v'(t) = 2t \cdot e^t + t^2 \cdot e^t = e^t \cdot t(2+t) \quad (54)$$

Exercício IV

$$y = x^5 \rightarrow y' = 5x^4$$

$$y = (2x^2 - 4)^5 \rightarrow y' = 5(2x^2 - 4)^4 \cdot 4x = 20x(2x^2 - 4)^4 \quad (55)$$

Exercício V

$$y = \sqrt{2x^3 - 1} = (2x^3 - 1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(2x^3 - 1)^{-\frac{1}{2}} \cdot 6x^2 = 3x^2(2x^3 - 1)^{-\frac{1}{2}} = \frac{3x^2}{(2x^3 - 1)^{\frac{1}{2}}} = \left(\frac{3x^2}{\sqrt{2x^3 - 1}} \right) \left(\frac{\sqrt{2x^3 - 1}}{\sqrt{2x^3 - 1}} \right) =$$

$$\frac{3x^2 \sqrt{2x^3 - 1}}{2x^3 - 1} \quad (56)$$

Exercício VI

$$y = \frac{5}{(3x^2 + 9)^4} = 5(3x^2 + 9)^{-4}$$

$$y' = -20(3x^2 + 9)^{-5} \cdot 6x = -120x(3x^2 + 9)^{-5} = -\frac{120x}{(3x^2 + 9)^5} \quad (57)$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 20](#)

Exercício I

$$y = \sqrt[3]{6x^2 + 7x + 2} = (6x^2 + 7x + 2)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(6x^2 + 7x + 2)^{-\frac{2}{3}} \cdot (12x + 7) = \frac{12x + 7}{3(6x^2 + 7x + 2)^{\frac{2}{3}}} = \left(\frac{12x + 7}{3\sqrt[3]{(6x^2 + 7x + 2)^2}} \right) \left(\frac{\sqrt[3]{6x^2 + 7x + 2}}{\sqrt[3]{6x^2 + 7x + 2}} \right)$$

$$\frac{(12x + 7)\sqrt[3]{6x^2 + 7x + 2}}{3(6x^2 + 7x + 2)} \quad (58)$$

Exercício II

$$y = e^{5x^2 + 4}$$

$$y' = e^{5x^2 + 4} \cdot 10x \quad (59)$$

Exercício III

$$y = e^{\frac{1}{x^2}} = e^{x^{-2}}$$

$$y' = e^{\frac{1}{x^2}} (-2x^{-3}) = e^{\frac{1}{x^2}} \left(-\frac{2}{x^3} \right) = -\frac{2e^{\frac{1}{x^2}}}{x^3} \quad (60)$$

Exercício IV

$$y = 3^{x^2}$$

$$y' = 3^{x^2} \cdot \ln(3) \cdot 2x \quad (61)$$

Exercício V

$$y = 5^{2x^2+3x-1}$$

$$y' = 5^{2x^2+3x-1} \cdot \ln(5) \cdot (4x+3) \quad (62)$$

Exercício VI

$$y = \left(\frac{1}{2}\right)^{\sqrt{x}} = \left(\frac{1}{2}\right)^{x^{\frac{1}{2}}}$$

$$y' = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{\frac{1}{2}-1} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{-\frac{1}{2}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2x^{\frac{1}{2}}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2\sqrt{x}} \quad (63)$$

Exercício VII

$$f = e^{\frac{x+1}{x-1}} \rightarrow g = \frac{x+1}{x-1} \rightarrow f = e^g$$

$$g' = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2} \quad (64)$$

$$f' = e^g \cdot g' \rightarrow f' = e^{\frac{x+1}{x-1}} \cdot \frac{-2}{(x-1)^2} = \frac{-2e^{\frac{x+1}{x-1}}}{(x-1)^2}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 21](#)

Exercício I

$$y = \ln(5x^2 - 4x)$$

$$y' = \frac{10x-4}{5x^2-4x} = \frac{2(5x-2)}{x(5x-4)} \quad (65)$$

Exercício II

$$y = \log_2(3x^2 - 7)$$

$$y' = \frac{6x}{3x^2-7} \cdot \log_2(e) \quad (66)$$

Exercício III

$$\begin{aligned}
 f &= \log_{10} \left(\frac{x+1}{x^2+1} \right) \rightarrow g = \frac{x+1}{x^2+1} \rightarrow f = \log_{10}(g) \\
 g' &= \frac{1(x^2+1) - (x+1)2x}{(x^2+1)^2} = \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2} = \frac{-(x^2+2x-1)}{(x^2+1)^2} \\
 f' &= \frac{g'}{g} \cdot \log_{10}(e) \rightarrow f' = \frac{\frac{-(x^2+2x-1)}{(x^2+1)^2}}{\frac{x+1}{x^2+1}} \cdot \log_{10}(e) = \frac{-(x^2+2x-1)(x^2+1)}{(x^2+1)^2(x+1)} \cdot \log_{10}(e) = \\
 &= \frac{-(x^2+2x-1)}{(x^2+1)(x+1)} \cdot \log_{10}(e) = \frac{-x^2-2x+1}{x^3+x^2+x+1} \cdot \log_{10}(e)
 \end{aligned} \tag{67}$$

$$\begin{aligned}
 f &= \log_{10} \left(\frac{x+1}{x^2+1} \right) = \log_{10}(x+1) - \log_{10}(x^2+1) \\
 f' &= \frac{1}{x+1} \cdot \log_{10}(e) - \frac{2x}{x^2+1} \cdot \log_{10}(e) = \log_{10}(e) \left(\frac{1}{x+1} - \frac{2x}{x^2+1} \right) = \\
 &= \log_{10}(e) \left(\frac{1(x^2+1) - 2x(x+1)}{x^3+x+x^2+1} \right) = \log_{10}(e) \left(\frac{x^2+1-2x^2-2x}{x^3+x^2+x+1} \right) = \\
 &= \log_{10}(e) \left(\frac{-x^2-2x+1}{x^3+x^2+x+1} \right)
 \end{aligned}$$

Exercício IV

$$\begin{aligned}
 y &= \ln \left(\frac{e^x}{x+1} \right) = \ln(e^x) - \ln(x+1) = x - \ln(x+1) \\
 y' &= 1 - \frac{1}{x+1} = \frac{x+1-1}{x+1} = \frac{x}{x+1}
 \end{aligned} \tag{68}$$

Exercício V

$$\begin{aligned}
 y &= \ln(2x-1)^3 \rightarrow f = (2x-1)^3 \rightarrow y = \ln(f) \\
 f' &= 3(2x-1)^2 \cdot 2 = 6(2x-1)^2 \\
 y' &= \frac{f'}{f} = \frac{6(2x-1)^2}{(2x-1)^3} = \frac{6}{2x-1} \\
 y &= \ln(2x-1)^3 = 3 \cdot \ln(2x-1) \\
 y' &= 0 \cdot \ln(2x-1) + 3 \cdot \frac{2}{2x-1} = \frac{6}{2x-1}
 \end{aligned} \tag{69}$$

Exercício VI

$$\begin{aligned}
 y &= \ln[(4x^2+3)(2x-1)] = \ln(4x^2+3) + \ln(2x-1) \\
 y' &= \frac{8x}{4x^2+3} + \frac{2}{2x-1} = \frac{8x(2x-1) + 2(4x^2+3)}{8x^3-4x^2+6x-3} = \frac{16x^2-8x+8x^2+6}{8x^3-4x^2+6x-3} = \\
 &= \frac{24x^2-8x+6}{8x^3-4x^2+6x-3} = \frac{2(12x^2-4x+3)}{8x^3-4x^2+6x-3}
 \end{aligned} \tag{70}$$

Exercício VII

$$\begin{aligned}
 f(x) &= \left(\frac{2x+1}{3x-1}\right)^4 \rightarrow g(x) = \frac{2x+1}{3x-1} \rightarrow f(x) = g(x)^4 \\
 g'(x) &= \frac{2(3x-1) - (2x+1)3}{(3x-1)^2} = \frac{6x-2-6x-3}{(3x-1)^2} = \frac{-5}{(3x-1)^2} \\
 f'(x) &= 4g(x)^{4-1} \cdot g'(x) = 4\left(\frac{2x+1}{3x-1}\right)^3 \cdot \frac{-5}{(3x-1)^2} = \frac{-20}{(3x-1)^2} \cdot \left(\frac{2x+1}{3x-1}\right)^3
 \end{aligned} \tag{71}$$

Derivação (ou diferenciação) Logarítmica – [Aula 22](#)

Exercício I

$$\begin{aligned}
 y &= x^x \rightarrow \ln(y) = \ln(x^x) \rightarrow \ln(y) = x \cdot \ln(x) \\
 \frac{y'}{y} &= 1 \cdot \ln(x) + x \cdot \frac{1}{x} \rightarrow \frac{y'}{y} = \ln(x) + 1 \rightarrow y' = y(\ln(x) + 1) = x^x(\ln(x) + 1)
 \end{aligned} \tag{72}$$

Exercício II

$$\begin{aligned}
 y &= c^f \rightarrow \ln(y) = \ln(c^f) \rightarrow \ln(y) = f \cdot \ln(c) \\
 \frac{y'}{y} &= f' \cdot \ln(c) + f \cdot \frac{0}{c} \rightarrow y' = y(f' \cdot \ln(c)) \rightarrow y' = c^f \cdot f' \cdot \ln(c) \\
 y &= e^f \rightarrow \ln(y) = \ln(e^f) \rightarrow \ln(y) = f \cdot \ln(e) \rightarrow \ln(y) = f \\
 \frac{y'}{y} &= f' \rightarrow y' = y \cdot f' \rightarrow y' = e^f \cdot f'
 \end{aligned} \tag{73}$$

Derivação (ou diferenciação) Logarítmica – [Aula 23](#)

Exercício I

$$\begin{aligned}
 y &= x^{\sqrt{x}} \rightarrow \ln(y) = \ln(x^{\sqrt{x}}) = \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x) \\
 \frac{y'}{y} &= \frac{1}{2} x^{\frac{1}{2}-1} \cdot \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{\frac{-1}{2}}}{2} \cdot \ln(x) + x^{\frac{1}{2}-1} = \frac{x^{\frac{-1}{2}} \cdot \ln(x)}{2} + x^{\frac{-1}{2}} = x^{\frac{-1}{2}} \left(\frac{\ln(x)}{2} + 1 \right) \rightarrow \\
 y' &= y \cdot x^{\frac{-1}{2}} \left(\frac{\ln(x)}{2} + 1 \right) = x^{\sqrt{x}} \cdot x^{\frac{-1}{2}} \left(\frac{\ln(x)}{2} + 1 \right) = x^{\sqrt{x}-\frac{1}{2}} \left(\frac{\ln(x)}{2} + 1 \right) = x^{\frac{x\sqrt{x}-1}{x}} \left(\frac{\ln(x)+2}{2} \right)
 \end{aligned} \tag{74}$$

Derivada de função composta – [Aula 24](#)

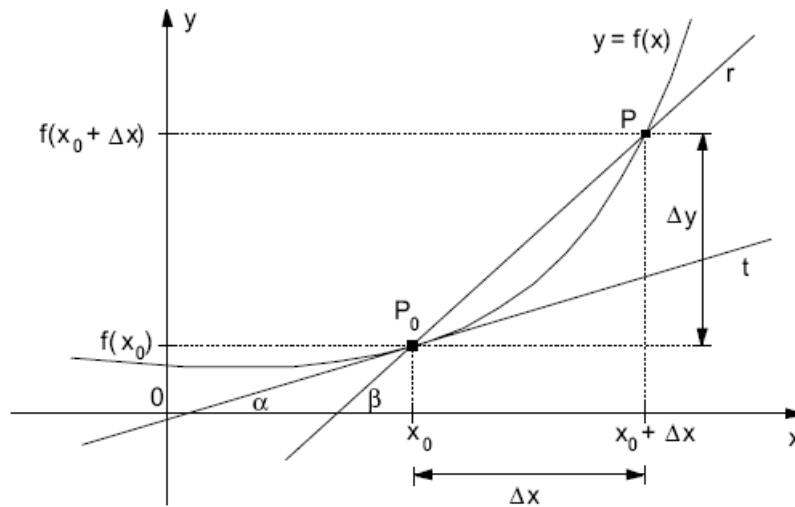
Exercício I

$$\begin{aligned}
 y &= \left(\frac{2x+1}{3x-1} \right)^4 \rightarrow f = \frac{2x+1}{3x-1} \rightarrow y = f^4 \\
 f' &= \frac{2(3x-1) - (2x+1)3}{(3x-1)^2} = \frac{6x-2-6x-3}{(3x-1)^2} = \frac{-5}{(3x-1)^2} \\
 y' &= 4f^{4-1} \cdot f' = 4 \left(\frac{2x+1}{3x-1} \right)^3 \cdot \frac{-5}{(3x-1)^2} = \frac{-20}{(3x-1)^2} \cdot \left(\frac{2x+1}{3x-1} \right)^3 = \frac{-20}{(3x-1)^2} \cdot \frac{(2x+1)^3}{(3x-1)^3} = \\
 &\quad \frac{-20(2x+1)^3}{(3x-1)^5}
 \end{aligned}$$

$$\begin{aligned}
 y &= \left(\frac{2x+1}{3x-1} \right)^4 \rightarrow \ln(y) = \ln \left(\frac{2x+1}{3x-1} \right)^4 = 4 \cdot \ln \left(\frac{2x+1}{3x-1} \right) = 4(\ln(2x+1) - \ln(3x-1)) \quad (75) \\
 \frac{y'}{y} &= 4 \cdot \left(\frac{2}{2x+1} - \frac{3}{3x-1} \right) \rightarrow y' = 4y \cdot \left(\frac{2}{2x+1} - \frac{3}{3x-1} \right) = \\
 &= 4 \cdot \left(\frac{2x+1}{3x-1} \right)^4 \cdot \left(\frac{2}{2x+1} - \frac{3}{3x-1} \right) = 4 \cdot \left(\frac{2x+1}{3x-1} \right)^4 \cdot \frac{2(3x-1) - 3(2x+1)}{(2x+1)(3x-1)} = \\
 &= \frac{4(2x+1)^4 \cdot 2(3x-1) - 3(2x+1)}{(3x-1)^4 \cdot (2x+1)(3x-1)} = \frac{4(2x+1)^3(2(3x-1) - 3(2x+1))}{(3x-1)^5} = \\
 &= \frac{4(2x+1)^3(6x-2-6x-3)}{(3x-1)^5} = \frac{4(2x+1)^3(-5)}{(3x-1)^5} = \frac{-20(2x+1)^3}{(3x-1)^5}
 \end{aligned}$$

O que é uma Derivada – [Aula 25](#)

Derivada é a principal ferramenta matemática utilizada para calcular e estudar taxas de variação.



$$\operatorname{tg} \beta = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

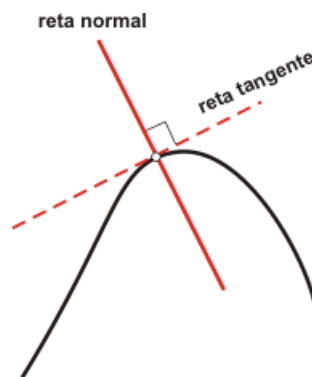
$$\operatorname{tg} \alpha = \lim_{\Delta x \rightarrow 0} \left[\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \right]$$

$$f(x_0) = x_0^2$$

$$\operatorname{tg} \alpha = f'(x_0) = 2x_0$$

$$\operatorname{tg} \alpha = \lim_{\Delta x \rightarrow 0} \left[\frac{(x_0 + \Delta x)^2 - (x_0)^2}{\Delta x} \right] = \frac{x_0^2 + 2x_0\Delta x + \Delta x^2 - x_0^2}{\Delta x} = \frac{\Delta x(2x_0 + \Delta x)}{\Delta x} = 2x_0 + \Delta x = 2x_0 + 0 = 2x_0$$

Derivadas: Reta Tangente e Reta Normal – [Aula 26](#)



Reta tangente: $y - y_0 = f'(x_0)(x - x_0) \rightarrow y = f'(x_0)(x - x_0) + y_0 = f'(x_0)(x - x_0) + f(x_0)$

Reta normal: $y - y_0 = \frac{-1}{f'(x_0)}(x - x_0) = \frac{-(x - x_0)}{f'(x_0)} \rightarrow y = \frac{-(x - x_0)}{f'(x_0)} + y_0 = \frac{-(x - x_0)}{f'(x_0)} + f(x_0)$

Exercício I

$$f(x_0) = \frac{1}{x_0} = x_0^{-1}$$

$$f'(x_0) = -x_0^{-2} = \frac{-1}{x_0^2}$$

$$y_t = \frac{-1}{x_0^2}(x - x_0) + \frac{1}{x_0} = \frac{-x + x_0}{x_0^2} + \frac{1}{x_0} = \frac{-x + x_0 + x_0}{x_0^2} = \frac{-x + 2x_0}{x_0^2}$$

$$y_n = \frac{-(x - x_0)}{\frac{-1}{x_0^2}} + \frac{1}{x_0} = x_0^2(x - x_0) + \frac{1}{x_0} = x_0^2 x - x_0^3 + \frac{1}{x_0} = \frac{x_0^3 x - x_0^4 + 1}{x_0}$$

$$x_0 = 1$$

$$y_0 = f(x_0) = \frac{1}{1} = 1$$

$$\operatorname{tg} \alpha = f'(x_0) = \frac{-1}{1^2} = -1$$

$$y_t = \frac{-x + 2 \cdot 1}{1^2} = -x + 2$$

$$y_n = \frac{1^3 x - 1^4 + 1}{1} = x$$

(76)

$$\lim_{x_0 \rightarrow 0} f(x_0) = \lim_{x_0 \rightarrow 0} \left[\frac{1}{x_0} \right] = \nexists$$

$$\lim_{x_0 \rightarrow 0^+} \left[\frac{1}{x_0} \right] = \frac{1}{0^+} = +\infty$$

$$\lim_{x_0 \rightarrow 0^+} \left[\frac{1}{x_0} \right] = \frac{1}{0^-} = -\infty$$

$$\lim_{x_0 \rightarrow +\infty} f(x_0) = \lim_{x_0 \rightarrow +\infty} \left[\frac{1}{x_0} \right] = \frac{1}{+\infty} = 0$$

$$\lim_{x_0 \rightarrow -\infty} f(x_0) = \lim_{x_0 \rightarrow -\infty} \left[\frac{1}{x_0} \right] = \frac{1}{-\infty} = 0$$

Exercício II

$$\begin{aligned}
 x_0 &= 1 \\
 f(x) &= \sqrt{x} = x^{\frac{1}{2}} \\
 f'(x) &= \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}} \\
 y_0 &= f(x_0) = \sqrt{1} = 1 \\
 \tan \alpha &= f'(x_0) = \frac{1}{2\sqrt{1}} = \frac{1}{2} \\
 y_t - y_0 &= f'(x_0)(x - x_0) \rightarrow y_t - 1 = \frac{1}{2}(x - 1) \rightarrow y_t = \frac{x-1}{2} + 1 = \frac{x-1+2}{2} = \frac{x+1}{2} \\
 y_n - y_0 &= \frac{-1}{f'(x_0)}(x - x_0) \rightarrow y_n - 1 = \frac{-1}{\frac{1}{2}}(x - 1) \rightarrow y_n = -2(x - 1) + 1 = -2x + 2 + 1 = -2x + 3
 \end{aligned} \tag{77}$$

Derivada pela Definição – [Aula 27](#)

Exercício I

$$\begin{aligned}
 f(x) &= 2x^2 - 3x + 4 \\
 f'(x) &= 4x - 3 \\
 \lim_{h \rightarrow 0} \left[\frac{(2(x+h)^2 - 3(x+h) + 4) - (2x^2 - 3x + 4)}{h} \right] &= \\
 \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 4 - 2x^2 + 3x - 4}{h} &= \\
 \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 4 - 2x^2 + 3x - 4}{h} &= \frac{2h^2 + 4xh - 3h}{h} = \frac{h(2h + 4x - 3)}{h} = \\
 2h + 4x - 3 &= 2 \cdot 0 + 4x - 3 = 4x - 3
 \end{aligned} \tag{78}$$

Exercício II

$$\begin{aligned}
 f(x) &= \frac{2}{x} = 2x^{-1} \\
 f'(x) &= -2x^{-2} = \frac{-2}{x^2} \\
 \lim_{h \rightarrow 0} \left[\frac{\left(\frac{2}{x+h} \right) - \left(\frac{2}{x} \right)}{h} \right] &= \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h} = \frac{\frac{2x - 2x - 2h}{x(x+h)}}{h} = \frac{-2h}{xh(x+h)} = \frac{-2}{x(x+h)} = \\
 \frac{-2}{x(x+0)} &= \frac{-2}{x^2}
 \end{aligned} \tag{79}$$

Derivada pela Regra da Cadeia – [Aula 1](#)

$$y=f(g(x)) \rightarrow y'=f'(g(x)) \cdot g'(x)$$

$$y=(2x^2-1)^3=a^3 \rightarrow a=2x^2-1$$

$$y=e^{5x^2-1}=e^a \rightarrow a=5x^2-1$$

$$y=\ln[\cos(5x^2-1)]=\ln[a] \rightarrow a=\cos(5x^2-1)=\cos(b) \rightarrow b=5x^2-1$$

Exercício I

$$\begin{aligned} y &= (2x^2-1)^4 = a^4 \rightarrow a = 2x^2-1 \\ a' &= 4x \\ y' &= 4a^{4-1} \cdot a' = 4(2x^2-1)^3 \cdot 4x = 16x(2x^2-1)^3 \end{aligned} \quad (80)$$

Exercício II

$$\begin{aligned} y &= e^{2x^2-1} = e^a \rightarrow a = 2x^2-1 \\ a' &= 4x \\ y' &= e^a \cdot a' = e^{2x^2-1} \cdot 4x \end{aligned} \quad (81)$$

Exercício III

$$\begin{aligned} y &= \cos(2x^2-9) = \cos(a) \rightarrow a = 2x^2-9 \\ a' &= 4x \\ y' &= -\sin(a) \cdot a' = -\sin(2x^2-9) \cdot 4x = -4x \cdot \sin(2x^2-9) \end{aligned} \quad (82)$$

Exercício IV

$$\begin{aligned} y &= \ln[\sin(x^3-1)] = \ln[a] \rightarrow a = \sin(x^3-1) = \sin(b) \rightarrow b = x^3-1 \\ b' &= 3x^2 \\ a' &= \cos(b) \cdot b' = \cos(x^3-1) \cdot 3x^2 \\ y' &= \frac{1}{a} \cdot a' = \frac{1}{\sin(x^3-1)} \cdot \cos(x^3-1) \cdot 3x^2 = 3x^2 \cdot \frac{\cos(x^3-1)}{\sin(x^3-1)} = 3x^2 \cdot \cotg(x^3-1) \end{aligned} \quad (83)$$

Exercício V

$$\begin{aligned} y &= \ln[\cos(\sin(2x^2-1))] = \ln[a] \\ a &= \cos(\sin(2x^2-1)) = \cos(b) \\ b &= \sin(2x^2-1) = \sin(c) \\ c &= 2x^2-1 \\ c' &= 4x \\ b' &= \cos(c) \cdot c' = \cos(2x^2-1) \cdot 4x \\ a' &= -\sin(b) \cdot b' = -\sin(\sin(2x^2-1)) \cdot \cos(2x^2-1) \cdot 4x \\ y' &= \ln[a] \cdot a' = \frac{1}{\cos(\sin(2x^2-1))} (-\sin(\sin(2x^2-1))) \cdot \cos(2x^2-1) \cdot 4x = \\ &= -4x \cdot \cos(2x^2-1) \cdot \frac{\sin(\sin(2x^2-1))}{\cos(\sin(2x^2-1))} = -4x \cdot \cos(2x^2-1) \cdot \tg(\sin(2x^2-1)) \end{aligned} \quad (84)$$

Derivada pela Regra da Cadeia – [Aula 2](#)

Exercício I

$$y = x^4 \rightarrow \frac{\partial y}{\partial x} = 4x^3$$

$$y = (x^2 + 1)^4$$
$$\frac{\partial y}{\partial x} = 4(x^2 + 1)^3 \cdot 2x = 8x(x^2 + 1)^3 \quad (85)$$

Exercício II

$$y = \cos(x^2 + 1)^4 \rightarrow \frac{\partial y}{\partial x} = -\operatorname{sen}(x^2 + 1)^4 \cdot 4(x^2 + 1)^3 \cdot 2x = -8x(x^2 + 1)^3 \operatorname{sen}(x^2 + 1)^4 \quad (86)$$

Exercício III

$$y = \cos^2 x \rightarrow \frac{\partial y}{\partial x} = 2 \cdot \cos x (-\operatorname{sen} x) = -2 \cdot \operatorname{sen} x \cdot \cos x \quad (87)$$

Exercício IV

$$y = \cos^2(2x^3 - 1) \rightarrow \frac{\partial y}{\partial x} = 2 \cos(2x^3 - 1) (-\operatorname{sen}(2x^3 - 1)) 6x^2 =$$
$$-12x^2 \operatorname{sen}(2x^3 - 1) \cos(2x^3 - 1) \quad (88)$$

Exercício V

$$y = (\ln x + 1)^3 \rightarrow \frac{\partial y}{\partial x} = 3(\ln x + 1)^2 \frac{1}{x} = \frac{3}{x} (\ln x + 1)^2 \quad (89)$$

Exercício VI

$$y = \cos(\ln x + 1)^3 \rightarrow \frac{\partial y}{\partial x} = -\operatorname{sen}(\ln x + 1)^3 3(\ln x + 1)^2 \frac{1}{x} = -\frac{3}{x} (\ln x + 1)^2 \operatorname{sen}(\ln x + 1)^3 \quad (90)$$

Exercício VII

$$y = \ln^3(2x^2 - 1) \rightarrow \frac{\partial y}{\partial x} = 3 \ln^2(2x^2 - 1) \frac{1}{2x^2 - 1} 4x = \frac{12x}{2x^2 - 1} \ln^2(2x^2 - 1) \quad (91)$$

Exercício VIII

$$y = e^{2x^2 - 1} \rightarrow \frac{\partial y}{\partial x} = e^{2x^2 - 1} 4x \quad (92)$$

Exercício IX

$$\begin{aligned}
 y &= \text{tg}^2(x^2-1) = [\text{tg}(x^2-1)]^2 = \left[\frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \right]^2 \rightarrow \\
 \frac{\partial y}{\partial x} &= 2 \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \frac{\cos(x^2-1) \cos(x^2-1) - \text{sen}(x^2-1)(-\text{sen}(x^2-1))}{\cos^2(x^2-1)} 2x = \\
 4x \frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \frac{\cos^2(x^2-1) + \text{sen}^2(x^2-1)}{\cos^2(x^2-1)} &= 4x \frac{\text{sen}(x^2-1) \cos^2(x^2-1) + \text{sen}^3(x^2-1)}{\cos^3(x^2-1)} = \\
 4x \left[\frac{\text{sen}(x^2-1) \cos^2(x^2-1)}{\cos^3(x^2-1)} + \frac{\text{sen}^3(x^2-1)}{\cos^3(x^2-1)} \right] &= 4x \left[\frac{\text{sen}(x^2-1)}{\cos(x^2-1)} + \left(\frac{\text{sen}(x^2-1)}{\cos(x^2-1)} \right)^3 \right] = \\
 4x [\text{tg}(x^2-1) + \text{tg}^3(x^2-1)] &= 4x \cdot \text{tg}(x^2-1) [1 + \text{tg}^2(x^2-1)] \\
 y = \text{tg}^2(x^2-1) \rightarrow \frac{\partial y}{\partial x} &= 2 \text{tg}(x^2-1) \sec^2(x^2-1) 2x = 4x \cdot \text{tg}(x^2-1) \sec^2(x^2-1)
 \end{aligned} \tag{93}$$

Exercício X

$$\begin{aligned}
 y &= \left(\frac{2x^2-1}{1-x^2} \right)^3 \rightarrow \frac{\partial y}{\partial x} = 3 \left(\frac{2x^2-1}{1-x^2} \right)^2 \frac{4x(1-x^2) - (2x^2-1)(-2x)}{(1-x^2)^2} = \\
 3 \left(\frac{2x^2-1}{1-x^2} \right)^2 \frac{4x-4x^3+4x^3-2x}{(1-x^2)^2} &= 3 \left(\frac{2x^2-1}{1-x^2} \right)^2 \frac{2x}{(1-x^2)^2} = \frac{3(2x^2-1)^2}{(1-x^2)^2} \frac{2x}{(1-x^2)^2} = \\
 \frac{6x(2x^2-1)^2}{(1-x^2)^4} &= 6x \left(\frac{2x^2-1}{(1-x^2)^2} \right)^2
 \end{aligned} \tag{94}$$

Exercício XI

$$\begin{aligned}
 y &= \ln[\cos(x^3-1)^4] \rightarrow \frac{\partial y}{\partial x} = \frac{1}{\cos(x^3-1)^4} (-\text{sen}(x^3-1)^4) 4(x^3-1)^3 3x^2 = \\
 -12x^2(x^3-1)^3 \frac{\text{sen}(x^3-1)^4}{\cos(x^3-1)^4} &= -12x^2(x^3-1)^3 \text{tg}(x^3-1)^4
 \end{aligned} \tag{95}$$

Derivada pela Regra da Cadeia – [Aula 3](#)

Exercício I

$$\begin{aligned}
 y &= e^{x\sqrt[3]{2x-1}} = e^a \\
 a &= x\sqrt[3]{2x-1} = xb \\
 b &= \sqrt[3]{2x-1} = (2x-1)^{\frac{1}{3}} = c^{\frac{1}{3}} \\
 c &= 2x-1 \\
 c' &= 2 \\
 b' &= \frac{1}{3}c^{\frac{1}{3}-1} \cdot c' = \frac{1}{3}(2x-1)^{-\frac{2}{3}} \cdot 2 = \frac{2}{3(2x-1)^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{(2x-1)^2}} \\
 a' &= 1 \cdot b + x \cdot b' = 1(\sqrt[3]{2x-1}) + x \left(\frac{2}{3\sqrt[3]{(2x-1)^2}} \right) = \sqrt[3]{2x-1} + \frac{2x}{3\sqrt[3]{(2x-1)^2}} = \\
 &= \frac{3\sqrt[3]{(2x-1)^3} + 2x}{3\sqrt[3]{(2x-1)^2}} = \frac{3(2x-1) + 2x}{3\sqrt[3]{(2x-1)^2}} = \frac{6x-3+2x}{3\sqrt[3]{(2x-1)^2}} = \frac{8x-3}{3\sqrt[3]{(2x-1)^2}} \\
 y' &= e^a \cdot a' = e^{x\sqrt[3]{2x-1}} \left(\frac{8x-3}{3\sqrt[3]{(2x-1)^2}} \right) = \frac{(8x-3)e^{x\sqrt[3]{2x-1}}}{3\sqrt[3]{(2x-1)^2}}
 \end{aligned} \tag{96}$$

Derivada pela Regra da Cadeia – [Aula 4](#)

Exercício I

$$\begin{aligned}
 f(x) &= \sin^3(2^{5x^4}) \rightarrow \frac{\partial f(x)}{\partial x} = 3\sin^2(2^{5x^4})\cos(2^{5x^4})2^{5x^4}\ln(2)20x^3 = \\
 &= \ln(2)60x^32^{5x^4}\sin^2(2^{5x^4})\cos(2^{5x^4})
 \end{aligned} \tag{97}$$

Derivada pela Regra da Cadeia – [Aula 5](#)

Exercício I

$$\begin{aligned}
 y &= 2^{-3x^2} \ln x = a \cdot b \\
 a &= 2^{-3x^2} = 2^c \\
 b &= \ln x \\
 c &= -3x^2 \\
 c' &= -6x \\
 b' &= \frac{1}{x} \\
 a' &= 2^c c' \ln(2) = 2^{-3x^2} (-6x) \ln(2) = -\ln(2) 6x \cdot 2^{-3x^2} \\
 y' &= a' b + a b' = (-\ln(2) 6x \cdot 2^{-3x^2}) \ln x + 2^{-3x^2} \frac{1}{x} = -\ln(2) \ln(x) 6x \cdot 2^{-3x^2} + \frac{2^{-3x^2}}{x} = \\
 &= \frac{-\ln(2) \ln(x) 6x^2 2^{-3x^2} + 2^{-3x^2}}{x} = \frac{-2^{-3x^2} (\ln(2) \ln(x) 6x^2 - 1)}{x}
 \end{aligned} \tag{98}$$

Taxas Relacionadas – [Aula 1](#)

$$\begin{aligned}
 y &= x^2 \\
 1 \frac{\partial y}{\partial x} &= 2x \frac{\partial x}{\partial x} \rightarrow \frac{\partial y}{\partial x} = 2x \\
 4y^3 &= x^2 + 1 \\
 12y^2 \frac{\partial y}{\partial x} &= 2x \frac{\partial x}{\partial x} \rightarrow 12y^2 \frac{\partial y}{\partial x} = 2x \rightarrow \frac{\partial y}{\partial x} = \frac{2x}{12y^2} = \frac{x}{6y^2} \\
 y^3 &= x^2 + y + 2x^3 + 3 = y^3 - y = 2x^3 + x^2 + 3 \\
 (3y^2 - 1) \frac{\partial y}{\partial x} &= (6x^2 + 2x) \frac{\partial x}{\partial x} \rightarrow \frac{\partial y}{\partial x} = \frac{6x^2 + 2x}{3y^2 - 1} = \frac{2x(3x + 1)}{3y^2 - 1} \\
 y^3 + 1 + z^2 &= x^2 \\
 3y^2 \frac{\partial y}{\partial x} + 2z \frac{\partial z}{\partial x} &= 2x \\
 3y^2 \frac{\partial y}{\partial z} + 2z &= 2x \frac{\partial x}{\partial z} \\
 3y^2 + 2z \frac{\partial z}{\partial y} &= 2x \frac{\partial x}{\partial y} \\
 3y^2 \frac{\partial y}{\partial t} + 2z \frac{\partial z}{\partial t} &= 2x \frac{\partial x}{\partial t}
 \end{aligned}$$

1. Identificar as variáveis.
2. Achar uma relação entre as variáveis.
3. Derivar em relação a variável de referência.
4. Substituir os valores conhecidos.
5. Isolar o que se quer calcular.

Exercício I

O volume do balão esférico cresce a uma taxa de 100 centímetros cúbicos por segundo, qual é a taxa de crescimento do raio quando o mesmo mede 50 cm.

1. Volume (v): cm^3 ; Raio (r): cm ; Tempo(t): s
2. $v = \frac{4\pi r^3}{3}$
3. $\frac{\partial v}{\partial t} = 3 \frac{4\pi r^{3-1}}{3} \frac{\partial r}{\partial t} = 4\pi r^2 \frac{\partial r}{\partial t}$
4. $100 \frac{cm^3}{s} = 4\pi (50cm)^2 \frac{\partial r}{\partial t} = 4\pi 2500 cm^2 \frac{\partial r}{\partial t} = 10000\pi cm^2 \frac{\partial r}{\partial t}$
5. $\frac{\partial r}{\partial t} = \frac{100 \frac{cm^3}{s}}{10000\pi cm^2} = \frac{100 cm^3}{10000\pi cm^2 s} = \frac{1}{100\pi} \frac{cm}{s}$

(99)

Exercício II

Uma mancha de óleo expande-se em forma de círculo onde a área cresce a uma taxa constante de 26 quilômetros quadrados/h. Com que rapidez variará o raio da mancha quando a área for 9 quilômetros quadrados.

1. Área (a): km^2 ; Raio (r): km ; Tempo (t): h
2. $a = \pi r^2 \rightarrow r = \sqrt{\frac{a}{\pi}} \rightarrow r = \sqrt{\frac{9km^2}{\pi}} = \frac{3km}{\sqrt{\pi}} = \frac{3\sqrt{\pi}km}{\pi}$
3. $\frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t}$
4. $26 \frac{km^2}{h} = 2\pi \frac{3\sqrt{\pi}km}{\pi} \frac{\partial r}{\partial t} = 6\sqrt{\pi}km \frac{\partial r}{\partial t}$
5. $\frac{\partial r}{\partial t} = \frac{26 \frac{km^2}{h}}{6\sqrt{\pi}km} = \frac{26 km^2}{6\sqrt{\pi}km h} = \frac{26 km}{6\sqrt{\pi}h} = \frac{13}{3\sqrt{\pi}} \frac{km}{h} = \frac{13\sqrt{\pi} km}{3\pi h}$

(100)

Exercício III

Um foguete sobe verticalmente e é acompanhado por uma estação no solo a 5 km da base de lançamento. Com que rapidez o foguete subirá, quando sua altura for 4 km e a sua distância da estação estiver crescendo a 2000 km/h.

1. Distância (d): km; Altura (a): km; Tempo (t): h

$$2. d = \sqrt{(5 \text{ km})^2 + a^2} = \sqrt{25 \text{ km}^2 + a^2} = (25 \text{ km}^2 + a^2)^{\frac{1}{2}}$$

$$3. \frac{\partial d}{\partial t} = \frac{1}{2} (25 \text{ km}^2 + a^2)^{-\frac{1}{2}} \cdot 2a \frac{\partial a}{\partial t} = \frac{a}{\sqrt{25 \text{ km}^2 + a^2}} \frac{\partial a}{\partial t}$$

$$4. 2000 \frac{\text{km}}{\text{h}} = \frac{4 \text{ km}}{\sqrt{25 \text{ km}^2 + (4 \text{ km})^2}} \frac{\partial a}{\partial t} = \frac{4 \text{ km}}{\sqrt{25 \text{ km}^2 + 16 \text{ km}^2}} \frac{\partial a}{\partial t} = \frac{4 \text{ km}}{\sqrt{41 \text{ km}^2}} \frac{\partial a}{\partial t} = \frac{4\sqrt{41}}{41} \frac{\partial a}{\partial t} \quad (101)$$

$$5. \frac{\partial a}{\partial t} = \frac{2000 \frac{\text{km}}{\text{h}}}{\frac{4\sqrt{41}}{41}} = \frac{2000 \text{ km}}{\frac{4\sqrt{41}}{41} h} = \frac{41 \cdot 500 \text{ km}}{\sqrt{41} h} = 500 \sqrt{41} \frac{\text{km}}{h}$$

Taxas Relacionadas – [Aula 2](#)

Exercício I

Um tanque de água tem a forma de um cone circular invertido com base de raio igual a 2 metros e altura igual a 4 metros. Se a água está sendo bombeada para dentro do tanque a uma taxa de 2 metros cúbicos por minuto, encontre a taxa na qual o nível de água estará elevado quando a água estiver a 3 metros de profundidade.

1. Volume do tanque (v): m³; Profundidade do tanque invertido (h): m; Tempo (t): min

$$2. v = \frac{\pi r^2 h}{3} \rightarrow \frac{2 \text{ m}}{4 \text{ m}} = \frac{r}{h} \rightarrow r = \frac{2h}{4} = \frac{h}{2} \rightarrow v = \frac{\pi \left(\frac{h}{2}\right)^2 h}{3} = \frac{\pi \frac{h^2}{4} h}{3} = \frac{\pi \frac{h^3}{4}}{3} = \frac{\pi h^3}{12}$$

$$3. \frac{\partial v}{\partial t} = \frac{3\pi h^2}{12} \frac{\partial h}{\partial t} = \frac{\pi h^2}{4} \frac{\partial h}{\partial t}$$

$$4. 2 \frac{\text{m}^3}{\text{min}} = \frac{\pi (3 \text{ m})^2}{4} \frac{\partial h}{\partial t} = \frac{9\pi \text{ m}^2}{4} \frac{\partial h}{\partial t}$$

$$5. \frac{\partial h}{\partial t} = \frac{2 \frac{\text{m}^3}{\text{min}}}{\frac{9\pi \text{ m}^2}{4}} = \frac{8 \text{ m}^3}{9\pi \text{ m}^2 \text{ min}} = \frac{8}{9\pi} \frac{\text{m}}{\text{min}} \quad (102)$$

Exercício II

Uma escada de 5 metros está apoiada em uma parede. A base é arrastada a 3 m/s. Qual a velocidade da escada a longo da parede, quando a base se encontra a 3 metros da parede.

1. Base (b): m; Altura (h): m; Tempo (t): s
2. $(5m)^2 = b^2 + h^2 \rightarrow b^2 + h^2 = 25m^2 \rightarrow$
 $h = \sqrt{25m^2 - b^2} = \sqrt{25m^2 - (3m)^2} = \sqrt{25m^2 - 9m^2} = \sqrt{16m^2} = 4m$
3. $2b \frac{\partial b}{\partial t} + 2h \frac{\partial h}{\partial t} = 0$
4. $2 \cdot 3m \cdot 3 \frac{m}{s} + 2 \cdot 4m \cdot \frac{\partial h}{\partial t} = 0 \rightarrow 18 \frac{m^2}{s} + 8m \frac{\partial h}{\partial t} = 0$
5. $\frac{\partial h}{\partial t} = \frac{-18 \frac{m^2}{s}}{8m} = \frac{-18m^2}{8ms} = \frac{-9m}{4s}$

(103)

Exercício III

Determine a taxa de variação instantânea da área em relação ao lado, quando esse mede 4 m.

1. Área (a): m^2 ; Lado (l): m
2. $a = l^2$
3. $\frac{\partial a}{\partial l} = 2l$
4. $\frac{\partial a}{\partial l} = 2(4m)$
5. $\frac{\partial a}{\partial l} = 8 \frac{m^2}{m}$

(104)

Exercício IV

Uma pedra foi lançada no lago e com isso, gerou ondas circulares que se propagam com a velocidade constante de 3 m/s. Qual a taxa de crescimento da área após transcorrido 10 segundos.

1. Área (a): m^2 , Raio (r): m; Tempo (t): s
2. $a = \pi r^2 \rightarrow r = 3 \frac{m}{s} \cdot 10s = 30m$
3. $\frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t}$
4. $\frac{\partial a}{\partial t} = 2\pi(30m)3 \frac{m}{s}$
5. $\frac{\partial a}{\partial t} = 180\pi \frac{m^2}{s}$

(105)

Exercício V

Acumula-se areia em um monte com a forma de um cone. A altura do cone é igual ao raio da base. O volume de areia cresce a uma taxa de 10 metros cúbicos por hora. Com que razão aumenta a área da base quando a altura for 4 metros.

$$\begin{aligned}
 &1. \text{ Volume (v): } m^3 \text{ Altura(r): } m; \text{ Área (a): } m^2; \text{ Tempo (t): } h \\
 &2. \text{ } a = \pi r^2 \rightarrow r = h \rightarrow v = \frac{\pi r^2 h}{3} = \frac{\pi r^3}{3} \\
 &3. \frac{\partial a}{\partial t} = 2\pi r \frac{\partial r}{\partial t} \\
 &\frac{\partial v}{\partial t} = \frac{3\pi r^2}{3} \frac{\partial r}{\partial t} = \pi r^2 \frac{\partial r}{\partial t} \\
 &4. 10 \frac{m^3}{h} = \pi (4m)^2 \frac{\partial r}{\partial t} = 16\pi m^2 \frac{\partial r}{\partial t} \quad (106) \\
 &\frac{\partial r}{\partial t} = \frac{10 \frac{m^3}{h}}{16\pi m^2} = \frac{10m^3}{16\pi m^2 h} = \frac{5}{8\pi} \frac{m}{h} \\
 &\frac{\partial a}{\partial t} = 2\pi r \frac{5m}{8\pi h} = \frac{r \cdot 5m}{4h} \\
 &5. \frac{\partial a}{\partial t} = \frac{(4m)5m}{4h} = \frac{20m^2}{4h} = 5 \frac{m^2}{h}
 \end{aligned}$$