Introdução as Derivadas – Aula 1

$$\frac{\partial f(x)}{\partial x} = D_x f(x) = f'(x)$$

1) $y = c \rightarrow y' = 0$
2) $y = x \rightarrow y' = 1$
3) $y = x^c \rightarrow y' = c \cdot x^{c-1}$
4) $y = e^x \rightarrow y' = e^x$
5) $y = \ln(x) \rightarrow y' = \frac{1}{x}$
6) $y = f \cdot h \rightarrow y' = f' \cdot h + f \cdot h'$
7) $y = \frac{f}{h} \rightarrow y' = \frac{f' \cdot h - f \cdot h'}{h^2}$
8) $y = f^c \rightarrow y' = c \cdot f^{c-1} \cdot f'$
9) $y = e^f \rightarrow y' = e^f \cdot f'$
10) $y = c^f \rightarrow y' = c^f \cdot f' \cdot \ln(c)$
11) $y = \ln(f) \rightarrow y' = \frac{f'}{f}$
12) $y = \log_c(f) \rightarrow y' = \frac{f'}{f} \cdot \log_c(e)$

Exercício I

$$\begin{array}{l} \text{a) } y = 8 \rightarrow y' = 0 \\ \text{b) } y = \sqrt{3} \rightarrow y' = 0 \\ \text{c) } f(x) = \pi \rightarrow f'(x) = 0 \\ \text{d) } g(x) = (\pi - 1)^\pi \rightarrow g'(x) = 0 \end{array} \tag{1}$$

Exercício II

$$\begin{array}{l} \text{a) } y = x^5 \rightarrow y' = 5x^{5-1} = 5x^4 \\ \text{b) } h(x) = x^{-5} \rightarrow h'(x) = -5x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = -\frac{5}{x^6} \\ \text{c) } g(x) = 5x^3 \rightarrow g'(x) = 5 \cdot 3x^{3-1} = 15x^2 \end{array} \tag{2}$$

Exercício III

$$h(x) = 8x \rightarrow h'(x) = 8 \cdot 1 = 8 \tag{3}$$

Exercício IV

$$\begin{array}{c} f(x) = 7x^3 - 2x - 400 \\ f'(x) = 7 \cdot 3x^{3-1} - 2 \cdot 1 - 0 = 21x^2 - 2 \end{array} \tag{4}$$

Derivada com X no Denominador – Aula 2

Exercício I

$$\begin{gathered} g(x)=\frac{3}{x^5}=3x^{-5} \\ g'(x)=3(-5x^{-5-1})=-15x^{-6}=-\frac{15}{x^6} \end{gathered} \tag{5}$$

Exercício II

$$\begin{gathered} h(x)=3x^5-\frac{2}{x^4}=3x^5-2x^{-4} \\ h'(x)=3(5x^{5-1})-2(-4x^{-4-1})=15x^4+8x^{-5}=15x^4+\frac{8}{x^5} \end{gathered} \tag{6}$$

Derivada de Função Raiz – Aula 3

Exercício I

$$\begin{gathered} y=\sqrt[3]{x^4}=x^{\frac{4}{3}} \\ y'=\frac{4}{3}x^{\frac{4}{3}-1}=\frac{4}{3}x^{\frac{1}{3}}=\frac{4}{3}\sqrt[3]{x} \end{gathered} \tag{7}$$

Exercício II

$$\begin{gathered} g(x)=7\sqrt[3]{x}=7x^{\frac{1}{3}} \\ g'(x)=7\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)=\frac{7}{3}x^{-\frac{2}{3}}=\frac{7}{3}\cdot\frac{1}{x^{\frac{2}{3}}}=\frac{7}{3\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right)=\frac{7\sqrt[3]{x}}{3x} \end{gathered} \tag{8}$$

Derivada de uma Função Potência – Aula 4

Exercício I

$$y=x^3 \rightarrow y'=3x^2 \tag{9}$$

Exercício II

$$\begin{gathered} y=\frac{5x^4x^3}{x^2}=5x^{4+3-2}=5x^5 \\ y'=5(5x^{5-1})=25x^4 \end{gathered} \tag{10}$$

Derivada de uma Função Potência – Aula 5

Exercício I

$$y=\frac{x^2\sqrt{x}}{\sqrt[3]{x}}=x^2\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}=x^{2+\frac{1}{2}-\frac{1}{3}}=x^{\frac{12+3-2}{6}}=x^{\frac{13}{6}}$$
$$y'=\frac{13}{6}x^{\frac{13}{6}-1}=\frac{13}{6}x^{\frac{7}{6}}=\frac{13}{6}\sqrt[6]{x^7} \tag{11}$$

Derivada de Função Exponencial e Logarítmica – Aula 6

Exercício I

$$f(x)=3e^x+10\cdot\ln(x)$$
$$f'(x)=3e^x+\frac{10}{x}=\frac{3xe^x+10}{x} \tag{12}$$

Exercício II

$$g(x)=7e^x+9\cdot\ln(x)+3x^4-4x+100$$
$$g'(x)=7e^x+\frac{9}{x}+12x^3-4=\frac{7xe^x+9+12x^4-4x}{x} \tag{13}$$

Derivada de um Produto de Funções – Aula 7

Exercício I

$$f(x)=x\cdot\ln(x)$$
$$\begin{cases} g(x)=x & \rightarrow \quad g'(x)=1 \\ h(x)=\ln(x) & \rightarrow \quad h'(x)=\frac{1}{x} \end{cases} \tag{14}$$
$$f(x)=g(x)\cdot h(x)\rightarrow f'(x)=g'(x)\cdot h(x)+g(x)\cdot h'(x)=1\cdot\ln(x)+x\cdot\frac{1}{x}=\ln(x)+1$$

Derivada de uma Divisão de Funções – Aula 8

Exercício I

$$
\begin{gathered}
f(x)=\frac{e^x}{3x}=e^x\cdot\frac{1}{3x}=e^x\cdot\frac{x^{-1}}{3}\\
\begin{cases}
g(x)=e^x & \rightarrow \quad g'(x)=e^x\\
h(x)=\frac{x^{-1}}{3}=\frac{1}{3x} & \rightarrow \quad h'(x)=\frac{-x^{-2}}{3}=-\frac{1}{3x^2}
\end{cases}\\
f(x)=g(x)\cdot h(x)\rightarrow {\color{red}f'(x)}=g'(x)\cdot h(x)+g(x)\cdot h'(x)=\\
e^x\cdot\frac{1}{3x}+e^x\cdot\left(-\frac{1}{3x^2}\right)=\frac{e^x}{3x}-\frac{e^x}{3x^2}=\frac{x\cdot e^x-e^x}{3x^2}={\color{red}\frac{e^x(x-1)}{3x^2}}\\
f(x)=\frac{e^x}{3x}\\
\begin{cases}
g(x)=e^x & \rightarrow \quad g'(x)=e^x\\
h(x)=3x & \rightarrow \quad h'(x)=3
\end{cases}\\
f(x)=\frac{g(x)}{h(x)}\rightarrow {\color{red}f'(x)}=\frac{g'(x)\cdot h(x)-g(x)\cdot h'(x)}{h(x)^2}=\\
\frac{e^x\cdot 3x-e^x\cdot 3}{(3x)^2}=\frac{3e^x(x-1)}{9x^2}={\color{red}\frac{e^x(x-1)}{3x^2}}
\end{gathered}
\tag{15}
$$

Derivadas Básicas – Aula 9

Exercício I

$$
\begin{gathered}
y=\frac{2x^2+5x}{x}=\frac{2x^2}{x}+\frac{5x}{x}=2x+5\\
y'=2
\end{gathered}
\tag{16}
$$

Exercício II

$$
\begin{gathered}
h(x)=\frac{3(x^2-1)}{x}=\frac{3x^2-3}{x}=\frac{3x^2}{x}-\frac{3}{x}=3x-3x^{-1}\\
h'(x)=3-(-3x^{-2})=3+\frac{3}{x^2}=\frac{3x^2+3}{x^2}=\frac{3(x^2+1)}{x^2}
\end{gathered}
\tag{17}
$$

Exercícios de Derivada – Aula 10

Exercício I

$$
\begin{gathered}
h(x)=3x^3(2+4x)=6x^3+12x^4\\
h'(x)=18x^2+48x^3=6x^2(8x+3)
\end{gathered}
\tag{18}
$$

Exercício II

$$g(x)=(x^2-1)(x^3+4)=x^5+4x^2-x^3-4$$
$$g'(x)=5x^4+8x-3x^2=x(5x^3-3x+8) \quad (19)$$

Curso básico de derivadas – Aula 12

Exercício I

$$g(x)=x^4+2e^x+e^2$$
$$g'(x)=4x^3+2e^x=2(2x^3+e^x) \quad (20)$$

Exercício II

$$g(x)=\sqrt[3]{x^7}+\frac{3}{x^2}+5=x^{\frac{7}{3}}+3x^{-2}+5$$
$$g'(x)=\frac{7x^{\frac{4}{3}}}{3}+(-6x^{-3})=\frac{7\sqrt[3]{x^4}}{3}-\frac{6}{x^3}=\frac{7x^3\sqrt[3]{x^4}-18}{3x^3}=\frac{7\sqrt[3]{x^{13}}-18}{3x^3} \quad (21)$$

Derivada de um Produto de Funções – Aula 13

Exercício I

$$y=8x\cdot\ln(x)$$
$$y'=8\cdot\ln(x)+8x\cdot\frac{1}{x}=8\cdot\ln(x)+8=8(\ln(x)+1) \quad (22)$$

Derivada de função composta, raiz, polinomial – Aula 14

Exercício I

$$y=x^3\rightarrow y'=3x^2 \quad (23)$$

Exercício II

$$f(x)=(2x^2-1)^3$$
$$g(x)=2x^2-1\rightarrow g'(x)=4x$$
$$f(x)=g(x)^p\rightarrow f'(x)=p\cdot g(x)^{p-1}\cdot g'(x)=3(2x^2-1)^2\cdot 4x=12x(2x^2-1)^2 \quad (24)$$

Exercício III

$$y=(3-x^2)^3$$
$$y'=3(3-x^2)^2(-2x)=-6x(3-x^2)^2 \quad (25)$$

Exercício IV

$$y=\frac{3}{(2x^2-1)^4}=3(2x^2-1)^{-4}$$
$$y'=3(-4)(2x^2-1)^{-5}\cdot 4x=-48x(2x^2-1)^{-5}=-\frac{48x}{(2x^2-1)^5} \tag{26}$$

Exercício V

$$y=\sqrt{(2x^2-1)^3}=(2x^2-1)^{\frac{3}{2}}$$
$$y'=\frac{3}{2}(2x^2-1)^{\frac{1}{2}}\cdot 4x=6x(2x^2-1)^{\frac{1}{2}}=6x\sqrt{2x^2-1} \tag{27}$$

Derivada de funções quociente, produto, polinomial – Aula 15

Exercício I

$$y=7x^4-2x^3+8x+5$$
$$y'=28x^3-6x^2+8=2(14x^3-3x^2+4) \tag{28}$$

Exercício II

$$y=(2x^3-4x^2)(3x^5+x^2)=6x^8+2x^5-12x^7-4x^4=6x^8-12x^7+2x^5-4x^4$$
$$y'=48x^7-84x^6+10x^4-16x^3=2x^3(24x^4-42x^3+5x-8) \tag{29}$$

Exercício III

$$h(x)=\frac{2x^3+4}{x^2-4x+1}$$
$$\begin{cases} f(x)=2x^3+4 & \rightarrow\ f'(x)=6x^2 \\ g(x)=x^2-4x+1 & \rightarrow\ g'(x)=2x-4 \end{cases}$$
$$h'(x)=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g(x)^2}=\frac{6x^2(x^2-4x+1)-(2x^3+4)(2x-4)}{(x^2-4x+1)^2}=$$
$$\frac{6x^4-24x^3+6x^2-(4x^4-8x^3+8x-16)}{(x^2-4x+1)^2}=\frac{6x^4-24x^3+6x^2-4x^4+8x^3-8x+16}{(x^2-4x+1)^2}=$$
$$\frac{2x^4-16x^3+6x^2-8x+16}{(x^2-4x+1)^2}=\frac{2(x^4-8x^3+3x^2-4x+8)}{(x^2-4x+1)^2} \tag{30}$$

Exercício IV

$$y=\frac{3}{x^5}=3x^{-5}\rightarrow y'=-15x^{-6}=-\frac{15}{x^6} \tag{31}$$

Exercício V

$$v(r)=\frac{4}{3}\pi r^3 \rightarrow v'(r)=4\pi r^2 \tag{32}$$

Exercício VI

$$\begin{gathered} f(s)=\sqrt{3}(s^3-s^2)=\sqrt{3}s^3-\sqrt{3}s^2 \\ f'(s)=3\sqrt{3}s^2-2\sqrt{3}s=s\sqrt{3}(3s-2) \end{gathered} \tag{33}$$

Exercício VII

$$\begin{gathered} y=(4x^2+3)^2=16x^4+12x^2+12x^2+9=16x^4+24x^2+9 \\ y'=64x^3+48x=16x(4x^2+3) \end{gathered} \tag{34}$$

Derivadas de função quociente e produto – Aula 16

Exercício I

$$\begin{gathered} y=\frac{x^4-2x^2+5x+1}{x^4}=\frac{x^4}{x^4}-\frac{2x^2}{x^4}+\frac{5x}{x^4}+\frac{1}{x^4}=1-2x^{-2}+5x^{-3}+x^{-4} \\ y'=4x^{-3}-15x^{-4}-4x^{-5}=\frac{4}{x^3}-\frac{15}{x^4}-\frac{4}{x^5}=\frac{4x^2-15x-4}{x^5} \end{gathered} \tag{35}$$

Exercício II

$$\begin{gathered} y=\frac{x}{x-1} \\ y'=\frac{1(x-1)-x\cdot 1}{(x-1)^2}=\frac{x-1-x}{(x-1)^2}=-\frac{1}{(x-1)^2} \end{gathered} \tag{36}$$

Exercício III

$$\begin{gathered} y=\left(\frac{2x+1}{x+5}\right)(3x-1)=\frac{6x^2-2x+3x-1}{x+5}=\frac{6x^2+x-1}{x+5} \\ y'=\frac{(12x+1)(x+5)-(6x^2+x-1)\cdot 1}{(x+5)^2}=\frac{12x^2+60x+x+5-6x^2-x+1}{(x+5)^2}= \\ \frac{6x^2+60x+6}{(x+5)^2}=\frac{6(x^2+10x+1)}{(x+5)^2} \end{gathered} \tag{37}$$

Exercício IV

$$y=\frac{1}{8}x^8-4x^4 \rightarrow y'=x^7-16x^3=x^3(x^4-16) \tag{38}$$

Exercício V

$$y = x^2 + 3x + \frac{1}{x^2} = x^2 + 3x + x^{-2}$$
$$y' = 2x + 3 - 2x^{-3} = 2x + 3 + \frac{2}{x^3} = \frac{2x^4 + 3x^3 + 2}{x^3} \qquad (39)$$

Exercício VI

$$y = \frac{3}{x^2} + \frac{5}{x^4} = 3x^{-2} + 5x^{-4} \rightarrow y' = -6x^{-3} - 20x^{-5} = -\frac{6}{x^3} - \frac{20}{x^5} = \frac{-6x^2 - 20}{x^5} \qquad (40)$$

Derivada de funções envolvendo raiz, quociente, produto – Aula 17

Exercício I

$$g(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{\frac{1}{2}}} = 3x^{-\frac{1}{2}}$$
$$g'(x) = 3\left(-\frac{1}{2}\right)x^{-\frac{3}{2}} = \frac{-3}{2x^{\frac{3}{2}}} = -\frac{3}{2\sqrt{x^3}} \qquad (41)$$

Exercício II

$$g(x) = 5\sqrt[3]{x^2} = 5x^{\frac{2}{3}}$$
$$g'(x) = 5 \cdot \frac{2}{3}x^{-\frac{1}{3}} = \frac{10}{3x^{\frac{1}{3}}} = \frac{10}{3\sqrt[3]{x}} \qquad (42)$$

Exercício III

$$h(r) = 6\sqrt[4]{r^3} + \frac{7}{r^3} = 6r^{\frac{3}{4}} + 7r^{-3}$$
$$h'(r) = 6 \cdot \frac{3}{4}r^{-\frac{1}{4}} + 7(-3)r^{-4} = \frac{9}{2}r^{-\frac{1}{4}} - 21r^{-4} = \frac{9}{2r^{\frac{1}{4}}} - \frac{21}{r^4} = \frac{9}{2\sqrt[4]{r}} - \frac{21}{r^4} = \qquad (43)$$
$$\frac{9r^4 - 42\sqrt[4]{r}}{2r} = \frac{3}{2r}\left(3r^4 - 14\sqrt[4]{r}\right)$$

Exercício IV

$$g(t)=\frac{5t^3}{6\sqrt{t}}=5t^3\cdot\frac{1}{6t^{\frac{1}{2}}}=5t^3\cdot\frac{t^{-\frac{1}{2}}}{6}$$

$$g'(t)=5\cdot 3t^{3-1}\cdot\frac{1}{6\sqrt{t}}+5t^3\left(\frac{1}{6}\left(-\frac{1}{2}\right)t^{-\frac{1}{2}-1}\right)=\frac{15t^2}{6\sqrt{t}}+5t^3\left(-\frac{t^{-\frac{3}{2}}}{12}\right)=\frac{5t^2}{2t^{\frac{1}{2}}}-\frac{5}{12}t^{3-\frac{3}{2}}=$$

$$\frac{5}{2}t^{2-\frac{1}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5}{2}t^{\frac{3}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5\sqrt{t^3}}{2}-\frac{5\sqrt{t^3}}{12}=\frac{30\sqrt{t^3}-5\sqrt{t^3}}{12}=\frac{5\sqrt{t^3}(6-1)}{12}=\frac{25\sqrt{t^3}}{12}$$

$$g(t)=\frac{5t^3}{6\sqrt{t}}=\frac{5t^3}{6t^{\frac{1}{2}}}$$

$$g'(t)=\frac{5\cdot 3t^{3-1}\cdot 6t^{\frac{1}{2}}-5t^3\cdot 6\left(\frac{1}{2}\right)t^{\frac{1}{2}-1}}{\left(6t^{\frac{1}{2}}\right)^2}=\frac{15t^2\cdot 6t^{\frac{1}{2}}-5t^3\cdot 3t^{-\frac{1}{2}}}{36t}=$$

$$\frac{90t^{2+\frac{1}{2}}-15t^{3-\frac{1}{2}}}{36t}=\frac{15\left(6t^{\frac{5}{2}}-t^{\frac{5}{2}}\right)}{36t}=\frac{5\sqrt{t^5}(6-1)}{12t}=\frac{30\sqrt{t^5}}{12t}-\frac{5\sqrt{t^5}}{12t}=$$

$$\frac{5}{2}t^{\frac{5}{2}-1}-\frac{5}{12}t^{\frac{5}{2}-1}=\frac{5}{2}t^{\frac{3}{2}}-\frac{5}{12}t^{\frac{3}{2}}=\frac{5\sqrt{t^3}}{2}-\frac{5\sqrt{t^3}}{12}=\frac{5\sqrt{t^3}(6-1)}{12}=\frac{25\sqrt{t^3}}{12} \tag{44}$$

$$g(t)=\frac{5t^3}{6\sqrt{t}}=\frac{5t^3}{6t^{\frac{1}{2}}}=\frac{5}{6}t^{3-\frac{1}{2}}=\frac{5}{6}t^{\frac{5}{2}}$$

$$g'(t)=\frac{5}{6}\cdot\frac{5}{2}t^{\frac{5}{2}-1}=\frac{25t^{\frac{3}{2}}}{12}=\frac{25\sqrt{t^3}}{12}$$

Exercício V

$$h(x)=7x\sqrt[3]{x^2}=7x\cdot x^{\frac{2}{3}}=7x^{1+\frac{2}{3}}=7x^{\frac{5}{3}}$$

$$h'(x)=7\cdot\frac{5}{3}x^{\frac{5}{3}-1}=\frac{35x^{\frac{2}{3}}}{3}=\frac{35\sqrt[3]{x^2}}{3} \tag{45}$$

Exercício VI

$$g(t)=\frac{8t^3\sqrt{t}}{t}=\frac{8t^3t^{\frac{1}{2}}}{t}=8t^{3+\frac{1}{2}-1}=8t^{\frac{5}{2}}$$

$$g'(t)=8\cdot\frac{5}{2}t^{\frac{5}{2}-1}=20t^{\frac{3}{2}}=20\sqrt{t^3} \tag{46}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 18

Exercício I

$$\begin{aligned} f(x) &= \frac{3x^4-8x^2+4}{x^2} = \frac{3x^4}{x^2} - \frac{8x^2}{x^2} + \frac{4}{x^2} = 3x^2 - 8 + 4x^{-2} \\ f'(x) &= 6x - 8x^{-3} = 6x - \frac{8}{x^3} = \frac{6x^4-8}{x^3} \end{aligned} \tag{47}$$

Exercício II

$$\begin{aligned} f(x) &= \frac{\pi x^3}{\sqrt{x^3}} = \frac{\pi x^3}{x^{\frac{3}{2}}} = \pi x^{3-\frac{3}{2}} = \pi x^{\frac{3}{2}} \\ f'(x) &= \pi \cdot \frac{3}{2} x^{\frac{3}{2}-1} = \frac{3\pi x^{\frac{1}{2}}}{2} = \frac{3\pi\sqrt{x}}{2} \end{aligned} \tag{48}$$

Exercício III

$$\begin{aligned} y &= \frac{x^2}{e^x} \\ y' &= \frac{2x \cdot e^x - x^2 \cdot e^x}{(e^x)^2} = \frac{e^x \cdot x(2-x)}{e^{2x}} = \frac{x(2-x)}{e^x} = \frac{x}{e^x}(2-x) \end{aligned} \tag{49}$$

Exercício IV

$$\begin{aligned} y &= x^5 \cdot e^x \\ y' &= 5x^4 \cdot e^x + x^5 \cdot e^x = e^x \cdot x^4 (5+x) \end{aligned} \tag{50}$$

Exercício V

$$\begin{aligned} y &= \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x) \\ y' &= \left(\frac{1}{2} x^{\frac{1}{2}-1}\right) \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{-\frac{1}{2}} \cdot \ln(x)}{2} + x^{\frac{1}{2}} \cdot x^{-1} = \frac{\ln(x)}{2x^{\frac{1}{2}}} + x^{\frac{1}{2}-1} = \frac{\ln(x)}{2\sqrt{x}} + x^{-\frac{1}{2}} = \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{x^{\frac{1}{2}}} = \\ & \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} = \left(\frac{1}{\sqrt{x}}\left(\frac{\ln(x)}{2} + 1\right)\right)\left(\frac{\sqrt{x}}{\sqrt{x}}\right) = \frac{\sqrt{x}}{x}\left(\frac{\ln(x)}{2} + 1\right) \end{aligned} \tag{51}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 19

Exercício I

$$f(t)=e^t\cdot\ln(t)$$
$$f'(t)=e^t\cdot\ln(t)+e^t\cdot\frac{1}{t}=e^t\left(\ln(t)+\frac{1}{t}\right)=\frac{e^t(t\cdot\ln(t)+1)}{t}=\frac{e^t}{t}(t\cdot\ln(t)+1) \tag{52}$$

Exercício II

$$f(r)=\frac{e^r}{\sqrt{r}}=\frac{e^r}{r^{\frac{1}{2}}}$$
$$f'(r)=\frac{e^r\sqrt{r}-e^r\cdot\frac{1}{2}r^{\frac{1}{2}-1}}{(\sqrt{r})^2}=\frac{e^r\sqrt{r}-e^r\cdot\frac{r^{-\frac{1}{2}}}{2}}{r}=\frac{e^r\sqrt{r}-\frac{e^r}{2r^{\frac{1}{2}}}}{r}=\frac{e^r\sqrt{r}-\frac{e_r}{2\sqrt{r}}}{r}=\frac{\frac{2r\cdot e^r-e^r}{2\sqrt{r}}}{r}=$$
$$\frac{2r\cdot e^r-e^r}{2\sqrt{r}\cdot r}=\frac{e^r(2r-1)}{2r^{\frac{1}{2}+1}}=\frac{e^r(2r-1)}{2r^{\frac{3}{2}}}=\left(\frac{e^r(2r-1)}{2\sqrt{r^3}}\right)\left(\frac{\sqrt{r^3}}{\sqrt{r^3}}\right)=\frac{e^r\sqrt{r^3}(2r-1)}{2r^3}=\frac{e^r\sqrt{r^3}}{2r^3}(2r-1)$$
$$f(r)=\frac{e^r}{\sqrt{r}}=\frac{e^r}{r^{\frac{1}{2}}}$$
$$f'(r)=\frac{e^r\sqrt{r}-e^r\cdot\frac{1}{2}r^{\frac{1}{2}-1}}{(\sqrt{r})^2}=\frac{e^r\sqrt{r}-e^r\cdot\frac{r^{-\frac{1}{2}}}{2}}{r}=\frac{e^r\sqrt{r}-\frac{e^r}{2r^{\frac{1}{2}}}}{r}=\frac{e^r\sqrt{r}-\frac{e_r}{2\sqrt{r}}}{r}=\frac{e^r\left(\sqrt{r}-\frac{1}{2\sqrt{r}}\right)}{r}=$$
$$\frac{e^r}{r}\left(\sqrt{r}-\frac{1}{2\sqrt{r}}\right)=\frac{e^r}{r}\left(\frac{2r-1}{2\sqrt{r}}\right)\left(\frac{\sqrt{r}}{\sqrt{r}}\right)=\frac{e^r\sqrt{r}}{2r^2}(2r-1) \tag{53}$$

Exercício III

$$v(t)=\frac{t^3\cdot e^t}{t}=t^{3-1}\cdot e^t=t^2\cdot e^t$$
$$v'(t)=2t\cdot e^t+t^2\cdot e^t=e^t\cdot t(2+t) \tag{54}$$

Exercício IV

$$y=x^5\rightarrow y'=5x^4$$
$$y=(2x^2-4)^5\rightarrow y'=5(2x^2-4)^4\cdot 4x=20x(2x^2-4)^4 \tag{55}$$

Exercício V

$$y=\sqrt{2x^3-1}=(2x^3-1)^{\frac{1}{2}}$$
$$y'=\frac{1}{2}(2x^3-1)^{-\frac{1}{2}}\cdot 6x^2=3x^2(2x^3-1)^{-\frac{1}{2}}=\frac{3x^2}{(2x^3-1)^{\frac{1}{2}}}=\left(\frac{3x^2}{\sqrt{2x^3-1}}\right)\left(\frac{\sqrt{2x^3-1}}{\sqrt{2x^3-1}}\right)=\frac{3x^2\sqrt{2x^3-1}}{2x^3-1} \tag{56}$$

Exercício VI

$$y=\frac{5}{(3x^2+9)^4}=5(3x^2+9)^{-4}$$
$$y'=-20(3x^2+9)^{-5}\cdot 6x=-120x(3x^2+9)^{-5}=-\frac{120x}{(3x^2+9)^5} \tag{57}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 20]

Exercício I

$$y=\sqrt[3]{6x^2+7x+2}=(6x^2+7x+2)^{\frac{1}{3}}$$
$$y'=\frac{1}{3}(6x^2+7x+2)^{-\frac{2}{3}}\cdot(12x+7)=\frac{12x+7}{3(6x^2+7x+2)^{\frac{2}{3}}}=\left(\frac{12x+7}{3\sqrt[3]{(6x^2+7x+2)^2}}\right)\left(\frac{\sqrt[3]{6x^2+7x+2}}{\sqrt[3]{6x^2+7x+2}}\right)=\frac{(12x+7)\sqrt[3]{6x^2+7x+2}}{3(6x^2+7x+2)} \tag{58}$$

Exercício II

$$y=e^{5x^2+4}$$
$$y'=e^{5x^2+4}\cdot 10x \tag{59}$$

Exercício III

$$y=e^{\frac{1}{x^2}}=e^{x^{-2}}$$
$$y'=e^{\frac{1}{x^2}}(-2x^{-3})=e^{\frac{1}{x^2}}\left(-\frac{2}{x^3}\right)=-\frac{2e^{\frac{1}{x^2}}}{x^3} \tag{60}$$

Exercício IV

$$y=3^{x^2}$$
$$y'=3^{x^2}\cdot\ln(3)\cdot 2x \tag{61}$$

Exercício V

$$y=5^{2x^2+3x-1}$$
$$y'=5^{2x^2+3x-1}\cdot\ln(5)\cdot(4x+3) \tag{62}$$

Exercício VI

$$y=\left(\frac{1}{2}\right)^{\sqrt{x}}=\left(\frac{1}{2}\right)^{x^{\frac{1}{2}}}$$
$$y'=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2}x^{\frac{1}{2}-1}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2}x^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2x^{\frac{1}{2}}}=\left(\frac{1}{2}\right)^{\sqrt{x}}\cdot\ln\left(\frac{1}{2}\right)\cdot\frac{1}{2\sqrt{x}} \tag{63}$$

Exercício VII

$$f=e^{\frac{x+1}{x-1}}\rightarrow g=\frac{x+1}{x-1}\rightarrow f=e^{g}$$
$$g'=\frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}=\frac{x-1-x-1}{(x-1)^2}=\frac{-2}{(x-1)^2} \tag{64}$$
$$f'=e^{g}\cdot g'\rightarrow f'=e^{\frac{x+1}{x-1}}\cdot\frac{-2}{(x-1)^2}=\frac{-2e^{\frac{x+1}{x-1}}}{(x-1)^2}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – Aula 21

Exercício I

$$y=\ln(5x^2-4x)$$
$$y'=\frac{10x-4}{5x^2-4x}=\frac{2(5x-2)}{x(5x-4)} \tag{65}$$

Exercício II

$$y=\log_2(3x^2-7)$$
$$y'=\frac{6x}{3x^2-7}\cdot\log_2(e) \tag{66}$$

Exercício III

$$f=\log_{10}\left(\frac{x+1}{x^2+1}\right) \to g=\frac{x+1}{x^2+1} \to f=\log_{10}(g)$$

$$g'=\frac{1(x^2+1)-(x+1)2x}{(x^2+1)^2}=\frac{x^2+1-2x^2-2x}{(x^2+1)^2}=\frac{-x^2-2x+1}{(x^2+1)^2}=\frac{-(x^2+2x-1)}{(x^2+1)^2}$$

$$f'=\frac{g'}{g}\cdot\log_{10}(e) \to f'=\frac{\frac{-(x^2+2x-1)}{(x^2+1)^2}}{\frac{x+1}{x^2+1}}\cdot\log_{10}(e)=\frac{-(x^2+2x-1)(x^2+1)}{(x^2+1)^2(x+1)}\cdot\log_{10}(e)=$$

$$\frac{-(x^2+2x-1)}{(x^2+1)(x+1)}\cdot\log_{10}(e)=\frac{-x^2-2x+1}{x^3+x^2+x+1}\cdot\log_{10}(e) \tag{67}$$

$$f=\log_{10}\left(\frac{x+1}{x^2+1}\right)=\log_{10}(x+1)-\log_{10}(x^2+1)$$

$$f'=\frac{1}{x+1}\cdot\log_{10}(e)-\frac{2x}{x^2+1}\cdot\log_{10}(e)=\log_{10}(e)\left(\frac{1}{x+1}-\frac{2x}{x^2+1}\right)=$$

$$\log_{10}(e)\left(\frac{1(x^2+1)-2x(x+1)}{x^3+x+x^2+1}\right)=\log_{10}(e)\left(\frac{x^2+1-2x^2-2x}{x^3+x^2+x+1}\right)=$$

$$\log_{10}(e)\left(\frac{-x^2-2x+1}{x^3+x^2+x+1}\right)$$

Exercício IV

$$y=\ln\left(\frac{e^x}{x+1}\right)=\ln(e^x)-\ln(x+1)=x-\ln(x+1)$$

$$y'=1-\frac{1}{x+1}=\frac{x+1-1}{x+1}=\frac{x}{x+1} \tag{68}$$

Exercício V

$$y=\ln(2x-1)^3 \to f=(2x-1)^3 \to y=\ln(f)$$

$$f'=3(2x-1)^2\cdot 2=6(2x-1)^2$$

$$y'=\frac{f'}{f}=\frac{6(2x-1)^2}{(2x-1)^3}=\frac{6}{2x-1} \tag{69}$$

$$y=\ln(2x-1)^3=3\cdot\ln(2x-1)$$

$$y'=0\cdot\ln(2x-1)+3\cdot\frac{2}{2x-1}=\frac{6}{2x-1}$$

Exercício VI

$$\begin{gathered}y=\ln[(4x^2+3)(2x-1)]=\ln(4x^2+3)+\ln(2x-1)\\ y'=\frac{8x}{4x^2+3}+\frac{2}{2x-1}=\frac{8x(2x-1)+2(4x^2+3)}{8x^3-4x^2+6x-3}=\frac{16x^2-8x+8x^2+6}{8x^3-4x^2+6x-3}=\\ \frac{24x^2-8x+6}{8x^3-4x^2+6x-3}=\frac{2(12x^2-4x+3)}{8x^3-4x^2+6x-3}\end{gathered}\tag{70}$$

Exercício VII

$$\begin{gathered}f(x)=\left(\frac{2x+1}{3x-1}\right)^4\rightarrow g(x)=\frac{2x+1}{3x-1}\rightarrow f(x)=g(x)^4\\ g'(x)=\frac{2(3x-1)-(2x+1)3}{(3x-1)^2}=\frac{6x-2-6x-3}{(3x-1)^2}=\frac{-5}{(3x-1)^2}\\ f'(x)=4g(x)^{4-1}\cdot g'(x)=4\left(\frac{2x+1}{3x-1}\right)^3\cdot\frac{-5}{(3x-1)^2}=\frac{-20}{(3x-1)^2}\cdot\left(\frac{2x+1}{3x-1}\right)^3\end{gathered}\tag{71}$$

Derivação (ou diferenciação) Logarítmica – Aula 22

Exercício I

$$\begin{gathered}y=x^x\rightarrow\ln(y)=\ln(x^x)\rightarrow\ln(y)=x\cdot\ln(x)\\ \frac{y'}{y}=1\cdot\ln(x)+x\cdot\frac{1}{x}\rightarrow\frac{y'}{y}=\ln(x)+1\rightarrow y'=y(\ln(x)+1)=x^x(\ln(x)+1)\end{gathered}\tag{72}$$

Exercício II

$$\begin{gathered}y=c^f\rightarrow\ln(y)=\ln(c^f)\rightarrow\ln(y)=f\cdot\ln(c)\\ \frac{y'}{y}=f'\cdot\ln(c)+f\cdot\frac{0}{c}\rightarrow y'=y(f'\cdot\ln(c))\rightarrow y'=c^f\cdot f'\cdot\ln(c)\\ \\ y=e^f\rightarrow\ln(y)=\ln(e^f)\rightarrow\ln(y)=f\cdot\ln(e)\rightarrow\ln(y)=f\\ \frac{y'}{y}=f'\rightarrow y'=y\cdot f'\rightarrow y'=e^f\cdot f'\end{gathered}\tag{73}$$

Derivação (ou diferenciação) Logarítmica – Aula 23

Exercício I

$$\begin{gathered}y=x^{\sqrt{x}}\rightarrow\ln(y)=\ln(x^{\sqrt{x}})=\sqrt{x}\cdot\ln(x)=x^{\frac{1}{2}}\cdot\ln(x)\\ \frac{y'}{y}=\frac{1}{2}x^{\frac{1}{2}-1}\cdot\ln(x)+x^{\frac{1}{2}}\cdot\frac{1}{x}=\frac{x^{\frac{-1}{2}}}{2}\cdot\ln(x)+x^{\frac{1}{2}-1}=\frac{x^{\frac{-1}{2}}\cdot\ln(x)}{2}+x^{\frac{-1}{2}}=x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)\rightarrow\\ y'=y\cdot x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)=x^{\sqrt{x}}\cdot x^{\frac{-1}{2}}\left(\frac{\ln(x)}{2}+1\right)=x^{\sqrt{x}-\frac{1}{x}}\left(\frac{\ln(x)}{2}+1\right)=x^{\frac{x\sqrt{x}-1}{x}}\left(\frac{\ln(x)+2}{2}\right)\end{gathered}\tag{74}$$

Derivada de função composta – Aula 24

Exercício I

$$y=\left(\frac{2x+1}{3x-1}\right)^4 \rightarrow f=\frac{2x+1}{3x-1} \rightarrow y=f^4$$

$$f'=\frac{2(3x-1)-(2x+1)3}{(3x-1)^2}=\frac{6x-2-6x-3}{(3x-1)^2}=\frac{-5}{(3x-1)^2}$$

$$y'=4f^{4-1}\cdot f'=4\left(\frac{2x+1}{3x-1}\right)^3\cdot\frac{-5}{(3x-1)^2}=\frac{-20}{(3x-1)^2}\cdot\left(\frac{2x+1}{3x-1}\right)^3=\frac{-20}{(3x-1)^2}\cdot\frac{(2x+1)^3}{(3x-1)^3}=$$

$$\frac{-20(2x+1)^3}{(3x-1)^5}$$

$$y=\left(\frac{2x+1}{3x-1}\right)^4 \rightarrow \ln(y)=\ln\left(\frac{2x+1}{3x-1}\right)^4=4\cdot\ln\left(\frac{2x+1}{3x-1}\right)=4(\ln(2x+1)-\ln(3x-1)) \qquad (75)$$

$$\frac{y'}{y}=4\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right)\rightarrow y'=4y\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right)=$$

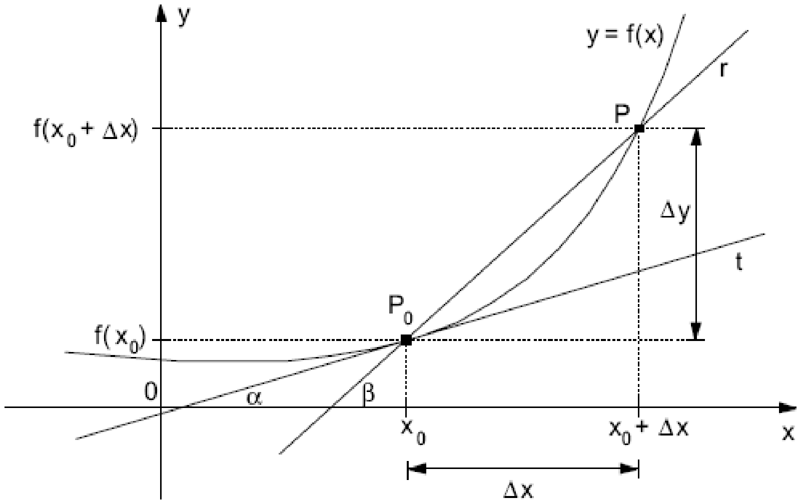
$$4\cdot\left(\frac{2x+1}{3x-1}\right)^4\cdot\left(\frac{2}{2x+1}-\frac{3}{3x-1}\right)=4\cdot\left(\frac{2x+1}{3x-1}\right)^4\cdot\frac{2(3x-1)-3(2x+1)}{(2x+1)(3x-1)}=$$

$$\frac{4(2x+1)^4}{(3x-1)^4}\cdot\frac{2(3x-1)-3(2x+1)}{(2x+1)(3x-1)}=\frac{4(2x+1)^3(2(3x-1)-3(2x+1))}{(3x-1)^5}=$$

$$\frac{4(2x+1)^3(6x-2-6x-3)}{(3x-1)^5}=\frac{4(2x+1)^3(-5)}{(3x-1)^5}=\frac{-20(2x+1)^3}{(3x-1)^5}$$

O que é uma Derivada – Aula 25

Derivada é a principal ferramenta matemática utilizada para calcular e estudar taxas de variação.



$$tg\beta=\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

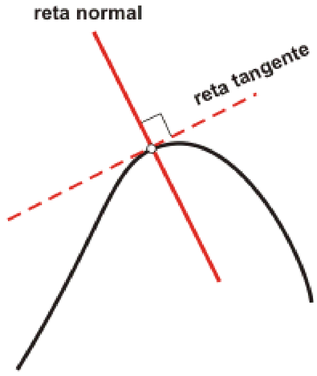
$$tg\alpha=\lim_{\Delta x\to 0}\left[\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}\right]$$

$$f(x_0)=x_0^2$$

$$tg\alpha=f'(x_0)=2x_0$$

$$tg\alpha=\lim_{\Delta x\to 0}\left[\frac{(x_0+\Delta x)^2-(x_0)^2}{\Delta x}\right]=\frac{x_0^2+2x_0\Delta x+\Delta x^2-x_0^2}{\Delta x}=\frac{\Delta x(2x_0+\Delta x)}{\Delta x}=2x_0+\Delta x=2x_0+0=2x_0$$

Derivadas: Reta Tangente e Reta Normal – Aula 26



Reta tangente: $y-y_0=f'(x_0)(x-x_0)\to y=f'(x_0)(x-x_0)+y_0=f'(x_0)(x-x_0)+f(x_0)$

Reta normal: $y-y_0=\frac{-1}{f'(x_0)}(x-x_0)=\frac{-(x-x_0)}{f'(x_0)}\to y=\frac{-(x-x_0)}{f'(x_0)}+y_0=\frac{-(x-x_0)}{f'(x_0)}+f(x_0)$

Exercício I

$$f(x_0)=\frac{1}{x_0}=x_0^{-1}$$
$$f'(x_0)=-x_0^{-2}=\frac{-1}{x_0^2}$$

$$y_t=\frac{-1}{x_0^2}(x-x_0)+\frac{1}{x_0}=\frac{-x+x_0}{x_0^2}+\frac{1}{x_0}=\frac{-x+x_0+x_0}{x_0^2}=\frac{-x+2x_0}{x_0^2}$$
$$y_n=\frac{-(x-x_0)}{\frac{-1}{x_0^2}}+\frac{1}{x_0}=x_0^2(x-x_0)+\frac{1}{x_0}=x_0^2x-x_0^3+\frac{1}{x_0}=\frac{x_0^3x-x_0^4+1}{x_0}$$

$$x_0=1$$
$$y_0=f(x_0)=\frac{1}{1}=1$$
$$tg\,\alpha=f'(x_0)=\frac{-1}{1^2}=-1 \qquad (76)$$
$$y_t=\frac{-x+2\cdot 1}{1^2}=-x+2$$
$$y_n=\frac{1^3x-1^4+1}{1}=x$$

$$\lim_{x_0\to 0}f(x_0)=\lim_{x_0\to 0}\left[\frac{1}{x_0}\right]=\nexists$$
$$\lim_{x_0\to 0^+}\left[\frac{1}{x_0}\right]=\frac{1}{0^+}=+\infty$$
$$\lim_{x_0\to 0^+}\left[\frac{1}{x_0}\right]=\frac{1}{0^-}=-\infty$$
$$\lim_{x_0\to +\infty}f(x_0)=\lim_{x_0\to +\infty}\left[\frac{1}{x_0}\right]=\frac{1}{+\infty}=0$$
$$\lim_{x_0\to -\infty}f(x_0)=\lim_{x_0\to -\infty}\left[\frac{1}{x_0}\right]=\frac{1}{-\infty}=0$$

Exercício II

$$\begin{gathered}x_0=1\\ f(x)=\sqrt{x}=x^{\frac{1}{2}}\\ f'(x)=\frac{1}{2}x^{\frac{-1}{2}}=\frac{1}{2\sqrt{x}}\\ y_0=f(x_0)=\sqrt{1}=1\\ tg\,\alpha=f'(x_0)=\frac{1}{2\sqrt{1}}=\frac{1}{2}\\ y_t-y_0=f'(x_0)(x-x_0)\to y_t-1=\frac{1}{2}(x-1)\to y_t=\frac{x-1}{2}+1=\frac{x-1+2}{2}=\frac{x+1}{2}\\ y_n-y_0=\frac{-1}{f'(x_0)}(x-x_0)\to y_n-1=\frac{-1}{\frac{1}{2}}(x-1)\to y_n=-2(x-1)+1=-2x+2+1=-2x+3\end{gathered}\tag{77}$$

Derivada pela Definição – Aula 27

Exercício I

$$\begin{gathered}f(x)=2x^2-3x+4\\ f'(x)=4x-3\\ \lim_{h\to 0}\left[\frac{(2(x+h)^2-3(x+h)+4)-(2x^2-3x+4)}{h}\right]=\\ \frac{2(x^2+2xh+h^2)-3x-3h+4-2x^2+3x-4}{h}=\\ \frac{2x^2+4xh+2h^2-3x-3h+4-2x^2+3x-4}{h}=\frac{2h^2+4xh-3h}{h}=\frac{h(2h+4x-3)}{h}=\\ 2h+4x-3=2\cdot 0+4x-3=4x-3\end{gathered}\tag{78}$$

Exercício II

$$\begin{gathered}f(x)=\frac{2}{x}=2x^{-1}\\ f'(x)=-2x^{-2}=\frac{-2}{x^2}\\ \lim_{h\to 0}\left[\frac{\left(\frac{2}{x+h}\right)-\left(\frac{2}{x}\right)}{h}\right]=\frac{\frac{2x-2(x+h)}{x(x+h)}}{h}=\frac{\frac{2x-2x-2h}{x(x+h)}}{h}=\frac{-2h}{xh(x+h)}=\frac{-2}{x(x+h)}=\\ \frac{-2}{x(x+0)}=\frac{-2}{x^2}\end{gathered}\tag{79}$$