

## Introdução as Derivadas – Aula 1

$$\frac{\partial f(x)}{\partial x} = D_x f(x) = f'(x)$$

- 1)  $f(x) = c \rightarrow f'(x) = 0$
- 2)  $f(x) = x \rightarrow f'(x) = 1$
- 3)  $f(x) = x^p \rightarrow f'(x) = p \cdot x^{p-1}$
- 4)  $f(x) = e^x \rightarrow f'(x) = e^x$
- 5)  $f(x) = \ln(x) \rightarrow f'(x) = \frac{1}{x}$
- 6)  $f(x) = g(x) \cdot h(x) \rightarrow f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$
- 7)  $f(x) = \frac{g(x)}{h(x)} \rightarrow f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$
- 8)  $f(x) = g(x)^p \rightarrow f'(x) = p \cdot g(x)^{p-1} \cdot g'(x)$
- 9)  $f(x) = e^{g(x)} \rightarrow f'(x) = e^{g(x)} \cdot g'(x)$
- 10)  $f(x) = c^{g(x)} \rightarrow f'(x) = c^{g(x)} \cdot g'(x) \cdot \ln(c)$
- 11)  $f(x) = \ln(g(x)) \rightarrow f'(x) = \frac{g'(x)}{g(x)}$
- 12)  $f(x) = \log_c(g(x)) \rightarrow f'(x) = \frac{g'(x)}{g(x)} \cdot \log_c(e)$

### Exercício I

- a)  $y = 8 \rightarrow y' = 0$
  - b)  $y = \sqrt{3} \rightarrow y' = 0$
  - c)  $f(x) = \pi \rightarrow f'(x) = 0$
  - d)  $g(x) = (\pi - 1)^\pi \rightarrow g'(x) = 0$
- (1)

### Exercício II

- a)  $y = x^5 \rightarrow y' = 5x^{5-1} = 5x^4$
  - b)  $h(x) = x^{-5} \rightarrow h'(x) = -5x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = -\frac{5}{x^6}$
  - c)  $g(x) = 5x^3 \rightarrow g'(x) = 5 \cdot 3x^{3-1} = 15x^2$
- (2)

### Exercício III

$$h(x) = 8x \rightarrow h'(x) = 8 \cdot 1 = 8$$

(3)

### Exercício IV

$$\begin{aligned} f(x) &= 7x^3 - 2x - 400 \\ f'(x) &= 7 \cdot 3x^{3-1} - 2 \cdot 1 - 0 = 21x^2 - 2 \end{aligned}$$

(4)

## Derivada com X no Denominador – [Aula 2](#)

### Exercício I

$$\begin{aligned}g(x) &= \frac{3}{x^5} = 3x^{-5} \\g'(x) &= 3(-5x^{-5-1}) = -15x^{-6} = -\frac{15}{x^6}\end{aligned}\tag{5}$$

### Exercício II

$$\begin{aligned}h(x) &= 3x^5 - \frac{2}{x^4} = 3x^5 - 2x^{-4} \\h'(x) &= 3(5x^{5-1}) - 2(-4x^{-4-1}) = 15x^4 + 8x^{-5} = 15x^4 + \frac{8}{x^5}\end{aligned}\tag{6}$$

## Derivada de Função Raiz – [Aula 3](#)

### Exercício I

$$\begin{aligned}y &= \sqrt[3]{x^4} = x^{\frac{4}{3}} \\y' &= \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4}{3}\sqrt[3]{x}\end{aligned}\tag{7}$$

### Exercício II

$$\begin{aligned}g(x) &= 7\sqrt[3]{x} = 7x^{\frac{1}{3}} \\g'(x) &= 7\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) = \frac{7}{3}x^{-\frac{2}{3}} = \frac{7}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = \frac{7}{3\sqrt[3]{x^2}}\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}}\right) = \frac{7\sqrt[3]{x}}{3x}\end{aligned}\tag{8}$$

## Derivada de uma Função Potência – [Aula 4](#)

### Exercício I

$$y = x^3 \rightarrow y' = 3x^2\tag{9}$$

### Exercício II

$$\begin{aligned}y &= \frac{5x^4x^3}{x^2} = 5x^{4+3-2} = 5x^5 \\y' &= 5(5x^{5-1}) = 25x^4\end{aligned}\tag{10}$$

## Derivada de uma Função Potência – [Aula 5](#)

### Exercício I

$$y = \frac{x^2 \sqrt[3]{x}}{\sqrt[3]{x}} = x^2 \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{2+\frac{1}{2}-\frac{1}{3}} = x^{\frac{12+3-2}{6}} = x^{\frac{13}{6}}$$
$$y' = \frac{13}{6} x^{\frac{13}{6}-1} = \frac{13}{6} x^{\frac{7}{6}} = \frac{13}{6} \sqrt[6]{x^7}$$
(11)

## Derivada de Função Exponencial e Logarítmica – [Aula 6](#)

### Exercício I

$$f(x) = 3e^x + 10 \cdot \ln(x)$$
$$f'(x) = 3e^x + \frac{10}{x} = \frac{3xe^x + 10}{x}$$
(12)

### Exercício II

$$g(x) = 7e^x + 9 \cdot \ln(x) + 3x^4 - 4x + 100$$
$$g'(x) = 7e^x + \frac{9}{x} + 12x^3 - 4 = \frac{7xe^x + 9 + 12x^4 - 4x}{x}$$
(13)

## Derivada de um Produto de Funções – [Aula 7](#)

### Exercício I

$$f(x) = x \cdot \ln(x)$$
$$\begin{cases} g(x) = x & \rightarrow g'(x) = 1 \\ h(x) = \ln(x) & \rightarrow h'(x) = \frac{1}{x} \end{cases}$$
(14)
$$f(x) = g(x) \cdot h(x) \rightarrow f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$$

## Derivada de uma Divisão de Funções – [Aula 8](#)

### Exercício I

$$f(x) = \frac{e^x}{3x} = e^x \cdot \frac{1}{3x} = e^x \cdot x^{-1}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = \frac{x^{-1}}{3} = \frac{1}{3x} & \rightarrow h'(x) = \frac{-x^{-2}}{3} = -\frac{1}{3x^2} \end{cases}$$

$$f'(x) = g(x) \cdot h'(x) + g'(x) \cdot h(x) = e^x \cdot \frac{1}{3x} + e^x \cdot \left(-\frac{1}{3x^2}\right) = \frac{e^x}{3x} - \frac{e^x}{3x^2} = \frac{x \cdot e^x - e^x}{3x^2} = \frac{e^x(x-1)}{3x^2}$$

(15)

$$f(x) = \frac{e^x}{3x}$$

$$\begin{cases} g(x) = e^x & \rightarrow g'(x) = e^x \\ h(x) = 3x & \rightarrow h'(x) = 3 \end{cases}$$

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2} = \frac{e^x \cdot 3x - e^x \cdot 3}{(3x)^2} = \frac{3e^x(x-1)}{9x^2} = \frac{e^x(x-1)}{3x^2}$$

## Derivadas Básicas – [Aula 9](#)

### Exercício I

$$y = \frac{2x^2 + 5x}{x} = \frac{2x^2}{x} + \frac{5x}{x} = 2x + 5$$

$$y' = 2$$

(16)

### Exercício II

$$h(x) = \frac{3(x^2 - 1)}{x} = \frac{3x^2 - 3}{x} = \frac{3x^2}{x} - \frac{3}{x} = 3x - 3x^{-1}$$

$$h'(x) = 3 - (-3x^{-2}) = 3 + \frac{3}{x^2} = \frac{3x^2 + 3}{x^2} = \frac{3(x^2 + 1)}{x^2}$$

(17)

## Exercícios de Derivada – [Aula 10](#)

### Exercício I

$$h(x) = 3x^3(2 + 4x) = 6x^3 + 12x^4$$

$$h'(x) = 18x^2 + 48x^3 = 6x^2(8x + 3)$$

(18)

### Exercício II

$$\begin{aligned}g(x) &= (x^2 - 1)(x^3 + 4) = x^5 + 4x^2 - x^3 - 4 \\g'(x) &= 5x^4 + 8x - 3x^2 = x(5x^3 - 3x + 8)\end{aligned}\quad (19)$$

### Curso básico de derivadas – [Aula 12](#)

#### Exercício I

$$\begin{aligned}g(x) &= x^4 + 2e^x + e^2 \\g'(x) &= 4x^3 + 2e^x = 2(2x^3 + e^x)\end{aligned}\quad (20)$$

#### Exercício II

$$\begin{aligned}g(x) &= \sqrt[3]{x^7} + \frac{3}{x^2} + 5 = x^{\frac{7}{3}} + 3x^{-2} + 5 \\g'(x) &= \frac{7x^{\frac{4}{3}}}{3} + (-6x^{-3}) = \frac{7\sqrt[3]{x^4}}{3} - \frac{6}{x^3} = \frac{7x^3\sqrt[3]{x^4} - 18}{3x^3} = \frac{7\sqrt[3]{x^{13}} - 18}{3x^3}\end{aligned}\quad (21)$$

### Derivada de um Produto de Funções – [Aula 13](#)

#### Exercício I

$$\begin{aligned}y &= 8x \cdot \ln(x) \\y' &= 8 \cdot \ln(x) + 8x \cdot \frac{1}{x} = 8 \cdot \ln(x) + 8 = 8(\ln(x) + 1)\end{aligned}\quad (22)$$

### Derivada de função composta, raiz, polinomial – [Aula 14](#)

#### Exercício I

$$y = x^3 \rightarrow y' = 3x^2 \quad (23)$$

#### Exercício II

$$\begin{aligned}f(x) &= (2x^2 - 1)^3 \\g(x) &= 2x^2 - 1 \rightarrow g'(x) = 4x \\f'(x) &= g(x)^p \rightarrow f'(x) = p \cdot g(x)^{p-1} \cdot g'(x) = 3(2x^2 - 1)^2 \cdot 4x = 12x(2x^2 - 1)^2\end{aligned}\quad (24)$$

#### Exercício III

$$\begin{aligned}y &= (3 - x^2)^3 \\y' &= 3(3 - x^2)^2(-2x) = -6x(3 - x^2)^2\end{aligned}\quad (25)$$

#### Exercício IV

$$y = \frac{3}{(2x^2 - 1)^4} = 3(2x^2 - 1)^{-4}$$

$$y' = 3(-4)(2x^2 - 1)^{-5} \cdot 4x = -48x(2x^2 - 1)^{-5} = -\frac{48x}{(2x^2 - 1)^5} \quad (26)$$

#### Exercício V

$$y = \sqrt{(2x^2 - 1)^3} = (2x^2 - 1)^{\frac{3}{2}}$$

$$y' = \frac{3}{2}(2x^2 - 1)^{\frac{1}{2}} \cdot 4x = 6x(2x^2 - 1)^{\frac{1}{2}} = 6x\sqrt{2x^2 - 1} \quad (27)$$

### Derivada de funções quociente, produto, polinomial – [Aula 15](#)

#### Exercício I

$$y = 7x^4 - 2x^3 + 8x + 5$$

$$y' = 28x^3 - 6x^2 + 8 = 2(14x^3 - 3x^2 + 4) \quad (28)$$

#### Exercício II

$$y = (2x^3 - 4x^2)(3x^5 + x^2) = 6x^8 + 2x^5 - 12x^7 - 4x^4 = 6x^8 - 12x^7 + 2x^5 - 4x^4$$

$$y' = 48x^7 - 84x^6 + 10x^4 - 16x^3 = 2x^3(24x^4 - 42x^3 + 5x - 8) \quad (29)$$

#### Exercício III

$$h(x) = \frac{2x^3 + 4}{x^2 - 4x + 1}$$

$$\begin{cases} f(x) = 2x^3 + 4 & \rightarrow f'(x) = 6x^2 \\ g(x) = x^2 - 4x + 1 & \rightarrow g'(x) = 2x - 4 \end{cases}$$

$$h'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2} = \frac{6x^2(x^2 - 4x + 1) - (2x^3 + 4)(2x - 4)}{(x^2 - 4x + 1)^2} =$$

$$\frac{6x^4 - 24x^3 + 6x^2 - (4x^4 - 8x^3 + 8x - 16)}{(x^2 - 4x + 1)^2} = \frac{6x^4 - 24x^3 + 6x^2 - 4x^4 + 8x^3 - 8x + 16}{(x^2 - 4x + 1)^2} =$$

$$\frac{2x^4 - 16x^3 + 6x^2 - 8x + 16}{(x^2 - 4x + 1)^2} = \frac{2(x^4 - 8x^3 + 3x^2 - 4x + 8)}{(x^2 - 4x + 1)^2} \quad (30)$$

#### Exercício IV

$$y = \frac{3}{x^5} = 3x^{-5} \rightarrow y' = -15x^{-6} = -\frac{15}{x^6} \quad (31)$$

Exercício V

$$v(r) = \frac{4}{3} \pi r^3 \rightarrow v'(r) = 4 \pi r^2 \quad (32)$$

Exercício VI

$$\begin{aligned} f(s) &= \sqrt{3}(s^3 - s^2) = \sqrt{3}s^3 - \sqrt{3}s^2 \\ f'(s) &= 3\sqrt{3}s^2 - 2\sqrt{3}s = s\sqrt{3}(3s - 2) \end{aligned} \quad (33)$$

Exercício VII

$$\begin{aligned} y &= (4x^2 + 3)^2 = 16x^4 + 12x^2 + 12x^2 + 9 = 16x^4 + 24x^2 + 9 \\ y' &= 64x^3 + 48x = 16x(4x^2 + 3) \end{aligned} \quad (34)$$

Derivadas de função quociente e produto – [Aula 16](#)

Exercício I

$$\begin{aligned} y &= \frac{x^4 - 2x^2 + 5x + 1}{x^4} = \frac{x^4}{x^4} - \frac{2x^2}{x^4} + \frac{5x}{x^4} + \frac{1}{x^4} = 1 - 2x^{-2} + 5x^{-3} + x^{-4} \\ y' &= 4x^{-3} - 15x^{-4} - 4x^{-5} = \frac{4}{x^3} - \frac{15}{x^4} - \frac{4}{x^5} = \frac{4x^2 - 15x - 4}{x^5} \end{aligned} \quad (35)$$

Exercício II

$$\begin{aligned} y &= \frac{x}{x-1} \\ y' &= \frac{1(x-1) - x \cdot 1}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2} \end{aligned} \quad (36)$$

Exercício III

$$\begin{aligned} y &= \left( \frac{2x+1}{x+5} \right) (3x-1) = \frac{6x^2 - 2x + 3x - 1}{x+5} = \frac{6x^2 + x - 1}{x+5} \\ y' &= \frac{(12x+1)(x+5) - (6x^2 + x - 1) \cdot 1}{(x+5)^2} = \frac{12x^2 + 60x + x + 5 - 6x^2 - x + 1}{(x+5)^2} = \\ &= \frac{6x^2 + 60x + 6}{(x+5)^2} = \frac{6(x^2 + 10x + 1)}{(x+5)^2} \end{aligned} \quad (37)$$

Exercício IV

$$y = \frac{1}{8} x^8 - 4x^4 \rightarrow y' = x^7 - 16x^3 = x^3(x^4 - 16) \quad (38)$$

Exercício V

$$y = x^2 + 3x + \frac{1}{x^2} = x^2 + 3x + x^{-2}$$

$$y' = 2x + 3 - 2x^{-3} = 2x + 3 + \frac{2}{x^3} = \frac{2x^4 + 3x^3 + 2}{x^3} \quad (39)$$

Exercício VI

$$y = \frac{3}{x^2} + \frac{5}{x^4} = 3x^{-2} + 5x^{-4} \rightarrow y' = -6x^{-3} - 20x^{-5} = -\frac{6}{x^3} - \frac{20}{x^5} = \frac{-6x^2 - 20}{x^5} \quad (40)$$

Derivada de funções envolvendo raiz, quociente, produto – [Aula 17](#)

Exercício I

$$g(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{\frac{1}{2}}} = 3x^{-\frac{1}{2}}$$

$$g'(x) = 3 \left( -\frac{1}{2} \right) x^{-\frac{3}{2}} = \frac{-3}{2x^{\frac{3}{2}}} = -\frac{3}{2\sqrt{x^3}} \quad (41)$$

Exercício II

$$g(x) = 5\sqrt[3]{x^2} = 5x^{\frac{2}{3}}$$

$$g'(x) = 5 \cdot \frac{2}{3} x^{-\frac{1}{3}} = \frac{10}{3x^{\frac{1}{3}}} = \frac{10}{3\sqrt[3]{x}} \quad (42)$$

Exercício III

$$h(r) = 6\sqrt[4]{r^3} + \frac{7}{r^3} = 6r^{\frac{3}{4}} + 7r^{-3}$$

$$h'(r) = 6 \cdot \frac{3}{4} r^{-\frac{1}{4}} + 7(-3)r^{-4} = \frac{9}{2} r^{-\frac{1}{4}} - 21r^{-4} = \frac{9}{2r^{\frac{1}{4}}} - \frac{21}{r^4} = \frac{9}{2\sqrt[4]{r}} - \frac{21}{r^4} =$$

$$\frac{9r^4 - 42\sqrt[4]{r}}{2r} = \frac{3}{2r} (3r^4 - 14\sqrt[4]{r}) \quad (43)$$



### Exercício IV

$$\begin{aligned}
 g(t) &= \frac{5t^3}{6\sqrt{t}} = 5t^3 \cdot \frac{1}{6t^{\frac{1}{2}}} = 5t^3 \cdot \frac{t^{-\frac{1}{2}}}{6} \\
 g'(t) &= 5 \cdot 3t^{3-1} \cdot \frac{1}{6\sqrt{t}} + 5t^3 \left( \frac{1}{6} \left( -\frac{1}{2} \right) t^{-\frac{1}{2}-1} \right) = \frac{15t^2}{6\sqrt{t}} + 5t^3 \left( -\frac{t^{-\frac{3}{2}}}{12} \right) = \frac{5t^2}{2\sqrt{t}} - \frac{5}{12} t^{3-\frac{3}{2}} = \\
 \frac{5}{2} t^{2-\frac{1}{2}} - \frac{5}{12} t^{\frac{3}{2}} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{30\sqrt{t^3} - 5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} \\
 g'(t) &= \frac{5 \cdot 3t^{3-1} \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 6 \left( \frac{1}{2} \right) t^{\frac{1}{2}-1}}{\left( 6t^{\frac{1}{2}} \right)^2} = \frac{15t^2 \cdot 6t^{\frac{1}{2}} - 5t^3 \cdot 3t^{-\frac{1}{2}}}{36t} = \\
 \frac{90t^{2+\frac{1}{2}} - 15t^{3-\frac{1}{2}}}{36t} &= \frac{15 \left( 6t^{\frac{5}{2}} - t^{\frac{5}{2}} \right)}{36t} = \frac{5\sqrt{t^5}(6-1)}{12t} = \frac{30\sqrt{t^5}}{12t} - \frac{5\sqrt{t^5}}{12t} = \\
 \frac{5}{2} t^{\frac{5}{2}-1} - \frac{5}{12} t^{\frac{5}{2}-1} &= \frac{5}{2} t^{\frac{3}{2}} - \frac{5}{12} t^{\frac{3}{2}} = \frac{5\sqrt{t^3}}{2} - \frac{5\sqrt{t^3}}{12} = \frac{5\sqrt{t^3}(6-1)}{12} = \frac{25\sqrt{t^3}}{12} \\
 g(t) &= \frac{5t^3}{6\sqrt{t}} = \frac{5t^3}{6t^{\frac{1}{2}}} = \frac{5}{6} t^{3-\frac{1}{2}} = \frac{5}{6} t^{\frac{5}{2}} \\
 g'(t) &= \frac{5}{6} \cdot \frac{5}{2} t^{\frac{5}{2}-1} = \frac{25t^{\frac{3}{2}}}{12} = \frac{25\sqrt{t^3}}{12}
 \end{aligned}
 \tag{44}$$

### Exercício V

$$\begin{aligned}
 h(x) &= 7x\sqrt[3]{x^2} = 7x \cdot x^{\frac{2}{3}} = 7x^{1+\frac{2}{3}} = 7x^{\frac{5}{3}} \\
 h'(x) &= 7 \cdot \frac{5}{3} x^{\frac{5}{3}-1} = \frac{35x^{\frac{2}{3}}}{3} = \frac{35\sqrt[3]{x^2}}{3}
 \end{aligned}
 \tag{45}$$

### Exercício VI

$$\begin{aligned}
 g(t) &= \frac{8t^3\sqrt{t}}{t} = \frac{8t^3t^{\frac{1}{2}}}{t} = 8t^{3+\frac{1}{2}-1} = 8t^{\frac{5}{2}} \\
 g'(t) &= 8 \cdot \frac{5}{2} t^{\frac{5}{2}-1} = 20t^{\frac{3}{2}} = 20\sqrt{t^3}
 \end{aligned}
 \tag{46}$$

## Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 18](#)

### Exercício I

$$f(x) = \frac{3x^4 - 8x^2 + 4}{x^2} = \frac{3x^4}{x^2} - \frac{8x^2}{x^2} + \frac{4}{x^2} = 3x^2 - 8 + 4x^{-2}$$

$$f'(x) = 6x - 8x^{-3} = 6x - \frac{8}{x^3} = \frac{6x^4 - 8}{x^3} \quad (47)$$

### Exercício II

$$f(x) = \frac{\pi x^3}{\sqrt{x^3}} = \frac{\pi x^3}{x^{\frac{3}{2}}} = \pi x^{3 - \frac{3}{2}} = \pi x^{\frac{3}{2}}$$

$$f'(x) = \pi \cdot \frac{3}{2} x^{\frac{3}{2} - 1} = \frac{3\pi x^{\frac{1}{2}}}{2} = \frac{3\pi \sqrt{x}}{2} \quad (48)$$

### Exercício III

$$y = \frac{x^2}{e^x}$$

$$y' = \frac{2x \cdot e^x - x^2 \cdot e^x}{(e^x)^2} = \frac{e^x \cdot x(2 - x)}{e^{2x}} = \frac{x(2 - x)}{e^x} = \frac{x}{e^x} (2 - x) \quad (49)$$

### Exercício IV

$$y = x^5 \cdot e^x$$

$$y' = 5x^4 \cdot e^x + x^5 \cdot e^x = e^x \cdot x^4 (5 + x) \quad (50)$$

### Exercício V

$$y = \sqrt{x} \cdot \ln(x) = x^{\frac{1}{2}} \cdot \ln(x)$$

$$y' = \left( \frac{1}{2} x^{\frac{1}{2} - 1} \right) \ln(x) + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{x^{-\frac{1}{2}} \cdot \ln(x)}{2} + x^{\frac{1}{2}} \cdot x^{-1} = \frac{\ln(x)}{2x^{\frac{1}{2}}} + x^{\frac{1}{2} - 1} = \frac{\ln(x)}{2\sqrt{x}} + x^{-\frac{1}{2}} = \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{x^{\frac{1}{2}}} =$$

$$\frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}} = \left( \frac{1}{\sqrt{x}} \left( \frac{\ln(x)}{2} + 1 \right) \right) \left( \frac{\sqrt{x}}{\sqrt{x}} \right) = \frac{\sqrt{x}}{x} \left( \frac{\ln(x)}{2} + 1 \right) \quad (51)$$

## Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 19](#)

### Exercício I

$$f(t) = e^t \cdot \ln(t)$$

$$f'(t) = e^t \cdot \ln(t) + e^t \cdot \frac{1}{t} = e^t \left( \ln(t) + \frac{1}{t} \right) = \frac{e^t(t \cdot \ln(t) + 1)}{t} = \frac{e^t}{t} (t \cdot \ln(t) + 1) \quad (52)$$

### Exercício II

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} =$$

$$\frac{2r \cdot e^r - e^r}{2\sqrt{r} \cdot r} = \frac{e^r(2r-1)}{2r^{\frac{1}{2}+1}} = \frac{e^r(2r-1)}{2r^{\frac{3}{2}}} = \left( \frac{e^r(2r-1)}{2\sqrt{r^3}} \right) \left( \frac{\sqrt{r^3}}{\sqrt{r^3}} \right) = \frac{e^r \sqrt{r^3}(2r-1)}{2r^3} = \frac{e^r \sqrt{r^3}}{2r^3} (2r-1) \quad (53)$$

$$f(r) = \frac{e^r}{\sqrt{r}} = \frac{e^r}{r^{\frac{1}{2}}}$$

$$f'(r) = \frac{e^r \sqrt{r} - e^r \cdot \frac{1}{2} r^{\frac{1}{2}-1}}{(\sqrt{r})^2} = \frac{e^r \sqrt{r} - e^r \cdot \frac{r^{-\frac{1}{2}}}{2}}{r} = \frac{e^r \sqrt{r} - \frac{e^r}{2r^{\frac{1}{2}}}}{r} = \frac{e^r \sqrt{r} - \frac{e_r}{2\sqrt{r}}}{r} = \frac{e^r \left( \sqrt{r} - \frac{1}{2\sqrt{r}} \right)}{r} =$$

$$\frac{e^r}{r} \left( \sqrt{r} - \frac{1}{2\sqrt{r}} \right) = \frac{e^r}{r} \left( \frac{2r-1}{2\sqrt{r}} \right) \left( \frac{\sqrt{r}}{\sqrt{r}} \right) = \frac{e^r \sqrt{r}}{2r^2} (2r-1)$$

### Exercício III

$$v(t) = \frac{t^3 \cdot e^t}{t} = t^{3-1} \cdot e^t = t^2 \cdot e^t$$

$$v'(t) = 2t \cdot e^t + t^2 \cdot e^t = e^t \cdot t(2+t) \quad (54)$$

### Exercício IV

$$y = x^5 \rightarrow y' = 5x^4$$

$$y = (2x^2 - 4)^5 \rightarrow y' = 5(2x^2 - 4)^4 \cdot 4x = 20x(2x^2 - 4)^4 \quad (55)$$

### Exercício V

$$y = \sqrt{2x^3 - 1} = (2x^3 - 1)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(2x^3 - 1)^{-\frac{1}{2}} \cdot 6x^2 = 3x^2(2x^3 - 1)^{-\frac{1}{2}} = \frac{3x^2}{(2x^3 - 1)^{\frac{1}{2}}} = \left( \frac{3x^2}{\sqrt{2x^3 - 1}} \right) \left( \frac{\sqrt{2x^3 - 1}}{\sqrt{2x^3 - 1}} \right) =$$

$$\frac{3x^2 \sqrt{2x^3 - 1}}{2x^3 - 1} \quad (56)$$

### Exercício VI

$$y = \frac{5}{(3x^2 + 9)^4} = 5(3x^2 + 9)^{-4}$$

$$y' = -20(3x^2 + 9)^{-5} \cdot 6x = -120x(3x^2 + 9)^{-5} = -\frac{120x}{(3x^2 + 9)^5} \quad (57)$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 20](#)

### Exercício I

$$y = \sqrt[3]{6x^2 + 7x + 2} = (6x^2 + 7x + 2)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}(6x^2 + 7x + 2)^{-\frac{2}{3}} \cdot (12x + 7) = \frac{12x + 7}{3(6x^2 + 7x + 2)^{\frac{2}{3}}} = \left( \frac{12x + 7}{3\sqrt[3]{(6x^2 + 7x + 2)^2}} \right) \left( \frac{\sqrt[3]{6x^2 + 7x + 2}}{\sqrt[3]{6x^2 + 7x + 2}} \right)$$

$$\frac{(12x + 7)\sqrt[3]{6x^2 + 7x + 2}}{3(6x^2 + 7x + 2)} \quad (58)$$

### Exercício II

$$y = e^{5x^2 + 4}$$

$$y' = e^{5x^2 + 4} \cdot 10x \quad (59)$$

### Exercício III

$$y = e^{\frac{1}{x^2}} = e^{x^{-2}}$$

$$y' = e^{\frac{1}{x^2}} (-2x^{-3}) = e^{\frac{1}{x^2}} \left( -\frac{2}{x^3} \right) = -\frac{2e^{\frac{1}{x^2}}}{x^3} \quad (60)$$

### Exercício IV

$$y = 3^{x^2}$$

$$y' = 3^{x^2} \cdot \ln(3) \cdot 2x \quad (61)$$

### Exercício V

$$y = 5^{2x^2+3x-1}$$

$$y' = 5^{2x^2+3x-1} \cdot \ln(5) \cdot (4x+3) \quad (62)$$

### Exercício VI

$$y = \left(\frac{1}{2}\right)^{\sqrt{x}} = \left(\frac{1}{2}\right)^{x^{\frac{1}{2}}}$$

$$y' = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{\frac{1}{2}-1} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2} x^{-\frac{1}{2}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2x^{\frac{1}{2}}} = \left(\frac{1}{2}\right)^{\sqrt{x}} \cdot \ln\left(\frac{1}{2}\right) \cdot \frac{1}{2\sqrt{x}} \quad (63)$$

### Exercício VII

$$f = e^{\frac{x+1}{x-1}} \rightarrow g = \frac{x+1}{x-1} \rightarrow f = e^g$$

$$g' = \frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2} \quad (64)$$

$$f' = e^g \cdot g' \rightarrow f' = e^{\frac{x+1}{x-1}} \cdot \frac{-2}{(x-1)^2} = \frac{-2e^{\frac{x+1}{x-1}}}{(x-1)^2}$$

Derivadas envolvendo funções quociente, produto, raiz, logaritmo, exponencial – [Aula 21](#)

### Exercício I

$$y = \ln(5x^2 - 4x)$$

$$y' = \frac{10x-4}{5x^2-4x} = \frac{2(5x-2)}{x(5x-4)} \quad (65)$$

### Exercício II

$$y = \log_2(3x^2 - 7)$$

$$y' = \frac{6x}{3x^2-7} \cdot \log_2(e) \quad (66)$$

### Exercício III

$$\begin{aligned}
 f &= \log_{10} \left( \frac{x+1}{x^2+1} \right) \rightarrow g = \frac{x+1}{x^2+1} \rightarrow f = \log_{10}(g) \\
 g' &= \frac{1(x^2+1) - (x+1)2x}{(x^2+1)^2} = \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2} = \frac{-(x^2+2x-1)}{(x^2+1)^2} \\
 f' &= \frac{g'}{g} \cdot \log_{10}(e) \rightarrow f' = \frac{\frac{-(x^2+2x-1)}{(x^2+1)^2}}{\frac{x+1}{x^2+1}} \cdot \log_{10}(e) = \frac{-(x^2+2x-1)(x^2+1)}{(x^2+1)^2(x+1)} \cdot \log_{10}(e) = \\
 &= \frac{-(x^2+2x-1)}{(x^2+1)(x+1)} \cdot \log_{10}(e) = \frac{-x^2-2x+1}{x^3+x^2+x+1} \cdot \log_{10}(e)
 \end{aligned} \tag{67}$$

$$\begin{aligned}
 f &= \log_{10} \left( \frac{x+1}{x^2+1} \right) = \log_{10}(x+1) - \log_{10}(x^2+1) \\
 f' &= \frac{1}{x+1} \cdot \log_{10}(e) - \frac{2x}{x^2+1} \cdot \log_{10}(e) = \log_{10}(e) \left( \frac{1}{x+1} - \frac{2x}{x^2+1} \right) = \\
 &= \log_{10}(e) \left( \frac{1(x^2+1) - 2x(x+1)}{x^3+x+x^2+1} \right) = \log_{10}(e) \left( \frac{x^2+1-2x^2-2x}{x^3+x^2+x+1} \right) = \\
 &= \log_{10}(e) \left( \frac{-x^2-2x+1}{x^3+x^2+x+1} \right)
 \end{aligned}$$

### Exercício IV

$$\begin{aligned}
 y &= \ln \left( \frac{e^x}{x+1} \right) = \ln(e^x) - \ln(x+1) = x - \ln(x+1) \\
 y' &= 1 - \frac{1}{x+1} = \frac{x+1-1}{x+1} = \frac{x}{x+1}
 \end{aligned} \tag{68}$$

### Exercício V

$$\begin{aligned}
 y &= \ln(2x-1)^3 \rightarrow f = (2x-1)^3 \rightarrow y = \ln(f) \\
 f' &= 3(2x-1)^2 \cdot 2 = 6(2x-1)^2 \\
 y' &= \frac{f'}{f} = \frac{6(2x-1)^2}{(2x-1)^3} = \frac{6}{2x-1} \\
 y &= \ln(2x-1)^3 = 3 \cdot \ln(2x-1) \\
 y' &= 0 \cdot \ln(2x-1) + 3 \cdot \frac{2}{2x-1} = \frac{6}{2x-1}
 \end{aligned} \tag{69}$$

Exercício VI

$$\begin{aligned}
 y &= \ln[(4x^2+3)(2x-1)] = \ln(4x^2+3) + \ln(2x-1) \\
 y' &= \frac{8x}{4x^2+3} + \frac{2}{2x-1} = \frac{8x(2x-1) + 2(4x^2+3)}{8x^3-4x^2+6x-3} = \frac{16x^2-8x+8x^2+6}{8x^3-4x^2+6x-3} = \\
 &= \frac{24x^2-8x+6}{8x^3-4x^2+6x-3} = \frac{2(12x^2-4x+3)}{8x^3-4x^2+6x-3}
 \end{aligned} \tag{70}$$

Exercício VII

$$\begin{aligned}
 f(x) &= \left(\frac{2x+1}{3x-1}\right)^4 \rightarrow g(x) = \frac{2x+1}{3x-1} \rightarrow f(x) = g(x)^4 \\
 g'(x) &= \frac{2(3x-1) - (2x+1)3}{(3x-1)^2} = \frac{6x-2-6x-3}{(3x-1)^2} = \frac{-5}{(3x-1)^2} \\
 f'(x) &= 4g(x)^{4-1} \cdot g'(x) = 4\left(\frac{2x+1}{3x-1}\right)^3 \cdot \frac{-5}{(3x-1)^2} = \frac{-20}{(3x-1)^2} \cdot \left(\frac{2x+1}{3x-1}\right)^3
 \end{aligned} \tag{71}$$