# Introdução às Derivadas Parciais de 1ª ordem – Aula 1

Exercício I

$$f(x,y)=4\frac{x^3}{y^2}-2xy-3x-4y-7=4x^3y^{-2}-2xy-3x-4y-7$$
$$\frac{\partial f(x,y)}{\partial x}=4y^{-2}\frac{\partial(x^3)}{\partial x}-2y\frac{\partial(x)}{\partial x}-3\frac{\partial(x)}{\partial x}-0-0=4y^{-2}3x^2-2y-3=\frac{12x^2}{y^2}-2y-3 \quad (1)$$
$$\frac{\partial f(x,y)}{\partial y}=4x^3\frac{\partial(y^{-2})}{\partial y}-2x\frac{\partial(y)}{\partial y}-0-4\frac{\partial(y)}{\partial y}-0=4x^3(-2y^{-3})-2x-4=\frac{-8x^3}{y^3}-2x-4$$

# Derivadas Parciais: Interpretação Geométrica – Aula 2

Exercício I

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano x = -1, no ponto P(-1, 1, -2).

$$f(x,y)=x^2+y^2-2x^3y+5xy^4-1$$
$$z=f(-1,1)=(-1)^2+(1)^2-2(-1)^3(1)+5(-1)(1)^4-1=1+1+2-5-1=4-6=-2$$
$$\frac{\partial f(x,y)}{\partial y}=0+\frac{\partial(y^2)}{\partial y}-2x^3\frac{\partial(y)}{\partial y}+5x\frac{\partial(y^4)}{\partial y}-0=2y-2x^3+5x4y^3=2y+20xy^3-2x^3 \quad (2)$$
$$\frac{\partial f(-1,1)}{\partial y}=2(1)+20(-1)(1)^3-2(-1)^3=2-20+2=-16$$

Exercício II

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano y = 2, no ponto P(2, 2, 8).

$$f(x,y)=x^2+y^2$$
$$z=f(2,2)=(2)^2+(2)^2=4+4=8$$
$$\frac{\partial f(x,y)}{\partial x}=\frac{\partial(x^2)}{\partial x}+0=2x \quad (3)$$
$$\frac{\partial f(2,2)}{\partial x}=2(2)=4$$

# Derivadas Parciais de 2ª ordem – Aula 3

Exercício I

$$\begin{aligned}
f(x,y) &= x^2 + y^2 - 2x^3y + 5xy^4 - 1\\
\frac{\partial f(x,y)}{\partial x} &= 2x + 0 - 2y3x^2 + 5y^4 - 0 = 2x - 6x^2y + 5y^4\\
\frac{\partial^2 f(x,y)}{\partial x^2} &= 2 - 6y2x = -12xy + 2\\
\frac{\partial^2 f(x,y)}{\partial y \partial x} &= 0 - 6x^2 + 5 \cdot 4y^3 = -6x^2 + 20y^3\\
\frac{\partial f(x,y)}{\partial y} &= 0 + 2y - 2x^3 + 5x4y^3 - 0 = -2x^3 + 20xy^3 + 2y\\
\frac{\partial^2 f(x,y)}{\partial y^2} &= -0 + 20x3y^2 + 2 = 60xy^2 + 2\\
\frac{\partial^2 f(x,y)}{\partial x \partial y} &= -2 \cdot 3x^2 + 20y^3 + 0 = -6x^2 + 20y^3
\end{aligned} \tag{4}$$

Exercício II

$$\begin{aligned}
z &= x^2y - xy^2 + 2x - y\\
\frac{\partial z}{\partial x} &= y \cdot 2x - y^2 + 2 - 0 = 2xy - y^2 + 2\\
\frac{\partial^2 z}{\partial x^2} &= 2y - 0 + 0 = 2y\\
\frac{\partial^2 z}{\partial y \partial x} &= 2x - 2y + 0 = 2x - 2y\\
\frac{\partial z}{\partial y} &= x^2 - x2y + 0 - 1 = x^2 - 2xy - 1\\
\frac{\partial^2 z}{\partial y^2} &= 0 - 2x - 0 = -2x\\
\frac{\partial^2 z}{\partial x \partial y} &= 2x - 2y - 0 = 2x - 2y
\end{aligned} \tag{5}$$

Exercício III

$$z = xy$$

$$\frac{\partial z}{\partial x} = y$$
$$\frac{\partial^2 z}{\partial x^2} = 0$$
$$\frac{\partial^2 z}{\partial y \partial x} = 1 \tag{6}$$

$$\frac{\partial z}{\partial y} = x$$
$$\frac{\partial^2 z}{\partial y^2} = 0$$
$$\frac{\partial^2 z}{\partial x \partial y} = 1$$

Exercício IV

$$z = \ln(xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy} y = \frac{1}{x} = x^{-1}$$
$$\frac{\partial^2 z}{\partial x^2} = -x^{-2} = \frac{-1}{x^2}$$
$$\frac{\partial^2 z}{\partial y \partial x} = 0 \tag{7}$$

$$\frac{\partial z}{\partial y} = \frac{1}{xy} x = \frac{1}{y} = y^{-1}$$
$$\frac{\partial^2 z}{\partial y^2} = -y^{-2} = \frac{-1}{y^2}$$
$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

## Derivadas Parciais de 2ª ordem – Aula 4

Exercício I

$$z = e^{-xy^2}$$

$$\frac{\partial z}{\partial x} = e^{-xy^2}\left(-y^2\right) = -y^2 e^{-xy^2}$$

$$\frac{\partial^2 z}{\partial x^2} = -y^2 e^{-xy^2}\left(-y^2\right) = y^4 e^{-xy^2}$$

$$\frac{\partial^2 z}{\partial y \partial x} = -\left[2ye^{-xy^2} + y^2 e^{-xy^2}(-x2y)\right] = -\left(2ye^{-xy^2} - 2xy^3 e^{-xy^2}\right) = 2ye^{-xy^2}\left(xy^2 - 1\right) \quad (8)$$

$$\frac{\partial z}{\partial y} = e^{-xy^2}(-x2y) = -2xye^{-xy^2}$$

$$\frac{\partial^2 z}{\partial y^2} = -\left[2xe^{-xy^2} + 2xye^{-xy^2}(-x2y)\right] = -\left(2xe^{-xy^2} - 4x^2y^2 e^{-xy^2}\right) = 2xe^{-xy^2}\left(2xy^2 - 1\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\left[2ye^{-xy^2} + 2xye^{-xy^2}\left(-y^2\right)\right] = -\left(2ye^{-xy^2} - 2xy^3 e^{-xy^2}\right) = 2ye^{-xy^2}\left(xy^2 - 1\right)$$

## Máximos, Mínimos e Sela através do Hessiano – Aula 5

1. Ache o **x** e o **y** crítico, igualando a **0** a derivada de **z** em relação a x e a y:

   $$\frac{\partial f(x,y)}{\partial x} = \frac{\partial z}{\partial x} = 0 \rightarrow x_c$$

   $$\frac{\partial f(x,y)}{\partial y} = \frac{\partial z}{\partial y} = 0 \rightarrow y_c$$

2. Calcule o determinante de **x** e **y** crítico: $h(x_c, y_c) = \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix}$

3. $h < 0 \rightarrow$ ponto de sela

   $h > 0 \rightarrow \frac{\partial^2 z}{\partial x^2} > 0 \rightarrow$ Mínimo, $\frac{\partial^2 z}{\partial x^2} < 0 \rightarrow$ Máximo

   $h = 0 \rightarrow$ NPA = Nada podemos afirmar

Exercício I

$$z=3x^4+8x^3-18x^2+6y^2+12y-4$$

1.

$$\frac{\partial z}{\partial x}=12x^3+24x^2-36x=12x(x^2+2x-3)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 12x(x^2+2x-3)=0$$

$$12x=0\rightarrow x=\frac{0}{12}=0\rightarrow x_{c1}=0$$

$$x^2+2x-3=0\rightarrow x^2+2x-3+1-1=0\rightarrow (x^2+2x+1)-4=0\rightarrow (x+1)^2-4=0\rightarrow$$

$$(x+1)^2=4\rightarrow x+1=\pm\sqrt{4}\rightarrow x=\pm 2-1\rightarrow x_{c2}=1,x_{c3}=-3$$

$$\frac{\partial z}{\partial y}=12y+12=12(y+1)$$

$$\frac{\partial z}{\partial y}=0\rightarrow 12(y+1)=0$$

$$y+1=0\rightarrow y_c=-1$$

$P_1(-3,-1),P_2(0,-1),P_1(1,-1)$

2.

$$\frac{\partial^2 z}{\partial x^2}=36x^2+48x-36=12(3x^2+4x-3)$$

$$\frac{\partial^2 f(-3,-1)}{\partial x^2}=12(3(-3)^2+4(-3)-3)=12(27-12-3)=12\cdot 12=144$$

$$\frac{\partial^2 f(0,-1)}{\partial x^2}=12(3(0)^2+4(0)-3)=12(-3)=-36 \tag{9}$$

$$\frac{\partial^2 f(1,-1)}{\partial x^2}=12(3(1)^2+4(1)-3)=12(3+4-3)=12\cdot 4=48$$

$$\frac{\partial^2 z}{\partial y\partial x}=0$$

$$\frac{\partial^2 z}{\partial x\partial y}=0$$

$$\frac{\partial^2 z}{\partial y^2}=12$$

$$h(-3,-1)=\begin{bmatrix}144 & 0\\ 0 & 12\end{bmatrix}=(144\cdot 12)-(0\cdot 0)=1728$$

$$h(0,-1)=\begin{bmatrix}-36 & 0\\ 0 & 12\end{bmatrix}=-36\cdot 12=-432$$

$$h(1,-1)=\begin{bmatrix}48 & 0\\ 0 & 12\end{bmatrix}=48\cdot 12=576$$

3.

$$P_1(-3,-1)\rightarrow h(-3,-1)>0\rightarrow\frac{\partial^2 f(-3,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

$$P_2(0,-1)\rightarrow h(0,-1)<0\rightarrow \text{é pto de sela}$$

$$P_1(1,-1)\rightarrow h(1,-1)>0\rightarrow\frac{\partial^2 f(1,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

Exercício II

$$z=x^3+3xy+y^2-2$$

1.

$$\frac{\partial z}{\partial x}=3x^2+3y=3(x^2+y)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 3(x^2+y)=0$$

$$x^2+y=0\rightarrow y=-x^2\rightarrow 3x+2(-x^2)=0\rightarrow 3x-2x^2=0\rightarrow x(3-2x)=0$$

$$x_{c1}=0$$

$$3-2x=0\rightarrow 3=2x\rightarrow x_{c2}=\frac{3}{2}$$

$$\frac{\partial z}{\partial y}=3x+2y$$

$$\frac{\partial z}{\partial y}=0\rightarrow 3x+2y=0\rightarrow 2y=-3x\rightarrow y=\frac{-3x}{2}$$

$$y=\frac{-3(0)}{2}\rightarrow y_{c1}=0$$

$$y=\frac{-3\left(\frac{3}{2}\right)}{2}\rightarrow y_{c2}=\frac{-9}{4}$$

$$P_1(0,0),P_2\left(\frac{3}{2},\frac{-9}{4}\right)$$

2.

$$\frac{\partial^2 z}{\partial x^2}=6x \tag{10}$$

$$\frac{\partial^2 f(0,0)}{\partial x^2}=6(0)=0$$

$$\frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}=6\left(\frac{3}{2}\right)=9$$

$$\frac{\partial^2 z}{\partial y\,\partial x}=3$$

$$\frac{\partial^2 z}{\partial x\,\partial y}=3$$

$$\frac{\partial^2 z}{\partial y^2}=2$$

$$h(0,0)=\begin{bmatrix}0 & 3\\ 3 & 2\end{bmatrix}=(0\cdot 2)-(3\cdot 3)=0-9=-9$$

$$h\left(\frac{3}{2},\frac{-9}{4}\right)=\begin{bmatrix}9 & 3\\ 3 & 2\end{bmatrix}=(9\cdot 2)-(3\cdot 3)=18-9=9$$

3.

$$P_1(0,0)\rightarrow h(0,0)<0\rightarrow \text{é pto de sela}$$

$$P_2\left(\frac{3}{2},\frac{-9}{4}\right)\rightarrow h\left(\frac{3}{2},\frac{-9}{4}\right)>0\rightarrow \frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$