

Introdução aos limites – [Aula 1](#)

Exercício I

$$\lim_{x \rightarrow 1} [2x+1] = 3 \quad (1)$$

Exercício II

$$\lim_{x \rightarrow 3} \left[\frac{2x+2}{x+1} \right] = 2 \quad (2)$$

Exercício III

$$f(x) = \begin{cases} 2x+1 & x \geq 1 \\ x^2+2 & x < 1 \end{cases} \quad (3)$$
$$\lim_{x \rightarrow 1} f(x) = 3$$

Exercício IV

$$f(x) = \begin{cases} x^2+3x & x \geq 2 \\ 3x+1 & x < 2 \end{cases} \quad (4)$$
$$\lim_{x \rightarrow 2^+} f(x) = 10$$
$$\lim_{x \rightarrow 2^-} f(x) = 7$$

Indeterminação de limites – [Aula 2](#)

Exercício I

$$\lim_{x \rightarrow 0} \left[\frac{x^2+2x}{x} \right] = \frac{0^2+2 \cdot 0}{0} = \frac{0}{0} \rightarrow (x=0)$$
$$\frac{(x^2+2x) \div x}{x \div x} = x+2 \quad (5)$$
$$\lim_{x \rightarrow 0} \left[\frac{x^2+2x}{x} \right] = 0+2 = 2$$

Exercício II

$$\lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{x - 2} \right] = \frac{2^2 - 4}{2 - 2} = \frac{0}{0} \rightarrow (x = 2 \rightarrow x - 2 = 0)$$

$$\frac{(x^2 - 4) \div (x - 2)}{(x - 2) \div (x - 2)} = x + 2 \quad (6)$$

$$\lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{x - 2} \right] = 2 + 2 = 4$$

Exercício III

$$\lim_{x \rightarrow 2} \left[\frac{2x^2 - 2x - 4}{x - 2} \right] = \frac{2 \cdot 2^2 - 2 \cdot 2 - 4}{2 - 2} = \frac{8 - 4 - 4}{0} = \frac{0}{0} \rightarrow (x = 2 \rightarrow x - 2 = 0)$$

$$\frac{(2x^2 - 2x - 4) \div (x - 2)}{(x - 2) \div (x - 2)} = 2x + 2 \quad (7)$$

$$\lim_{x \rightarrow 2} \left[\frac{2x^2 - 2x - 4}{x - 2} \right] = 2 \cdot 2 + 2 = 6$$

Indeterminação de limites – [Aula 3](#)

Exercício I

$$\lim_{x \rightarrow 3} \left[\frac{x^2 - 9}{x - 3} \right] = \frac{3^2 - 9}{3 - 3} = \frac{0}{0} \rightarrow (x = 3 \rightarrow x - 3 = 0)$$

$$\frac{(x^2 - 9) \div (x - 3)}{(x - 3) \div (x - 3)} = x + 3 \quad (8)$$

$$\lim_{x \rightarrow 3} \left[\frac{x^2 - 9}{x - 3} \right] = 3 + 3 = 6$$

Exercício II

$$\lim_{x \rightarrow -2} \left[\frac{x + 2}{x^2 - 4} \right] = \frac{-2 + 2}{(-2)^2 - 4} = \frac{0}{0} \rightarrow (x = -2 \rightarrow x + 2 = 0)$$

$$\frac{(x + 2) \div (x + 2)}{(x^2 - 4) \div (x + 2)} = \frac{1}{x - 2} \quad (9)$$

$$\lim_{x \rightarrow -2} \left[\frac{x + 2}{x^2 - 4} \right] = \frac{1}{-2 - 2} = -\frac{1}{4}$$

Exercício III

$$\lim_{x \rightarrow 3} \left[\frac{2x^3 - 6x^2 + x - 3}{x - 3} \right] = \frac{54 - 54 + 3 - 3}{3 - 3} = \frac{0}{0} \rightarrow (x = 3 \rightarrow x - 3 = 0)$$

$$\frac{(2x^3 - 6x^2 + x - 3) \div (x - 3)}{(x - 3) \div (x - 3)} = 2x^2 + 1$$

$$\lim_{x \rightarrow 3} \left[\frac{2x^3 - 6x^2 + x - 3}{x - 3} \right] = 2 \cdot 3^2 + 1 = 19$$
(10)

Indeterminação de limites 0/0 – [Aula 3a](#)

Exercício I

$$\lim_{x \rightarrow 1} \left[\frac{x^2 - x}{2x^2 + 5x - 7} \right] = \frac{1^2 - 1}{2 \cdot 1^2 + 5 \cdot 1 - 7} = \frac{0}{0} \rightarrow (x = 1 \rightarrow x - 1 = 0)$$

$$\frac{(x^2 - x) \div (x - 1)}{(2x^2 + 5x - 7) \div (x - 1)} = \frac{x}{2x + 7}$$

$$\lim_{x \rightarrow 1} \left[\frac{x^2 - x}{2x^2 + 5x - 7} \right] = \frac{1}{2 \cdot 1 + 7} = \frac{1}{9}$$
(11)

Exercício II

$$\lim_{x \rightarrow 2} \left[\frac{x^3 - 8}{x^2 - 4} \right] = \frac{2^3 - 8}{2^2 - 4} = \frac{0}{0} \rightarrow (x = 2 \rightarrow x - 2 = 0)$$

$$\frac{(x^3 - 8) \div (x - 2)}{(x^2 - 4) \div (x - 2)} = \frac{x^2 + 2x + 4}{x + 2}$$

$$\lim_{x \rightarrow 2} \left[\frac{x^3 - 8}{x^2 - 4} \right] = \frac{2^2 + 2 \cdot 2 + 4}{2 + 2} = \frac{12}{4} = 3$$
(12)

Indeterminação polinomial de limites – [Aula 4](#)

Exercício I

$$\begin{aligned}\lim_{h \rightarrow 0} \left[\frac{(x+h)^3 - x^3}{h} \right] &= \frac{(x+0)^3 - x^3}{0} = \frac{x^3 - x^3}{0} = \frac{0}{0} \\ \frac{(x+h)^3 - x^3}{h} &= \frac{(x+h)^2(x+h) - x^3}{h} = \frac{(x^2 + 2xh + h^2)(x+h) - x^3}{h} = \\ \frac{x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 - x^3}{h} &= \frac{3x^2h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2 \\ \lim_{h \rightarrow 0} \left[\frac{(x+h)^3 - x^3}{h} \right] &= 3x^2 + 3x \cdot 0 + 0^2 = 3x^2\end{aligned}\tag{13}$$

Exercício II

$$\begin{aligned}\lim_{x \rightarrow -1} \left[\frac{x^3 + 1}{x^2 - 1} \right] &= \frac{(-1)^3 + 1}{(-1)^2 - 1} = \frac{-1 + 1}{1 - 1} = \frac{0}{0} \rightarrow (x = -1 \rightarrow x + 1 = 0) \\ \frac{(x^3 + 1) \div (x + 1)}{(x^2 - 1) \div (x + 1)} &= \frac{x^2 - x + 1}{x - 1} \\ \lim_{x \rightarrow -1} \left[\frac{x^3 + 1}{x^2 - 1} \right] &= \frac{(-1)^2 - (-1) + 1}{-1 - 1} = \frac{1 + 1 + 1}{-2} = -\frac{3}{2}\end{aligned}\tag{14}$$

Indeterminação polinomial de limites – [Aula 5](#)

Exercício I

$$\begin{aligned}\lim_{t \rightarrow -2} \left[\frac{t^3 + 4t^2 + 4t}{(t+2)(t-3)} \right] &= \frac{(-2)^3 + 4 \cdot (-2)^2 + 4 \cdot (-2)}{(-2+2)(-2-3)} = \frac{-8 + 16 - 8}{0 \cdot (-5)} = \frac{0}{0} \rightarrow (x = -2 \rightarrow x + 2 = 0) \\ \frac{(t^3 + 4t^2 + 4t) \div (t+2)}{[(t+2)(t-3)] \div (t+2)} &= \frac{t^2 + 2t}{t-3} \\ \lim_{t \rightarrow -2} \left[\frac{t^3 + 4t^2 + 4t}{(t+2)(t-3)} \right] &= \frac{(-2)^2 + 2 \cdot (-2)}{-2 - 3} = \frac{4 - 4}{-5} = \frac{0}{-5} = 0\end{aligned}\tag{15}$$

Exercício II

$$\begin{aligned}\lim_{t \rightarrow 0} \left[\frac{(4+t)^2 - 16}{t} \right] &= \frac{(4+0)^2 - 16}{0} = \frac{16 - 16}{0} = \frac{0}{0} \rightarrow (t = 0) \\ \frac{(4+t)^2 - 16}{t} &= \frac{16 + 8t + t^2 - 16}{t} = \frac{t(8+t)}{t} = 8 + t \\ \lim_{t \rightarrow 0} \left[\frac{(4+t)^2 - 16}{t} \right] &= 8 + 0 = 8\end{aligned}\tag{16}$$

Exercício III

$$\lim_{x \rightarrow a} \left[\frac{x^2 + (1-a)x - a}{x-a} \right] = \frac{a^2 + (1-a)a - a}{a-a} = \frac{a^2 + a - a^2 - a}{0} = \frac{0}{0} \rightarrow (x=a \rightarrow x-a=0)$$

$$\frac{[x^2 + (1-a)x - a] \div (x-a)}{(x-a) \div (x-a)} = x+1$$

$$\lim_{x \rightarrow a} \left[\frac{x^2 + (1-a)x - a}{x-a} \right] = a+1$$
(17)

Indeterminação de limites com raiz – [Aula 6](#)

Exercício I

$$\lim_{x \rightarrow 1} \left[\frac{x-1}{\sqrt{x}-1} \right] = \frac{1-1}{\sqrt{1}-1} = \frac{0}{0} \rightarrow (x=1 \rightarrow x-1=0)$$

$$\left(\frac{x-1}{\sqrt{x}-1} \right) \left(\frac{\sqrt{x}+1}{\sqrt{x}+1} \right) = \frac{(x-1)(\sqrt{x}+1)}{x-1} = \sqrt{x}+1$$

$$\lim_{x \rightarrow 1} \left[\frac{x-1}{\sqrt{x}-1} \right] = \sqrt{1}+1 = 2$$
(18)

Exercício II

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{x+2} - \sqrt{2}}{x} \right] = \frac{\sqrt{0+2} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0} = \frac{0}{0} \rightarrow (x=0)$$

$$\left(\frac{\sqrt{x+2} - \sqrt{2}}{x} \right) \left(\frac{\sqrt{x+2} + \sqrt{2}}{\sqrt{x+2} + \sqrt{2}} \right) = \frac{x+2-2}{x(\sqrt{x+2} + \sqrt{2})} = \frac{x}{x(\sqrt{x+2} + \sqrt{2})} = \frac{1}{\sqrt{x+2} + \sqrt{2}}$$

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{x+2} - \sqrt{2}}{x} \right] = \frac{1}{\sqrt{0+2} + \sqrt{2}} = \frac{1}{\sqrt{2} + \sqrt{2}} = \left(\frac{1}{2\sqrt{2}} \right) \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{\sqrt{2}}{4}$$
(19)

Exercício III

$$\lim_{x \rightarrow 4} \left[\frac{x^2 - 16}{\sqrt{x} - 2} \right] = \frac{4^2 - 16}{\sqrt{4} - 2} = \frac{16 - 16}{2 - 2} = \frac{0}{0} \rightarrow (x=4 \rightarrow x-4=0)$$

$$\left(\frac{x^2 - 16}{\sqrt{x} - 2} \right) \left(\frac{\sqrt{x} + 2}{\sqrt{x} + 2} \right) = \frac{(x-4)(x+4)(\sqrt{x}+2)}{x-4} = (x+4)(\sqrt{x}+2)$$

$$\lim_{x \rightarrow 4} \left[\frac{x^2 - 16}{\sqrt{x} - 2} \right] = (4+4)(\sqrt{4}+2) = 8 \cdot 4 = 32$$
(20)

Exercício IV

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{4+x}-2}{x} \right] = \frac{\sqrt{4+0}-2}{0} = \frac{2-2}{0} = \frac{0}{0} \rightarrow (x=0)$$

$$\left(\frac{\sqrt{4+x}-2}{x} \right) \left(\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} \right) = \frac{4+x-4}{x(\sqrt{4+x}+2)} = \frac{x}{x(\sqrt{4+x}+2)} = \frac{1}{\sqrt{4+x}+2}$$

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{4+x}-2}{x} \right] = \frac{1}{\sqrt{4+0}+2} = \frac{1}{2+2} = \frac{1}{4}$$
(21)

Indeterminação de limites com raiz – [Aula 7](#)

Exercício I

$$\lim_{x \rightarrow 7} \left[\frac{2-\sqrt{x-3}}{x^2-49} \right] = \frac{2-\sqrt{7-3}}{7^2-49} = \frac{2-\sqrt{4}}{49-49} = \frac{0}{0} \rightarrow (x=7 \rightarrow x-7=0)$$

$$\left(\frac{2-\sqrt{x-3}}{x^2-49} \right) \left(\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}} \right) = \frac{4-(x-3)}{(x+7)(x-7)(2+\sqrt{x-3})} = \frac{4-x+3}{(x+7)(x-7)(2+\sqrt{x-3})} = \frac{-1}{(x+7)(x-7)(2+\sqrt{x-3})}$$

$$\lim_{x \rightarrow 7} \left[\frac{2-\sqrt{x-3}}{x^2-49} \right] = \frac{-1}{(7+7)(2+\sqrt{7-3})} = \frac{-1}{14 \cdot (2+\sqrt{4})} = \frac{-1}{14 \cdot 4} = -\frac{1}{56}$$
(22)

Exercício II

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} \right] = \frac{\sqrt{0^2+a^2}-a}{\sqrt{0^2+b^2}-b} = \frac{\sqrt{a^2}-a}{\sqrt{b^2}-b} = \frac{0}{0} \rightarrow (x=0)$$

$$\left(\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} \right) \left(\frac{\sqrt{x^2+b^2}+b}{\sqrt{x^2+b^2}+b} \right) = \left(\frac{(\sqrt{x^2+a^2}-a)(\sqrt{x^2+b^2}+b)}{x^2} \right) \left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}+a} \right) = \frac{x^2(\sqrt{x^2+b^2}+b)}{x^2(\sqrt{x^2+a^2}+a)}$$

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} \right] = \frac{\sqrt{0^2+b^2}+b}{\sqrt{0^2+a^2}+a} = \frac{\sqrt{b^2}+b}{\sqrt{a^2}+a} = \frac{b+b}{a+a} = \frac{2b}{2a} = \frac{b}{a}$$
(23)

Indeterminação de limites com raiz – [Aula 8](#)

Exercício I

$$\begin{aligned}
 \lim_{x \rightarrow 4} \left[\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right] &= \frac{3 - \sqrt{5+4}}{1 - \sqrt{5-4}} = \frac{3 - \sqrt{9}}{1 - \sqrt{1}} = \frac{0}{0} \rightarrow (x=4 \rightarrow x-4=0) \\
 &\left(\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right) \left(\frac{1 + \sqrt{5-x}}{1 + \sqrt{5-x}} \right) = \frac{(3 - \sqrt{5+x})(1 + \sqrt{5-x})}{1 - (5-x)} = \\
 &\left(\frac{(3 - \sqrt{5+x})(1 + \sqrt{5-x})}{-4+x} \right) \left(\frac{3 + \sqrt{5+x}}{3 + \sqrt{5+x}} \right) = \frac{(9 - (5+x))(1 + \sqrt{5-x})}{(-4+x)(3 + \sqrt{5+x})} = \\
 &\frac{(4-x)(1 + \sqrt{5-x})}{(-4+x)(3 + \sqrt{5+x})} = \frac{-(x-4)(1 + \sqrt{5-x})}{(x-4)(3 + \sqrt{5+x})} = \frac{-(1 + \sqrt{5-x})}{3 + \sqrt{5+x}} = \frac{-1 - \sqrt{5-x}}{3 + \sqrt{5+x}} \\
 \lim_{x \rightarrow 4} \left[\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right] &= \frac{-1 - \sqrt{5-4}}{3 + \sqrt{5+4}} = \frac{-1 - \sqrt{1}}{3 + \sqrt{9}} = \frac{-2}{6} = -\frac{1}{3}
 \end{aligned} \tag{24}$$

Indeterminação de limites com raiz – [Aula 9](#)

Exercício I

$$\begin{aligned}
 \lim_{x \rightarrow 0} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right] &= \frac{\sqrt{1+0} - \sqrt{1-0}}{0} = \frac{\sqrt{1} - \sqrt{1}}{0} = \frac{0}{0} \rightarrow (x=0) \\
 &\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right) \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{1+x - (1-x)}{x(\sqrt{1+x} + \sqrt{1-x})} = \frac{1+x-1+x}{x(\sqrt{1+x} + \sqrt{1-x})} = \\
 &\frac{2x}{x(\sqrt{1+x} + \sqrt{1-x})} = \frac{2}{\sqrt{1+x} + \sqrt{1-x}} \\
 \lim_{x \rightarrow 0} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right] &= \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{\sqrt{1} + \sqrt{1}} = \frac{2}{2} = 1
 \end{aligned} \tag{25}$$

Limites com módulo – [Aula 10](#)

Exercício I

$$\lim_{x \rightarrow 4} \left[\frac{|x-4|}{x-4} \right] = \nexists$$

$$|x-4| = \begin{cases} (x-4) & \text{para } (x-4) \geq 0 \\ -(x-4) & \text{para } (x-4) < 0 \end{cases}$$

$$|x-4| = \begin{cases} (x-4) & \text{para } x \geq 4 \\ -(x-4) & \text{para } x < 4 \end{cases} \quad (26)$$

$$\lim_{x \rightarrow 4^+} \left[\frac{x-4}{x-4} \right] = 1$$

$$\lim_{x \rightarrow 4^-} \left[\frac{-(x-4)}{x-4} \right] = -1$$

Exercício II

$$\lim_{x \rightarrow 0} \left[\frac{|x|}{x} \right] = \nexists$$

$$|x| = \begin{cases} x & \text{para } x \geq 0 \\ -x & \text{para } x < 0 \end{cases} \quad (27)$$

$$\lim_{x \rightarrow 0^+} \left[\frac{x}{x} \right] = 1$$

$$\lim_{x \rightarrow 0^-} \left[\frac{-x}{x} \right] = -1$$

Exercício III

$$\lim_{x \rightarrow 0} \left[\frac{|x|}{x^2} \right] = +\infty$$

$$|x| = \begin{cases} x & \text{para } x \geq 0 \\ -x & \text{para } x < 0 \end{cases} \quad (28)$$

$$\lim_{x \rightarrow 0^+} \left[\frac{x}{x^2} \right] = \frac{1}{x} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \rightarrow 0^-} \left[\frac{-x}{x^2} \right] = \frac{-1}{x} = \frac{-1}{0^-} = +\infty$$

Limites no infinito – [Aula 11](#)

Exercício I

$$\lim_{x \rightarrow +\infty} \left[\frac{2x-5}{x+8} \right] = 2$$

$$\frac{x \left(\frac{2x-5}{x} - \frac{5}{x} \right)}{x \left(\frac{x}{x} + \frac{8}{x} \right)} = \frac{2 - \frac{5}{x}}{1 + \frac{8}{x}} = \frac{2-0}{1+0} = 2 \quad (29)$$

Exercício II

$$\lim_{x \rightarrow -\infty} \left[\frac{2x^3-3x+5}{4x^5-2} \right] = 0$$

$$\frac{x^3 \left(\frac{2x^3}{x^3} - \frac{3x}{x^3} + \frac{5}{x^3} \right)}{x^3 \left(\frac{4x^5}{x^3} - \frac{2}{x^3} \right)} = \frac{2 - \frac{3}{x^2} + \frac{5}{x^3}}{4x^2 - \frac{2}{x^3}} = \frac{2-0+0}{4x^2-0} = \frac{1}{2x^2} = 0 \quad (30)$$

Exercício III

$$\lim_{x \rightarrow +\infty} \left[\frac{5-x^3}{8x+2} \right] = \frac{-(+\infty)^2}{8} = \frac{-\infty}{8} = -\infty$$

$$\frac{x \left(\frac{5}{x} - \frac{x^3}{x} \right)}{x \left(\frac{8x}{x} + \frac{2}{x} \right)} = \frac{\frac{5}{x} - x^2}{8 + \frac{2}{x}} = \frac{0 - x^2}{8 + 0} = \frac{-x^2}{8} \quad (31)$$

Limites com x tendendo ao infinito – [Aula 11a](#)

Exercício I

$$\lim_{x \rightarrow +\infty} \left[\frac{2x+5}{\sqrt{2x^2-5}} \right] = \sqrt{2}$$

$$\frac{x \left(\frac{2x}{x} + \frac{5}{x} \right)}{x \left(\frac{\sqrt{2x^2-5}}{x} \right)} = \frac{2 + \frac{5}{x}}{\frac{\sqrt{2x^2-5}}{\sqrt{x^2}}} = \frac{2 + \frac{5}{x}}{\sqrt{\frac{2x^2}{x^2} - \frac{5}{x^2}}} = \frac{2 + \frac{5}{x}}{\sqrt{2 - \frac{5}{x^2}}} = \frac{2+0}{\sqrt{2-0}} = \left(\frac{2}{\sqrt{2}} \right) \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{2\sqrt{2}}{2} = \sqrt{2} \quad (32)$$

Exercício II

$$\lim_{x \rightarrow -\infty} \left[\frac{2x+5}{\sqrt{2x^2-5}} \right] = -\sqrt{2}$$

$$\frac{x \left(\frac{2x+5}{x} \right)}{x \left(\frac{\sqrt{2x^2-5}}{x} \right)} = \frac{2+\frac{5}{x}}{\frac{\sqrt{2x^2-5}}{-\sqrt{x^2}}} = \frac{2+\frac{5}{x}}{-\sqrt{\frac{2x^2}{x^2}-\frac{5}{x^2}}} = \frac{2+\frac{5}{x}}{-\sqrt{2-\frac{5}{x^2}}} = \frac{2+0}{-\sqrt{2-0}} = \left(\frac{2}{-\sqrt{2}} \right) \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = -\sqrt{2} \quad (33)$$

Limites no infinito – [Aula 12](#)

Exercício I

$$\lim_{x \rightarrow +\infty} \left[\frac{2x^2-3x}{x+1} \right] = 2 \cdot (+\infty) = +\infty$$

$$\frac{x \left(\frac{2x^2-3x}{x} \right)}{x \left(\frac{x+1}{x} \right)} = \frac{2x-3}{1+\frac{1}{x}} = \frac{2x-3}{1+0} = 2x \quad (34)$$

Exercício II

$$\lim_{x \rightarrow +\infty} \left[\frac{2x^4+3x^2+2x+1}{4-x^4} \right] = -2$$

$$\frac{x^4 \left(\frac{2x^4}{x^4} + \frac{3x^2}{x^4} + \frac{2x}{x^4} + \frac{1}{x^4} \right)}{x^4 \left(\frac{4}{x^4} - \frac{x^4}{x^4} \right)} = \frac{2+\frac{3}{x^2}+\frac{2}{x^3}+\frac{1}{x^4}}{\frac{4}{x^4}-1} = \frac{2+0+0+0}{0-1} = \frac{2}{-1} = -2 \quad (35)$$

Exercício III

$$\lim_{x \rightarrow +\infty} \left[\frac{x^2+3x-1}{x^3-2} \right] = 0$$

$$\frac{x^2 \left(\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2} \right)}{x^2 \left(\frac{x^3}{x^2} - \frac{2}{x^2} \right)} = \frac{1+\frac{3}{x}-\frac{1}{x^2}}{x-\frac{2}{x^2}} = \frac{1+0-0}{x-0} = \frac{1}{x} = 0 \quad (36)$$

Limites infinitos – [Aula 13](#)

Exercício I

$$\begin{aligned}\lim_{x \rightarrow 3} \left[\frac{1}{x-3} \right] &= \frac{1}{3-3} = \frac{1}{0} \\ \lim_{x \rightarrow 3^+} \left[\frac{1}{x-3} \right] &= \frac{1}{3^+-3} = \frac{1}{0^+} = +\infty \\ \lim_{x \rightarrow 3^-} \left[\frac{1}{x-3} \right] &= \frac{1}{3^--3} = \frac{1}{0^-} = -\infty \\ \lim_{x \rightarrow 3} \left[\frac{1}{x-3} \right] &= \nexists\end{aligned}\tag{37}$$

Exercício II

$$\begin{aligned}\lim_{x \rightarrow 1} \left[\frac{1}{(x-1)^2} \right] &= \frac{1}{(1-1)^2} = \frac{1}{0^2} = \frac{1}{0} \\ \lim_{x \rightarrow 1^+} \left[\frac{1}{(x-1)^2} \right] &= \frac{1}{(1^+-1)^2} = \frac{1}{(0^+)^2} = \frac{1}{0^+} = +\infty \\ \lim_{x \rightarrow 1^-} \left[\frac{1}{(x-1)^2} \right] &= \frac{1}{(1^--1)^2} = \frac{1}{(0^-)^2} = \frac{1}{0^+} = +\infty \\ \lim_{x \rightarrow 1} \left[\frac{1}{(x-1)^2} \right] &= +\infty\end{aligned}\tag{38}$$

Exercício III

$$\begin{aligned}\lim_{x \rightarrow 3} \left[\frac{5-6x}{3x-9} \right] &= \frac{5-6 \cdot 3}{3 \cdot 3-9} = \frac{5-18}{9-9} = \frac{-13}{0} \\ \lim_{x \rightarrow 3^+} \left[\frac{5-6x}{3x-9} \right] &= \frac{5-6(3^+)}{3(3^+)-9} = \frac{5-18^+}{9^+-9} = \frac{-13^+}{0^+} = -\infty \\ \lim_{x \rightarrow 3^-} \left[\frac{5-6x}{3x-9} \right] &= \frac{5-6(3^-)}{3(3^-)-9} = \frac{5-18^-}{9^--9} = \frac{-13^-}{0^-} = +\infty \\ \lim_{x \rightarrow 3} \left[\frac{5-6x}{3x-9} \right] &= \nexists\end{aligned}\tag{39}$$

Limites – aplicações das propriedades – [Aula 14](#)

Exercício I

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} f(x) &= 3 \\
 \lim_{x \rightarrow +\infty} g(x) &= 5 \\
 \lim_{x \rightarrow +\infty} h(x) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} [f(x) + 3 \cdot g(x)] &= \\
 \lim_{x \rightarrow +\infty} f(x) + \lim_{x \rightarrow +\infty} 3 \cdot g(x) &= \\
 \lim_{x \rightarrow +\infty} f(x) + \lim_{x \rightarrow +\infty} [3] \cdot \lim_{x \rightarrow +\infty} g(x) &= 3 + 3 \cdot 5 = 3 + 15 = \mathbf{18}
 \end{aligned}$$
(40)

Exercício II

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} f(x) &= 3 \\
 \lim_{x \rightarrow +\infty} g(x) &= 5 \\
 \lim_{x \rightarrow +\infty} h(x) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} [f(x) \cdot g(x)] &= \\
 \lim_{x \rightarrow +\infty} f(x) \cdot \lim_{x \rightarrow +\infty} g(x) &= 3 \cdot 5 = \mathbf{15}
 \end{aligned}$$
(41)

Exercício III

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} f(x) &= 3 \\
 \lim_{x \rightarrow +\infty} g(x) &= 5 \\
 \lim_{x \rightarrow +\infty} h(x) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} \left[\frac{3 \cdot h(x) + 4}{x} \right] &= \\
 \frac{\lim_{x \rightarrow +\infty} [3 \cdot h(x) + 4]}{\lim_{x \rightarrow +\infty} [x]} &= \\
 \frac{\lim_{x \rightarrow +\infty} [3 \cdot h(x)] + \lim_{x \rightarrow +\infty} [4]}{\lim_{x \rightarrow +\infty} [x]} &= \\
 \frac{\lim_{x \rightarrow +\infty} [3] \cdot \lim_{x \rightarrow +\infty} h(x) + \lim_{x \rightarrow +\infty} [4]}{\lim_{x \rightarrow +\infty} [x]} &= \frac{3 \cdot 0 + 4}{+\infty} = \frac{0 + 4}{+\infty} = \frac{4}{+\infty} = \mathbf{0}
 \end{aligned}$$
(42)

Exercício IV

$$\begin{aligned}
 f(x) &= \begin{cases} k \cdot x - 1 & x \geq 3 \\ 3x - 7 & x < 3 \end{cases} \\
 \lim_{x \rightarrow 3^+} [k \cdot x - 1] &= 3k - 1 \\
 \lim_{x \rightarrow 3^-} [3x - 7] &= 3 \cdot 3 - 7 = 9 - 7 = 2 \\
 3k - 1 = 2 &\rightarrow 3k = 2 + 1 \rightarrow 3k = 3 \rightarrow k = \frac{3}{3} \rightarrow \mathbf{k = 1}
 \end{aligned} \tag{43}$$

Exercício V

$$\begin{aligned}
 f(x) &= \begin{cases} \frac{x^2 - 9}{x + 3} & x \geq -3 \\ k + 1 & x < -3 \end{cases} \\
 \lim_{x \rightarrow -3^+} \left[\frac{x^2 - 9}{x + 3} \right] &= \frac{(-3)^2 - 9}{-3 + 3} = \frac{9 - 9}{0} = \frac{0}{0} \rightarrow (x = -3 \rightarrow x + 3 = 0) \\
 \frac{x^2 - 9}{x + 3} &= \frac{(x - 3)(x + 3)}{x + 3} = x - 3 \\
 \lim_{x \rightarrow -3^+} [x - 3] &= -3 - 3 = -6 \\
 \lim_{x \rightarrow -3^-} [k + 1] &= k + 1 \\
 k + 1 = -6 &\rightarrow k = -6 - 1 \rightarrow \mathbf{k = -7}
 \end{aligned} \tag{44}$$