Introdução aos limites – Aula 1

Exercício I

$$\lim_{x\to 1}[2x+1]=3 \tag{1}$$

Exercício II

$$\lim_{x\to 3}\left[\frac{2x+2}{x+1}\right]=2 \tag{2}$$

Exercício III

$$f(x)=\begin{cases} 2x+1 & x\geq 1 \\ x^2+2 & x<1 \end{cases}$$
$$\lim_{x\to 1} f(x)=3 \tag{3}$$

Exercício IV

$$f(x)=\begin{cases} x^2+3x & x\geq 2 \\ 3x+1 & x<2 \end{cases}$$
$$\lim_{x\to 2^+} f(x)=10 \tag{4}$$
$$\lim_{x\to 2^-} f(x)=7$$

Indeterminação de limites – Aula 2

Exercício I

$$\lim_{x\to 0}\left[\frac{x^2+2x}{x}\right]=\frac{0^2+2\cdot 0}{0}=\frac{0}{0}\to(x=0)$$
$$\frac{(x^2+2x)\div x}{x\div x}=x+2 \tag{5}$$
$$\lim_{x\to 0}\left[\frac{x^2+2x}{x}\right]=0+2=2$$

Exercício II

$$\lim_{x\to 2}\left[\frac{x^2-4}{x-2}\right]=\frac{2^2-4}{2-2}=\frac{0}{0}\to(x=2\to x-2=0)$$

$$\frac{(x^2-4)\div(x-2)}{(x-2)\div(x-2)}=x+2 \tag{6}$$

$$\lim_{x\to 2}\left[\frac{x^2-4}{x-2}\right]=2+2=4$$

Exercício III

$$\lim_{x\to 2}\left[\frac{2x^2-2x-4}{x-2}\right]=\frac{2\cdot 2^2-2\cdot 2-4}{2-2}=\frac{8-4-4}{0}=\frac{0}{0}\to(x=2\to x-2=0)$$

$$\frac{(2x^2-2x-4)\div(x-2)}{(x-2)\div(x-2)}=2x+2 \tag{7}$$

$$\lim_{x\to 2}\left[\frac{2x^2-2x-4}{x-2}\right]=2\cdot 2+2=6$$

Indeterminação de limites – Aula 3

Exercício I

$$\lim_{x\to 3}\left[\frac{x^2-9}{x-3}\right]=\frac{3^2-9}{3-3}=\frac{0}{0}\to(x=3\to x-3=0)$$

$$\frac{(x^2-9)\div(x-3)}{(x-3)\div(x-3)}=x+3 \tag{8}$$

$$\lim_{x\to 3}\left[\frac{x^2-9}{x-3}\right]=3+3=6$$

Exercício II

$$\lim_{x\to -2}\left[\frac{x+2}{x^2-4}\right]=\frac{-2+2}{(-2)^2-4}=\frac{0}{0}\to(x=-2\to x+2=0)$$

$$\frac{(x+2)\div(x+2)}{(x^2-4)\div(x+2)}=\frac{1}{x-2} \tag{9}$$

$$\lim_{x\to -2}\left[\frac{x+2}{x^2-4}\right]=\frac{1}{-2-2}=-\frac{1}{4}$$

Exercício III

$$\lim_{x\to 3}\left[\frac{2x^3-6x^2+x-3}{x-3}\right]=\frac{54-54+3-3}{3-3}=\frac{0}{0}\rightarrow(x=3\rightarrow x-3=0)$$

$$\frac{(2x^3-6x^2+x-3)\div(x-3)}{(x-3)\div(x-3)}=2x^2+1 \qquad (10)$$

$$\lim_{x\to 3}\left[\frac{2x^3-6x^2+x-3}{x-3}\right]=2\cdot 3^2+1=19$$

Indeterminação de limites 0/0 – Aula 3a

Exercício I

$$\lim_{x\to 1}\left[\frac{x^2-x}{2x^2+5x-7}\right]=\frac{1^2-1}{2\cdot 1^2+5\cdot 1-7}=\frac{0}{0}\rightarrow(x=1\rightarrow x-1=0)$$

$$\frac{(x^2-x)\div(x-1)}{(2x^2+5x-7)\div(x-1)}=\frac{x}{2x+7} \qquad (11)$$

$$\lim_{x\to 1}\left[\frac{x^2-x}{2x^2+5x-7}\right]=\frac{1}{2\cdot 1+7}=\frac{1}{9}$$

Exercício II

$$\lim_{x\to 2}\left[\frac{x^3-8}{x^2-4}\right]=\frac{2^3-8}{2^2-4}=\frac{0}{0}\rightarrow(x=2\rightarrow x-2=0)$$

$$\frac{(x^3-8)\div(x-2)}{(x^2-4)\div(x-2)}=\frac{x^2+2x+4}{x+2} \qquad (12)$$

$$\lim_{x\to 2}\left[\frac{x^3-8}{x^2-4}\right]=\frac{2^2+2\cdot 2+4}{2+2}=\frac{12}{4}=3$$

Indeterminação polinomial de limites – Aula 4

Exercício I

$$\lim_{h\to 0}\left[\frac{(x+h)^3-x^3}{h}\right]=\frac{(x+0)^3-x^3}{0}=\frac{x^3-x^3}{0}=\frac{0}{0}$$

$$\frac{(x+h)^3-x^3}{h}=\frac{(x+h)^2(x+h)-x^3}{h}=\frac{(x^2+2xh+h^2)(x+h)-x^3}{h}=$$
$$\frac{x^3+2x^2h+xh^2+x^2h+2xh^2+h^3-x^3}{h}=\frac{3x^2h+3xh^2+h^3}{h}=\frac{h(3x^2+3xh+h^2)}{h}=3x^2+3xh+h^2 \quad (13)$$
$$\lim_{h\to 0}\left[\frac{(x+h)^3-x^3}{h}\right]=3x^2+3x\cdot 0+0^2=3x^2$$

Exercício II

$$\lim_{x\to -1}\left[\frac{x^3+1}{x^2-1}\right]=\frac{(-1)^3+1}{(-1)^2-1}=\frac{-1+1}{1-1}=\frac{0}{0}\rightarrow(x=-1\rightarrow x+1=0)$$

$$\frac{(x^3+1)\div(x+1)}{(x^2-1)\div(x+1)}=\frac{x^2-x+1}{x-1} \quad (14)$$
$$\lim_{x\to -1}\left[\frac{x^3+1}{x^2-1}\right]=\frac{(-1)^2-(-1)+1}{-1-1}=\frac{1+1+1}{-2}=-\frac{3}{2}$$

Indeterminação polinomial de limites – Aula 5

Exercício I

$$\lim_{t\to -2}\left[\frac{t^3+4t^2+4t}{(t+2)(t-3)}\right]=\frac{(-2)^3+4\cdot(-2)^2+4\cdot(-2)}{(-2+2)(-2-3)}=\frac{-8+16-8}{0\cdot(-5)}=\frac{0}{0}\rightarrow(x=-2\rightarrow x+2=0)$$

$$\frac{(t^3+4t^2+4t)\div(x+2)}{[(t+2)(t-3)]\div(x+2)}=\frac{t^2+2t}{t-3} \quad (15)$$
$$\lim_{t\to -2}\left[\frac{t^3+4t^2+4t}{(t+2)(t-3)}\right]=\frac{(-2)^2+2\cdot(-2)}{-2-3}=\frac{4-4}{-5}=\frac{0}{-5}=0$$

Exercício II

$$\lim_{t\to 0}\left[\frac{(4+t)^2-16}{t}\right]=\frac{(4+0)^2-16}{0}=\frac{16-16}{0}=\frac{0}{0}\rightarrow(t=0)$$

$$\frac{(4+t)^2-16}{t}=\frac{16+8t+t^2-16}{t}=\frac{t(8+t)}{t}=8+t \quad (16)$$
$$\lim_{t\to 0}\left[\frac{(4+t)^2-16}{t}\right]=8+0=8$$

Exercício III

$$\lim_{x\to a}\left[\frac{x^2+(1-a)x-a}{x-a}\right]=\frac{a^2+(1-a)a-a}{a-a}=\frac{a^2+a-a^2-a}{0}=\frac{0}{0}\to(x=a\to x-a=0)$$

$$\frac{[x^2+(1-a)x-a]\div(x-a)}{(x-a)\div(x-a)}=x+1 \tag{17}$$

$$\lim_{x\to a}\left[\frac{x^2+(1-a)x-a}{x-a}\right]=a+1$$

Indeterminação de limites com raiz – Aula 6

Exercício I

$$\lim_{x\to 1}\left[\frac{x-1}{\sqrt{x}-1}\right]=\frac{1-1}{\sqrt{1}-1}=\frac{0}{0}\to(x=1\to x-1=0)$$

$$\left(\frac{x-1}{\sqrt{x}-1}\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)=\frac{(x-1)(\sqrt{x}+1)}{x-1}=\sqrt{x}+1 \tag{18}$$

$$\lim_{x\to 1}\left[\frac{x-1}{\sqrt{x}-1}\right]=\sqrt{1}+1=2$$

Exercício II

$$\lim_{x\to 0}\left[\frac{\sqrt{x+2}-\sqrt{2}}{x}\right]=\frac{\sqrt{0+2}-\sqrt{2}}{0}=\frac{\sqrt{2}-\sqrt{2}}{0}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{x+2}-\sqrt{2}}{x}\right)\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right)=\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}=\frac{x}{x(\sqrt{x+2}+\sqrt{2})}=\frac{1}{\sqrt{x+2}+\sqrt{2}} \tag{19}$$

$$\lim_{x\to 0}\left[\frac{\sqrt{x+2}-\sqrt{2}}{x}\right]=\frac{1}{\sqrt{0+2}+\sqrt{2}}=\frac{1}{\sqrt{2}+\sqrt{2}}=\left(\frac{1}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{4}$$

Exercício III

$$\lim_{x\to 4}\left[\frac{x^2-16}{\sqrt{x}-2}\right]=\frac{4^2-16}{\sqrt{4}-2}=\frac{16-16}{2-2}=\frac{0}{0}\to(x=4\to x-4=0)$$

$$\left(\frac{x^2-16}{\sqrt{x}-2}\right)\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right)=\frac{(x-4)(x+4)(\sqrt{x}+2)}{x-4}=(x+4)(\sqrt{x}+2) \tag{20}$$

$$\lim_{x\to 4}\left[\frac{x^2-16}{\sqrt{x}-2}\right]=(4+4)(\sqrt{4}+2)=8\cdot 4=32$$

Exercício IV

$$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x}\right]=\frac{\sqrt{4+0}-2}{0}=\frac{2-2}{0}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{4+x}-2}{x}\right)\left(\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right)=\frac{4+x-4}{x(\sqrt{4+x}+2)}=\frac{x}{x(\sqrt{4+x}+2)}=\frac{1}{\sqrt{4+x}+2} \qquad (21)$$

$$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x}\right]=\frac{1}{\sqrt{4+0}+2}=\frac{1}{2+2}=\frac{1}{4}$$

Indeterminação de limites com raiz – Aula 7

Exercício I

$$\lim_{x\to 7}\left[\frac{2-\sqrt{x-3}}{x^2-49}\right]=\frac{2-\sqrt{7-3}}{7^2-49}=\frac{2-\sqrt{4}}{49-49}=\frac{0}{0}\to(x=7\to x-7=0)$$

$$\left(\frac{2-\sqrt{x-3}}{x^2-49}\right)\left(\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}\right)=\frac{4-(x-3)}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{4-x+3}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{}{(x+7)(x} \qquad (22)$$

$$\frac{-(x-7)}{(x+7)(x-7)(2+\sqrt{x-3})}=\frac{-1}{(x+7)(2+\sqrt{x-3})}$$

$$\lim_{x\to 7}\left[\frac{2-\sqrt{x-3}}{x^2-49}\right]=\frac{-1}{(7+7)(2+\sqrt{7-3})}=\frac{-1}{14\cdot(2+\sqrt{4})}=\frac{-1}{14\cdot 4}=-\frac{1}{56}$$

Exercício II

$$\lim_{x\to 0}\left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right]=\frac{\sqrt{0^2+a^2}-a}{\sqrt{0^2+b^2}-b}=\frac{\sqrt{a^2}-a}{\sqrt{b^2}-b}=\frac{0}{0}\to(x=0)$$

$$\left(\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right)\left(\frac{\sqrt{x^2+b^2}+b}{\sqrt{x^2+b^2}+b}\right)=\left(\frac{(\sqrt{x^2+a^2}-a)(\sqrt{x^2+b^2}+b)}{x^2}\right)\left(\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}+a}\right)=\frac{x^2(\sqrt{x^2+b^2}+b)}{x^2(\sqrt{x^2+a^2}+a)}= \qquad (23)$$

$$\lim_{x\to 0}\left[\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}\right]=\frac{\sqrt{0^2+b^2}+b}{\sqrt{0^2+a^2}+a}=\frac{\sqrt{b^2}+b}{\sqrt{a^2}+a}=\frac{b+b}{a+a}=\frac{2b}{2a}=\frac{b}{a}$$

Indeterminação de limites com raiz – Aula 8

Exercício I

$$\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]=\frac{3-\sqrt{5+4}}{1-\sqrt{5-4}}=\frac{3-\sqrt{9}}{1-\sqrt{1}}=\frac{0}{0}\rightarrow(x=4\rightarrow x-4=0)$$

$$\begin{gathered}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)\left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right)=\frac{(3-\sqrt{5+x})(1+\sqrt{5-x})}{1-(5-x)}=\\ \left(\frac{(3-\sqrt{5+x})(1+\sqrt{5-x})}{-4+x}\right)\left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)=\frac{(9-(5+x))(1+\sqrt{5-x})}{(-4+x)(3+\sqrt{5+x})}=\\ \frac{(4-x)(1+\sqrt{5-x})}{(-4+x)(3+\sqrt{5+x})}=\frac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}=\frac{-(1+\sqrt{5-x})}{3+\sqrt{5+x}}=\frac{-1-\sqrt{5-x}}{3+\sqrt{5+x}}\\ \lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]=\frac{-1-\sqrt{5-4}}{3+\sqrt{5+4}}=\frac{-1-\sqrt{1}}{3+\sqrt{9}}=\frac{-2}{6}=-\frac{1}{3}\end{gathered}\tag{24}$$