Introdução às Derivadas Parciais de 1ª ordem – Aula 1

Exercício I

$$\begin{aligned}
f(x,y)&=4\frac{x^3}{y^2}-2xy-3x-4y-7=4x^3y^{-2}-2xy-3x-4y-7\\
\frac{\partial f(x,y)}{\partial x}&=4y^{-2}\frac{\partial(x^3)}{\partial x}-2y\frac{\partial(x)}{\partial x}-3\frac{\partial(x)}{\partial x}-0-0=4y^{-2}3x^2-2y-3=\frac{12x^2}{y^2}-2y-3\\
\frac{\partial f(x,y)}{\partial y}&=4x^3\frac{\partial(y^{-2})}{\partial y}-2x\frac{\partial(y)}{\partial y}-0-4\frac{\partial(y)}{\partial y}-0=4x^3(-2y^{-3})-2x-4=\frac{-8x^3}{y^3}-2x-4
\end{aligned} \tag{1}$$

Derivadas Parciais: Interpretação Geométrica – Aula 2

Exercício I

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano x = -1, no ponto P(-1, 1, -2).

$$\begin{aligned}
f(x,y)&=x^2+y^2-2x^3y+5xy^4-1\\
z=f(-1,1)&=(-1)^2+(1)^2-2(-1)^3(1)+5(-1)(1)^4-1=1+1+2-5-1=4-6=-2\\
\frac{\partial f(x,y)}{\partial y}&=0+\frac{\partial(y^2)}{\partial y}-2x^3\frac{\partial(y)}{\partial y}+5x\frac{\partial(y^4)}{\partial y}-0=2y-2x^3+5x4y^3=2y+20xy^3-2x^3\\
\frac{\partial f(-1,1)}{\partial y}&=2(1)+20(-1)(1)^3-2(-1)^3=2-20+2=-16
\end{aligned} \tag{2}$$

Exercício II

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano y = 2, no ponto P(2, 2, 8).

$$\begin{aligned}
f(x,y)&=x^2+y^2\\
z=f(2,2)&=(2)^2+(2)^2=4+4=8\\
\frac{\partial f(x,y)}{\partial x}&=\frac{\partial(x^2)}{\partial x}+0=2x\\
\frac{\partial f(2,2)}{\partial x}&=2(2)=4
\end{aligned} \tag{3}$$

Derivadas Parciais de 2ª ordem – Aula 3

Exercício I

$$\begin{aligned}
f(x,y) &= x^2+y^2-2x^3y+5xy^4-1\\
\frac{\partial f(x,y)}{\partial x} &= 2x+0-2y3x^2+5y^4-0=2x-6x^2y+5y^4\\
\frac{\partial^2 f(x,y)}{\partial x^2} &= 2-6y2x=-12xy+2\\
\frac{\partial^2 f(x,y)}{\partial y\partial x} &= 0-6x^2+5\cdot 4y^3=-6x^2+20y^3\\
\frac{\partial f(x,y)}{\partial y} &= 0+2y-2x^3+5x4y^3-0=-2x^3+20xy^3+2y\\
\frac{\partial^2 f(x,y)}{\partial y^2} &= -0+20x3y^2+2=60xy^2+2\\
\frac{\partial^2 f(x,y)}{\partial x\partial y} &= -2\cdot 3x^2+20y^3+0=-6x^2+20y^3
\end{aligned} \tag{4}$$

Exercício II

$$\begin{aligned}
z &= x^2y-xy^2+2x-y\\
\frac{\partial z}{\partial x} &= y\cdot 2x-y^2+2-0=2xy-y^2+2\\
\frac{\partial^2 z}{\partial x^2} &= 2y-0+0=2y\\
\frac{\partial^2 z}{\partial y\partial x} &= 2x-2y+0=2x-2y\\
\frac{\partial z}{\partial y} &= x^2-x2y+0-1=x^2-2xy-1\\
\frac{\partial^2 z}{\partial y^2} &= 0-2x-0=-2x\\
\frac{\partial^2 z}{\partial x\partial y} &= 2x-2y-0=2x-2y
\end{aligned} \tag{5}$$

Exercício III

$$z = xy$$

$$\frac{\partial z}{\partial x} = y$$
$$\frac{\partial^2 z}{\partial x^2} = 0$$
$$\frac{\partial^2 z}{\partial y \partial x} = 1 \tag{6}$$

$$\frac{\partial z}{\partial y} = x$$
$$\frac{\partial^2 z}{\partial y^2} = 0$$
$$\frac{\partial^2 z}{\partial x \partial y} = 1$$

Exercício IV

$$z = \ln(xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy} y = \frac{1}{x} = x^{-1}$$
$$\frac{\partial^2 z}{\partial x^2} = -x^{-2} = \frac{-1}{x^2}$$
$$\frac{\partial^2 z}{\partial y \partial x} = 0 \tag{7}$$

$$\frac{\partial z}{\partial y} = \frac{1}{xy} x = \frac{1}{y} = y^{-1}$$
$$\frac{\partial^2 z}{\partial y^2} = -y^{-2} = \frac{-1}{y^2}$$
$$\frac{\partial^2 z}{\partial x \partial y} = 0$$

Derivadas Parciais de 2ª ordem – Aula 4

Exercício I

$$z=e^{-xy^2}$$

$$\frac{\partial z}{\partial x}=e^{-xy^2}(-y^2)=-y^2e^{-xy^2}$$

$$\frac{\partial^2 z}{\partial x^2}=-y^2e^{-xy^2}(-y^2)=y^4e^{-xy^2}$$

$$\frac{\partial^2 z}{\partial y\partial x}=-\left[2ye^{-xy^2}+y^2e^{-xy^2}(-x2y)\right]=-\left(2ye^{-xy^2}-2xy^3e^{-xy^2}\right)=2ye^{-xy^2}\left(xy^2-1\right) \tag{8}$$

$$\frac{\partial z}{\partial y}=e^{-xy^2}(-x2y)=-2xye^{-xy^2}$$

$$\frac{\partial^2 z}{\partial y^2}=-\left[2xe^{-xy^2}+2xye^{-xy^2}(-x2y)\right]=-\left(2xe^{-xy^2}-4x^2y^2e^{-xy^2}\right)=2xe^{-xy^2}\left(2xy^2-1\right)$$

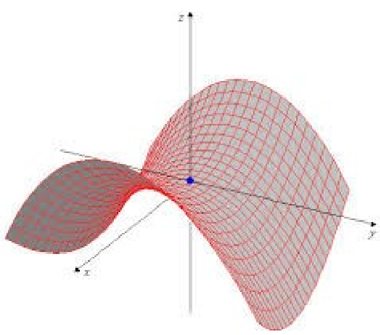
$$\frac{\partial^2 z}{\partial x\partial y}=-\left[2ye^{-xy^2}+2xye^{-xy^2}(-y^2)\right]=-\left(2ye^{-xy^2}-2xy^3e^{-xy^2}\right)=2ye^{-xy^2}\left(xy^2-1\right)$$

Máximos, Mínimos e Sela através do Hessiano – Aula 5

1. Ache o **x** e o **y** crítico, igualando a **0** a derivada de **z** em relação a x e a y:
   $$\frac{\partial f(x,y)}{\partial x}=\frac{\partial z}{\partial x}=0\rightarrow x_c$$
   $$\frac{\partial f(x,y)}{\partial y}=\frac{\partial z}{\partial y}=0\rightarrow y_c$$

2. Calcule o determinante de **x** e **y** crítico: $h(x_c,y_c)=\begin{vmatrix}\dfrac{\partial^2 z}{\partial x^2} & \dfrac{\partial^2 z}{\partial y\partial x}\\ \dfrac{\partial^2 z}{\partial x\partial y} & \dfrac{\partial^2 z}{\partial y^2}\end{vmatrix}$

3. $h<0\rightarrow$ ponto de sela
   $h>0\rightarrow\dfrac{\partial^2 z}{\partial x^2}>0\rightarrow\text{Mínimo},\ \dfrac{\partial^2 z}{\partial x^2}<0\rightarrow\text{Máximo}$
   $h=0\rightarrow$ NPA = Nada podemos afirmar



Exercício I

$$z=3x^4+8x^3-18x^2+6y^2+12y-4$$

1.

$$\frac{\partial z}{\partial x}=12x^3+24x^2-36x=12x(x^2+2x-3)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 12x(x^2+2x-3)=0$$

$$12x=0\rightarrow x=\frac{0}{12}=0\rightarrow x_{c1}=0$$

$$x^2+2x-3=0\rightarrow x^2+2x-3+1-1=0\rightarrow(x^2+2x+1)-4=0\rightarrow(x+1)^2-4=0\rightarrow$$

$$(x+1)^2=4\rightarrow x+1=\pm\sqrt{4}\rightarrow x=\pm 2-1\rightarrow x_{c2}=1,x_{c3}=-3$$

$$\frac{\partial z}{\partial y}=12y+12=12(y+1)$$

$$\frac{\partial z}{\partial y}=0\rightarrow 12(y+1)=0$$

$$y+1=0\rightarrow y_c=-1$$

$P_1(-3,-1),P_2(0,-1),P_1(1,-1)$

2.

$$\frac{\partial^2 z}{\partial x^2}=36x^2+48x-36=12(3x^2+4x-3)$$

$$\frac{\partial^2 f(-3,-1)}{\partial x^2}=12(3(-3)^2+4(-3)-3)=12(27-12-3)=12\cdot 12=144$$

$$\frac{\partial^2 f(0,-1)}{\partial x^2}=12(3(0)^2+4(0)-3)=12(-3)=-36 \qquad (9)$$

$$\frac{\partial^2 f(1,-1)}{\partial x^2}=12(3(1)^2+4(1)-3)=12(3+4-3)=12\cdot 4=48$$

$$\frac{\partial^2 z}{\partial y\partial x}=0$$

$$\frac{\partial^2 z}{\partial x\partial y}=0$$

$$\frac{\partial^2 z}{\partial y^2}=12$$

$$h(-3,-1)=\begin{bmatrix}144 & 0\\ 0 & 12\end{bmatrix}=(144\cdot 12)-(0\cdot 0)=1728$$

$$h(0,-1)=\begin{bmatrix}-36 & 0\\ 0 & 12\end{bmatrix}=-36\cdot 12=-432$$

$$h(1,-1)=\begin{bmatrix}48 & 0\\ 0 & 12\end{bmatrix}=48\cdot 12=576$$

3.

$$P_1(-3,-1)\rightarrow h(-3,-1)>0\rightarrow\frac{\partial^2 f(-3,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

$$P_2(0,-1)\rightarrow h(0,-1)<0\rightarrow \text{é pto de sela}$$

$$P_1(1,-1)\rightarrow h(1,-1)>0\rightarrow\frac{\partial^2 f(1,-1)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

Exercício II

1.

$$z=x^3+3xy+y^2-2$$

$$\frac{\partial z}{\partial x}=3x^2+3y=3(x^2+y)$$

$$\frac{\partial z}{\partial x}=0\rightarrow 3(x^2+y)=0$$

$$x^2+y=0\rightarrow y=-x^2\rightarrow 3x+2(-x^2)=0\rightarrow 3x-2x^2=0\rightarrow x(3-2x)=0$$

$$x_{c1}=0$$

$$3-2x=0\rightarrow 3=2x\rightarrow x_{c2}=\frac{3}{2}$$

$$\frac{\partial z}{\partial y}=3x+2y$$

$$\frac{\partial z}{\partial y}=0\rightarrow 3x+2y=0\rightarrow 2y=-3x\rightarrow y=\frac{-3x}{2}$$

$$y=\frac{-3(0)}{2}\rightarrow y_{c1}=0$$

$$y=\frac{-3\left(\frac{3}{2}\right)}{2}\rightarrow y_{c2}=\frac{-9}{4}$$

$$P_1(0,0),P_2\left(\frac{3}{2},\frac{-9}{4}\right)$$

2.

$$\frac{\partial^2 z}{\partial x^2}=6x \tag{10}$$

$$\frac{\partial^2 f(0,0)}{\partial x^2}=6(0)=0$$

$$\frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}=6\left(\frac{3}{2}\right)=9$$

$$\frac{\partial^2 z}{\partial y\,\partial x}=3$$

$$\frac{\partial^2 z}{\partial x\,\partial y}=3$$

$$\frac{\partial^2 z}{\partial y^2}=2$$

$$h(0,0)=\begin{bmatrix}0 & 3\\ 3 & 2\end{bmatrix}=(0\cdot 2)-(3\cdot 3)=0-9=-9$$

$$h\left(\frac{3}{2},\frac{-9}{4}\right)=\begin{bmatrix}9 & 3\\ 3 & 2\end{bmatrix}=(9\cdot 2)-(3\cdot 3)=18-9=9$$

3.

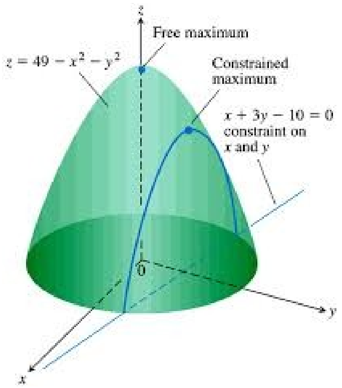
$$P_1(0,0)\rightarrow h(0,0)<0\rightarrow \text{é pto de sela}$$

$$P_2\left(\frac{3}{2},\frac{-9}{4}\right)\rightarrow h\left(\frac{3}{2},\frac{-9}{4}\right)>0\rightarrow \frac{\partial^2 f\left(\frac{3}{2},\frac{-9}{4}\right)}{\partial x^2}>0\rightarrow \text{é pto de mínimo}$$

Máximos e Mínimos Condicionados com Multiplicadores de Lagrange – Aula 6

Função: $f(x,y)$
Restrição: $r(x,y)=0$

Função de Lagrange: $$L(x,y,\lambda)=f(x,y)-\lambda r(x,y)$$
$$\frac{\partial L}{\partial x}=0,\frac{\partial L}{\partial y}=0,r(x,y)=0$$



Exercício I

Ache o ponto de máximo ou mínimo da função a seguir:
$f(x,y)=x^2+y^2$, sujeito a restrição $x+y=4$

$$\begin{gathered}
r(x,y)=0\rightarrow x+y-4=0\rightarrow r(x,y)=x+y-4\\
L(x,y,\lambda)=f(x,y)-\lambda r(x,y)=(x^2+y^2)-\lambda(x+y-4)=x^2+y^2-\lambda x-\lambda y+4\lambda\\
\frac{\partial L}{\partial x}=2x+0-\lambda-0+0=2x-\lambda\\
\frac{\partial L}{\partial x}=0\rightarrow 2x-\lambda=0\rightarrow 2x=\lambda\rightarrow x=\frac{\lambda}{2}\\
\frac{\partial L}{\partial y}=0+2y-0-\lambda+0=2y-\lambda\\
\frac{\partial L}{\partial y}=0\rightarrow 2y-\lambda=0\rightarrow 2y=\lambda\rightarrow y=\frac{\lambda}{2}\\
x+y-4=0\rightarrow\frac{\lambda}{2}+\frac{\lambda}{2}-4=0\rightarrow\frac{2\lambda}{2}=4\rightarrow\lambda=4\\
x=\frac{4}{2}=2,\ y=\frac{4}{2}=2\rightarrow P(2,2)\\
f(2,2)=(2)^2+(2)^2=4+4=8\\
x=0\rightarrow x+y=4\rightarrow y=4-x=4-(0)=4\rightarrow f(0,4)=(0)^2+(4)^2=16\\
P(2,2)\rightarrow f(0,4)>f(2,2)\rightarrow \text{pto de mínimo}
\end{gathered}\tag{11}$$

Exercício II

Função: $f(x,y)=9-x^2-y^2$
Restrição: $x+y=2 \to x+y-2=0 \to r(x,y)=x+y-2$

$$\begin{gathered}
L(x,y,\lambda)=(9-x^2-y^2)-\lambda(x+y-2)=9-x^2-y^2-\lambda x-\lambda y+2\lambda \\
\frac{\partial L}{\partial x}=0-2x-0-\lambda-0+0=-2x-\lambda \\
\frac{\partial L}{\partial x}=0 \to -2x-\lambda=0 \to -\lambda=2x \to x=\frac{-\lambda}{2} \\
\frac{\partial L}{\partial y}=0-0-2y-0-\lambda+0=-2y-\lambda \\
\frac{\partial L}{\partial y}=0 \to -2y-\lambda=0 \to -\lambda=2y \to y=\frac{-\lambda}{2} \\
x+y=2 \to -\frac{\lambda}{2}+\left(\frac{-\lambda}{2}\right)=2 \to \frac{-2\lambda}{2}=2 \to \lambda=-2 \\
x=\frac{-\lambda}{2}=\frac{-(-2)}{2}=1,\ y=\frac{-\lambda}{2}=\frac{-(-2)}{2}=1 \to P(1,1) \\
f(1,1)=9-(1)^2-(1)^2=9-1-1=7 \\
x=0 \to x+y=2 \to y=2-x=2-(0)=2 \to f(0,2)=9-(0)^2-(2)^2=9-4=5 \\
P(1,1) \to f(1,1)>f(0,2) \to \text{pto de máximo}
\end{gathered} \tag{12}$$

Exercício III

Seja a função lucro de uma indústria, $f(x,y)=-2x^2-y^2+32x+20y$ que produz e comercializa dois produtos em quantidades x e y. Calcule o lucro máximo, sabendo que a produção da indústria é limitada em 24 unidades.

Função: $f(x,y)=-2x^2-y^2+32x+20y$

Restrição: $x+y=24 \rightarrow x+y-24=0 \rightarrow r(x,y)=x+y-24$

$$
\begin{gathered}
L(x,y,\lambda)=(-2x^2-y^2+32x+20y)-\lambda(x+y-24)= \\
-2x^2-y^2+32x+20y-\lambda x-\lambda y+24\lambda \\
\frac{\partial L}{\partial x}=-4x-0+32+0-\lambda-0+0=-4x+32-\lambda \\
\frac{\partial L}{\partial x}=0 \rightarrow -4x+32-\lambda=0 \rightarrow 32-\lambda=4x \rightarrow x=\frac{32-\lambda}{4} \\
\frac{\partial L}{\partial y}=-0-2y+0+20-0-\lambda+0=-2y+20-\lambda \\
\frac{\partial L}{\partial y}=0 \rightarrow -2y+20-\lambda=0 \rightarrow 20-\lambda=2y \rightarrow y=\frac{20-\lambda}{2} \\
x+y=24 \rightarrow \frac{32-\lambda}{4}+\frac{20-\lambda}{2}=24 \rightarrow 32-\lambda+2(20-\lambda)=96 \rightarrow 32-\lambda+40-2\lambda=96 \rightarrow \\
72-3\lambda=96 \rightarrow -3\lambda=96-72 \rightarrow 3\lambda=-24 \rightarrow \lambda=\frac{-24}{3}=-8 \\
x=\frac{32-\lambda}{4}=\frac{32-(-8)}{4}=\frac{40}{4}=10,\ y=\frac{20-\lambda}{2}=\frac{20-(-8)}{2}=\frac{28}{2}=14 \rightarrow P(10,14) \\
f(10,14)=-2(10)^2-(14)^2+32(10)+20(14)= \\
-200-196+320+280=204 \\
x=0 \rightarrow x+y=24 \rightarrow y=24-x=24-(0)=24 \rightarrow f(0,24)=-2(0)^2-(24)^2+32(0)+20(24)= \\
-576+480=-96 \\
P(10,14) \rightarrow f(10,14)>f(0,24) \rightarrow \text{pto de máximo}
\end{gathered}
\tag{13}
$$