Introdução às Derivadas Parciais de 1ª ordem – Aula 1

Exercício I

$$
\begin{gathered}
f(x,y)=4\frac{x^3}{y^2}-2xy-3x-4y-7=4x^3y^{-2}-2xy-3x-4y-7\\
\frac{\partial f(x,y)}{\partial x}=4y^{-2}\frac{\partial(x^3)}{\partial x}-2y\frac{\partial(x)}{\partial x}-3\frac{\partial(x)}{\partial x}-0-0=4y^{-2}3x^2-2y-3=\frac{12x^2}{y^2}-2y-3\\
\frac{\partial f(x,y)}{\partial y}=4x^3\frac{\partial(y^{-2})}{\partial y}-2x\frac{\partial(y)}{\partial y}-0-4\frac{\partial(y)}{\partial y}-0=4x^3(-2y^{-3})-2x-4=\frac{-8x^3}{y^3}-2x-4
\end{gathered}
\tag{1}
$$

Derivadas Parciais: Interpretação Geométrica – Aula 2

Exercício I

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano x = -1, no ponto P(-1, 1, -2).

$$
\begin{gathered}
f(x,y)=x^2+y^2-2x^3y+5xy^4-1\\
z=f(-1,1)=(-1)^2+(1)^2-2(-1)^3(1)+5(-1)(1)^4-1=1+1+2-5-1=4-6=-2\\
\frac{\partial f(x,y)}{\partial y}=0+\frac{\partial(y^2)}{\partial y}-2x^3\frac{\partial(y)}{\partial y}+5x\frac{\partial(y^4)}{\partial y}-0=2y-2x^3+5x4y^3=2y+20xy^3-2x^3\\
\frac{\partial f(-1,1)}{\partial y}=2(1)+20(-1)(1)^3-2(-1)^3=2-20+2=-16
\end{gathered}
\tag{2}
$$

Exercício II

Encontre a declividade da reta tangente a curva abaixo, resultante da intersecção da função f(x, y) com o plano y = 2, no ponto P(2, 2, 8).

$$
\begin{gathered}
f(x,y)=x^2+y^2\\
z=f(2,2)=(2)^2+(2)^2=4+4=8\\
\frac{\partial f(x,y)}{\partial x}=\frac{\partial(x^2)}{\partial x}+0=2x\\
\frac{\partial f(2,2)}{\partial x}=2(2)=4
\end{gathered}
\tag{3}
$$

Derivadas Parciais de 2ª ordem – Aula 3

Exercício I

$$
\begin{aligned}
f(x,y) &= x^2+y^2-2x^3y+5xy^4-1\\
\frac{\partial f(x,y)}{\partial x} &= 2x+0-2y3x^2+5y^4-0=2x-6x^2y+5y^4\\
\frac{\partial^2 f(x,y)}{\partial x^2} &= 2-6y2x=-12xy+2\\
\frac{\partial^2 f(x,y)}{\partial y\partial x} &= 0-6x^2+5\cdot 4y^3=-6x^2+20y^3\\
\\
\frac{\partial f(x,y)}{\partial y} &= 0+2y-2x^3+5x4y^3-0=-2x^3+20xy^3+2y\\
\frac{\partial^2 f(x,y)}{\partial y^2} &= -0+20x3y^2+2=60xy^2+2\\
\frac{\partial^2 f(x,y)}{\partial x\partial y} &= -2\cdot 3x^2+20y^3+0=-6x^2+20y^3
\end{aligned}
\tag{4}
$$

Exercício II

$$
\begin{aligned}
z &= x^2y-xy^2+2x-y\\
\frac{\partial z}{\partial x} &= y\cdot 2x-y^2+2-0=2xy-y^2+2\\
\frac{\partial^2 z}{\partial x^2} &= 2y-0+0=2y\\
\frac{\partial^2 z}{\partial y\partial x} &= 2x-2y+0=2x-2y\\
\\
\frac{\partial z}{\partial y} &= x^2-x2y+0-1=x^2-2xy-1\\
\frac{\partial^2 z}{\partial y^2} &= 0-2x-0=-2x\\
\frac{\partial^2 z}{\partial x\partial y} &= 2x-2y-0=2x-2y
\end{aligned}
\tag{5}
$$

Exercício III

$$z = xy$$

$$\frac{\partial z}{\partial x} = y$$
$$\frac{\partial^2 z}{\partial x^2} = 0$$
$$\frac{\partial^2 z}{\partial y \partial x} = 1 \qquad (6)$$

$$\frac{\partial z}{\partial y} = x$$
$$\frac{\partial^2 z}{\partial y^2} = 0$$
$$\frac{\partial^2 z}{\partial x \partial y} = 1$$

Exercício IV

$$z = \ln(xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy} y = \frac{1}{x} = x^{-1}$$
$$\frac{\partial^2 z}{\partial x^2} = -x^{-2} = \frac{-1}{x^2}$$
$$\frac{\partial^2 z}{\partial y \partial x} = 0 \qquad (7)$$

$$\frac{\partial z}{\partial y} = \frac{1}{xy} x = \frac{1}{y} = y^{-1}$$
$$\frac{\partial^2 z}{\partial y^2} = -y^{-2} = \frac{-1}{y^2}$$
$$\frac{\partial^2 z}{\partial x \partial y} = 0$$