Trigonometrische Funktionen

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot x$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$

Additionstheoreme

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

doppelter Winkel

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$
$$\sin 2x = 2 \sin x \cos x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

halber Winkel

$$\cos\frac{x}{2} =^* \pm\sqrt{\frac{1}{2}(1+\cos x)}$$
$$\sin\frac{x}{2} =^* \pm\sqrt{\frac{1}{2}(1-\cos x)}$$
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} =^* \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$\cot\frac{x}{2} = \frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x} =^* \pm\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Symmetrie

$\cos(-x)$	$= \cos x$	gerade Funktion
$\sin(-x)$	$= -\sin x$	ungerade Funktion
$\tan(-x)$	$= -\tan x$	ungerade Funktion
$\cot(-x)$	$= -\cot x$	ungerade Funktion

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x) \qquad \sin x =^* \frac{\tan x}{\pm\sqrt{1+\tan^2 x}}$$
$$\sin^2 x = \frac{1}{2}(1-\cos 2x) \qquad \cos x =^* \frac{1}{\pm\sqrt{1+\tan^2 x}}$$
$$\cos x = \sin\left(\frac{\pi}{2} \pm x\right) \qquad \tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sin x + \sin y = 2 \sin\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos\frac{x+y}{2} \sin\frac{x-y}{2}$$
$$\sin x \cdot \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$
$$\cos x + \cos y = 2 \cos\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin\frac{x+y}{2} \sin\frac{x-y}{2}$$
$$\cos x \cdot \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$
$$\sin x \cdot \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

* Vorzeichen je nach Quadranten!

Hyperbelfunktionen

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x}-1}{e^{2x}+1}$$
$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \coth x = \frac{\cosh x}{\sinh x} = \frac{e^{2x}+1}{e^{2x}-1}$$

$$\cosh 0 = 1, \quad \sinh 0 = 0, \quad \tanh 0 = 0$$
$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$\cosh(-x) = \cosh x\,, \quad \sinh(-x) = -\sinh x\,, \quad \tanh(-x) = -\tanh x\,, \quad \coth(-x) = -\coth x$$

Additionstheoreme

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh\frac{x}{2} = \sqrt{\frac{1}{2}(\cosh x + 1)}$$
$$\sinh\frac{x}{2} = \pm\sqrt{\frac{1}{2}(\cosh x - 1)}, \quad \text{für } \begin{cases} x \geq 0 \\ x < 0 \end{cases}$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$

$$\operatorname{arsinh} x = \ln(x + \sqrt{x^2+1})$$
$$\operatorname{arcosh} x = \ln(x + \sqrt{x^2-1}), \quad \text{für } x \geq 1$$

Überlagerung von Schwingungen

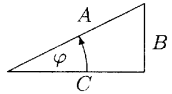
$$\boxed{A_1 \sin(\omega t + \varphi_1) + A_2 \sin(\omega t + \varphi_2) = A \sin(\omega t + \varphi)}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1 - \varphi_2)}$$
$$\tan\varphi = \frac{A_1 \sin\varphi_1 + A_2 \sin\varphi_2}{A_1 \cos\varphi_1 + A_2 \cos\varphi_2} \quad \text{(Quadranten beachten!)}$$

Spezialfall: $\boxed{B \cos\omega t + C \sin\omega t = A \sin(\omega t + \varphi)}$

$B = A \sin\varphi$

$C = A \cos\varphi$



$A = \sqrt{B^2 + C^2}$

$\tan\varphi = \frac{B}{C}$ Quadranten beachten!

Quadratische Gleichung

$$x^2 + px + q = 0$$
$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}\,.$$

allgemeine Binomialkoeffizienten

$r \in \mathbb{R}$ und $k = 1, 2, \ldots$

$$\binom{r}{k} = \frac{r(r-1)\cdots(r-k+1)}{k!}$$
$$\binom{r}{0} = \binom{r}{r} = 1, \quad \binom{r}{1} = r$$

Polarkoordinaten

$x = r\cos\varphi$

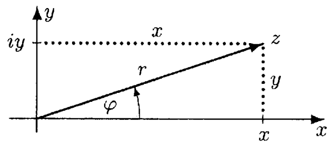
$y = r\sin\varphi$

$dF = r\,dr\,d\varphi$

$r = \sqrt{x^2 + y^2}$

$\tan\varphi = \frac{y}{x}$ Quadranten beachten!

$$z = x + iy = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$



Rechnen mit Potenzen und Logarithmen

a: Basis, mit $0 < a \neq 1$

$a^{x+y} = a^x a^y$	$\log_a xy = \log_a x + \log_a y$
$a^{-x} = \frac{1}{a^x}$	$\log_a \frac{1}{x} = -\log_a x$
$a^0 = 1$	$\log_a 1 = 0$
$(a^x)^r = a^{xr}$	$\log_a x^r = r \log_a x$

Logarithmen zu verschiedenen Basen:

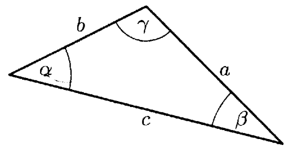
$$\log_a x = \frac{\log_b x}{\log_b a}, \text{ speziell: } \log_a x = \frac{\ln x}{\ln a}$$

Kosinussatz

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Pythagoras

$$c^2 = a^2 + b^2, \text{ falls } \gamma = 90^0.$$

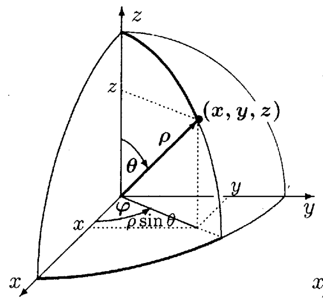


Sinussatz

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Kugelkoordinaten

θ : Polabstand



$x = \rho\sin\theta\cos\varphi$

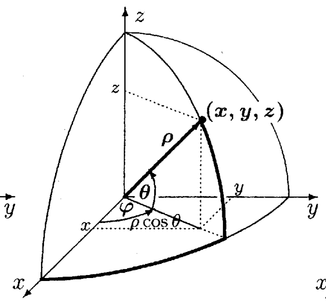
$y = \rho\sin\theta\sin\varphi$

$z = \rho\cos\theta$

$dV = \rho^2 \sin\theta\, d\rho\, d\theta\, d\varphi$

Kugelkoordinaten

θ : geographische Breite



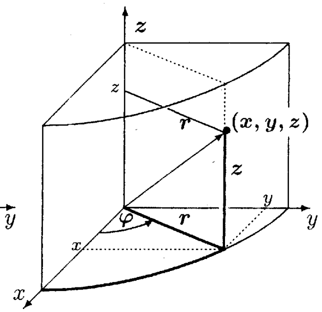
$x = \rho\cos\theta\cos\varphi$

$y = \rho\cos\theta\sin\varphi$

$z = \rho\sin\theta$

$dV = \rho^2 \cos\theta\, d\rho\, d\theta\, d\varphi$

Zylinderkoordinaten



$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$dV = r\, dr\, d\varphi\, dz$

Potenzreihen

e^x	$= \sum_{n=0}^{\infty} \frac{1}{n!}x^n$	$= 1+\frac{1}{1!}x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\cdots$	für	$x \in \mathbb{R}$
$\sin x$	$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$	$= x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-+\cdots$	für	$x \in \mathbb{R}$
$\cos x$	$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$	$= 1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-+\cdots$	für	$x \in \mathbb{R}$
$\sinh x$	$= \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}x^{2n+1}$	$= x+\frac{1}{3!}x^3+\frac{1}{5!}x^5+\cdots$	für	$x \in \mathbb{R}$
$\cosh x$	$= \sum_{n=0}^{\infty} \frac{1}{(2n)!}x^{2n}$	$= 1+\frac{1}{2!}x^2+\frac{1}{4!}x^4+\cdots$	für	$x \in \mathbb{R}$
$\arctan x$	$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}x^{2n+1}$	$= x-\frac{1}{3}x^3+\frac{1}{5}x^5-\frac{1}{7}x^7+-\cdots$	für	$\lvert x\rvert \le 1$
$\ln(1+x)$	$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}x^n$	$= x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+-\cdots$	für	$-1 < x \le 1$
$\ln(1-x)$	$= -\sum_{n=1}^{\infty} \frac{1}{n}x^n$	$= -(x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\cdots)$	für	$-1 \le x < 1$
$\sqrt{1+x}$	$= \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} x^n$	$= 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+-\cdots$	für	$\lvert x\rvert \le 1$
$\frac{1}{\sqrt{1+x}}$	$= \sum_{n=0}^{\infty} \binom{-\frac{1}{2}}{n} x^n$	$= 1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{5}{16}x^3+\frac{35}{128}x^4-+\cdots$	für	$\lvert x\rvert < 1$

geometrische Reihe	$\sum_{n=0}^{\infty} x^n$	$= 1+x+x^2+x^3+\cdots$	$= \frac{1}{1-x}$,	für $\lvert x\rvert < 1$
endliche geom. Reihe	$\sum_{n=0}^{k} x^n$	$= 1+x+x^2+\cdots+x^k$	$= \frac{1-x^{k+1}}{1-x}$,	für $x \ne 1$
harmonische Reihe	$\sum_{n=1}^{\infty} \frac{1}{n^x}$	$= 1+\frac{1}{2^x}+\frac{1}{3^x}+\cdots$	konvergent $\iff$	$x > 1$
binomische Reihe	$\sum_{n=0}^{\infty} \binom{r}{n} x^n$	$= 1+rx+\binom{r}{2}x^2+\binom{r}{3}x^3+\cdots = (1+x)^r$,		$\lvert x\rvert \le 1,\ r>0$ $\lvert x\rvert < 1,\ r<0$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$	$= \infty$
$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+-\cdots$	$= \ln 2$
$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots$	$= e$
$1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+-\cdots$	$= \frac{1}{e}$
$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$	$= 2$
$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-\cdots$	$= \frac{\pi}{4}$
$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots$	$= \frac{\pi^2}{6}$
$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+-\cdots$	$= \frac{\pi^2}{12}$
$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$	$= \frac{\pi^2}{8}$

wichtige Grenzwerte
$(n \to \infty)$

$\sqrt[n]{a} \to 1$	$\left(\frac{n+1}{n}\right)^n \to e$	$\binom{a}{n} \to 0,\ a > -1$
$\sqrt[n]{n} \to 1$	$\left(1+\frac{1}{n}\right)^n \to e$	$\frac{a^n}{n!} \to 0$
$\sqrt[n]{n!} \to \infty$	$\left(1-\frac{1}{n}\right)^n \to e^{-1}$	$\frac{n^n}{n!} \to \infty$
$\frac{n}{\sqrt[n]{n!}} \to e$	$\left(1+\frac{x}{n}\right)^n \to e^x$	$\frac{a^n}{n^k} \to \infty \begin{cases} a>1 \\ k \text{ fest} \end{cases}$
$\frac{1}{n}\sqrt[n]{n!} \to \frac{1}{e}$	$\left(1-\frac{x}{n}\right)^n \to e^{-x}$	$a^n n^k \to 0 \begin{cases} \lvert a\rvert<1 \\ k \text{ fest} \end{cases}$
		$n(\sqrt[n]{a}-1) \to \ln a,\ a>0$

Differentiations- und Integrationsregeln

Produktregel:	$(u\cdot v)' = u'\cdot v + u\cdot v'$ $(uvw)' = u'vw + uv'w + uvw'$
partielle Integration:	$\int u'v\,dx = uv - \int uv'\,dx$
Quotientenregel:	$\left(\frac{u}{v}\right)' = \frac{u'\cdot v - u\cdot v'}{v^2}$

Vektorfunktionen

$$(\lambda\vec{u})' = \lambda'\vec{u} + \lambda\vec{u}'$$
$$(\vec{u}\cdot\vec{v})' = \vec{u}'\cdot\vec{v} + \vec{u}\cdot\vec{v}'$$
$$(\vec{u}\times\vec{v})' = \vec{u}'\times\vec{v} + \vec{u}\times\vec{v}'$$
$$(\vec{u}(\lambda(t)))' = \vec{u}'(\lambda(t))\cdot\lambda'(t)$$

Kettenregel: $(y(x(t)))' = \frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = y'(x(t))\cdot x'(t)$

Substitutionsregel: $\int f(x)\,dx = \int f(g(t))\,g'(t)\,dt$, dabei ist $\begin{cases} x = g(t) \\ dx = g'(t)\,dt \end{cases}$

f	f'
x^n	nx^{n-1}
$\frac{1}{x^n}$	$\frac{-n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
e^x	e^x
$\ln x$	$\frac{1}{x}$
a^x	$a^x \ln a$
x^x	$x^x(1+\ln x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{x^2+1}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}},\quad x>1$
$\operatorname{artanh} x$	$\frac{1}{1-x^2},\quad \lvert x\rvert<1$
$\operatorname{arcoth} x$	$\frac{1}{1-x^2},\quad \lvert x\rvert>1$
$\int g\,dx$	g

$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad (n \ne -1)$ | $\int \frac{f'}{f}\,dx = \ln\lvert f\rvert$

$\int \frac{1}{x}\,dx = \ln\lvert x\rvert$	$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$
$\int \frac{dx}{x+a} = \ln\lvert x+a\rvert$	$\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{3}{2}\sqrt[3]{x^2}$
$\int \frac{dx}{(x+a)^2} = -\frac{1}{x+a}$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int xe^{ax}\,dx = \frac{ax-1}{a^2}e^{ax}$
$\int \sin^2 ax\,dx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax$	$\int \ln x\,dx = x\ln x - x$
$\int \cos^2 ax\,dx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax$	$\int x\ln x\,dx = x^2\left(\frac{\ln x}{2}-\frac{1}{4}\right)$

$$\int \ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$$
$$\int \sin ax\cos ax\,dx = \frac{1}{2a}\sin^2 ax$$
$$\int \frac{dx}{\sin ax\cos ax} = \frac{1}{a}\ln\lvert\tan ax\rvert$$
$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$
$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$
$$\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$
$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

Bezeichnungen: $X = ax^2+bx+c, \quad \Delta = 4ac-b^2, \quad a \ne 0$

$$\int \frac{dx}{X} = \begin{cases} \frac{2}{\sqrt{\Delta}}\arctan\frac{2ax+b}{\sqrt{\Delta}} & (\Delta>0) \\ \frac{-2}{\sqrt{-\Delta}}\operatorname{artanh}\frac{2ax+b}{\sqrt{-\Delta}} \\ \frac{1}{\sqrt{-\Delta}}\ln\frac{2ax+b-\sqrt{-\Delta}}{2ax+b+\sqrt{-\Delta}} & (\Delta<0) \\ \frac{-2}{2ax+b} & (\Delta=0) \end{cases}$$

$$\int \frac{dx}{X^2} = \frac{2ax+b}{\Delta X} + \frac{2a}{\Delta}\int\frac{dx}{X}$$
$$\int \frac{x\,dx}{X} = \frac{1}{2a}\ln\lvert X\rvert - \frac{b}{2a}\int\frac{dx}{X}$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\operatorname{arsinh}\frac{x}{a}\right) = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\ln(x+\sqrt{x^2+a^2})\right)$$
$$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\operatorname{arcosh}\frac{x}{a}\right) = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\ln(x+\sqrt{x^2-a^2})\right)$$
$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a}\right)$$