What's (kinda) Missing?

Degree of the M.F.	Degree of the D.F.	
...	⋮	
4	3	
3	2	
2	1	
1	0	-1 ?
0	0	
-1	-2	
-2	-3	
-3	-4	
-4	-5	
...	⋮	

5.2.1: <u>The Derivative of the Natural Log, $y = \ln x$</u> (pg 571) Date: ______________

Ex1. Find $f'(x)$ if $f(x) = \ln x$.

$f(x) = \ln x$

$y = \ln x$

$e^y = x$

$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$

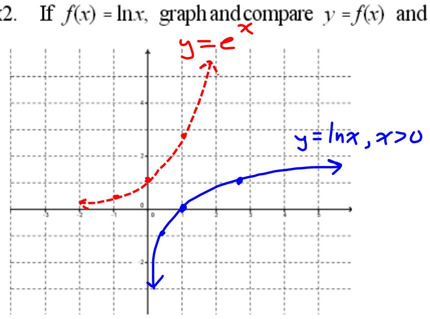
$e^y \cdot y' = 1$

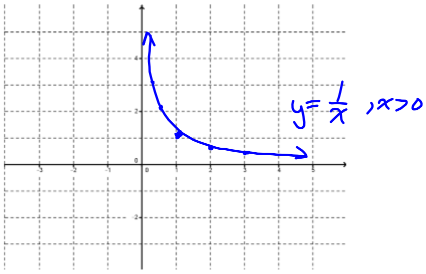
$y' = \frac{1}{e^y}$

$y' = \frac{1}{x}$

So...	IF $f(x) = \ln x$	THEN $f'(x) = \frac{1}{x}$
	IF $f(x) = \ln(g(x))$	THEN $f'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$

Ex2. If $f(x) = \ln x$, graph and compare $y = f(x)$ and $y = f'(x)$.





- when $x \to 0^+$,
  - the shape of $y = \ln x \to$ uphill + almost vertical
  - the value of $y = \frac{1}{x} \to +\infty$

- when $x \to \infty$,
  - the shape of $y = \ln x \to$ uphill + horizontal
  - the value of $y = \frac{1}{x} \to 0^+$

Ex3. Find all critical numbers.

a. $y = \ln(3x + 4)$

$y' = \frac{3}{3x+4}$

$\therefore$ No C.N.'s

b. $y = \frac{\ln x}{x^2}$

$y' = \frac{\left(\frac{1}{x}\right)(x^2) - (\ln x)(2x)}{x^4}$

$= \frac{x - 2x\ln x}{x^4}$

$= \frac{1 - 2\ln x}{x^3}$

$y' = 0$ when $1 - 2\ln x = 0$

$1 = 2\ln x$

$\ln x = \frac{1}{2}$

$x = e^{\frac{1}{2}}$

$x = \sqrt{e}$

Ex4. Use the 1st derivative to make an accurate sketch of each function.

a. $y = \dfrac{\ln x^2}{3x}$ HA @ $y=0$ VA @ $x=0$

$\ln x^2 = 0$
$x^2 = e^0$
$x^2 = 1$
$x = \pm 1$

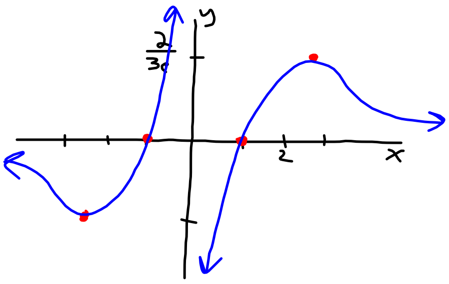
x	y
-1	0
1	0

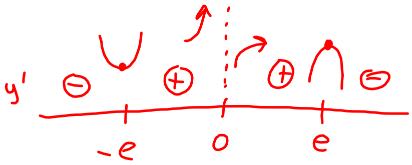
$$y' = \frac{\left(\frac{2x}{x^2}\right)(3x) - (\ln x^2)(3)}{9x^2}$$

$$= \frac{6 - 3\ln x^2}{9x^2}$$

$$= \frac{2 - \ln x^2}{3x^2}$$

$y' = 0$ when $2 - \ln x^2 = 0$
$\ln x^2 = 2$
$x^2 = e^2$
$x = \pm e$





max @ $\left(e, \frac{2}{3e}\right)$

min @ $\left(-e, \frac{-2}{3e}\right)$

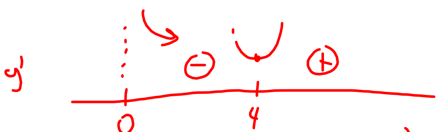
b. $y = \sqrt{x} - \ln x$ $x > 0$, VA @ $x = 0$

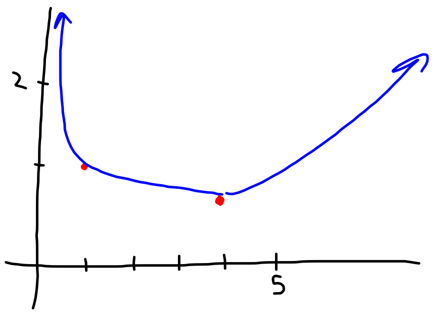
$$y' = \frac{1}{2\sqrt{x}} - \frac{1}{x}$$

$$= \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{x} \cdot \frac{2}{2}$$

$$= \frac{\sqrt{x} - 2}{2x}$$

$y' = 0$
$\sqrt{x} - 2 = 0$
$x = 4$





min @ $(4, 2 - \ln 4)$

Ex6. Find the equation of the tangent to $y = \ln(x^2 + e^x)$ when $x = 0$.

$$y' = \frac{2x + e^x}{x^2 + e^x}$$

when $x = 0$
$y = \ln(0 + 1)$
$= 0$

$$y' = \frac{0+1}{0+1}$$
$= 1$

$P_0 = (0,0)$
$m = 1$ $\Longrightarrow \therefore$ $y = x$ ITRE

Homefun: Page 575 #1, 3, 4a→d, 5ab, 6, 7a, 8→11, 13 {Ans 4d $g'(z) = \frac{-z}{1+z}$ }

Desmos Investigation of End-Behaviour

1. Graph $y = \frac{\ln x^2}{3x}$ compared with $y = \frac{1}{x}$.
2. Graph $y = \sqrt{x} - \ln x$ compared with $y = \sqrt{x}$. Or use $y = \sqrt{x} - 7$ for a really good match.
3. **Order of Domination**: Exponential, Polynomial, Logarithmic

   Graph $y = e^x$

   $y = x^{1/4}$, $y = x$, $y = x^4$

   $y = \ln x$

   To see the domination of $y = e^x$ over $y = x^4$, scale max y-value to 10 000.
   To see the domination of $y = x^{1/4}$ over $y = \ln x$, scale max x-value to 10 000.

   Graph $y = x^{0.1}$ compared with $y = \ln x$.

   To see the polynomial function $y = x^{0.1}$ dominate the logarithmic function $y = \ln x$, change your window so that $0 \le x \le 5 \times 10^{15}$, $0 \le y \le 40$.