

# Calculus I Review

## 1.0 Prerequisite Skills

- using pascals triangle
- factoring
  - difference of squares
  - decomposition/aussie method
  - factoring by grouping
  - difference of cubes
  - **synthetic division?** ...
  - factor theorem ..
  - recognizing domain (where a function is continuous)

$$f(x) = \frac{2 + \sqrt{x-3}}{5x-3}$$

the line above is continuous from

$$x > 3$$

$$x \neq \frac{3}{5}$$

so ...

$$D = \{x \in \mathbb{R} | x > 3\}$$

example 2

$$y = \sqrt{60 + 14x - 2x^2}$$

$$60 + 14x - 2x^2 \geq 0$$

$$2x^2 - 14x - 60 \geq 0$$

$$(x-10)(2x+6) \geq 0$$

$$\text{thus } D = \{x \in \mathbb{R} | -3 \leq x \leq 10\}$$

- finding average and instantaneous rates of change

## 1.1 Radical expressions / rationalizing Denominators

- simplify by rationalizing denominators and numerators

## 1.2 Slope of a Tangent

- solving for a tangent with

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- can stop solving for a solution if the top is zero
- solve for lines of tangents by using the slope and a  $p_0$  with the symmetric equation cross multiplication shortcut

$$\frac{x-2}{1} = y - 1013$$

$$13x - 26 = y - 10$$

$$13x - y - 14 = 0$$

## 1.3 Rates of Change

- solving for average rates of change with a table test
  - calculating average rate of change over a given time
  - calculate instantaneous rate of change using limit formula
  - be aware of the parameters of an application function
    - \* a function that dictates height would have a slope that dictates velocity
  - can solve for zeros and times by backwards substituting the slope or wanted value into the equation

$$\lim_{h \rightarrow 0} \frac{H(t+h) - H(t)}{h} = 0$$

**can solve for t**

- solve for the average rate of change of applications (furballs)
- solve for instantaneous rate of change in applications (furballs)

## 1.4 The Limit of A Function

- a limit means that we can determine the value of a function at  $a$  by choosing a value  $x$  which is sufficiently close to  $a$  but is not equal to  $a$
- this limit only exists if the limiting value from the left equals the limiting value from the right
- and so the value of a limit can be determined by checking if the left and right sides of the limit are equal

answers to true or false questions

- the limit of a function may exist even if the value of a function does not equal the limit, or does not even exist
- if the limit of both sides of an x value are equal, the limit is real,
- the limit does not have to always be equal to the function value
- the limit is effectively flowing throw a point
- the limit of an asymptote cannot be determined

### 1.5 Properties of limits

- the limit of a constant is equal to the constant (if a function will always return the same value)
- the limit of x as x approaches a is equal to a
- the limits can be broken up into sums and differences

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

- The limit of a constant times a function is analagous to the constant times the limit of the function

$$\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} f(x)$$

- limits can also be multiplied and divided if that's something you are into ... as long as neither of the limits are equal to 0
- finally the limit of a power or root is equal to the power or root of the limit

$$\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$$

*also works with roots*

- when a limit cannot be found by direct substitution it in indeterminate form (0/0)
- these functions are said to have a **removable discontinuity**
- eliminating removable discontinuities are cared for with simplifying, factoring, rationalizing, and changing the variable
- again, if no removable discontinuity, there is no answer

when checking if roots exist, you must sometimes break up a function into two different functions, a function for when approaching the limit from above and a function for when approaching the limit from below, based on these separate functions the original function value can be substituted in to check if both limit sides are equal

## 1.6 Continuity

- a function is continuous at point  $a$  if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

- in other words the limit is the same as the value of the function

**Example 1** determine the range of continuity of the function

$$g(x) = \frac{(5-x)}{\sqrt{x^2-16}}$$

so

$$x^2 - 16 \geq 0$$

$$(x-4)(x+4) \geq 0$$

therefore

$$x \in \mathbb{R} \mid x \geq 4, x \leq -4$$

- a function are not continuous at holes or wherever the function is not defined