5.2.3: **Logarithmic Differentiation** (pg 579) Date: ________________

Ex1. Find $f'(x)$ if $f(x) = x^x$.

$$y = x^x$$
$$\ln y = \ln(x^x)$$
$$\ln y = x \ln x$$

$$\frac{y'}{y} = (1)(\ln x) + (x)\left(\frac{1}{x}\right)$$
$$= \ln x + 1$$

$$\to y' = y(\ln x + 1)$$
$$y' = x^x(\ln x + 1)$$

Ex2. Use logarithmic differentiation (take the ln of both sides, then differentiate) to differentiate each function. Do NOT use the Power Rule.

a. $y = x^5$

$$\ln y = \ln x^5$$
$$= 5 \ln x$$

$$\frac{y'}{y} = \frac{5}{x}$$
$$y' = \frac{5y}{x}$$
$$= \frac{5x^5}{x}$$
$$= 5x^4$$

b. $y = x^n,\ n \in R$

$$y = x^n$$
$$\ln y = \ln x^n$$
$$= n \ln x$$

$$\frac{y'}{y} = \frac{n}{x}$$
$$y' = \frac{ny}{x}$$
$$= \frac{nx^n}{x^1}$$

$$\to y' = nx^{n-1}$$

Ex3. Differentiate $y = (x^2 + 3)^x$.

So $\ln y = \ln(x^2+3)^x$

$$= x \ln(x^2+3)$$

$$\frac{y'}{y} = (1)(\ln(x^2+3)) + (x)\left(\frac{2x}{x^2+3}\right)$$
$$y' = y\left(\ln(x^2+3) + \frac{2x^2}{x^2+3}\right)$$
$$= (x^2+3)^x\left[\ln(x^2+3) + \frac{2x^2}{x^2+3}\right]$$

Ex4. Find $f'(-1)$ if $f(x) = \dfrac{(x^4+1)\sqrt{x+2}}{2x^2+2x+1}$.

$$\log(10^4 \cdot 10^3) = \log 10^4 + \log 10^3$$
$$= 4 + 3$$
$$= 7$$

$$y = \frac{A \cdot B}{C}$$

$$\ln y = \ln\left(\frac{A \cdot B}{C}\right) = \ln A + \ln B - \ln C$$

$$\ln y = \ln(x^4+1) + \tfrac{1}{2}\ln(x+2) - \ln(2x^2+2x+1)$$

$$\frac{y'}{y} = \frac{4x^3}{x^4+1} + \frac{1}{2(x+2)} - \frac{4x+2}{2x^2+2x+1}$$

$$f'(-1) = f(-1) \cdot \left[\frac{-4}{2} + \frac{1}{2} - \frac{-2}{1}\right]$$

$$= \left[\frac{2(1)}{1}\right]\left[\frac{1}{2}\right]$$

$$\therefore f'(-1) = 1$$

Homefun: Page 582 # 1→3, 5→7, 9, 12 (Note: Do **NOT** use Logarithmic Differentiation if not needed or helpful. As always, **MAKE GOOD CHOICES!**)

Ex5. Prove $e^{\pi} > \pi^{e}$.

$$e^{\pi} \ ? \ \pi^{e} \iff \ln(e^{\pi}) \ ? \ \ln(\pi^{e})$$

$$\iff \pi \ln e \ ? \ e \ln \pi$$

$$\iff \frac{\ln e}{e} \ ? \ \frac{\ln \pi}{\pi}$$

Hmmm.... Consider $y = \frac{\ln x}{x}$

$$\therefore \quad y' = \frac{\left(\frac{1}{x}\right)x - (\ln x)(1)}{x^2}$$

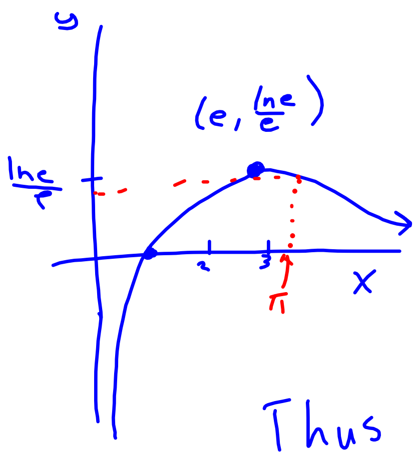
$$= \frac{1 - \ln x}{x^2}$$

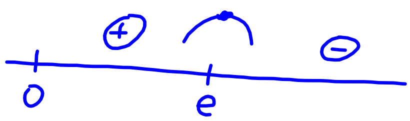
So $y' = 0$ when

$1 - \ln x = 0$

$\ln x = 1$

$x = e$





Thus $\frac{\ln e}{e} > \frac{\ln \pi}{\pi}$ $\therefore e^{\pi} > \pi^{e}$ ∎