5.1 Q9 Prove $y'' = \frac{y}{25}$ if $y = \frac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)$

Well, $y' = \frac{5}{2}\left[e^{\frac{x}{5}} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \left(-\frac{1}{5}\right)\right]$

$$\therefore y'' = \frac{5}{2}\left[e^{\frac{x}{5}} \cdot \frac{1}{5} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\right]$$

$$= \frac{\frac{5}{2}\left(e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right)}{25}$$

$$= \frac{y}{25}$$

Less exciting, but effective...

Prove: $y'' = \frac{y}{25}$

$$LS = y''$$
$$= \frac{5}{2}\left[e^{\frac{x}{5}} \cdot \frac{1}{5} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \frac{-1}{5} \cdot \frac{-1}{5}\right]$$
$$= \frac{1}{10} e^{\frac{x}{5}} + \frac{1}{10} e^{-\frac{x}{5}}$$

$$RS = \frac{y}{25}$$
$$= \frac{\frac{5}{2}\left[e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right]}{25}$$
$$= \frac{5}{2} \cdot \frac{1}{25} \cdot \left[e^{\frac{x}{5}} + e^{-\frac{x}{5}}\right]$$
$$= \frac{1}{10} e^{\frac{x}{5}} + \frac{1}{10} e^{-\frac{x}{5}}$$
$$= LS \checkmark$$

Q13 $s=160\left(\frac{1}{4}t-1+e^{-\frac{t}{4}}\right)$

a) $v=160\left(\frac{1}{4}+e^{-\frac{t}{4}}\cdot\left(-\frac{1}{4}\right)\right)$

$=40\left(1-e^{-\frac{t}{4}}\right)$

b) $v=40-40e^{-\frac{t}{4}}$ and $a=40\left(-e^{-\frac{t}{4}}\cdot\left(-\frac{1}{4}\right)\right)$

$e^{-\frac{t}{4}}=\frac{40-v}{40}$

$=1-\frac{1}{40}v$

$=40\left(\frac{1}{4}e^{-\frac{t}{4}}\right)$

$=10e^{-\frac{t}{4}}$

$=10\left(1-\frac{1}{40}v\right)$

$=10-\frac{1}{4}v$

Again, less exciting but effective...

Prove: $a=10-\frac{1}{4}v$

$LS=a$

$=10e^{-\frac{t}{4}}$

$RS=10-\frac{1}{4}v$

$=10-\frac{1}{4}\left(40-40e^{-\frac{t}{4}}\right)$

$=10-10+10e^{-\frac{t}{4}}$

$=10e^{-\frac{t}{4}}$

$=LS$ ✓

c) $V_T = \lim_{t\to\infty} v$

$= \lim_{t\to\infty} \left(40 - 40e^{-\frac{t}{4}}\right)$

$= \lim_{t\to\infty} \left(40 - \dfrac{40}{e^{\frac{t}{4}}}\right)$

$= 40 - 0$

$= 40\ m/s$

d) $40\left(1 - e^{-\frac{t}{4}}\right) = 0.95(40)$

$1 - e^{-\frac{t}{4}} = 0.95$

$e^{-\frac{t}{4}} = 0.05$

$e^{\frac{t}{4}} = 20$

$\frac{t}{4} = \ln 20$

$t = 4\ln 20$

$\doteq 12\ s$

$S = 160\left[\frac{1}{4}4\ln 20 - 1 + e^{\frac{-4\ln 20}{4}}\right]$

$= 160\left[\ln 20 - 1 + e^{\ln\frac{1}{20}}\right]$

$= 160\left[\ln 20 - \frac{20}{20} + \frac{1}{20}\right]$

$= 160\left[\ln 20 - \frac{19}{20}\right]$

$= 160\ln 20 - 152$

$\doteq 327.3\ m$

Q16

$y = Ae^{mt}$

$y' = Ae^{mt} \cdot m$

$= mAe^{mt}$

$y'' = mAe^{mt} \cdot m$

$= m^2Ae^{mt}$

Since $y'' + y' + 6y = 0$

$\therefore m^2Ae^{mt} + mAe^{mt} - 6Ae^{mt} = 0$

$\therefore Ae^{mt}(m^2 + m - 6) = 0$

$\therefore Ae^{mt}(m+3)(m-2) = 0$

$\therefore m = -3$ OR 2.