Warm Up

Find $f'(x)$ if $f(x) = \sqrt{g(x)}$

$$\boxed{f(x) = \sqrt{g(x)}}$$

$$= [g(x)]^{\frac{1}{2}}$$

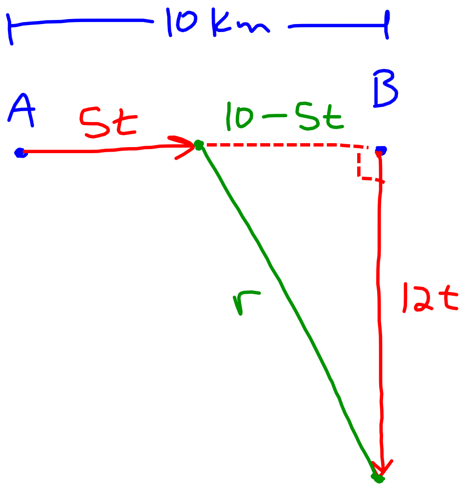
$$\therefore f'(x) = \frac{1}{2}[g(x)]^{-\frac{1}{2}} \cdot g'(x)$$

$$\boxed{f'(x) = \frac{g'(x)}{2\sqrt{g(x)}}}$$

3.3: Optimization Problems II

Ex1. A is 10 km west of B. At 12 pm, A moves east at 5 km/h and B moves south at 12 km/h.

When are A and B closest together?



$$r = \sqrt{(10-5t)^2 + (12t)^2}$$

$$= \sqrt{25t^2 - 100t + 100 + 144t^2}$$

$$= \sqrt{169t^2 - 100t + 100}$$

$$r^2 = (10-5t)^2 + (12t)^2$$

$$\frac{d}{dt}\left(r^2 = 169t^2 - 100t + 100\right)$$

$$2r \cdot \frac{dr}{dt} = 338t - 100$$

$$\frac{dr}{dt} = \frac{338t - 100}{2r} = 0$$

$$\frac{dr}{dt} = \frac{338t - 100}{2\sqrt{}} = 0$$

Thus $338t - 100 = 0$

$$t = \frac{100}{338}$$

$$= 0.2958\ldots \text{ h}$$

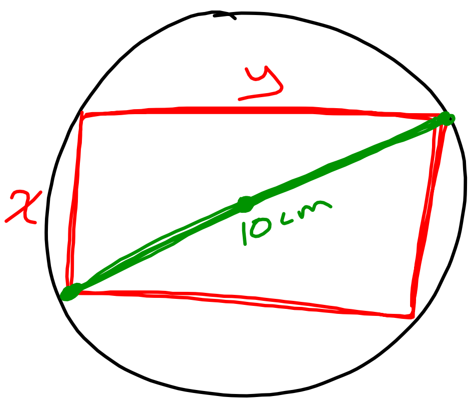
$$= 17.75\ldots \text{ min}$$

$$\doteq 17 \text{ min } 45.1 \text{ sec}$$

$\therefore$ Time required

$= 12{:}17{:}45$ pm

Ex2. Determine the largest rectangle that can be inscribed inside a circle of radius 5 cm.



$$x^2 + y^2 = 10^2$$

$$x^2 + y^2 = 100$$

$$y^2 = 100 - x^2$$

$$y = \sqrt{100 - x^2}$$

$$A = xy$$

$$A(x) = x\sqrt{100 - x^2}$$

$$A(x) = \sqrt{x^2(100 - x^2)}$$

$$= \sqrt{100x^2 - x^4}$$

$$A'(x) = (1)\sqrt{100 - x^2} + (x)\frac{-\cancel{2}x}{\cancel{2}\sqrt{100 - x^2}} = 0$$

$$\therefore \sqrt{100 - x^2} = \frac{x^2}{\sqrt{100 - x^2}}$$

$$100 - x^2 = x^2$$

$$2x^2 = 100$$

$$x^2 = 50$$

$$x = \sqrt{50}$$

$$y = \sqrt{100 - x^2}$$

$$= \sqrt{100 - 50}$$

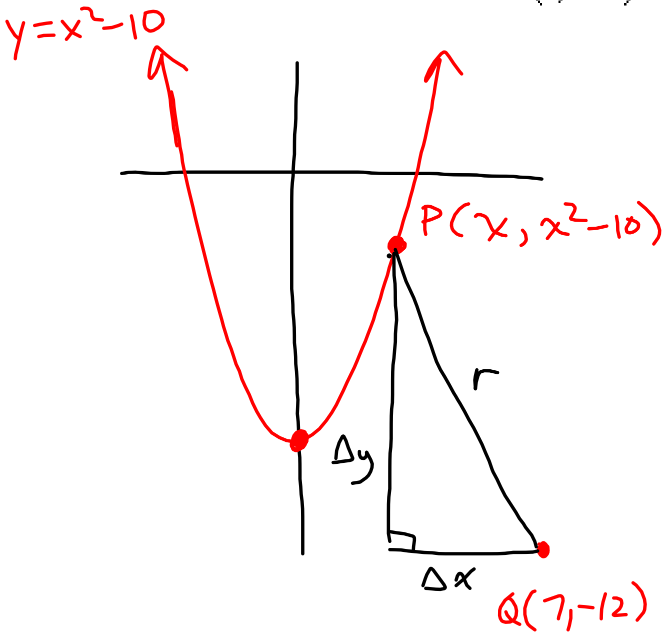
$$= \sqrt{50}$$

Thus,

$$\text{Max } A = \sqrt{50}\sqrt{50}$$

$$= 50 \text{ cm}^2$$

Ex3. Find the minimum distance from $(7, -12)$ to $y = x^2 - 10$.



$$\Delta y = x^2 - 10 - (-12)$$
$$= x^2 + 2$$

$$\Delta x = x - 7$$

$$r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$r = \sqrt{(x-7)^2 + (x^2+2)^2}$$

$$\frac{dr}{dx} = \frac{2(x-7)'(1) + 2(x^2+2)'(2x)}{2\sqrt{\quad}} = 0$$

Thus, $2x - 14 + 4x(x^2+2) = 0$

$$2x - 14 + 4x^3 + 8x = 0$$
$$4x^3 + 10x - 14 = 0$$
$$2x^3 + \underline{5x} - 7 = 0$$
$$(x-1)(2x^2 + 2x + 7) = 0$$

$b^2 - 4ac < 0$

So $x = 1$ is the ONLY sol'n!!!

So....

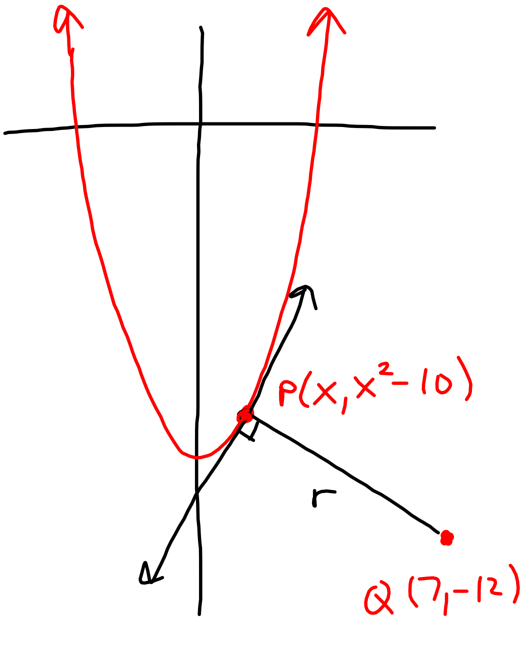
$P(1, -9)$

$Q(7, -12)$

$\therefore$ MIN Distance

$$d = \sqrt{6^2 + 3^2}$$
$$= \sqrt{45}$$

Ex3. Find the minimum distance from $(7, -12)$ to $y = x^2 - 10$.



slope of tangent $= y'$

$= 2x$

$\therefore m_{PQ} = \dfrac{-1}{2x}$

$$\frac{\Delta y}{\Delta x} = \frac{-1}{2x}$$

$$\frac{x^2+2}{x-7} = \frac{-1}{2x}$$

$$2x^3 + 4x = -x + 7$$

$$2x^3 + 5x - 7 = 0$$

Homefun:				Note:
Page 147 #	9, 11	←	Measurement Type Problems a la Optimization I	- Similar type questions are listed on the same row.
	10, **<u>13</u>**	←	Measurement Type Problems a la Optimization II	- Bold/underlined question is the trickier of the pair. So, if you're comfortable solving that one, you're probably fine with the other.
	14, **<u>19</u>**	←	Measurement Type Problems a la Optimization II	
	15, **<u>16</u>**	←	Speed Problems	
	<u>12</u>	←	Similar Triangle Problem	
	20, 22	←	Grid/Graph/Function Problems	
	<u>23</u>	←	Grid/Graph/Function Problems	