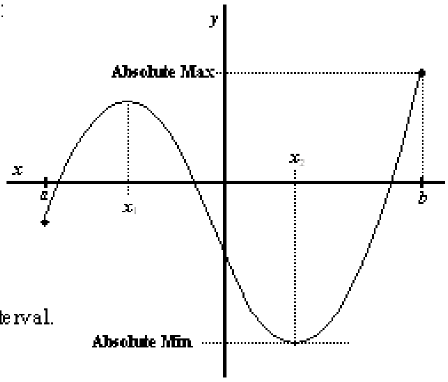
3.2: Maximum and Minimum Values on an Interval (i.e. Extreme Values) Date: ____________

The extreme values of a function f on an interval $[a, b]$ occur where:

- ➜ $f'(x) = 0$ provided that $x \in (a, b)$ or
- ➜ when $x = a$ or $x = b$.

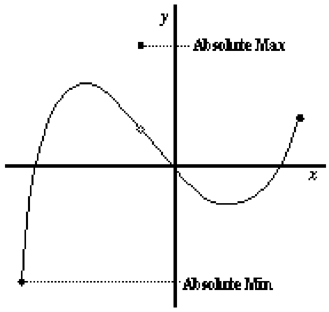
This is only the case when $f(x)$ is continuous on the interval $[a, b]$.

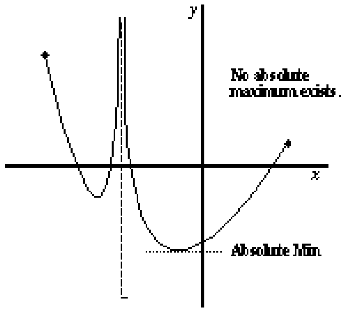


Note:
$f(x_1)$ is a **local maximum**.
$f(x_2)$ is a **local minimum** and the **absolute minimum** value of the function on the interval.
$f(b)$ is the **absolute maximum** value of the function on the interval.

If the function $f(x)$ is NOT continuous on the interval,

then those areas of discontinuity must also be checked for potential extreme values.





Ex1. Find the extreme values of the following functions and provide a quick sketch on the given interval.

a. $f(x) = -x^3 + 6x^2 + 5$ on $x \in [-3, 5]$

$f'(x) = -3x^2 + 12x$

$= -3x(x-4)$

$\therefore f'(x) = 0$ when

$x = 0$ or 4

Check

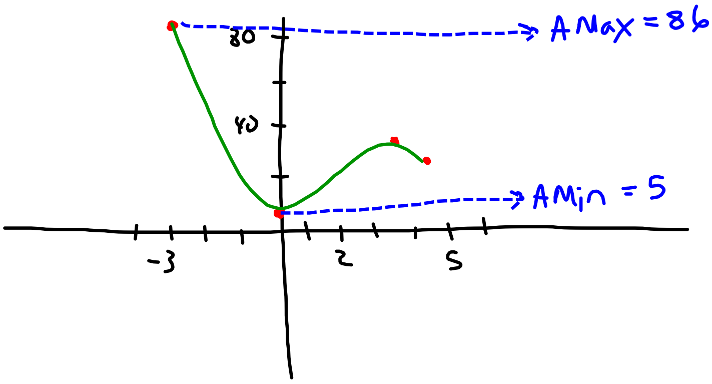
$f(-3) = 86 \leftarrow$ A Max

$f(0) = 5 \leftarrow$ A Min

$f(4) = 37$

$f(5) = 30$

$\therefore$ A Max $= 86$, A Min $= 5$



b. $C(t) = -t^3 + t^2 + 21t, \quad 0 \le t \le 5$

$c'(t) = -3t^2 + 2t + 21$

$= -(3t^2 - 2t - 21)$

$= -\dfrac{(3t-9)(3t+7)}{3}$

$= -(t-3)(3t+7)$

$c'(t) = 0$ when

$t = 3$ or $\cancel{t = -\frac{7}{3}}$

Check

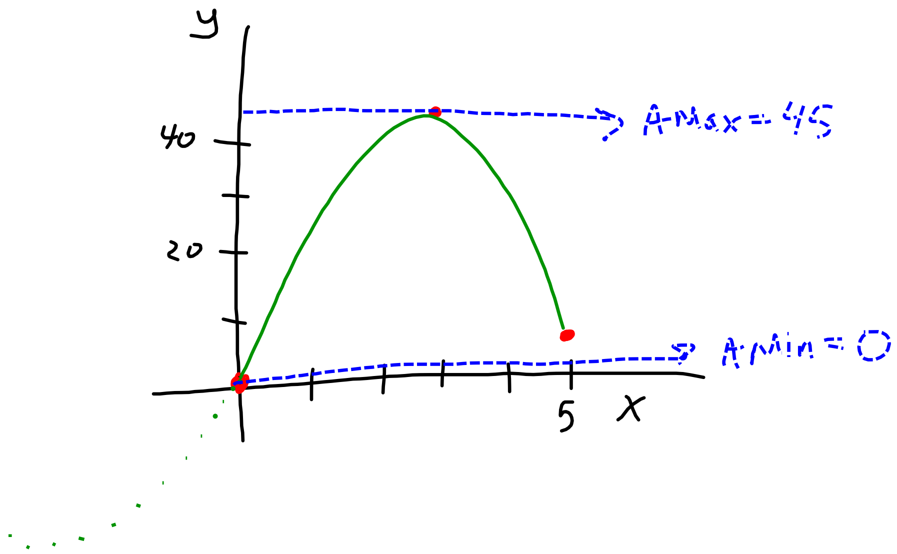
$c(0) = 0$

$c(3) = 45$

$c(5) = 5$

$\therefore$ A Max = 45

A Min = 0



c. $I(t) = \dfrac{t^2 + 2t + 16}{t + 2}$, $t \in [-10, 6]$

$$I'(t) = \frac{(2t+2)(t+2) - (t^2+2t+16)(1)}{(t+2)^2}$$

$$= \frac{2t^2+6t+4-t^2-2t-16}{(t+2)^2}$$

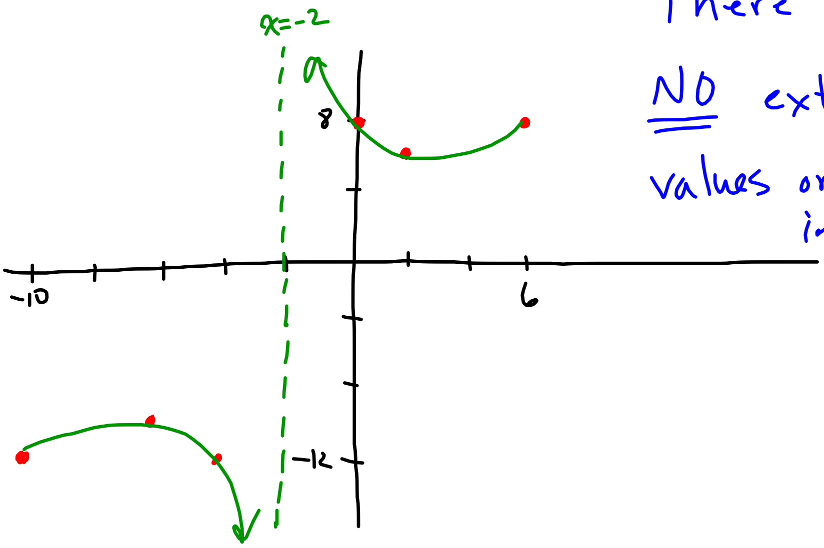
$$= \frac{t^2+4t-12}{(t+2)^2}$$

$$= \frac{(t+6)(t-2)}{(t+2)^2}$$

t	I
−10	−12
−6	−10
−4	−12
−2	X
0	8
2	6
6	8

$I'(t) = 0$ when $t = -6, 2$

VA @ $x = -2$



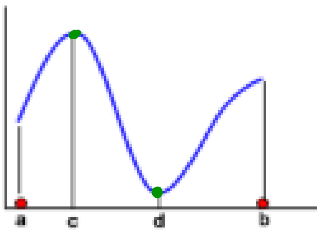
There are **NO** extreme values on this interval.

Ex2.

Extreme Value Theorem:

If f is continuous over a closed interval, then f has a maximum and minimum value over that interval.

Identify where the absolute minimum and maximum values are located in each of the three cases above.

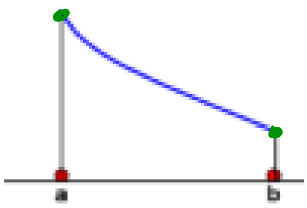


Where?

AMax is at $(c, f(c))$

AMin is at $(d, f(d))$

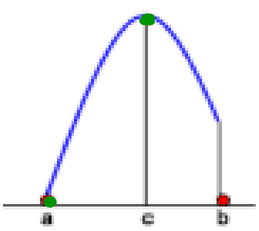
Don't usually care about "where"



When?

AMax when $x=a$

AMin when $x=b$



What?

AMax is $f(c)$

AMin is $f(a)$

Homefun: Page 135 #2abc, 4abdf, 6→12, 14, 5b