Ch 2 Review: Derivatives

I. Practice Test

Page 114 # 3, 4, [4e alternate answer $\frac{2x(8x^2-7)}{\left[\sqrt[3]{6x^2-7}\right]^2}$], 5→11

II. Implicit Differentiation

1. Find $\frac{dy}{dx}$ for the relation $9x^2 + y^2 = 36$ using each method.

   a. Solve for y explicitly as a function of x. Then, differentiate with respect to x.

   b. Use implicit differentiation to differentiate with respect to x.

   c. For what type of relations is implicit differentiation the only possible way to find $\frac{dy}{dx}$ (without technology)?

2. Find a formula for the slope of the tangent at any point (x, y) on each curve.

   a. $x^2 + y^2 = 25$

   b. $x^2y^3 + 2xy = 20$

   c. $(x-2)^2 + (y+7)^3 = 64$

   d. $x^2 + xy^3 - 6x + 8y = 9$

3. Find the slope and equation of the tangent to each curve at the given point.

   a. $(x+1)^2 + (y+4)^2 = 13, \quad (2, -2)$

   b. $x^3y - y^3 = 60, \quad (1, -4)$

III. Related Rates

1. Given $V = \frac{1}{3}\pi r^2 h$, and $r = h$, find $\frac{dr}{dt}$ if $\frac{dV}{dt} = 4$ when $r = 6$.

2. A spotlight on the ground shines on the outside wall of a parking garage 12 m away. If a 2-m tall man walks toward the garage at a speed of 0.75 m/s, how fast is the height of the man's shadow on the garage wall decreasing when he is 4 m from the building?

3. Cassandra and Marissa leave at the same time from their homes and head toward the park for their weekly soccer game. Cassandra rides her scooter west at 32 m/min, and Marissa skateboards south at 26 m/min. As the teammates approach the park, at what rate is the distance between them changing when Cassandra is 1.8 km from the park and Marissa is 1.2 km from the park?

4. After a fun morning in the snow, Emily props her 1.5 m aluminum sled up against the house and goes in for lunch. When the bottom of the sled is 1.0 m from the wall, it is slipping farther away from the wall at 15 cm/s. How fast is the top of the sled moving down the wall?

5. In a medical procedure called balloon angioplasty, a long tube with a balloon on the end is inserted into a patient's artery that has narrowed because of plaque deposits. The balloon is then expanded for a short time to make the passageway for the blood wider. For a particular patient, the doctor conducting the procedure decides that the best results will be achieved if the balloon's radius increases at a rate of 0.15 mm/s. Determine the rate at which air should be pumped into the balloon when its radius is 1 mm,

   a. assuming that the balloon is spherical

   b. assuming that the balloon is cylindrical with length 1 cm

6. Boyle's law states that when a sample of gas is compressed at a constant temperature, the pressure, P, and volume, V, satisfy the equation $PV = C$, where C is a constant. At a certain instant, the volume is 450 cm^3, the pressure is 150 kPa, and the pressure is increasing at a rate of 15 kPa/min. At what rate is the volume decreasing at this instant?

7. A spherical weather balloon is losing air at a rate of 3 m^3/min when its radius is 12 m. At this moment, what is the rate of change of the balloon's radius with respect to time?

8. A comet passing near the sun evaporates and the evaporated material forms the tail of the comet. Assume that the comet always maintains a spherical shape and that its surface area is decreasing at 250 m^2/min. Find the rate at which the radius decreases when the radius is 5 km.

9. A water tank at a filtration plant is built in the shape of a circular cone with height 4 m and diameter 5 m at the top. Water is being pumped into the tank at a rate of 1.2 m^3/min. Find the rate at which the water level is rising when the water is 3 m deep.

Answers

Implicit Differentiation

1. a. $y = \pm\sqrt{36 - 9x^2}$

   $\frac{dy}{dx} = \frac{9x}{\pm\sqrt{36 - 9x^2}}$

   b. $\frac{dy}{dx} = -\frac{9x}{y}$

   c. **Relations whose equations cannot be solved for y explicitly in terms of x.**

2. a. $-\frac{x}{y}$ b. $-\frac{2xy^3 + 2y}{3x^2y^2 + 2x}$

   c. $\frac{2(2 - x)}{3(y + 7)^2}$ d. $\frac{6 - 2x - y^3}{3xy^2 + 8}$

3. a. -1.5; $3x + 2y - 2 = 0$

   b. $-\frac{12}{47}$; $12x + 47y + 176 = 0$

Related Rates

1. $\frac{1}{9\pi}$
2. **0.281 25 m/s**
3. **41.05 m/min**
4. **13.42 cm/s**
5. a. **1.885 mm^3/s**

   b. **9.425 mm^3/s**
6. **45 cm^3/min**
7. $-\frac{1}{192\pi}$ **m/min**
8. **0.001 99 m/min**
9. **0.109 m/min**

IV. Additional Review; Page 111

1st Principles:

2, 22, 23 [**23c) f is not differentiable at x =1,6 too, so f is not differentiable if $1 \le x \le 6$**]

Differentiating Skills:

3cef, 4adf, 5acde, 28 [28f $\frac{-6x^3(7x - 16)}{\sqrt{(4x - 8)^3}}$ or $\frac{-3x^3(7x - 16)}{4\sqrt{(x - 2)^3}}$; 28g $\frac{8(2x + 5)^3(x + 2)(x + 3)}{(6 - x^2)^5}$]

Applications:

16, 17[17a $N'(t) = \frac{30t}{\left(\sqrt{9 + t^2}\right)^3}$], 18→21, 24, 30abc

Tangents:

8, 10a, 11a, 12, 13, 14cd, 15, 29

Chain Rule/Composite Functions:

6, 7, 26, 27 [**for 26a and 27a, the u function in the back should be $u = 2x - 3$**]