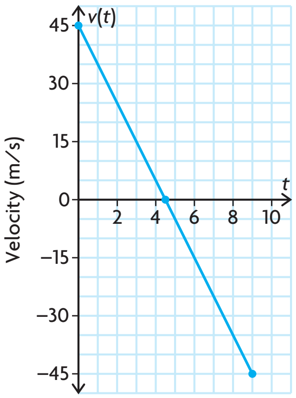
Review Exercise, pp. 156–159

1. $f'(x) = 4x^3 + 4x^{-5}$,
   $f''(x) = 12x^2 - 20x^{-6}$
2. $\dfrac{d^2y}{dx^2} = 72x^7 - 42x$
3. $v = 2t + (2t - 3)^{\frac{1}{2}}$,
   $a = 2 - (2t - 3)^{\frac{3}{2}}$
4. $v(t) = 1 - 5t^{-2}$,
   $a(t) = 10t^{-3}$
5. The upward velocity is positive for $0 \leq t \leq 4.5$ s, zero for $t = 4.5$ s, and negative for $t > 4.5$ s.



6. **a.** min: -52,
   max: 0
   b. min: -65,
   max: 16
   c. min: 12,
   max: 20
7. **a.** 62 m
   b. Yes, 2 m beyond the stop sign
   c. Stop signs are located two or more metres from an intersection. Since the car only went 2 m beyond the stop sign, it is unlikely the car would hit another vehicle travelling perpendicular.
8. min is 2, max is $2 + 3\sqrt{3}$
9. 250
10. **a.** **i.** \$2200
    ii. \$5.50
    iii. \$3.00; \$3.00
    b. **i.** \$24 640
    ii. \$61.60
    iii. \$43.20; \$43.21
    c. **i.** \$5020
    ii. \$12.55
    iii. \$0.03; \$0.03
    d. **i.** \$2705
    ii. \$6.88
    iii. \$5.01; \$5.01
11. 2000
12. **a.** moving away from its starting point
    b. moving away from the origin and towards its starting position
13. **a.** $t = \dfrac{2}{3}$ **b.** yes
14. 27.14 cm by 27.14 cm for the base and height 13.57 cm
15. length 190 m, width approximately 63 m
16. 31.6 cm by 11.6 cm by 4.2 cm
17. radius 4.3 cm, height 8.6 cm
18. Run the pipe 7.2 km along the river shore and then cross diagonally to the refinery.
19. 10:35 p.m.
20. \$204 or \$206
21. The pipeline meets the shore at a point C, 5.7 km from point A, directly across from P.
22. 11.35 cm by 17.02 cm
23. 34.4 m by 29.1 m
24. 2:23 p.m.
25. 3.2 km from point C
26. **a.** absolute maximum: $f(7) = 41$,
    absolute minimum: $f(1) = 5$
    b. absolute maximum: $f(3) = 36$,
    absolute minimum: $f(-3) = -18$
    c. absolute maximum: $f(5) = 67$,
    absolute minimum: $f(-5) = -63$
    d. absolute maximum: $f(4) = 2752$,
    absolute minimum: $f(-2) = -56$
27. **a.** 62.9 m **c.** 3.6 m/s^2
    b. 4.7 s
28. **a.** $f''(2) = 60$ **d.** $f''(1) = -\dfrac{5}{16}$
    b. $f''(-1) = 26$ **e.** $f''(4) = -\dfrac{1}{108}$
    c. $f''(0) = 192$ **f.** $f''(8) = -\dfrac{1}{72}$
29. **a.** position: 1, velocity: $\frac{1}{6}$,
    acceleration: $-\left(\frac{1}{18}\right)$, speed: $\frac{1}{6}$
    b. position: $\frac{8}{3}$, velocity: $\frac{4}{9}$,
    acceleration: $\frac{10}{27}$, speed: $\frac{4}{9}$
30. **a.** $v(t) = \dfrac{2}{3}(t^2 + t)^{-\frac{1}{3}}(2t + 1)$,
    $a(t) = \dfrac{2}{9}(t^2 + t)^{-\frac{4}{3}}(2t^2 + 2t - 1)$
    b. 1.931 m/s
    c. 2.36 m/s
    d. undefined
    e. 0.141 m/s^2

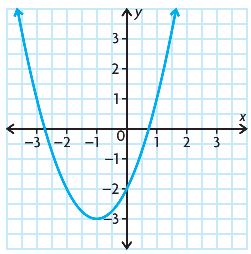
Chapter 3 Test, p. 160

1. **a.** $y'' = 14$
   b. $f''(x) = -180x^3 - 24x$
   c. $y'' = 60x^{-5} + 60x$
   d. $f''(x) = 96(4x - 8)$
2. **a.** $v(3) = -57$,
   $a(3) = -44$
   b. $v(2) = 6$,
   $a(2) = -24$
3. **a.** $v(t) = 2t - 3$,
   $a(t) = 2$
   b. -0.25 m
   c. 1 m/s, 1 m/s
   d. between $t = 0$ s and $t = 1.5$ s
   e. 2 m/s^2
4. **a.** min: -63, max: 67
   b. min: 7.5, max: 10
5. **a.** 2.1 s
   b. about 22.9
6. 250 m by 166.7 m
7. 162 mm by 324 mm by 190 mm
8. \$850/month

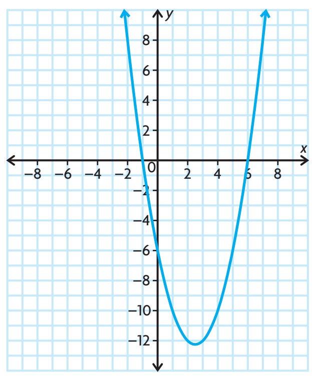
Chapter 4

Review of Prerequisite Skills, pp. 162–163

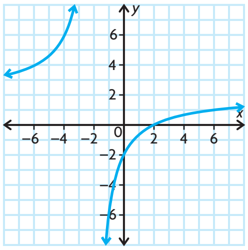
1. **a.** $y = -\dfrac{3}{2}$ or $y = 1$
   b. $x = 7$ or $x = -2$
   c. $x = -\dfrac{5}{2}$
   d. $y = 1$ or $y = -3$ or $y = -2$
2. **a.** $x < -\dfrac{7}{3}$
   b. $x \leq 2$
   c. $-1 < t < 3$
   d. $x < -4$ or $x > 1$
3. **a.**



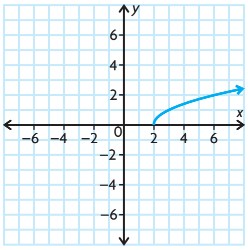
b.



c.



d.



4. **a.** 0
b. 7
c. 27
d. 3

5. **a.** $x^3 + 6x + x^{-2}$
b. $-\dfrac{x^2 + 2x + 3}{(x^2 - 3)^2}$
c. $2(3x^2 - 6x)(6x - 6)$
d. $\dfrac{t - 8}{(t - 4)^{\frac{3}{2}}}$

6. **a.** $x - 8 + \dfrac{28}{x + 3}$
b. $x + 7 - \dfrac{2}{x - 1}$

7. $\left(\dfrac{2}{3}, 2.19\right), (-1, 4.5)$

8. **a.** If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$.
b. If $f(x) = k$, where k is a constant, then $f'(x) = 0$.
c. If $k(x) = f(x)g(x)$, then $k'(x) = f'(x)g(x) - f(x)g'(x)$
d. If $h(x) = \dfrac{f(x)}{g(x)}$, then
$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2},$$
$g(x) \neq 0$.
e. If f and g are functions that have derivatives, then the composite function $h(x) = f(g(x))$ has a derivative given by $h'(x) = f'(g(x)) \cdot g'(x)$.
f. If u is a function of x, and n is a positive integer, then $\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$.

9. **a.** As $x \to \pm\infty, f(x) \to \infty$.
b. As $x \to -\infty, f(x) \to -\infty$.
As $x \to \infty, f(x) \to \infty$.
c. As $x \to -\infty, f(x) \to -\infty$.
As $x \to \infty, f(x) \to -\infty$.

10. **a.** $\dfrac{1}{2x}$; $x = 0$
b. $\dfrac{1}{-x + 3}$; $x = 3$
c. $\dfrac{1}{(x + 4)^2 + 1}$; no vertical asymptote
d. $\dfrac{1}{(x + 3)^2}$; $x = -3$

11. **a.** $y = 0$
b. $y = 4$
c. $y = \dfrac{1}{2}$
d. $y = 2$

12. **a.** **i.** no x-intercept; $(0, 5)$
ii. $(0, 0)$; $(0, 0)$
iii. $\left(\dfrac{5}{3}, 0\right)$; $\left(0, \dfrac{5}{3}\right)$
iv. $\left(\dfrac{2}{5}, 0\right)$; no y-intercept
b. **i.** Domain: $\{x \in \mathbf{R} \mid x \neq -1\}$,
Range: $\{y \in \mathbf{R} \mid y \neq 0\}$
ii. Domain: $\{x \in \mathbf{R} \mid x \neq 2\}$,
Range: $\{y \in \mathbf{R} \mid y \neq 4\}$
iii. Domain: $\left\{x \in \mathbf{R} \mid x \neq \dfrac{1}{2}\right\}$,
Range: $\left\{y \in \mathbf{R} \mid y \neq \dfrac{1}{2}\right\}$
iv. Domain: $\{x \in \mathbf{R} \mid x \neq 0\}$,
Range: $\{y \in \mathbf{R} \mid y \neq 2\}$

Section 4.1, pp. 169–171

1. **a.** $(0, 1), (-4, 33)$
b. $(0, 2)$
c. $\left(\dfrac{1}{2}, 0\right), (2.25, -48.2), (-2, -125)$
d. $\left(1, \dfrac{5}{2}\right), \left(-1, -\dfrac{5}{2}\right)$

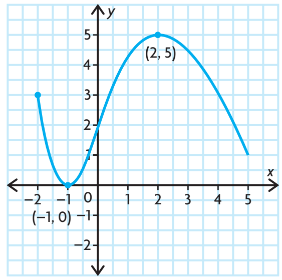
2. A function is increasing when $f'(x) > 0$ and is decreasing when $f'(x) < 0$.

3. **a.** **i.** $x < -1, x > 2$
ii. $-1 < x < 2$
iii. $(-1, 4), (2, -1)$
b. **i.** $-1 < x < 1$
ii. $x < -1, x > 1$
iii. $(-1, 2), (2, 4)$
c. **i.** $x < -2$
ii. $-2 < x < 2, 2 < x$
iii. none
d. **i.** $-1 < x < 2, 3 < x$
ii. $x < -1, 2 < x < 3$
iii. $(2, 3)$

4. **a.** increasing: $x < -2, x > 0$;
decreasing: $-2 < x < 0$
b. increasing: $x < 0, x > 4$;
decreasing: $0 < x < 4$
c. increasing: $x < -1, x > 1$;
decreasing: $-1 < x < 0$,
$0 < x < 1$
d. increasing: $-1 < x < 3$;
decreasing: $x < -1, x > 3$
e. increasing: $-2 < x < 0, x > 1$;
decreasing: $x < -2, 0 < x < 1$
f. increasing: $x > 0$;
decreasing: $x < 0$

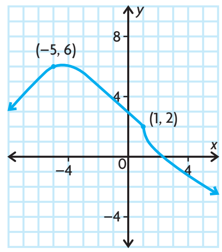
5. increasing: $-3 < x < -2, x > 1$;
decreasing: $x < -3, -2 < x < 1$

6.

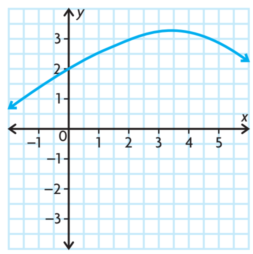


7. $a = 3, b = -9, c = -9$

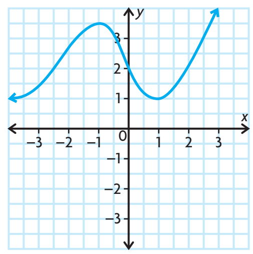
8.



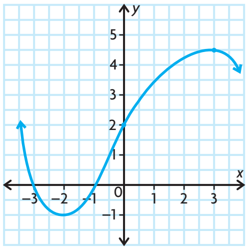
9. **a.** **i.** $x < 4$
ii. $x > 4$
iii. $x = 4$



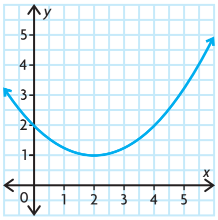
b. **i.** $x < -1, x > 1$
ii. $-1 < x < 1$
iii. $x = -1, x = 1$



c. **i.** $-2 < x < 3$
ii. $x < -2, x > 3$
iii. $x = -2, x = 3$



d. **i.** $x > 2$
ii. $x < 2$
iii. $x = 2$



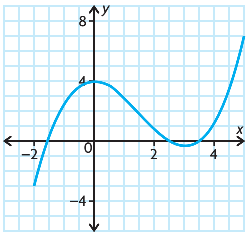
10. $f(x) = ax^2 + bx + c$
$f'(x) = 2ax + b$

Let $f'(x) = 0$, then $x = \frac{-b}{2a}$.

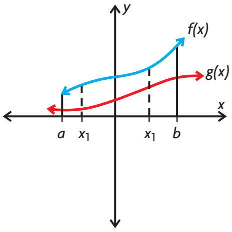
If $x < \frac{-b}{2a}, f'(x) < 0$, therefore the function is decreasing.
If $x > \frac{-b}{2a}, f'(x) > 0$, therefore the function is increasing.

11. $f'(x) = 0$ for $x = 2$,
increasing: $x > 2$,
decreasing: $x < 2$,
local minimum: $(2, -44)$

12.



13. Let $y = f(x)$ and $u = g(x)$.
Let x_1 and x_2 be any two values in the interval $a \le x \le b$ so that $x_1 < x_2$.
Since $x_1 < x_2$, both functions are increasing:
$f(x_2) > f(x_1)$ (1)
$g(x_2) > g(x_1)$ (2)
$yu = f(x) \cdot g(x)$
(1) $\times$ (2) results in
$f(x_2) \cdot g(x_2) > f(x_1)g(x_1)$
The function yu or $f(x) \cdot g(x)$ is strictly increasing.

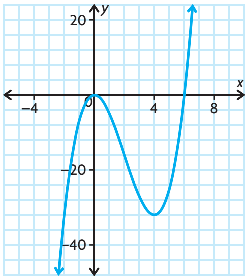


14. strictly decreasing

Section 4.2, pp. 178–180

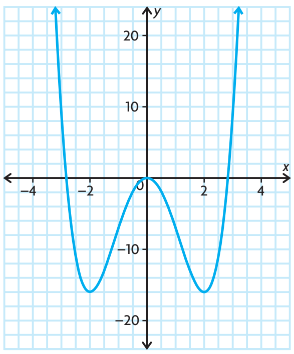
1. Determining the points on the graph of the function for which the derivative of the function at the x-coordinate is 0

2. **a.** Take the derivative of the function. Set the derivative equal to 0. Solve for x. Evaluate the original function for the values of x. The (x, y) pairs are the critical points.
b. $(0, 0), (4, -32)$

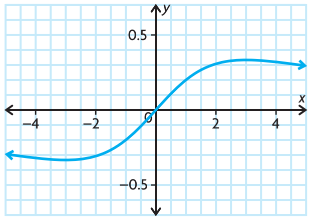


3. **a.** local minima: $(-2, -16), (2, -16)$,
local maximum: $(0, 0)$
b. local minimum: $(-3, -0.3)$,
local maximum: $(3, 0.3)$
c. local minimum: $(-2, 5)$,
local maximum: $(0, 1)$

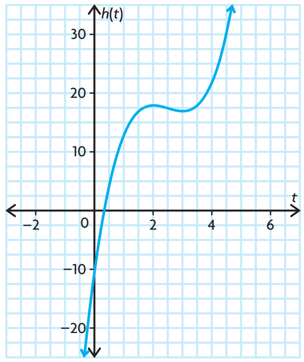
4. **a.** $(0, 0), (2\sqrt{2}, 0), (-2\sqrt{2}, 0)$



b. $(0, 0)$

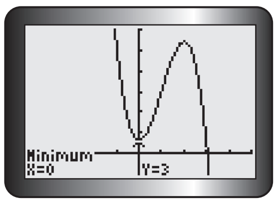


c. $(-3, 1, 0), (0, 1)$

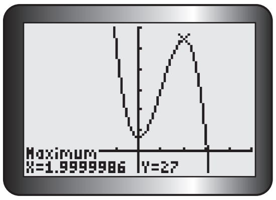


5. **a.** local minimum: $(0, 3)$,
local maximum: $(2, 27)$,
Tangent is parallel to the horizontal axis for both.
b. $(0, 0)$ neither maximum nor minimum,
Tangent is parallel to the horizontal axis.
c. $(5, 0)$; neither maximum nor minimum,
Tangent is not parallel to the horizontal axis.
d. local minimum: $(0, -1)$,
Tangent is parallel to the horizontal axis.
$(-1, 0)$ and $(1, 0)$ are neither maxima or minima.
Tangent is not parallel to the horizontal axis for either.

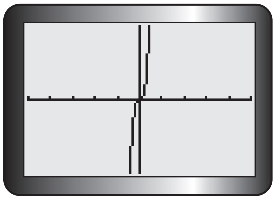
6. **a.**



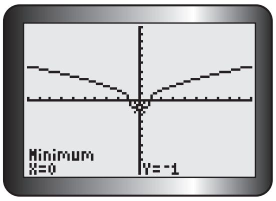
b.



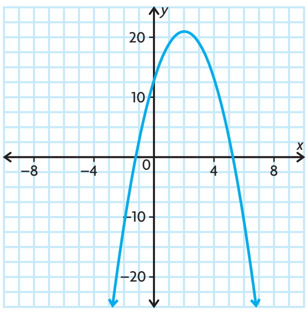
c.



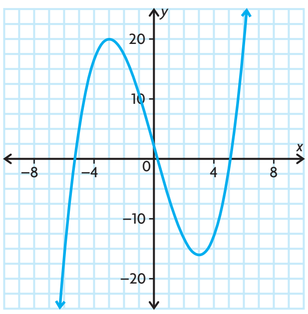
d.



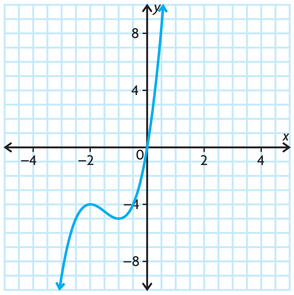
7. **a.** $(2, 21)$ local maximum



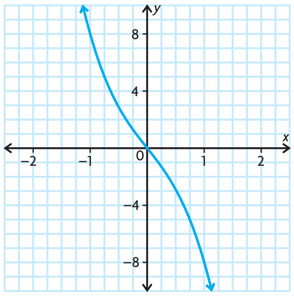
b. $(-3, 20)$ local maximum, $(3, -16)$ local minimum



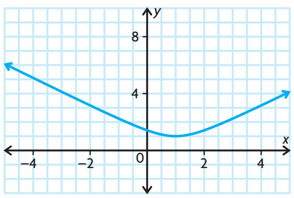
c. $(-2, -4)$ local maximum, $(-1, -5)$ local minimum



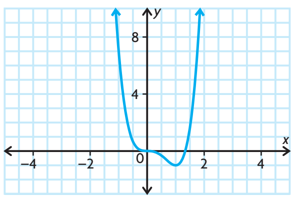
d. no critical points



e. $(1, 1)$ local minimum

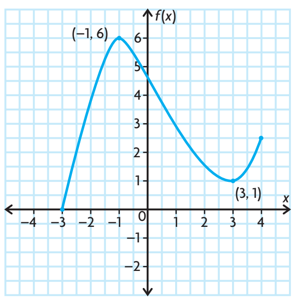


f. $(0, 0)$ neither maximum nor minimum, $(1, -1)$ local minimum



8. local minima at $x = -6$ and $x = 2$; local maximum at $x = -1$

9.



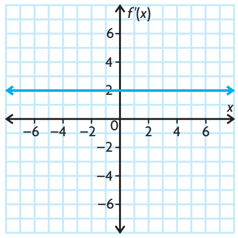
10. $a = -\frac{11}{9}, b = \frac{22}{3}, c = 1$

11. $p = -2, q = 6$
minimum; the derivative is negative to the left and positive to the right

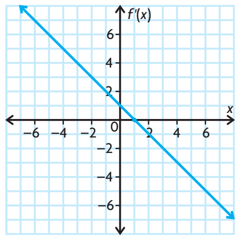
12. **a.** $k < 0$
b. $k = 0$
c. $k > 0$

13. $a = -1, b = 3, c = 0, d = 0$

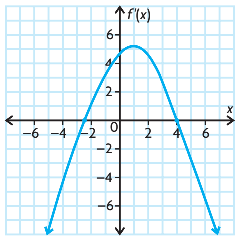
14. **a.**



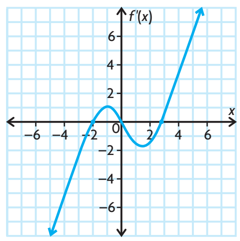
b.



c.

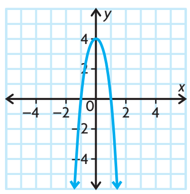


d.

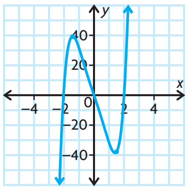


15. **a.** $a = -4, b = -36, c = 0$
b. $(3, -198)$
c. local minimum: $(-2, -73)$ and $(3, -198)$,
local maximum: $(0, -9)$

16. **a.** local maximum: $(0, 4)$



b. local minimum: $(1.41, -39.6)$, local maximum: $(1.41, 39.6)$



17. $h(x) = \dfrac{f(x)}{g(x)}$

Since $f(x)$ has a local maximum at $x = c$, then $f'(x) > 0$ for $x < c$ and $f'(x) < 0$ for $x > c$. Since $g(x)$ has a local minimum at $x = c$, then $g'(x) < 0$ for $x < c$ and $g'(x) > 0$ for $x > c$.

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

If $x < c, f'(x) > 0$ and $g'(x) < 0$, then $h'(x) > 0$.
If $x > c, f'(x) < 0$ and $g'(x) > 0$, then $h'(x) < 0$.
Since for $x < c$, $h'(x) > 0$ and for $x > c$, $h'(x) < 0$. Therefore, $h(x)$ has a local maximum at $x = c$.

Section 4.3, pp. 193–195

1. **a.** vertical asymptotes at $x = -2$ and $x = 2$; horizontal asymptote at $y = 1$
b. vertical asymptote at $x = 0$; horizontal asymptote at $y = 0$

2. $f(x) = \dfrac{g(x)}{h(x)}$

Conditions for a vertical asymptote: $h(x) = 0$ must have at least one solution s, and $\lim_{x \to \infty} f(x) = \infty$.

Conditions for a horizontal asymptote: $\lim_{x \to \infty} f(x) = k$, where $k \in \mathbf{R}$, or $\lim_{x \to \infty} f(x) = k$, where $k \in \mathbf{R}$.

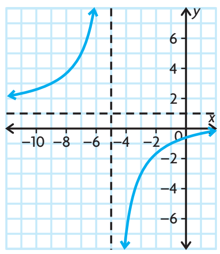
Condition for an oblique asymptote: The highest power of $g(x)$ must be one more than the highest power of $k(x)$.

3. **a.** 2
b. 5
c. $-\dfrac{5}{2}$
d. ∞

4. **a.** $x = -5$; large and positive to left of asymptote, large and negative to right of asymptote
b. $x = 2$; large and negative to left of asymptote, large and positive to right of asymptote
c. $x = 3$; large and positive to left of asymptote, large and positive to right of asymptote
d. hole at $x = 3$, no vertical asymptote
e. $x = -3$; large and positive to left of asymptote, large and negative to right of asymptote
$x = 1$; large and negative to left of asymptote, large and positive to right of asymptote
f. $x = -1$; large and positive to left of asymptote, large and negative to right of asymptote
$x = 1$; large and negative to left of asymptote, large and positive to right of asymptote

5. **a.** $y = 1$; large negative: approaches from above, large positive: approaches from below
b. $y = 0$; large negative: approaches from below, large positive: approaches from above
c. $y = 3$; large negative: approaches from above, large positive: approaches from above
d. no horizontal asymptotes

6. **a.**



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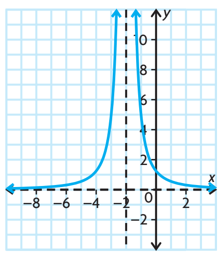
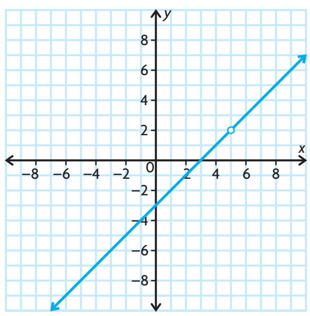


c.



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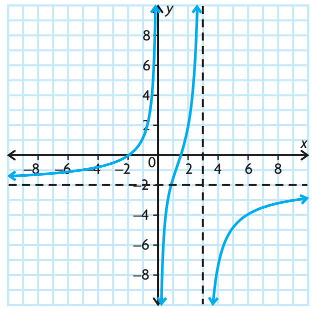
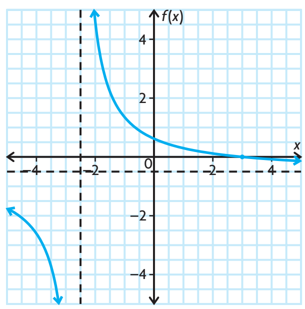


7. **a.** $y = 3x - 7$
b. $y = x + 3$
c. $y = x - 2$
d. $y = x + 3$

8. **a.** large negative: approaches from below, large positive: approaches from above
b. large negative: approaches from above, large positive: approaches from below

9. **a.** $x = -5$; large and positive to left of asymptote, large and negative to right of asymptote
$y = 3$
b. $x = 1$; large and positive to left of asymptote, large and positive to right of asymptote
$y = 1$
c. $x = -2$; large and negative to left of asymptote, large and positive to right of asymptote
$y = 1$
d. $x = 2$; large and negative to left of asymptote, large and positive to right of asymptote
$y = 1$

10. **a.**



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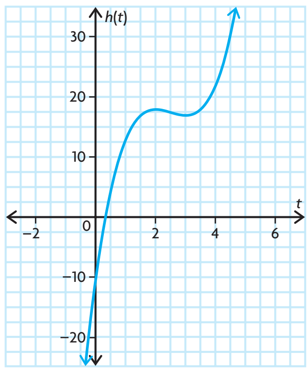
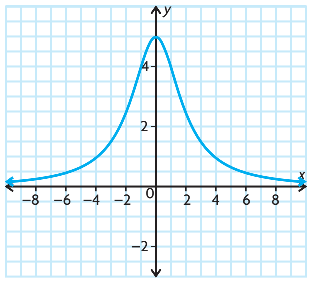


c.



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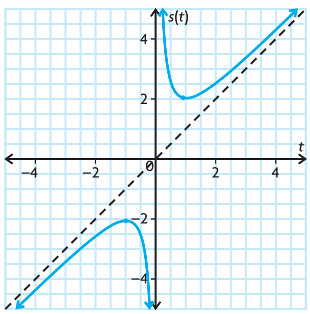

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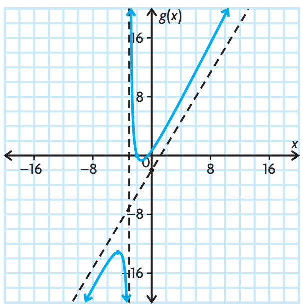

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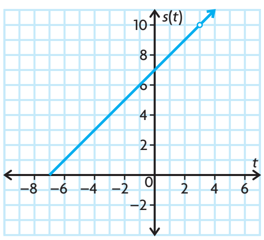
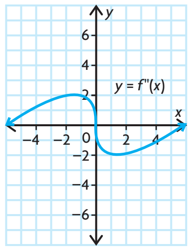


11. **a.** $y = \dfrac{a}{c}$

b. $x = -\dfrac{d}{c}$

12. **a.**



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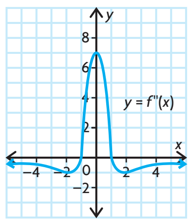
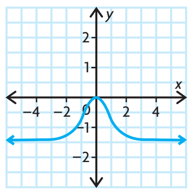
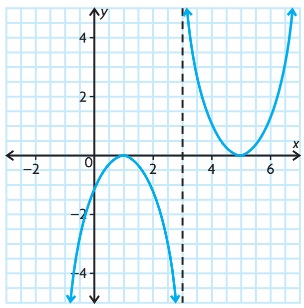


13. **a.**



b.



14. **a.** $f(x)$ and $r(x)$: $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to\infty} r(x)$ exist.

b. $h(x)$: the highest degree of x in the numerator is exactly one degree higher than the highest degree of x in the denominator.

c. $h(x)$: the denominator is defined for all $x \in \mathbf{R}$.

$f(x) = \dfrac{-x-3}{(x-7)(x+2)}$ has vertical Asymptotes at $x = 7$ and $x = -2$.

As $x \to -2^-$, $f(x) \to -\infty$.
As $x \to -2^+$, $f(x) \to \infty$.
As $x \to 7^-$, $f(x) \to \infty$.
As $x \to 7^+$, $f(x) \to -\infty$.

$f(x)$ has a horizontal asymptote at $y = 0$.

$g(x)$ has a vertical asymptote at $x = 3$.

As $x \to 3^-$, $g(x) \to \infty$.
As $x \to 3^+$, $g(x) \to -\infty$.

$y = x$ is an oblique asymptote for $h(x)$.

$r(x) = \dfrac{(x+3)(x-2)}{(x-4)(x+4)}$ has vertical asymptotes at $x = -4$ and $x = 4$.

As $x \to -4^-$, $r(x) \to \infty$.
As $x \to -4^+$, $r(x) \to -\infty$.
As $x \to 4^-$, $r(x) \to -\infty$.
As $x \to 4^+$, $r(x) \to \infty$.

$r(x)$ has a horizontal asymptote at $y = 1$.

15. $a = \dfrac{9}{5}$, $b = \dfrac{3}{5}$

16. **a.** $\lim\limits_{x\to\infty} \dfrac{x^2+1}{x+1} = \lim\limits_{x\to\infty} \dfrac{x+\frac{1}{x}}{1+\frac{1}{x}}$

$= \infty$

$\lim\limits_{x\to\infty} \dfrac{x^2+2x+1}{x+1}$

$= \lim\limits_{x\to\infty} \dfrac{(x+1)(x+1)}{(x+1)}$

$= \lim\limits_{x\to\infty} (x+1)$

$= \infty$

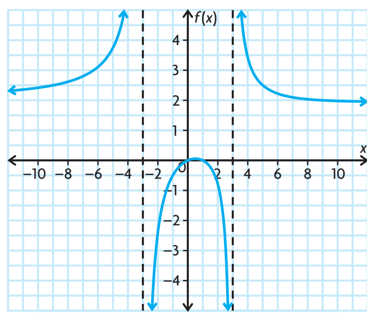
b. $\lim\limits_{x\to\infty} \left[\dfrac{x^2+1}{x+1} - \dfrac{x^2+2x+1}{x+1}\right]$

$= \lim\limits_{x\to\infty} \dfrac{x^2+1-x^2-2x-1}{x+1}$

$= \lim\limits_{x\to\infty} \dfrac{-2x}{x+1}$

$= \lim\limits_{x\to\infty} \dfrac{-2}{1} = -2$

17.



Mid-Chapter Review, pp. 196–197

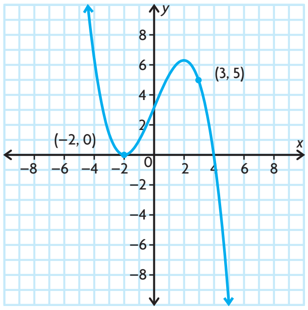
1. **a.** decreasing: $(-\infty, 2)$, increasing: $(2, \infty)$

b. decreasing: $(0, 2)$, increasing: $(-\infty, 0)$, $(2, \infty)$

c. increasing: $(-\infty, -3)$, $(3, \infty)$

d. decreasing: $(-\infty, 0)$, increasing: $(0, \infty)$

2. increasing: $x < -1$ and $x > 2$, decreasing: $-1 < x < 2$

3. 

4. a. -4
   b. $0, \pm 3\sqrt{3}$
   c. $0, \pm\sqrt{2}$
   d. $\pm 1, \pm 2$
   e. 0
   f. $\pm\sqrt{2}$

5. a. $x = 1$ local maximum, $x = 2$ local minimum
   b. $x = -\frac{2}{3}$ local maximum, $x = 2$ local minimum

6. ± 2

7. $x = 2$ local minimum, increasing: $x > 2$, decreasing: $x < 2$

8. a. $x = -2$; large and positive to left of asymptote, large and negative to right of asymptote
   b. $x = -3$; large and negative to left of asymptote, large and positive to right of asymptote
   $x = 3$; large and positive to left of asymptote, large and negative to right of asymptote
   c. $x = -3$; large and negative to left of asymptote, large and positive to right of asymptote
   d. $x = -\frac{2}{3}$; large and positive to left of asymptote, large and negative to right of asymptote
   $x = 5$; large and positive to left of asymptote, large and negative to right of asymptote

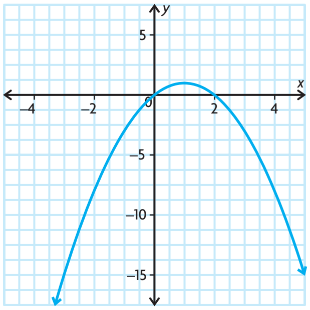
9. a. $y = 3$; large negative: approaches from above, large positive: approaches from below
   b. $y = 1$; large negative: approaches from below, large positive: approaches from above

10. a. $x = 5$; large and positive to left of asymptote, large and positive to right of asymptote
   b. no discontinuities
   c. $x = 6 + 2\sqrt{6}$; large and negative to left of asymptote, large and positive to right of asymptote
   $x = 6 - 2\sqrt{6}$; large and negative to left of asymptote, large and positive to right of asymptote

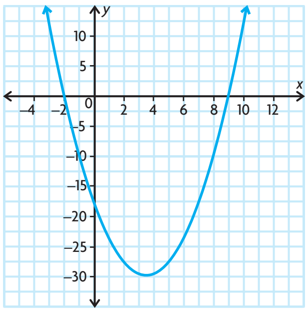
11. a. $f(x)$ is increasing.
   b. $f(x)$ is decreasing.

12. increasing: $0 < t < 0.97$, decreasing: $t > 0.97$

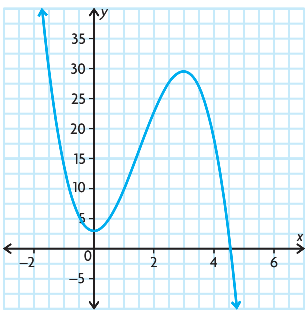
13. increasing: $t > 2.5198$, decreasing: $t < 2.5198$

14. 

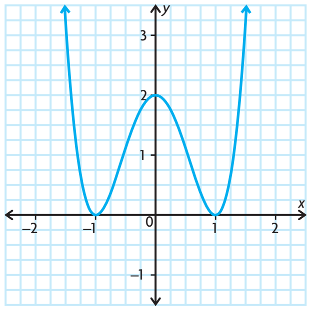
15. a. i. $x = \frac{7}{2}$
   ii. increasing: $x > \frac{7}{2}$, decreasing: $x < \frac{7}{2}$
   iii. local minimum at $x = \frac{7}{2}$
   iv.



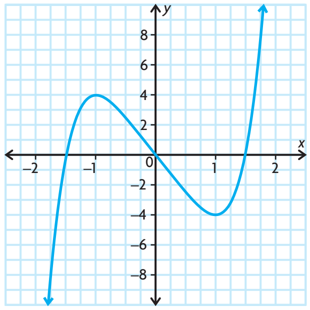
   b. i. $x = 0, x = 3$
   ii. increasing: $0 < x < 3$, decreasing: $x < 0, x > 3$
   iii. local minimum at $x = 0$, local maximum at $x = 3$
   iv.



   c. i. $x = -1, x = 0, x = 1$
   ii. increasing: $-1 < x < 0, x > 1$; decreasing: $x < -1, 0 < x < 1$
   iii. local maximum at $x = 0$, local minimum at $x = 1, -1$
   iv.



   d. i. $x = -1, x = 1$
   ii. increasing: $x < -1, x > 1$; decreasing: $-1 < x < 1$
   iii. local maximum at $x = -1$, local minimum at $x = 1$
   iv.

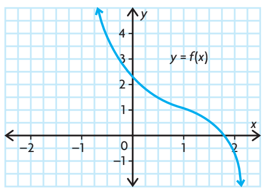


16. a. $x = -\frac{1}{2}$; large and positive to left of asymptote, large and negative to right of asymptote; $y = \frac{1}{2}$
   b. $x = -2$; large and positive to left of asymptote, large and positive to right of asymptote; $y = 1$
   c. $x = -3$; large and positive to left of asymptote, large and negative to right of asymptote; $y = -1$
   d. $x = -4$; large and negative to left of asymptote, large and positive to right of asymptote; $y = 2$

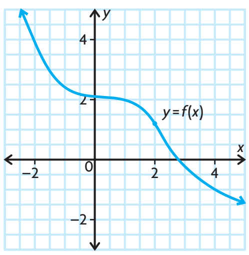
17. a. $-\frac{2}{3}$
   b. $\frac{1}{6}$
   c. -3
   d. 0
   e. ∞
   f. 1
   g. 1
   h. ∞

Section 4.4, pp. 205–206

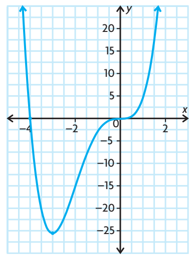
1. **a.** A: negative, B: negative, C: positive, D: positive
   b. A: negative, B: negative, C: positive, D: negative
2. **a.** local minimum: $(5, -105)$, local maximum: $(-1, 20)$
   b. local maximum: $\left(0, \frac{25}{48}\right)$
   c. local maximum: $(-1, -2)$, local minimum: $(1, 2)$
   d. $(3, 8)$ is neither a local maximum or minimum.
3. **a.** $\left(\frac{4}{3}, -14\frac{20}{27}\right)$
   b. $\left(-4, \frac{25}{64}\right)\left(4, \frac{25}{64}\right)$
   c. no points of inflection
   d. $(3, 8)$
4. **a.** 24; above
   b. 4; above
   c. $-\dfrac{9}{100\sqrt{10}}$; below
   d. $-\frac{2}{27}$; below
5. **a.** **i.** concave up on $x < 1$, concave down on $x > 1$
   ii. $x = 1$
   iii.



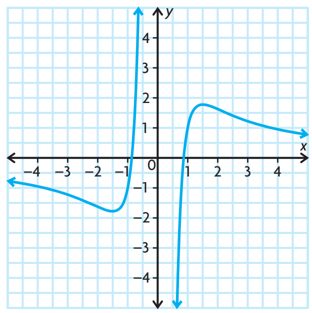
   b. **i.** concave up on $x < 0$ or $x > 2$, concave down on $0 < x < 2$
   ii. $x = 0$ and $x = 2$
   iii.



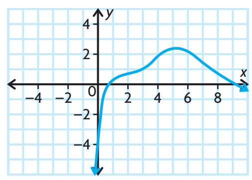
6. For any function $y = f(x)$, find the critical points, i.e., the values of x such that $f'(x) = 0$ or $f'(x)$ does not exist. Evaluate $f''(x)$ for each critical value. If the value of the second derivative at a critical point is positive, the point is a local minimum. If the value of the second derivative at a critical point is negative, the point is a local maximum.
7. Use the first derivative test or the second derivative test to determine the type of critical points that may be present.
8. **a.** **i.** $(-2, -16)$, $(0, 0)$
   ii.



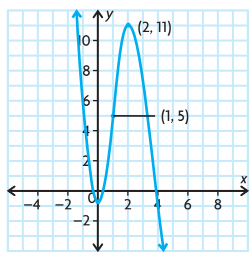
   b. **i.** $\left(-\frac{3}{\sqrt{2}}, -\frac{8\sqrt{2}}{9}\right), \left(\frac{3}{\sqrt{2}}, \frac{8\sqrt{2}}{9}\right)$
   ii.



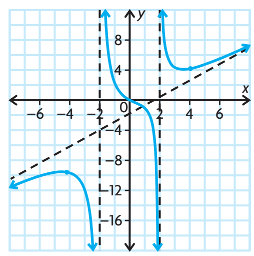
9.



10. $a = -3$, $b = 9$, $c = -1$



11. $\dfrac{27}{64}$
12. $f(x) = ax^4 + bx^3$
    $f'(x) = 4ax^3 + 3bx^2$
    $f''(x) = 12ax^2 + 6bx$
    For possible points of inflection, we solve $f''(x) = 0$:
    $12ax^2 + 6bx = 0$
    $6x(2ax + b) = 0$
    $x = 0$ or $x = -\frac{b}{2a}$.
    The graph of $y = f''(x)$ is a parabola with x-intercepts 0 and $-\frac{b}{2a}$.
    We know the values of $f''(x)$ have opposite signs when passing through a root. Thus, at $x = 0$ and at $x = -\frac{b}{2a}$, the concavity changes as the graph goes through these points. Thus, $f(x)$ has points of inflection at $x = 0$ and $x = -\frac{b}{2a}$.
    To find the x-intercepts, we solve $f(x) = 0$
    $x^3(ax + b) = 0$
    $x = 0$ or $x = -\frac{b}{a}$
    The point midway between the x-intercepts has x-coordinate $-\frac{b}{2a}$.
    The points of inflection are $(0, 0)$ and $\left(-\frac{b}{2a}, -\frac{b^4}{16a^3}\right)$.
13. **a.**



    b. Answers may vary. For example, there is a section of the graph that lies between the two sections of the graph that approaches the asymptote.
14. $n = 1$, $n = 2$: no inflection points; $n = 3$, $n = 4$: inflection point at $x = c$; The graph of f has an inflection point at $x = c$ when $n \geq 3$.

Section 4.5, pp. 212–213

1. A cubic polynomial that has a local minimum must also have a local maximum. If the local minimum is to the left of the local maximum, then $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. If the local minimum is to the right of the local maximum, then $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.
2. A polynomial of degree three has at most two local extremes. A polynomial of degree four has at most three local extremes. Since each local maximum and minimum of a function corresponds

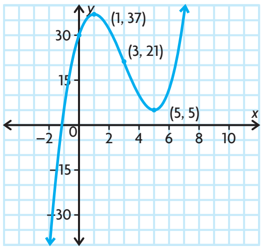
to a zero of its derivative, the number of zeros of the derivative is the maximum number of local extreme values that the function can have. For a polynomial of degree n, the derivative has degree $n - 1$, so it has at most $n - 1$ zeros, and thus at most $n - 1$ local extremes.

3. **a.** $x = -3$ or $x = -1$

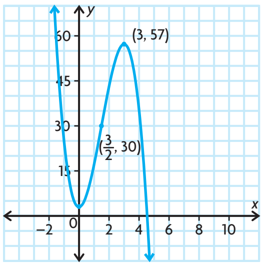
b. no vertical asymptotes

c. $x = 3$

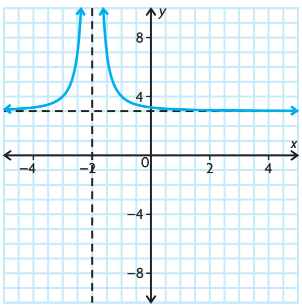
4. **a.**



b.



c.



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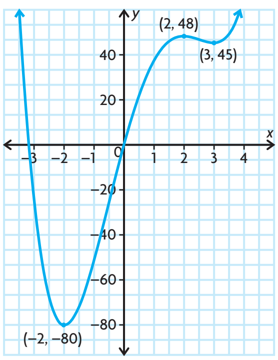

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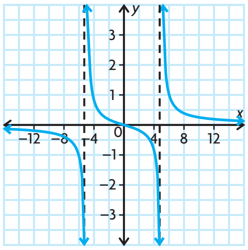
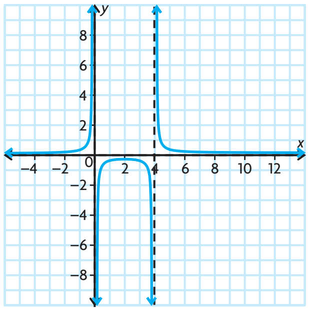
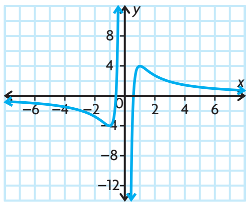


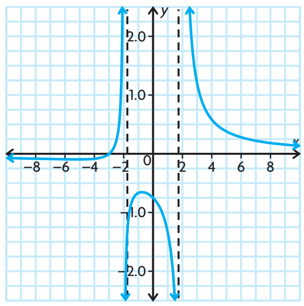
f.



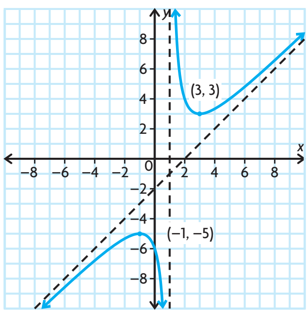
g.



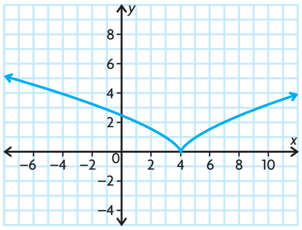
h.



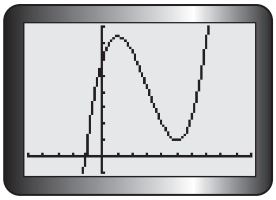
i.



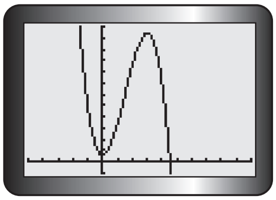
j.



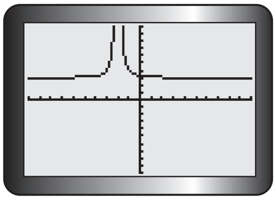
5. **a.**



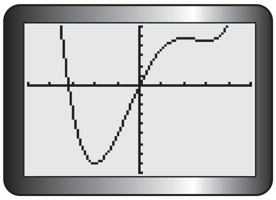
b.



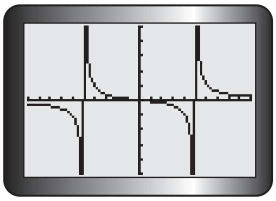
c.



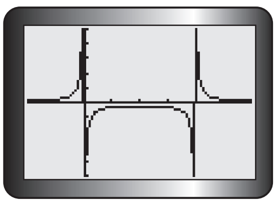
d.



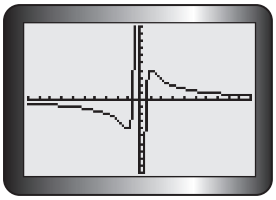
e.



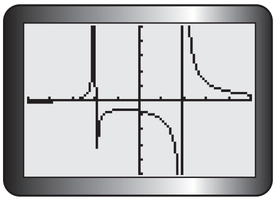
f.



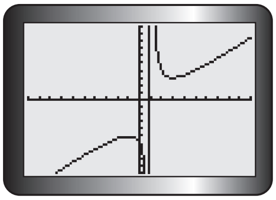
g.



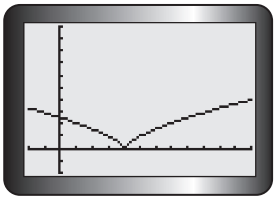
h.



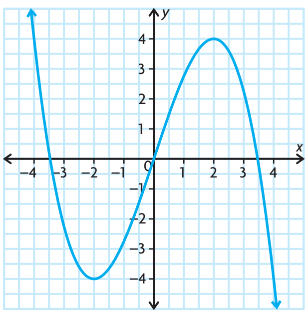
i.



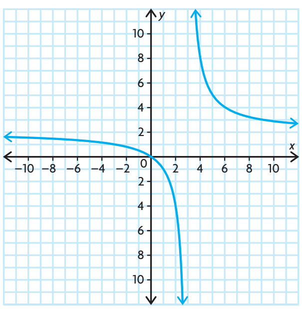
j.



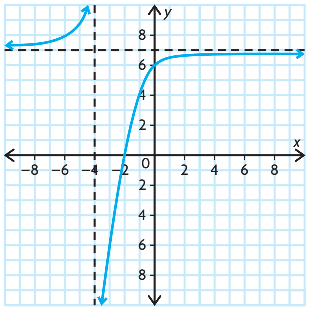
6. $a = -\frac{1}{4}, b = 0, c = 3, d = 0$



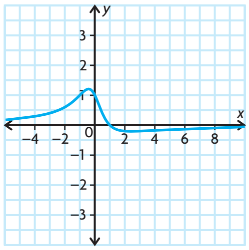
7. a. Answers may vary. For example:



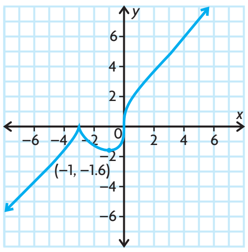
b. Answers may vary. For example:



8.



9.



10. a. $y = 1$ is a horizontal asymptote to the right-hand branch of the graph. $y = -1$ is a horizontal asymptote to the left-hand branch of the graph.

b. $y = \frac{3}{2}$ and $y = -\frac{3}{2}$ are horizontal asymptotes.

11. $y = ax^3 + bx^2 + cx + d$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

$$\frac{d^2y}{dx^2} = 6ax + 2b = 6a\left(x + \frac{b}{3a}\right)$$

For possible points of inflection, we solve $\frac{d^2y}{dx^2} = 0$:

$$x = -\frac{b}{3a}$$

The sign of $\frac{d^2y}{dx^2}$ changes as x goes from values less than $\frac{-b}{3a}$ to values greater than $\frac{-b}{3a}$. Thus, there is a point of inflection at $x = \frac{-b}{3a}$.

At $x = \frac{b}{3a}, \frac{dy}{dx} = 3a\left(\frac{-b}{3a}\right)^2 +$

$2b\left(\frac{-b}{3a}\right) + c = c - \frac{b^2}{3a}$

Review Exercise, pp. 216–219

1. a. i. $x < 1$

ii. $x > 1$

iii. $(1, 20)$

b. i. $x < -3, -3 < x < 1, x > 6.5$

ii. $1 < x < 3, 3 < x < 6.5$

iii. $(1, -1), (6.5, -1)$

2. No, a counter example is sufficient to justify the conclusion. The function $f(x) = x^3$ is always increasing, yet the graph is concave down for $x < 0$ and concave up for $x > 0$.

3. a. $(0, 20)$, local minimum; tangent is horizontal

b. $(0, 6)$, local maximum; tangent is horizontal

$(3, 33)$, neither local maximum nor minimum; tangent is horizontal

c. $\left(-1, -\frac{1}{2}\right)$, local minimum;

$\left(7, \frac{1}{14}\right)$, local maximum; tangents at both points are parallel

d. $(1, 0)$, neither local maximum nor minimum; tangent is not horizontal

4. a. $a < x < b, x > e$

b. $b < x < c$

c. $x < a, d < x < e$

d. $c < x < d$

5. a. $x = 3$; large and negative to left of asymptote, large and positive to right of asymptote

b. $x = -5$; large and positive to left of asymptote, large and negative to right of asymptote

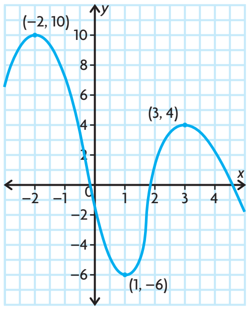
c. hole at $x = -3$

d. $x = -4$; large and positive to left of asymptote, large and negative to right of asymptote

$x = 5$; large and negative to left of asymptote, large and positive to right of asymptote

6. $(0, 5)$; Since the derivative is 0 at $x = 0$, the tangent line is parallel to the x-axis at that point. Because the derivative is always positive, the function is always increasing and, therefore, must cross the tangent line instead of just touching it.

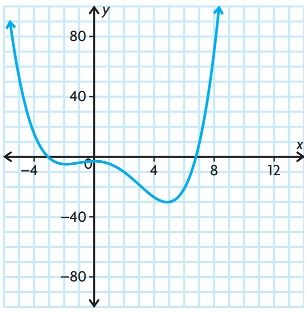
7.



8. a. i. concave up: $-1 < x < 3$; concave down: $x < -1$, $3 < x$

ii. points of inflection: $x = -1$, $x = 5$

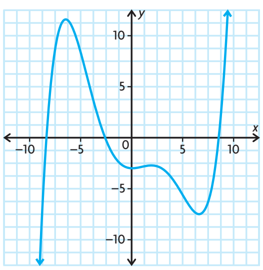
iii.



b. i. concave up: $-4.5 < x < 1$, $5 < x$
concave down: $x < -4.5$, $1 < x < 5$

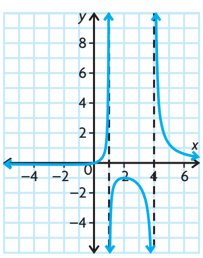
ii. points of inflection: $x = -4.5$, $x = 1$, and $x = 5$

iii.

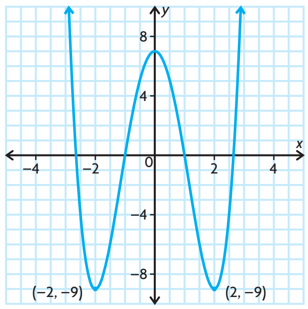


9. a. $a = 1, b = 0$

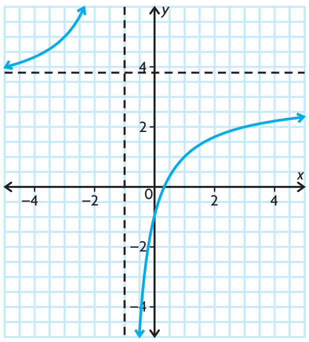
b.



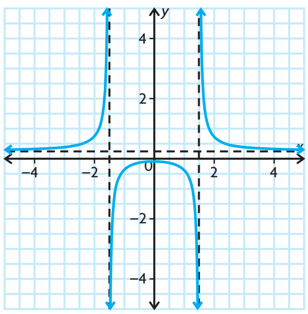
10. a.



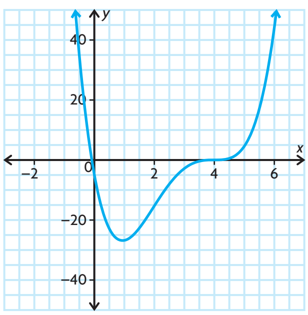
b.



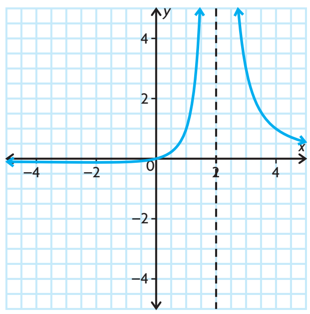
c.



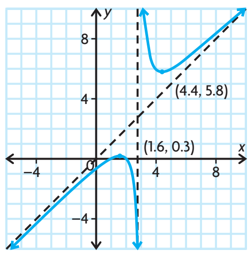
d.



e.

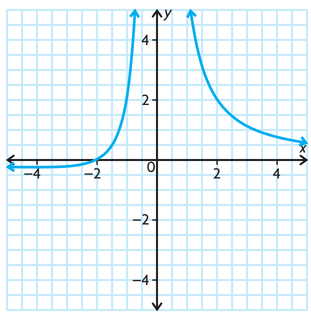


f.

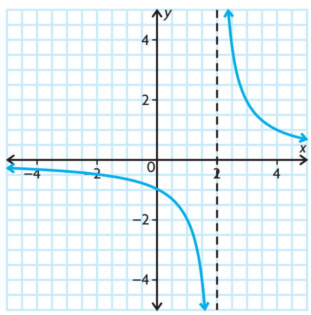


11. a. $-2 \le k \le 2, x \ne \pm k$

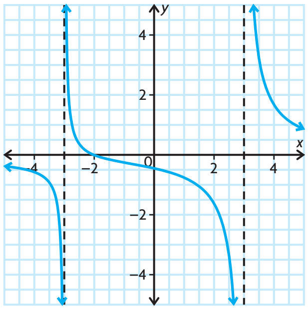
b. There are three different graphs that result for values of k chosen.
$k = 0$



$k = 2$



For all other values of k, the graph will be similar to the graph below.



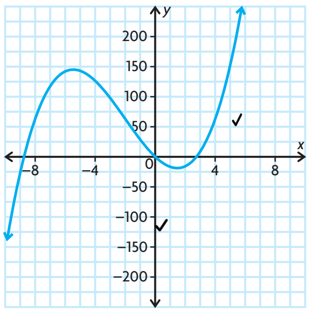
12. **a.** $y = x - 3$

b. $y = 4x + 11$

13. $x = -2, x = 0, x = 2;$
increasing: $-2 < x < 0, x > 2;$
decreasing: $x < -2, 0 < x < 2$

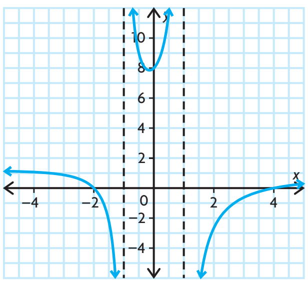
14. local maximum: $(-2.107, 17.054)$,
local minimum: $(1.107, 0.446)$,
absolute maximum: $(3, 24.5)$,
absolute minimum: $(-4, -7)$

15.

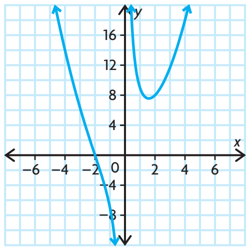


16. **a.** $p(x)$: oblique asymptote, $y = 0.75x$
$q(x)$: vertical asymptotes at $x = -1$ and $x = 3$; horizontal asymptote at $y = 0$
$r(x)$: vertical asymptotes at $x = -1$ and $x = 1$; horizontal asymptote at $y = 1$
$s(x)$: vertical asymptote at $y = 2$

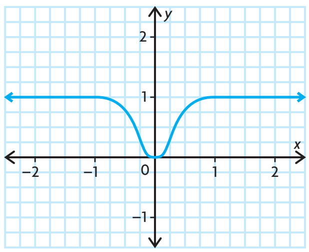
b.



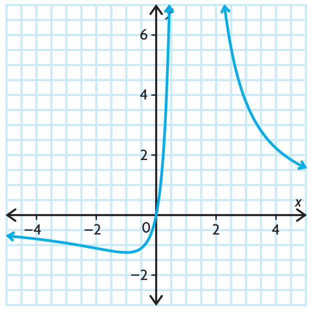
17. Domain: $\{x \in \mathbf{R} | x \neq 0\}$;
x-intercept: -2;
y-intercept: 8;
vertical asymptote: $x = 0$; large and negative to the left of the asymptote, large and positive to the right of the asymptote;
no horizontal or oblique asymptote;
increasing: $x > 1.59$;
decreasing: $x < 0, 0 < x < 1.59$;
concave up: $x < -2, x > 0$;
concave down: $-2 < x < 0$;
local minimum at $(1.59, 7.56)$;
point of inflection at $(-2, 0)$



18. If $f(x)$ is increasing, then $f'(x) > 0$. From the graph of f', $f'(x) > 0$ for $x > 0$. If $f(x)$ is decreasing, then $f'(x) < 0$. From the graph of f', $f'(x) < 0$ for $x < 0$. At a stationary point, $x = 0$. From the graph, the zero for $f'(x)$ occurs at $x = 0$. At $x = 0$, $f'(x)$ changes from negative to positve, so f has a local minimum point there. If the graph of f is concave up, then f'' is positive. From the slope of f', the graph of f is concave up for $-0.6 < x < 0.6$. If the graph of f is concave down, then f'' is negative. From the slope of f', the graph of f is concave down for $x < -0.6$ and $x > 0.6$. Graphs will vary slightly.



19. domain: $\{x \in \mathbf{R} | x \neq 1\}$;
x-intercept and y-intercept: $(0, 0)$;
vertical asymptote: $x = 1$; large and positive on either side of the asymptote;
horizontal asymptote: $y = 0$;
increasing: $-1 < x < 1$;
decreasing: $x < -1, x > 1$;
concave down: $x < -2$;
concave up: $-2 < x < 1, x > 1$;
local minimum at $(-1, -1.25)$;
point of inflection: $(-2, -1.11)$



20. **a.** Graph A is f, graph C is f', and graph B is f''. We know this because when you take the derivative, the degree of the denominator increases by one. Graph A has a squared term in the denominator, graph C has a cubic term in the denominator, and graph B has a term to the power of four in the denominator.

b. Graph F is f, graph E is f' and graph D is f''. We know this because the degree of the denominator increases by one degree when the derivative is taken.

Chapter 4 Test, p. 220

1. **a.** $x < -9$ or $-6 < x < -3$ or $0 < x < 4$ or $x > 8$

b. $-9 < x < -6$ or $-3 < x < 0$ or $4 < x < 8$

c. $(-9, 1), (-6, -2), (0, 1), (8, -2)$

d. $x = -3, x = 4$

e. $f''(x) > 0$

f. $-3 < x < 0$ or $4 < x < 8$

g. $(-8, 0), (10, -3)$

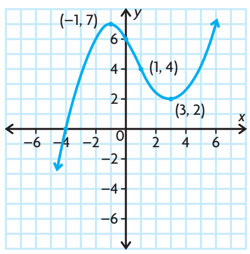
2. **a.** $x = 3$ or $x = -\frac{1}{2}$ or $x = \frac{1}{2}$

b. $\left(-\frac{1}{2}, -\frac{17}{8}\right)$: local maximum

$\left(\frac{1}{2}, \frac{15}{8}\right)$: local maximum

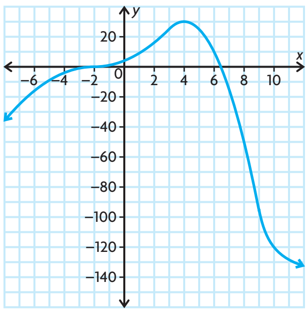
$(3, -45)$: local minimum

3.



4. hole at $x = -2$; large and negative to left of asymptote, large and positive to right of asymptote;
$y = 1$;
Domain: $\{x \in \mathbf{R} \mid x \neq -2, x \neq 3\}$

5.



6. There are discontinuities at $x = -3$ and $x = 3$.

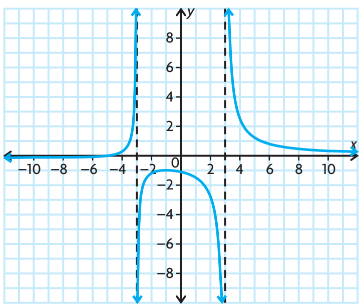
$$\left.\begin{array}{l}\lim\limits_{x\to 3^-} f(x) = \infty \\ \lim\limits_{x\to 3^+} f(x) = -\infty\end{array}\right\} x = -3 \text{ is a vertical asymptote.}$$

$$\left.\begin{array}{l}\lim\limits_{x\to 3^-} f(x) = -\infty \\ \lim\limits_{x\to 3^+} f(x) = \infty\end{array}\right\} x = 3 \text{ is a vertical asymptote.}$$

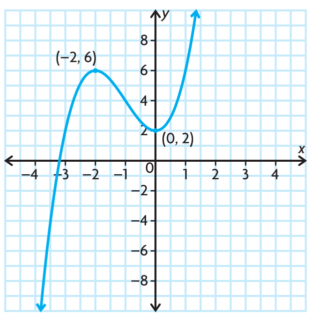
The y-intercept is $-\frac{10}{9}$ and x-intercept is -5.

$\left(-9, -\frac{1}{9}\right)$ is a local minimum and $(-1, -1)$ is a local maximum.

$y = 0$ is a horizontal asymptote.



7. $b = 3$, $c = 2$



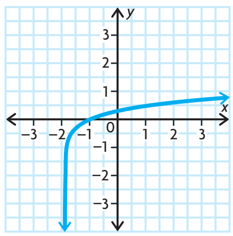
Chapter 5

Review of Prerequisite Skills, pp. 224–225

1. **a.** $\frac{1}{9}$ **c.** $\frac{1}{9}$
b. 4 **d.** $\frac{9}{4}$

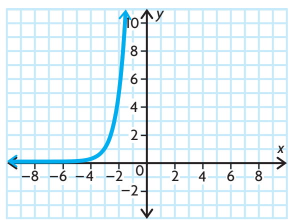
2. **a.** $\log_5 625 = 4$
b. $\log_4 \frac{1}{16} = -2$
c. $\log_x 3 = 3$
d. $\log_{10} 450 = w$
e. $\log_3 z = 8$
f. $\log_a T = b$

3. **a.**



x-intercept: $(-1, 0)$

b.



no x-intercept

4. **a.** $\frac{y}{r}$ **b.** $\frac{x}{r}$ **c.** $\frac{y}{x}$

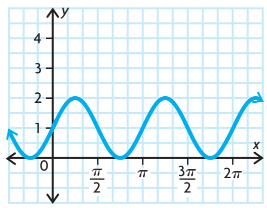
5. **a.** 2π **e.** $\frac{3\pi}{2}$
b. $\frac{\pi}{4}$ **f.** $-\frac{2\pi}{3}$
c. $-\frac{\pi}{2}$ **g.** $\frac{5\pi}{4}$
d. $\frac{\pi}{6}$ **h.** $\frac{11\pi}{6}$

6. **a.** b **c.** a **e.** b
b. $\frac{b}{a}$ **d.** a **f.** $-b$

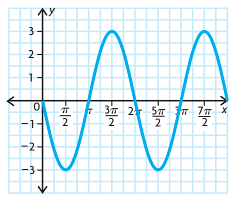
7. **a.** $\cos\theta = -\frac{12}{13}$,
$\tan\theta = -\frac{5}{12}$
b. $\sin\theta = -\frac{\sqrt{5}}{3}$,
$\tan\theta = \frac{\sqrt{5}}{2}$
c. $\sin\theta = -\frac{2}{\sqrt{5}}$,
$\cos\theta = \frac{1}{\sqrt{5}}$
d. $\cos\frac{\pi}{2} = 0$,
$\tan\frac{\pi}{2}$ is undefined

8. **a.** period: π, amplitude: 1
b. period: 4π, amplitude: 2
c. period: 2, amplitude: 3
d. period: $\frac{\pi}{6}$, amplitude: $\frac{2}{7}$
e. period: 2π, amplitude: 5
f. period: 2π, amplitude: $\frac{3}{2}$

9. **a.**



b.



10. **a.** $\tan x + \cot x = \sec x \csc x$

$$\begin{aligned}\text{LS} &= \tan x + \cot x \\ &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x + \sin x} \\ &= \frac{1}{\cos x + \sin x} \\ \text{RS} &= \sec x \csc x \\ &= \frac{1}{\cos x} \times \frac{1}{\sin x} \\ &= \frac{1}{\cos x \sin x}\end{aligned}$$

Therefore, $\tan x + \cot x = \sec x \csc x$.