

Ch 5 Test

Mock Test

~~1. 6~~ ~~2.~~

① $f(x) = 5e^{-4x-9\pi}$

$$f'(x) = 5e^{-4x-9\pi} \cdot (-4) = -20e^{-4x-9\pi}$$

② $y = 12^{-4x+9}$

$$\frac{dy}{dx} = 12^{-4x+9} \cdot \ln 12 \cdot (-4) = -4 \cdot 12^{-4x+9} \cdot \ln 12$$

③ $f(x) = -0.5x^2 \cdot 2^x$

$\therefore f'(x) = 0$ when

$$f'(x) = -x \cdot 2^x - 0.5x^2 [2^x \cdot \ln 2]$$

$x=0$ or $x = \frac{-2}{\ln 2}$

$$= -0.5x2^x [2 + x \ln 2]$$

④ $y = 3 \cos(8x+6)$

$$\frac{dy}{dx} = 3 [-\sin(8x+6)] (8) = -24 \sin(8x+6)$$

⑤ $y = \tan^2(e^x)$

$$\frac{dy}{dx} = 2 \tan(e^x) \cdot \sec^2(e^x) \cdot e^x = 2e^x \tan(e^x) \sec^2(e^x)$$

⑥ $f(x) = 4xe^x$

$\therefore f'(0) = 4 + 4(0) = 4$

$$f'(x) = 4e^x + 4xe^x$$

⑦ $f(x) = 7^x \cdot x^7$

$$f'(x) = 7^x \ln 7 \cdot x^7 + 7^x [7x^6]$$

$$= 7^x \cdot x^6 [x \ln 7 + 7]$$

⑧ $f(x) = 3x3^x - 1$

$$f'(x) = 3 \cdot 3^x + 3x [3^x \ln 3]$$

$$= 3 \cdot 3^x [1 + x \ln 3]$$

$\therefore f'(x) = 0$ when $x = -\frac{1}{\ln 3}$

$$\begin{array}{ccc} f'(x) < 0 & \vee & f'(x) > 0 \\ & \downarrow & \\ & -\frac{1}{\ln 3} & \end{array}$$

As $x \rightarrow \infty$, $f(x) \rightarrow \infty$

$f(-\frac{1}{\ln 3}) \approx -2.00$ is the min. value of $f(x)$.

⑨ $f(x) = (-2)^x$

as $x \rightarrow \infty$ $f(x) \rightarrow \pm \infty$

as $x \rightarrow \infty$ $f(x) \rightarrow 0$, but 0^+ and 0^-

$f(x)$ is not continuous, and not diff

⑩ $F(x) = 5^{\tan(\sqrt{x})}$

$$f'(x) = 5^{\tan(\sqrt{x})} \cdot \ln 5 \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$\approx \tan \sqrt{x} \sec^2(\sqrt{x}) \ln 5$

11. $v(t) = 60(1 - 0.7^t)$

a) $a(t) = v'(t)$
 $= 60(0 - 0.7^t \ln(0.7))$
 $= -60(0.7)^t \ln(0.7)$

b) $a(2) = -60(0.7)^2 \ln(0.7)$
 $\approx 10.5 \text{ m/s}^2$

c) $v(0) = 0 \text{ m/s}$

The car initially was stationary

d) $-60(0.7)^t \ln(0.7) = 3$
 $(0.7)^t = \frac{-3}{60 \ln(0.7)}$

$t \ln(0.7) = \ln\left(\frac{-1}{20 \ln(0.7)}\right)$

$t \approx 5.5 \text{ s}$

12. $P(t) = 1200(2^{-t})$

a) $P(0) = 1200$

b) $P(1) = 600$

(c) $P'(t) = 1200(2^{-t})(\ln 2)(-1)$
 $= -1200(2^{-t}) \ln 2$
 $P'(1) \approx -416 \text{ people/week}$

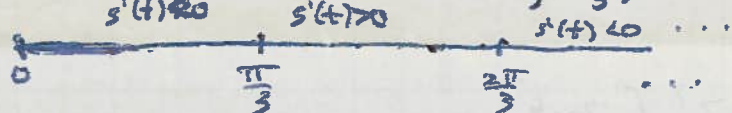
13. $s(t) = 11 \cos(3t)$, $t \geq 0$

a) $s'(t) = -11 \sin(3t) \cdot 3$
 $= -33 \sin(3t)$

$s'(t) = 0$ when $\sin(3t) = 0$

$3t = 0, \pi, 2\pi, \dots$

$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \dots$



\therefore particle switches direction
 @ $t = \frac{\pi}{3}$ for the first
 time.

(b) The particle changes direction
 for $t = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \dots$

(c) $a(t) = -33 \cos(3t) \cdot 3$
 $= -99 \cos(3t)$

$a(t) = 0$ when $\cos(3t) = 0$

$3t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \dots$

$v(\frac{\pi}{6}) = -33$, $v(\frac{\pi}{2}) = +33$, ...

\therefore max velocity is 33

14. $f(x) = \cot x$

$f(x) = \frac{1}{\tan x}$
 $= (\tan x)^{-1}$

$\therefore f'(x) = -(\tan x)^{-2} (\sec^2 x)$

$= -\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$

$= -\frac{1}{\sin^2 x}$

$= -\csc^2 x$

OR $f(x) = \frac{\cos x}{\sin x}$

USE QUOTIENT RULE TO
 FIND $f'(x)$

15. a) $y = \sqrt{\tan x}$
 $= (\tan x)^{\frac{1}{2}}$

$y' = \frac{1}{2} (\tan x)^{-\frac{1}{2}} \sec^2 x$

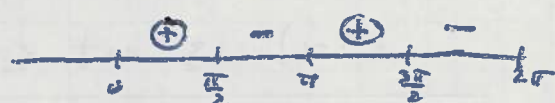
$= \frac{\sec^2 x}{2\sqrt{\tan x}}$

b) $\tan x > 0$ for y to be diff
 Let $\tan x = 0$

$x = 0, \pi, 2\pi, \dots$

and $\tan x$ is undefined if

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$



So, y is not diff if

$\frac{\pi}{2} \leq x \leq \pi$ or $-\frac{3\pi}{2} \leq x \leq -2\pi$