4.1

Q7. $f(x) = x^3 + ax^2 + bx + c$ and $\begin{cases} f(-3)=18, & f'(-3)=0 \\ f(1)=-14, & f'(1)=0 \end{cases}$

- $f'(x) = 3x^2 + 2ax + b$

$f'(3)=0 \rightarrow 27 - 6a + b = 0$
$b = 6a - 27$

$f'(1)=0 \rightarrow 3 + 2a + b = 0$
$b = -3 - 2a$

So, $6a - 27 = -3 - 2a$
$8a = 24$
$a = 3$

$\therefore b = 18 - 27$
$b = -9$

$f(1) = -14 \rightarrow 1 + a + b + c = -14$
$a + b + c = -15$

So, $3 - 9 + c = -15$
$c = -9$

Q10. $f(x) = ax^2 + bx + c, \quad a > 0$

$f'(x) = 2ax + b$

$f'(x) = 0$ when $x = \frac{-b}{2a}$

Clearly $f'(x) = 2ax + b$ is an increasing function (i.e. $f''(x) = 2a > 0$)

$f'(x) < 0 \qquad f'(x) = 0 \qquad f'(x) > 0$

$-\frac{b}{2a}$

Thus $f(x)$ is decreasing when $x < -\frac{b}{2a}$ and increasing when $x > \frac{-b}{2a}$

Q13 f and g are both
- increasing on $[a,b]$
- $f(x)>0$, $g(x)>0$ on $[a,b]$

Thus, whenever $x_2>x_1$ on $[a,b]$

$\therefore$ $0<f(x_1)<f(x_2)$

and $0<g(x_1)<g(x_2)$

Thus $f(x_1)g(x_1)<f(x_2)g(x_2)$

$\therefore$ $f(x)g(x)$ is increasing on $[a,b]$ too.

OR Let $h=fg$

$\therefore$ $h'=f'g+fg'$

$= \oplus\oplus + \oplus\oplus$

$= \oplus$

$\therefore$ $h'>0$ on $[a,b]$

$\therefore$ fg is increasing on $[a,b]$.

Q14 f and g are both increasing on $[a,b]$
and $f(x)<0$ and $g(x)<0$ on $[a,b]$

Thus whenever $x_2>x_1$ on $[a,b]$

Then $f(x_1)<f(x_2)<0$

$g(x_1)<g(x_2)<0$

Thus $f(x_1)g(x_1)>f(x_2)g(x_2)>0$

So, $f(x)g(x)$ must be decreasing on $[a,b]$.

OR Let $h=fg$

$h'=f'g+fg'$

$= \oplus\ominus + \ominus\oplus$

$= \ominus + \ominus$

$= \ominus$

Since $h'<0$ on $[a,b]$

$\therefore$ fg must be decreasing on $[a,b]$