3.1: Higher-Order Derivatives, Velocity and Acceleration

Date: _______________

Ex1. Find the 2nd derivative of each.

a. $y = 2x^{-3} - 5x^2$ $y' = -6x^{-4} - 10x$ $y'' = 24x^{-5} - 10$

b. $f(x) = \sqrt{4x-3}$

$f(x) = (4x-3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(\quad)^{-\frac{1}{2}}(4)$

$= \frac{4}{2\sqrt{4x-3}}$

$= \frac{2}{\sqrt{4x-3}}$

$f'(x) = 2(4x-3)^{-\frac{1}{2}}$

$f''(x) = -(4x-3)^{-\frac{3}{2}}(4)$

$= \frac{-4}{\sqrt{(4x-3)^3}}$

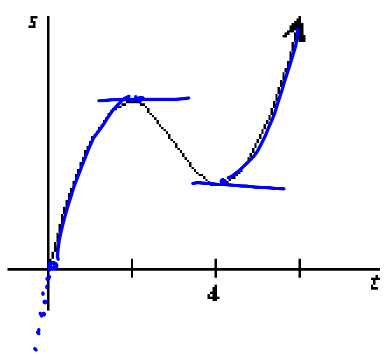
Motion on a Straight Line

When s represents the location of an object when moving in a straight line after t units of time,

Then the velocity of the object is
$v(t) = s'(t)$ or $v = \frac{ds}{dt}$

And its acceleration is
$a(t) = v'(t) = s''(t)$ or $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$

Ex2. Consider the graph of $y = s(t)$ given to the right showing the motion of an object moving in a straight line.



a. When is the velocity zero?

$s'(t) = 0 \rightarrow$ graph horizontal

$\therefore t = 2, 4$

b. When is the object moving in the positive direction?

$s'(t) > 0 \rightarrow$ graph uphill

$\therefore 0 < t < 2, t > 4$

When is the object moving in the negative direction?

$s'(t) < 0 \rightarrow$ graph downhill

$\therefore 2 < t < 4$

Ex3. A particle moves s metres along a straight line in t seconds according to $s = 6t^2 - t^3$, $t \geq 0$.

a. Find the particle's initial position, initial velocity and initial acceleration.

$s(t) = 6t^2 - t^3$ $\qquad$ $s(0) = 0\text{ m}$

$v(t) = 12t - 3t^2$ $\qquad$ $v(0) = 0\text{ m/s}$

$a(t) = 12 - 6t$ $\qquad$ $a(0) = 12\text{ m/s}^2$

b. Find the particle's position when the velocity is 12 m/s.

$$12t - 3t^2 = 12$$
$$12t - 3t^2 - 12 = 0$$
$$t^2 - 4t + 4 = 0$$
$$(t-2)^2 = 0$$
$$t = 2\text{ s}$$

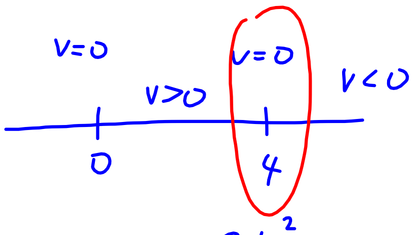
$$s(2) = 6(2)^2 - 2^3$$
$$= 24 - 8$$
$$= 16\text{ m}$$

c. Find the time and position when the object is at rest.

$$12t - 3t^2 = 0$$
$$3t(4-t) = 0$$
$$t = 0, 4$$

$$s(0) = 0\text{ m}$$
$$s(4) = 6(4)^2 - 4^3$$
$$= 4^2(6-4)$$
$$= 32\text{ m}$$

d. When does the particle change direction?



$$v = 12t - 3t^2$$
$$= 3t(4-t)$$

$\therefore$ the particle changed direction when $t = 4$ s.

e. When does the particle return to its initial position?

$s(0)=0$

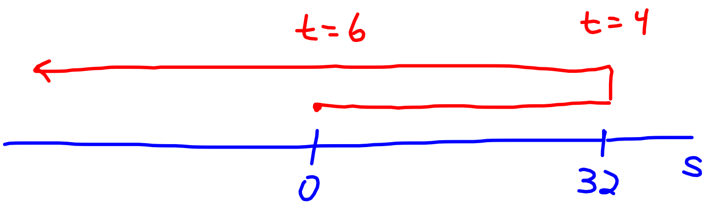
$\therefore\ 6t^2-t^3=0$

$t^2(6-t)=0$

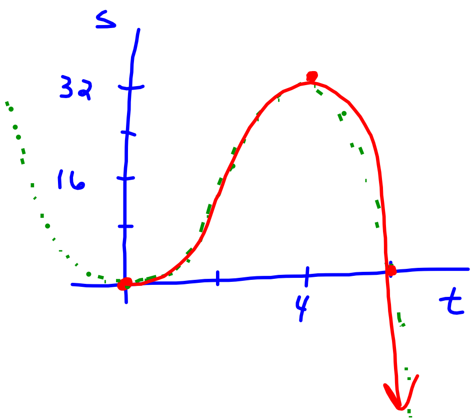
$t=0,\ 6$

$\therefore$ the particle returns after 6 s.

f. Illustrate the particle's motion on a horizontal line, *s*.



g. Graph $s=6t^2-t^3$, $t\geq 0$. Compare your graph with the answers in a→f.



Homefun: Page 127 #

2, 3acdf, 4, 5[(5c) t=3], 6ab, 8→12,

13[(13a) Instead of "It passes the origin..." state "It passes the starting position after 6 s." (13b) Instead of "It passes the origin..." state "It passes the starting position after ≈3.5 s."],

14, 15a