2.2: **The Derivatives of Polynomial Functions** Date: ______________

RULE	FUNCTION NOTATION	LEIBNIZ NOTATION
Constant Function Rule **If** $f(x) = k$, **then**	$f'(x) = 0$	$\frac{d}{dx}(k) = 0$
Power Rule **If** $f(x) = x^n$, **then**	$f'(x) = nx^{n-1}$	$\frac{d}{dx}(x^n) = nx^{n-1}$
Constant Multiple Rule **If** $f(x) = kg(x)$, **then**	$f'(x) = k\,g'(x)$	$\frac{d}{dx}(ky) = k\frac{dy}{dx}$
Sum/Difference Rule **If** $f(x) = p(x) \pm q(x)$, **then**	$f'(x) = p'(x) \pm q'(x)$,	$\frac{d}{dx}(f(x)) = \frac{d}{dx}(p(x)) \pm \frac{d}{dx}(q(x))$

Ex1. Prove the 1st, 3rd and 4th derivative rules. The 2nd was *sorta* proved last class and we'll prove the rule in its entirety later. Without logarithmic differentiation, the proof is long and nasty.

IF $f(x) = p(x) + q(x)$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{p(x+h) + q(x+h) - [p(x) + q(x)]}{h}$$

$$= \lim_{h \to 0} \frac{p(x+h) - p(x)}{h} + \lim_{h \to 0} \frac{q(x+h) - q(x)}{h}$$

$$= p'(x) + q'(x)$$

Ex2. Apply the derivative rules to differentiate each function.

Constant Function Rule

a. $f(x) = -3$

$f'(x) = 0$

b. $y = \pi^2$

$y' = 0$

Power Rule

c. $f(x) = x^8$

$f'(x) = 8x^7$

d. $g(x) = \frac{1}{x^2} = x^{-2}$

$g'(x) = -2x^{-3}$

$= \frac{-2}{x^3}$

e. $s = \sqrt{t^5} = t^{\frac{5}{2}}$

$\therefore \frac{ds}{dt} = \frac{5}{2}t^{\frac{3}{2}}$

$= \frac{5}{2}\sqrt{t^3}$

f. $y = x'$

$\frac{dy}{dx} = 1 \cdot x^0$

$= 1$

$y = 7x$

$y' = 7$

Constant Multiple Rule / Power Rule

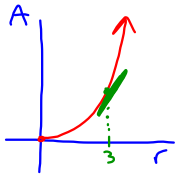
g. $f(x) = 6x^5$

$f'(x) = 6(5x^4)$

$= 30x^4$

i. $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$



h. $y = 21\sqrt[7]{x^2} = 21 \cdot x^{\frac{2}{7}}$

$y' = 21 \cdot \frac{2}{7} \cdot x^{-\frac{5}{7}}$

$= 6x^{-\frac{5}{7}}$

$\therefore y' = \frac{6}{\sqrt[7]{x^5}}$

Ex3. Differentiate the following functions.

a. $y = \frac{2}{x^2} - \frac{x^3}{3} + \sqrt{9x^3}$

$y = 2x^{-2} - \frac{1}{3}x^3 + 3x^{\frac{3}{2}}$

$y' = -4x^{-3} - x^2 + \frac{9}{2}x^{\frac{1}{2}}$

b. $y = (2x + 1)^3$

$y = (2x)^3 + 3(2x)^2 + 3(2x) + 1$

$y = 8x^3 + 12x^2 + 6x + 1$

$y' = 24x^2 + 24x + 6$

c. $f(x) = -3\left(\frac{x}{2}\right)^4 + \frac{4 + \sqrt{x}}{\sqrt{x^3}}$

$f(x) = -3\left(\frac{x^4}{16}\right) + \frac{4}{x^{\frac{3}{2}}} + \frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}$

$= -\frac{3}{16}x^4 + 4x^{-\frac{3}{2}} + x^{-1}$

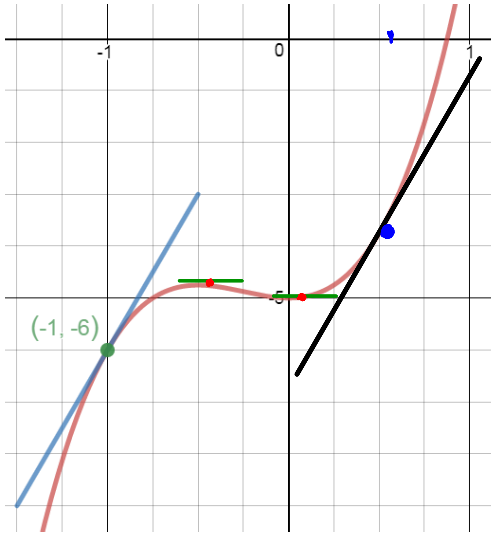
$f'(x) = -\frac{3}{4}x^3 - 6x^{-\frac{5}{2}} - x^{-2}$

Ex4. The graph of $f(x) = 4x^3 + 3x^2 - 5$ is given to the right.

a. Determine the equation of the tangent of $f(x)$ at $x = -1$.

b. Determine points on the graph with tangents parallel to the tangent found in (a).

c. Determine points on the graph where the tangents are horizontal.



a) $f(-1) = -6 \quad \therefore P_0 = (-1, -6)$

$f'(x) = 12x^2 + 6x$

$f'(-1) = 12 - 6 \qquad \therefore m = 6$

$= 6$

So $\dfrac{x-(-1)}{1} = \dfrac{y-(-6)}{6} \rightarrow 6x + 6 = y + 6$

$y = 6x$ 1TRE

(b) $f'(x) = 6$

$\therefore 12x^2 + 6x = 6$

$12x^2 + 6x - 6 = 0$

$2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0$

$\therefore x = -1$ OR $\frac{1}{2}$

So $P_1 = (-1, -6)$

$P_2 = \left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$

(c) $f'(x) = 0$

$12x^2 + 6x = 0$

$6x(2x+1) = 0$

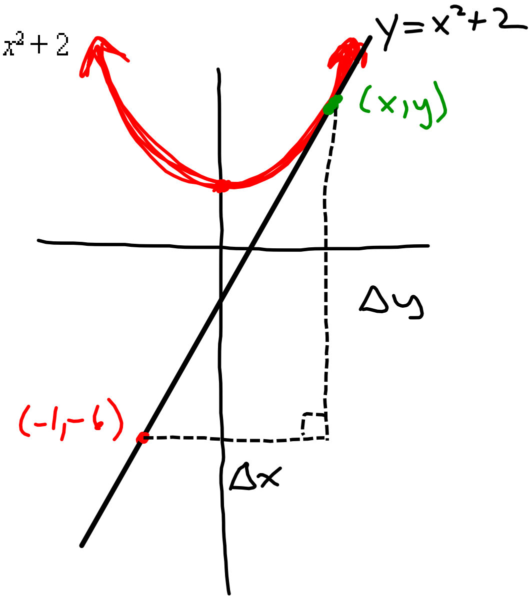
$x = 0$ OR $-\frac{1}{2}$

$P_3 = (0, -5)$

$P_4 = \left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right)$

Ex5. Determine the equation of the tangents to the curve $y = x^2 + 2$ that pass through $(-1, -6)$.

Note: $(-1, -6)$ is NOT on the curve $y = x^2 + 2$.



$$\frac{\Delta y}{\Delta x} = y'$$

$$\frac{y-(-6)}{x-(-1)} = 2x$$

$$\frac{x^2+2+6}{x+1} = 2x$$

$$\frac{x^2+8}{x+1} = 2x$$

$$x^2+8 = 2x(x+1)$$

$$x^2+8 = 2x^2+2x$$

$$x^2+2x-8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4 \text{ or } 2$$

Tangent #1 : $x = 2$

$P_0 = (-1,-6)$ $\quad m = 2(2) = 4$

$$\frac{x+1}{1} = \frac{y+6}{4} \rightarrow 4x+4 = y+6$$

$$y = 4x-2$$

Tangent #2 : $x = -4$

$P_0 = (-1,-6)$ $\quad m = 2(-4) = -8$

$$\frac{x+1}{1} = \frac{y+6}{-8} \rightarrow -8x-8 = y+6$$

$$y = -8x-14$$

* Homefun: Page 82 #2→5, 7, 8d, 9df, 11→25, [27, 28]