5.1: **The Derivative of $y = e^x$** $A = P(1+i)^n$ Date: ______________

Ex1. Find the amount after 1 year if \$1 is invested at 100% compounded as follows.

a. annually

$A = 1(1+1)^1$
$= \$2$

b. semi-annually

$A = 1(1+\frac{1}{2})^2$
$= \$2.25$

c. monthly

$A = 1(1+\frac{1}{12})^{12}$
$= \$2.61$

d. daily

$A = 1(1+\frac{1}{365})^{365}$
$= \$2.71$

e. every instant

$A = \lim_{x\to\infty} (1+\frac{1}{x})^x$
$= 2.71828\ldots$
$= \$e$

The Natural Number (a.k.a. Euler's Number)

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

Note: $e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots$ and $e^x = \frac{1}{0!}+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$ *

The above are Taylor Series Expansions of e^x. You'll learn more about these in University.

Ex2. Find $f'(x)$ from first principles if $\boxed{f(x) = e^x}$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

$$= \lim_{h\to 0} e^x\left[\frac{e^h-1}{h}\right]$$

$$= \lim_{h\to 0} e^x\left[\frac{1+h-1}{h}\right]$$

$$= \lim_{h\to 0} e^x$$

$$= e^x$$

As $h\to 0$

$e = (1+h)^{\frac{1}{h}}$

$e^h = \left[(1+h)^{\frac{1}{h}}\right]^h$

$e^h = 1+h$

$f(x) = e^x$
$f'(x) = e^x$
$f''(x) = e^x$
$\vdots$

http://www.youtube.com/watch?v=qu_Y1wQ923g https://www.youtube.com/watch?v=zeIsxXDyjlc#t=69

Ex3. Differentiate $f(x) = e^x$
using the Taylor Series

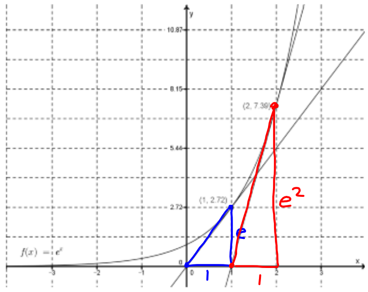
$$\frac{d}{dx}\left(e^x = \frac{1}{0!}+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots\right)$$

$$\frac{d}{dx}e^x = 0+1+\frac{2x}{2!}+\frac{3x^2}{3!}+\frac{4x^3}{4!}+\ldots$$

$$= \frac{1}{0!}+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$$

$$= e^x$$

Ex4. The given graph is of $f(x) = e^x$ with its tangents at $(1, e)$ and $(2, e^2)$



Determine the slope of the tangents at

a. $(1, e)$

b. $(2, e^2)$

c. $(3, e^3)$

d. (x, e^x)

x	y	y'
1	e	e
2	e^2	e^2
3	e^3	e^3
x	e^x	e^x

Ex5. Differentiate each function.

a. $y = e^{\sqrt{x}}$

$y = e^u$, $u = \sqrt{x}$

$\frac{dy}{du} = e^u$, $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$y = e^{\sqrt{x}}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= e^u \cdot \frac{1}{2\sqrt{x}}$

$= e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

So...	IF $f(x) = e^x$	THEN $f'(x) = e^x$
	IF $f(x) = e^{g(x)}$	THEN $f'(x) = e^{g(x)} \cdot g'(x)$

b. $f(x) = 2e^{3x - x^2}$

$f'(x) = 2 \cdot e^{3x - x^2} \cdot (3 - 2x)$

c. $g(x) = \boxed{-4x^2} e^{2x}$

$g'(x) = (-8x)(e^{2x}) + (-4x^2)(e^{2x} \cdot 2)$

$= -8xe^{2x}(1 + x)$

d. $h(x) = \frac{3e^x}{x - 1}$

$h'(x) = \frac{3e^x(x-1) - 3e^x(1)}{(x-1)^2}$

$= \frac{3e^x[x - 2]}{(x-1)^2}$

Ex6. Find the equation of the tangent when $x = -1$ for functions a, b, and c from the previous question.

a) $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

NO!

c) $g'(-1) = 0$

$g(-1) = -4e^{-2}$

$= \frac{-4}{e^2}$

Thus,

$y = \frac{-4}{e^2}$ ITRE

b) $f'(-1) = 2 \cdot e^{-4} \cdot (5)$

$= \frac{10}{e^4}$

$f(-1) = 2e^{-4}$

$= \frac{2}{e^4}$

$\therefore m = \frac{10}{e^4}$, $P_0 = \left(-1, \frac{2}{e^4}\right)$

$\frac{x+1}{e^4} = \frac{y - \frac{2}{e^4}}{10}$

$10x + 10 = e^4 y - 2$

$10x - e^4 y + 12 = 0$ ITRE

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