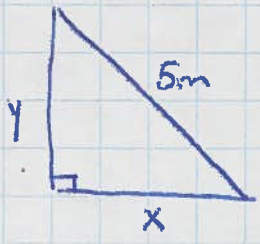
Related Rates Exercises Pg 569 Day 2

8.



$\frac{dx}{dt} = \frac{1}{3}$ m/s Find $\frac{dy}{dx}$ at $y = 3$ m

$x^2 + y^2 = 5^2$

$\therefore \quad 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

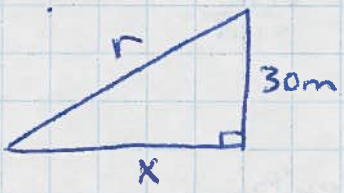
$\frac{2x}{3} + y \cdot \frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x}{3y}$ when $y = 3$, $x = 4$

$= \frac{-4}{3(3)}$

$= -\frac{4}{9}$ m/s

9.



Given: $\frac{dx}{dt} = 10$m/min

Find: $\frac{dr}{dt}$ @ $x = 40$m, $r = \sqrt{30^2 + 40^2} = 50$m

$x^2 + 30^2 = r^2$

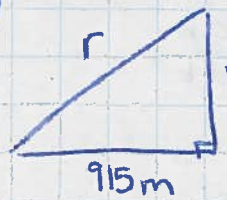
$2x \frac{dx}{dt} + 0 = 2r \cdot \frac{dr}{dt}$

$\frac{10x}{r} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10(40)}{50}$

$= 8$ m/min.

10.



$\frac{dy}{dt} = 268$ m/s @ $y = 1220$; Find $\frac{dr}{dt}$

$915^2 + y^2 = r^2$

$0 + 2y \cdot \frac{dy}{dt} = 2r \cdot \frac{dr}{dt}$

When $y = 1220$

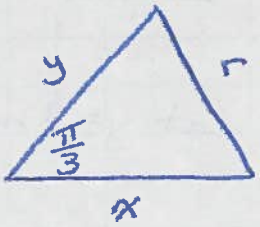
$r = 1525$

$\frac{dr}{dt} = \frac{y}{r} \cdot \frac{dy}{dt}$

$= \frac{1220}{1525} \cdot (268)$

$= 214.4$ m/s

11.



$\frac{dx}{dt} = 15$ km/h $\frac{dy}{dt} = 20$ km/h

Find $\frac{dr}{dt}$ @ $t = 2$h.

$r^2 = x^2 + y^2 - 2xy\cos\frac{\pi}{3}$
$= x^2 + y^2 - xy$

$2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} - \frac{dx}{dt}y - x\frac{dy}{dt}$

At $t = 2$
$x = 15(2) = 30$ km
$y = 20(2) = 40$ km

$r^2 = 30^2 + 40^2 - 2(30)(40)\cos\frac{\pi}{3}$
$= 1300$
$r = \sqrt{1300}$

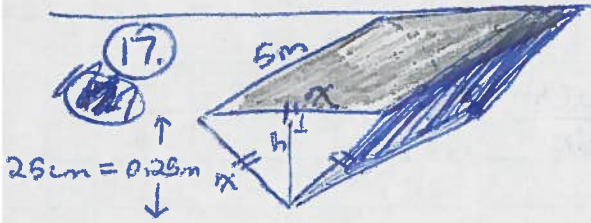
$2\sqrt{1300} \cdot \frac{dr}{dt} = 2(30)(15) + 2(40)(20) - 15(40) - 30(20)$

$2\sqrt{1300} \cdot \frac{dr}{dt} = 1300 \longrightarrow \therefore \frac{dr}{dt} = \frac{1300}{2\sqrt{1300}} \cdot \frac{\sqrt{1300}}{\sqrt{1300}}$

$= \frac{\sqrt{1300}}{2}$

$= 5\sqrt{13}$ Km/h

17.



$\frac{dV}{dt} = 0.25$ m³/min ; Find $\frac{dh}{dt}$ @ $h = 10$cm $= 0.1$m

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$V = \frac{xh}{2} \cdot 5$

$= \frac{2h^2}{2\sqrt{3}} \cdot 5$

$= \frac{5h^2}{\sqrt{3}}$

$\frac{dV}{dh} = \frac{10h}{\sqrt{3}}$

But $\left(\frac{x}{2}\right)^2 + h^2 = x^2$

$x^2 + 4h^2 = 4x^2$

$4h^2 = 3x^2$

$x^2 = \frac{4}{3}h^2$

$x = \frac{2}{\sqrt{3}}h$

So, $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

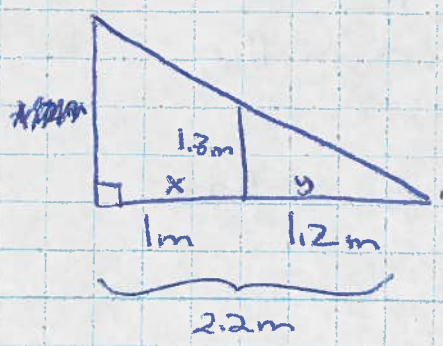
$0.25 = \frac{10h}{\sqrt{3}} \cdot \frac{dh}{dt}$

$\frac{\sqrt{3}}{40h} = \frac{dh}{dt}$

When $h = 0.1$m,

$\frac{dh}{dt} = \frac{\sqrt{3}}{40(0.1)} = \frac{\sqrt{3}}{4}$ m/min

(18.)



$\frac{dx}{dt} = 120\ m/min$

$= 2\ m/s$

Find $\frac{dy}{dt}$ @ $t = 5\ s$

$\frac{x+y}{y} = \frac{2.2}{1.2}$

$1.2x + 1.2y = 2.2y$

$1.2x = y$

Method #1

Well, $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= 1.2\,(2)$

$= 2.4\ m/s$

$= 144\ m/min$

Method #2

Thus $\frac{d\,(1.2x)}{dt} = \frac{dy}{dt}$

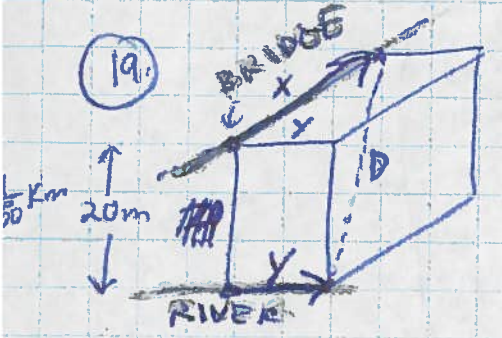
$1.2 \cdot \frac{dx}{dt} = \frac{dy}{dt}$

$\frac{dy}{dt} = 1.2\,(2)$

$= 2.4\ m/s$

$= 144\ m/min.$

(19.)



$\frac{dx}{dt} = 60\ km/h$

$\frac{dy}{dt} = 20\ km/h$

Find $\frac{dD}{dt}$ @ $t = 10s = \frac{1}{360}\ h$

$D^2 = x^2 + y^2 + \left(\frac{1}{50}\right)^2$ ← $20m = \frac{1}{50}\ Km$

At $t = \frac{1}{360}\ h$

$x = \frac{1}{6}\ Km$

$y = \frac{1}{18}\ Km$

$D = \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{18}\right)^2 + \left(\frac{1}{50}\right)^2}$

$2D\ \frac{dD}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} + 0$

$\frac{dD}{dt} = \frac{60x + 20y}{D} = 62.8\ Km/h$

$= 0.176816847\ Km$

2a) $T(x) = \frac{200}{1+x^2}$ $T(x)$ = temp perceived x metres from fire

a) $\frac{dx}{dt} = 2 m/s$ Find $\frac{d}{dt}T(x)$ @ $x = 5m$

$$\frac{d}{dt}T(x) = \frac{dT(x)}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{-200}{(1+x^2)^{+2}} \cdot \frac{2x}{1} \cdot [2]$$

$$= \frac{-800x}{(1+x^2)^{+2}}$$

$$= \frac{-800(5)}{(1+25)^2}$$

$$= \frac{-4000}{676}$$

$$= -5.9°C/s$$

At 5m, the perceived temp decreases 5.9°C every second when walking away at 2 m/s.