MCV4U Chapter Five Practice Test – Exponential, Log and Trig Derivatives DWOM

KNOWLEDGE SECTION.

1. State the value of each limit. No work is necessary.

   a) $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h}$ b) $\lim\limits_{t \to 0} \ln(1 + t)^{\frac{1}{t}}$ c) $\lim\limits_{h \to 0} \dfrac{\sin h}{h}$ d) $\lim\limits_{h \to 0} \dfrac{\cos h - 1}{h}$

2. Differentiate.

   a) $y = \sin x + e^x$ b) $f(x) = 2^x \tan(2x)$ c) $y = \dfrac{4x}{\ln \sqrt{x}}$ d) $y = \log_2(\sin x)$

3. Use logarithmic differentiation to find y' where $y = \left(\sqrt{2x - 3}\right)^{2x^2}$.

4. Determine the equation of tangent to $s(t) = \dfrac{\cos t}{2 + \sin t}$ at $t = \dfrac{\pi}{2}$.

APPLICATION SECTION.

5. Prove that if $y = \cos x$, that $y' = -\sin x$.
   Use a proof from ***first principles***.

6. If $f(x) = \dfrac{e^x}{x^2}$:
   a) Solve $f'(x) = 0$.
   b) Explain what your calculations tell you about the graph of $y = f(x)$.

7. If $s(t) = 4\sin t + \cos(2t)$ where x is in the interval $[0, 2\pi]$:
   a) Determine all critical numbers for $s(t)$ on the given interval.
   b) Determine all intervals (on the given interval) where $s(t)$ is increasing.
   c) Determine any points of inflection on the given interval.
   d) Sketch the curve.

Answers: 1a)1 b)1 c)1 d)0 2.a) $y' = \cos x + e^x$
b) $f'(x) = 2^x \ln 2 \tan 2x + 2^{x+1} sec^2(2x)$
c) $y' = \dfrac{8[\ln x - 1]}{[\ln x]^2}$ d) $y' = \dfrac{\cot x}{\ln 2}$
3. $y' = y\left[2x\ln(2x - 3) + \dfrac{2x^2}{2x - 3}\right]$
4. $2x + 18y - \pi = 0$ 5. proof – see sol'n
6.a) x= 2 b) critical point – minimum
7a) $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ b) $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{3\pi}{2}, 2\pi\right]$ c) $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$
d) See sol'ns 8a) $e^3 \approx 20.08$; $R \approx 100428$
b)$000/month; R is increasing by $5000 per dollar price raise
9.a) $h'(t) = -0.065 \sin 13t$ b) max of 0.065 m/s @ $t = \dfrac{3\pi}{26}, \dfrac{7\pi}{26}$; min of -0.065m/s @ $t = \dfrac{\pi}{26}, \dfrac{5\pi}{26}$
10. $y = ae^{2x} + be^{-2x}$ for any constants *a*, *b*. The most obvious is a = 1, b = 0.

8. Lasertronics, a manufacturer of MP3 players, find that its monthly revenue *R*, is given by the equation $R(x) = 100[200x - 50x\ln x]$ where x is the selling price of its product in dollars. The model is valid for values of x between 15 and 25 inclusive.
   a) At what price should the company sell its product to maximize monthly revenue? What is the maximum monthly revenue?
   b) Calculate $\dfrac{dR}{dx}$ when $x = e^2$. Interpret your answer.

THINKING SECTION.

9. A piston in an engine oscillates up and down from rest. The height of the piston from rest can be approximated by $h(t) = 0.005\cos(13t)$ where t is time in seconds and $h(t)$ is the height in metres above rest position after time t.
   a) Determine an equation for the velocity of the piston head as a function of time.
   b) Find the maximum and minimum velocities and the ***first two*** times (after $t = 0$) at which they occur.

10. A differential equation is an equation involving a function and one or more of its derivatives. Determine a function that satisfies the differential equation $\dfrac{d^2y}{dx^2} = 4y$. Explain briefly how you arrived at your solution. Are there other possible solutions? Discuss briefly.

MCV4U Chapter Five Test – Exponential, Log and Trig Derivatives Name:

K	A	T	C	INSTRUCTIONS - Complete all parts on foolscap showing all work. Use correct mathematical communication to demonstrate your understanding.

KNOWLEDGE SECTION.

1. State each derivative. No work is necessary.
   a) $f(x) = e^x$ b) $y = \log_2 x$ c) $s(t) = \sin t$ d) $f(x) = \tan x$

2. Take the derivative.
   a) $y = e^{x^2 - 2x}$ b) $f(x) = \ln(\cos x)$ c) $s(t) = \dfrac{e^t}{\sin t}$

3. Find the absolute maximum and minimum values of $y = -2\sin x$ over the interval $[-\pi, \pi]$.

4. Consider $f(x) = \dfrac{(x+2)^4\sqrt{2x+3}}{4x^3 - 8x}$.
   a) Take the natural logarithm of each side and simplify the right hand side.
   b) Use implicit differentiation to take the derivative of the left hand and right hand expressions you wrote in part a). You may leave your derivative unsimplified.
   c) What is the name given to this technique of differentiation?

APPLICATION SECTION.

$$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

$$\lim_{t\to 0}\ln(1+t)^{\frac{1}{t}} = 1$$

$$\lim_{h\to 0}\frac{\sin h}{h} = 1$$

$$\lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

5. Use the limit facts in the box to the right to develop the derivative for ***ONE*** of the following functions from ***first principles***. You choose which derivative to develop.
   $f(x) = e^x,\ g(x) = \ln x,\ h(x) = \sin x,\ j(x) = \cos x$

6. If $f(x) = \dfrac{\ln(x-1)}{e^2 x}$:
   a) Find the equation of the tangent at $x = 2$.
   b) Explain what your calculation in a) tells you about the graph of $y = f(x)$.

7. If the position of a particle from a fixed position is given by $s(t) = \cos t \sin t$ where $t \geq 0$; s is in cm, t is in seconds:
   a) Find the maximum distance the particle is away from the fixed position over the interval $[0, \pi]$.
   b) What is the initial velocity of the particle?
   c) What is the velocity when the acceleration is equal to zero for the first time (on the interval $[0, \pi]$)?

THINKING SECTION.

8. The value of Ferd's Pokemon card collection is given by $V(t) = e\left(2^{-\frac{t}{12}}\right) + 1.6$ where $V(t)$ is the value of the collection in hundreds of dollars and t is the number of years since 2002. Calculate $V(6)$ and $V'(6)$ and explain what it means in terms of Ferd and his card collection.

9. If $y = A\cos kt + B\sin kt$ where A, B, and k are constants, show that $y'' + k^2 y = 0$.