Chapter 2 Derivatives

2.1 The Derivative Function

- to find the derivative function of a function, it can be found with first principles the formula

$$\lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}$$

where the literal x and h variables are substituted into the function

- determine derivative of irrational and rational functions
- be aware when a function asks for the equation of a normal to a graph, the normal will simply be the equation of a line at a tangent, with a slope of the inverse reciprocal of the line slope
- Certain functions are not differentiable at different points
  - Absolute value functions are not differentiable at their switchover points,
    - these are called a **CUSP** or **POINT** as no derivative exists
  - asymptotes or Discontinuities are not differentiable, as they would yield direct vertical tangents
  - Direct vertical tangents in graphs are not differentiable
- not all continuous points are differentiable,
- all differentiable points are continuous

2.2 The Derivatives of polynomial Equations

- The Constant Function Rule

$$f(x) = k \to f\prime(x) = 0$$

- The Power Rule

$$f(x) = x^n \to f\prime(x) = nx^{n-1}$$

- The Constant Multiple Rule

$$f(x) = kg(x) \to f\prime(x) = kg\prime(x)$$

- Sum Difference Rule

$$f(x) = p(x) \pm q(x) \rightarrow f\prime(x) = p\prime(x) \pm q\prime(x)$$

- solve for points where the tangent is a certain direction by setting the derivative to the slope, then solving for x
- determining the equation of a tangent to a given curve that passes through a point not on the curve

Ex. Determine the equation of the tangent to $y = x^2 + 2$ that passes through (-1, -6)

set the slope of that tangent equal to the value of the derivative function of the x value it intersects

$$\frac{dy}{dx} = y\prime$$
$$\frac{y_2 - y_1}{x_2 - x_1} = y\prime$$

$$\frac{y_2 + 6}{x_2 + 1} = y\prime$$

$$\frac{x^2 + 2 + 6}{x + 1} = 2x$$

$$x^2 + 8 = 2x^2 + 2x$$

$$0 = x^2 + 2x - 8$$
$$(x + 4)(x - 2) = 0$$

$\therefore$ the tangent that passes through the given point also intersects the curve at x = 2 and - 4, so the tangent equations can be found from this information

- **HOMEWORK FROM THIS UNIT IS HIGHLIGHTED**

2.3 The Chain Rule

$$P(x) = f(x)g(x)$$

$$P\prime(x) = f\prime(x)g(x) + f(x)g\prime(x)$$

- able to solve these questions wth negative exponents

2.4 The Quotient Rule

be able to find the slope of tangents at given points and able to find the equation of tangents of rational functions

2.5 The Chain Rule

if $h(x) = f(g(x))$

then $h\prime(x) = f\prime(g(x))g\prime(x)$

- remember all other derivative rules when using the chain rule and finding f(x) and g(x) derivatives, especially product and quotient rule

Leibniz Notation

$$\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \cdot \frac{\delta u}{\delta x}$$

- able to use the chain rule
- able to use the chain rule in leipniz notation

2.5.1 Implicit Differentiation

- be able to implicitly differentiate standard functions using leipniz notation and the chain rule
- be able to find the slope at a given point with a implicit derivative

$$\frac{x^2}{9} + \frac{2y^2}{7} = 1$$

remove the common denominator

$$7x^2 + 18y^2 = 63$$

- remember the product rule while implicitly differentiating

$$3x^3 + x^2y + (x+y)^2 + y^3 = 100$$

- spend time with the mother function before you differentiate

$$\frac{x+y}{2x-y} = 1$$
$$x + y = 2x - y$$

2.5.2 Related Rates

- related rates questions involving
  - 2d circles
  - 2d squares
  - perimeter
  - cylinders
  - cones
  - triangles
    - pythagorean theorem
    - similar triangles

GENERAL

- Don't use product rule when an expression can be easily expanded into a polynomial
- don't use quotient rule when the numerator is a constant
- use product rule when a factored derivative function is required and the denominator is a radical.