5.3: Optimization Problems Involving Exponential Functions Date: ______________

Ex1. Identify the local extrema of the function $f(x) = x^2 e^x$.

Sketch the function.

$f(x) = x^2 e^x$

$f'(x) = 2x e^x + x^2 e^x$ $\quad f'(x) = 0$ when $x = -2, 0$

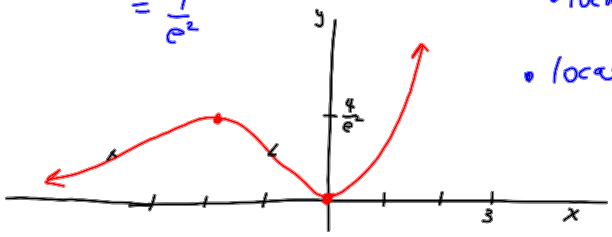
$= x e^x (2 + x)$

$\oplus \ \cap \ \ominus \ \cup \ \oplus$ (number line: -2, 0)

$f(-2) = (-2)^2 e^{-2}$ $\quad f(0) = 0$

$= \frac{4}{e^2}$

Thus
- local max is $\frac{4}{e^2}$
- local min is 0



Ex2. Ferd has bought a new scooter for \$4 000. The value of the scooter depreciates over time.

The value of the scooter after t years is ~~$V(t) = 4000e^{-\frac{t}{4}}$~~.

a. At what rate is the value of the scooter depreciating the instant Ferd drives it off the dealer's lot?

Find $V'(0)$

$V'(t) = 4000 e^{-\frac{t}{4}} \cdot \left(-\frac{1}{4}\right)$ $\quad V'(0) = -\$1000/yr$

$= -1000 e^{-\frac{t}{4}}$

b. How long will it take for the scooter to depreciate to one quarter of its initial value?
What rate is the scooter depreciating at this time?

$V(0) = \$4000$

$V(t) = \$1000$

$4000 e^{-\frac{t}{4}} = 1000$

$e^{-\frac{t}{4}} = \frac{1}{4}$

$-\frac{t}{4} = \ln\left(\frac{1}{4}\right)$

$\frac{t}{4} = \ln 4$

$t = 4 \ln 4$

$t \doteq 5.5$ yrs

$V'(4\ln 4)$

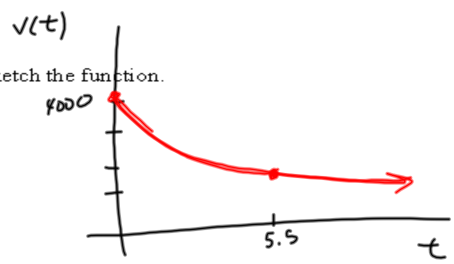
$= -1000 e^{-\frac{4\ln 4}{4}}$

$= -1000 e^{\ln\left(\frac{1}{4}\right)}$

$= -1000\left(\frac{1}{4}\right)$

$= -\$250/yr$

c. Sketch the function.



Ex3. Some experiments show that the effectiveness of studying for an exam is $E(t) = 0.5[10 + te^{-\frac{t}{20}}]$, where t is the number of hours spent studying for the exam. E is put on a scale of 0 to 10.

If a student has up to 30 hours for studying, how many hours are needed for maximum effectiveness?

Sketch the function.

$$E'(t) = 0.5\left[(1)e^{-\frac{t}{20}} + t \cdot e^{-\frac{t}{20}} \cdot \frac{-1}{20}\right]$$

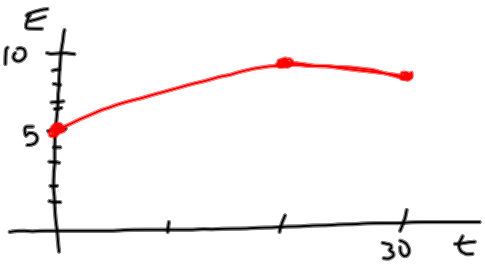
$$= \frac{1}{2} \cdot e^{-\frac{t}{20}} \cdot \frac{1}{20}\left[20 - t\right]$$

Thus $E'(t) = 0$ when $t = 20$

$E(0) = 5$

$E(20) = 8.7$

$E(30) \doteq 8.3$



Ex4. A mathematical consultant determines that the rate of people who will have responded to the advertisement of a new product after it has been marketed for t days is given by $r(t) = 0.7(1 - e^{-0.2t})$.

The area covered by the advertisement contains 10 million potential customers, and each response to the advertisement results in \$0.50 (on average) revenue to the company.

The advertising costs \$30 000 to produce and a further \$5000 per day to run.

a. Determine $\lim\limits_{t \to \infty} r(t)$, and interpret the result.

$$\lim_{t \to \infty} r(t) = \lim_{t \to \infty} 0.7(1 - e^{-0.2t})$$

$$= 0.7(1 - 0)$$

$$= 0.7$$

So, after marketing the product forever, 70% of the potential customers will have responded.

b. How many customers will respond after 7 days of advertising?
Determine the revenue generated.

# of customers

$= 10000000\, r(7)$

$= 10000000\,(0.52738\ldots)$

$= 5273821$

$R(7)$

$= 0.5\,(5273821)$

$= \$2636910.50$

c. Write the function $P(t)$ that represents the profit after t days of advertising.
What is the profit after 7 days of advertising?

$P(t) = R(t) - C(t)$

$= [10000000\, r(t)][0.5] - (30000 + 5000t)$

$= 5000000\, r(t) - 5000t - 30000$

$P(7) = 2636910.50 - 35000 - 30000$

$= \$2571910.50$

d. For how many full days should the advertising campaign be run in order to maximize profit?

$P'(t) = 5000000\, r'(t) - 5000$

$= 700000\, e^{-0.2t} - 5000$

$r'(t) = 0.7\left[-e^{-0.2t}(-0.2)\right]$

$= 0.14\, e^{-0.2t}$

$P'(t) = 0$ when $e^{-0.2t} = \dfrac{5\cancel{000}}{700\cancel{000}}$

$e^{-0.2t} = \dfrac{1}{140}$

$e^{0.2t} = 140$

$\frac{1}{5}t = \ln 140$

$t = 5 \ln 140$

$t \doteq 24.7$

Check

MAX

$P(24)$ ≟ $P(25)$

Thus the ad should run for 25 days.

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