Ch 5 Test : Mock Test

~~① & ②~~

1. $f(x) = 5e^{-4x-9\pi}$

$f'(x) = 5e^{4x-9\pi} \cdot 4$
$= 20e^{4x-9\pi}$

2. $y = 12^{-4x+9}$

$\frac{dy}{dx} = 12^{-4x+9} \cdot \ln 12 \cdot (-4)$
$= -4 \cdot 12^{-4x+9} \cdot \ln 12$

3. $f(x) = -0.5x^2 \cdot 2^x$

$f'(x) = -x \cdot 2^x - 0.5x^2 [2^x \cdot \ln 2]$
$= -0.5x 2^x [2 + x\ln 2]$

$\therefore f'(x) = 0$ when $x = 0$ OR $x = \frac{-2}{\ln 2}$

4. $y = 3\cos(8x+6)$

$\frac{dy}{dx} = 3[-\sin(8x+6)](8)$
$= -24\sin(8x+6)$

5. $y = \tan^2(e^x)$

$\frac{dy}{dx} = 2\tan(e^x) \cdot \sec^2(e^x) \cdot e^x$
$= 2e^x \tan(e^x)\sec^2(e^x)$

6. $f(x) = 4xe^x$

$f'(x) = 4e^x + 4xe^x$

$\therefore f'(0) = 4 + 4(0)$
$= 4$

7. $f(x) = 7^x \cdot x^7$

$f'(x) = 7^x \ln 7 \cdot x^7 + 7^x [7x^6]$
$= 7^x \cdot x^6 [x\ln 7 + 7]$

8. $f(x) = 3x3^x - 1$

$f'(x) = 3 \cdot 3^x + 3x[3^x \ln 3]$
$= 3 \cdot 3^x [1 + x\ln 3]$

$\therefore f'(x) = 0$ when $x = -\frac{1}{\ln 3}$

$f'(x) < 0$ $\quad -\frac{1}{\ln 3} \quad$ $f'(x) > 0$

As $x \to \infty$, $f(x) \to \infty$

$f(-\frac{1}{\ln 3}) \doteq -2.00$ is the min. value of $f(x)$.

9. $f(x) = (-2)^x$

as $x \to \infty$ $f(x) \to \pm\infty$

as $x \to -\infty$ $f(x) \to 0$, but 0^+ and 0^-

$f(x)$ is not continuous. and not diff

10. $f(x) = 5^{\tan(\sqrt{x})}$ $\quad f'(x) = 5^{\tan(\sqrt{x})} \cdot \ln 5 \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= 5^{\tan\sqrt{x}} \sec^2(\sqrt{x}) \ln 5$

11. $v(t) = 60(1 - 0.7^t)$

a) $a(t) = v'(t)$
$= 60(0 - 0.7^t \ln(0.7))$
$= -60(0.7)^t \ln(0.7)$

b) $a(2) = -60(0.7)^2 \ln(0.7)$
$\doteq 10.5 \text{ m/s}^2$

c) $v(0) = 0$ m/s
The car initially was stationary

d) $-60(0.7)^t \ln(0.7) = 3$
$(0.7)^t = \dfrac{-3}{60\ln(0.7)}$
$t\ln(0.7) = \ln\left(\dfrac{-1}{20\ln(0.7)}\right)$
$t \doteq 5.5$ s

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12. $P(t) = 1200(2^{-t})$

a) $P(0) = 1200$

b) $P(1) = 600$

(c) $P'(t) = 1200(2^{-t})(\ln 2)(-1)$
$= -1200(2^{-t})\ln 2$
$P'(1) \doteq -416$ people/week

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13. $s(t) = 11\cos(3t)$, $t \geq 0$

a) $s'(t) = -11\sin(3t)\cdot 3$
$= -33\sin(3t)$

$s'(t) = 0$ when $\sin(3t) = 3$
$3t = 0, \pi, 2\pi, \ldots$
$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \ldots$

$s'(t) < 0$ $\quad$ $s'(t) > 0$ $\quad$ $s'(t) < 0$ $\quad \ldots$
$0 \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \ldots$

$\therefore$ particle switches direction @ $t = \frac{\pi}{3}$ for the first time.

(b) The particle changes direction for $t = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \ldots$

(c) $a(t) = -33\cos(3t)(3)$
$= -99\cos(3t)$
$a(t) = 0$ when $\cos(3t) = 0$
$3t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \ldots$

$v(\frac{\pi}{6}) \doteq -33$, $v(\frac{\pi}{2}) = +33$, ...
$\therefore$ max velocity is 33

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14. $f(x) = \cot x$ $\quad$ $f(x) = \dfrac{1}{\tan x} = (\tan x)^{-1}$

$\therefore f'(x) = -(\tan x)^{-2}(\sec^2 x)$
$= -\dfrac{\cos^2 x}{\sin^2 x} \cdot \dfrac{1}{\cos^2 x}$
$= -\dfrac{1}{\sin^2 x}$
$= -\csc^2 x$

OR $f(x) = \dfrac{\cos x}{\sin x}$
USE QUOTIENT RULE TO FIND $f'(x)$

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15. a) $y = \sqrt{\tan x}$
$= (\tan x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(\tan x)^{-\frac{1}{2}} \sec^2 x$

$= \dfrac{\sec^2 x}{2\sqrt{\tan x}}$

b) $\tan x > 0$ for y to be diff
Let $\tan x = 0$
$x = 0, \pi, 2\pi, \ldots$
and $\tan$ is undefined if
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

S $\quad$ A

$+ \quad - \quad + \quad -$
$0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

So, y is not diff if
$\frac{\pi}{2} \leq x \leq \pi$ or $\frac{3\pi}{2} \leq x \leq 2\pi$