4.4: Concavity and Points of Inflection

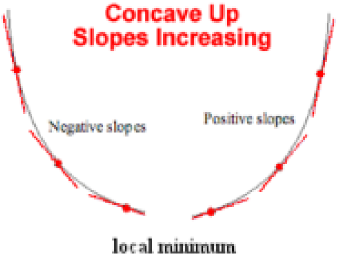
Date: ______________

Test for Concavity

IF $f(x)$ is a differentiable function and $f''(x)$ exists on $x \in [a,b]$, THEN

i. IF $f''(x) > 0$

THEN the graph of $f(x)$ is **CONCAVE UP**



ii. IF $f''(x) < 0$

THEN the graph of $f(x)$ is **CONCAVE DOWN**

The 2[nd] Derivative Test [Used to test if critical points are local maxima or minima. Not used to test for points of inflection.]

IF $(c, f(c))$ exists and $f'(c) = 0$, THEN

i. $f''(c) > 0 \rightarrow f(c)$ is a local min

ii. $f''(c) < 0 \rightarrow f(c)$ is a local max

iii. $f''(c) = 0 \rightarrow$ *FAIL!* Use the 1[st] Derivative Test!!

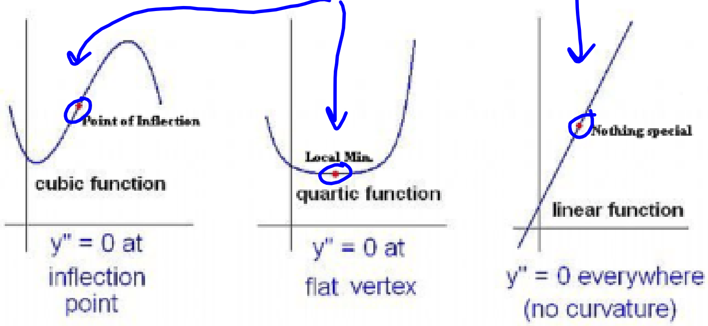
"linear"

When $f''(c) = 0$, the derivative is stationary and $(c, f(c))$ could be

i.e. slope is constant

- an inflection point
- a local maximum or minimum
- neither

function is linear



Points of Inflection

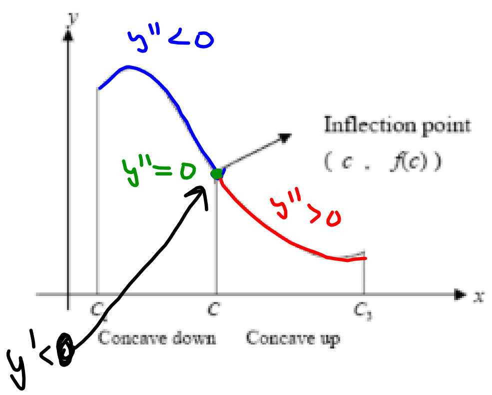
A POINT OF INFLECTION occurs at $(c, f(c))$ if $f''(x)$ changes sign at $x = c$. That is, the curve changes from concave down to concave up, or vice versa.

IF $(c, f(c))$ is a point of inflection,

THEN

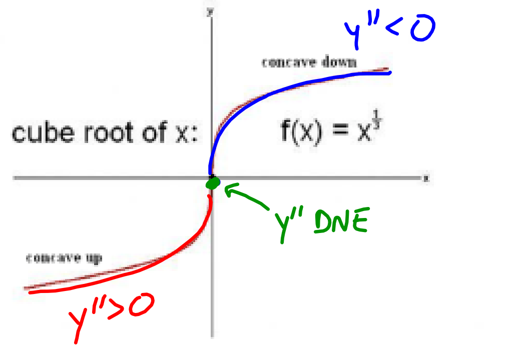
> So, to find points of inflection:
> 1. Find potential candidates $(c, f(c))$ where $f''(c) = 0$ or $f''(c)$ does not exist.
> 2. Check if $f''(x)$ switches signs on either side of c.

$f''(c) = 0$



OR

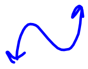
$f''(c)$ does not exist.



inflection point at (0,0), but both f'(x) and f''(x) are undefined at (0,0)

Ex1. Find any points of inflection and use the 2nd derivative to test for local minimum or maximum values. Then, sketch the graph using your results along with more information if needed.

a. $y = x^3 - 3x^2 - 9x + 10$



x	y
0	10
?	0

$y' = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x+1)(x-3)$

$y'' = 6x - 6$

$= 6(x-1)$

2nd D Test

IF $x = -1$

$y = 15$, $y' = 0$

Since $y'' < 0$

$\therefore$ local max @ $(-1, 15)$

IF $x = 3$

$\therefore$ $y = -17$, $y' = 0$

Since $y'' > 0$

$\therefore$ min @ $(3, -17)$

Points of Inflection

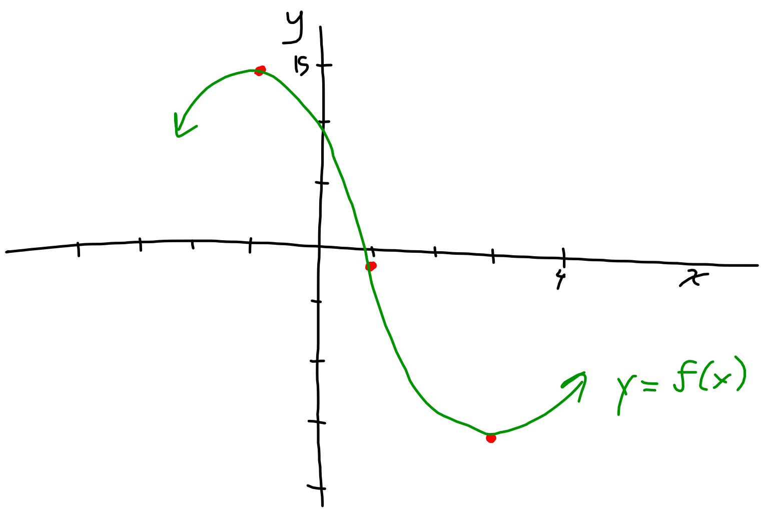
$y'' = 6x - 6$

$= 6(x-1)$

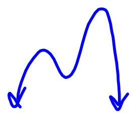
y'' $\ominus$ $\oplus$

1

Thus there is a point of inflection at $(1, -1)$.



b. $y = -(x-2)^4 + 3$



x	y
2	3
0	−13
	0

$(x-2)^4 = 3$

$x - 2 = \pm\sqrt[4]{3}$

$x = 2 \pm \sqrt[4]{3}$

$y' = -4(x-2)^3(1)$

$= -4(x-2)^3$

$y'' = -12(x-2)^2(1)$

$= -12(x-2)^2$

2nd D Test

IF $x = 2$

$\therefore y = 3$, $y' = 0$

Since $y'' = 0$

$\therefore$ FAIL

So use 1st D Test

1st D Test

y' $\oplus$ $\ominus$

2

$\therefore$ max @ $(2, 3)$

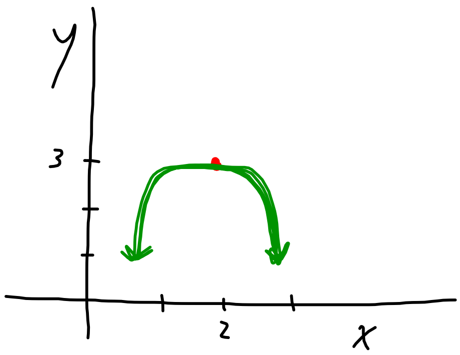
Inflection Points?

$y'' = -12(x-2)^2$

y'' $\ominus$ $\ominus$

2

$\therefore$ there are no points of inflection.



c. $y = 9(x+1)^{\frac{1}{3}}$

$y = 9\sqrt[3]{x+1}$

x	y
-1	0
0	9

$y' = 3(x+1)^{-\frac{2}{3}}(1)$

$= \dfrac{3}{\sqrt[3]{(x+1)^2}}$

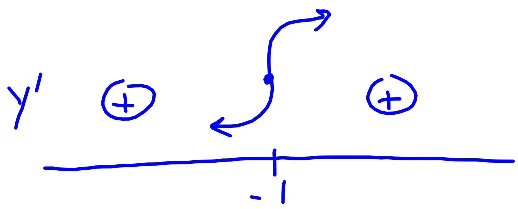
$y'' = -2(x+1)^{-\frac{5}{3}}$

$= \dfrac{-2}{\sqrt[3]{(x+1)^5}}$

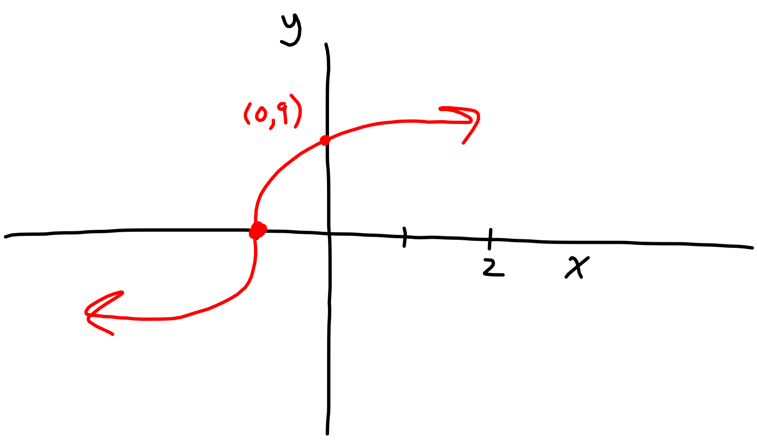
2nd D Test

FAIL!!!

1st D Test



$\therefore$ inflection @ $(-1, 0)$



d. $y = \dfrac{1}{x^2+3}$ HA @ $y=0$

x	y
0	$\frac{1}{3}$
	~~0~~

$y=(x^2+3)^{-1}$

$$y' = -(x^2+3)^{-2}(2x)$$

$$= \frac{-2x}{(x^2+3)^2}$$

$$y'' = \frac{-2(x^2+3)^2 - (-2x)\left(2(x^2+3)'(2x)\right)}{(x^2+3)^4}$$

$$= \frac{-2(x^2+3)+8x^2}{(x^2+3)^3}$$

$$= \frac{6x^2-6}{(x^2+3)^3}$$

$$= \frac{6(x-1)(x+1)}{(x^2+3)^3}$$

2nd D Test

IF $x=0$

$\therefore$ $y=\frac{1}{3}$, $y'=0$

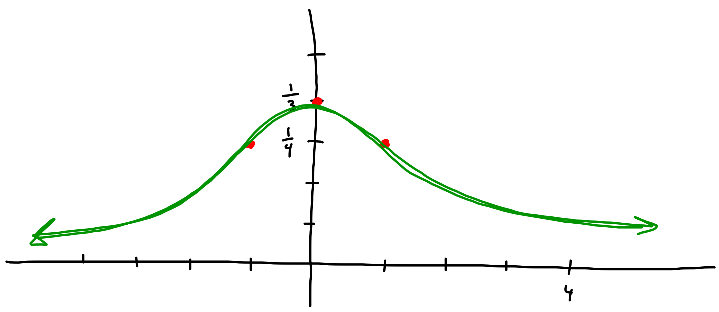
Since $y''<0$

$\therefore$ max @ $(0, \frac{1}{3})$

Inflection ???

y'': $\oplus$ ($x<-1$), $\ominus$ ($-1<x<1$), $\oplus$ ($x>1$)

$\therefore$ $(-1, \frac{1}{4})$ and $(1, \frac{1}{4})$ are points of inflection.



Homefun: Page 205 #1→4,

5[ans. 5a and 5b graphs should pass through (0, 2) and 5b is quartic, so right side of graph should show increase to ∞],

6, 8[ans. 8b $\left(\frac{\pm 3}{\sqrt{7}}, \frac{\pm 10\sqrt{7}}{9}\right)$], 9[ans. since $f'(2)=0$, graph must be flat at $x=2$], 10, 11