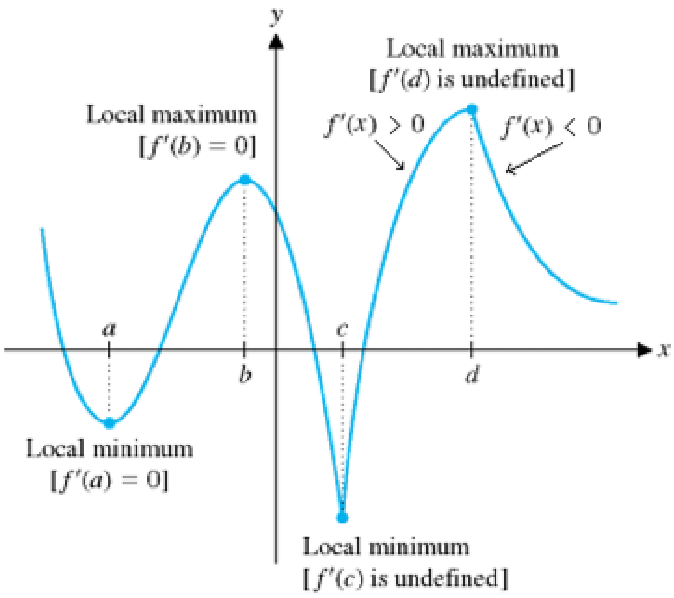
4.2: <u>Critical Points, Local Maxima, and Local Minima</u> Date: _______________

A. For a function f, c is a **CRITICAL NUMBER** if $f(c)$ exists and $f'(c) = 0$ or $f'(c)$ is undefined.

As a result, the point $(c, f(c))$ is called a **CRITICAL POINT**.



a, b, c, d **are the critical number of this function**

At those x-values,
- the function is defined, AND
- the derivative is zero or does not exist.

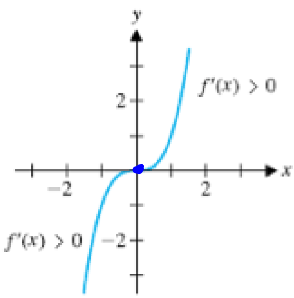
B. **EVERY LOCAL MAXIMUM OR MINIMUM VALUE OF A FUNCTION WILL OCCUR AT A CRITICAL POINT** of the function since peaks and valleys of a graph occur where the tangent is horizontal, or does not exist.

<u>FIRST DERIVATIVE TEST</u>
If $f(c)$ is a local maximum or minimum, then $f'(x)$ will change its sign at c.
If $f'(x)$ changes from positive to negative, then $f(c)$ is a local maximum.
Otherwise $f(c)$ is a local minimum.

C. **SOME CRITICAL POINTS ARE NEITHER LOCAL MAXIMA OR LOCAL MINIMA.**
This occurs when $f'(x)$ does **NOT** change its sign at c.
Examples of critical points which are NOT local maxima or minima are given below.

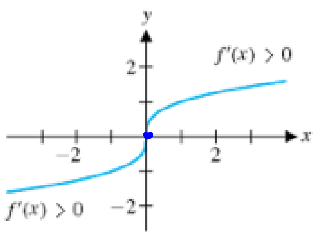
A critical point exists at (0, 0) since $f'(0) = 0$,

but (0, 0) is neither a local minimum or maximum.



A critical point exists at (0, 0) since $f'(0)$ does not exist,

but (0, 0) is neither a local minimum or maximum.



Ex1. Determine all critical numbers. Use the **FIRST DERIVATIVE TEST** to see if critical points are local maxima, local minima, or neither. Then, provide a quick sketch of each graph.

a. $y = x^5 - 5x$

$= x(x^4 - 5)$

x	y
0	0
$\pm\sqrt[4]{5}$	0

$y' = 5x^4 - 5$

$= 5(x^4 - 1)$

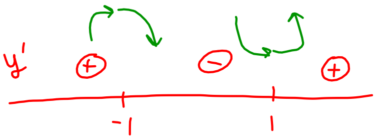
$= 5(x^2 - 1)(x^2 + 1)$

$y' = 5(x-1)(x+1)(x^2+1)$

Critical Numbers

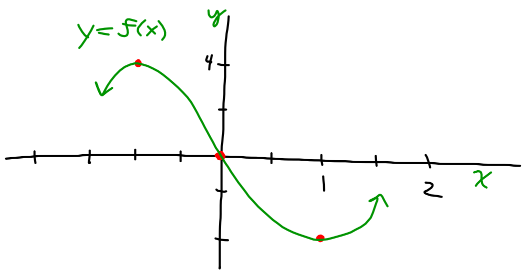
$x = \pm 1$

1st D Test



Critical Points

- local max @ $(-1, 4)$
- local min @ $(1, -4)$



b. $f(x) = x^4 - 8x^3 + 18x^2$

$= x^2(x^2 - 8x + 18)$

x	y
0	0

$f'(x) = 4x^3 - 24x^2 + 36x$

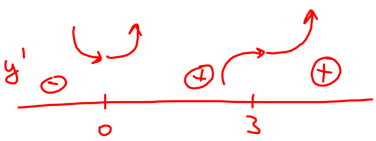
$= 4x(x^2 - 6x + 9)$

$y' = 4x(x-3)^2$

C.N.'s

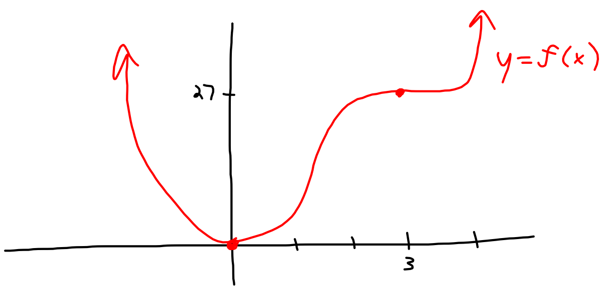
$x = 0, 3$

1st D Test



C.P.'s

- local min @ $(0,0)$
- inflection @ $(3, 27)$



c. $f(x) = (x+2)^{\frac{2}{3}} = \sqrt[3]{(x+2)^2}$

x	y
-2	0

$$f'(x) = \frac{2}{3}(x+2)^{-\frac{1}{3}}(1)$$

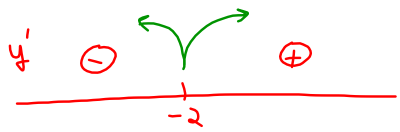
$$= \frac{2}{3\sqrt[3]{x+2}}$$

C.N.'s

$x=-2$ $f'(-2)$ DNE ✓

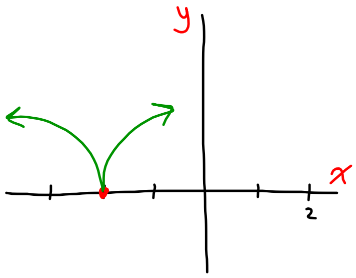
$f(-2)=0$ ✓

1st D Test



C.P.

- local min @ $(-2, 0)$



d. $f(x) = (x+2)^{\frac{1}{3}} = \sqrt[3]{x+2}$

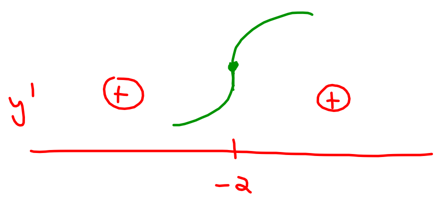
x	y
-2	0

$$f'(x) = \frac{1}{3}(x+2)^{-\frac{2}{3}}(1)$$

$$= \frac{1}{3\sqrt[3]{(x+2)^2}}$$

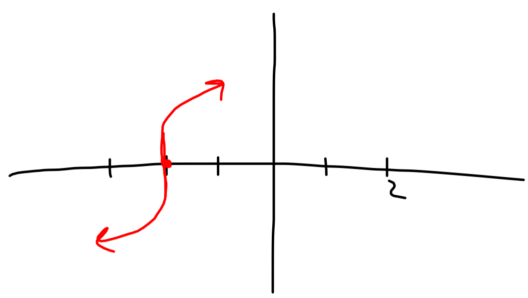
C.N.'s

$x=-2$



C.P.

- inflection @ $(-2,0)$



Ex2. Find all critical points of $y = \dfrac{2x^2}{x+2}$ and determine whether the points are local maxima or local minima.

Then, explain why -2 is not a critical number even though $f'(2)$ does not exist.

$$y = \frac{2x^2}{x+2}$$

VA @ $x = -2$

x	y
0	0
-4	-16

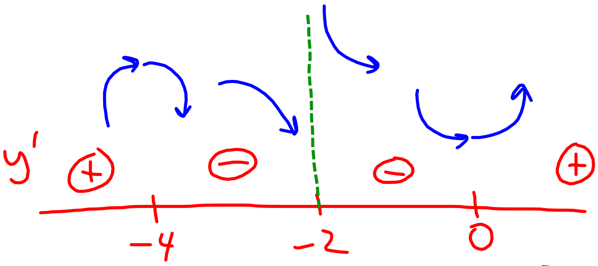
$$y' = \frac{4x(x+2) - 2x^2(1)}{(x+2)^2}$$

$$= \frac{2x^2 + 8x}{(x+2)^2}$$

$$= \frac{2x(x+4)}{(x+2)^2}$$

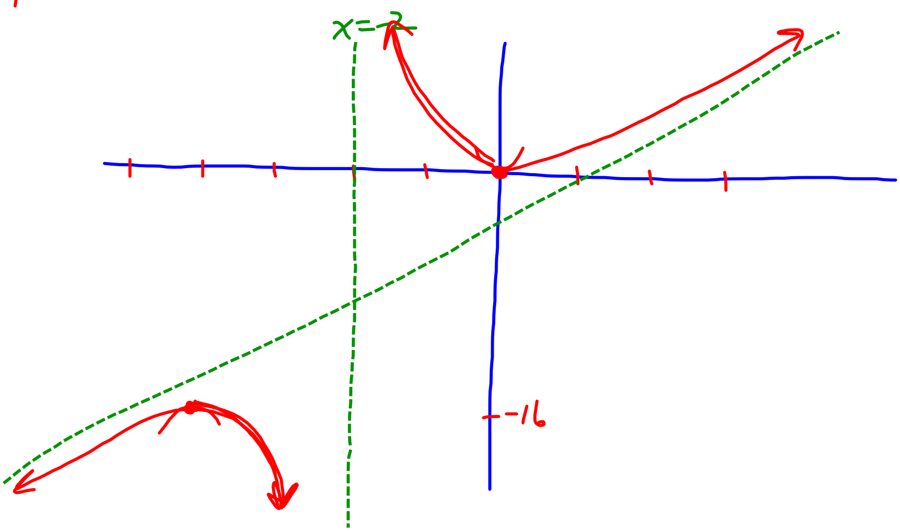
C.N.'s

$x = 0, -4, \cancel{-2}$



C.P.'s

- local max @ $(-4, -16)$
- local min @ $(0, 0)$



Homefun: Page 178 # 2b, 3[3b&3c ans: max/min labeled backwards], 5cd, 6cd(sketch graphs), 7acef, 8, 9, 14[14c ans: parabola should have zeros at -1 and 2], 10→13