Calculate the following limits.

(1.) $\lim_{x \to 2} (x+3) = 2+3$

$= 5$

(2.) $\lim_{x \to 0} (x^2+2x+7) = 0^2+2(0)+7$

$= 7$

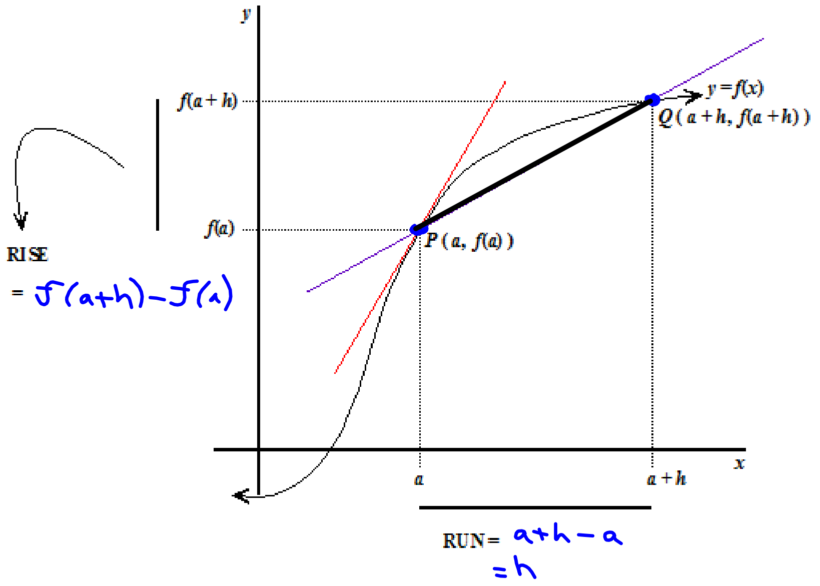
Evaluate and justify your answer by writing the equivalent multiplication statement.

$\frac{24}{3} = 8$ $\quad 8 \times 3 = 24$ ✓

$\frac{0}{3} = 0$ $\quad 0 \times 3 = 0$ ✓

$\frac{24}{0} = ?$ $\quad ? \times 0 = 24$ Nothing

$\frac{0}{0} = ?$ $\quad ? \times 0 = 0$ Anything



AROC from $x = a$ to $x = a + h$ equals the slope of the secant from P to Q.

$$\boxed{\textbf{AROC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(a+h)-f(a)}{h}}$$

Whereas the IROC at $x = a$ equals the slope of the tangent at P.

$$\boxed{\textbf{IROC} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

Ex1. Determine the slope of the tangent at

a. $(5, 10)$ to $f(x) = x^2 - 3x$

$$m = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h}$$

$$= \lim_{h\to 0} \frac{h^2+7h+10-10}{h}$$

$$= \lim_{h\to 0} \frac{h^2+7h}{h}$$

$$= \lim_{h\to 0} (h+7)$$

$$= 0+7$$

$$= 7$$

Aside

$f(5+h)$

$= (5+h)^2 - 3(5+h)$

$= 25+10h+h^2-15-3h$

$= h^2+7h+10$

$f(5) = 10$

b. $x = 1$ to $y = \dfrac{2}{x+3}$

$$m = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{h+4} - \frac{1}{2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\left[\frac{4-(h+4)}{2(h+4)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\frac{\cancel{4} - h - \cancel{4}}{2(h+4)}$$

$$= \lim_{h\to 0} \frac{-1}{2(h+4)}$$

$$= \frac{-1}{2(0+4)}$$

$$= -\frac{1}{8}$$

Aside

$$f(1+h) = \frac{2}{(1+h)+3}$$

$$= \frac{2}{h+4}$$

$$f(1) = \frac{2}{1+3}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

c. $(0, 3)$ to $f(x) = \frac{3}{4}\sqrt{16 - x^2}$ ← include a quick sketch of the function and tangent

$$m = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3\sqrt{16-h^2}}{4} - \frac{3\times 4}{4}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\frac{3\sqrt{16-h^2} - 12}{4}\cdot\frac{3\sqrt{\quad} + 12}{3\sqrt{\quad} + 12}$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\frac{9(16-h^2) - 144}{4(3\sqrt{\quad} + 12)}$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\frac{-9h^2}{4(3\sqrt{\quad} + 12)}$$

$$= \lim_{h\to 0} \frac{-9h}{4(3\sqrt{16-h^2} + 12)}$$

$$= \frac{-9(0)}{\text{NOT ZERO}}$$

$$= 0$$

$$f(h) = \frac{3}{4}\sqrt{16-h^2}$$

$$= \frac{3\sqrt{16-h^2}}{4}$$

$$f(0) = 3$$

Ex2. Given $y = x^3 + x$, find the equation of the tangent at $x = 2$.

1
1 1
1 2 1
1 3 3 1

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 6h^2 + 13h + 10 - 10}{h}$$

$$= \lim_{h \to 0} \left(h^2 + 6h + 13\right)$$

$$= 13$$

$f(2+h)$

$= (2+h)^3 + (2+h)$

$= 2^3 + 3 \cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2 + h^3 + 2 + h$

$= h^3 + 6h^2 + 13h + 10$

$f(2) = 2^3 + 2$

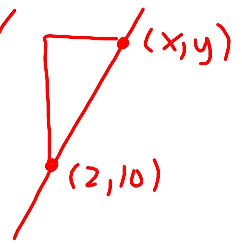
$= 10$

So $m = 13$ and $P_0 = (2, 10)$

$$\therefore \frac{x - 2}{1} = \frac{y - 10}{13}$$

$$13x - 26 = y - 10$$

$$13x - y - 16 = 0 \quad \text{1TRE}$$



$$\frac{y - 10}{x - 2} = \frac{13}{1}$$

Homefun: Page 18 #(1b, 3ad, 4be, 5c)(basic skills), (8a, 10b, 11ecf, 12, 13, 16, 18abdf, 21)(key questions)