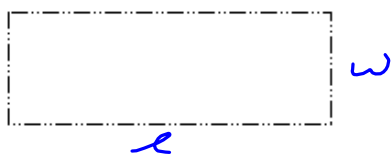


### 3.3: Optimization Problems I

Date: \_\_\_\_\_

Ex1. Maximize the area of the given pens if 40 m of fencing can be used.

a.



$$2l + 2w = 40$$

$$2w = 40 - 2l$$

$$w = 20 - l$$

$$A = lw$$

$$A(l) = l(20 - l) \\ = 20l - l^2$$

$$A'(l) = 20 - 2l$$

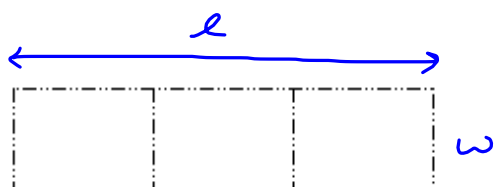
$$\text{If } A'(l) = 0$$

$$\therefore 20 = 2l \\ l = 10$$

$$w = 20 - l \\ = 20 - 10 \\ = 10$$

$$\therefore \text{Max } A = 10(10) \\ = 100 \text{ m}^2$$

b.



$$2l + 4w = 40$$

$$2l = 40 - 4w$$

$$l = 20 - 2w$$

$$A = lw$$

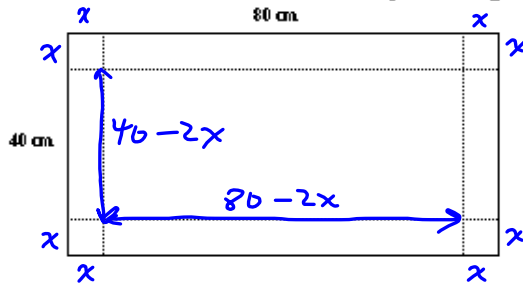
$$A(w) = (20 - 2w)(w) \\ = 20w - 2w^2$$

$$A'(w) = 20 - 4w = 0$$

$$\therefore 4w = 20 \\ w = 5$$

$$\text{Max } A = (20 - 2w)(w) \\ = (20 - 10)(5) \\ = (10)(5) \\ = 50 \text{ m}^2$$

Ex2. Maximize the volume of an open box given the net below. The corners are squares.



$$V = lwh$$

$$V(x) = (80 - 2x)(40 - 2x)(x)$$

$$= (-2)(x - 40)(-2)(x - 20)(x)$$

$$= 4x(x - 40)(x - 20)$$

$$= 4x(x^2 - 60x + 800)$$

$$= 4(x^3 - 60x^2 + 800x)$$

Thus,

Max  $V$

$$= (80 - 2x)(40 - 2x)(x)$$

$$\approx \underline{\underline{12317 \text{ cm}^3}}$$

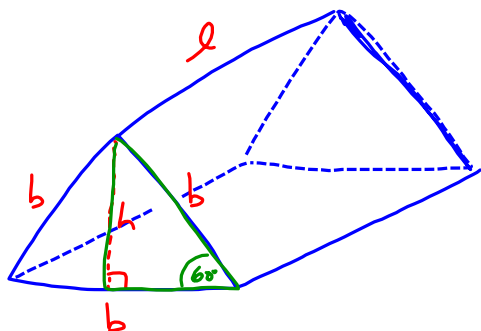
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$$V'(x) = 4(3x^2 - 120x + 800) = 0$$

$$\therefore x = \frac{120 \pm \sqrt{4800}}{6}$$

$$x \approx \frac{\cancel{31.547}}{\cancel{6}} \text{ or } \underline{8.453}$$

Ex3. Minimize the surface area of a triangular prism open at one end if the volume is  $500 \text{ cm}^3$ .



$$\begin{aligned}\sin 60 &= \frac{h}{b} & BA &= \frac{1}{2} b h \\ \frac{\sqrt{3}}{2} &= \frac{h}{b} & &= \frac{1}{2} b \frac{\sqrt{3}}{2} b \\ & & &= \frac{\sqrt{3}}{4} b^2 \\ h &= \frac{\sqrt{3}}{2} b\end{aligned}$$

$$SA = BA + 3bl$$

$$= \frac{1}{2} b h + 3bl$$

$$= \frac{\sqrt{3}}{4} b^2 + 3b \left( \frac{2000}{\sqrt{3} b^2} \right)$$

$$= \frac{\sqrt{3}}{4} b^2 + \frac{6000}{\sqrt{3}} b^{-1}$$

$$BA \cdot l = 500$$

$$\frac{\sqrt{3}}{4} b^2 \cdot l = 500$$

$$l = \frac{2000}{\sqrt{3} b^2}$$

$$\frac{d}{db}(SA) = \frac{\sqrt{3}}{2} b - \frac{6000}{\sqrt{3}} b^{-2} = 0$$

$$\frac{\sqrt{3} b}{2} = \frac{6000}{\sqrt{3} b^2}$$

$$b^3 = \frac{12000}{3}$$

$$b^3 = 4000$$

$$\rightarrow b = \sqrt[3]{4000}$$

$$\therefore \text{MIN } SA$$

$$= \frac{\sqrt{3}}{4} ( )^2 + \frac{6000}{\sqrt{3}} ( )^{-1}$$

$$= \text{_____ } \text{cm}^2$$

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← Application of Skills