

Relativistic Mass And Momentum

Time dilation and length contraction change the momentum of an object

At low speeds $p = mv$ works, but it fails to work at higher speeds

Relativistic Momentum

We can use a similar transformation as time dilation and length contraction to calculate momentum. As velocity approaches the speed of light, momentum approaches infinity, thus no force could accelerate a body to the speed of light as that would take an infinite amount of force.

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

thus the momentum of a proton traveling at 0.75 the speed of light would

$$p = \frac{mv}{\sqrt{1 - \frac{(0.75c)^2}{c^2}}}$$

$$p = \frac{mv}{\sqrt{0.4375}}$$

$$p = \frac{(1.67 \times 10^{-27} \text{kg})(0.75c)}{\sqrt{0.4375}}$$

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$$p = 5.68 \times 10^{-19} \frac{\text{kg} * m}{s}$$

Relative Mass

We find that mass increases as we approach the speed of light

Einstein's equation describes an object at rest

$$E_{rest} = mc^2$$

The total energy of an object must account for the kinetic energy of the object. The total energy of an object relates to its relativistic mass, not just its rest mass

$$E_{total} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Kinetic Energy

To find the kinetic energy of an object we can find the total amount of energy in an object, and subtract the amount of energy it has at rest

$$\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$