Inverse Operations

$\frac{(20)}{5} = 4 \longleftrightarrow 5\cdot(4) = 20$

$20 \to \boxed{\frac{x}{5}} \to 4$ $\qquad$ $4 \to \boxed{5x} \to 20$

$\sqrt{(9)} = 3$ $\qquad$ $(3)^2 = 9$

$9 \to \boxed{\sqrt{x}} \to 3$ $\qquad$ $3 \to \boxed{x^2} \to 9$

$\sin(30) = \frac{1}{2}$ $\qquad$ $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

$30^\circ \to \boxed{\sin x} \to \frac{1}{2}$ $\qquad$ $\frac{1}{2} \to \boxed{\sin^{-1}(x)} \to 30^\circ$

angle $\qquad$ ratio $\qquad$ ratio $\qquad$ angle

$5^{(3)} = 125$ $\qquad$ $\log_5(125) = 3$

$3 \to \boxed{5^x} \to 125$ $\qquad$ $125 \to \boxed{\log_5 x} \to 3$

exponent $\qquad$ power $\qquad$ power $\qquad$ exponent

5.2: **The Derivative of $y = b^x$** Date: ______________

Recall the Logarithm Definitions and Logarithm Laws

Definitions:

1. $\log_a c = b \Leftrightarrow$ $a^b = c$ Ex: $\log_2 8 = 3 \Leftrightarrow$ $2^3 = 8$

2. The Common Log, $\log_{10} M =$ $\log M$ The Natural Log, $\log_e M =$ $\ln M$

Laws:

1. $\log_a(MN) =$ $\log_a M + \log_a N$ Ex: $\log(1000 \times 10) =$ $\log 1000 + \log 10 = 3 + 1 = 4$

2. $\log_a\left(\frac{M}{N}\right) =$ $\log_a M - \log_a N$ Ex: $\log(1000 \div 10) =$ $\log 1000 - \log 10 = 3 - 1 = 2$

3. $\log_a(M^n) =$ $n(\log_a M)$ Ex: $\log(10^4) =$ $4(\log 10) = 4(1) = 4$

Ex1. Find x.

a. $\log x = 2$

$x = 10^2$

$x = 100$

b. $\ln x = 3$

$x = e^3$

c. $\boxed{x = \ln(e^7)}$

$x = 7(\ln e)$

$= 7(1)$

$= 7$

d. $\boxed{x = e^{\ln 7}}$

$\ln x = \ln(e^{\ln 7})$

$\ln x = (\ln 7)(\ln e)$

$\ln x = \ln 7$

$x = 7$

Ex2. Express each power in base e.

a. 2^4

let $e^x = 2^4$

$\ln(e^x) = \ln(2^4)$

$x = 4\ln 2$

Thus, $e^{4\ln 2} = 2^4$

b. 3^7

$e^{\ln(3^7)} = 3^7$

Thus $e^{7\ln 3} = 3^7$

c. b^x

So...

$b^x = e^{\ln(b^x)}$

$= e^{x\ln b}$

Ex3. Find $f'(x)$ if $f(x) = b^x$.

IF $f(x) = b^x$

$\therefore f(x) = e^{x\ln b}$

$\therefore f'(x) = e^{x\ln b} \cdot \ln b$

$= b^x \cdot \ln b$

$= \ln b \cdot b^x$

Ex4. Differentiate.

a. $f(x) = 7^x$

$f'(x) = \ln 7 \cdot 7^x$

b. $y = 5^{x - 3x^2}$

$y' = \ln 5 \cdot 5^{x-3x^2} \cdot (1 - 6x)$

So... IF $f(x) = b^x$ THEN $f'(x) =$ $\ln b \cdot b^x$

IF $f(x) = b^{g(x)}$ THEN $f'(x) =$ $\ln b \cdot b^{g(x)} \cdot g'(x)$

c. $h(x) = 2 \cdot 3^{x^2}$

$h'(x) = 2 \cdot \ln 3 \cdot 3^{x^2} \cdot 2x$

$= 4x \ln 3 \cdot 3^{x^2}$

d. $P(t) = 500(1.02)^{\frac{t}{2}}$

$P'(t) = 500 \cdot \ln(1.02) \cdot (1.02)^{\frac{t}{2}} \cdot \frac{1}{2}$

$= 250 \ln(1.02) \cdot (1.02)^{\frac{t}{2}}$

Ex5. Find x when $f'(x) = 0$ if $f(x) = \frac{\sqrt{5^x}}{x}$.

$$f(x) = \frac{(5^x)^{\frac{1}{2}}}{x} = \frac{5^{\frac{x}{2}}}{x}$$

$$f'(x) = \frac{\left[\ln 5 \cdot 5^{\frac{x}{2}} \cdot \frac{1}{2}\right](x) - 5^{\frac{x}{2}}[1]}{x^2} = 0$$

$$\therefore \frac{1}{2} 5^{\frac{x}{2}}\left[x\ln 5 - 2\right] = 0$$

So $x\ln 5 - 2 = 0$

$$x\ln 5 = 2$$

$$x = \frac{2}{\ln 5}$$

Ex6. Determine $\lim\limits_{h \to 0} \frac{b^h - 1}{h}$.

IF $f(x) = b^x$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\ln b \cdot b^x = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$$

$$\ln b \cdot b^x = \lim_{h \to 0} \frac{b^x(b^h - 1)}{h}$$

$$\underline{\ln b} \cdot b^x = \left[\underline{\lim_{h \to 0} \frac{b^h - 1}{h}}\right] \cdot b^x$$

Thus,

$$\boxed{\lim_{h \to 0} \frac{b^h - 1}{h} = \ln b}$$

So...	$\lim\limits_{h \to 0} \frac{b^h - 1}{h} = \ln b$	$\lim\limits_{h \to 0} \frac{e^h - 1}{h} = \ln e = 1$

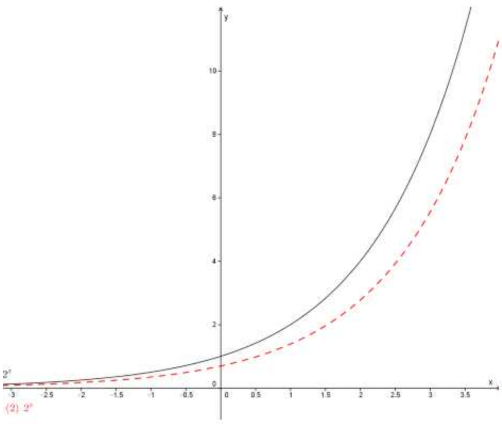
Key Ideas for $f(x) = b^x$ and $f'(x) = \ln b \cdot b^x$ and the natural birth of e.

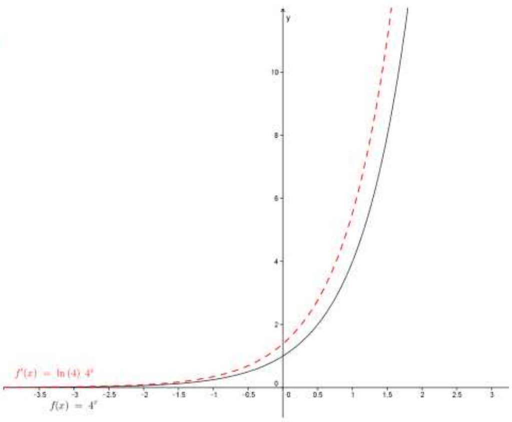
If $0 < b < e$, then $0 < \ln b < 1$.

So the graph of $f'(x) = \ln b \cdot b^x$ will be **below** the graph of $f(x) = b^x$.

If $b > e$, then $\ln b > 1$.

So the graph of $f'(x) = \ln b \cdot b^x$ will be **above** the graph of $f(x) = b^x$.





Homefun: Page 240 # 1→4, 6, 7ab, 8, 9 {Ans 2d $\frac{3^{\frac{x}{2}}[x\ln 3 - 4]}{2x^3}$ }