2.2

Q17a $y=2x^2+3$

$P_1(2,3)$, $P_2(x, 2x^2+3)$

$\frac{\Delta y}{\Delta x} = y'$

$\frac{2x^2+3-3}{x-2} = 4x$

$2x^2 = 4x^2 - 8x$

$2x^2 - 8x = 0$

$2x(x-4) = 0$

$\therefore x = 0$ or 4

When $x=0$, $y=3$

$y'=0$

$\therefore$ eq'n is $y=3$ ITRE

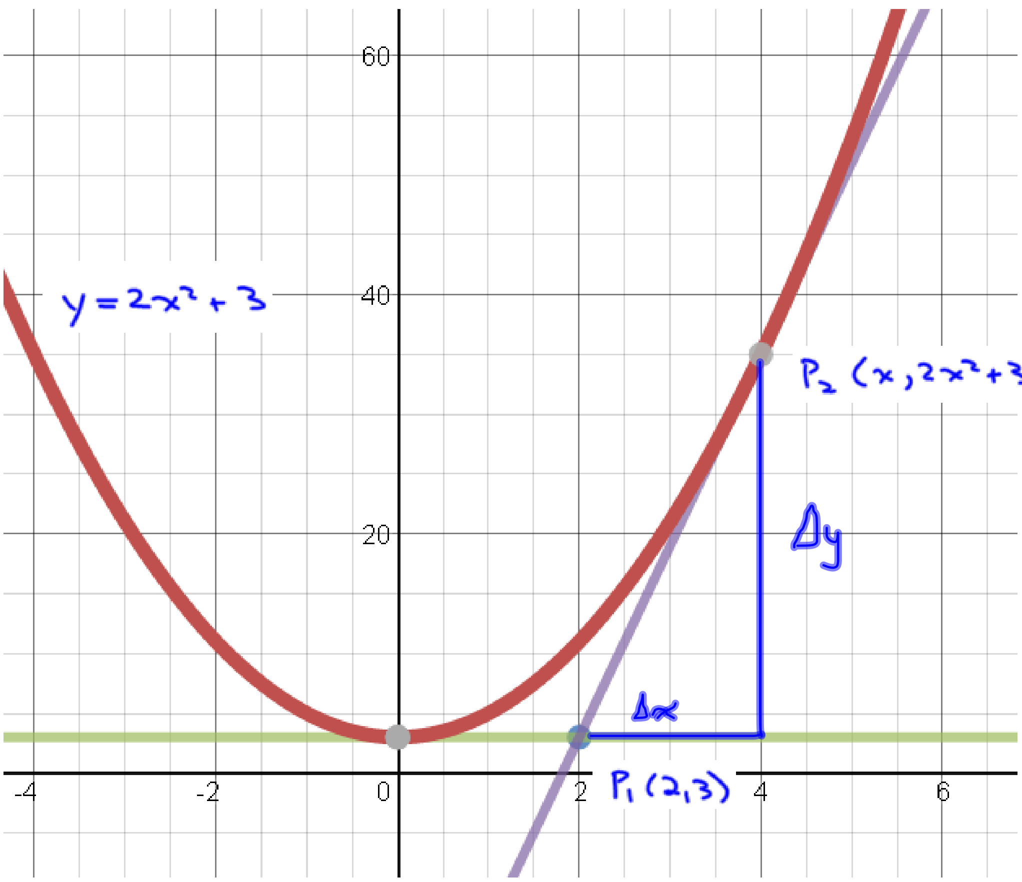
When $x=4$, $y' = 4(4)$

$= 16$

and tangent passes through $(2,3)$

$\therefore \frac{x-2}{1} = \frac{y-3}{16}$

$\therefore 16x - y - 24 = 0$ ITRE



Q18 Since $ax-4y+21=0$ is tangent to $y=\frac{a}{x^2}$ @ $x=-2$

$\therefore$ the point $\left(-2, \frac{a}{(-2)^2}\right) = \left(-2, \frac{a}{4}\right)$ is on tangent and curve

So, $a(-2)-4\left(\frac{a}{4}\right)+21=0$

$-2a-a=-21$

$-3a=-21$

$a=7$

Note:

Finding *y*' and setting *y*' equal to the slope of the line would normally be a good approach, but it won't help us find *a* for this particular question. Try it and find out why!

Q24 $y=x^3-6x^2+8x \longrightarrow y'=3x^2-12x+8$

Slope of tangent @ $A(3,-3) = 3(3)^2-12(3)+8$
$= -1$

So, eq'n of tangent @ $A(3,-3)$ is $y=-x$.

$\boxed{\frac{x-3}{1} = \frac{y+3}{-1} \longrightarrow -x+3 = y+3 \rightarrow y=-x}$

Now, find intersection of $y=-x$ & $y=x^3-6x^2+8x$

So, $-x=x^3-6x^2+8x$

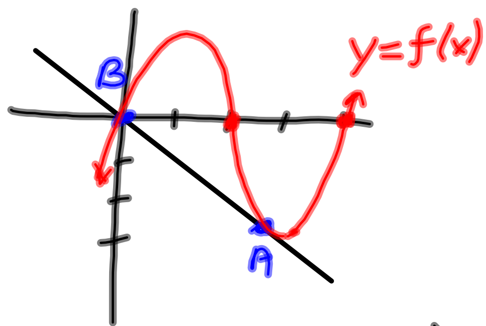
$x^3-6x^2+9x=0$

$x(x^2-6x+9)=0$

$x(x-3)^2=0$

When $x=3$, $A=(3,-3)$

$x=0$, $B=(0,0)$



Note: $y=x(x^2-6x+8)$
$=x(x-4)(x-2)$

Q27 $f(x) = x^n \longrightarrow f'(x) = n \cdot x^{n-1}$

So @ $(1,1)$, $f'(1) = n(1)^{n-1}$
$= n$

Thus eq'n of tangent @ $(1,1)$

$$\frac{x-1}{1} = \frac{y-1}{n} \quad \text{i.e. } y = nx - n + 1$$

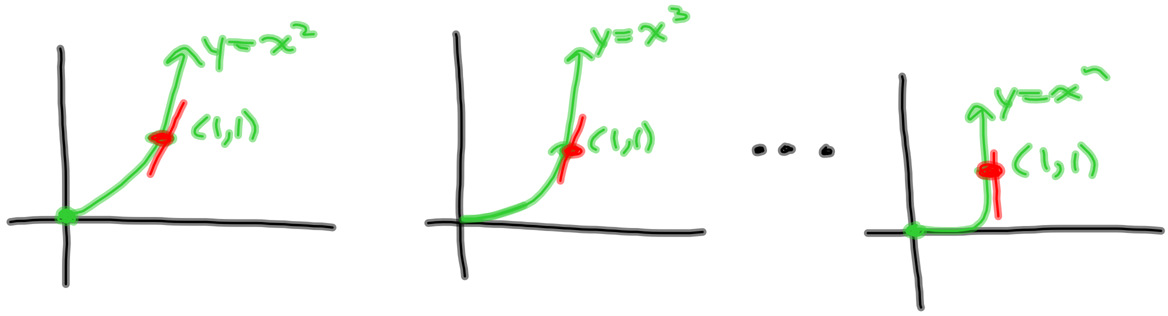
To find the x-cept, let $y = 0$

Thus $x - 1 = -\frac{1}{n} \longrightarrow x = 1 - \frac{1}{n}$

However, $\lim_{n\to\infty} \left(1 - \frac{1}{n}\right) = 1$

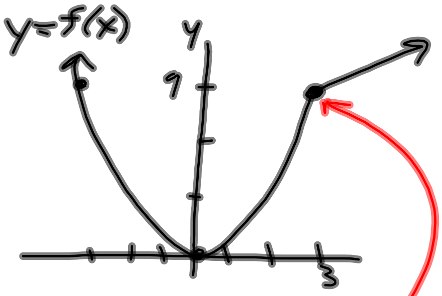
So, as $n \to \infty$, the x-cept approaches $(1,0)$

That is, the tangent approaches a vertical line b/c the function is becoming very steep at $(1,1)$



(28) a) $f(x) = \begin{cases} x^2 , & x < 3 \\ x+6 , & x \geq 3 \end{cases}$

$\therefore f'(x) = \begin{cases} 2x , & x < 3 \\ 1 , & x > 3 \end{cases}$



Note: $\lim_{x\to 3^-} f'(x) = 6$

$\lim_{x\to 3^+} f'(x) = 1$

$\therefore f'(3)$ does not exist!

y=f(x) is NOT diff @ x=3 (cusp)

i.e. $f'(3)$ does NOT exist.

(b) $f(x) = |3x^2-6|$

$3x^2-6=0$

$x^2=2$

$x=\pm\sqrt{2}$

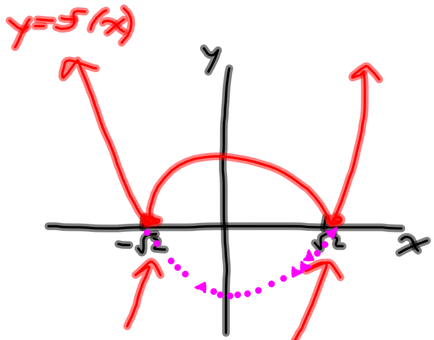
$\therefore 3x^2-6 < 0$ if $|x|<\sqrt{2}$

i.e. $-\sqrt{2}<x<\sqrt{2}$

$3x^2-6 \geq 0$ if $|x|\geq\sqrt{2}$

i.e. $x\leq-\sqrt{2}$ or $x\geq\sqrt{2}$

So, $f(x) = \begin{cases} -3x^2+6 , & |x|<\sqrt{2} \\ 3x^2-6 , & |x| \geq \sqrt{2} \end{cases}$



$f'(x) = \begin{cases} -6x , & |x| < \sqrt{2} \\ 6x , & |x| > \sqrt{2} \end{cases}$

Clearly the function is NOT diff at $\pm\sqrt{2}$,

$\therefore f'(\pm\sqrt{2})$ does not exist.

Note $\lim_{x\to\pm\sqrt{2}^-} f(x) \neq \lim_{x\to\pm\sqrt{2}^+} f(x)$

c) $f(x) = \left| |x| - 1 \right|$

$$\therefore f(x) = \begin{cases} -x-1, & x < -1 \\ x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ x-1, & x \ge 1 \end{cases}$$

$$\text{So, } f'(x) = \begin{cases} -1, & x < -1 \\ 1, & -1 < x < 0 \\ -1, & 0 < x < 1 \\ 1, & x > 1 \end{cases}$$

clearly $f'(-1)$, $f'(0)$ and $f'(1)$ do not exist.

Aside: $f(x) = \left| |x| - 1 \right|$

When $x < 0$, $f(x) = |-x-1|$

$$= \begin{cases} -x-1, & x < -1 \\ x+1, & -1 \le x < 0 \end{cases}$$

When $x \geq 0$, $f(x) = |x-1|$

$$= \begin{cases} -x+1 & 0 \le x < 1 \\ x-1 & x \ge 1 \end{cases}$$