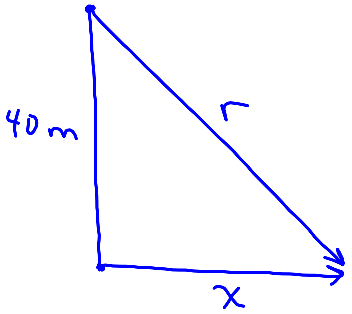
Ex6. A bird of prey is perched at the top of a tree that is 40 m high. A squirrel runs away from the base of the tree at a rate of 2 m/s. What is the rate of change of the distance between the bird and the squirrel when the squirrel is 30 m from the tree?



$\frac{dx}{dt} = 2\,m/s$

Find $\frac{dr}{dt}$ when $x = 30\,m$

Ⓜ1 $\overset{?}{\frac{dr}{dt}} = \frac{dr}{dx} \cdot \overset{\checkmark}{\frac{dx}{dt}}$

$r^2 = x^2 + 40^2$

$r^2 = x^2 + 1600$

$r = (x^2+1600)^{\frac{1}{2}}$

$\frac{dr}{dx} = \frac{1}{2}(x^2+1600)^{-\frac{1}{2}} \cdot (2x)$

$= \frac{x}{\sqrt{x^2+1600}}$

So, $\frac{dr}{dt} = \frac{x}{\sqrt{x^2+1600}} \cdot 2$

$= \frac{2x}{\sqrt{x^2+1600}}$

So when $x=30$, $\frac{dr}{dt} = \frac{60}{50} = \frac{6}{5}\,m/s$

Ⓜ2 $\frac{d}{dt}\left(x^2 + 1600 = r^2\right)$

$2x \cdot \frac{dx}{dt} + 0 = 2r \cdot \frac{dr}{dt}$

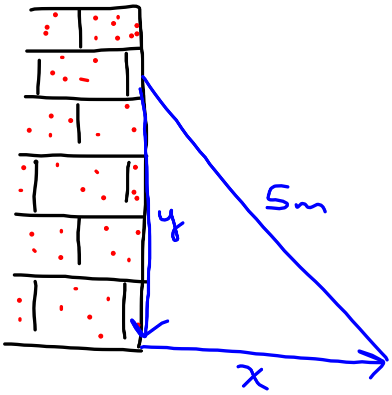
Thus, $\frac{dr}{dt} = \frac{x}{r} \cdot \frac{dx}{dt}$

$= \frac{2x}{r}$

So when $x = 30\,m$, $r = 50\,m$

$\therefore \frac{dr}{dt} = \frac{2(30)}{50} = \frac{6}{5}\,m/s.$

Ex7. A 5-m ladder rests in a vertical position against the side of a building. The base of the ladder begins to slip at a constant rate of 0.5m/min. How fast is the top of the ladder sliding down the building at 240 s?



Given: $\frac{dx}{dt} = 0.5$ m/min

Find $\frac{dy}{dt}$ when $t = 240$s

$$\frac{d}{dt}\left(x^2 + y^2 = 25\right)$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$2y \cdot \frac{dy}{dt} = -2x \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{-x}{y} \cdot \frac{1}{2}$$

$$= \frac{-x}{2y}$$

When $t = 240$s
$= 4$ min.

$\therefore\ x = \left(\frac{1}{2}\right)(4)$
$= 2$ m

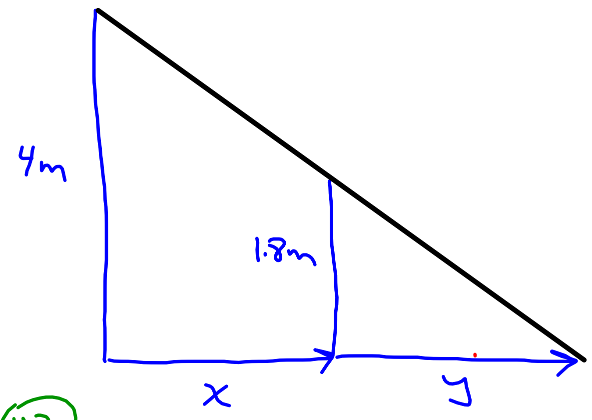
$y^2 = 21 \quad y = \sqrt{21}$

Thus,

$$\frac{dy}{dt} = \frac{-2}{2\sqrt{21}}$$

$$= \frac{-1}{\sqrt{21}} \text{ m/min.}$$

Ex8. A person is walking away from a streetlight at a rate of 2 m/s. The person is 1.8 m tall and the light is 4 m high. How fast is the length of the person's shadow increasing when she is 3 m from the base of the streetlight?



Given: $\frac{dx}{dt} = 2 m/s$

Find $\frac{dy}{dt}$ when $x = 3$ m

M2

$$\frac{4}{x+y} = \frac{1.8}{y}$$

$$4y = 1.8x + 1.8y$$

$$2.2y = 1.8x$$

$$22y = 18x$$

$$y = \frac{9}{11}x$$

$$\frac{d}{dt}\left(y = \frac{9}{11}x\right)$$

$$\frac{dy}{dt} = \frac{9}{11} \cdot \frac{dx}{dt}$$

$$= \frac{9}{11}(2)$$

$$= \frac{18}{11} m/s$$

M1

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{9}{11} \cdot 2$$

Homefun: Page 569 #8, 9[Hint: Height remains constant at 30 m], 10, 11, 17, 18, 19[Hint: Picture the actions occurring on the edges of a box], 2a

[**WARNING**: watch units for 17, 18, 19]; Answers: [8a. $\frac{4}{9}$ not 4]