19. **a.** $y = 7$
b. $y = -5x - 5$
c. $y = 18x + 9$
d. $y = -216x + 486$

20. **a.** 700 000
b. 109 000/h

Chapter 1 Test, p. 60

1. $\lim_{x\to 1^+}\frac{1}{x-1} = +\infty \neq \lim_{x\to 1^-}\frac{1}{x-1} = -\infty$

2. -13

3. **a.** $\lim_{x\to 1} f(x)$ does not exist.
b. 1
c. 1
d. $x = 1$ and $x = 2$

4. **a.** 1 km/h
b. 2 km/h

5. $\frac{\sqrt{16+h} - \sqrt{16}}{h}$

6. -31

7. **a.** 12 **d.** $-\frac{3}{4}$
b. $\frac{7}{5}$ **e.** $\frac{1}{6}$
c. 4 **f.** $\frac{1}{12}$

8. **a.** $a = 1, b = -\frac{18}{5}$

Chapter 2

Review of Prerequisite Skills, pp. 62–63

1. **a.** a^8 **d.** $\frac{1}{a^2b^7}$
b. $-8a^6$ **e.** $48e^{18}$
c. $2p$ **f.** $-\frac{b}{2a^6}$

2. **a.** $x^{\frac{7}{6}}$ **b.** $4x^4$ **c.** $a^{\frac{1}{3}}$

3. **a.** $-\frac{3}{2}$ **c.** $-\frac{3}{5}$
b. 2 **d.** 1

4. **a.** $x - 6y - 21 = 0$
b. $3x - 2y - 4 = 0$
c. $4x + 3y - 7 = 0$

5. **a.** $2x^2 - 5xy - 3y^2$
b. $x^3 - 5x^2 + 10x - 8$
c. $12x^2 + 36x - 21$
d. $-13x + 42y$
e. $29x^2 - 2xy + 10y^2$
f. $-13x^3 - 12x^2y + 4xy^2$

6. **a.** $\frac{15}{2}x; x \neq 0, -2$
b. $\frac{y-5}{4y^2(y+2)}; y \neq -2, 0, 5$
c. $\frac{8}{9}; h \neq -k$
d. $\frac{2}{(x+y)^2}; x \neq -y, +y$
e. $\frac{11x^2 - 8x + 7}{2x(x-1)}; x \neq 0, 1$
f. $\frac{4x+7}{(x+3)(x-2)}; x \neq -3, 2$

7. **a.** $(2k+3)(2k-3)$
b. $(x-4)(x+8)$
c. $(a+1)(3a-7)$
d. $(x^2+1)(x+1)(x-1)$
e. $(x-y)(x^2+xy+y^2)$
f. $(r+1)(r-1)(r+2)(r-2)$

8. **a.** $(a-b)(a^2+ab+b^2)$
b. $(a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$
c. $(a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$
d. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + a^3b^{n-3} + ab^{n-2} + b^{n-1})$

9. **a.** -17 **c.** $\frac{53}{8}$
b. 10 **d.** about 7.68

10. **a.** $\frac{3\sqrt{2}}{2}$
b. $\frac{4\sqrt{3} - \sqrt{6}}{3}$
c. $-\frac{30 + 17\sqrt{2}}{23}$
d. $-\frac{11 - 4\sqrt{6}}{5}$

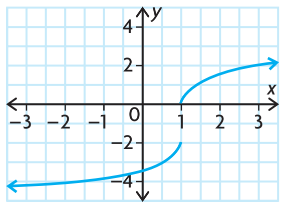
11. **a.** $3h + 10$; expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2 + h, f(2 + h))$
b. For $h = 0.01$: 10.03
c. value represents the slope of the secant line through $(2, 8)$ and $(2.01, 8.1003)$

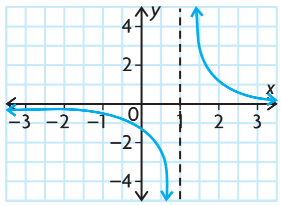
Section 2.1, pp. 73–75

1. **a.** $\{x \in \mathbf{R} \mid x \neq -2\}$
b. $\{x \in \mathbf{R} \mid x \neq 2\}$
c. $\{x \in \mathbf{R}\}$
d. $\{x \in \mathbf{R} \mid x \neq 1\}$
e. $\{x \in \mathbf{R}\}$
f. $\{x \in \mathbf{R} \mid x > 2\}$

2. The derivative of a function represents the slope of the tangent line at a give value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3. Answers may vary. For example:





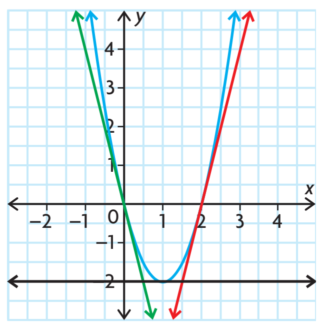
4. **a.** $5a + 5h - 2; 5h$
b. $a^2 + 2ah + h^2 + 3a + 3h - 1; 2ah + h^2 + 3h$
c. $a^3 + 3a^{2h} + 3ah^2 + h^3 - 4a - 4h + 1; 3a^2h + 3ah^2 + h^3 - 4h$
d. $a^2 + 2ah + h^2 + a + h - 6; 2ah + h^2 + h$
e. $-7a - 7h + 4; -7h$
f. $4 - 2a - 2h - a^2 - 2ah - h^2; -2h - h^2 - 2ah$

5. **a.** 2 **c.** $\frac{1}{2}$
b. 9 **d.** -5

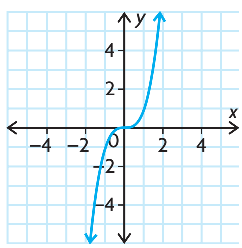
6. **a.** -5 **c.** $18x^2 - 7$
b. $4x + 4$ **d.** $\frac{3}{2\sqrt{3x+2}}$

7. **a.** -7
b. $-\frac{2}{(x-1)^2}$ **c.** $6x$

8. $f'(0) = -4; f'(1) = 0; f'(2) = 4$

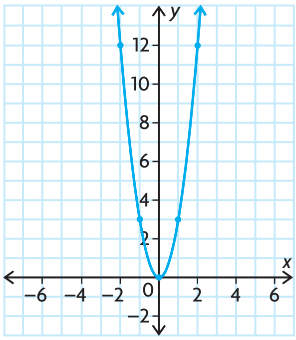


9. **a.**



b. $f'(-2) = 12; f'(-1) = 3;$
$f'(0) = 0; f'(1) = 3; f'(2) = 12$

c.



d. graph of $f(x)$ is cubic; graph of $f'(x)$ seems to be a parabola

10. $s'(0) = 8$ m/s; $s'(4) = 0$ m/s; $s'(6) = -4$ m/s

11. $x - 6y + 10 = 0$

12. **a.** 0 **c.** m
b. 1 **d.** $2ax + b$

13. Since $3x^2$ is nonnegative for all x, the original function never has a negative slope.

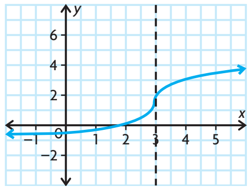
14. **a.** -1.6 m/s
b. $h'(2)$ measures the rate of change in the height of the ball with respect to time when $t = 2$.

15. **a.** e. **b.** f. **c.** d.

16.
$$\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-}\frac{-(0+h)^2-(-0^2)}{h} = \lim_{h\to 0^-}\frac{-h^2}{h} = \lim_{h\to 0^-}(-h) = 0$$
$$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\frac{(0+h)^2-(0^2)}{h} = \lim_{h\to 0^+}\frac{h^2}{h} = \lim_{h\to 0^+}(h) = 0$$
Since the limits are equal for both sides, the derivative exists and $f'(0) = 0$.

17. 3

18. Answers may vary. For example:



19. $(3, -8)$

20. $2x + y + 1 = 0$ and $6x - y - 9 = 0$

Section 2.2, pp. 82–84

1. Answers may vary. For example:
constant function rule: $\frac{d}{dx}(5) = 0$
power rule: $\frac{d}{dx}(x^3) = 3x^2$
constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$
sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$
difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

2. **a.** 4 **d.** $\frac{1}{3\sqrt[3]{x^2}}$
b. $3x^2 - 2x$ **e.** $\frac{x^3}{4}$
c. $-2x + 5$ **f.** $-3x^{-4}$

3. **a.** $4x + 11$ **d.** $x^4 + x^2 - x$
b. $6x^2 + 10x - 4$ **e.** $40x^7$
c. $4t^3 - 6t^2$ **f.** $2t^3 - \frac{3}{2}$

4. **a.** $5x^{\frac{2}{3}}$
b. $-2x^{-\frac{3}{2}} + 6x^{-2}$
c. $\frac{-18}{x^4} - \frac{4}{x^3}$
d. $-18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}$
e. $\frac{1}{2}(x^{-\frac{1}{2}}) + 9x^{\frac{1}{2}}$
f. $-x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}$

5. **a.** $-4t + 7$ **b.** $5 - t^2$ **c.** $2t - 6$

6. **a.** 47.75 **b.** $\frac{11}{24}$

7. **a.** 12 **c.** $-\frac{1}{2}$
b. 5 **d.** 12

8. **a.** 9 **c.** 4
b. $\frac{1}{2}$ **d.** -7

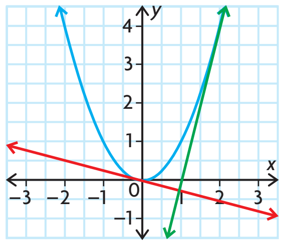
9. **a.** $6x - y - 4 = 0$
b. $18x - y + 25 = 0$
c. $9x - 2y - 9 = 0$
d. $x + y - 3 = 0$
e. $7x - 2y - 28 = 0$
f. $5x - 6y - 11 = 0$

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point; $x + 18y - 125 = 0$

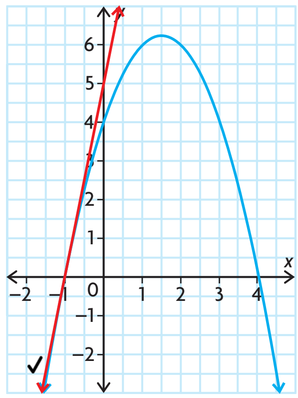
11. 8

12. no

13. $y = x^2, \frac{dy}{dx} = 2x$
The slope of the tangent at $A(2, 4)$ is 4 and at $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ is $-\frac{1}{4}$.
Since the product of the slopes is -1, the tangents at $A(2, 4)$ and $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ will be perpendicular.



14. $(-1, 0)$



15. $(2, 10)$ and $(-2, -6)$

16. $y = \frac{1}{5}x^5 - 10x$, slope is 6
$\frac{dy}{dx} = x^4 - 10 = 6$
$x^4 = 16$
$x^2 = 4$ or $x^2 = -4$
$x = \pm 2$ non-real
Tangents with slope 6 are at the points $\left(2, -\frac{68}{5}\right)$ and $\left(-2, \frac{68}{5}\right)$.

17. **a.** $y - 3 = 0$; $16x - y - 29 = 0$
b. $20x - y - 47 = 0$; $4x + y - 1 = 0$

18. 7

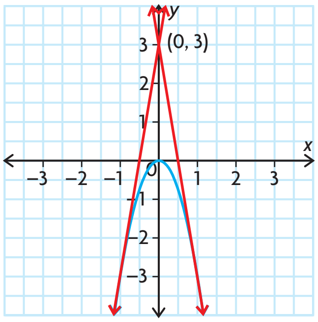
19. **a.** 49.9 km
b. 0.12 km/m

20. **a.** 34.3 m/s
b. 39.2 m/s
c. 54.2 m/s

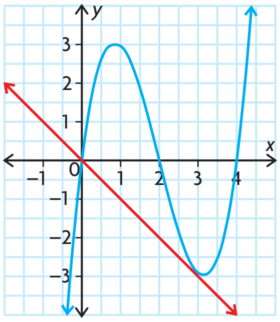
21. 0.29 min and 1.71 min

22. -20 m/s

23. $(1, -3)$ and $(-1, -3)$



24. $B(0, 0)$



25. **a.** **i.** $\left(\frac{1}{5}, \frac{1}{5}\right)$

ii. $\left(-\frac{1}{4}, -\frac{13}{4}\right)$

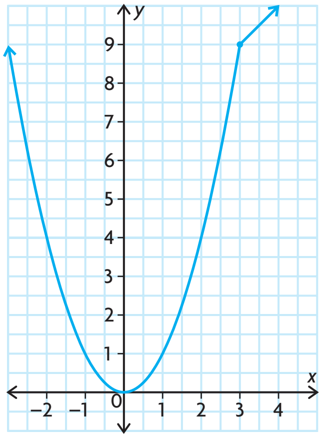
iii. $\left(\frac{1}{3}, \frac{103}{27}\right)$ and $(5, -47)$

b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum and minimum points.

26. $\sqrt{x} + \sqrt{y} = 1$
$P(a, b)$ is on the curve; therefore, $a \geq 0, b \geq 0$.
$\sqrt{y} = 1 - \sqrt{x}$
$y = 1 - 2\sqrt{x} + x$
$\frac{dy}{dx} = -\frac{1}{2}\left(2x^{-\frac{1}{2}} + 1\right)$
At $x = a$. Slope is
$-\frac{1}{\sqrt{a}} + 1 = \frac{-1 + \sqrt{a}}{\sqrt{a}}$.
But, $\sqrt{a} + \sqrt{b} = 1$
$-\sqrt{b} = \sqrt{a} - 1$
Therefore, slope is $-\frac{\sqrt{b}}{\sqrt{a}} = -\sqrt{\frac{b}{a}}$.

27. The x-intercept is $1 - \frac{1}{n}$, as $n \to \infty$, $\frac{1}{n} \to 0$, and the x-intercept approaches 1. As $n \to \infty$, the slope of the tangent at (1, 1) increase without bound, and the tangent approaches a vertical line having equation $x - 1 = 0$.

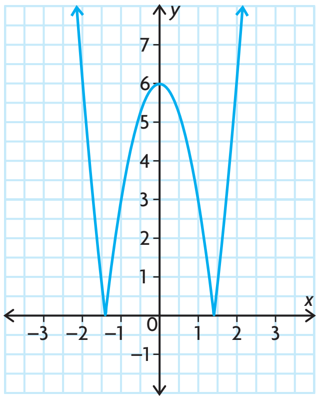
28. **a.** $f'(x) = \begin{cases} 2x, \text{ if } x < 3 \\ 1, \text{ if } x \geq 3 \end{cases}$
$f'(3)$ does not exist.



b.

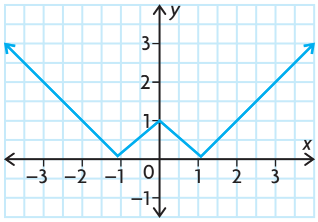
$$f'(x) = \begin{cases} 6x, \text{ if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ -6x, \text{ if } -\sqrt{2} \leq x \leq \sqrt{2} \end{cases}$$

$f'\left(\sqrt{2}\right)$ and $f'\left(-\sqrt{2}\right)$ do not exist.



c. $f'(x) = \begin{cases} 1, \text{ if } x > 1 \\ -1, \text{ if } 0 < x < 1 \\ 1, \text{ if } -1 < x < 0 \\ -1, \text{ if } x < -1 \end{cases}$

$f'(0), f'(-1)$, and $f'(1)$ do not exist.



Section 2.3, pp. 90–91

1. **a.** $2x - 4$
b. $6x^2 - 2x$
c. $12x - 17$
d. $45x^8 - 80x^7 + 2x - 2$
e. $-8t^3 + 2t$
f. $\frac{6}{(x + 3)^2}$

2. **a.** $(5x + 1)^3 + 15(5x + 1)^2(x - 4)$
b. $15x^2(3x^2 + 4)(3 + x^3)^4 + 6x(3 + x^3)^5$
c. $-8x(1 - x^2)^3(2x + 6)^3 + 6(1 - x^2)^4(2x + 6)^2$
d. $6(x^2 - 9)^4(2x - 1)^2 + 8x(x^2 - 9)^3(2x - 1)^3$

3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for $f(x) = 3(x^2 + 1)$ or $g(x) = (x + 1)(x - 1)$.

4. $F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$

5. **a.** 9 **d.** -36
b. -4 **e.** 22
c. -9 **f.** 671

6. $10x + y - 8 = 0$

7. **a.** $(14, -450)$
b. $(-1, 0)$

8. **a.** $3(x + 1)^2(x + 4)(x - 3)^2 + (x + 1)^3(1)(x - 3)^2 + (x + 1)^3(x + 4)[2(x - 3)]$
b. $2x(3x^2 + 4)^2(3 - x^3)^4 + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 + x^2(3x^2 + 4)^2 \times [4(3 - x^3)^3(-3x^2)]$

9. -4.84 L/h

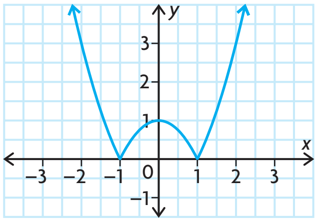
10. -30; Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.

11. **a.** $f'(x) = g_1'(x)g_2(x)g_3(x) \ldots g_{n-1}(x)g_n(x) + g_1(x)g_2'(x)g_3(x) \ldots g_{n-1}(x)g_n(x) + g_1(x)g_2(x)g_3'(x) \ldots g_{n-1}(x)g_n(x) + \ldots + g_1(x)g_2(x)g_3(x) \ldots g_{n-1}(x)g_n'(x)$

b. $\dfrac{n(n+1)}{2}$

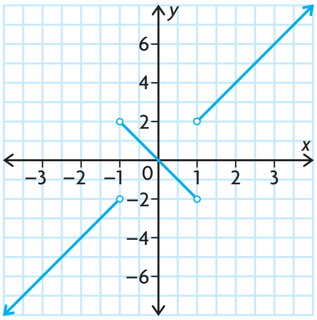
12. $y = 3x^2 + 6x - 5$

13.



a. $x = 1$ or $x = -1$

b. $f'(x) = -2x, -1 < x < 1$



c. $f'(-2) = -4, f'(0) = 0, f'(3) = 6$

14. $y = \dfrac{16}{x^2} - 1$

$\dfrac{dy}{dx} = -\dfrac{32}{x^3}$

Slope of this line is 4.

$-\dfrac{32}{x^3} = 4$

$x = -2$

$y = 3$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$y = 4x + 11$

Substitute,

$4x + 11 = \dfrac{16}{x^2} - 1$

$4x^3 + 11x^2 - 16 - x^2$ or

$4x^3 + 12x^2 - 16 = 0$

Let $x = -2$

$\text{RS} = 4(-2) + 12(-2)^2 - 16$

$= 0$

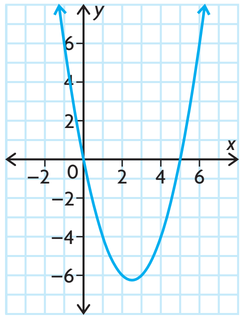
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2$, $y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

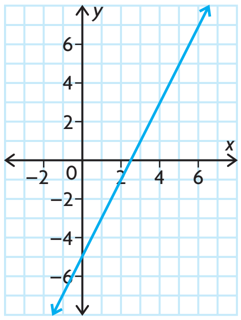
Mid-Chapter Review, pp. 92–93

1. **a.**



b. $f'(0) = -5, f'(1) = -3, f'(2) = -1, f'(3) = 1, f'(4) = 3, f'(5) = 5$

c.



d. $f(x)$ is quadratic; $f'(x)$ is linear.

2. **a.** 6

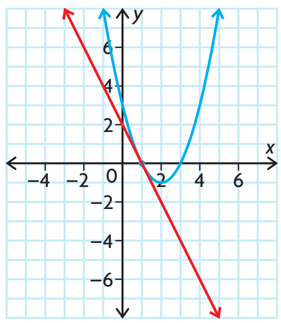
b. $4x$

c. $\dfrac{-5}{(x-5)^2}$

d. $\dfrac{1}{2\sqrt{x-2}}$

3. **a.** $y = -2x + 2$

b.



4. **a.** $24x^3$

b. $\dfrac{5}{\sqrt{x}}$

c. $-\dfrac{6}{x^4}$

d. $5 - \dfrac{6}{x^3}$

e. $242t + 22$

f. $\dfrac{1}{x^2}$

5. $y = x - \dfrac{3}{8}$

6. **a.** $8x - 7$

b. $-6x^2 + 8x + 5$

c. $-\dfrac{10}{x^3} + \dfrac{9}{x^4}$

d. $\dfrac{1}{2x^{\frac{1}{2}}} + \dfrac{1}{3x^{\frac{2}{3}}}$

e. $-\dfrac{14}{x^3} - \dfrac{3}{2x^{\frac{1}{2}}}$

f. $\dfrac{4}{x^2} + 5$

7. **a.** $y = 7$

b. $y = -\dfrac{1}{3}x$

c. $y = -128x + 297$

8. **a.** $48x^3 - 81x^2 + 40x - 45$

b. $-36t^2 - 50t + 39$

c. $24x^3 + 24x^2 - 78x - 36$

d. $-162x^2 + 216x^5 - 72x^8$

9. $76x - y - 28 = 0$

10. $(3, 8)$

11. $10x - 8$

12. **a.** $\dfrac{500}{9}$ L

b. $-\dfrac{200}{27}$ L/min

c. $-\dfrac{200}{27}$ L/min

13. **a.** $\dfrac{1900}{3}\pi\ \text{cm}^3/\text{cm}$

b. $256\,\pi\ \text{cm}^3/\text{cm}$

14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d, a \neq 0$. So, the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$ and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f'(x) = x^3 + x^2 + 1$, we get $f'(x) = 3x^2 + 2x$, and if $f(x) = 2x^3 + 3x^2 + 6x + 2$, we get $f'(x) = 6x^2 + 6x + 6$.

15. $y = \dfrac{x^{2a+3b}}{x^{a-b}}, a, b \in I$

Simplifying,

$y = x^{2a+3b-(a-b)} = x^{a+4b-1}$

Then,

$y'(a + 4b)^{a+4b-1}$

16. **a.** -188

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

17. **a.** 100 bacteria
b. 1200 bacteria
c. 370 bacteria/h

18. $C'(t) = -\frac{100}{t^2}$; The values of the derivative are the rates of change of the percent with respect to time at 5, 50, and 100 min. The percent of carbon dioxide that is released per unit of time from the soft drink is decreasing. The soft drink is getting flat.

Section 2.4, pp. 97–98

1. For x, a, b real numbers,
$x^a x^b = x^{a+b}$
For example,
$x^9 x^{-6} = x^3$
Also,
$(x^a)^b = x^{ab}$
For example,
$(x^2)^3 = x^6$
Also,
$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$
For example,
$\frac{x^5}{x^3} = x^2$

2.

Function	Rewrite	Differentiate and Simplify, if Necessary
$f(x) = \frac{x^2 + 3x}{x}$, $x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{5}{3}}}{x}$, $x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}$, $x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}$, $x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}$, $t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

3. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification. A second approach would be to rewrite a rational example
$h(x) = \frac{f(x)}{g(x)}$
using the exponent rules as
$h(x) = f(x)(g(x))^{-1}$, and then apply the product rule for differentiation (together with the power of a function rule) to find $h'(x)$. A third (an perhaps easiest) approach would be to just apply the quotient rule to find $h'(x)$.

4. **a.** $\frac{1}{(x+1)^2}$
b. $\frac{13}{(t+5)^2}$
c. $\frac{2x^4 - 3x^2}{(2x^2 - 1)^2}$
d. $\frac{-2x}{(x^2 + 3)^2}$
e. $\frac{5x^2 + 6x + 5}{(1 - x^2)^2}$
f. $\frac{x^2 + 4x - 3}{(x^2 + 3)^2}$

5. **a.** $\frac{13}{4}$ **c.** $\frac{200}{841}$
b. $\frac{7}{25}$ **d.** $-\frac{7}{3}$

6. -9

7. $\left(9, \frac{27}{5}\right)$ and $\left(-1, \frac{3}{5}\right)$

8. Since $(x + 2)^2$ is positive or zero for all $x \in \mathbf{R}$, $\frac{8}{(x+2)^2} > 0$ for $x \neq -2$. Therefore, tangents to the graph of $f(x) = \frac{5x + 2}{x + 2}$ do not have a negative slope.

9. **a.** (0, 0) and (8, 32)
b. no horizontal tangents

10. 75.4 bacteria per hour at $t = 1$ and 63.1 bacteria per hour at $t = 2$

11. $5x - 12y - 4 = 0$

12. **a.** 20 m
b. $\frac{10}{9}$ m/s

13. **a.** **i.** 1 cm
ii. 1 s
iii. 0.25 cm/s
b. No, the radius will never reach 2 cm because $y = 2$ is a horizontal asymptote of the graph of the function. Therefore, the radius approaches but never equals 2 cm.

14. $a = 1, b = 0$

15. 1.87 h

16. 2.83 s

17. $ad - bc > 0$

Section 2.5, pp. 105–106

1. **a.** 0 **d.** $\sqrt{15}$
b. 0 **e.** $\sqrt{x^2 - 1}$
c. -1 **f.** $x - 1$

2. **a.** $(f \circ g) = x$,
$(g \circ f) = |x|$,
$\{x \geq 0\}$, $\{x \in \mathbf{R}\}$; not equal
b. $(f \circ g) = \frac{1}{(x^2 + 1)}$,
$(g \circ f) = \left(\frac{1}{x^2}\right) + 1$,
$\{x \neq 0\}$, $\{x \in \mathbf{R}\}$; not equal
c. $(f \circ g) = \frac{1}{\sqrt{x + 2}}$,
$(g \circ f) = \sqrt{\frac{1}{x} + 2}$,
$\{x > -2\}$, $\left\{x \leq -\frac{1}{2}, x > 0\right\}$; not equal

3. If $f(x)$ and $g(x)$ are two differentiable functions of x, and
$h(x) = (f \circ g)(x)$
$= f(g(x))$
is the composition of these two functions, then $h'(x) = f'(g(x)) \times g'(x)$.
This is known as the "chain rule" for differentiation of composite functions.
For example, if $f(x) = x^{10}$ and $g(x) = x^2 + 3x + 5$, then $h(x) = (x^2 + 3x + 5)^{10}$, and so
$h'(x) = f'(g(x)) \times g'(x)$
$= 10(x^2 + 3x + 5)^9(2x + 3)$
As another example, if $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^2 + 1$, then $h(x) = (x^2 + 1)^{\frac{2}{3}}$, and so $h'(x) = \frac{2}{3}(x^2 + 1)^{-\frac{1}{3}}(2x)$.

4. **a.** $8(2x + 3)^2$
b. $6x(x^2 - 4)^2$
c. $4(2x^2 + 3x - 5)^3(4x + 3)$
d. $-6x(\pi^2 - x^2)^2$
e. $\frac{x}{\sqrt{x^2 - 3}}$
f. $\frac{-10x}{(x^2 - 16)^6}$

5. **a.** $-2x^{-3}$; $\frac{6}{x^4}$
b. $(x + 1)^{-1}$; $\frac{-1}{(x + 1)^2}$
c. $(x^2 - 4)^{-1}$; $\frac{-2x}{(x^2 - 4)^2}$
d. $3(9 - x^2)^{-1}$; $\frac{6x}{(9 - x^2)^2}$
e. $(5x^2 + x)^{-1}$; $-\frac{10x + 1}{(5x^2 + x)^2}$
f. $(x^2 + x + 1)^{-4}$; $-\frac{8x + 4}{(x^2 + x + 1)^5}$

6. $h(-1) = -4$; $h'(-1) = 35$

7. $-\frac{2}{x^2}\left(\frac{1}{x} - 3\right)$

8. **a.** $(x + 4)^2(x - 3)^5(9x + 15)$
b. $6x(x^2 + 3)^2(x^3 + 3)$
$(2x^3 + 3x + 3)$
c. $\dfrac{-2x^2 + 6x + 2}{(x^2 + 1)^2}$
d. $15x^2(3x - 5)(x - 1)$
e. $4x^3(1 - 4x^2)^2(1 - 10x^2)$
f. $\dfrac{48x(x^2 - 3)^3}{(x^2 + 3)^5}$

9. **a.** $\dfrac{91}{36}$ **b.** $-\dfrac{5\sqrt[3]{2}}{24\pi}$

10. $x = 0$ or $x = 1$

11. $\dfrac{1}{4}$

12. $60x - y - 119 = 0$

13. **a.** 52 **b.** 54 **c.** 878 **d.** 78

14. -6

15. 2222 L/min

16. 2.75 m/s

17. **a.** $p'(x)q(x)r(x) + p(x)q'(x)r(x) + p(x)q(x)r'(x)$
b. -344

18. $\dfrac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$
At the point (1, 3), slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.
Equation of the tangent at (1, 3) is
$y = 3$
$(x^2 + x - 2)^3 + 3 = 3$
$(x + 2)^3(x - 1)^3 = 0$
$x = -2$ or $x = 1$
Since -2 and 1 are both triple roots, the line with equation $y = 3$ is also a tangent at $(-2, 3)$.

19. $-\dfrac{2x(x^2 + 3x - 1)(1 - x)^2}{(1 + x)^4}$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not exist at this particular value of x. As an alternative to this limit, we could also find $f'(x)$ from the definition by computing the equivalent limit $f'(x) = \lim\limits_{z \to x} \dfrac{f(z) - f(x)}{z - x}$. These two limits are seen to be equivalent by substituting $z = x + h$.

2. **a.** $4x - 5$ **c.** $\dfrac{4}{(4 - x)^2}$
b. $\dfrac{1}{2\sqrt{x - 6}}$

3. **a.** $2x - 5$
b. $\dfrac{3}{4x^{\frac{1}{4}}}$
c. $-\dfrac{28}{3x^5}$
d. $-\dfrac{2x}{(x^2 + 5)^2}$
e. $\dfrac{12x}{(3 - x^2)^3}$
f. $\dfrac{7x + 2}{\sqrt{7x^2 + 4x + 1}}$

4. **a.** $2 + \dfrac{2}{x^3}$
b. $\dfrac{\sqrt{x}}{2}(7x^2 - 3)$
c. $-\dfrac{5}{(3x - 5)^2}$
d. $\dfrac{3x - 1}{2\sqrt{x - 1}}$
e. $-\dfrac{1}{3\sqrt{x}(\sqrt{x} + 2)^{\frac{5}{3}}}$
f. 1

5. **a.** $20x^3(2x - 5)^5(x - 1)$
b. $\dfrac{x^2}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1}$
c. $\dfrac{(2x - 5)^3(2x + 23)}{(x + 1)^4}$
d. $\dfrac{318(10x - 1)^5}{(3x + 5)^7}$
e. $(x - 2)^2(x^2 + 9)^3 \times (11x^2 - 16x + 27)$
f. $-\dfrac{3(1 - x^2)^2(x^2 + 6x + 1)}{8(3 - x)^4}$

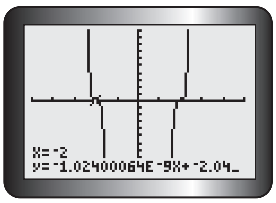
6. **a.** $f(x^2) \times 2x$
b. $2xf'(x) + 2f(x)$

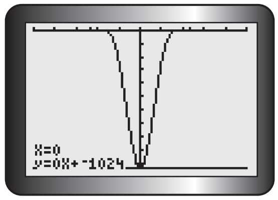
7. **a.** $-\dfrac{184}{9}$ **b.** $\dfrac{25}{289}$ **c.** $-\dfrac{8}{5}$

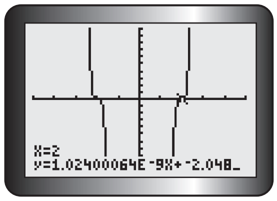
8. $-\dfrac{2}{3}$

9. $x = 2 \pm 2\sqrt{2}$;
$x = 5, x = -1$

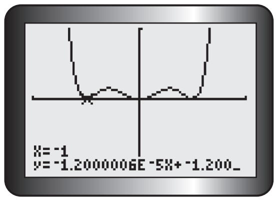
10. **a.** **i.** $x = 0, x = \pm 2$
ii. $x = 0, x = \pm 1, x = \pm\dfrac{\sqrt{3}}{3}$
b. **i.**

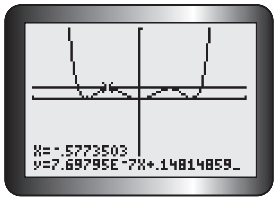


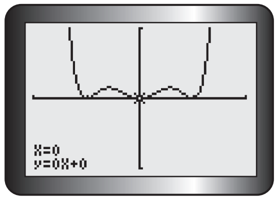


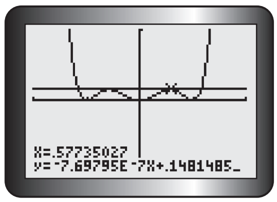


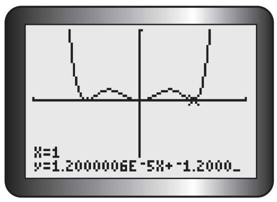
ii.









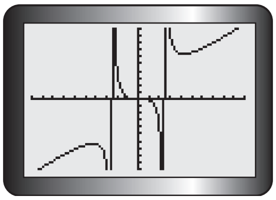


11. **a.** $160x - y + 16 = 0$
b. $60x + y - 61 = 0$

12. $5x - y - 7 = 0$

13. $(2, 8); b = -8$

14. **a.**



b. $y = 0, y = 6.36, y = -6.36$

c. $(0, 0), \left(3\sqrt{2}, \frac{9\sqrt{2}}{2}\right),$

$\left(-3\sqrt{2}, -\frac{9\sqrt{2}}{2}\right)$

d. -14

15. **a.** $\sqrt[3]{50}$
b. 1

16. **a.** When $t = 10$, 9; when $t = 15$, 19
b. At $t = 10$, the number of words memorized is increasing by 1.7 words/min. At $t = 15$, the number of words memorized is increasing by 2.325 words/min.

17. **a.** $\dfrac{30t}{(9 + t^2)^{\frac{1}{2}}}$
b. No; since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier's productivity is always increasing. However, these increases must be small, since, according to the model, the cashier's productivity can never exceed 20.

18. **a.** $x^2 + 40$
b. 6 gloves/week

19. **a.** $750 - \dfrac{x}{3} - 2x^2$
b. \$546.67

20. $-\dfrac{5}{4}$

21. **a.** $B(0) = 500, B(30) = 320$
b. $B'(0) = 0, B'(30) = -12$
c. $B(0)$ = blood sugar level with no insulin
$B(30)$ = blood sugar level with 30 mg of insulin
$B'(0)$ = rate of change in blood sugar level with no insulin
$B'(30)$ = rate of change in blood sugar level with 30 mg of insulin
d. $B'(50) = -20, B(50) = 0$
$B'(50) = -20$ means that the patient's blood sugar level is decreasing at 20 units/mg of insulin 1 h after 50 mg of insulin is injected.
$B(50) = 0$ means that the patient's blood sugar level is *zero* 1 h after 50 mg of insulin is injected. These values are not logical because a person's blood sugar level can never reach zero and continue to decrease.

22. **a.** $f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).
b. $g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).
c. The graph has a cusp at (2, 0) but is differentiable at $x = 1$.
d. The graph has a corner at $x = 1$, so $m(x)$ is not differentiable at $x = 1$.

23. **a.** $f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$. Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.
b. $f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.
c. $f(x)$ is not defined for $1 < x < 6$. Therefore, $f(x)$ is not differentiable for $1 < x < 6$.

24. $\dfrac{25}{(t + 1)^2}$

25. Answers may vary. For example:
$f(x) = 2x + 3$

$$y = \frac{1}{2x + 3}$$
$$y' = \frac{(2x + 3)(0) - (1)(2)}{(2x + 3)^2}$$
$$= -\frac{2}{(2x + 3)^2}$$

$f(x) = 5x + 10$

$$y = \frac{1}{5x + 10}$$
$$y' = \frac{(5x + 10)(0) - (1)(5)}{(5x + 10)^2}$$
$$= -\frac{5}{(5x + 10)^2}$$

Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$, then

$$y' = \frac{-a}{(ax + b)^2}$$
$$y' = \lim_{h \to 0} \frac{1}{h}\left[\frac{1}{a(x + h) + b} - \frac{1}{ax + b}\right]$$
$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{ax + b - [a(x + h) + b]}{[a(x + h) + b](ax + h)}\right]$$
$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-ah}{[a(x + h) + b](ax + b)}\right]$$
$$= \lim_{h \to 0} \left[\frac{-a}{[a(x + h) + b](ax + b)}\right]$$
$$= \frac{-a}{(ax + b)^2}$$

26. **a.** $y = u + 5u^{-1}$
b. $2(1 - 5(2x - 3)^{-2})$

27. **a.** $y = \sqrt{u} + 5u$
b. $(2x - 3)^{-\frac{1}{2}} + 10$

28. **a.** $6(2x - 5)^2(3x^2 + 4)^4 \times (13x^2 - 25x + 4)$
b. $8x^2(4x^2 + 2x - 3)^4$ $(52x^2 + 16x - 9)$
c. $2(5 + x)(4 - 7x^3)^5 \times (4 - 315x^2 - 70x^3)$
d. $\dfrac{6(-9x + 7)}{(3x + 5)^5}$
e. $\dfrac{2(2x^2 - 5)^2(4x^2 + 48x + 5)}{(x + 8)^3}$
f. $\dfrac{-3x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$
g. $8\left(\dfrac{2x + 5}{6 - x^2}\right)^3\left(\dfrac{(x + 2)(x + 3)}{(6 - x^2)^2}\right)$
h. $-9(4x + x^2)^{-10}(4 + 2x)$

29. $a = -4, b = 32, c = 0$

30. **a.** $-3t^2 + 5$
b. -7000 ants/h
c. 75 000 ants
d. 9.27 h

Chapter 2 Test, p. 114

1. You need to use the chain rule when the derivative for a given function cannot be found using the sum, difference, product, or quotient rules or when writing the function in a form that would allow the use of these rules is tedious. The chain rule is used when a given function is a composition of two or more functions.

2. f is the blue graph (it's cubic). f' is the red graph (it is quadratic). The derivative of a polynomial function has degree one less than the derivative of the function. Since the red graph is a quadratic (degree 2) and the blue graph is cubic (degree 3), the blue graph is f and the red graph is f'.

3. $1 - 2x$

4. a. $x^2 + 15x^{-6}$
   b. $60(2x - 9)^4$
   c. $-x^{-\frac{3}{2}} + \dfrac{1}{\sqrt{3}} + 2x^{-\frac{2}{3}}$
   d. $\dfrac{5(x^2 + 6)^4(3x^2 + 8x - 18)}{(3x + 4)^6}$
   e. $2x(6x^2 - 7)^{-\frac{2}{3}}(8x^2 - 7)$
   f. $\dfrac{4x^5 - 18x + 8}{x^5}$

5. 14

6. $-\dfrac{40}{3}$

7. $60x + y - 61 = 0$

8. $\dfrac{75}{32}$ ppm/year

9. $\left(-\dfrac{1}{4}, \dfrac{1}{256}\right)$

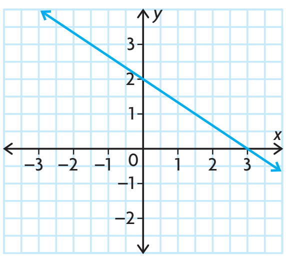
10. $\left(-\dfrac{1}{3}, \dfrac{32}{27}\right)$, $(1, 0)$

11. $a = 1, b = -1$

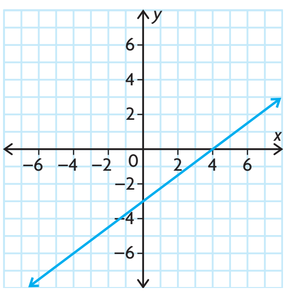
Chapter 3

Review of Prerequisite Skills, pp. 116–117

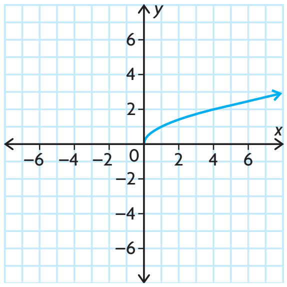
1. a.



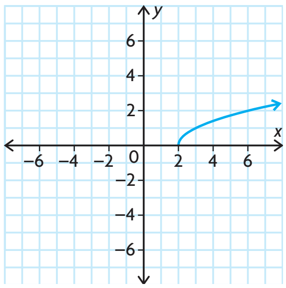
b.



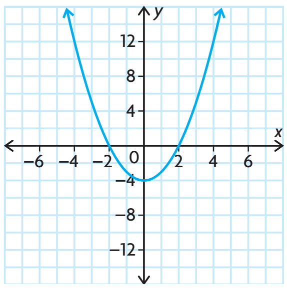
c.



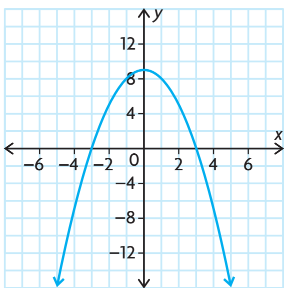
d.



e.



f.



2. a. $x = \dfrac{14}{5}$
   b. $x = -13$
   c. $t = 3$ or $t = 1$
   d. $t = -\dfrac{1}{2}$ or $t = 3$
   e. $t = 3$ or $t = 6$
   f. $x = 0$ or $x = -3$ or $x = 1$
   g. $x = 0$ or $x = 4$
   h. $t = -3$ or $t = \dfrac{1}{2}$ or $t = -\dfrac{1}{2}$
   i. $t = \pm\dfrac{9}{4}$ or $t = \pm 1$

3. a. $x > 3$
   b. $x < 0$ or $x > 3$
   c. $0 < x < 4$

4. a. 25 cm^2
   b. 48 cm^2
   c. $49\pi\text{ cm}^2$
   d. $36\pi\text{ cm}^2$

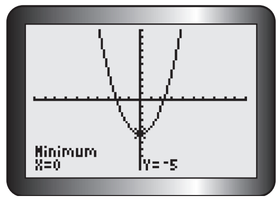
5. a. $SA = 56\pi\text{ cm}^2$, $V = 48\pi\text{ cm}^3$
   b. $h = 6$ cm, $SA = 80\pi\text{ cm}^2$
   c. $r = 6$ cm, $SA = 144\pi\text{ cm}^2$
   d. $h = 7$ cm, $V = 175\pi\text{ cm}^3$

6. a. $SA = 54\text{ cm}^2$, $V = 27\text{ cm}^3$
   b. $SA = 30\text{ cm}^2$, $V = 5\sqrt{5}\text{ cm}^3$
   c. $SA = 72\text{ cm}^2$, $V = 24\sqrt{3}\text{ cm}^3$
   d. $SA = 24k^2\text{ cm}^2$, $V = 8k^3\text{ cm}^3$

7. a. $(3, \infty)$
   b. $(-\infty, -2]$
   c. $(-\infty, 0)$
   d. $[-5, \infty)$
   e. $(-2, 8]$
   f. $(-4, 4)$

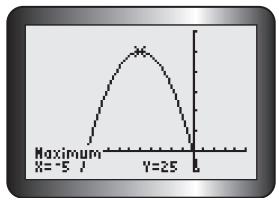
8. a. $\{x \in \mathbf{R} \mid x > 5\}$
   b. $\{x \in \mathbf{R} \mid x \leq -1\}$
   c. $\{x \in \mathbf{R}\}$
   d. $\{x \in \mathbf{R} \mid -10 \leq x \leq 12\}$
   e. $\{x \in \mathbf{R} \mid -1 < x < 3\}$
   f. $\{x \in \mathbf{R} \mid 2 \leq x < 20\}$

9. a.



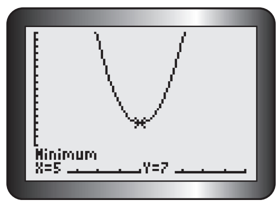
The function has a minimum value of -5 and no maximum value.

b.



The function has a maximum value of 25 and no minimum value.

c.



The function has a minimum value of 7 and no maximum value.