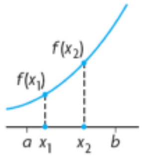
4.1: Increasing and Decreasing Functions

Date: _______________

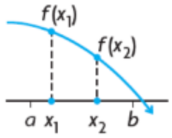
f is **INCREASING** on an interval $x \in [a,b]$, when

- $f(x_2) > f(x_1)$ whenever $x_2 > x_1$
- and $f'(x)$ > 0



f is **DECREASING** on an interval $x \in [a,b]$, when

- $f(x_2) < f(x_1)$ whenever $x_2 > x_1$
- and $f'(x)$ < 0



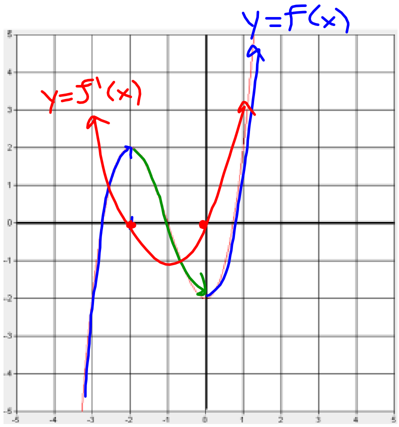
Ex1. For each graph, state the value(s) of x where the function is

i. increasing

ii. decreasing

iii. stationary

a.

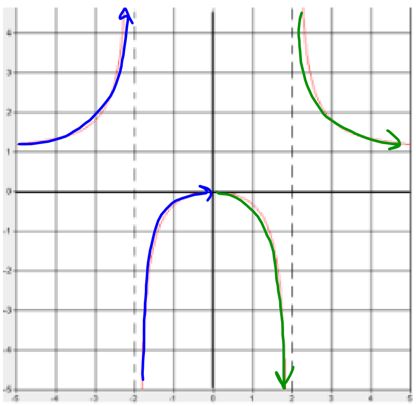


(i) $f'(x) > 0$ when $x < -2$ or $x > 0$

ii) $f'(x) < 0$ when $-2 < x < 0$

iii) $f'(x) = 0$ when $x = -2$ or 0.

b.



(i) $f'(x) > 0$ when $x < -2$ or $-2 < x < 0$

ii) $f'(x) < 0$ when $x > 0$, $x \neq 2$.

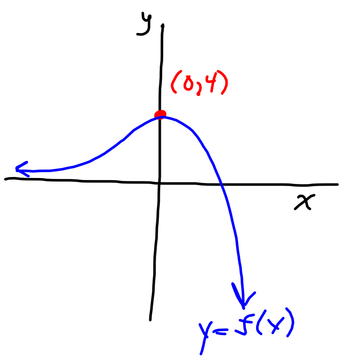
iii) $f'(x) = 0$ when $x = 0$.

Ex2. Sketch a graph of a function f that is differentiable and satisfies the following conditions.

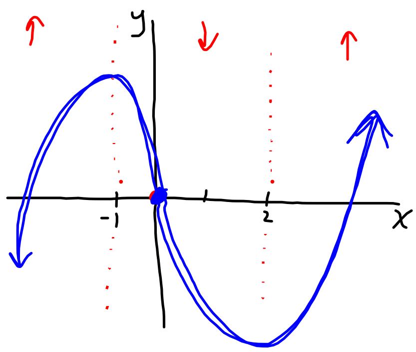
a. $f'(x) > 0$ when $x < 0$, $f'(x) < 0$ when $x > 0$, $f(0) = 4$

b. $f'(x) > 0$ when $x < -1$ and when $x > 2$, $f'(x) < 0$ when $-1 < x < 2$, $f(0) = 0$

a)



b)



Ex3. The graph represents the derivative function $f'(x)$ of a function $f(x)$.

a. Determine

i. the intervals where $f(x)$ is increasing

$f'(x) > 0$ when

$x > -1$

ii. the intervals where $f(x)$ is decreasing

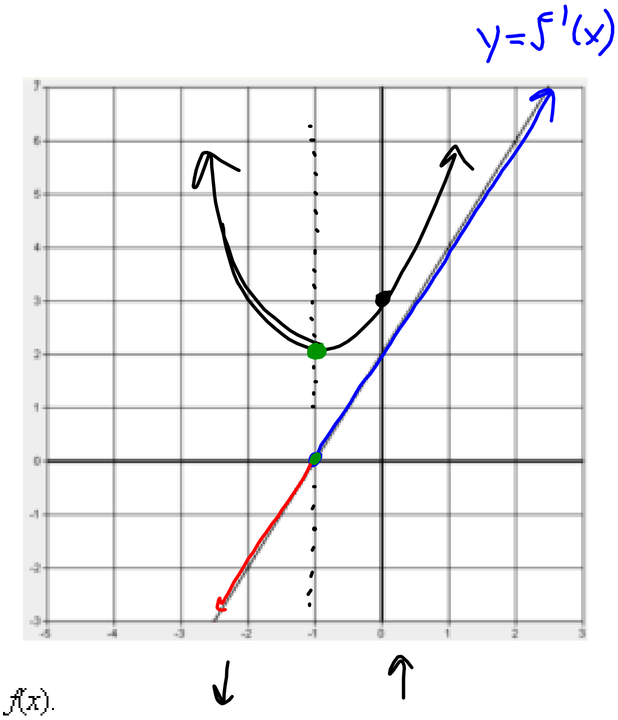
$f'(x) < 0$ when

$x < -1$

iii. the x-coordinate of all local extrema of $f(x)$

$f'(x) = 0$ when

$x = -1$



b. If $f(0) = 3$, make a sketch of a possible graph of $f(x)$.

Ex4. Sketch the graph of $f'(x)$ using the graph of $f(x)$ from part a in Ex1.

Ex5. Determine algebraically where each function is increasing and where it is decreasing. Sketch $y = f(x)$.

a. $f(x) = (2x+3)^3(x^2+3)^2$

x	$-\frac{3}{2}$	0
y	0	3^5

$$f'(x) = 3(2x+3)^2(2)(x^2+3)^2 + (2x+3)^3\left(2(x^2+3)(2x)\right)$$

$$= 2(2x+3)^2(x^2+3)\left[3(x^2+3) + 2x(2x+3)\right]$$

$$\boxed{f'(x) = 2(2x+3)^2(x^2+3)\left[7x^2+6x+9\right]}$$

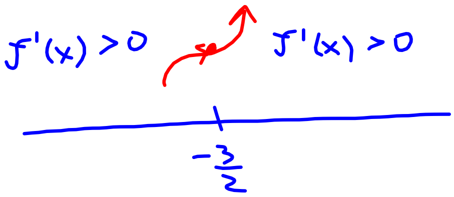
63
1 63
3 21
7 9

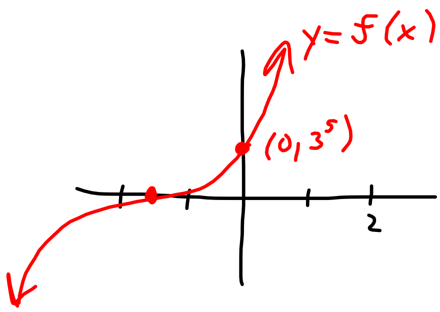
$f'(x) = 0$ when

$2x+3=0$ $\quad$ $x^2+3=0$ $\quad$ $7x^2+6x+9=0$

$x = \frac{-3}{2}$ $\quad$ NO SOL'N $\quad$ $b^2-4ac = 36-4(7)(9) < 0$

$\therefore$ NO SOL'N





b. $f(x) = \dfrac{x}{(x-2)^2} = \dfrac{x}{x^2-4x+4}$

x	0
y	0

VA @ $x = 2$

$$f'(x) = \frac{(1)(x-2)^2 - (x)\left(2(x-2)(1)\right)}{(x-2)^4}$$

$$= \frac{(x-2) - 2x}{(x-2)^3}$$

$$= \frac{-x-2}{(x-2)^3}$$

$$\boxed{f'(x) = \frac{-(x+2)}{(x-2)^3}}$$

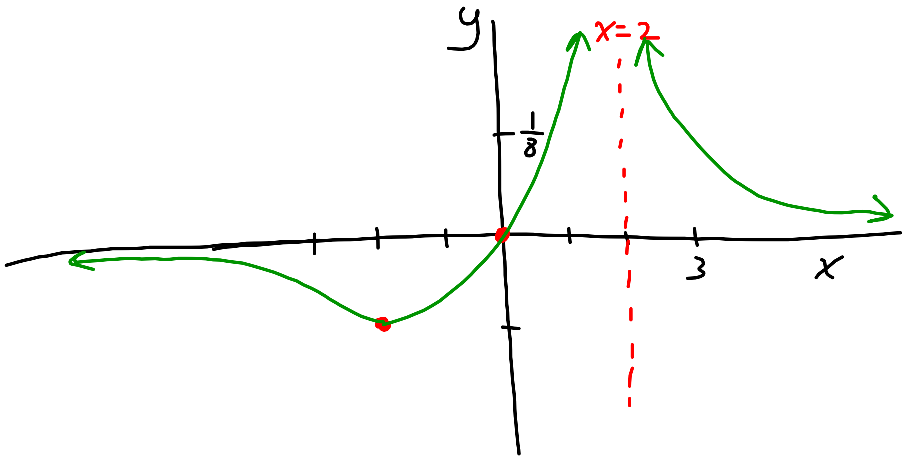
$f'(x) = 0$ when $x = -2$

$f'(x)$ DNE when $x = +2$

$f'(x) < 0$ $f'(x) > 0$ $f'(x) < 0$

-2 2

↓ →   ↑   ×   ↓



x	y
-2	$-\frac{1}{8}$
0	0

Homefun: Page 169 # (1, 4, 5, 7, 11)(algebraic concepts), (3, 6, 8, 9, 12)(graphing concepts), (10, 13, 14)(tricky-wickies)