2.3: The Product Rule Date: ______________

Ex1. The **PRODUCT RULE** says, "the derivative of the product of two functions is equal to the derivative of the first function times the second function plus the first function times the derivative of the second function." Show this is true.

In other words, if $P(x) = f(x)g(x)$, then show $P'(x) = f'(x)g(x) + f(x)g'(x)$.

$P(x) = f(x)g(x)$

$$P'(x) = \lim_{h\to 0} \frac{P(x+h) - P(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \left(\left[\frac{f(x+h) - f(x)}{h}\right] g(x+h) + f(x) \left[\frac{g(x+h) - g(x)}{h}\right] \right)$$

$$= f'(x)g(x) + f(x)g'(x)$$

Ex2. Apply the product rule to differentiate $P(x) = (x^2 + 4)(3x - 5)$. Then, differentiate $P(x)$ using a different method.

Moral of the Story : Make Good Choices

Instead of using the product rule, it is sometimes easier to expand and simplify FIRST, then differentiate.

$P(x) = (x^2+4)(3x-5)$

$P'(x) = 2x(3x-5) + (x^2+4)(3)$

$= 6x^2 - 10x + 3x^2 + 12$

$= 9x^2 - 10x + 12$

$P(x) = (x^2+4)(3x-5)$

$= 3x^3 - 5x^2 + 12x - 20$

$P'(x) = 9x^2 - 10x + 12$

Ex3. Use the product rule to find an expression for the **EXTENDED PRODUCT RULE** and continue the pattern.

IF $P(x) = f(x)g(x)h(x)$, THEN $P'(x) = f'gh + fg'h + fgh'$

IF $P(x) = f(x)g(x)h(x)i(x)$, THEN $P'(x) =$

…

$P = f(gh)$

$P' = f'gh + f(g'h + gh')$

$= f'gh + fg'h + fgh'$

$P = f(ghi)$

$P' = f'ghi + f(g'hi + gh'i + ghi')$

$= f'ghi + fg'hi + fgh'i + fghi'$

Ex4. Use the product rule (including the extended product rule) to differentiate the following functions.

a. $P(x) = [g(x)]^2$

b. $P(x) = (x^2 - 3)^2$

c. $P(x) = [g(x)]^3$

d. $P(x) = (x^2 + 3x + 5)^3$

e. $P(x) = [g(x)]^4$ ← CONTINUE THE PATTERN. DO NOT USE THE EXTENDED PRODUCT RULE.

a) $P(x) = [g(x)]^2$ $\quad P'(x) = g'g + gg'$

$= g \cdot g$ $\quad = 2\,g(x) \cdot g'(x)$

c) $P(x) = [g(x)]^3$ $\quad P'(x) = g'gg + gg'g + ggg'$

$= ggg$ $\quad = 3[g(x)]^2 \cdot g'(x)$

e) $P(x) = [g(x)]^4 \longrightarrow P'(x) = 4[g(x)]^3 \cdot g'(x)$

b) $P(x) = (x^2-3)^2 \longrightarrow P'(x) = 2(x^2-3)^1 \cdot (2x)$

d) $P(x) = (x^2+3x+5)^3 \longrightarrow P'(x) = 3(x^2+3x+5)^2(2x+3)$

Ex5. Use the results from the previous example to summarize the **POWER OF A FUNCTION RULE**.

IF $P(x) = [f(x)]^n$, THEN $P'(x) = n\left[f(x)\right]^{n-1} \cdot f'(x)$

Ex6. Differentiate. Do not simplify.

a. $f(x) = (x^2 - 5x + 7)^6$

$$f'(x) = 6(x^2-5x+7)^5(2x-5)$$

$f'g + fg'$

b. $g(x) = (x^2 - 16)^5(1 - 2x)^3$

$$g'(x) = \left[5(x^2-16)^4(2x)\right](1-2x)^3 + (x^2-16)^5\left[3(1-2x)^2(-2)\right]$$

Ex7. Differentiate and simplify.

$$h(x) = \frac{3x+4}{2x-1} = (3x+4)(2x-1)^{-1}$$

$$h'(x) = 3(2x-1)^{-1} + (3x+4)\left[-(2x-1)^{-2}(2)\right]$$

$$= \frac{3(2x-1)}{(2x-1)^2} + \frac{-2(3x+4)}{(2x-1)^2}$$

$$= \frac{6x-3-6x-8}{(2x-1)^2}$$

$$= \frac{-11}{(2x-1)^2}$$

Ex8. Determine the slope of the tangent at $x = 1$ to the curve $y = -3x(x-1)^5(x^2+3x-4)^3$. Then, make sure your answer is consistent with a rough sketch of the function.

$f'gh + fg'h + fgh'$

$y = (-3x)(x-1)^5(x^2+3x-4)^3$

$y' = [-3](x-1)^5(x^2+3x-4)^3$

$+ (-3x)\left[5(x-1)^4(1)\right](x^2+3x-4)^3$

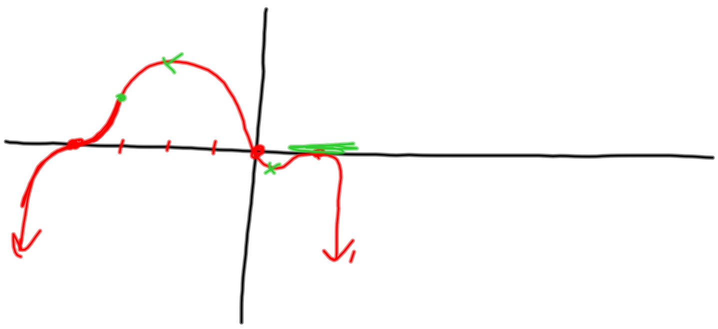
$+ (-3x)(x-1)^5\left[3(x^2+3x-4)^2(2x+3)\right]$

So when $x=1$, $y'=0$

$y = -3x(x-1)^5(x^2+3x-4)^3$

$= -3x(x-1)^5(x+4)^3(x-1)^3$

$= -3x(x-1)^8(x+4)^3$



Homefun: Page 90 # 1 (differentiate using any method), 2cd, 3, 4, 5bcdf, 6, 7, 8b, 9, 11b, 12, 13, 14

Solution in text for question 14 finishes unnecessarily difficult. Read the following AFTER trying the question.
After showing the slope of line is 4, and the slope of the tangent on the curve is 4 at (−2, 3), **then simply show that (−2, 3) is also on the line**. Finding the intersection is not necessary.