2.4: **The Quotient Rule** Date: ______________

Ex1. Use the product rule to derive the **QUOTIENT RULE**. That is, find $Q'(x)$ if $Q(x) = \frac{f(x)}{g(x)}$.

THE QUOTIENT RULE

IF $Q(x) = \frac{f(x)}{g(x)}$,

THEN $Q'(x) = \frac{f'g - fg'}{g^2}$

In Leibniz notation,

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}\cdot v - u\cdot\frac{dv}{dx}}{v^2}$$

$$\text{IF } Q(x) = \frac{f(x)}{g(x)} = \underline{f(x)}\cdot\underline{(g(x))^{-1}}$$

$$\therefore\ Q'(x) = f'(x)\cdot(g(x))^{-1} + f(x)\cdot\left[-(g(x))^{-2}\cdot g'(x)\right]$$

$$= \frac{f'(x)\cdot g(x)}{(g(x))^2} + \frac{-f(x)\cdot g'(x)}{(g(x))^2}$$

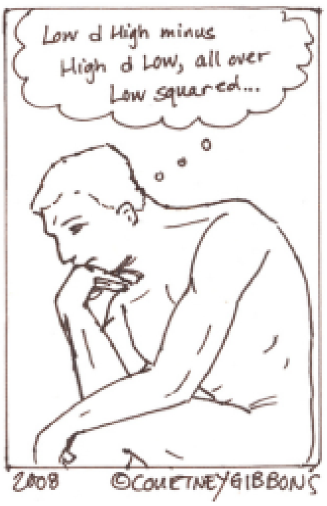
$$= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Ex2. Apply the quotient rule to differentiate $f(x) = \frac{5x - 1}{x^3 + 2}$.

$$f'(x) = \frac{5(x^3+2) - (5x-1)(3x^2)}{(x^3+2)^2}$$

$$= \frac{5x^3 + 10 - 15x^3 + 3x^2}{(x^3+2)^2}$$

$$= \frac{10 + 3x^2 - 10x^3}{(x^3+2)^2}$$



Ex3. Find the derivative of $g(x) = \frac{50}{x} + \frac{(x^2 - 5x)^2}{4x + 1}$ at $x = 5$.

$$g(x) = 50x^{-1} + \frac{x^4 - 10x^3 + 25x^2}{4x+1}$$

So $$g'(x) = -50x^{-2} + \frac{2(x^2-5x)' \cdot (2x-5)(4x+1) - (x^2-5x)^2(4)}{(4x+1)^2}$$

$$\therefore\ g'(5) = \frac{-50}{25} + \frac{2(0)(\text{whocares}) - (0)^2(4)}{\text{NOT ZERO}}$$

$$= -2$$

Ex4. Find the equation of the tangent to $y = \frac{x(2x + 1)}{(x + 1)(x + 3)}$ at $x = 1$.

$$y = \frac{2x^2 + x}{x^2 + 4x + 3}$$

So $$y' = \frac{(4x+1)(x+1)(x+3) - x(2x+1)(2x+4)}{(x+1)^2(x+3)^2}$$

So when $x = 1$,

$$y' = \frac{(5)(2)(4) - (1)(3)(6)}{(4)(16)}$$

$$= \frac{5(4) - (3)(3)}{2(16)}$$

$$= \frac{11}{32}$$

$$y = \frac{2+1}{1+4+3}$$

$$= \frac{3}{8}$$

So $m = \frac{11}{32}$, $P_0 = \left(1, \frac{3}{8}\right)$

$$\therefore\ \frac{x-1}{32} = \frac{y - \frac{3}{8}}{11} \rightarrow 11x - 11 = 32y - 12$$

$$11x - 32y + 1 = 0 \quad \text{ITRE}$$

Ex5. Determine the coordinates on $h(x) = \dfrac{9x+4}{3\sqrt{x}}$ where the tangent is horizontal.

$$h(x) = \frac{9x}{3x^{\frac{1}{2}}} + \frac{4}{3x^{\frac{1}{2}}}$$

$$= 3x^{\frac{1}{2}} + \frac{4}{3}x^{-\frac{1}{2}}$$

$$h'(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{2}{3}x^{-\frac{3}{2}}$$

let $h'(x) = 0$

$$\frac{3}{2}x^{-\frac{1}{2}} - \frac{2}{3}x^{-\frac{3}{2}} = 0$$

$$\frac{3}{2x^{\frac{1}{2}}} = \frac{2}{3x^{\frac{3}{2}}}$$

$$\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = \frac{4}{9}$$

$$x = \frac{4}{9}$$

$$\left(\frac{3}{2}x^{-\frac{1}{2}} - \frac{2}{3}x^{-\frac{3}{2}} = 0\right) \times 6$$

$$9x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}} = 0$$

$$x^{-\frac{3}{2}}\left(9x^{\frac{2}{2}} - 4\right) = 0$$

$$\frac{9x-4}{\sqrt{x^3}} = 0$$

$$\therefore x = \frac{4}{9}$$

$$h\left(\frac{4}{9}\right) = \frac{9\left(\frac{4}{9}\right)+4}{3\sqrt{\frac{4}{9}}}$$

$$= \frac{4+4}{2}$$

$$= 4$$

Thus the required is $\left(\frac{4}{9}, 4\right)$.

MEMORY AID FOR THE PRODUCT AND QUOTIENT RULES

1. The product rule and quotient rule are similar in that both have $f'(x)g(x)$ and $f(x)g'(x)$ terms.
2. For the product rule, we put a $+$ between the terms.
3. For the quotient rule, we put a $-$ between the terms and divide by the square of the original denominator.
4. For the quotient rule, the $f'(x)g(x)$ term must come first. This isn't the case for the product rule.

Homefun:

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[We haven't directly covered the ideas covered in 15 and 16, but you do have the tools to solve these anyway.]