1.4: **The Limit of a Function** Date: ______________

The notation $\lim_{x \to a} f(x) = L$ means that

the value of $f(x)$ can be made <u>arbitrarily</u> close to L

by choosing x <u>sufficiently</u> close to a (but not equal to a).

However, $\lim_{x \to a} f(x)$ exists if and only if

the limiting value from the <u>left</u> equals the limiting value from the <u>right</u>.

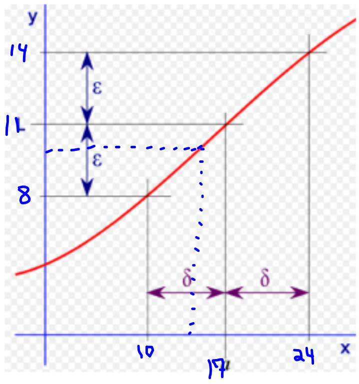
Note: The above is an intuitive explanation of the limit of a function. A rigorous definition is important for more advanced work, but is not necessary for our purposes.

Just for fun, here is the rigorous definition of a limit:

$\lim_{x \to a} f(x) = L$ means that

for each real $\xi > 0$, there exists a real $\delta > 0$ such that

for all x with $0 < |x - a| < \delta$ we have $|f(x) - L| < \xi$.



Symbolically we write:

$$\forall \varepsilon > 0, \exists \delta > 0 \mid$$

$$\forall x \in 0 < |x-a| < \delta \longrightarrow |f(x) - L| < \varepsilon$$

Ex1. Consider the function $f(x) = \sqrt{25 - x^2}$.

x	-5	-4	-3	0	3	4	5
y	0	3	4	5	4	3	0

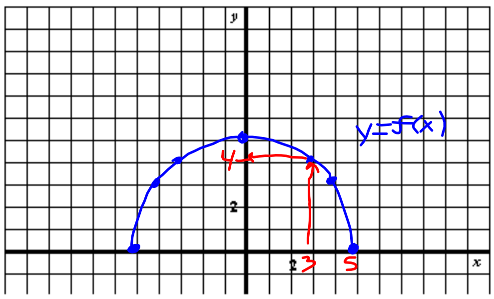
Determine each limit by graphing.

a. $\lim_{x \to 3} f(x) = 4$

b. $\lim_{x \to 5^-} f(x) = 0$

c. $\lim_{x \to 5} f(x)$ DNE

b/c $\lim_{x \to 5^+} f(x)$ DNE.



Ex2. Determine $\lim_{x \to 2} \frac{x-2}{x^2-4}$ by using a table.

x approaches 2 from the left | x approaches 2 from the right

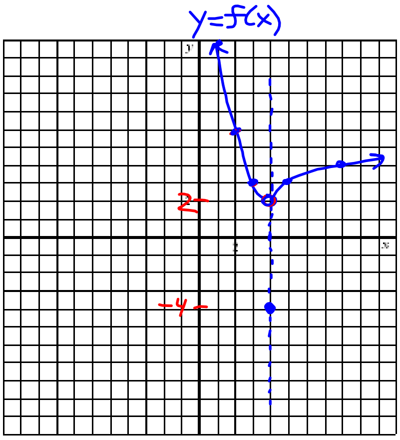
x	1.9	1.99	1.999	2	2.001	2.01	2.1
$\frac{x-2}{x^2-4}$	0.256	0.251	0.250	undefined	0.250	0.249	0.244

$\frac{x-2}{x^2-4}$ approaches $\frac{1}{4}$ from above | $\frac{x-2}{x^2-4}$ approaches $\frac{1}{4}$ from below

Since $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \frac{1}{4}$

$\therefore \lim_{x \to 2} f(x) = \frac{1}{4}$

Ex3. Sketch the graph of the piecewise function $f(x) = \begin{cases} (x-4)^2 + 2, & x < 4 \\ -4, & x = 4 \\ \sqrt{x-4}+2, & x > 4 \end{cases}$ and determine $\lim_{x \to 4} f(x)$.



$\lim_{x \to 4^-} f(x) = (4-4)^2+2$
$= 2$

$\lim_{x \to 4^+} f(x) = \sqrt{4-4}+2$
$= 2$

Thus, $\lim_{x \to 4} f(x) = 2$

Even though $f(4) = $ NOT 2

Ex4. Answer true or false.

a. $\lim_{x \to a} f(x)$ may exist even if $f(a) \neq \lim_{x \to a} f(x)$ and even if $f(a)$ does not exist. T

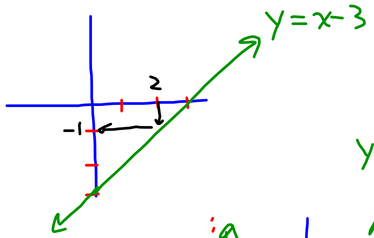
b. If $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x) = L$. T

c. If $\lim_{x \to a} f(x) = L$, then $f(a) = L$. F

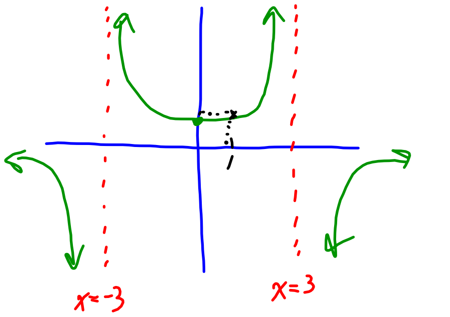
d. If $\lim_{x \to a} f(x) = f(a)$, then the graph of $y = f(x)$ "flows" through the point $(a, f(a))$. T

Ex5. Calculate each limit and include a quick sketch of the corresponding function to support your answer.

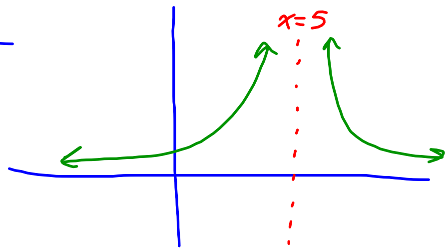
a. $\lim_{x \to 2^+} (x-3)$ $= 2-3$
$= -1$



b. $\lim_{x \to 1^+} \dfrac{-2}{x^2-9} = \dfrac{-2}{1-9}$
$= \dfrac{-2}{-8}$
$= \dfrac{1}{4}$



c. $\lim_{x \to 5} \dfrac{1}{(x-5)^2}$ DNE



Q11. $t=\sqrt{\frac{s}{5}}$ ~~[illegible]~~ $s=125$

$$\text{IROC}=\lim_{h\to 0}\frac{\sqrt{\frac{125+h}{5}}-5}{h}\times\frac{\sqrt{}+5}{\sqrt{}+5}$$

$$=\lim_{h\to 0}\frac{\frac{125+h}{5}-\frac{125}{5}}{h(\qquad)}$$

$$=\lim_{h\to 0}\frac{\frac{1}{5}\cancel{h}}{\cancel{h}(\qquad)}$$

Q15 a) $f(x)=-x^2+2x+3$, $(-2,-5)$

$$m=\lim_{x\to -2}\frac{f(x)-f(-2)}{x-(-2)}$$

$$=\lim\frac{-x^2+2x+\cancel{3}\cancel{+5}+8}{x+2}$$

$$=\lim\frac{-(x^2-2x-8)}{(x+2)}$$

$$=\lim\frac{-(x-4)\cancel{(x+2)}}{\cancel{(x+2)}}$$