2.5.1

Q5 $5x^2 - 6xy + 5y^2 = 16$

$\therefore 10x - [6y + 6x \cdot y'] + 10y \cdot y' = 0$

$\therefore y' \cdot [-6x + 10y] = -10x + 6y$

So, $y' = \dfrac{6y - 10x}{10y - 6x}$

Q5a @ $(1, -1)$

$y' = \dfrac{-6-10}{-10-6}$

$= \dfrac{-16}{-16}$

$= 1$

Q5b Since $y' = 0$

$\therefore \dfrac{6y - 10x}{10y - 6x} = 0$

$\therefore 6y - 10x = 0$

$6y = 10x$

$y = \dfrac{5}{3}x$

But $5x^2 - 6xy + 5y^2 = 16$

So, $5x^2 - 6x\left(\frac{5}{3}x\right) + 5\left(\frac{5}{3}x\right)^2 = 16$

$\left(5x^2 - \frac{30}{3}x^2 + \frac{125}{9}x^2 = 16\right) \times 9$

$45x^2 - 90x^2 + 125x^2 = 16 \cdot 9$

$80x^2 = 16 \cdot 9$

$x = \pm\sqrt{\dfrac{16 \cdot 9}{16 \cdot 5}}$

$= \pm\dfrac{3}{\sqrt{5}}$

$y = \frac{5}{3}x$

$= \frac{5}{3}\left(\pm\frac{3}{\sqrt{5}}\right)$

$= \pm\sqrt{5}$

Thus, the points are $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(\frac{-3}{\sqrt{5}}, -\sqrt{5}\right)$.

Q10 $4x^2y - 3y = x^3$

a) $8xy + 4x^2 \cdot y' - 3 \cdot y' = 3x^2$

$y' \cdot (4x^2 - 3) = 3x^2 - 8xy$

$y' = \dfrac{3x^2 - 8xy}{4x^2 - 3}$

b) $4x^2y - 3y = x^3$

$y(4x^2 - 3) = x^3$

$y = \dfrac{x^3}{4x^2 - 3}$

$\therefore y' = \dfrac{3x^2(4x^2 - 3) - x^3(8x)}{(4x^2 - 3)^2}$

$= \dfrac{12x^4 - 9x^2 - 8x^4}{*}$

$= \dfrac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

c) $y' = \dfrac{3x^2 - 8x\left(\dfrac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$

$= \left[\dfrac{3x^2(4x^2 - 3)}{4x^2 - 3} - \dfrac{8x(x^3)}{4x^2 - 3}\right] \cdot \dfrac{1}{4x^2 - 3}$

$= \dfrac{4x^4 - 9x^2}{(4x^2 - 3)^2}$