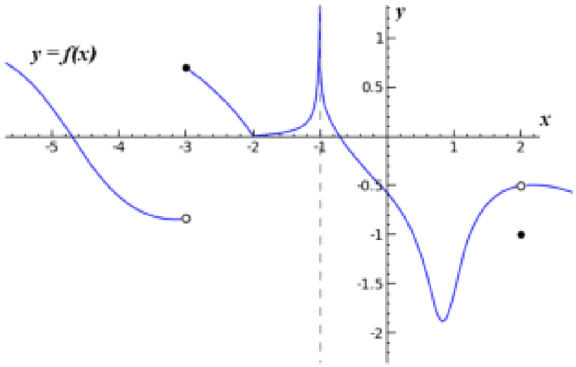
The function $y = f(x)$ is continuous at $x = a$

IF $\lim_{x \to a} f(x) = f(a)$

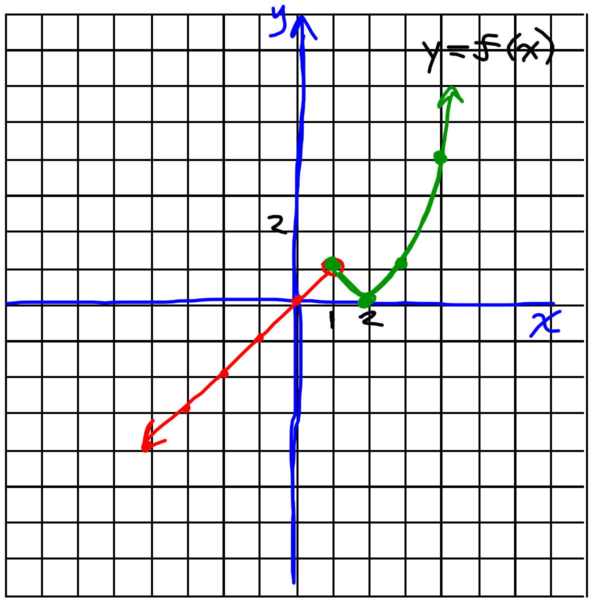
OTHERWISE $y = f(x)$ is discontinuous at $x = a$.

For the graph to right, $y = f(x)$ is continuous for all $x \in R$ provided that $x = -3, -1, 2$



Ex1. Consider the function: $f(x) = \begin{cases} x, & x < 1 \\ (x-2)^2, & x \geq 1 \end{cases}$.

a. Graph $y = f(x)$.



b. Find $\lim_{x \to 1} f(x)$.

$\lim_{x \to 1^-} f(x) = (1)$
$= 1$

$\lim_{x \to 1^+} f(x) = (1-2)^2$
$= 1$

Thus $\lim_{x \to 1} f(x) = 1$

this equals that

c. Find $f(1)$. $f(1) = (1-2)^2$
$= 1$

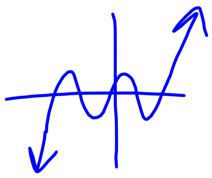
d. Is $y = f(x)$ continuous at $x = 1$? Explain.

Yes b/c $\lim_{x \to 1} f(x) = f(1)$

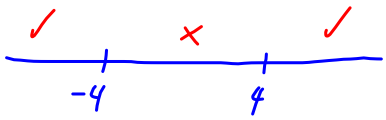
Ex2. Determine all values of x for which each function is continuous.

a. $f(x) = x^5 - 9x$

$f(x)$ is continuous $x \in R$.



b. $g(x) = \dfrac{5 - x}{\sqrt{x^2 - 16}}$



So $g(x)$ is continuous for $x \in R \mid x < -4$ or $x > 4$

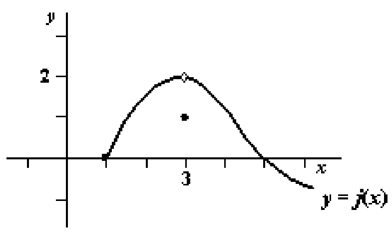
c. $h(x) = \begin{cases} 4 + x, & x < 5 \\ 2x, & x \geq 5 \end{cases}$

$\lim\limits_{x \to 5^-} h(x) = 4 + 5 = 9$

$\lim\limits_{x \to 5^+} h(x) = 2(5) = 10$

$\therefore h(x)$ is cont. for $x \in R \mid x \neq 5$.

d.



$j(x)$ is continuous for $x \in R \mid x > 1,\ x \neq 3$

$j(1) = 0$

$\lim\limits_{x \to 1} j(x)$ DNE

$\therefore y = j(x)$ is NOT continuous when $x = 1$.

Ex3. Find a and b such that $f(x) = \begin{cases} x+3 & x < -2 \\ ax^2+5 & -2 \leq x < 2 \\ x+b & x \geq 2 \end{cases}$ is continuous for all $x \in R$. Then, sketch $y = f(x)$.

Since $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^+} f(x)$

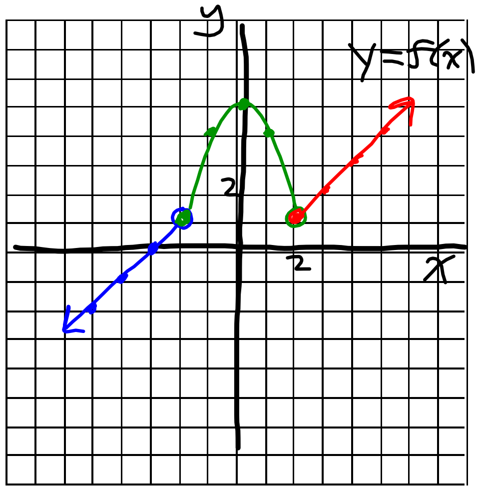
$\therefore$ $x+3 = ax^2+5$ when $x=-2$

$-2+3 = a(-2)^2+5$

$1 = 4a+5$

$4a = -4$

$a = -1$



Similarily,

$ax^2+5 = x+b$ when $x=2$

$-(2)^2+5 = 2+b$

$1 = 2+b$

$b = -1$

$$f(x) = \begin{cases} x+3 & x < -2 \\ -x^2+5 & -2 \leq x < 2 \\ x-1 & x \geq 2 \end{cases}$$

Homefun: Page 51 #1→5(Ans. 5d $x \in R \mid x > -2$), 7, 8, 10→14, 15(Ans. $A = B - 3,\ B \neq 1$ is less awkward), 16, 17