1.0: Review of Prerequisite Skills

Date: ______________

Ex1. Use Pascal's triangle to expand and simplify each of the following.

a. $(1 + h)^5$

b. $(2x + 1)^4$

c. $(3a - 5)^3$

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
⋮

a) $(1+h)^5 = 1(1)^5(h)^0 + 5(1)^4(h)^1 + 10(1)^3(h)^2$
$+10(1)^2(h)^3 + 5(1)^1(h)^4 + 1(1)^0(h)^5$

$= 1 + 5h + 10h^2 + 10h^3 + 5h^4 + h^5$

b) $(2x+1)^4 = (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x) + 1$
$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$

c) $(3a-5)^3 = (3a)^3 + 3(3a)^2(-5)^1 + 3(3a)^1(-5)^2 + (-5)^3$
$= 27a^3 - 135a^2 + 225a - 125$

Ex2. Factor.

$a^2 - b^2 = (a+b)(a-b)$ $\quad$ $a^2 + b^2$ can NOT be factored!

$x^2 + Bx + C = (x+p)(x+q)$ where $p+q = B$ and $pq = C$

$Ax^2 + Bx + C = \frac{(Ax+p)(Ax+q)}{A}$ where $p+q = B$ and $pq = AC$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Factor Theorem: If $P(x)$ is a polynomial and $P(a) = 0$, then $x-a$ is a factor.

a. $16x^5 - x$

$= x(16x^4 - 1)$

$= x(4x^2+1)(4x^2-1)$

$= x(4x^2+1)(2x+1)(2x-1)$

b. $-x^3 - 3x^2 + 4x$

$= -x(x^2+3x-4)$

$= -x(x+4)(x-1)$

c. $6x^2 + 7x - 3$

$= \frac{(\underbrace{6x+9}_{3})(\underbrace{6x-2}_{2})}{\cancel{6}}$

$= (2x+3)(3x-1)$

d. $8x^3 + 27$

$= (2x+3)(4x^2-6x+9)$

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

e. $\underbrace{x^3 - 2x^2}\underbrace{- 9x + 18}$

$= x^2(x-2) - 9(x-2)$

$= (x-2)[x^2 - 9]$

$= (x-2)(x+3)(x-3)$

f. $4x^3 + 16x^2 + 9x - 9$

Try $x = \pm 1, \pm 3, \pm 9$

Since $P(-3) = 0$, $\therefore x+3$ is a factor

$\therefore 4x^3 + 16x^2 + 9x - 9$

$= (x+3)(4x^2 + 4x - 3)$

$= (x+3)\frac{(4x+6)(4x-2)}{4}$

$= (x+3)(2x+3)(2x-1)$

Ex3. State the domain.

a. $f(x) = \frac{2+\sqrt{x-3}}{5x-3}$

$x-3 \geq 0$ and $5x-3 \neq 0$

$x \geq 3$ $\qquad x \neq \frac{3}{5}$

$\therefore D = \{x \in R \mid x \geq 3\}$

b. $y = \sqrt{60 + 14x - 2x^2}$

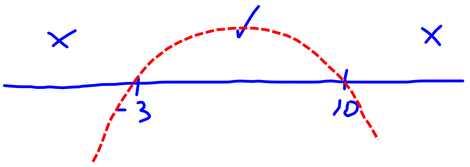
$60 + 14x - 2x^2 \geq 0$

let $60 + 14x - 2x^2 = 0$

$\therefore x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$

$\therefore x = 10$ or $x = -3$



Thus $\underline{-3 \leq x \leq 10}$.

Ex4. Consider $\underline{h(t) = 3(t-1)^2 + 7}$ where $h(t)$ is the height in metres after t seconds.

a. Find the average rate of change (AROC) during the 3rd second.

b. Estimate the instantaneous rate of change (IROC) at exactly 3 s.

a)

t	h(t)
2	h(2) =10
3	h(3) =19

+1

+9

$$AROC = \frac{h(3) - h(2)}{3-2}$$

$$= \frac{9 \text{ m}}{1 \text{ sec}}$$

$$= 9 \text{ m/s}$$

b)

$$IROC = \frac{h(3.0001) - h(3)}{0.0001}$$

$$= \frac{19.00120... - 19}{0.0001}$$

$$\doteq \frac{0.0012 \text{ m}}{0.0001 \text{ s}}$$

$\therefore \underline{IROC \doteq 12 \text{m/s}}$

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