3.4: Optimization Problems in Economics and Science

Date: ______________

Ex1. The beverage industry in Canada produces over \$10 billion worth of product annually. Based on a 10-year study of production costs, a winery in the Niagara region has determined that the cost of producing x bottles of wine is $C(x) = 12000 + 4x + 0.0002x^2$.

Market research shows that the demand for the wine is given by the price function $p(x) = 12 - 0.0001x$.

a. Interpret $p(10000) = 11$ and $p(20000) = 10$.

IF supply is 10 000 bottles,

$\therefore$ the price should be \$11/bottle to create the demand to consume that supply.

(i.e. @ \$11/bottle, they can sell 10 000 bottles)

b. Determine the production level and price that maximizes the revenue.

Revenue = (sales)(price)

$$R(x) = x \cdot p(x)$$
$$= x(12 - 0.0001x)$$
$$= 12x - 0.0001x^2$$

$R'(x) = 12 - 0.0002x$

So $R'(x) = 0$ when $x = 60\,000$ bottles

And $p(60\,000) = 12 - 0.0001(60\,000)$
$= \$6$/bottle

Thus, Max Revenue $= (60000)(6)$
$= \$360\,000$

c. Determine the production level and price that maximizes the profit.

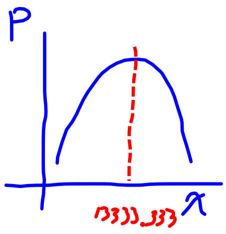
Profit = Revenue − Cost

$$P(x) = R(x) - C(x)$$

$$P'(x) = R'(x) - C'(x)$$

$$= 12 - 0.0002x - (4 + 0.0004x)$$

$$= 8 - 0.0006x$$



So $P'(x) = 0$ when $x = 13333.333\ldots$

$x \doteq 13333$ bottles

So $p(13333) = 12 - 0.0001(13333)$

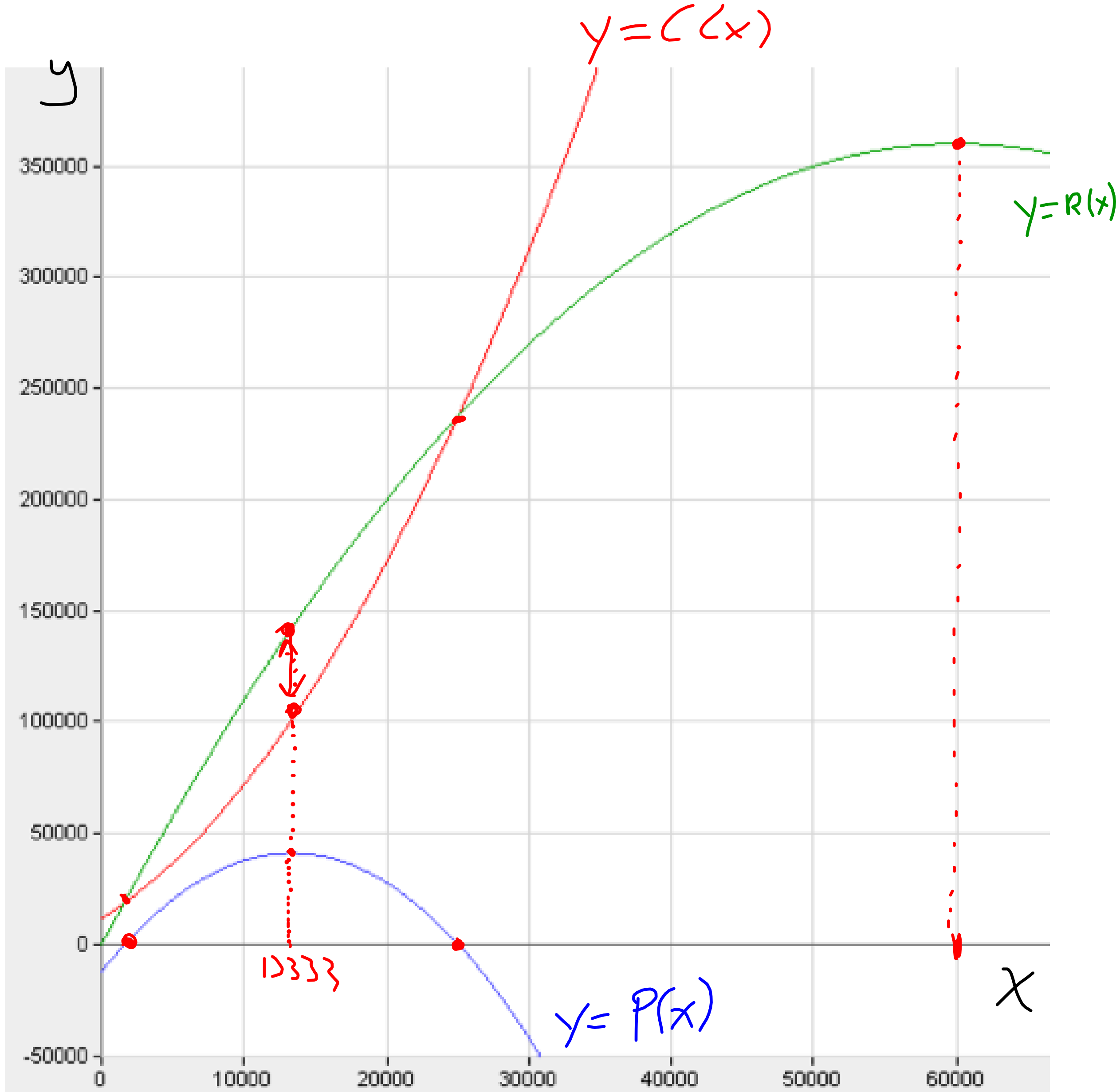
$= \$10.67$

Max Profit $= R(13333) - C(13333)$

$= \$142219.11 - \$100885.78$

$= \underline{\$41,333.33}$

d. Label the graphs to show the relationship between cost, revenue, and maximum profit.



Ex2. Market research has shown that for every drop in price of a product there is usually an increase in sales. Similarly, an increase in price usually leads to a decrease in sales. A large retailer selling mountain bicycles has found that, for every $20 reduction in price on the Rockhopper model, 2 more bicycles are sold per month. The Rockhopper usually sells for $900 and at that price the store sells 50 bicycles per month.

a. Determine the price and monthly sales which will maximize the revenue.

Gr10: Revenue = (sales)(price)

$R(x) = (50+2x)(90-20x)$

BAD FUNCTION

x	$p(x)$
50	900
52	880

+2, −20

$m = \frac{-20}{2}$

$m = -10$

$\frac{x-50}{1} = \frac{p(x)-900}{-10}$

$-10x + 500 = p(x) - 900$

$p(x) = -10x + 1400$

So $R(x) = x \cdot p(x)$

$= -10x^2 + 1400x$

$R'(x) = -20x + 1400$

$R'(x) = 0$ when $x = 70$ bikes

Thus, $p(70) = -10(70) + 1400$

$= \$700/\text{bike}$

Max Revenue

$= (70)(700)$

$= \$49\,000$

b. Determine the price and monthly sales which will maximize profits if the bicycles can be purchased from a supplier for $500/bicycle.

Use the table below to compare this scenario with the scenario from (a).

Sales	Price	Revenue	Cost	Profit
70	$ 700/bike	$ 49000	$35 000	$14 000
45	$950/bike	$ 42750	$ 22500	$20 250

Gr10: Profit $= (50+2x)(900-20x) - (50+2x)(500)$

$= (50+2x)(900-20x-500)$

$= (50+2x)(400-20x)$

BAD !!

$P(x) = R(x) - C(x)$, $C(x) = 500x$

$P'(x) = R'(x) - C'(x)$

$= -20x + 1400 - 500$

$= -20x + 900$

$P'(x) = 0$ when $x = 45$ bikes

$p(45) = -10(45) + 1400$

$= \$950/\text{bike}$

Homefun: Page 150 # {1, 4}, {6, 13, 14}, {7, 11a [$1169.92 @ 138 sales], 16}, {11b [$1072.50 @ 145 sales]}, {15} [sets ordered from easy-peasy to backyard-hard]