Related Rates Exercises Pg 569 Day 1

3. $\frac{ds}{dt} = 5$ cm/s

Find $\frac{dA}{dt}$ @ $s=10$cm

$\frac{dP}{dt}$ @ $s=10$cm

$\frac{dA}{dt} = \frac{dA}{ds} \cdot \frac{ds}{dt}$

$= \frac{d}{ds}(s^2) \cdot 5$

$= 2s(5)$

$= 10s$

$= 10(10)$

$= 100$ cm^2/s

$\frac{dP}{dt} = \frac{dP}{ds} \cdot \frac{ds}{dt}$

$= \frac{d}{ds}(4s) \cdot 5$

$= 4(5)$

$= 20$ cm/s

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4. $\frac{ds}{dt} = 4$ cm/s

Find $\frac{dV}{dt}$ @ $s=5$cm

$\frac{dSA}{dt}$ @ $s=7$cm

$\frac{dV}{dt} = \frac{dV}{ds} \cdot \frac{ds}{dt}$

$= \frac{d}{ds}(s^3) \cdot 4$

$= 3s^2(4)$

$= 12s^2$

$= 12(5)^2$

$= 300$ cm^3/s

$\frac{dSA}{dt} = \frac{dSA}{ds} \cdot \frac{ds}{dt}$

$= \frac{d}{ds}[6s^2] \cdot 4$

$= 12s(4)$

$= 48s$

$= 48(7)$

$= 336$ cm^2/s

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5. $\frac{dw}{dt} = 2$ cm/s

$\frac{dl}{dt} = -3$ cm/s

Find $\frac{dA}{dt}$ @ $w=20$, $l=50$

Since $A = lw$

$\therefore \frac{dA}{dt} = \frac{dl}{dt}(w) + l\frac{dw}{dt}$

$= -3w + 2l$

$= -3(20) + 2(50)$

$= 40$ cm^2/s

(6.) $\frac{dA}{dt} = 5 m^2/s$ @ $r = 3$ m

(a) Find $\frac{dr}{dt}$ @ $r = 3m$

(b) Find $\frac{dD}{dt}$ @ $r = 3m$, $D = 6m$

(a) $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$5 = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt}$

$5 = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{2\pi r}$

$= \frac{5}{2\pi(3)}$

$= \frac{5}{6\pi}$ m/s

(b) $\frac{dA}{dt} = \frac{dA}{dD} \cdot \frac{dD}{dt}$

$5 = \frac{d}{dD}\left(\pi\left(\frac{D}{2}\right)^2\right) \cdot \frac{dD}{dt}$

$5 = \frac{d}{dD}\left(\frac{\pi D^2}{4}\right) \cdot \frac{dD}{dt}$

$5 = \frac{\pi D}{2} \cdot \frac{dD}{dt}$

$\frac{dD}{dt} = \frac{10}{\pi D}$

$= \frac{10}{\pi(6)}$

$= \frac{5}{3\pi}$ m/s

(7.) $\frac{dA}{dt} = 6 km^2/h$

Find $\frac{dr}{dt}$ @ $A = 9\pi\ km^2$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$6 = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt}$

$6 = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{\pi r}$

$= \frac{3}{\pi(3)}$

$= \frac{1}{\pi}$ Km/h

when $A = 9\pi$

$9\pi = \pi r^2$

$r^2 = 9$

$r = 3$

Related Rates Part 1 Continued Exercise (Pg 570)

12. $\frac{dV}{dt} = 8\ cm^3/s$

Find $\frac{dr}{dt}$ when

a) $r = 12$ cm

b) $V = 1435\ cm^3$

c) $t = 33.5$ s

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$8 = \frac{d}{dr}\left(\frac{4\pi r^3}{3}\right) \cdot \frac{dr}{dt}$$

$$8 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{2}{\pi r^2}$$

a) when $r = 12$ cm

$$\frac{dr}{dt} = \frac{2}{\pi(12)^2} = \frac{1}{72\pi}\ cm/s$$

b) when $V = 1435\ cm^3$

Then $\frac{4\pi r^3}{3} = 1435$

$$r^3 = \frac{4305}{4\pi}$$

$$r \doteq 6.9971486...$$

$$\therefore \frac{dr}{dt} = \frac{2}{\pi(6.99...)^2} \doteq 0.01\ cm/s$$

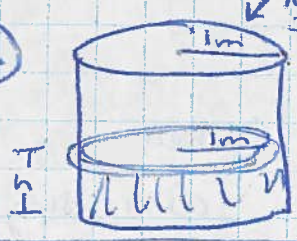
c) when $t = 33.5$ s, $A = 8(33.5)$

$= 268\ cm^3$

$$\therefore r = \sqrt[3]{\frac{3(268)}{4\pi}} \doteq 3.999589...$$

$$\therefore \frac{dr}{dt} = \frac{2}{\pi(3.99...)^2} \doteq 0.04\ cm/s$$

13.



$\frac{dV}{dt} = 500\ L/min$

$= 500000\ cm^3/min$

Find $\frac{dh}{dt}$ and h when full

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$500000 = \frac{d}{dh}(\pi(100)^2 h) \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{500000}{10000\pi}\ cm/min$$

$$\therefore \frac{dh}{dt} = \frac{50}{\pi}\ cm/min$$

when full $V = \pi(100)^2(1500)$

$= 15000000\pi\ cm^3$

$$\therefore t = \frac{15000000\pi}{500000} = 30\pi\ min$$

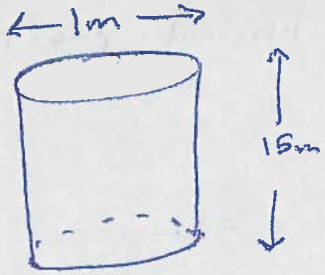
$\therefore$ Time to fill

$= 1500 \div \frac{50}{\pi}$

$\doteq 94.25$ min

$\doteq 1$ h 34 min 15 s

15.



$\frac{dr}{dt} = 0.003\ m/y$ Find $\frac{dV}{dt}$

$\frac{dh}{dt} = 0.4\ m/y$ at $r = 0.5m$, $h = 15m$

$V = (\pi r^2)h$

$\frac{dV}{dt} = \left(2\pi r \cdot \frac{dr}{dt}\right)(h) + (\pi r^2)\left(\frac{dh}{dt}\right)$

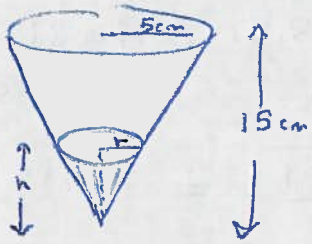
$= 2\pi(0.5)(0.003)(15) + \pi(0.5)^2(0.4)$

$= 0.045\pi + 0.1\pi$

$= 0.145\pi\ m^3/y$

$\doteq 0.456\ m^3/y$

16.



$\frac{dV}{dt} = -2\ cm^3/min$; Find $\frac{dh}{dt}$ @ $h = 3cm$

$\frac{r}{h} = \frac{5}{15}$

$r = \frac{1}{3}h$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$-2 = \frac{d}{dh}\left(\frac{\pi(\frac{1}{3}h)^2 h}{3}\right) \cdot \frac{dh}{dt}$

$-2 = \frac{d}{dh}\left(\frac{\pi h^3}{27}\right) \cdot \frac{dh}{dt}$

$-2 = \frac{\pi h^2}{9} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-18}{\pi h^2}$

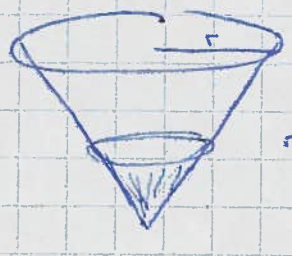
$= \frac{-18}{\pi(3)^2}$

$= -\frac{2}{\pi}\ cm/min$

∴ height decreasing @ $\frac{2}{\pi}$ cm/min.

Pg 3 Related Rates Exercises Continued Pg 570 Day 1

20.



$\frac{dV}{dt} = 200\,cm^3/s - 20\,cm^3/s$

$= 180\,cm^3/s$

a) Find $\frac{dh}{dt}$ @ $h = 15$ cm

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$180 = \frac{d}{dh}\left(\frac{\pi (h)^2 (h)}{3}\right) \cdot \frac{dh}{dt}$

$180 = \frac{d}{dh}\left(\frac{\pi h^3}{3}\right) \cdot \frac{dh}{dt}$

$180 = \pi h^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{180}{\pi h^2}$

$= \frac{180}{\pi (15)^2}$

$= \frac{4}{5\pi}$ cm/s

b) when $h = 25$ cm

$\frac{dV}{dt} = 200\,cm^3/s$

Find $\frac{dh}{dt}$ @ $h = 25$ cm

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$200 = \pi h^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{200}{\pi h^2}$

$= \frac{200}{\pi (25)^2}$

$= \frac{8}{25\pi}$ cm/s