2.3

Q9

$V(t) = 60\%$ of 75
$= 0.6(75)$

$\therefore\ 75\left(1-\frac{t}{24}\right)^2 = 0.6(75)$

$\left(1-\frac{t}{24}\right)^2 = 0.6$

$1-\frac{t}{24} \doteq 0.7746$

$t \doteq 5.4097\,h$

$V'(t) = 150\left(1-\frac{t}{24}\right)^1\left(-\frac{1}{24}\right)$

So, $V'(5.4097)$
$\doteq -4.841\ L/h$

Thus the volume of gas is decreasing by $4.841\,L/h$ when the tank is 60% full.

11b) $f(x) = (1+x)(1+2x)(1+3x)\cdots(1+nx)$

$f'(x) = (1)(1+2x)(1+3x)\cdots$
$+ (1+x)(2)(1+3x)\cdots$
$+ (1+x)(1+2x)(3)(1+4x)\cdots$
$\vdots$
$+ (1+x)(1+2x)\cdots(n)$

$f'(0) = (1)(1)(1)\cdots + (1)(2)(1)(1)\cdots + (1)(1)(3)(1)(1)\cdots$
$+ (1)(1)(1)\cdots(n)$

$f'(0) = 1+2+3+\cdots+n$

$= \frac{n(n+1)}{2}$

Q12 $f(2)=19,\ f(-1)=-8,\ f'(-1)=0$

$f(x)=ax^2+bx+c \longrightarrow f'(x)=2ax+b$

$f'(-1)=0 \longrightarrow 2a(-1)+b=0$
$b=2a$

$f(2)=19 \longrightarrow a(2)^2+(2a)(2)+c=19$
$8a+c=19 \quad (1)$

$f(-1)=-8 \longrightarrow a(-1)^2+(2a)(-1)+c=-8$
$-a+c=-8$
$a-c=8 \quad (2)$

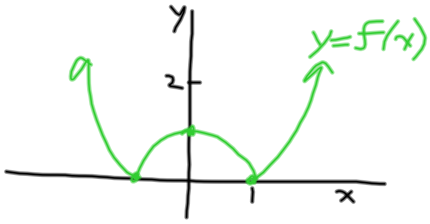
$$\begin{array}{r} 8a+c=19 \\ a-c=8 \\ \hline 9a=27 \\ a=3 \end{array}$$

$\therefore b=2(3)=6,\quad c=3-8=-5$

$\therefore \underline{f(x)=3x^2+6x-5}$

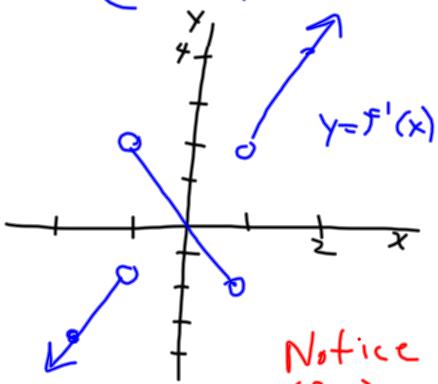
Q13 $f(x)=|x^2-1|$

$$=\begin{cases} x^2-1, & x\le -1 \\ -x^2+1, & -1<x<1 \\ x^2-1, & x\geq 1 \end{cases}$$



a) $x=\pm 1$

b) $f'(x)=\begin{cases} 2x, & x<-1 \\ -2x, & -1<x<1 \\ 2x, & x>1 \end{cases}$



(c) $f'(-2)=2(-2)$
$=-4$

$f'(0)=0$

$f'(3)=2(3)$
$=6$

Notice that $(-2,-4)$, $(0,0)$ and $(3,6)$ are all points on the graph of $y=f'(x)$.

Q14 $4x - y + 11 = 0$

$y = 4x + 11$

$\therefore m = 4$

So, $y' = 4$

Thus, $\frac{d}{dx}\left(\frac{16}{x^2} - 1\right) = 4$

$\frac{d}{dx}\left(16x^{-2} - 1\right) = 4$

$-32x^{-3} = 4$

$\frac{-32}{x^3} = \frac{4}{1}$

$x^3 = -8$

$x = -2$

When $x = 2$

- $y = \frac{16}{(-2)^2} - 1$
  $= 3$
- $y = 4(-2) + 11$
  $= 3$

So, $(-2, 3)$ is on $y = 4x + 11$ and $y = \frac{16}{x^2} - 1$

AND the slope of the tangent of $y = \frac{16}{x^2} - 1$ @ $(-2, 3)$ is 4.

Thus, $y = 4x + 11$ is tangent to $y = \frac{16}{x^2} - 1$ @ $(-2, 3)$