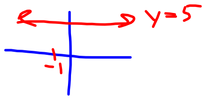
1.5: <u>Properties of Limits</u> Date: ______________



Properties of Limits

For any real number a where f and g both have limits that exist at $x = a$.

1. The limit of a constant **is equal to** the constant.

$$\lim_{x \to a} k = K$$

Example:

$$\lim_{x \to -1} 5 = 5$$

2. The limit of x as x approaches a **is equal to** a.

$$\lim_{x \to a} x = a$$

Example:

$$\lim_{x \to -1} x = -1$$

3. The limit of a sum (or difference) **is equal to** the sum (or difference) of the limits.

$$\lim_{x \to a} [f(x) + g(x)]$$

$$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

Example:

$$\lim_{x \to 2} [3x^2 - x]$$

$$= \lim_{x \to 2} (3x^2) - \lim_{x \to 2} (x)$$

$$= 3(2)^2 - 2$$

$$= 10$$

4. The limit of a constant times a function **is equal to** the constant times the limit of the function.

$$\lim_{x \to a} [c \cdot f(x)]$$

$$= c\left(\lim_{x \to a} f(x)\right)$$

Example:

$$\lim_{x \to 3} 10(x+2)$$

$$= 10\left(\lim_{x \to 3} (x+2)\right)$$

$$= 10(5)$$

$$= 50$$

5. The limit of a product (or quotient) **is equal to** the product (or quotient – provided $g(x) \neq 0$) of the limits.

$$\lim_{x \to a} [f(x) \cdot g(x)]$$

$$= \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$$

Example:

$$\lim_{x \to 0} \frac{x+1}{5x^2+2}$$

$$= \frac{\lim_{x \to 0} (x+1)}{\lim_{x \to 0} (5x^2+2)}$$

$$= \frac{1}{2}$$

6. The limit of a power (or root) **is equal to** the power (or root) of the limit – provided the exponent is rational (or the root exists).

$$\lim_{x \to a} [f(x)]^n$$

$$= \left[\lim_{x \to a} f(x)\right]^n$$

Example:

$$\lim_{x \to 2} \sqrt{2x+5}$$

$$= \sqrt{\lim_{x \to 2} (2x+5)}$$

$$= \sqrt{9}$$

$$= 3$$

→ When limits cannot be found by direct substitution (direct substitution gives an **INDETERMINATE FORM** $\frac{0}{0}$), we look for an equivalent function that agrees with f except at $x = a$.

→ These functions are said to have a **REMOVABLE DISCONTINUITY**.

Helpful Techniques to Eliminate a Removable Discontinuity:

- Simplifying ✓
- Factoring ✓
- Rationalizing ✓
- Change of Variable

Ex1. If possible, evaluate each limit.

a. $\lim_{x \to -4} \frac{2x^2 + 5x - 12}{x + 4} = \lim_{x \to -4} \frac{\frac{(2x+8)(2x-3)}{2}}{(x+4)}$

$= \lim_{x \to -4} \frac{\cancel{(x+4)}(2x-3)}{\cancel{(x+4)}}$

$= 2(-4) - 3$

$= -11$

b. $\lim_{x \to 0} \left(\frac{\sqrt{4+x} - 2}{x}\right) \cdot \left(\frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2}\right)$

$= \lim_{x \to 0} \frac{\cancel{4} + \cancel{x} - \cancel{4}}{\cancel{x}\left(\sqrt{4+x} + 2\right)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{4+x} + 2}$

$= \frac{1}{2+2}$

$= \frac{1}{4}$

c. $\lim_{x \to 8} \frac{2 - \sqrt[3]{x}}{8 - x}$

let $\sqrt[3]{x} = y$

$\therefore x = y^3$

As $x \to 8$

$y \to 2$

$= \lim_{y \to 2} \frac{2 - y}{8 - y^3}$

$= \lim_{y \to 2} \frac{y - 2}{y^3 - 8}$

$= \lim_{y \to 2} \frac{\cancel{(y-2)}}{\cancel{(y-2)}(y^2 + 2y + 4)}$

$= \frac{1}{4 + 4 + 4}$

$= \frac{1}{12}$

$\lim_{x \to 8} \frac{2 - \sqrt[3]{x}}{8 - x}$

$= \lim_{x \to 8} \frac{\sqrt[3]{x} - 2}{x - 8}$

$= \lim_{x \to 8} \frac{\cancel{(\sqrt[3]{x} - 2)}}{\cancel{(\sqrt[3]{x} - 2)}(\sqrt[3]{x^2} + 2\sqrt[3]{x} + 4)}$

$= \frac{1}{4 + 4 + 4}$

$= \frac{1}{12}$

d. $\lim_{x \to 1} \frac{1}{x-1}$ DNE

Ex2. Does $\lim_{x \to 3} \frac{x^2 - x - 6}{|x-3|}$ exist? Illustrate your answer by sketching a graph of the function.

let $f(x) = \frac{x^2 - x - 6}{|x-3|} = \begin{cases} -x-2, & x<3 \\ x+2, & x>3 \end{cases}$

if $x>3$

$\therefore f(x) = \frac{(x-3)(x+2)}{x-3}$

$= x+2$

if $x<3$

$\therefore f(x) = \frac{\cancel{(x-3)}(x+2)}{-\cancel{(x-3)}}$

$= -(x+2)$

$= -x-2$

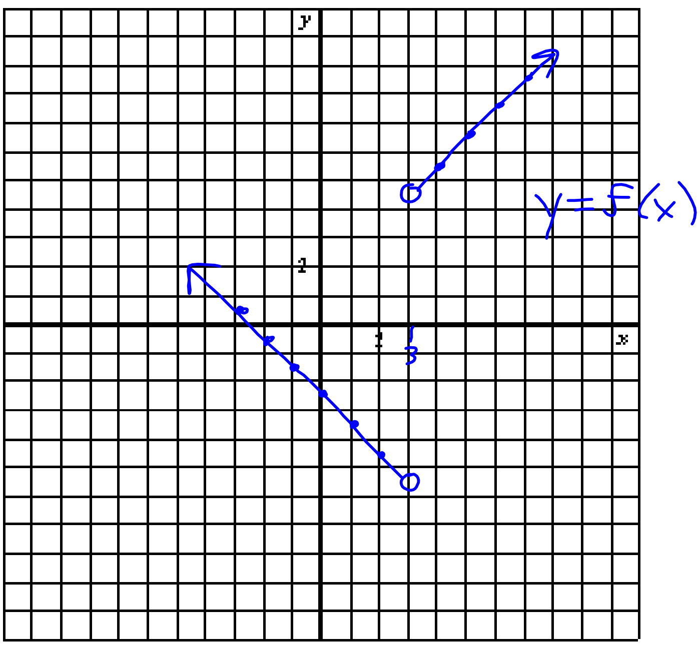
$\lim_{x \to 3^+} f(x) = 3+2$

$= 5$

$\lim_{x \to 3^-} f(x) = -3-2$

$= -5$

Thus, $\lim_{x \to 3} f(x)$ DNE $\therefore$



Homefun: Page 45 #1, 3, 4dcf, 6, 7, 8b→f, 9, 10acd, 17, 12, 13, [16 – Hint: Use BOTH conjugates]