28 $s(t) = 1 + 2t - 8(t^2+1)^{-1}$, $\underline{0 \le t \le 2}$

$v(t) = 2 + \dfrac{8(2t)}{(t^2+1)^2} = 2 + \dfrac{16t}{(t^2+1)^2}$

$a(t) = \dfrac{16(t^2+1)^2 - 16t\left[2(t^2+1)(2t)\right]}{*} = 0$

$\therefore 16(t^2+1)\left[t^2+1 - t(4t)\right] = 0$

$\therefore 16(t^2+1)(1-3t^2) = 0$

So, $3t^2 = 1$

$t = \dfrac{1}{\sqrt{3}}$

So, $v(0) = 2$ is ___ AND
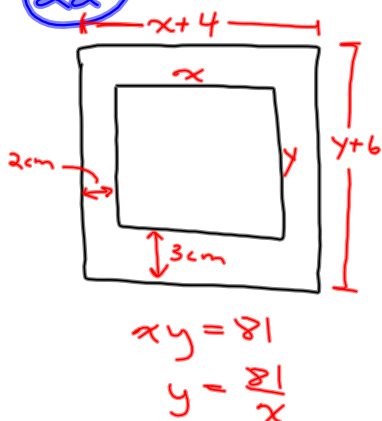
the __MIN__

velocity

(Note: $v(2) = 3.28$)

$v\left(\dfrac{1}{\sqrt{3}}\right) = 2 + \dfrac{\frac{16}{\sqrt{3}}}{\left(\frac{1}{3} + \frac{3}{3}\right)^2}$

$= 2 + \dfrac{16}{\sqrt{3}} \cdot \dfrac{9}{16}$

$= 2 + 3\sqrt{3}$ is

the __MAX__ velocity.

22

$x+4$

x

2cm

y $y+6$

3cm

$xy = 81$

$y = \dfrac{81}{x}$

$A = (x+4)(y+6)$

$A(x) = (x+4)\left(\dfrac{81}{x} + 6\right)$

$= 81 + 6x + 324x^{-1} + 24$

$A'(x) = 6 - \dfrac{324}{x^2} = 0$

$x^2 = 54$

$\therefore x \doteq 7.35$

$y \doteq 11.02$

So dimensions are 11.35 cm by 17.02 cm

⑲ Cost 60% more under water

← 8 km →
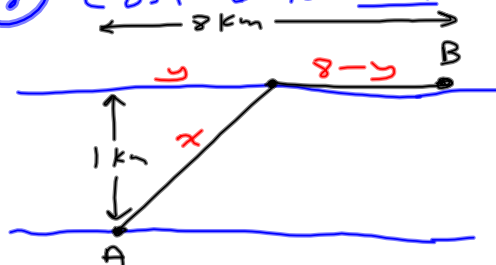
$$\text{Cost} = 1.6K[x] + K[8-y]$$

$$C(y) = 1.6K\sqrt{y^2+1} + 8K - Ky$$
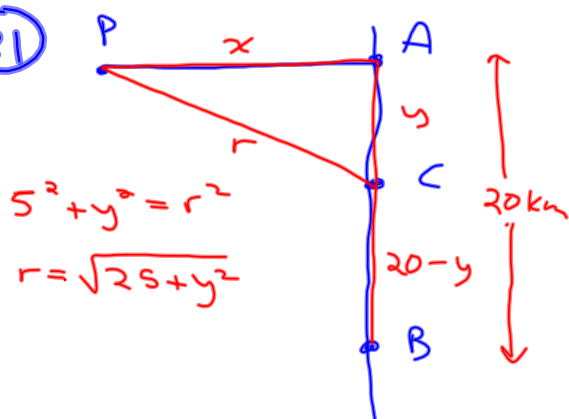
$$C'(y) = \frac{0.8K(2y)}{\sqrt{y^2+1}} - K = 0$$

$$\therefore \quad 1.6y = \sqrt{y^2+1}$$

$$2.56y^2 = y^2+1$$

$$y^2 = 0.64102\ldots$$

$$y \doteq 0.8006 \text{ km}$$

$$x^2 = y^2 + 1^2$$

$$x = \sqrt{y^2+1}$$

㉑

$$5^2 + y^2 = r^2$$

$$r = \sqrt{25+y^2}$$

20 km

$$\text{Cost} = 100000r + 75000(20-y)$$

$$C(y) = 100000\sqrt{25+y^2} + 1500000 - 75000y$$

$$C'(y) = \frac{500000(2y)}{\sqrt{25+y^2}} - 75000 = 0$$
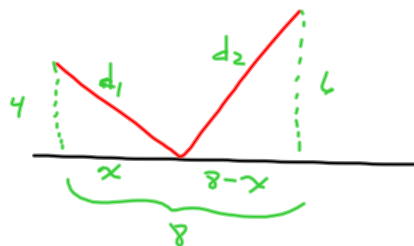
So, $\sqrt{25+y^2} = \dfrac{4y}{3}$

$$25 + y^2 = \frac{16y^2}{9}$$

$$225 + 9y^2 = 16y^2$$

$$y^2 = 32.1428\ldots$$

$$y \doteq 5.67 \text{ km}$$

2

(25)

$D = d_1 + d_2$

$D(x) = \sqrt{x^2 + 16} + \sqrt{(8-x)^2 + 6^2}$

$ = \sqrt{x^2 + 16} + \sqrt{x^2 - 16x + 100}$

$D'(x) = \dfrac{2x}{2\sqrt{}} + \dfrac{2x - 16}{2\sqrt{}} = 0$

$\therefore \quad x\sqrt{x^2 - 16x + 100} = -(x-8)\sqrt{x^2 + 16}$

$x^2(x^2 - 16x + 100) = (x^2 - 16x + 64)(x^2 + 16)$

$x^4 - 16x^3 + 100x^2$
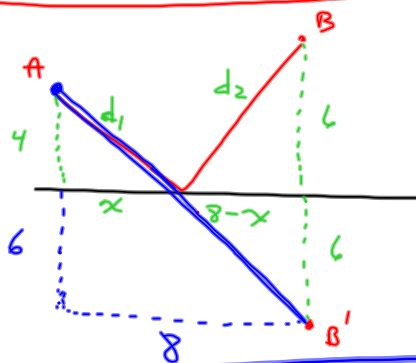$\qquad = x^4 - 16x^3 + 64x^2 + 16x^2$
$\qquad \qquad - 256x + 1024$

$20x^2 + 256x - 1024 = 0$

$5x^2 + 64x - 256 = 0$

$(x + 16)(5x - 16) = 0$

$\therefore x = -\cancel{16} \text{ OR } \dfrac{16}{5}$

Fancy Non-Calculus Sol'n

Min. distance from A to B equals straight line distance from A to B'.

30b) DANGER!

To find average velocity during the first 5 seconds, you need AROC of $s(t)$, not $v(t)$.

That is, $AROC = \dfrac{s(5) - s(0)}{5 - 0}$