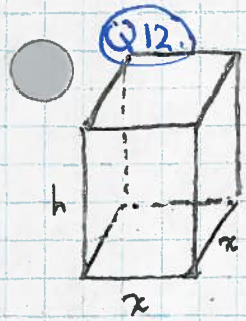
(3.4) Day 2, Hometown

Q12.



$x^2h = 4000\ cm^3$

$h = \frac{4000}{x^2}$

Note: $\$20/m^2 = \$0.02/cm^2$, $\$30/m^2 = \$0.03/cm^2$

$\text{Cost} = 0.02[2x^2] + 0.03[4xh]$

$C(x) = 0.04x^2 + 0.12x\left(\frac{4000}{x^2}\right)$

$= 0.04x^2 + 480x^{-1}$

Thus, $C'(x) = 0.08x - \frac{480}{x^2}$

$C'(x) = 0 \longrightarrow 0.08x = \frac{480}{x^2}$

$x^3 = 6000$

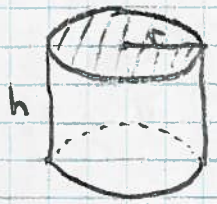
$x \doteq 18.17$

$h = \frac{4000}{x^2}$

$h \doteq 12.11$ cm

∴ the dimensions to minimize cost are 18.2 cm × 18.2 cm × 12.1 cm.

Q10.



$\text{Cost} = 2K(2\pi r^2) + K(2\pi rh)$

$\therefore C(r) = 4K\pi r^2 + 2K\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 4K\pi r^2 + 2000Kr^{-1}$

$\therefore C'(r) = 8K\pi r - \frac{2000K}{r^2}$

$C'(r) = 0$ when

$8K\pi r = \frac{2000K}{r^2}$

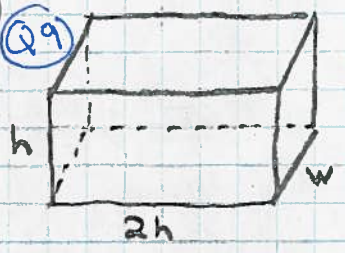
$r^3 = \frac{2000}{8\pi}$

So, $r \doteq 4.3$ cm, $h \doteq 17.2$ cm

$\pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

Q9



$(2h)(h)(w) = 20000$

$2h^2w = 20000$

$w = \frac{10000}{h^2}$

$\text{Cost} = 40[2hw] + 100[2hw + 2(2h^2)] + 200(2hw)$

$= 240(2hw) + 100(2hw + 4h^2)$

$\therefore C(h) = 480h\left[\frac{10000}{h^2}\right] + 200h\left[\frac{10000}{h^2}\right] + 400h^2$

$= 4800000h^{-1} + 2000000h^{-1} + 400h^2$

$= 6800000h^{-1} + 400h^2$

$\therefore C'(h) = -\frac{6800000}{h^2} + 800h$

So, $C'(h) = 0$ when $\frac{6800000}{h^2} = 800h$

$h^3 = \frac{6800000}{800}$

$h^3 = 8500$

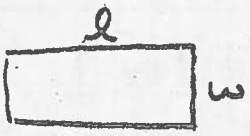
$h \doteq 20.4$ m

Thus,

$h \doteq 20.4$ m, $l = 40.8$ m,

$w \doteq 24.0$ m

(Q5)



$6(2\ell) + 9(2w) = 9000$

$12\ell + 18w = 9000$

$\ell = \frac{900 - 18w}{12}$

$\ell = 750 - \frac{3}{2}w$

$A = \ell w$

$\therefore A(w) = (750 - \frac{3}{2}w)w$

$= 750w - \frac{3}{2}w^2$

$A'(w) = 750 - 3w$

So, $A'(w) = 0$ when $w = 250$ m

Thus, $\ell = 750 - \frac{3}{2}(250)$, $w = 250$ m

$= 375$ m

(Q17)

$\pi r^2 h + \frac{4\pi r^3}{3} = 200$

$\therefore \pi r^2 h = 200 - \frac{4\pi r^3}{3}$

$h = \frac{200}{\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$

$h = \frac{200}{\pi r^2} - \frac{4}{3}r$

$\text{Cost} = 2K\left[4\pi r^2\right] + K\left[2\pi r\left(\frac{200}{\pi r^2} - \frac{4}{3}r\right)\right]$

$= K\left[8\pi r^2 + \frac{400}{r} - \frac{8\pi}{3}r^2\right]$

$= K\left[\frac{24\pi - 8\pi}{3}r^2 + 400r^{-1}\right]$

$C(r) = K\left[\frac{16\pi}{3}r^2 + 400r^{-1}\right]$

Thus, $C'(r) = K\left[\frac{32\pi}{3}r - \frac{400}{r^2}\right]$

So, $C'(r) = 0$ $\quad \frac{32\pi r}{3} = \frac{400}{r^2}$

$r^3 = \frac{1200}{32\pi}$

$r \doteq 2.285$m and $h \doteq 914$ cm

$\doteq 229$m

Q8 Fuel Cost Per Hour $= \frac{1}{2}v^3 + 216$

$\therefore$ Total Fuel Cost $= \left[\frac{1}{2}v^3 + 216\right] \cdot t$

$\therefore F(v) = \left[\frac{1}{2}v^3 + 216\right]\left[\frac{500}{v}\right]$

$= 250v^2 + 108000v^{-1}$

So, $F'(v) = 500v - 108000v^{-2}$

$\therefore F'(v) = 0 \longrightarrow 500v = \frac{108000}{v^2}$

$v^3 = 216$

$v = 6$ Knots

Note:

$d = 500m$

$\therefore t = \frac{500}{v}$

Q18

Speed (Km/h)	Mileage (Km/L)
110	8
110+x	8 − 0.1x

Gas Cost = \$1.15/L

Pay = \$35/h

Fixed costs = \$15.50/h

Distance = 450 Km

$\therefore t = \frac{450}{\text{speed}} = \frac{450}{110+x}$

So, Operating Expenses $= 1.15(\#\text{ of litres}) + 35t + 15.50t$

$= 1.15\left(\frac{\text{Distance}}{\text{Mileage}}\right) + 50.5t$

$\therefore E(x) = 1.15\left(\frac{450}{8-0.1x}\right) + 50.5\left(\frac{450}{110+x}\right)$

$= 517.5(8-0.1x)^{-1} + 22725(110+x)^{-1}$

Thus, $E'(x) = \frac{-517.5(-0.1)}{(8-0.1x)^2} - \frac{22725}{(110+x)^2} = 0$

$\therefore \frac{51.75}{(8-0.1x)^2} = \frac{22725}{(110+x)^2}$

$\therefore \left(\frac{110+x}{8-0.1x}\right)^2 = \frac{10100}{23}$

$\therefore \frac{110+x}{8-0.1x} = \pm\sqrt{\frac{10100}{23}}$

$\therefore 110 + x = \pm\sqrt{\frac{10100}{23}}(8-0.1x)$

$\therefore x \pm 0.1\sqrt{\frac{10100}{23}}x = \pm 8\sqrt{\frac{10100}{23}} - 110$

So,

$x = \frac{8\sqrt{\frac{10100}{23}} - 110}{1 + 0.1\sqrt{\frac{10100}{23}}}$ or $x = \frac{-8\sqrt{\frac{10100}{23}} - 110}{1 - 0.1\sqrt{\frac{10100}{23}}}$

$\doteq 18.62$ $\doteq -253.4$

$\therefore$ speed $= 110 + x$

$\doteq 128.6$ or ~~363~~ ← silly!

Ergo, speed required $= 128.6$ Km/h

(Q19) $V(r) = Ar^2(r_0 - r)$

$= Ar_0r^2 - Ar^3$, $0 \leq r \leq r_0$

$\therefore V'(r) = 2Ar_0r - 3Ar^2$

$V'(r) = 0 \longrightarrow 2Ar_0r = 3Ar^2$

$$\frac{2Ar_0}{3A} = \frac{r^2}{r}$$

$$r = \frac{2}{3}r_0$$

So, $V\left(\frac{2}{3}r_0\right) = A\left(\frac{2}{3}r_0\right)^2\left(\frac{3}{3}r_0 - \frac{2}{3}r_0\right)$

$$= \left(\frac{4A}{9}r_0^2\right)\left(\frac{1}{3}r_0\right)$$

$$= \frac{4A}{27}r_0^3$$