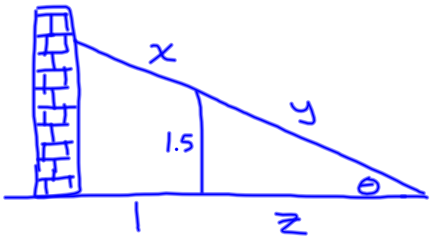
Ch 5 Review Exercises

Q20



$\sin\theta = \frac{1.5}{y} \rightarrow y = \frac{1.5}{\sin\theta}$

$y = 1.5(\sin\theta)^{-1}$

$\frac{x+y}{y} = \frac{1+z}{z}$

$xz + yz = y + yz$

$xz = y$

$z = \frac{y}{x}$

$\cos\theta = \frac{1+z}{x+y}$

$= \frac{1+\frac{y}{x}}{x+y}$

$= \frac{x+y}{x} \cdot \frac{1}{x+y}$

$= \frac{1}{x}$

Thus $\cos\theta = \frac{1}{x} \rightarrow x = \frac{1}{\cos\theta}$

$= (\cos\theta)^{-1}$

$l = x + y$

$= (\cos\theta)^{-1} + 1.5(\sin\theta)^{-1}$

$\therefore \frac{dl}{d\theta} = \frac{-1}{\cos^2\theta} \cdot (-\sin\theta)$

$+ \frac{-1.5}{\sin^2\theta}(\cos\theta)$

So, $\frac{dl}{d\theta} = \frac{\sin\theta}{\cos^2\theta} - \frac{1.5\cos\theta}{\sin^2\theta}$

$= \frac{\sin^3\theta - 1.5\cos^3\theta}{\cos^2\theta\sin^2\theta}$

Thus, $\frac{dl}{d\theta} = 0$ when

$\sin^3\theta - 1.5\cos^3\theta = 0$

$\frac{\sin^3\theta}{\cos^3\theta} = 1.5$

$\tan^3\theta = 1.5$

$\tan\theta = 1.144714\ldots$

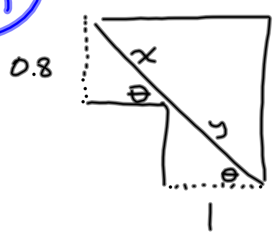
$\theta \doteq 48.9°$

Thus, the min. length of ladder required is

$l = \frac{1.5}{\sin\theta} + \frac{1}{\cos\theta}$

$\doteq 3.5\text{m}$

(21)



$\sin\theta = \dfrac{0.8}{x}$ $\qquad \cos\theta = \dfrac{1}{y}$

$x = \dfrac{0.8}{\sin\theta}$ $\qquad y = \dfrac{1}{\cos\theta}$

$$\ell = x + y$$
$$= 0.8(\sin\theta)^{-1} + (\cos\theta)^{-1}$$

$$\frac{d\ell}{d\theta} = \frac{-0.8\cos\theta}{\sin^2\theta} - \frac{-\sin\theta}{\cos^2\theta}$$

$$= \frac{-0.8\cos^3\theta + \sin^3\theta}{\sin^2\theta\cos^2\theta}$$

So, $\dfrac{d\ell}{d\theta} = 0$ when

$$\sin^3\theta = 0.8\cos^3\theta$$
$$\tan^3\theta = 0.8$$
$$\tan\theta = 0.9283\ldots$$
$$\theta \doteq 0.7482 \text{ radians}$$

Thus $\ell = \dfrac{0.8}{\sin\theta} + \dfrac{1}{\cos\theta}$

$$\doteq 2.54$$