Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$\div \cos^2\theta$ $\div \sin^2\theta$

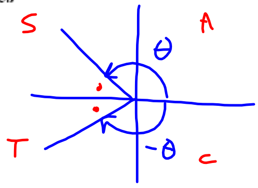
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

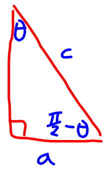
Reflection Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$



Cofunction Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$



Angle Sum Identities

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

$\cos^2 a = 1 - \sin^2 a$

Double Angle Identities

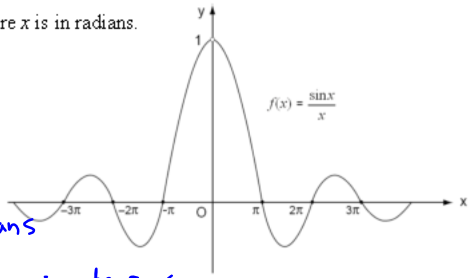
$$\sin(2a) = 2\sin a \cos a$$
$$\cos(2a) = \cos^2 a - \sin^2 a$$
$$\cos(2a) = 2\cos^2 a - 1$$
$$\cos(2a) = 1 - 2\sin^2 a$$

$\sin^2 a = 1 - \cos^2\theta$

$$\tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$$

Ex1. The graph of $f(x) = \frac{\sin x}{x}$ is given to the right where x is in radians.

Use the graph to determine $\lim_{h \to 0} \frac{\sin h}{h} = 1$

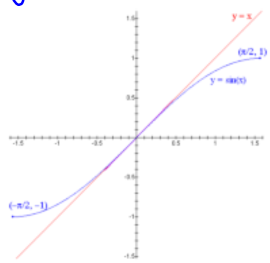


Ex2. Use your calculator to determine

$\lim_{h \to 0} \frac{\sin h}{h}$ in degrees and radians.

$\lim_{h \to 0} \frac{\sin h}{h} = 1$ in radians

$\lim_{h \to 0} \frac{\sin h}{h} = 0.0174\ldots$ in degrees



Ex3. Determine $\lim_{h \to 0} \frac{\cos h - 1}{h}$.

$$\lim_{h \to 0} \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}$$

$$= \lim_{h \to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)}$$

$$= \lim_{h \to 0} \frac{1 - \sin^2 h - 1}{h(\cos h + 1)}$$

$$= \lim_{h \to 0} \left[\left(\frac{\sin h}{h}\right) \cdot \left(\frac{-\sin h}{\cos h + 1}\right)\right]$$

$$= (1) \cdot \frac{-0}{1+1}$$

$$= (1)(0)$$

$$= 0$$

$$\boxed{\lim_{h \to 0} \frac{\sin h}{h} = 1 \qquad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0}$$

Ex4. Find the derivative $f(x) = \sin x$ from first principles.

IF $f(x) = \sin x$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\sin x \cdot \frac{(\cos h - 1)}{h} + \cos x \cdot \frac{\sin h}{h} \right]$$

$$= (\sin x)(0) + \cos x(1)$$

$$= \cos x$$

$$\boxed{\begin{array}{l} \text{IF } y = \sin x \\ \therefore y' = \cos x \end{array}}$$

Ex5. Use $\cos x = \sin\left(\frac{\pi}{2} - x\right)$ and $\sin x = \cos\left(\frac{\pi}{2} - x\right)$ to find the derivative of $g(x) = \cos x$.

IF $f(x) = \cos x$

$$= \sin\left(\frac{\pi}{2} - x\right)$$

$$\therefore f'(x) = \cos\left(\frac{\pi}{2} - x\right) \cdot (-1)$$

$$= \sin x \cdot (-1)$$

$$= -\sin x$$

$$\boxed{\begin{array}{l} \text{So... IF } y = \cos x \\ \therefore y' = -\sin x \end{array}}$$

Ex6. Determine the derivative of each function.

a. $y = \sin 4x$

b. $y = x\cos x$

c. $y = \cos x^2$

d. $y = \sin^2 x$

e. $y = \sin(2 + x^3)$

f. $y = e^{\sin x + \cos x}$

a) $y = \sin 4x$

$y' = \cos(4x) \cdot 4$

$= 4\cos 4x$

b) $y = x\cos x$

$y' = (1)(\cos x) + (x)(-\sin x)$

$= \cos x - x\sin x$

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c) $y = \cos x^2$

$y = \cos(x^2)$

$y' = -\sin(x^2) \cdot 2x$

$= -2x\sin(x^2)$

d) $y = \sin^2 x$

$y = (\sin x)^2$

$y' = 2(\sin x)^1 \cdot \cos x$

$y' = 2\sin x\cos x$ ✓

$y = \sin(2x)$

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e) $y = \sin(2+x^3)$

$y' = \cos(2+x^3) \cdot (3x^2)$

$= 3x^2 \cdot \cos(2+x^3)$

f) $y = e^{\sin x + \cos x}$

$y' = e^{\sin x + \cos x} \cdot (\cos x - \sin x)$

Ex7. Determine the equation of the tangent to the graph of $y = x\sin 2x$ at $x = \frac{3\pi}{4}$.

$y' = (1)(\sin 2x) + (x)(\cos 2x \cdot 2)$

$= \sin 2x + 2x\cos 2x$

So when $x = \frac{3\pi}{4}$

$y = \frac{3\pi}{4} \cdot \sin\frac{3\pi}{2}$

$= \frac{3\pi}{4} \cdot (-1)$

$= -\frac{3\pi}{4}$

$y' = \sin\frac{3\pi}{2} + \frac{3\pi}{2}\cos\frac{3\pi}{2}$

$= -1 + \frac{3\pi}{2}(0)$

$= -1$

$P_0 = \left(\frac{3\pi}{4}, -\frac{3\pi}{4}\right)$

$$\frac{x - \frac{3\pi}{4}}{1} = \frac{y + \frac{3\pi}{4}}{-1}$$

$m = -1$

$\therefore -x + \frac{3\pi}{4} = y + \frac{3\pi}{4}$

$\therefore y = -x$ ITRE.

Ex8. Determine the maximum and minimum values of the function $y = \sin^2 2x$ on the interval $x \in [0, \pi]$.

$y = \sin^2(2x)$

$y = (\sin(2x))^2$

$y' = 2(\sin(2x)) \cdot \cos(2x) \cdot 2$

$y' = 4\sin(2x)\cos(2x)$

$y' = 2[2\sin A \cos A]$

$= 2[\sin 2A]$

$y' = 2\sin(4x)$

So.... $y' = 0$ when

$\sin(4x) = 0$

$4x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	1	0	1	0

max = 1

min = 0

Note:

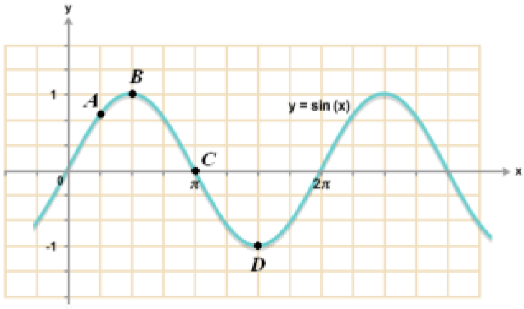
This is stoooooooooopid!!! Calculus is sooooo NOT needed to find the max and min of sinusoidal functions. Right? As always, make good choices!!

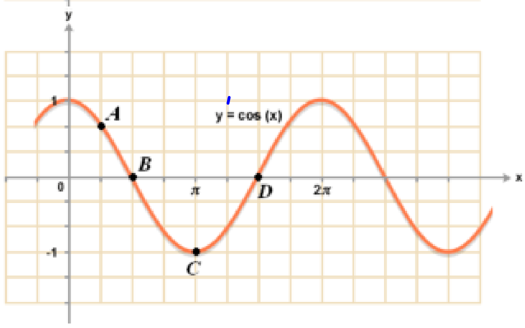
Ex9. Compare the graphs of

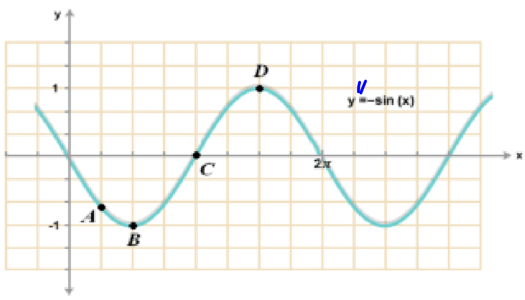
$y = \sin x$ and $y = \frac{d}{dx}\sin x$

and of

$y = \cos x$ and $y = \frac{d}{dx}\cos x$.







@ A

y is inc

$y' > 0$

y is concave down

y' is decreasing

$y'' < 0$

@ B

y is constant

$y' = 0$

y is concave down

y' is dec.

$y'' < 0$

@ C

y is dec

$y' < 0$

y is linear

y' is constant

$y'' = 0$

@ D

y is constant

$y' = 0$

y is concave up

y' is inc

$y'' > 0$

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