5.5: The Derivative of the Tangent Function Date: ________________

Ex1. Find $f'(x)$ if $f(x) = \tan x$.

$f(x) = \tan x$

$= \dfrac{\sin x}{\cos x}$

$f'(x) = \dfrac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \sec^2 x$

So....

IF $f(x) = \tan x$

$\therefore f'(x) = \sec^2 x$

Ex2. Find $\frac{dy}{dx}$.

a. $y = \tan(x^2 + x)$

$y' = \sec^2(x^2+x) \cdot (2x+1)$

b. $y = (\sin x + \tan x)^4$

$y' = 4(\sin x + \tan x)^3 \cdot (\cos x + \sec^2 x)$

c. $y = x\tan(2x - 1)$

$y' = (1)\tan(2x-1) + (x)[\sec^2(2x-1) \cdot 2]$

$= \tan(2x-1) + 2x\sec^2(2x-1)$

d. $y = \tan^2(2x)$

$y = [\tan(2x)]^2$

$y' = 2[\tan(2x)]^1 \cdot (\sec^2(2x) \cdot 2)$

$= 4\tan(2x)\sec^2(2x)$

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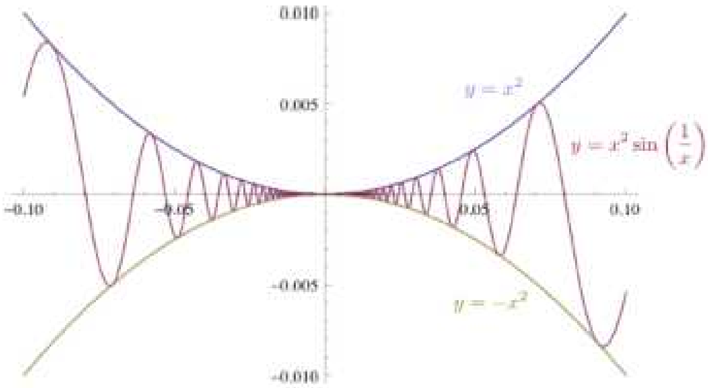
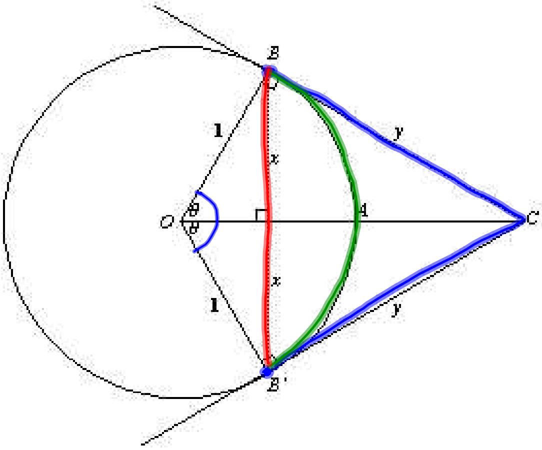
Page 260 # 1→4, 5a, 6→9 (9d should only be graphed for $x \in [-\pi, \pi]$.)

And finally, we can tie up our last loose end...

But first.... Here's a nice example of how the Squeeze (Sandwich) Theorem can be used.



Ex3. Use the unit circle with tangents at B and B' which meet at C such that $\angle BOC = \angle B'OC = \theta$ to prove that $\lim_{h \to 0} \frac{\sin h}{h} = 1$. Hint: Just make a sandwich.



$$2x \le \text{arc } BAB' \le 2y$$

$$2x \le 2\theta \le 2y$$

$$x \le \theta \le y$$

$$\sin\theta \le \theta \le \tan\theta$$

$$\left(\frac{1}{\sin\theta} \ge \frac{1}{\theta} \ge \frac{\cos\theta}{\sin\theta}\right) \cdot \sin\theta$$

$$1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta$$

$$\theta = \frac{a}{r}$$

$$2\theta = \frac{\text{arc } BAB'}{1}$$

$$\sin\theta = \frac{x}{1}$$

$$\tan\theta = \frac{y}{1}$$

As $\theta \to 0$, $\cos\theta \to 1$

Thus, by sandwich logic,

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \quad \blacksquare$$

Can you hear her singing?