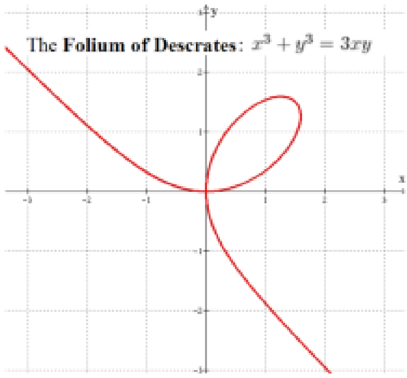
Implicit Differentiation (Calculus Appendix – Page 561) Date: ____________

PROCEDURE FOR IMPLICIT DIFFERENTIATION

If an equation defines y implicitly as a differentiable function of x, then you can determine y' as follows:

1. Differentiate both sides of the equation with respect to x.
   (Remember to use the chain rule when differentiating terms containing y.)
2. Solve for y'.



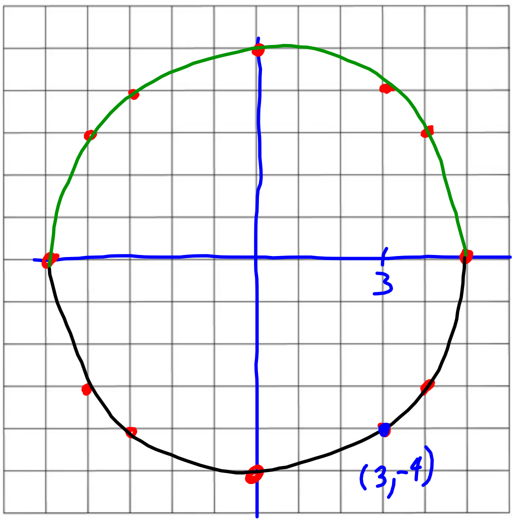
Ex1. Implicit differentiation is really an application of the chain rule. Use the chain rule to differentiate the function in (a) and implicit differentiation to differentiate the function in (b). (Notice these functions are the same function.) Do not solve for y'.

a. $f(x)=[f(x)]^3$ $\quad f'(x)=3[f(x)]^2\cdot f'(x)$ $\qquad$ b. $y=y^3$ $\quad y'=3y^2\cdot y'$

Ex2. Graph the relation $x^2+y^2=25$. (Note: In this equation, y is defined ***implicitly*** as a function of x).

a. Define y ***explicitly*** as a function of x.

Then find $\dfrac{dy}{dx}$ and the slope of the tangent at $(3,-4)$.



$x^2+y^2=25$

$y^2=25-x^2$

$y=\pm\sqrt{25-x^2}$

$$y=\begin{cases}-\sqrt{25-x^2}, & y<0\\ \sqrt{25-x^2}, & y>0\end{cases}$$

Well @ $(3,-4)$

$y=-(25-x^2)^{\frac{1}{2}}$

$y'=-\frac{1}{2}(25-x^2)^{-\frac{1}{2}}\cdot(-2x)$

$$y'=\frac{x}{\sqrt{25-x^2}}$$

Thus,

$$y'=\frac{3}{\sqrt{25-9}}$$

$$=\frac{3}{4}$$

b. Use implicit differentiation to find $\frac{dy}{dx}$ and the slope of the tangent at $(3, -4)$.

$$\frac{d}{dx}\left(x^2+y^2=25\right)$$

$$2x + 2y\cdot y' = 0$$

$$2y\cdot y' = -2x$$

$$y' = \frac{-2x}{2y}$$

$$y' = -\frac{x}{y}$$

So @ $(3,-4)$

$$\therefore\ y' = -\frac{3}{-4}$$

$$= \frac{3}{4}$$

> In Example 2, the derivative could be determined by either:
> - solving for y in terms of x (and using one of the methods introduced earlier in the unit) or by
> - using implicit differentiation.
>
> There are many situations in which solving for y in terms of x is very difficult and, in some cases, impossible. In such cases, implicit differentiation is the only algebraic method available to us.

Ex3. Determine $\frac{dy}{dx}$ for each of the following.

a. $\frac{d}{dx}\left(2xy^4 - y^3 = 4\right)$

$$2y^4 + 2x\left(4y^3\cdot y'\right) - 3y^2\cdot y' = 0$$

$$y'\cdot\left(8xy^3 - 3y^2\right) = -2y^4$$

$$y' = \frac{-2y^4}{8xy^3-3y^2}$$

$$y' = \frac{2y^4}{3y^2-8xy^3}$$

b. $\left(\frac{x^2}{9} + \frac{2y^2}{7} = 1\right) \times 63$

$$\frac{d}{dx}\left(7x^2 + 18y^2 = 63\right)$$

$$14x + 36y\cdot y' = 0$$

$$36y\cdot y' = -14x$$

$$y' = \frac{-14x}{36y} = \frac{-7x}{18y}$$

c. $\frac{d}{dx}\left(x^3 + x^2y + (x+y)^2 + y^3 = 100\right)$

$$3x^2 + 2xy + x^2\cdot y' + 2(x+y)^1\cdot(1+y') + 3y^2\cdot y' = 0$$

$$(2x+2y)(1+y')$$

$$2x + 2x\cdot y' + 2y + 2y\cdot y'$$

$$y'\cdot\left(x^2 + 2x + 2y + 3y^2\right) = -3x^2 - 2xy - 2x - 2y$$

$$y' = \frac{-\left(3x^2+2xy+2x+2y\right)}{x^2+2x+2y+3y^2}$$

$$\frac{d}{dx}\left(\frac{x+y}{2x-y}=1\right)$$

$$\frac{(1+y')(2x-y)-(x+y)(2-y')}{(2x-y)^2}=0$$

$$2x-y+\underline{2x\cdot y'}\boxed{-y\cdot y'}-2x+\underline{x\cdot y'}-2y\boxed{+y\cdot y'}=0$$

$$-3y+3x\cdot y'=0$$

$$3x\cdot y'=3y$$

$$y'=\frac{y}{x}$$

Wait a sec!!

IF $\frac{x+y}{2x-y}=1$

$\therefore\ x+y=2x-y$ $\quad y'=\frac{1}{2}$

$2y=x$

$y=\frac{1}{2}x$

Hey!! $y'=\frac{y}{x}$

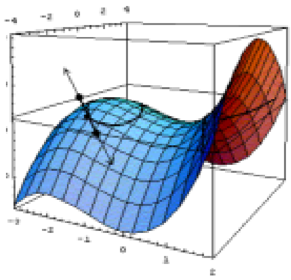
$=\frac{\frac{1}{2}x}{x}$

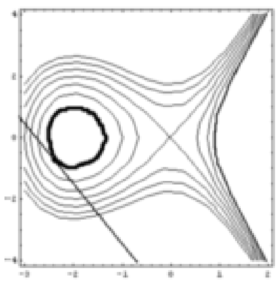
$=\frac{1}{2}$

Make Good Choices!!!!!....,

Homefun:

Page 564 # 2[the answer not simplified: $\frac{y}{2x+y}$],
3[as an exercise, try to match your equation with the form in the back],
4→9, 10(tricky; don't skip this one),
11d(also show algebraically there are 2 tangents)





Question 5b gets a bit nutty. Have fun?