Review

Q28f) $f(x) = \dfrac{-3x^4}{\sqrt{4x-8}} = -3x^4(4x-8)^{-\frac{1}{2}}$

$$f'(x) = -12x^3(4x-8)^{-\frac{1}{2}} - 3x^4\left[-\tfrac{1}{2}(4x-8)^{-\frac{3}{2}}(4)\right]$$

$$= -12x^3(4x-8)^{-\frac{1}{2}} + 6x^4(4x-8)^{-\frac{3}{2}}$$

$$= -6x^3(4x-8)^{-\frac{3}{2}}\left[2(4x-8)^{\frac{2}{2}} - x\right]$$

$$= \frac{-6x^3(7x-16)}{\sqrt{(4x-8)^3}}$$

OR

$$= -\frac{6x^3(7x-16)}{\sqrt{4^3}\sqrt{(x-2)^3}}$$

$$= \frac{-3x^3(7x-16)}{4\sqrt{(x-2)^3}}$$

Q15 $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$

a) IF $y=0 \rightarrow 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}} = 0$

Method #1: Factor

$$2x^{\frac{5}{3}} - 5x^{\frac{2}{3}} = 0$$

$$x^{\frac{2}{3}}\left(2x^{\frac{3}{3}} - 5\right) = 0$$

$$\sqrt[3]{x^2}\left(2x - 5\right) = 0$$

$$\therefore x = 0 \text{ or } \frac{5}{2}$$

Method #2: Isolate "x"

$$2x^{\frac{5}{3}} - 5x^{\frac{2}{3}} = 0$$

$$2x^{\frac{5}{3}} = 5x^{\frac{2}{3}}$$

$$\frac{x^{\frac{5}{3}}}{x^{\frac{2}{3}}} = \frac{5}{2} \quad \text{OR} \quad x = 0$$

$$x = \frac{5}{2} \quad \text{OR} \quad 0$$

Now we just need $f'(0)$ and $f'(\frac{5}{2})$.

$$f'(x) = \frac{10}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}}$$

$$= \frac{10}{3}x^{-\frac{1}{3}}(x-1)$$

$$= \frac{10(x-1)}{3\sqrt[3]{x}}$$

$$= \frac{10x^{\frac{2}{3}}}{3} \cdot \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{10}{3x^{\frac{1}{3}}}$$

$$= \frac{10x - 10}{3\sqrt[3]{x}}$$

$$= \frac{10(x-1)}{3\sqrt[3]{x}}$$

So, $f'(0)$ DNE $\rightarrow$ $\therefore$ there is no slope at $(0,0)$

$$f'\left(\frac{5}{2}\right) = \frac{10\left(\frac{5}{2}-1\right)}{3\sqrt[3]{\frac{5}{2}}}$$

$$= \frac{25-10}{3} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{5}}$$

$$= \frac{5\sqrt[3]{2}}{\sqrt[3]{5}} \times \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$$

$$\rightarrow = \frac{5\sqrt[3]{2\times 25}}{5}$$

$$= \sqrt[3]{50}$$

(b) Since $f'(x) = 0$

$$\therefore \frac{10(x-1)}{3\sqrt[3]{x}} = 0$$

$$\therefore x = 1$$

$$\rightarrow f(1) = 2-5$$

$$= -3$$

$$\therefore (a, f(a)) = (1, -3)$$