

5.1 Q9 Prove  $y'' = \frac{y}{25}$  if  $y = \frac{5}{2} (e^{\frac{x}{5}} + e^{-\frac{x}{5}})$

Well,  $y' = \frac{5}{2} \left[ e^{\frac{x}{5}} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \left(-\frac{1}{5}\right) \right]$

$$\begin{aligned} \therefore y'' &= \frac{5}{2} \left[ e^{\frac{x}{5}} \cdot \frac{1}{5} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \left(-\frac{1}{5}\right) \cdot \left(-\frac{1}{5}\right) \right] \\ &= \frac{\frac{5}{2} (e^{\frac{x}{5}} + e^{-\frac{x}{5}})}{25} \\ &= \frac{y}{25} \end{aligned}$$

Less exciting, but effective...

Prove:  $y'' = \frac{y}{25}$

$$\begin{aligned} LS &= y'' \\ &= \frac{5}{2} \left[ e^{\frac{x}{5}} \cdot \frac{1}{5} \cdot \frac{1}{5} + e^{-\frac{x}{5}} \cdot \frac{1}{5} \cdot \frac{1}{5} \right] \\ &= \frac{1}{10} e^{\frac{x}{5}} + \frac{1}{10} e^{-\frac{x}{5}} \end{aligned}$$

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$$\begin{aligned} RS &= \frac{y}{25} \\ &= \frac{\frac{5}{2} \left[ e^{\frac{x}{5}} + e^{-\frac{x}{5}} \right]}{25} \\ &= \frac{5}{2} \cdot \frac{1}{25} \cdot \left[ e^{\frac{x}{5}} + e^{-\frac{x}{5}} \right] \\ &= \frac{1}{10} e^{\frac{x}{5}} + \frac{1}{10} e^{-\frac{x}{5}} \\ &= LS \checkmark \end{aligned}$$

$$\textcircled{Q13} \quad s = 160 \left( \frac{1}{4}t - 1 + e^{-\frac{t}{4}} \right)$$

$$\begin{aligned} \text{a)} \quad v &= 160 \left( \frac{1}{4} + e^{-\frac{t}{4}} \cdot \left(-\frac{1}{4}\right) \right) \\ &= 40 \left( 1 - e^{-\frac{t}{4}} \right) \end{aligned}$$

$$\begin{aligned} \text{b)} \quad v &= 40 - 40 e^{-\frac{t}{4}} \quad \underline{\text{and}} \quad a = 40 \left( -e^{-\frac{t}{4}} \cdot \left(-\frac{1}{4}\right) \right) \\ e^{-\frac{t}{4}} &= \frac{40 - v}{40} &= 40 \left( \frac{1}{4} e^{-\frac{t}{4}} \right) \\ &= 1 - \frac{1}{40}v &= 10 e^{-\frac{t}{4}} \\ & &= 10 \left( 1 - \frac{1}{40}v \right) \\ & &= 10 - \frac{1}{4}v \end{aligned}$$

Again, less exciting but effective...

$$\text{Prove: } a = 10 - \frac{1}{4}v$$

$$\begin{aligned} \text{LS} &= a \\ &= 10 e^{-\frac{t}{4}} \\ \text{RS} &= 10 - \frac{1}{4}v \\ &= 10 - \frac{1}{4}(40 - 40 e^{-\frac{t}{4}}) \\ &= 10 - 10 + 10 e^{-\frac{t}{4}} \\ &= 10 e^{-\frac{t}{4}} \\ &= \text{LS} \quad \checkmark \end{aligned}$$

$$\begin{aligned}
 c) \quad V_T &= \lim_{t \rightarrow \infty} v \\
 &= \lim_{t \rightarrow \infty} (40 - 40e^{-\frac{t}{4}}) \\
 &= \lim_{t \rightarrow \infty} \left( 40 - \frac{40}{e^{\frac{t}{4}}} \right)
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} = 40 - 0 \\ = 40 \text{ m/s} \end{array}$$

$$\begin{aligned}
 d) \quad 40(1 - e^{-\frac{t}{4}}) &= 0.95(40) & s &= 160 \left[ \frac{1}{4} 4 \ln 20 - 1 + e^{-\frac{4 \ln 20}{4}} \right] \\
 1 - e^{-\frac{t}{4}} &= 0.95 & &= 160 \left[ \ln 20 - 1 + e^{\ln \frac{1}{20}} \right] \\
 e^{-\frac{t}{4}} &= 0.05 & &= 160 \left[ \ln 20 - \frac{20}{20} + \frac{1}{20} \right] \\
 e^{\frac{t}{4}} &= 20 & &= 160 \left[ \ln 20 - \frac{19}{20} \right] \\
 \frac{t}{4} &= \ln 20 & &= 160 \ln 20 - 152 \\
 t &= 4 \ln 20 & &= 327.3 \text{ m} \\
 &\doteq 12 \text{ s}
 \end{aligned}$$

Q16

$$y = A e^{mt}$$

$$\begin{aligned}
 y' &= A e^{mt} \cdot m \\
 &= m A e^{mt}
 \end{aligned}$$

$$\begin{aligned}
 y'' &= m A e^{mt} \cdot m \\
 &= m^2 A e^{mt}
 \end{aligned}$$

$$\text{Since } y'' + y' + by = 0$$

$$\therefore m^2 A e^{mt} + m A e^{mt} - 6 A e^{mt} = 0$$

$$\therefore A e^{mt} (m^2 + m - 6) = 0$$

$$\therefore A e^{mt} (m+3)(m-2) = 0$$

$$\therefore m = -3 \text{ or } 2.$$