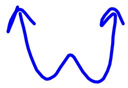
Ex1. Sketch an accurate graph for each. Use the first derivative and any other techniques as needed. Don't use techniques which require lots of work if only to produce marginal benefits. Be efficient and make good choices. For example, don't steal from babies. They don't have enough money to make it worth it.

a. $y = x^4 - 3x^2 + 2x$



| x | y |
|---|---|
| 0 | 0 |
| 1 | 0 |
| -2 | 0 |

$$y = x(x^3 - 3x + 2)$$
$$= x(x-1)(x^2+x-2)$$
$$= x(x-1)(x+2)(x-1)$$
$$= x(x-1)^2(x+2)$$

$$y' = 4x^3 - 6x + 2$$
$$= 2(2x^3 - 3x + 1)$$
$$\boxed{y' = 2(x-1)(2x^2+2x-1)}$$

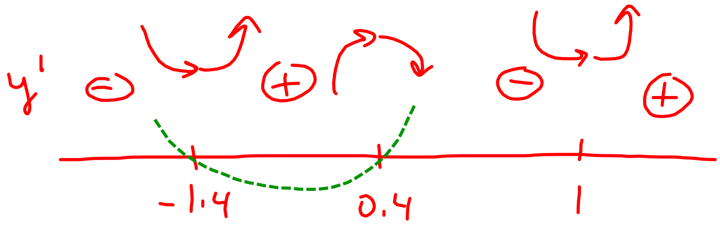
So $y' = 0$ when

$x = 1$ or $2x^2+2x-1=0$

$$x = \frac{-2 \pm \sqrt{12}}{4}$$

$x \doteq -1.4, 0.4$

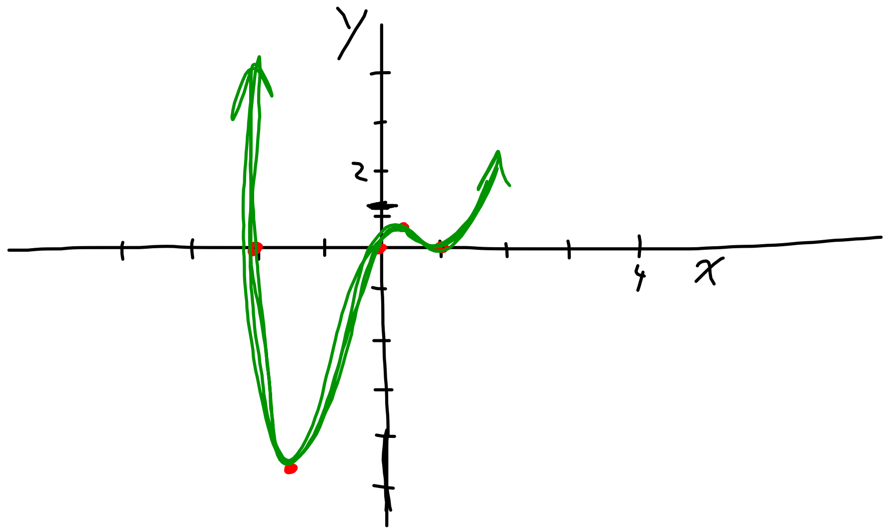
C.N.'s: $x = 1, -1.4, 0.4$




min @ $(-1.4, -4.8)$

max @ $(0.4, 0.3)$

min @ $(1, 0)$



b. $y = \dfrac{1-x}{1+x^2}$ HA @ $y=0$ $D=\{x \in R\}$

| $x$ | $y$ |
|---|---|
| 1 | 0 |
| 0 | 1 |

$$y = \frac{-(x-1)}{x^2+1}$$

---

$$y' = \frac{-(1+x^2)-(1-x)(2x)}{(1+x^2)^2}$$

$$= \frac{-1-x^2-2x+2x^2}{*}$$

$$\boxed{y' = \frac{x^2-2x-1}{(1+x^2)^2}}$$

$y'=0$ when $x^2-2x-1=0$

$$\therefore x = \frac{2 \pm \sqrt{8}}{2}$$

$$= 1 \pm \sqrt{2}$$

$$\doteq -0.6,\ 2.4$$

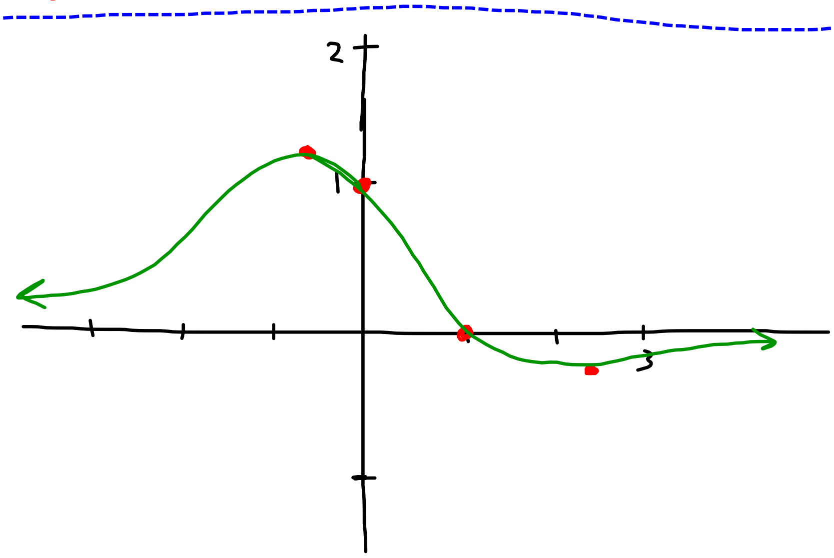
C.N: $x = -0.6,\ 2.4$

max @ $(-0.6, 1.2)$

min @ $(2.4, -0.2)$

$y'$ ⊕ ⊖ ⊕

$-0.6$ $2.4$



c. $g(x) = x^{\frac{1}{3}}(x+3)^{\frac{2}{3}}$

$D = \{x \in \mathbb{R}\}$

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| $-3$ | 0 |

$$g(x) = \sqrt[3]{x}\ \sqrt[3]{(x+3)^2}$$
$$= \sqrt[3]{x(x+3)^2}$$
$$= \sqrt[3]{x^3+6x^2+9x}$$

As $x \to \infty$, $y \to x$

---

$$g'(x) = \left[\frac{1}{3}x^{-\frac{2}{3}}\right](x+3)^{\frac{2}{3}} + \left(x^{\frac{1}{3}}\right)\left[\frac{2}{3}(x+3)^{-\frac{1}{3}}(1)\right]$$

$$= \frac{1}{3}x^{-\frac{2}{3}}(x+3)^{-\frac{1}{3}}\left[(x+3) + 2x\right]$$

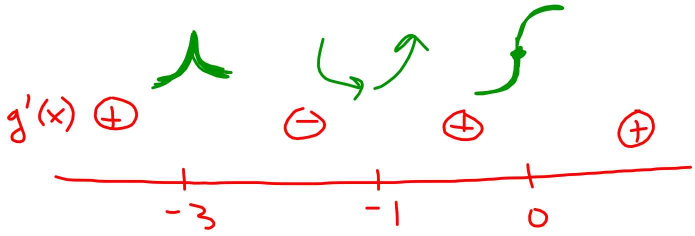
$$= \frac{3x+3}{3\sqrt[3]{x^2}\ \sqrt[3]{x+3}}$$

$$\boxed{g'(x) = \frac{x+1}{\sqrt[3]{x^2}\ \sqrt[3]{x+3}}}$$

$g'(x) = 0$ when $x = -1$

$g'(x)$ DNE when $x = 0, -3$

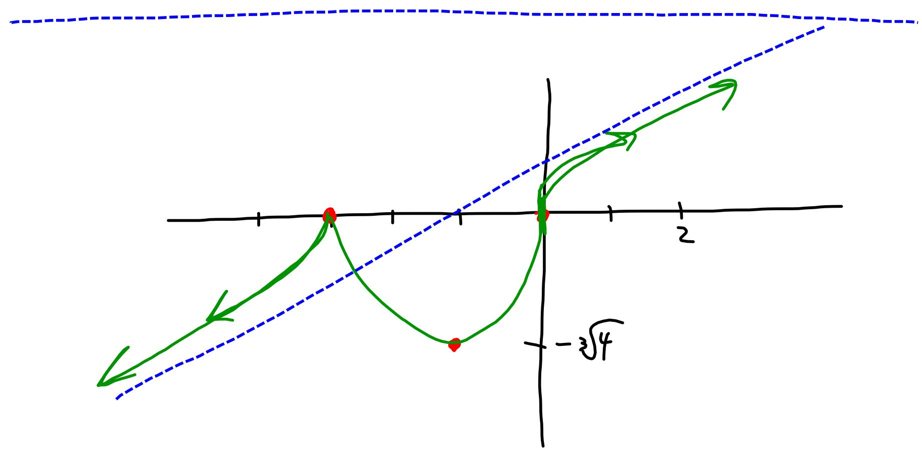
C.N.'s: $x = -1, 0, -3$




max cusp @ $(-3, 0)$

min horiz @ $(-1, -\sqrt[3]{4})$

inflection vert. @ $(0, 0)$



Ex2. Use desmos.com to examine the end behaviour of the following functions by ZOOMING out.

a. $g(x) = x^{\frac{1}{3}}(x+3)^{\frac{2}{3}}$

$y = x^{\frac{1}{3}}x^{\frac{2}{3}} = x$

b. $g(x) = x^{\frac{1}{3}}(x+3)^2$

$y = x^{\frac{1}{3}}x^2 = x^{\frac{7}{3}}$

c. $g(x) = x^{\frac{1}{3}}(x+3)^3$

$y = x^{\frac{1}{3}}x^3 = x^{\frac{10}{3}}$

Homefun: Page 212 # 2, 4→7

---

## Chapter 4 Review

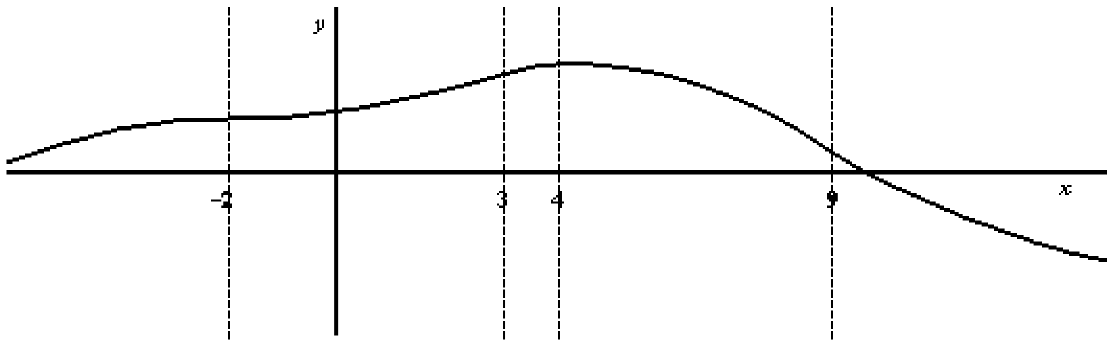
**Practice Test: Page 220 # 1→7**

Corrections:

2b $\left(-\frac{1}{2}, \frac{17}{8}\right)$ is a minimum.

5 The graph should:

- start concave down and ONLY switch concavity at inflection points when $x = -2, 3, 9$
- absolute maximum occurs when $x = 4$ and no other local max. or min. points exist
- be horizontal when $x = -2$ ( and $x = 4$ )




**Review Exercises: Page 217**

**4.1** **Q1**

**4.2** **Q3** {3b (0, 6) is a MIN, not a MAX}, **6, 7, 13, 18**

**4.3** **Q5, 9, 11, 12, 16, 19** $\left\{ f''(x) = \frac{10(x+2)}{(x-1)^4} \right\}$

**4.4** **Q2, 8** {8aiii graph should be up-side-down ( i.e. reflected in $y = -3$ ) }, **14, 15**

**4.5** **Q4, 10, 17, 20a**