Related Rates Review — Solutions

1. $V = \frac{1}{3}\pi r^2 h$, $r = h$

$V = \frac{1}{3}\pi r^2 (r)$

$V = \frac{1}{3}\pi r^3$

So, $\frac{dV}{dr} = \pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$4 = (\pi r^2)\left(\frac{dr}{dt}\right)$

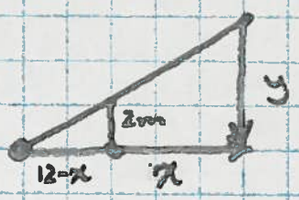
$\frac{dr}{dt} = \frac{4}{\pi r^2}$

When $r = 6$

$\frac{dr}{dt} = \frac{4}{\pi(6)^2}$

$= \frac{1}{9\pi}$

2.



$\frac{dx}{dt} = -0.75$ m/s

So, $\frac{2}{y} = \frac{12-x}{12}$

$y(12-x) = 24$

$y = 24(12-x)^{-1}$

So, $\frac{dy}{dx} = -24(12-x)^{-2}(-1)$

$= \frac{24}{(12-x)^2}$

So... $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= \frac{24}{(12-x)^2} \cdot -\frac{3}{4}$

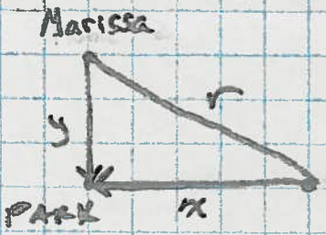
When $x = 4$, $\frac{dy}{dt} = \frac{24}{64} \cdot -\frac{3}{4}$

$= \frac{3}{8} \cdot -\frac{3}{4}$

$= -\frac{9}{32}$ m/s

∴ the shadow decreases in height by 0.28125 m every second.

3.



$\frac{dx}{dt} = -32$ m/min

$\frac{dy}{dt} = -26$ m/min

Find $\frac{dr}{dt}$ when $x = 1800$ m, $y = 1200$ m

$r = \sqrt{1800^2 + 1200^2}$

$= \sqrt{4680000}$

$x^2 + y^2 = r^2$

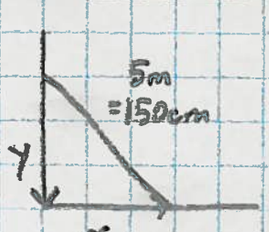
$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2r \cdot \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{1800(-32) + (1200)(-26)}{\sqrt{4680000}}$

$= -41.05$ m/min

∴ the girls are getter 41.05 m closer every min.

4.



$\frac{dx}{dt} = 15$ cm/s

$x^2 + y^2 = 150^2$

$\therefore 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x}{y} \cdot \frac{dx}{dt}$

So, when $x = 100$ cm, $y = \sqrt{12500}$ cm

$\therefore \frac{dy}{dx} = \frac{-100}{\sqrt{12500}} \cdot \frac{15}{1}$

$\doteq -13.42$ cm/s

∴ the top of the sled is moving 13.42 cm/s down the wall

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6. $PV = C$

$\therefore \frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$

$\frac{dV}{dt} = \frac{-V\frac{dP}{dt}}{P}$

So when $V = 450$ cm^3, $P = 150$ kPa

$\therefore \frac{dV}{dt} = \frac{-450(15)}{150}$

$= -45$ cm^3/min

∴ the volume is decreasing by 45 cm^3/min at this instant

5. a) $V = \frac{4}{3}\pi r^3 \quad \therefore \frac{dV}{dr} = 4\pi r^2$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$
$$= (4\pi r^2) \cdot (0.15)$$
$$= 0.6\pi r^2$$

When $r = 1$ mm, $\frac{dV}{dt} = 0.6\pi(1)^2$
$= 1.885$ mm^3/s

b) 1 cm = 10 mm

$V = \pi r^2 (10) \quad \frac{dV}{dr} = 20\pi r$
$= 10\pi r^2$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$
$$= (20\pi r) \cdot (0.15)$$
$$= 3\pi r$$

When $r = 1$ mm, $\frac{dV}{dt} = 3\pi(1)$
$= 9.42$ mm^3/s

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7. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$
$$-3 = (4\pi r^2) \cdot \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{-3}{4\pi r^2}$$

When $r = 12$ m
$$\frac{dr}{dt} = \frac{-3}{4\pi(12)^2}$$
$$= \frac{-1}{192\pi} \text{ m/min}$$

∴ the balloons radius is decreasing by ≈ 0.00166 m/min when $r = 12$ m

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8. $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$
$$-250 = (8\pi r) \cdot \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{-125}{4\pi r}$$

When $r = 5$ km $= 5000$ m
$$\frac{dr}{dt} = \frac{-125}{4\pi(5000)}$$
$$\doteq -0.00199 \text{ m/min}$$

∴ the radius is decreasing by ≈ 0.00199 m/min when $r = 5$ km

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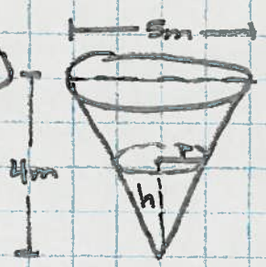
9.



$$\frac{2r}{h} = \frac{5}{4}$$
$$r = \frac{5}{8}h$$

$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi\left(\frac{5}{8}h\right)^2 h$
$= \frac{25}{192}\pi h^3$

So, $\frac{dV}{dh} = \frac{25}{64}\pi h^2$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$
$$1.2 = \left(\frac{25}{64}\pi h^2\right)\left(\frac{dh}{dt}\right)$$
$$\frac{dh}{dt} = \frac{6}{5} \cdot \frac{64}{25\pi h^2}$$
$$= \frac{384}{125\pi h^2}$$

So, when $h = 3$ m
$$\frac{dh}{dt} = \frac{384}{125\pi(3)^2}$$
$$= \frac{128}{375\pi}$$
$$\doteq 0.109 \text{ m/min}$$

∴ the water level is rising by about 0.109 m/min when the height is 3 m.