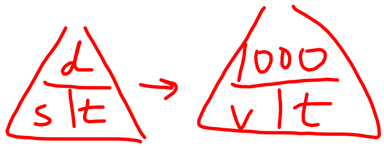
3.4: Optimization Problems in Economics and Science Continued Date: ________________

Ex3. The cost of fuel **per kilometre** for a truck traveling v km/h is given by the equation $C(v) = \frac{v}{400} + \frac{12}{v}$.

Assume the driver is paid \$20/h. What speed would give the lowest cost, including fuel and wages, for a 1000-km trip?



$$\text{Cost} = 20t + (C(v))(1000)$$

$$= 20t + \left(\frac{1}{400}v + 12v^{-1}\right)(1000)$$

$$= 20t + \frac{5}{2}v + 12000v^{-1}$$

$vt = 1000$

$t = \frac{1000}{v}$

$$C(v) = 20\left(\frac{1000}{v}\right) + \frac{5}{2}v + 12000v^{-1}$$

$$= 20000v^{-1} + \frac{5}{2}v + 12000v^{-1}$$

$$= 32000v^{-1} + \frac{5}{2}v$$

$$C'(v) = \frac{-32000}{v^2} + \frac{5}{2} = 0$$

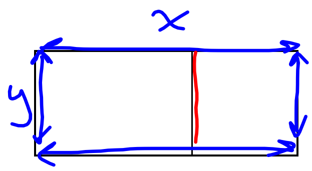
$$\therefore\ v^2 = \frac{64000}{5}$$

$$v^2 = 12800$$

$$\therefore\ v \doteq 113 \text{ km/h.}$$

Thus...

Ex4. A 5000 m^2 rectangular area of a field is to be enclosed by a fence, with a moveable inner fence built across the narrow part of the field, as shown. The perimeter fence costs \$10/m and the inner fence costs \$4/m. Determine the dimensions of the field to minimize the cost.



$$xy = 5000$$

$$y = \frac{5000}{x}$$

$$\text{Cost} = 10(2x+2y) + 4y$$

$$= 20x + 24y$$

$$C(x) = 20x + 24\left(\frac{5000}{x}\right)$$

$$= 20x + 120000x^{-1}$$

$$C'(x) = 20 - \frac{120000}{x^2} = 0$$

$$\therefore \; x^2 = \frac{120000}{20}$$

$$x^2 = 6000$$

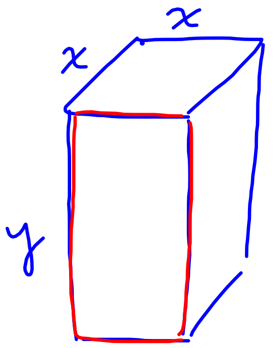
$$x \doteq 77.46\text{ m}$$

$$y = \frac{5000}{x}$$

$$y \doteq 64.55\text{ m}$$

Thus. . . .

Ex5. A cardboard box with a square base is to have a volume of 8000 cm^3. The cardboard for the square bottom is thicker, so it costs three times as much as the rest of the cardboard. Find the dimensions that will minimize the cost of the cardboard.



$$\text{Cost} = 3K(x^2) + K(4xy + x^2)$$

$$= K[3x^2 + 4xy + x^2]$$

$$= K[4x^2 + 4xy]$$

$$x^2y = 8000$$

$$y = \frac{8000}{x^2}$$

$$C(x) = K\left[4x^2 + 4x\left(\frac{8000}{x^2}\right)\right]$$

$$= K[4x^2 + 32000x^{-1}]$$

Thus, $C'(x) = \cancel{K}\left[8x - \frac{32000}{x^2}\right] = 0$

So, $x^3 = \frac{32000}{8}$

$= 4000$

$x = \sqrt[3]{4000}$

$x \doteq \underline{15.87}$ m

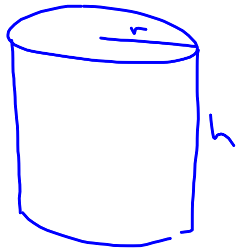
$y = \frac{8000}{x^2}$

$y \doteq \underline{31.75}$ m

Thus, ~~~~~~~~ .

Ex6. A cylindrical chemical storage tank is going to be constructed with a capacity of 1000 m^3. The specifications call for the base to be made of sheet steel that costs \$100/m^2, the top to be made of sheet steel that costs \$50/m^2, and the wall to be made of sheet steel that costs \$80/m^2.

Determine the proportions that minimize the cost of the steel for construction. All calculations should be accurate to two decimal places.



$$\pi r^2 h = 1000$$

$$h = \frac{1000}{\pi r^2}$$

$$\text{Cost} = 100(\pi r^2) + 50(\pi r^2) + 80(2\pi r h)$$

$$= 150\pi r^2 + 160\pi r h$$

$$\text{So, } C(r) = 150\pi r^2 + 160\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$= 150\pi r^2 + 160000 r^{-1}$$

$$C'(r) = 300\pi r - \frac{160000}{r^2} = 0$$

$$\therefore r^3 = \frac{160000}{300\pi}$$

$$= \frac{1600}{3\pi}$$

$$r \doteq 5.54 \text{ m}$$

$$h = \frac{1000}{\pi r^2}$$

$$h \doteq 10.38 \text{ m}$$

Ergo....

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Measurement Problems (ordered from medium pedium to [illegible]ing-hard): 12, 10, 9, 5, 17[r = 229 cm, h = 914 cm]

Speed Problems (ordered from medium-pedium to [illegible]ing-hard): 8, 18[128.6 km/h]

Another problem: 19

<u>Ch 3 Review</u>

1. Practice Test;
   Page 160 # 1→8 [ans 3 e) 4 m/s]

2. Review Exercises; Page 156
   3.1 # 1→4, **<u>5</u>**, **<u>7</u>**, **<u>12</u>**, **<u>13</u>**, **<u>27</u>**, 28, 29, **<u>30</u>**
   [ans 12 b) away from origin, but toward starting position]
   3.2 # 6, **<u>8</u>**, 26
   3.3 # (14→17)(measurement),
   (19, 24)(speed),
   25(misc)
   3.4 # (9, 10b[ans 10bii \$41.60/item])(function given),
   20(revenue/profit),
   (22, 23[ans. 40 m by 25 m])(measurement),
   (18, 21)(pythagorean)