5.2.2: **The Derivative of the Logarithmic Functions** (pg 576) Date: ________________

Ex1. Find $f'(x)$ if $f(x) = \log_a x$.

$f(x) = \log_a x$

$y = \log_a x$

$a^y = x$

$\frac{d}{dx}(a^y) = \frac{d}{dx}(x)$

$\ln a \cdot a^y \cdot y' = 1$

$y' = \frac{1}{a^y \cdot \ln a}$

$y' = \frac{1}{x \ln a}$

So...	IF $f(x) = \log_a x$.	THEN $f'(x) = \frac{1}{x \ln a}$
	IF $f(x) = \log_a(g(x))$	THEN $f'(x) = \frac{1}{g(x) \ln a} \cdot g'(x) = \frac{g'(x)}{g(x) \ln a}$

Ex2. Find the equation of the tangent to $y = \log_2 x$ at $x = 8$.

$y = \log_2 x$

$y' = \frac{1}{x \ln 2}$

So when $x = 8$

$y = \log_2 8$

$= 3$

$y' = \frac{1}{8 \ln 2}$

$P_0 = (8, 3)$

$m = \frac{1}{8 \ln 2}$

$\frac{x-8}{8 \ln 2} = \frac{y-3}{1}$

$x - 8 = (8 \ln 2) y - 24 \ln 2$

$x - 8y \ln 2 + 24 \ln 2 - 8 = 0$

Ex3. Find y' if $y = \log_4 (2x + 3)^5$.

$$y = \log_4 (2x+3)^5 \qquad \text{OR} \qquad y = 5 \log_4 (2x+3)$$

$$y' = \frac{5(2x+3)^4 (2)}{(2x+3)^5 \ln 4} \qquad\qquad y' = 5\left(\frac{2}{(2x+3)\ln 4}\right)$$

$$= \frac{10}{(2x+3)\ln 4} \qquad\qquad = \frac{10}{(2x+3)\ln 4}$$

Ex4. Show $\log_a c = \dfrac{\ln c}{\ln a}$. Then, differentiate $y = 2^x \log_2 x$.

Let $\underline{\log_a c} = \underline{b}$

$\therefore \quad a^b = c$

$\therefore \quad \ln(a^b) = \ln c$

$\underline{b} \ln a = \ln c$

$b = \dfrac{\ln c}{\ln a}$

$\therefore \quad \log_a c = \dfrac{\ln c}{\ln a}$

$$y = 2^x \cdot \log_2 x$$

$$y' = (\ln 2 \cdot 2^x)(\log_2 x) + (2^x)\left(\frac{1}{x \ln 2}\right)$$

$$= (\cancel{\ln 2} \cdot 2^x)\left(\frac{\ln x}{\cancel{\ln 2}}\right) + \frac{2^x}{x \ln 2}$$

$$= \frac{x \cdot 2^x \cdot \ln x \cdot \ln 2 + 2^x}{x \ln 2}$$

$$= \frac{2^x [x \ln x \cdot \ln 2 + 1]}{x \ln 2}$$

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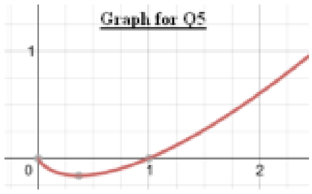

Ans 4b $h'(8) = \dfrac{1}{24(\ln 3)(\ln 2)}$

10 There are no critical numbers. Rest of the answer is fine.

5 Just graph function to show the critical points. Don't worry about graphing the tangent at $x = 10$. That part of the graph is boring.