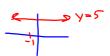
## 1.5: Properties of Limits

## Date:

## Properties of Limits

For any real number  $\alpha$  where f and g both have limits that exist at  $x = \alpha$ .



1. The limit of a constant is equal to the constant.

$$\lim_{x \to a} k = K$$

2. The limit of x as x approaches a is equal to a.

$$\lim_{x \to a} x = \mathbf{c}$$

 The limit of a sum (or difference) is equal to the sum (or difference) of the limits.

$$\lim_{x \to a} [f(x) + g(x)]$$

= 
$$\lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

4. The limit of a constant times a function is equal to the constant times the limit of the function.

$$\lim_{x \to a} [c \cdot f(x)]$$

Example:

$$\lim_{x \to -1} 5 = 5$$

Example:

$$\lim_{x \to -1} x = - /$$

Example:

$$\lim_{x \to 2} [3x^2 - x]$$

$$= \lim_{X \to 2} (3x^2) - \lim_{X \to 2} (x)$$

$$= 3(2)^2 - 2$$
$$= |0$$

Example:

$$\lim_{x \to 3} 10(x+2)$$

$$= 10 \left( \lim_{x \to 3} (x + 2) \right)$$
$$= 10 (5)$$

 The limit of a product (or quotient) is equal to the product (or quotient - provided g(x)+0) of the limits.

$$\lim_{x \to a} [f(x) \cdot g(x)]$$

$$= \left(\lim_{x \to \infty} f(x)\right) \left(\lim_{x \to \infty} g(x)\right)$$

Example:

$$\lim_{x \to 0} \frac{x+1}{5x^2+2}$$

$$= \frac{\lim_{x \to 0} (x+1)}{\lim_{x \to 0} (5x^2+2)}$$

6. The limit of a power (or root) is equal to
the power (or root) of the limit
- provided the exponent is rational (or the root exists)

$$\lim_{x \to a} [f(x)]^n$$

Example:

$$\lim_{x \to 2} \sqrt{2x + 5}$$

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When limits cannot be found by direct substitution (direct substitution gives an INDETERMINATE FORM \frac{0}{2}),
we look for an equivalent function that agrees with f except at x = a.
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These functions are said to have a REMOVABLE DISCONTINUITY

Helpful Techniques to Eliminate a Removable Discontinuity:

• Factoring / • Rationalizing /

• Change of Variable

If possible, evaluate each limit.

a. 
$$\lim_{x \to -4} \frac{2x^2 + 5x - 12}{x + 4} = \lim_{x \to -4} \frac{(2x + 8)(2x - 3)}{(x + 4)}$$

b. 
$$\lim_{x \to 0} \left( \frac{\sqrt{4+x}-2}{x} \right) \cdot \left( \frac{\sqrt{4+x}}{x} + 2 \right)$$

$$= \lim_{x \to 0} \frac{4 + x - x}{x \left(\sqrt{4 + x} + 2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{4+x}+2}$$

$$= \frac{\sqrt{2+2}}{2+2}$$

$$= \frac{1}{4}$$
c.  $\lim_{x \to 8} \frac{2-\sqrt{x}}{8-x}$ 

c. 
$$\lim_{x \to 8} \frac{2 - \sqrt{x}}{8 - x}$$

$$= \lim_{x \to 8} \frac{8 - x}{8 - x}$$

$$= \lim_{y \to 2} \frac{3 - y}{8 - y^{3}}$$

$$= \lim_{y \to 2} \frac{(y \to 3)}{(y \to 3)(y^2 + 3y + 4)} = \lim_{x \to 8} \frac{2 - 3\sqrt{x}}{8 - x}$$

$$= \lim_{x \to 8} \frac{3\sqrt{x} - 2}{x - 8}$$

$$= \lim_{x \to 8} \frac{(3x - 2)(x^2 + 3x + 4)}{(x^2 + 3x + 4)}$$

$$\begin{cases} |z + \sqrt[3]{x} = y \\ \therefore & x = y^3 \\ As & x \to 8 \\ & y \to 2 \end{cases}$$

$$| \lim_{x \to 8} \frac{8 - x}{3 - 3/x}$$

$$=\frac{1}{12}$$

d. 
$$\lim_{x \to 1} \frac{1}{x-1} DNE$$

Ex2. Does 
$$\lim_{x \to 3} \frac{x^2 - x - 6}{|x - 3|}$$
 exist? Illustrate your answer by sketching a graph of the function.

$$|e+f(x) = \frac{x^2 - x - 6}{|x-3|} = \begin{cases} -x - 2, & x < 3 \\ x + 2, & x > 3 \end{cases}$$

if 
$$x > 3$$
  

$$f(x) = \frac{(x-3)(x+2)}{x-3}$$

$$= x+2$$

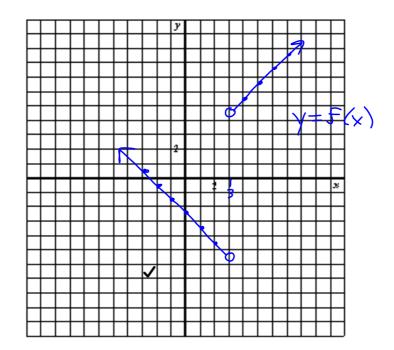
if 
$$x > 3$$
  

$$f(x) = \frac{(x-3)(x+2)}{x-3}$$

$$f(x) = \frac{(x-3)(x+2)}{-(x-3)}$$

$$\lim_{x \to 3^+} f(x) = 3+2$$
  $\lim_{x \to 3^+} f(x) = -3-2$ 

$$\lim_{x\to 3^{-}} f(x) = -3-\lambda$$



Homefun: Page 45 #1, 3, 4dcf, 6, 7, 8b -+f, 9, 10acd, 17, 12, 13, [16 - Hint: Use BOTH conjugates]