EQ2341 Pattern Recognition and Machine Learning

Assignment 3 Forward Algorithm

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Contents

1. Description

This assignment focuses on implementing and verifying the forward algorithm in HMM. And also with the design of log probabilities of the complete observed sequence.

This algorithm calculates conditional state probabilities, given an observed feature sequence $(x_1 \dots x_t \dots)$ and an HMM $\lambda = ((q, A), B)$, as

$$\hat{\alpha}_{j,t} = P(S_t = j | x_1 \dots x_t, \lambda)$$

Also the forward scale factors sequence $(c_1,\dots,c_T)$ or $(c_1,\dots,c_T,c_{T+1})$, derived by the algorithm, can be used to calculate the total probability $P(X = x|\lambda)$ of the observed sequence with given HMM.

The "forward", the "logprob" functions, and the test file are in the repertory "Changed_code".

2. Main tasks

2.1 Implement the Forward Algorithm

The implement of the algorithm is based on the textbook, which can be divided into 3 steps according the calculation procedure of the algorithm.

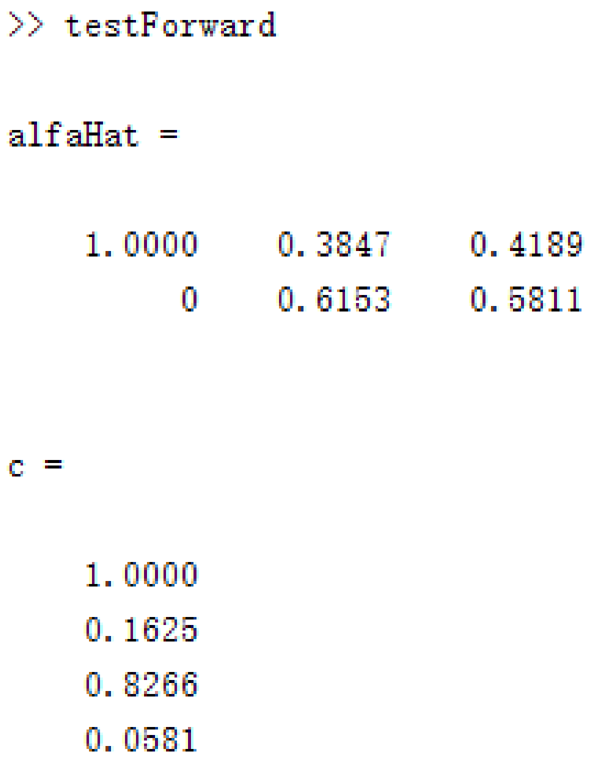
First step is initialization, which uses the Eqs. (5.42)–(5.44). Second step is calculating the forward process using Eqs. (5.50)–(5.52). And the last one is the Termination step, only for the finite duration HMMs. The equations are from the Pattern Recognition book of Leijon and Henter.

2.2 Verify the Implementation

After designing the forward algorithm, we use the given example to verify the function. The test HMM is given by

$$q = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \end{pmatrix}$$

And the observed finite-duration feature sequence is $x = (-0.2, 2.6, 1.3)$. Using the example to test function of forward algorithm, the output is shown in Fig. 2.1. This result in correspond to the given one in the textbook.



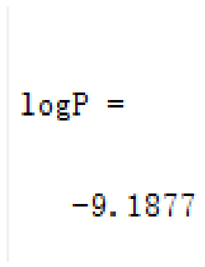
```
>> testForward

alfaHat =

    1.0000    0.3847    0.4189
         0    0.6153    0.5811



c =

    1.0000
    0.1625
    0.8266
    0.0581
```

Figure 2.1: Verify the forward algorithm

2.3 Probability of a Feature Sequence

The next task is to build a function to calculate the log-probability $log(P)$ of an observed sequence based on given HMM instance λ. To design the logprob function, it is useful to use forward algorithm function designed before, as the forward scale factors sequence c_t means the observation probabilities based on the given HMM. Actually, it has strong correlation with log(P).

First we get the HMM object using HMM class, then we derive the pX matrix with state-conditional likelihood values. The probability of an observed sequence P[$x_1..x_t$ | HMM] is equal to c_1*c_2*...*c_t. If take the logarithm of P, it becomes the sum of $ln(c_t)$.



```
logP =

   -9.1877
```

Figure 2.2: Verify the logprob function

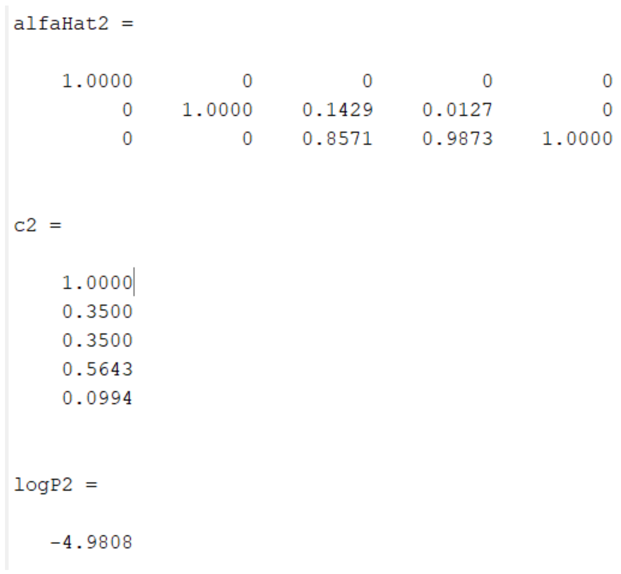
Using the natural logarithm, the result of designed logprob function is equal to the text answer, which can be seen in Fig. 2.2.

2.4 Test with an infinite duration HMM

For this test, we took the data from the problem 5.1 of the coursebook with the following Hidden Markov Chain :

$$q = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}; \qquad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{pmatrix};$$

In one test run the observed sequence was $\underline{z} = (1, 2, 4, 4, 1)$. For the logprob results, we have to find $ln(P(z|\lambda)) = ln(0.0069) \approx -4.980$. Using this example, the outputs are shown in Fig. 2.3. These results correspond to the given ones in the textbook and the tutorials.



```
alfaHat2 =

    1.0000         0         0         0         0
         0    1.0000    0.1429    0.0127         0
         0         0    0.8571    0.9873    1.0000


c2 =

    1.0000
    0.3500
    0.3500
    0.5643
    0.0994


logP2 =

   -4.9808
```

Figure 2.3: Verify with the infinite duration HMM

So our implementation works for the infinite duration HMM.