EQ2341 Pattern Recognition and Machine Learning

Assignment 4 Backward Algorithm

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1. Description

This assignment focuses on implementing and verifying the backward algorithm in HMM.

This algorithm calculates conditional state probabilities, based on an observed feature sequence $(x_{t+1} \dots x_T)$ and an HMM $\lambda = ((q, A), B)$, also the forward scale factors sequence $(c_1,\dots,c_T)$ or $(c_1,\dots,c_T,c_{T+1})$, derived by the forwars algorithm, as

$$\hat{\beta_{i,t}} = P(X_{t+1} = x_{t+1} \dots X_t = x_T \mid S_t = i, \lambda)$$

The "backward" function and the test file are in the repertory "Changed_code".

2. Main tasks

2.1 Implement the Backward Algorithm

The implement of the algorithm is based on the textbook, which can be divided into 2 steps according the calculation procedure of the algorithm.
First step is initialization, an infinite-duration HMM uses the Eqs. (5.64) and for an finite-duration HMM uses (5.65). Second step is the backward step using Eqs. (5.66). The equations are from the Pattern Recognition book of Leijon and Henter.

2.2 Verify the Implementation

After designing the backward algorithm, we use the given example to verify the function. The test HMM is given by

$$q = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \end{pmatrix}$$

And the observed finite-duration feature sequence is $x = (-0.2, 2.6, 1.3)$. The forward scale factors sequence calculated by forward algorithm is c = (1, 0.1625, 0.8266, 0.0581). Using the example to test function of forward algorithm, the output is shown in Fig. 2.1. This result in correspond to the given one in the textbook.

```
betaHat =


    1.0003    1.0393         0
    8.4182    9.3536    2.0822
```

Figure 2.1: Verify the backward algorithm

2.3 Test with an infinite duration HMM

For this test, we took the data from the problem 5.1 of the coursebook with the following Hidden Markov Chain :

$$q = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \qquad A = \begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}; \qquad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{pmatrix};$$

In one test run the observed sequence was $\underline{z} = (1, 2, 4, 4, 1)$. Using this example, the outputs are shown in Fig. 2.2. To verify the results, we check the last column of betaHat2. All the terms have the same scaled backward variable value 10.0637. It is valid because at $t = T$, according to the problem 5.1:

$$c_T = 0.0994; \widehat{\beta_{i,T}} = 1/c_T = \frac{1}{0.0994} = 10.0637, \forall i \in 1, 2, 3$$

```
betaHat2 =

    1.0000    0.3312    0.1783    5.3503   10.0637
    0.9763    2.8571    1.6561    0.8917   10.0637
    0.5241    5.2411    3.0573    1.7834   10.0637
```

Figure 2.2: Verify with the infinite duration HMM

So our implementation works for the infinite duration HMM.