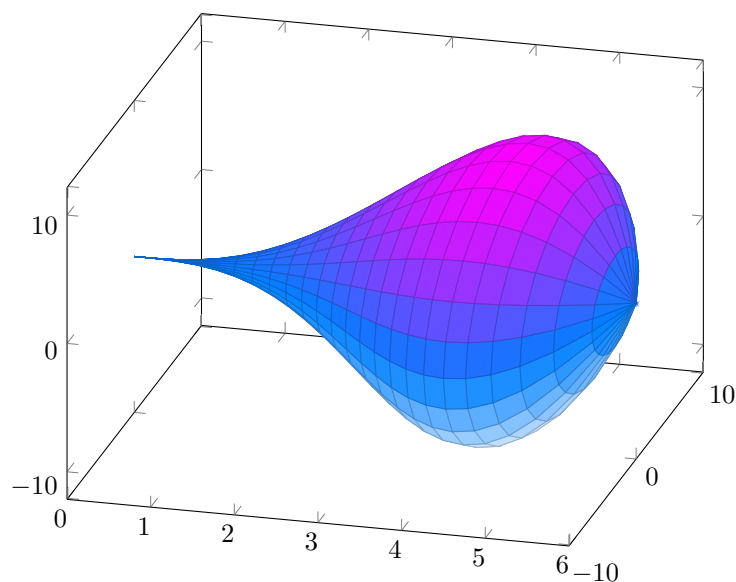
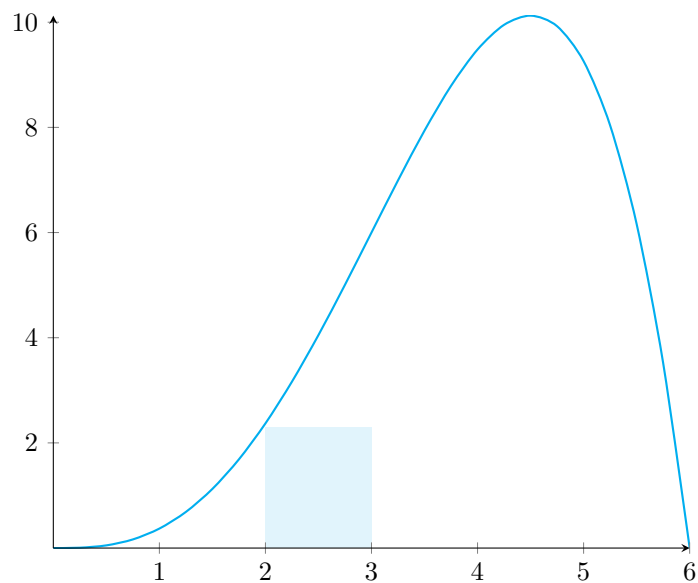


The disc method is a way to calculate the volume (yes volume, not area) of a cylindrical object. We do not know the volume formula for an odd graph like this:



But we can try and approximate it. Just like how we can use rectangles to approximate 2-d area, we can use cylinders to approximate cylindrical shapes. If we can find the two-dimensional cross-section of a revolved shape, we can calculate the volume of the revolved shape by taking an infinite sum of infinitely small discs knowing their radius (height in 2-dimensions). This means that we can find a Riemann-style rectangle in a cross-section like this:



...and revolve it around the x -axis to create a cylinder. We can revolve infinitely small cylinders over the axis just like taking the integral. In fact, the volume of a cylinder is just $r^2 * \pi * \text{height}$. Because we have the rectangle, we know $r = f(x)$ and $\text{height} = dx$, so we end up with a formula for revolving a cross-section about the x -axis:

$$\pi \int_a^b f(x)^2 dx$$

And if it is hollow (there is a smaller function $g(x)$ that forms the inside of the cross section, the area is

$$\begin{aligned} & \pi \int_a^b f(x)^2 dx - \pi \int_a^b g(x)^2 dx \\ &= \pi \int_a^b (f(x)^2 - g(x)^2) dx \end{aligned}$$

An easier way to remember this is big radius R minus small radius r :

$$\pi \int_a^b (R^2 - r^2) dx$$