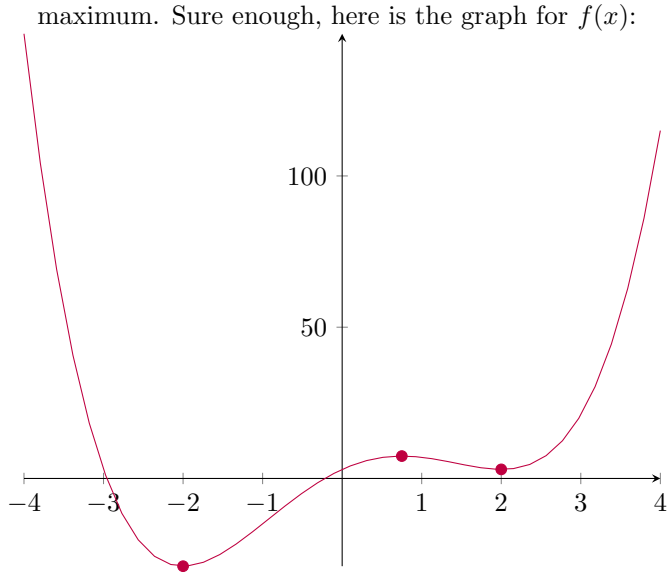
Example 1: Find the relative maxima and minima of the function $f(x) = x^4 - x^3 - 8x^2 + 12x + 3$. Since we have no bounds, we will evaluate this over $(-\infty, \infty)$

$$\begin{aligned}
f(x) &= x^4 - x^3 - 8x^2 + 12x + 3 \\
f'(x) &= 4x^3 - 3x^2 - 16x + 12 \\
f'(x) &= (4x-3)(x^2) - (4x-3)(4) \\
f'(x) &= (4x-3)(x^2-4) \\
f'(x) &= (4x-3)(x-2)(x+2) \\
f'(x) &\text{ must be zero at } x = \frac{3}{4}, \pm 2
\end{aligned}$$

Writing out our first derivative test table, we get:

(-∞, -2)	-2	(-2, 0.75)	0.75	(0.75,2)	2	(2, ∞)
-	0	+	0	-	0	+

Because when $x = \pm 2$ the derivative changes signs from negative to positive, we know that these two points are relative minimums. Because at $x = 0.75$ the derivative changes signs from positive to negative, we know that it is a local maximum. Sure enough, here is the graph for $f(x)$:



Example 2: Find the absolute maximum and minimum of the function $f(x) = \sin(\pi x)$ over the interval [0,0.75]

$$\begin{aligned} f(x) &= \sin(\pi x) \\ f'(x) &= \pi \cos(\pi x) \\ 0 &= \pi \cos(\pi x) \quad \{0 \leq x \leq 0.75\} \\ x &= 0.5 \end{aligned}$$

[0,0.5)	0.5	(0.5,0.75]
+	0	-

So $x = 0.5$ must be an absolute maximum. However, we are also constrained to an interval, so we must check the bounds as well:

$$f(0) = 0$$

$$f(0.5) = 1$$

$$f(0.75) = \frac{\sqrt{2}}{2}$$

From this, we can see that $x = 0.5$ is indeed our absolute maximum but $x = 0$ is our absolute minimum on this interval as well.