

Example For $f(x) = x^2$, the derivative is:

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} 2x + h \\ &= 2x + 0 \\ &= 2x \end{aligned}$$

So for any point x on the graph of $f(x) = x^2$, the slope of the tangent line will be $y = 2x$. Be familiar with the limit of the difference quotient as the definition of the derivative. You will prove more of these in the exercises but the math required is pre-Calculus¹ and generally unnecessary in the understanding of Calculus.

¹All math before Calculus will be referred to as pre-Calculus in this textbook