Example 1: Find the instantaneous rate of change at $t = 3$ and the average rate of change over the interval $2 \leq t \leq 6$ for the function $f(t) = \dfrac{t}{t^2 - 3}$

$$
\begin{aligned}
f(t) &= \frac{t}{t^2 - 3} \\
f(t) &= t * (t^2 - 3)^{-1} \\
\frac{df}{dt} &= (t^2 - 3)^{-1} + t * -1(t^2 - 3)^{-2}(2t) \\
\frac{df}{dt} &= \frac{1}{t^2 - 3} - \frac{2t^2}{(t^2 - 3)^2} \\
\frac{df}{dt} &= \frac{t^2 - 3}{(t^2 - 3)^2} - \frac{2t^2}{(t^2 - 3)^2} \\
\frac{df}{dt} &= -\frac{t^2 + 3}{(t^2 - 3)^2} \\
\frac{df}{dt}(3) &= -\frac{12}{(6)^2} \\
\frac{df}{dt}(3) &= -\frac{1}{3}
\end{aligned}
$$

So the instantaneous rate of change is $-\frac{1}{3}$ when $t = 3$. Next, the average rate of change:

$$
\frac{f(6) - f(2)}{6 - 2}
$$

$$
\frac{\frac{6}{33} - \frac{2}{1}}{4}
$$

$$
\frac{\frac{2}{11} - \frac{22}{11}}{4}
$$

$$
\frac{-\frac{20}{11}}{4}
$$

$$
-\frac{20}{11} * \frac{1}{4}
$$

$$
-\frac{5}{11}
$$

Remember, *instantaneous rate of change* means the rate of change *at a single point*. The derivative finds the rate of change at a point if the function is differentiable at that point. It's just a complicated way of saying "take the derivative".