Example 1: We already know how to find the slope of a linear equation. We can prove the difference quotient gives the slope for any linear equation. The general slope-intercept form for linear equations is $f(x) = mx + b$ where m is the slope and b is the y-intercept.

$$m \stackrel{?}{=} \lim_{h\to 0} \frac{(m(x+h)+b)-(mx+b)}{h}$$
$$m \stackrel{?}{=} \lim_{h\to 0} \frac{mx+mh+b-mx-b}{h}$$
$$m \stackrel{?}{=} \lim_{h\to 0} \frac{mh}{h}$$
$$m \stackrel{?}{=} \lim_{h\to 0} m$$
$$m = m$$
$$\checkmark$$

Example 2: For $f(x) = x^2$, the derivative is:

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$$
$$= \lim_{h\to 0} \frac{2xh+h^2}{h}$$
$$= \lim_{h\to 0} 2x+h$$
$$= 2x+0$$
$$= 2x$$

So for any point x on the graph of $f(x) = x^2$, the slope of the tangent line will be $\frac{\Delta y}{\Delta x} = 2x$. Be familiar with the limit of the difference quotient as the definition of the derivative. You will prove more of these in the exercises but the math required is pre-Calculus[1] and generally unneccesary in the understanding of Calculus.

Example 3: Working off of the previous problem, we can calculate the slope of the tangent line at any point on the graph. For example, the slope at the point (3.5,12.25) is:

$$\frac{\Delta y}{\Delta x} = 2x$$
$$\frac{\Delta y}{\Delta x} = 2(3.5)$$
$$\frac{\Delta y}{\Delta x} = 7$$

[1]All math before Calculus will be referred to as pre-Calculus in this textbook

Knowing the slope, we can calculate the equation itself. In Calculus, it is often the easiest when finding a tangent line equation to use the point-slope form of a line which is:

$$(y - y_0) = m(x - x_0)$$

where m is the slope at a point (x_0, y_0). The tangent line to the graph of $f(x) = x^2$ at the point (3.5,12.5) is therefore

$$y - 12.25 = 7(x - 3.5)$$

or

$$y = 7(x - 3.5) + 12.25$$

See the Tips about the AP Test section for more information.