

Okay, let's slow down a bit. Here's what we just found out:

1. The derivative of an indefinite integral of $f(x)$ is $f(x)$ itself
2. The integral from a to b of $f(x)$ is $f(b) - f(a)$

This shouldn't be *too* surprising. There's also something else we learned:

We can now calculate the (change in) area under a curve without needing to know the bounds!

As you will see later, this is very handy. Note that we can't actually find the exact value of the integral given only the derivative because of the $+C$.

This is really cool! The limit of the difference quotient of an *infinite* sum of a function is the function itself! It may seem obvious knowing the rules of derivatives and integrals, but think about it this way:

$$\lim_{h \rightarrow 0} \frac{\lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + i \frac{b-a}{n} + h\right) - \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + i \frac{b-a}{n}\right)}{h} = f(x)$$

How cool is that?! Really cool. Math rocks.