Example 1: Find the average value of $\sin(x)$ over $(0, \frac{\pi}{4})$

$$\frac{1}{\frac{\pi}{4}} \int_0^{\frac{\pi}{4}} \sin(x) dx$$
$$\frac{4}{\pi} (-\cos(x)) \Big|_0^{\frac{\pi}{4}}$$
$$\frac{4}{\pi} \left(\frac{-\sqrt{2}}{2} + 1 \right)$$
$$\frac{2 - 2\sqrt{2}}{2\pi}$$

Example 2: Find the average value of $x^3 - 2x + 1$ over (5,9)

$$\frac{1}{9-5} \int_{5}^{9} x^{3} - 2x + 1 dx$$

$$\frac{1}{4} \left(\frac{1}{4} x^{4} - x^{2} + x \right) \Big|_{5}^{9}$$

$$\frac{1}{4} (1568.25 - 136.25)$$

$$358$$