Example 1: Find when the function $f(x) = 1/4x^4 + x^3 - 1/2x^2 - 3x + 17$ is: Concave up increasing, concave down increasing, concave up decreasing, and concave down decreasing.

$$f(x) = 1/4x^4 + x^3 - 1/2x^2 - 3x + 17$$
$$f'(x) = x^3 + 3x^2 - x - 3$$
$$f''(x) = 3x^2 + 6x - 1$$

First, we will find when the first derivative is positive or negative to determine where f(x) is increasing or decreasing.

$$f'(x) = x^{3} + 3x^{2} - x - 3$$

$$f'(x) = x^{3} - x + 3x^{2} - 3$$

$$f'(x) = x(x^{2} - 1) + 3(x^{2} - 1)$$

$$f'(x) = (x + 3)(x^{2} - 1)$$

$$f'(x) = (x + 3)(x - 1)(x + 1)$$

$$0 = (x + 3)(x - 1)(x + 1)$$

$$x = -3, \pm 1$$

$(-\infty, -3)$	(-3, -1)	(-1,1)	$(1,\infty)$
-	+	-	+

So f(x) is increasing over (-3, -1) and $(1, \infty)$, and decreasing over $(-\infty, -3)$ and (-1, 1). Now, we will find where the function is concave up and down:

$$f''(x) = 3x^{2} + 6x - 1$$

$$0 = 3x^{2} + 6x - 1$$

$$x = \frac{-6 \pm \sqrt{6^{2} - 4 * 3 * - 1}}{2 * 3}$$

$$x = \frac{-6 \pm \sqrt{48}}{6}$$

$$x = \frac{-3 \pm 2\sqrt{3}}{3}$$

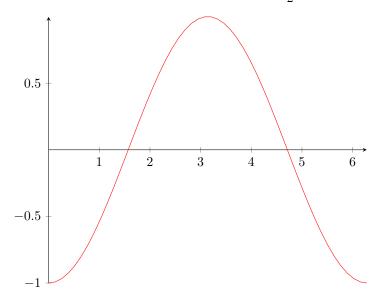
$$x \approx -2.155, 0.155$$

$(-\infty, -2.155)$	(-2.155, -0.155)	$(-0.155, \infty)$
+	-	+

So f(x) is concave up over $(-\infty, -2.155)$ and $(-0.155, \infty)$, and concave down over (-2.155, -0.155).

Example 2: Find where $\sin(x - \frac{\pi}{2})$ is concave up and down:

$$f(x) = \sin(x - \frac{\pi}{2})$$
$$f'(x) = \cos(x - \frac{\pi}{2})$$
$$f''(x) = -\sin(x - \frac{\pi}{2})$$



 $\sin(x-1)$

Interestingly, the concavity is just the inverse of the function itself. $\mathrm{Sin}(x-\frac{\pi}{2})$ is concave up when it is negative, written out mathematically: $(-\frac{\pi}{2},\frac{\pi}{2})+n*2\pi$, where n is any integer.