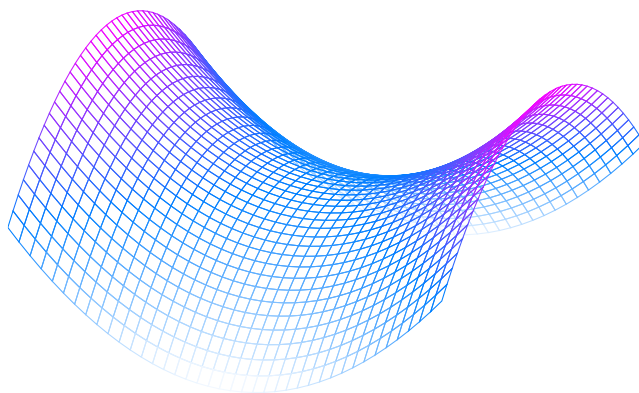


Noah's Guide to Calculus

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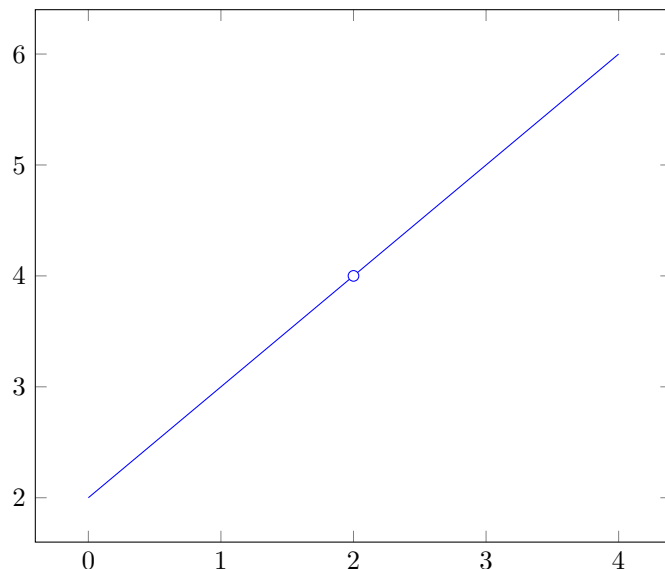


Contents

1	Limits	3
1.1	Existence of Limits	4
1.2	Types of Limits	4
1.3	Estimating Limits	5
1.4	The Squeeze Theorem	5
1.5	L'hôpital's Rule	5
1.6	Asymptotes	5
1.7	Relative Magnitudes	5
1.8	Continuity	5
2	Derivatives	5
2.1	Difference Quotient	5
2.2	Estimating Derivatives	5
2.3	Rules	5
2.4	Implicit Differentiation	5
2.5	Maxima & Minima	5
2.6	Concavity	5
2.7	Graphical Relations	5
2.8	Relationship with Continuity	5
3	Applications of Derivatives	5
3.1	Units	5
3.2	Instantaneous Rate of Change	5
3.3	Tangent Line Approximations	5
3.4	Physics	5
3.5	Related Rates	5
3.6	Optimization	5
3.7	Differential Equations	5
3.8	Slope Fields	5
3.9	Mean Value Theorem	5

1 Limits

Limits are a way of skirting the normal rules of math. Without the knowledge of limits, whenever a function divides by 0 or involves ∞ in any way, calculations become impossible. Limits take the rules of math a little less seriously and can be used to calculate what a value “should be”. A simple example of where limits come in handy is when there is a “hole” in a graph:



$$f(x) = \frac{x^2 - 4}{x - 2}$$

Because $f(x)$ divides by 0 when $x = 2$, there can be no answer here. However, we can tell that $f(2)$ should be 4 ignoring the division by zero. We can tell this because as x becomes greater and nearer to 2 (approaching $x = 2$ from the left), the value of $f(x)$ approaches 4. Similarly, when x decreases and becomes nearer to $x = 2$ (approaching $x = 2$ from the right), the value of $f(x)$ approaches 4. Therefore, as both sides of $x = 2$ become closer and closer, they converge upon a single point: $f(2) = 4$.

1.1 Types of Limits

There are more than one type of limits. There is the traditional limit:

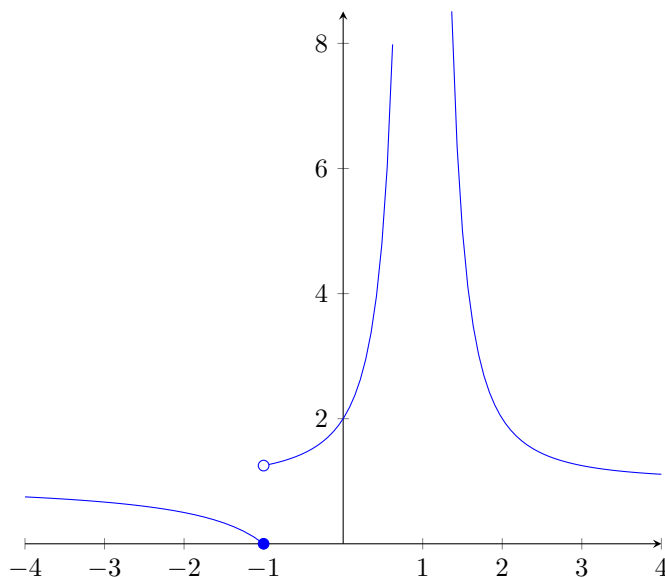
$$\lim_{x \rightarrow c} f(x)$$

which simply is read as the limit of the function $f(x)$ as x approaches c . There are more than just this first type, however.

Types We define $\lim_{x \rightarrow c}$ to be:

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x)$$

which should be read as: *the limit of $f(x)$ as x approaches c is equal to the limit of $f(x)$ as x approaches c from the right side and equal to the limit of $f(x)$ as x approaches c from the left side.* If $\lim_{x \rightarrow c^+} f(x) \neq \lim_{x \rightarrow c^-} f(x)$, we say that the limit of $f(x)$ at c must not exist. Let's take a look at a graph:



- 1.2 Estimating Limits
- 1.3 The Squeeze Theorem
- 1.4 L'hôpital's Rule
- 1.5 Asymptotes
- 1.6 Relative Magnitudes
- 1.7 Continuity
- 2 Derivatives
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 - 2.3 Rules
 - 2.4 Implicit Differentiation
 - 2.5 Maxima & Minima
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 - 2.7 Graphical Relations
 - 2.8 Relationship with Continuity
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 - 3.2 Instantaneous Rate of Change
 - 3.3 Tangent Line Approximations
 - 3.4 Physics
 - 3.5 Related Rates
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 - 3.7 Differential Equations
 - 3.8 Slope Fields
 - 3.9 Mean Value Theorem