Example 1: The squeeze theorem is uncommon and only applies (to Calculus) in a small number of situations. As a result, we will only prove the previous problem and then move on. For the squeeze theorem to work, we need to show that two *intersecting* graphs will be the maximum and minimum boundaries for a third function. In this case:

$$\begin{aligned} f(x) &\leq g(x) \\ -x^2 &\leq x^2 * \sin\left(\frac{4}{x}\right) \\ -1 &\leq \sin\left(\frac{4}{x}\right) \end{aligned}$$

That statement is true. The smallest value that $\sin(t)$ for some t can be is -1. Going the other way:

$$\begin{aligned} h(x) &\geq g(x) \\ x^2 &\geq x^2 * \sin\left(\frac{4}{x}\right) \\ 1 &\geq \sin\left(\frac{4}{x}\right) \end{aligned}$$

Again, a true statement. Because $f(x)$ and $h(x)$ intersect at $x = 0$, $g(x)$ must be between $f(0) = 0$ and $h(0) = 0$ where the only solution is $\lim_{x \to 0} g(x) = 0$.