

**The Slope of a Secant Line** The slope of a secant line through a point  $x$  and another point  $h$  units away will be the rise ( $\Delta y$ ) divided by the run ( $\Delta x$ ). Plugging in for the points  $(x, f(x))$  and  $(x + h, f(x + h))$ , we get

$$\frac{f(x + h) - f(x)}{x + h - x}$$

which solves to

$$\frac{f(x + h) - f(x)}{h}$$

## 0.1 The Derivative

The derivative is a tool that can find the slope of a tangent line at (almost) any point on a graph. While we can't find the slope between one point and itself because there is no difference, we can find the slope between some point and another that is **really** close to that point. We can minimize the difference between these two points by using a limit! We will find the difference between a point and a point whose position is almost zero units away:

$$\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

This is the derivative! If you were to plug in 0 for  $h$  immediately, you would divide by zero. Therefore, you must use the reduction technique for solving equations of this type. The derivative is often abbreviated  $\frac{d}{dt}f(t)$  with respect to some variable  $t$  and some function  $f$ ; or simply  $f'(x)$ . If you take  $n$  more derivatives of the same function past the first one, we call it the  $n^{th}$  order derivative. Derivatives beyond the first order are labeled  $\frac{d^2}{dt^2}f(t)$  or  $f''(t)$  for the second order;  $\frac{d^3}{dt^3}f(t)$  or  $f'''(t)$  for the third order; etc. We say the derivative is taken "with respect to" something else. In a pretty standard Calculus case, the objective is to find the change in  $y$  *with respect to*  $x$ . We use the notation to describe this relationship as  $\frac{dy}{dx}$ . Extending this, the second order derivative of  $y$  *with respect to*  $x$  is denoted  $\frac{d^2y}{dx^2}$ . When the derivative of a variable is taken with respect to itself, i.e.  $\frac{dx}{dx}$ , we do not write it as it is equal to 1.