Example 1: Find $\lim_{x\to\infty} \frac{x^3-x+1}{4x^3+2x^2+x-1}$. A method for solving this particular type of problem is shown before Calculus, but here is the reason why it works:

$$\lim_{x\to\infty} \frac{x^3-x+1}{4x^3+2x^2+x-1} = \frac{\infty}{\infty}$$

This is an indeterminate form so we can apply L'hôpital's rule:

$$\lim_{x\to\infty} \frac{x^3-x+1}{4x^3+2x^2+x-1} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{x^3-x+1}{4x^3+2x^2+x-1} = \lim_{x\to\infty} \frac{3x^2-1}{12x^2+4x+1}$$

$$\lim_{x\to\infty} \frac{3x^2-1}{12x^2+4x+1} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{3x^2-1}{12x^2+4x+1} = \lim_{x\to\infty} \frac{6x}{24x+4}$$

$$\lim_{x\to\infty} \frac{6x}{24x+4} = \lim_{x\to\infty} \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{6x}{24x+4} = \frac{6}{24}$$

$$\lim_{x\to\infty} \frac{6}{24} = \frac{6}{24} = \frac{1}{4}$$