Example 1: Find the volume of the solid bounded by $y = x^2$, $y = \sin\left(\pi x^2\right) + x$, x=0, and x=1 revolved around the y-axis.
The only way to calculate this is by finding the difference between x^2 and $\sin\left(\pi x^2\right) + x$, so we must use the shell method. Remember, the shell of a cylinder is $2 * \pi * r * height * width$. r is the distance away from the axis of revolution, which in this case is $x - 0 = x$. *height* will be the height of the rectangle, or $\sin\left(\pi x^2\right) + x - x^2$. *width* is going to be infinitely small, or dx. That leaves us with the volume being:

$$dV = 2\pi * x * \left(\sin\left(\pi x^2\right) + x - x^2\right) * dx$$

which simplifies down to

$$dV = 2\pi\left(x\sin\left(\pi x^2\right) + x^2 - x^3\right)dx$$

Now, we will add up all of these volumes between $x = 0$ and $x = 1$:

$$\begin{gathered}
V = \int_0^1 2\pi\left(x\sin(\pi x^2) + x^2 - x^3\right)dx \\
V = 2\pi\left(\int_0^1 x\sin(\pi x^2)dx + \int_0^1 x^2 dx - \int_0^1 x^3 dx\right) \\
u = x^2 \\
du = 2x\,dx \\
x = 0 \mapsto u = 0 \\
x = 1 \mapsto u = 1 \\
V = 2\pi\left(.5\int_0^1 \sin(\pi u)du + \int_0^1 x^2 dx - \int_0^1 x^3 dx\right) \\
V = 2\pi\left(\frac{1}{2\pi}(-\cos(\pi * 1) + \cos(\pi * 0)\right) + \frac{1}{3} - \frac{0}{3} - \frac{1}{4} + \frac{0}{4} \\
V = 2\pi\left(\frac{1}{2\pi}(1+1) + \frac{1}{3} - \frac{1}{4}\right) \\
V = 2 + \frac{\pi}{6}
\end{gathered}$$

Example 2: Find the volume of the solid bounded by $y = x^2$, $y = 4$, and $x = 0$ revolved around the y-*axis*.
It's probably best to approach this problem using the washer method. First, we have to find everything in terms of y:

$$\begin{gathered}
y = x^2 \\
\sqrt{y} = x
\end{gathered}$$

Now for the boundary points $x = 0$ and $x = 1$

$$x = 0 \mapsto y = 0$$
$$x = 2 \mapsto y = 4$$

Here's what we have so far: the boundaries are $y = 0$ and $y = 4$, the function is $x = \sqrt{y}$, and we are revolving around $x = 0$. Our infinitesimal volume is $\pi * r^2 * height$ which is:

$$dV = \pi \left(\sqrt{y}\right)^2 * dy$$

Therefore, our total sum of all of the volumes is:

$$V = \int_0^4 \pi * (x)^2 * dy$$
$$V = \pi \int_0^4 (\sqrt{y})^2 dy$$
$$V = \pi \int_0^4 y\, dy$$
$$V = \pi \left(.5(4)^2 - .5(0)^2\right)$$
$$V = 8\pi$$