u-Substitution is a very important tool to have in your integral toolbox. It works a bit like this:

> One of the most basic elements in math is the variable. Variables can represent *anything*, from a simple constant $r = 2$ to a complex system of other variables like $y = \ln(2x)$. These statements are true simply because we say they are. The only thing that we're doing when we assign a variable is essentially re-naming a phrase. In the previous example, we take $\ln(2x)$ and give it the name y.

Let's look at this equation:

$$f(x) = \int_0^2 x\cos(x^2+1)dx$$

Here, we are using x as our variable of choice. Unfortunately, there is no integral form that this falls under. We can change x to a different variable and the equation would still be true as long as the equality is kept:

$$x = t$$
$$f(t) = \int_0^2 x\cos(t^2+1)dt$$

Although this change doesn't change the value of the equality, it's not too helpful. We can assign an entire phrase to a variable though, and we will call that phrase stand-in u. In this case, let's say

$$u = x^2 + 1$$

That helps clear up the bottom. Our equation is now

$$f(x) = \int_0^2 t\cos(u)dx$$

That wasn't helpful at all. We now have one integral and two variables. A mess! That being said, we can start putting things in terms of u:

$$x = 0 \mapsto u = 1$$
$$x = 2 \mapsto u = 5$$

So we can rewrite the equation as

$$f(u) = \int_1^5 x\cos(u)dx$$

Again, unhelpful. One more thing, though. dx is the infinitesimal value of x that we're using in this integral. Because u is just x changed slightly, we can find du in terms of dx!

$$\begin{aligned} u &= x^2 + 1 \\ du &= 2x \quad dx \end{aligned}$$

Which can be re-written as

$$dx = \frac{du}{2x}$$

which we can substitute back into the integral:

$$f(u) = \int_1^5 x\cos(u)\frac{du}{2x}$$

which simplifies down to

$$f(u) = \frac{1}{2}\int_1^5 \cos(u)du$$

which is an easy integral! $f(u) = \sin(5) - \sin(1)$.