

Example 1: Find the rate of change in the volume of a sphere if the radius is $\sin^2(t)$ at time t . We will start with finding the derivative of the volume equation, implicitly:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \frac{dV}{dt} &= \frac{4}{3}\pi 3r^2 \frac{dr}{dt} \\ \frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \end{aligned}$$

Next, we will find the derivative of the radius equation:

$$\begin{aligned} r &= \sin^2(t) \\ r &= \sin(t) * \sin(t) \\ \frac{dr}{dt} &= \sin(t) * \cos(t) + \cos(t) * \sin(t) \\ \frac{dr}{dt} &= 2 \sin(t) \cos(t) \end{aligned}$$

Knowing these two equations and the equation for r , we can substitute:

$$\begin{aligned} \frac{dV}{dt} &= 4\pi r^2 * 2 \sin(t) \cos(t) \\ \frac{dV}{dt} &= 8\pi (\sin^2(t))^2 \sin(t) \cos(t) \\ \frac{dV}{dt} &= 8\pi \sin^5(t) \cos(t) \end{aligned}$$