Example 1: Find when the function $f(x) = 1/4x^4 + x^3 - 1/2x^2 - 3x + 17$ is: Concave up increasing, concave down increasing, concave up decreasing, and concave down decreasing.

$$f(x) = 1/4x^4 + x^3 - 1/2x^2 - 3x + 17$$
$$f'(x) = x^3 + 3x^2 - x - 3$$
$$f''(x) = 3x^2 + 6x - 1$$

First, we will find when the first derivative is positive or negative to determine where $f(x)$ is increasing or decreasing.

$$\begin{aligned} f'(x) &= x^3 + 3x^2 - x - 3 \\ f'(x) &= x^3 - x + 3x^2 - 3 \\ f'(x) &= x(x^2-1) + 3(x^2-1) \\ f'(x) &= (x+3)(x^2-1) \\ f'(x) &= (x+3)(x-1)(x+1) \\ 0 &= (x+3)(x-1)(x+1) \\ x &= -3, \pm 1 \end{aligned}$$

$(-\infty,-3)$	$(-3,-1)$	$(-1,1)$	$(1,\infty)$
-	+	-	+

So $f(x)$ is increasing over $(-3,-1)$ and $(1,\infty)$, and decreasing over $(-\infty,-3)$ and $(-1, 1$. Now, we will find where the function is concave up and down:

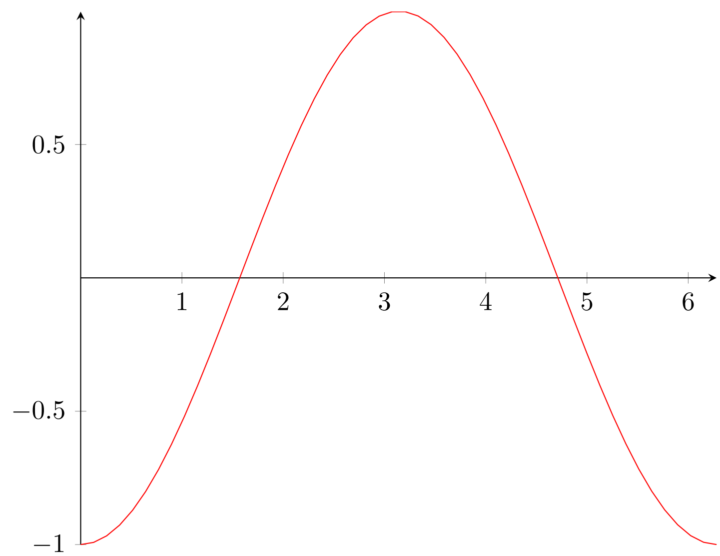
$$\begin{aligned} f''(x) &= 3x^2 + 6x - 1 \\ 0 &= 3x^2 + 6x - 1 \\ x &= \frac{-6 \pm \sqrt{6^2 - 4*3*-1}}{2*3} \\ x &= \frac{-6 \pm \sqrt{48}}{6} \\ x &= \frac{-3 \pm 2\sqrt{3}}{3} \\ x &\approx -2.155, 0.155 \end{aligned}$$

$(-\infty,-2.155)$	$(-2.155,-0.155)$	$(-0.155,\infty)$
+	-	+

So $f(x)$ is concave up over $(-\infty,-2.155)$ and $(-0.155,\infty)$, and concave down over $(-2.155,-0.155)$.

Example 2: Find where $\sin(x - \frac{\pi}{2})$ is concave up and down:

$$f(x) = \sin(x - \frac{\pi}{2})$$
$$f'(x) = \cos(x - \frac{\pi}{2})$$
$$f''(x) = -\sin(x - \frac{\pi}{2})$$



$\sin(x - 1)$

Interestingly, the concavity is just the inverse of the function itself. $\text{Sin}(x - \frac{\pi}{2})$ is concave up when it is negative, written out mathematically: $(-\frac{\pi}{2}, \frac{\pi}{2}) + n * 2\pi$, where n is any integer.