

Example 1: Find the derivative of $f(x) = \sqrt{x^3}$
 This can be rewritten as

$$f(x) = x^{1.5}$$

which fits into derivative form #2:

$$f(x) = x^{1.5}$$

$$f'(x) = 1.5 * x^{1.5-1} \frac{dx}{dx}$$

$$f'(x) = 1.5x^{.5}$$

$$f'(x) = 1.5\sqrt{x}$$

Example 2: Find the derivative of $f(x) = \ln(x^2 + 2x + 1)$
 This can be rewritten using rules of logarithms as:

$$f(x) = \ln(x^2 + 2x + 1)$$

$$f(x) = \ln((x+1)^2)$$

$$f(x) = 2\ln(x+1)$$

$$f'(x) = 2 * \frac{1}{x+1} \frac{dx}{dx}$$

$$f'(x) = \frac{2}{x+1}$$

Example 3: Find the derivative of $f(x) = \sin\left(\frac{\cos(x)}{x^2 + 1}\right)$

This will require the chain rule (twice) and the product rule. First, let's do the chain rule:

$$f(x) = \sin\left(\frac{\cos(x)}{x^2 + 1}\right)$$

$$\cos\left(\frac{\cos(x)}{x^2 + 1}\right) * \frac{d}{dx} \left(\frac{\cos(x)}{x^2 + 1}\right)$$

Now we have to find the still unfinished part:

$$\begin{aligned}
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \frac{d}{dx} \left(\frac{\cos(x)}{x^2+1}\right) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \frac{d}{dx} (\cos(x)(x^2+1)^{-1}) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \left(\cos(x) * \frac{d}{dx} ((x^2+1)^{-1}) + (-\sin(x)) * (x^2+1)^{-1}\right) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * (\cos(x) * -(x^2+1)^{-2} * 2x - \sin(x) * (x^2+1)^{-1}) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \left(\frac{2x \cos(x)}{(x^2+1)^2} - \frac{\sin(x)}{x^2+1}\right) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \left(\frac{2x \cos(x)}{(x^2+1)^2} - \frac{\sin(x)(x^2+1)}{(x^2+1)^2}\right) \\
& \cos\left(\frac{\cos(x)}{x^2+1}\right) * \left(\frac{2x \cos(x) - \sin(x)(x^2+1)}{(x^2+1)^2}\right) \\
& f'(x) = \cos\left(\frac{\cos(x)}{x^2+1}\right) * \frac{2x \cos(x) - \sin(x)(x^2+1)}{(x^2+1)^2}
\end{aligned}$$

It's not pretty but Calculus rarely is.