Noah's Guide to Calculus

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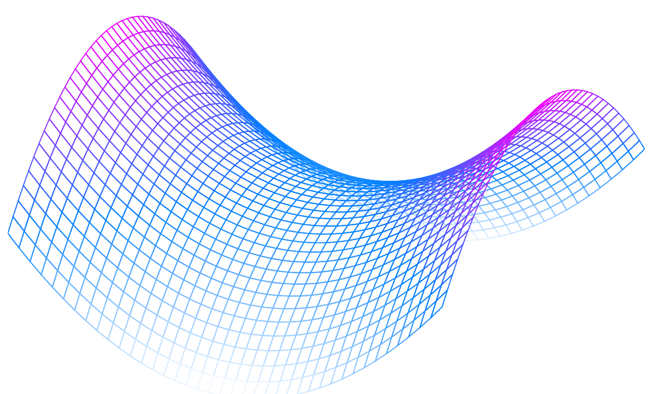

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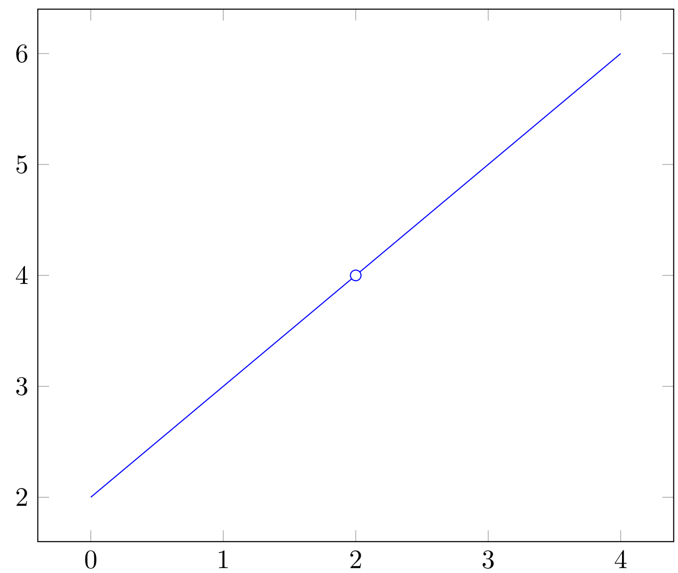
1 Limits

Limits are a way of skirting the normal rules of math. Without the knowledge of limits, whenever a function divides by 0 or involves ∞ in any way, calculations become impossible. Limits take the rules of math a little less seriously and can be used to calculate what a value "should be". A simple example of where limits come in handy is when there is a "hole" in a graph:



$$f(x) = \frac{x^2 - 4}{x - 2}$$

Because $f(x)$ divides by 0 when $x = 2$, there can be no answer here. However, we can tell that $f(2)$ should be 4 ignoring the division by zero. We can tell this because as x becomes greater and nearer to 2 (approaching $x = 2$ from the left), the value of $f(x)$ approaches 4. Similarly, when x decreases and becomes nearer to $x = 2$ (approaching $x = 2$ from the right), the value of $f(x)$ approaches 4. Therefore, as both sides of $x = 2$ become closer and closer, they converge upon a single point: $f(2) = 4$.

1.1 Types of Limits

There are more than one type of limits. There is the traditional limit:

$$\lim_{x \to c} f(x)$$

which simply is read as the limit of the function $f(x)$ as x approaches c. There are more than just this first type, however.

Types We define $\lim\limits_{x \to c}$ to be:

$$\lim_{x \to c} f(x) = \lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$$

which should be read as: *the limit of $f(x)$ as x approaches c* **is equal to** *the limit of $f(x)$ as x approaches c from the right side* **and equal to** *the limit of $f(x)$ as x approaches c from the left side.* If $\lim\limits_{x \to c^+} f(x) \neq \lim\limits_{x \to c^-} f(x)$, we say that the limit of $f(x)$ at c must not exist. Let's take a look at a graph: