

Example 1: Using implicit differentiation, find the derivative of the circle $(x - 4)^2 + (y + 3)^2 = 16$

$$\begin{aligned}(x - 4)^2 + (y + 3)^2 &= 16 \\ 2(x - 4)^1 \frac{dx}{dt} + 2(y + 3)^1 \frac{dy}{dt} &= 0 \\ (2x - 8) \frac{dx}{dt} + (2y + 6) \frac{dy}{dt} &= 0\end{aligned}$$

Pretty simple.

Example 2: Using implicit differentiation, find the derivative of the function $y^2 = 2xy + x^2$

$$\begin{aligned}y^2 &= 2xy + x^2 \\ 2y \frac{dy}{dt} &= 2x \frac{dy}{dt} + 2y \frac{dx}{dt} + 2x \frac{dx}{dt} \\ -2y \frac{dy}{dt} + 2y \frac{dy}{dt} &= 2x \frac{dx}{dt} + 2x \frac{dx}{dt} \\ 0 &= 4x \frac{dx}{dt}\end{aligned}$$