

1. Equations of Tangent & Secant Lines

1.1 Essential Questions

Essential Questions 1.1

1. How do we find the equation of the tangent line of a given function at the point $P(a, b)$?
2. How do we find the equation of the tangent line of a given function at $x = a$?

1.2 Finding the Equation of the Tangent Line

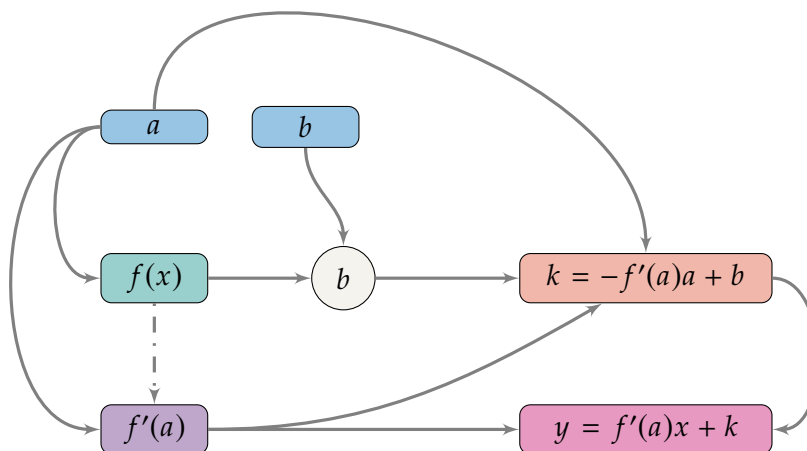


Figure 1.1: Finding the Equation of a Tangent Line Workflow

For a given function, $f(x)$, our goal is to find the equation of the tangent line at the point $P(a, b)$, which can be expressed in slope-intercept form as:

$$y = f'(a)x + k$$

Where $f'(a)$ is the value of the derivative, slope, at $x = a$ and k is the y -intercept. We therefore need to:

1. Find the derivative of the function $f(x)$: $f'(x)$
2. Find the value of the derivative at $x = a$: $f'(a)$
3. Find the value of the y -intercept: $k = -f'(a)a + b$:
4. If the ordinate b is not explicitly given, then find $f(a) = b$

Example 1.1 – id:20151011-154209.

Find the equation of the line tangent to the curve of the function $f(x) = 2x^2 + 3x + 7$ at the point $P(2, 21)$.

**Solution:**

Find the derivative of $f(x)$

$$f'(x) = 4x + 3 \text{ goto ??}$$

Evaluate the derivative at $x = 2$

$$f'(2) = 4[2] + 3 \quad \text{SPE(??)}$$

$$f'(2) = 8 + 3 \quad \text{OOM(??)}$$

$$f'(2) = 11 \quad \text{OOA(??)}$$

Find the y -intercept, k , of the equation of the tangent line.

$$y = f'(x)x + k$$

$$[21] = [11][2] + k \quad \text{SPE(??)}$$

$$21 = 22 + k \quad \text{OOM(??)}$$

$$-22 + [21] = -22 + [22 + k] \quad \text{SPE+AI}$$

$$-22 + 21 = (-22 + 22) + k \quad \text{APA(??)}$$

$$-1 = 0 + k \quad \text{OOA(??)}$$

$$-1 = k \quad \text{AId(??)}$$

$$-1 = k \quad \text{ONeg(??)}$$

$$k = -1 \quad \text{SyPE(??)}$$

The equation of the tangent line is

$$y = f'(2)x + k$$

$$y = 11x - 1 \quad \text{SPE(??)}$$