1. Solving Quadratic Equations

1.1 Power Inverse

Example 1.1 – id:20141107-131748.

Solve the equation $2 - x^2 = 0$ for x

S

Solution:

$$
\begin{aligned}
2 - 1x^2 &= 0 && \text{MId(??)}\\
2 + \neg 1x^2 &= 0 && \text{DOS(??)}\\
\left[2 + \neg 1x^2\right] + 1x^2 &= [0] + 1x^2 && \text{SPE(??)} + \text{AI(??)}\\
2 + \left(\neg 1x^2 + 1x^2\right) &= 0 + 1x^2 && \text{APA(??)}\\
2 + 0 &= 0 + 1x^2 && \text{OOA(??)}\\
2 &= 1x^2 && \text{AId(??)}\\
2 &= x^2 && \text{MId(??)}\\
\pm [2]^{\frac{1}{2}} &= \left[x^2\right]^{\frac{1}{2}} && \\
\pm 2^{\frac{1}{2}} &= x && \text{PoPo(??)}\\
\pm\sqrt{2} &= x && \text{PoTR(??)}\\
x &= \pm\sqrt{2} && \text{SyPE(??)}
\end{aligned}
$$

■

Example 1.2 – id:20151012-192313.

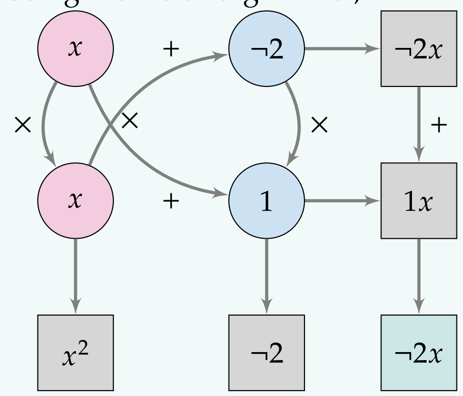
Solve the equation $6x^2 - 6x - 12 = 0$

S

Solution:

$$
\begin{aligned}
6x^2 + \neg 6x + \neg 12 &= 0 && \text{DOS(??)}\\
6(1x^2 \neg 1x + \neg 2) &= 0 && \text{DPF(??)}
\end{aligned}
$$

Using the factor orgnanizer,



Solving the linear equations using ZFP (**??**)

$$
\begin{aligned}
x_1 + \neg 2 &= 0 && \text{Case I}\\
[x_1 + \neg 2] + 2 &= [0] + \neg 2 && \text{AI(??), SPE(??)}\\
x_1 + (\neg 2 + 2) &= 0 + \neg 2 && \text{APA(??)}\\
x_1 + 0 &= \neg 2 && \text{OOA(??)}\\
x_1 &= \neg 2 && \text{AId(??)}\\
x_1 &= -2 && \text{ONeg(??)}
\end{aligned}
$$

$$
\begin{aligned}
x_2 + 1 &= 0 && \text{Case II}\\
[x_2 + 1] + \neg 1 &= [0] + \neg 1 && \text{AI(??), SPE(??)}\\
x_2 + (1 + \neg 1) &= 0 + \neg 1 && \text{APA(??)}\\
x_2 + 0 &= \neg 2 && \text{OOA(??)}\\
x_2 &= \neg 2 && \text{AId(??)}\\
x_2 &= -2 && \text{ONeg(??)}
\end{aligned}
$$

1.2 Completing The Square

Completing the square is an algebraic algorithm used to find the solutions of quadratic equations of the form, $ax^2 + bx + c = 0$. Essentially, we want to manipulate this equation such that $x =$ some values. To gain some understanding of how this algorithm works, we will consider each step individually. Let's begin with a quadratic equation in the general form: $ax^2 + bx + c = 0$.

Since we are trying to manipulate the equation $ax^2+bx+c = 0$ such that $x =$ some value, we first want the coefficient factor a to be equal to 1. This is done by multiplying both expressions by the multiplicative inverse of a (step 1) followed by simplifying both expressions.

$$
\begin{aligned}
\frac{1}{a}[ax^2 + bx + c] &= \frac{1}{a}[0] && \text{SPE(??) + MI(??)}\\
\frac{1}{a}\cdot ax^2 + \frac{1}{a}\cdot bx + \frac{1}{a}\cdot c &= \frac{1}{a}[0] && \text{DPE(??)}\\
\frac{1}{a}\cdot a\cdot x^2 + \frac{1}{a}\cdot b\cdot x + \frac{1}{a}\cdot c &= \frac{1}{a}[0] && \text{JTC(??)}\\
x^2 + \frac{b}{a}\cdot x + \frac{c}{a} &= 0 && \text{OOM(??)}\\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 && \text{CTJ(??)}
\end{aligned}
$$

We now have three terms in the left hand expression where the first two terms have at least one variable factor, x. The problem is that the third term is a constant and we want $x =$ some value. This text step is to get rid of the $\frac{c}{a}$ term, which can be done by using the additive inverse followed by simplifying both expressions.

$$
\begin{aligned}
\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] + \neg\frac{c}{a} &= [0] + \neg\frac{c}{a} && \text{SPE(??) + AI(??)}\\
x^2 + \frac{b}{a}x + \frac{c}{a} + \neg\frac{c}{a} &= 0 + \neg\frac{c}{a} && \text{APA(??)}\\
x^2 + \frac{b}{a}x &= \neg\frac{c}{a} && \text{OOA(??)}
\end{aligned}
$$

The next step is called completing the square. The idea is to add a *NeW* constant, k, to the left-hand expression, $x^2 + \frac{b}{a}x + k$, such that the quadratic expression can then be factored as two identical factors, $(x + m)(x + m) = (x + m)^2$, where $k = m \cdot m$. Notice that since we are adding a constant term, k, to the left-hand expression, then we must also add this constant, k, to the right-hand expression, $x^2 + \frac{b}{a}x + k = \neg\frac{c}{a} + k$. To determine the values of both m and k we should refer to the the organization of the two factors that make up the product of a quadratic expression, $x^2 + \frac{b}{a}x + k = (x + m)^2$.

Since both factors of this new quadratic expression are the same, both terms that make up the middle term, must also be the same. We know that $mx + mx = \frac{b}{a}x$, so we should be able to determine the value of m from this equation. If we can determine the value of m, then we can determine the value of k.

$$
\begin{aligned}
\frac{b}{a}x &= mx + mx\\
&= 2mx && \text{OOA(??)}
\end{aligned}
$$

Solving for m,

$$
\begin{aligned}
2mx &= \frac{b}{a}x \\
2 \cdot m \cdot x &= \frac{b}{a} \cdot x && \text{JTC}(\textbf{??}) \\
\frac{1}{2}\left[2 \cdot m \cdot x\right] &= \frac{1}{2}\left[\frac{b}{a} \cdot x\right] && \text{SPE}(\textbf{??}) + \text{MI}(\textbf{??}) \\
m \cdot x &= \frac{b}{2a} \cdot x && \text{OOM}(\textbf{??}) \\
\left[m \cdot x\right]\frac{1}{x} &= \left[\frac{b}{2a} \cdot x\right]\frac{1}{x} && \text{SPE}(\textbf{??}) + \text{MI}(\textbf{??}) \\
m &= \frac{b}{2a} && \text{OOM}(\textbf{??})
\end{aligned}
$$

$$\begin{aligned}
\left[x^2 + \frac{b}{a}x\right] + \left(\frac{b}{2a}\right)^2 &= \left[\neg\frac{c}{a}\right] + \left(\frac{b}{2a}\right)^2 && \text{SPE(\textbf{??})}\\
x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= \neg\frac{c}{a} + \left(\frac{b}{2a}\right)^2 && \text{APA(\textbf{??})}\\
\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) &= \neg\frac{c}{a} + \left(\frac{b}{2a}\right)^2 && \text{DPF(\textbf{??})}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c}{a} + \left(\frac{b}{2a}\right)^2 && \text{PoTF(\textbf{??})}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c}{a} + \frac{(b)^2}{(2a)^2} && \text{PoQPo(\textbf{??})}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c}{a} + \frac{b^2}{2^2a^2} && \text{PoPrPo}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c}{a} + \frac{b^2}{4a^2} && \text{OOE}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{4a^2} && \text{MId}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{c \cdot 4 \cdot a}{a \cdot 4 \cdot a} + \frac{b^2}{4a^2} && \text{JTC}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{4 \cdot a \cdot c}{4 \cdot a \cdot a} + \frac{b^2}{4a^2} && \text{CPM}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{4 \cdot a \cdot c}{4 \cdot a^2} + \frac{b^2}{4a^2} && \text{PrCBPo}\\
\left(x + \frac{b}{2a}\right)^2 &= \neg\frac{4ac}{4a^2} + \frac{b^2}{4a^2} && \text{CTJ}\\
\left(x + \frac{b}{2a}\right)^2 &= \frac{\neg 4ac + b^2}{4a^2} && \text{CD}\\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 + \neg 4ac}{4a^2} && \text{CPA}\\
\left[\left(x + \frac{b}{2a}\right)^2\right]^{\frac{1}{2}} &= \pm\left[\frac{b^2 + \neg 4ac}{4a^2}\right]^{\frac{1}{2}} && \text{PoI}
\end{aligned}$$

$$x + \frac{b}{2a} = \pm \left[\frac{b^2 + \neg 4ac}{4a^2} \right]^{\frac{1}{2}} \qquad \text{PoPrPo}$$

$$x + \frac{b}{2a} = \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{[4a^2]^{\frac{1}{2}}} \qquad \text{PoQPo}$$

$$x + \frac{b}{2a} = \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{4^{\frac{1}{2}}a} \qquad \text{PoPrPo}$$

$$x + \frac{b}{2a} = \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{OOE}$$

$$\left[x + \frac{b}{2a}\right] + \neg \frac{b}{2a} = \left[\pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a}\right] + \neg \frac{b}{2a} \qquad \text{AI}$$

$$x + \frac{b}{2a} + \neg \frac{b}{2a} = \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} + \neg \frac{b}{2a} \qquad \text{APA}$$

$$x + \frac{b}{2a} + \neg \frac{b}{2a} = \neg \frac{b}{2a} \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{CPA}$$

$$x = \neg \frac{b}{2a} \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{OOA}$$

$$x = \frac{\neg b \pm \left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{CD}$$

$$x = \frac{\neg b \pm \left[b^2 - 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{DOS}$$

$$x = \frac{\neg b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{ETR}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{ONeg}$$