Math Reference Sheet

Number Systems

$\mathbb{N}$	Natural numbers	$\mathbb{N} = \{1, 2, 3, \ldots\}$
$\mathbb{Z}$	Integers	$\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Q}$	Rational	$\mathbb{Q} = \left\{ \frac{m}{n} \mid m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0 \right\}$
$\mathbb{R}$	Real numbers	
$\mathbb{C}$	Complex numbers	$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$

Prime Numbers 2-997

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743
751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997

Prime Divisor rules

2	the 1's digit is even
3	sum of digits is divisible by 3
5	the 1's digit is 0 or 5

Reducing Fractions Process - RF

Reduce the fraction $\frac{m}{n}$

1. Simplify by factoring m
2. Simplify by factoring n
3. Find the $\gcd(m, n)$
4. If the $\gcd(m, n) = 1$ the fraction is reduced.
5. $\gcd(m, n)$ is the MId

Operations

DELIM	Delimiters
DO	Dyadic Operations
OOA	Operation of Addition
OOD	Operation of Division
OOE	Operation of Exponentiation
OON	Operation of Negation
OOS	Operation of Subtraction
OOO	Order of Operations
UO	Unary Operations

Order Operations

1. DELIM
2. DO (OOE, OOM, OOD, OOA, OOS)
3. UO (OON)

Operation of Negation

ONeg	Operation of Negation Notation $-a = \neg a$

Operation of Subtraction

DOS	Definition of Subtraction $a + \neg b = a - b$

Operation of Addition

APA	Associative Property of Addition $(a + b) + c = a + (b + c)$
CPA	Commutative Property of Addition $a + b = b + a$
DPF	Distributive Property Factoring $a \cdot b + a \cdot c = a(b + c)$ $b \cdot a + c \cdot a = (b + c)a$
CD	Common Denominator $\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$

Operation of Multiplicaiton

APM	Associative Property of Multiplication $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
CPM	Commutative Property of Multiplication $a \cdot b = b \cdot a$
CTJ	Center-Dot to Juxtaposition $a \cdot b = ab$
DPE	Distributive Property Expanding $a(b + c) = a \cdot b + a \cdot c$ $(b + c)a = b \cdot a + c \cdot a$
JTC	Juxtaposition to Center-Dot $ab = a \cdot b$
MC	Center-Dot Notation $a \cdot b$
MJ	Juxtaposition Notation $ab, a(b), (a)b, (a)(b), a[b], [a]b, [a][b]$
MT	Times Notation $a \times b$

Operation of Division

DOD	Definition of Division $a \div b = \frac{a}{b} = a \cdot b^{-1}, b \neq 0$
FN	Fraction Numerator (upstaris)
FD	Fraction Denominator (downstairs)
RF	Reduce Fraction

Powers

FTPo — Factor to Power
$$a_n \cdot a_{n-1} \cdot \ldots \cdot a_2 \cdot a_1 = a^n$$

PoNegE — Power Negative Exponent
$$b^{-k} = \frac{1}{b^k}$$

PoPo — Power of a Power
$$(b^m)^k = b^{m \cdot k}$$

PoQ — Power of a Quotient
$$\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}, b \neq 0$$

PoPr — Power of a Product
$$(a \cdot b)^k = a^k \cdot b^k$$

PoQPo — Power of a Quotient of Powers
$$\left(\frac{a^m}{b^n}\right)^k = \frac{a^{m \cdot k}}{b^{n \cdot k}}, b \neq 0$$

PoPrPo — Power of a Product of Powers
$$(a^m b^n)^k = a^{m \cdot k} b^{n \cdot k}$$

PoTR — Power to Radical
$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

PoTL — Power to Logarithm
$$y = b^x \Rightarrow x = \log_b y$$

PoTF — Power to Factor
$$(a)^n = a_1 \cdot a_2 \cdot \ldots \cdot a_n$$

PrCBPo — Product of Common Base Powers
$$b^m \cdot b^n = b^{m+n}$$

QCBPo — Quotient of Common Base Powers
$$\frac{b^m}{b^n} = b^{m-n}$$

RTPo — Radical to Power
$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Identities

AId — Additive Identity
$$a + 0 = a$$

MId — Multiplicative Identity
$$a \cdot 1 = a$$

PoId — Power Identity
$$b^0 = 1, \text{ given } b > 0$$

Inverses

ArcCos — Cosine Inverse
$$\cos^{-1}(\cos \theta) = \theta$$

ArcSin — Sine Inverse
$$\sin^{-1}(\sin \theta) = \theta$$

ArcTan — Tangent Inverse
$$\tan^{-1}(\tan \theta) = \theta$$

AI — Additive Inverse
$$a + (-a) = 0$$

EI — Exponential Inverse
$$\log_a(a^x) = x$$

LI — Logarithmic Inverse
$$a^{\log_a x} = x$$

MI — Multiplicative Inverse
$$a \cdot \frac{1}{a} = 1 = a \cdot a^{-1}, a \neq 0$$

PoI — Power Inverse
$$\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = x$$

Equality & Inequality

RPE — Reflexive Property of Equality
$$a = a$$

SPE — Substitution Property of Equality
$$a = b \text{ then } F(a) = F(b)$$

SPIn — Substitution Property of Inequality
$$a < b, \text{ then } a + c < b + c$$
$$a < b \text{ and } c > 0, \text{ then } ca < cb$$
$$a < b \text{ and } c < 0, \text{ then } ca > cb$$

SyPE — Symmetric Property of Equality
$$a = b \text{ then } b = a$$

TPE — Transitive Property of Equality
$$\text{if } a = b \text{ and } b = c, \text{ then } a = c$$

TPIn — Transitive Property of Inequality
$$\text{if } a < b \text{ and } b < c, \text{ then } a < c$$

ZPr — Zero Product Property
$$\text{if } a \cdot b = 0, \text{ then } a = 0 \text{ or } b = 0$$

Simplify Expressions Workflow

1. MId
2. ONeg
3. DOS
4. DELIM Goto 36, 21
5. DPE
6. JTC
7. CPM Goto 25
8. APM
9. OOM
10. RF
11. CTJ
12. CPA
13. DPF
14. APA
15. RF
16. OOA
17. AId Goto 4
18. DOS
19. ONeg
20. MId **DONE!**
21. PoId
22. PoTF
23. RTPo
24. PoNegE Goto 4
25. PoPr
26. PoQ
27. PoPrPo
28. PoQPo
29. PrCBPo
30. QCBPo
31. PoPo
32. PoNegE
33. OOE
34. PoTR
35. PoId Goto 8
36. LPoPo
37. LPrCBPo
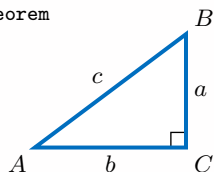38. LQCBPo
39. LEF Goto 4

Logarithms

LEV — Logarithm Exponent Visible
$$\log_b y \Rightarrow \log_b y = x$$

LPoPo — Logarithm Power of a Power
$$\log_b x^n = n \log_b x$$

LPrCBPo — Logarithm Product of Common Base Powers
$$\log_b(mn) = \log_b m + \log_b n$$

LQCBPo — Logarithm Quotient of Common Base Powers
$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

LTPo — Logarithm to Power
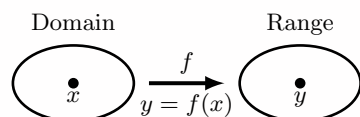$$x = \log_b y \Rightarrow y = b^x$$

Pascal's Triangle

```
            1
         1     1
      1     2     1
   1     3     3     1
1     4     6     4     1
1   5    10    10    5    1
```

Horizontal Line Test

FHLT   Function Horizontal Line Test

$y = f(x)$ $y = f(x)$
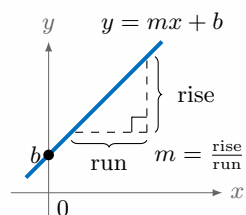
f is one-to-one f is not one-to-one

Quadratic Functions

If $ax^2 + bx + c = 0$, where $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $b^2 - 4ac > 0$ Two distinct real solutions
2. $b^2 - 4ac = 0$ Two repeated real solutions
3. $b^2 - 4ac < 0$ Two distinct complex solutions
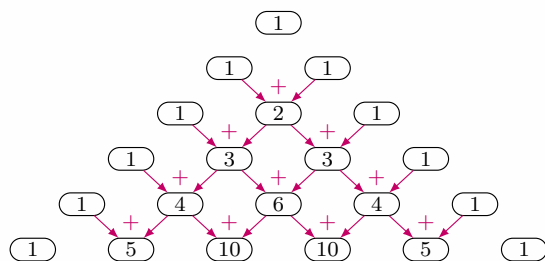
Pythagorean Theorem

PyThm   Pythagorean Theorem
$a^2 + b^2 = c^2$

Linear Functions

$y = mx + b$

rise

$m = \frac{\text{rise}}{\text{run}}$

run

DBP   Distance betweent $P_1 = (x_1, y_1)$
        & $P_2 = (x_2, y_2)$
$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MBP   Midpoint between $P_1 = (x_1, y_1)$
        & $P_2 = (x_2, y_2)$
$$\text{Midpoint of } P_1 P_2 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{3} \right)$$

Line Slope   Line Slope through $P_1 = (x_1, y_1)$
        & $P_2 = (x_2, y_2)$
$m = \frac{y_2 - y_1}{x_2 - x_1}$

PSE   Point slope equation though $P(x_1, y_2)$
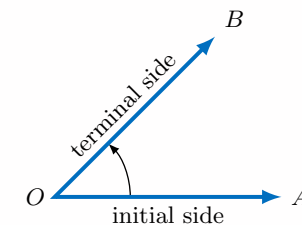$y - y_1 = m(x - x_1)$

SIE   Slope-intercept equation
$y = mx + b$

PrSPL   Product of slopes - Perpendicual Lines
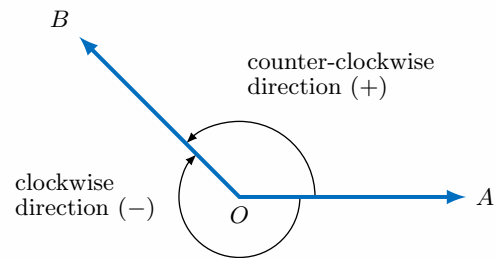$m_1 m_2 = -1$

$y = m_2 x + b_2$ $y = m_1 x + b_1$

Function

Domain   Range

f

$y = f(x)$
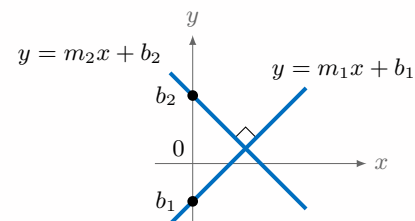
Greek Alphabet

Letters		Name	Letters		Name
A	α	alpha	N	ν	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ϵ	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	Υ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

Function Vertical Line Test

FVLT   Function Vertical Line Test

$y = f(x)$ $y = f(x)$

f is a function f is not a function

Angles: Components

terminal side

initial side

Angle Direction & Magnitude

B

counter-clockwise direction $(+)$

clockwise direction $(-)$

O

A

Classification of Angles

AA   Acute Angle
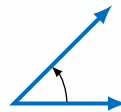
RA   Right Angle
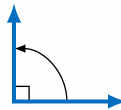
OA   Obtuse Angle

SA   Straight Angle

Trigonometric Function Definitions

hypotenuse c B
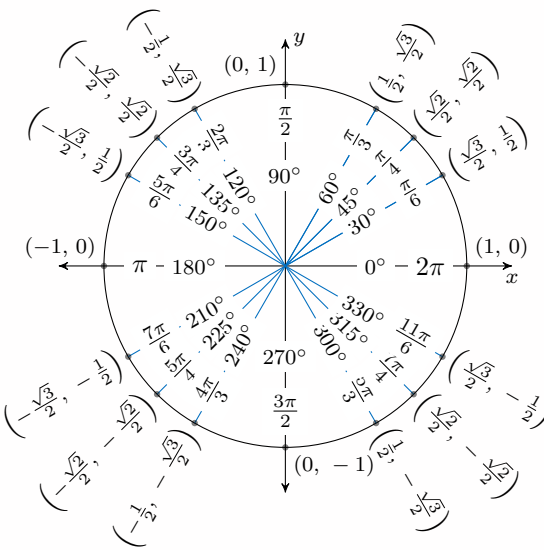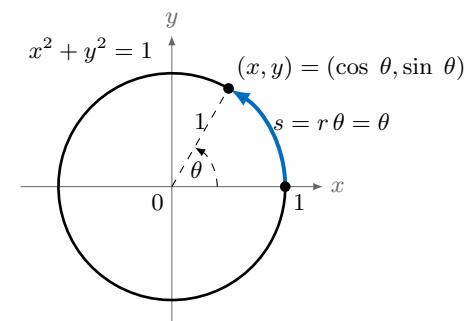
opposite a

A b C

adjacent

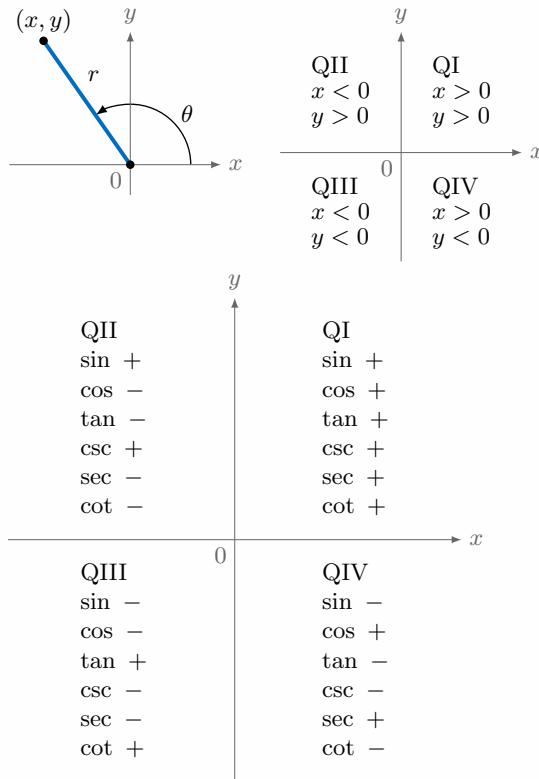$$\sin A = \frac{\text{opp}}{\text{hyp}} \qquad \csc A = \frac{\text{hyp}}{\text{opp}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} \qquad \sec A = \frac{\text{hyp}}{\text{adj}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} \qquad \cot A = \frac{\text{adj}}{\text{hyp}}$$

Radians

Rad   Radian

$1 \text{ radian} = \frac{180}{\pi} \approx 57.3^\circ$

r

θ

O r

$\theta = 1$ radian

Unit Circle

$x^2 + y^2 = 1$

$(x, y) = (\cos\ \theta, \sin\ \theta)$

$s = r\,\theta = \theta$

1

θ

0 1
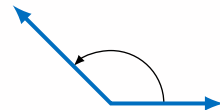
Angle Pairings

ComA   Complimentary Angles

$\angle B$ $\angle A$
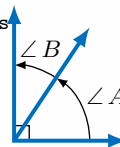
SA   Supplementary Angles

$\angle A$ $\angle B$

ConA   Conjugate Angles

$\angle B$ $\angle A$

Arc Length and Sector Area

AL   Arc Length

$s = r\theta$

ASect   Area of a Sector

$A = \frac{1}{2}r^2\theta$

r s

θ A

O

Quadrants

QII $x < 0$ $y > 0$	**QI** $x > 0$ $y > 0$
QIII $x < 0$ $y < 0$	**QIV** $x > 0$ $y < 0$

QII	**QI**
sin $+$	sin $+$
cos $-$	cos $+$
tan $-$	tan $+$
csc $+$	csc $+$
sec $-$	sec $+$
cot $-$	cot $+$

QIII	**QIV**
sin $-$	sin $-$
cos $-$	cos $+$
tan $+$	tan $-$
csc $-$	csc $-$
sec $-$	sec $+$
cot $+$	cot $-$

Trigonometric Identities

EOId — **Trigonometric Even/Odd Identities**
$$\sin -\theta = -\sin\theta \qquad \cos -\theta = \cos\theta$$
$$\csc -\theta = -\csc\theta \qquad \sec -\theta = \sec\theta$$
$$\tan -\theta = -\tan\theta \qquad \cot -\theta = -\tan\theta$$

RId — **Trigonometric Reciprocal Identities**
$$\sin\theta = \tfrac{1}{\csc\theta} \qquad \cos\theta = \tfrac{1}{\sec\theta} \qquad \cot\theta = \tfrac{1}{\tan\theta}$$
$$\csc\theta = \tfrac{1}{\sin\theta} \qquad \sec\theta = \tfrac{1}{\cos\theta} \qquad \tan\theta = \tfrac{1}{\cot\theta}$$

PyId — **Trigonometric Pythagorean Identities**
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = \tan^2\theta + 1 \qquad \csc^2\theta = 1 + \cot^2\theta$$

TanId — **Tangent Identity**
$$\tan\theta = \tfrac{\sin\theta}{\cos\theta}$$

CotId — **Cotangent Identity**
$$\cot\theta = \tfrac{\cos\theta}{\sin\theta}$$

SinDAId — **Sine Double Angle Identity**
$$\sin 2\theta = 2\sin\theta\cos\theta$$

CosDAId — **Cosine Double Angle Identity**
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

TanDAId — **Tangent Double Angle Identity**
$$\tan 2\theta = \tfrac{2\tan\theta}{1-\tan^2\theta}$$

SinSAId — **Sine Sum of Angles Identity**
$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

SinDiffAId — **Sine Difference of Angles Identity**
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

CosSAId — **Cosine Sum of Angles Identity**
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

CosDAId — **Cosine Difference of Angles Identity**
$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

TanSAId — **Tangent Sum of Angles Identity**
$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

TanDiffAId — **Tangent Difference of Angles Identity**
$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

Cosine Law

CL — **Cosine Law**
$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
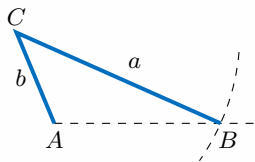$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
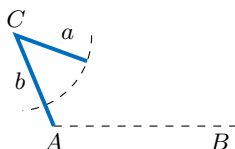

Sine Law

SL — **Sine Law**
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
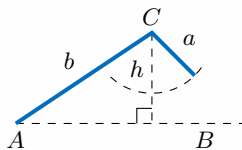
Summary of the Ambiguous Case
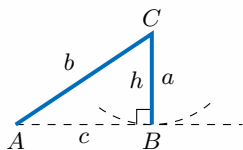
$90° \le A < 180°$, $a > b$: One solution
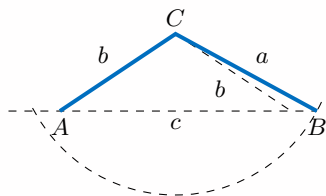
$90° \le A < 180°$, $a \le b$: No solution

$0° < A < 90°$, $a < b \sin A$: No solution

$0° < A < 90°$, $a = b \sin A$: One solution

$0° < A < 90°$, $a \ge b$: One solution

$0° < A < 90°$, $b \sin A < a < b$: Two solutions
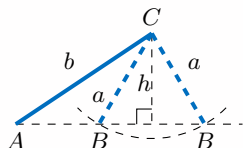
Graphs Trigonometric Functions
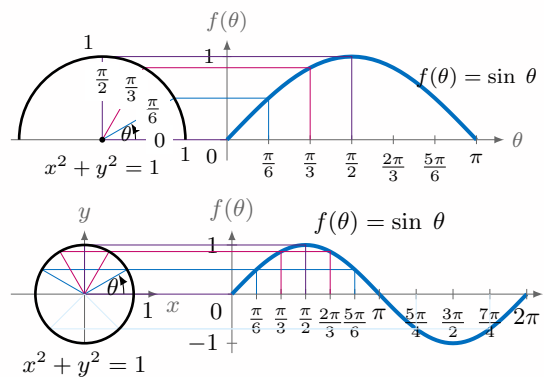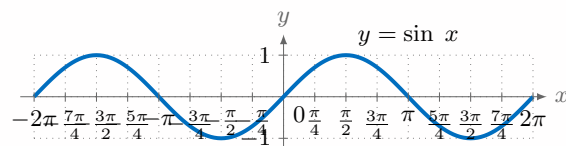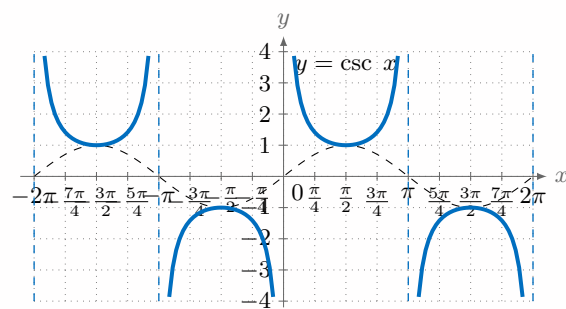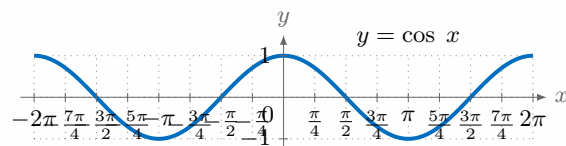
$x^2 + y^2 = 1$

$f(\theta) = \sin \theta$

$x^2 + y^2 = 1$

$f(\theta) = \sin \theta$

Sine Function

$y = \sin x$

Cosecant Function

$y = \csc x$

Cosine Function

$y = \cos x$

Secant Function

$y = \sec x$

Tangent Function

$y = \tan x$

Cotangent Function

$y = \cot x$
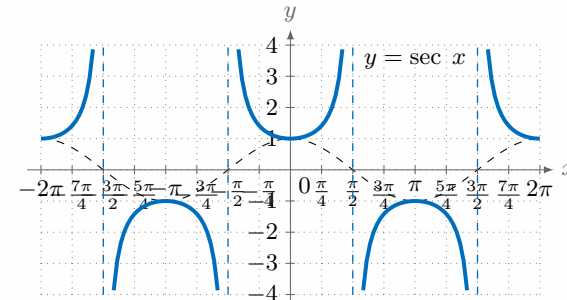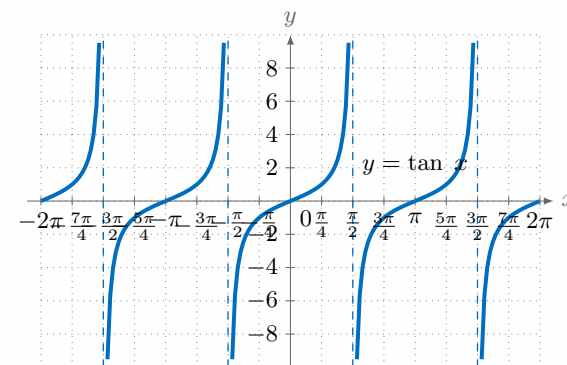
Differentiation by First Principles

DFP   Differentiation by first principles
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Notations

Leibniz's first derivative
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{d\,[f(x)]}{\mathrm{d}x} = \frac{d}{\mathrm{d}x}\,[f(x)]$$
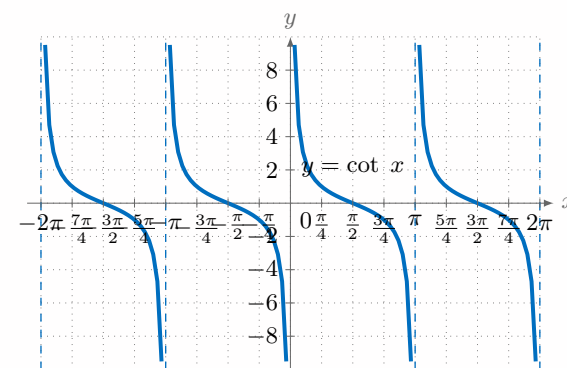
Leibniz's second derivative
$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$

Leibniz's nth derivative
$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$$

Leibniz's evaluate derivative at $x = a$
$$\frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=a} = \frac{\mathrm{d}y}{\mathrm{d}x}(a)$$

LaGrange's first derivative
$$f'(x)$$

LaGrange's second derivative
$$f''(x)$$

LaGrange's nth derivative
$$f^{(n)}(x)$$

LaGrange's evaluate derivative at $x = a$
$$f'(a)$$

Euler's first derivative
$$Df = D_x f$$

Euler's second derivative
$$D^2 f = D_x^2 f$$

Euler's nth derivative
$$D^n f = D_x^n f$$

Differentiation Structural Rules

DS   Derivative of a sum
$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[f(x) + g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}\,[f(x)] + \frac{\mathrm{d}}{\mathrm{d}x}\,[g(x)]$$

DD   Derivative of a difference
$$[f(x) - g(x)]' = f'(x) - g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[f(x) - g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}\,[f(x)] - \frac{\mathrm{d}}{\mathrm{d}x}\,[g(x)]$$

DPr   Derivative of a product "Product Rule"
$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[f(x)g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}\,[f(x)]\,g(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}\,[g(x)]$$

DQ   Derivative of a quotient "Quotient Rule"
$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{\dfrac{\mathrm{d}}{\mathrm{d}x}\,[f(x)]\,g(x) - f(x)\dfrac{\mathrm{d}}{\mathrm{d}x}\,[g(x)]}{[g(x)]^2}$$

DCF   Derivative of a composite function
$$[f(g(x))]' = [g(x)]'\,[f(g(x))]'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[f(g(x))] = \frac{\mathrm{d}}{\mathrm{d}x}\,[g(x)]\,\frac{\mathrm{d}}{\mathrm{d}x}\,[f(g(x))]$$

Differentiation Monomial Rules

DC   Derivative of a constant
$$[c]' = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[c] = 0$$

DCM   Derivative of a constant multiple
$$[cf(x)]' = c\,[f(x)]'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[cf(x)] = c\frac{\mathrm{d}}{\mathrm{d}x}\,[f(x)]$$

DPo   Derivative of a power "Power Rule"
$$[x^n]' = nx^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[x^n] = nx^{n-1}$$

Differentiation Exponential and Logarithmic Function Rules

DExp   Derivative of an exponential function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,[a^x] = a^x \ln a$$

DNExp   Derivative of a natural exponential function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,[e^x] = e^x$$

DL   Derivative of a logarithmic function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,[\log_a x] = \frac{1}{x \ln a}$$

DNL   Derivative of a natural logarithmic function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,[\ln x] = \frac{1}{x}$$

Differentiation Trigonometric Function Rules

DSin   Derivative of a sine function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\sin x) = \cos x$$

DCos   Derivative of a cosine function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\cos x) = -\sin x$$

DTan   Derivative of a tangent function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\tan x) = \sec^2 x$$

DCsc   Derivative of a cosecant function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\csc x) = -\csc x \cot x$$

DSec   Derivative of a secant function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\sec x) = \sec x \tan x$$

DCot   Derivative of a cotangent function
$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\cot x) = -\csc^2 x$$