

1. Arithmetic

1.1 Number Systems

Essential Questions 1.1

1. What is the difference between a numeral system and a numbers system?

Definition 1.1.1 – Natural Numbers.

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

- R** It is not uncommon for zero to be excluded from the natural numbers. In fact, some exclude zero from the natural numbers and then describe the set of natural numbers that include zero the whole numbers.

$$\mathbb{W} = \{0, 1, 2, 3, \dots\}$$

For the purposes of these notes, zero will be included within the set of natural numbers.

Definition 1.1.2 – Operation of Addition (OOA).

$$\underbrace{a}_{\text{Augend}} + \underbrace{b}_{\text{Addend}} = \underbrace{\hspace{1.5cm}}_{\text{Sum}} \quad (1.1)$$

More generally,

$$\underbrace{\overbrace{a}^{\text{Summand}} + \overbrace{b}^{\text{Summand}}}_{\text{Sum}} \quad (1.2)$$

Definition 1.1.3 – Operation of Multiplication (OOM).

$$\underbrace{\overbrace{a}^{\text{Multiplicand}} \times \overbrace{b}^{\text{Multiplier}}}_{\text{Product}} \quad (1.3)$$

More generally,

$$\underbrace{\overbrace{a}^{\text{Factor}} \times \overbrace{b}^{\text{Factor}}}_{\text{Product}} \quad (1.4)$$

Definition 1.1.4 – Integers.

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Definition 1.1.5 – Positive Integers.

$$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$$

Definition 1.1.6 – Greatest Common Divisor. Suppose that m and n are positive integers. The greatest common divisor is the largest divisor (factor) common to both m and n .

Definition 1.1.7 – Relatively Prime. Two integers m and n are relatively prime to each other, $m \perp n$, if they share no common positive integer divisors (factors) except 1.

$$m \perp n \text{ if } \gcd(m, n) = 1.$$

Definition 1.1.8 – Rational Numbers.

$$\mathbb{Q} = \{m/n \mid m, n \in \mathbb{Z}, n \neq 0\}$$

Definition 1.1.9 – Proper Fraction. Given $m < n$, then the fraction m/n is called **proper**.

Definition 1.1.10 – Improper Fraction. Given $m > n$, then the fraction m/n is called **improper**.

Definition 1.1.11 – Common Denominator (CD).

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad (1.5a)$$

$$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b} \quad (1.5b)$$

Rule 1.1.1 – Fraction Operation of Addition (FOOA).

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \quad (1.6a)$$

$$\frac{ad+bc}{bd} = \frac{a}{b} + \frac{c}{d} \quad (1.6b)$$