Math Reference Sheet

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Number Systems

$\mathbb{N}$	Natural numbers	$\mathbb{N} = \{1, 2, 3, \ldots\}$
$\mathbb{Z}$	Integers	$\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Q}$	Rational	$\mathbb{Q} = \{\frac{m}{n} \mid m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0\}$
$\mathbb{R}$	Real numbers	
$\mathbb{C}$	Complex numbers	

Prime Numbers 2-997

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743
751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997

Prime Divisor rules

2	the 1's digit is even
3	sum of digits is divisible by 3
5	the 1's digit is 0 or 5

Reducing Fractions Process - RF

Reduce the fraction $\frac{m}{n}$

1. Simplify by factoring m
2. Simplify by factoring n
3. Find the $\gcd(m, n)$
4. If the $\gcd(m, n) = 1$ the fraction is reduced.
5. $\gcd(m, n)$ is the MId

Operations

DELIM	Delimiters
DO	Dyadic Operations
OOA	Operation of Addition
OOD	Operation of Division
OOE	Operation of Exponentiation
OON	Operation of Negation
OOS	Operation of Subtraction
OOO	Order of Operations
UO	Unary Operations

Order Operations

1. DELIM
2. DO (OOE, OOM, OOD, OOA, OOS)
3. UO (OON)

Operation of Negation

ONeg Operation of Negation Notation
$-a = \neg a$

Operation of Subtraction

DOS Definition of Subtraction
$a + \neg b = a - b$

Operation of Addition

APA Associative Property of Addition
$(a + b) + c = a + (b + c)$

CPA Commutative Property of Addition
$a + b = b + a$

DPF Distributive Property Factoring
$a \cdot b + a \cdot c = a(b + c)$
$b \cdot a + c \cdot a = (b + c)a$

CD Common Denominator
$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$

Operation of Multiplicaiton

APM Associative Property of Multiplication
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

CPM Commutative Property of Multiplication
$a \cdot b = b \cdot a$

CTJ Center-Dot to Juxtaposition
$a \cdot b = ab$

DPE Distributive Property Expanding
$a(b + c) = a \cdot b + a \cdot c$
$(b + c)a = b \cdot a + c \cdot a$

JTC Juxtaposition to Center-Dot
$ab = a \cdot b$

MC Center-Dot Notation
$a \cdot b$

MJ Juxtaposition Notation
$ab, a(b), (a)b, (a)(b), a[b], [a]b, [a][b]$

MT Times Notation
$a \times b$

Operation of Division

DOD Definition of Division
$a \div b = \frac{a}{b} = a \cdot b^{-1}, b \neq 0$

FN Fraction Numerator (upstaris)

FD Fraction Denominator (downstairs)

RF Reduce Fraction

Powers

FTPo **Factor to Power**
$a_n \cdot a_{n-1} \cdot \ldots \cdot a_2 \cdot a_1 = a^n$

PoNE **Power with a Negative Exponent**
$b^{-k} = \frac{1}{b^k}$

PoPo **Power of a Power**
$(b^m)^k = b^{m \cdot k}$

PoQ **Power of a Quotient**
$\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}, b \neq 0$

PoPr **Power of a Product**
$(a \cdot b)^k = a^k \cdot b^k$

PoQPo **Power of a Quotient of Powers**
$\left(\frac{a^m}{b^n}\right)^k = \frac{a^{m \cdot k}}{b^{n \cdot k}}, b \neq 0$

PoPrPo **Power of a Product of Powers**
$(a^m b^n)^k = a^{m \cdot k} b^{n \cdot k}$

PoTR **Power to Radical**
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

PoTL **Power to Logarithm**
$y = b^x \Rightarrow x = \log_b y$

PoTF **Power to Factor**
$(a)^n = a_1 \cdot a_2 \cdot \ldots \cdot a_n$

PrCBPo **Product of Common Base Powers**
$b^m \cdot b^n = b^{m+n}$

QCBPo **Quotient of Common Base Powers**
$\frac{b^m}{b^n} = b^{m-n}$

RTPo **Radical to Power**
$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Identities

AId **Additive Identity**
$a + 0 = a$

MId **Multiplicative Identity**
$a \cdot 1 = a$

PoId **Power Identity**
$b^0 = 1$, given $b > 0$

Inverses

AI **Additive Inverse**
$a + (-a) = 0$

CI **Cosine Inverse**
$\cos^{-1}(\cos\theta) = \theta$

EI **Exponential Inverse**
$\log_a(a^x) = x$

LI **Logarithmic Inverse**
$a^{\log_a x} = x$

MI **Multiplicative Inverse**
$a \cdot \frac{1}{a} = 1 = a \cdot a^{-1}, a \neq 0$

PoI **Power Inverse**
$\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = x$

SI **Sine Inverse**
$\sin^{-1}(\sin\theta) = \theta$

TI **Tangent Inverse**
$\tan^{-1}(\tan\theta) = \theta$

Equality & Inequality

RPE **Reflexive Property of Equality**
$a = a$

SPE **Substitution Property of Equality**
$a = b$ then $F(a) = F(b)$

SPIn **Substitution Property of Inequality**
$a < b$, then $a + c < b + c$
$a < b$ and $c > 0$, then $ca < cb$
$a < b$ and $c < 0$, then $ca > cb$

SyPE **Symmetric Property of Equality**
$a = b$ then $b = a$

TPE **Transitive Property of Equality**
if $a = b$ and $b = c$, then $a = c$

TPIn **Transitive Property of Inequality**
if $a < b$ and $b < c$, then $a < c$

ZFP **Zero Factor Property**
if $a \cdot b = 0$, then $a = 0$ or $b = 0$

Simplify Expressions Workflow

1. MId
2. ONeg
3. DOS
4. DELIM Goto 36, 21
5. DPE
6. JTC
7. CPM Goto 25
8. APM
9. OOM
10. RF
11. CTJ
12. CPA
13. DPF
14. APA
15. RF
16. OOA
17. AId Goto 4
18. DOS
19. ONeg
20. MId **DONE!**
21. PoId
22. PoTF
23. RTPo
24. PoNE Goto 4
25. PoPr
26. PoQ
27. PoPrPo
28. PoQPo
29. PrCBPo
30. QCBPo
31. PoPo
32. PoNE
33. OOE
34. PoTR
35. PoId Goto 8
36. LPoPo
37. LPrCBPo
38. LQCBPo
39. LEF Goto 4

Logarithms

LEV **Logarithm Exponent Visible**
$\log_b y \Rightarrow \log_b y = x$

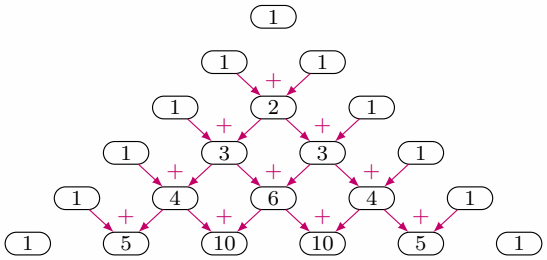
LPoPo **Logarithm Power of a Power**
$\log_b x^n = n \log_b x$

LPrCBPo **Logarithm Product of Common Base Powers**
$\log_b(mn) = \log_b m + \log_b n$

LQCBPo **Logarithm Quotient of Common Base Powers**
$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

LTPo **Logarithm to Power Form**
$x = \log_b y \Rightarrow y = b^x$

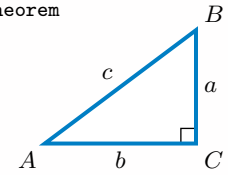
Pascal's Triangle



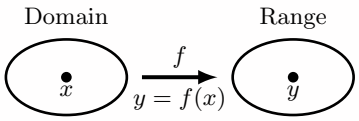
Pythagorean Theorem

PyThm `Pythagorean Theorem`

$a^2 + b^2 = c^2$

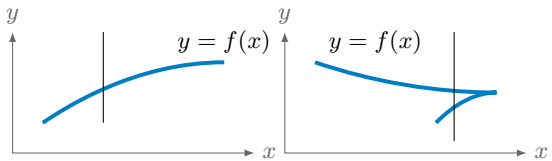


Function



Function Vertical Line Test

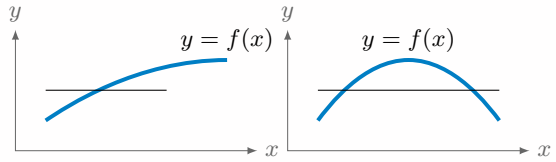
FVLT `Function Vertical Line Test`



f is a function f is not a function

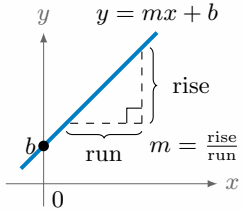
Horizontal Line Test

FHLT `Function Horizontal Line Test`



f is one-to-one f is not one-to-one

Linear Functions



DBP `Distance betweent` $P_1 = (x_1, y_1)$ `&` $P_2 = (x_2, y_2)$

$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

MBP `Midpoint between` $P_1 = (x_1, y_1)$ `&` $P_2 = (x_2, y_2)$

Midpoint of $P_1P_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{3}\right)$

Line Slope `Line Slope through` $P_1 = (x_1, y_1)$ `&` $P_2 = (x_2, y_2)$

$m = \frac{y_2-y_1}{x_2-x_1}$

PSE `Point slope equation though` $P(x_1, y_2)$

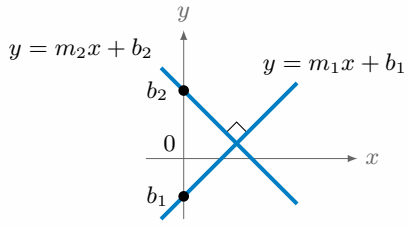
$y - y_1 = m(x - x_1)$

SIE `Slope-intercept equation`

$y = mx + b$

PrSPL `Product of slopes - Perpendicual Lines`

$m_1 m_2 = -1$



Quadratic Functions

If $ax^2 + bx + c = 0$, where $a \neq 0$, then

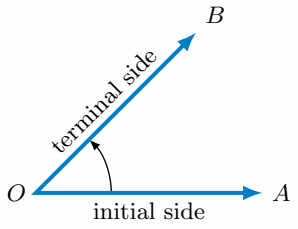
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $b^2 - 4ac > 0$ Two distinct real solutions
2. $b^2 - 4ac = 0$ Two repeated real solutions
3. $b^2 - 4ac < 0$ Two distinct complex solutions

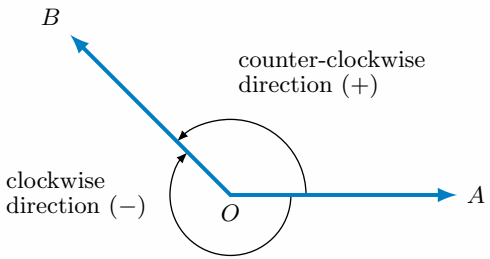
Greek Alphabet

Letters		Name	Letters		Name
A	α	`alpha`	N	ν	`nu`
B	β	`beta`	Ξ	ξ	`xi`
Γ	γ	`gamma`	O	o	`omicron`
Δ	δ	`delta`	Π	π	`pi`
E	ϵ	`epsilon`	P	ρ	`rho`
Z	ζ	`zeta`	Σ	σ	`sigma`
H	η	`eta`	T	τ	`tau`
Θ	θ	`theta`	Υ	υ	`upsilon`
I	ι	`iota`	Φ	ϕ	`phi`
K	κ	`kappa`	X	χ	`chi`
Λ	λ	`lambda`	Ψ	ψ	`psi`
M	μ	`mu`	Ω	ω	`omega`

Angles: Components

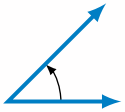


Angle Direction & Magnitude

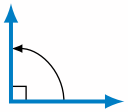


Classification of Angles

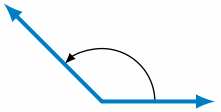
AA Acute Angle



RA Right Angle

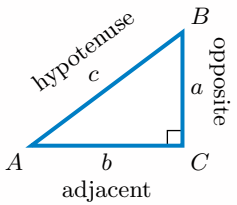


OA Obtuse Angle



SA Straight Angle

Trigonometric Function Definitions



$\sin A = \frac{\text{opp}}{\text{hyp}}$ $\quad \csc A = \frac{\text{hyp}}{\text{opp}}$

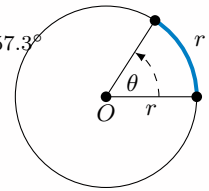
$\cos A = \frac{\text{adj}}{\text{hyp}}$ $\quad \sec A = \frac{\text{hyp}}{\text{adj}}$

$\tan A = \frac{\text{opp}}{\text{adj}}$ $\quad \cot A = \frac{\text{adj}}{\text{hyp}}$

Radians

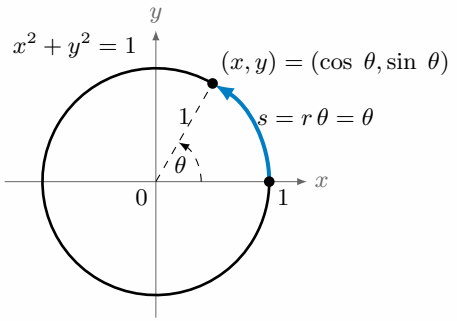
Rad Radian

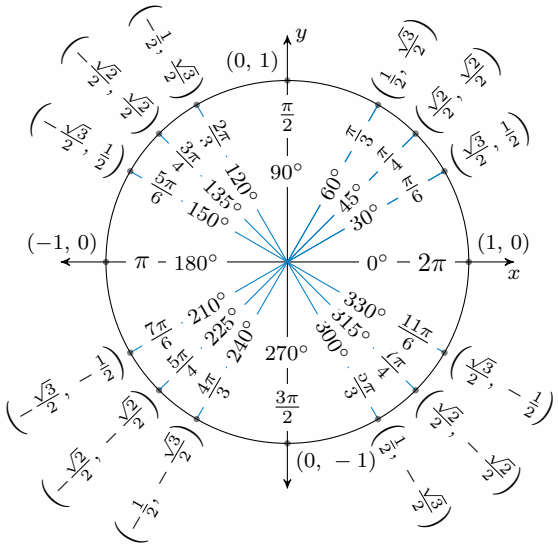
1 radian $= \frac{180}{\pi} \approx 57.3°$



$\theta = 1$ radian

Unit Circle





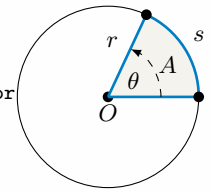
Arc Length and Sector Area

AL Arc Length

$s = r\theta$

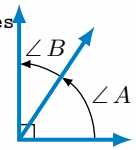
ASect Area of a Sector

$A = \frac{1}{2}r^2\theta$

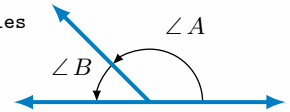


Angle Pairings

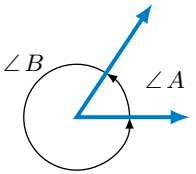
ComA Complimentary Angles



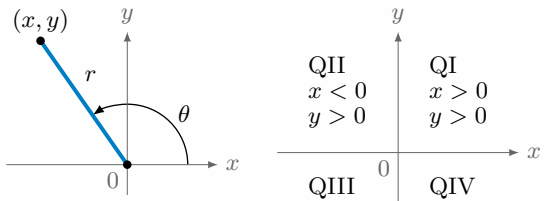
SA Supplementary Angles

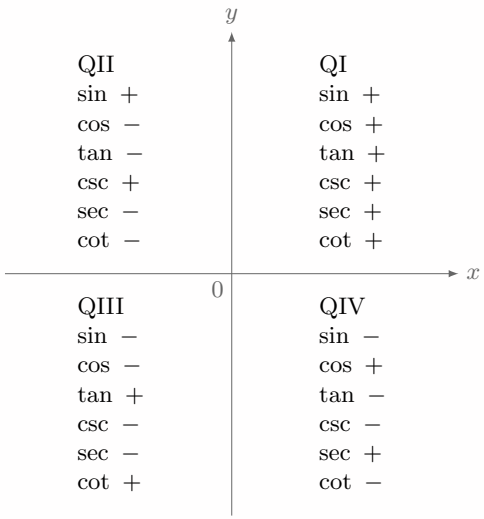


ConA Conjugate Angles



Quadrants





Trigonometric Identities

EOId **Trigonometric Even/Odd Identities**

$$\sin -\theta = -\sin\theta \qquad \cos -\theta = \cos\theta$$

$$\csc -\theta = -\csc\theta \qquad \sec -\theta = \sec\theta$$

$$\tan -\theta = -\tan\theta \qquad \cot -\theta = -\tan\theta$$

RId **Trigonometric Reciprocal Identities**

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PyId **Trigonometric Pythagorean Identities**

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta = \tan^2\theta + 1 \qquad \csc^2\theta = 1 + \cot^2\theta$$

TanId **Tangent Identity**

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

CotId **Cotangent Identity**

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

SDAId **Sine Double Angle Identity**

$$\sin 2\theta = 2\sin\theta\cos\theta$$

CDAId **Cosine Double Angle Identity**

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1\end{aligned}$$

TDAId **Tangent Double Angle Identity**

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

SSAId **Sine Sum of Angles Identity**

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

SDAId **Sine Difference of Angles Identity**

$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

CSAId **Cosine Sum of Angles Identity**

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

CDAId **Cosine Difference of Angles Identity**

$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

TSAId **Tangent Difference of Angles Identity**

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

TDAId **Tangent Difference of Angles Identity**

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

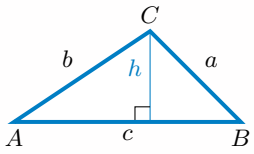
Cosine Law

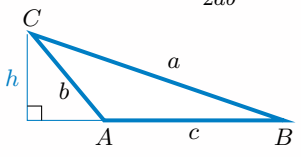
CL **Cosine Law**

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

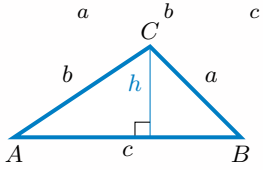


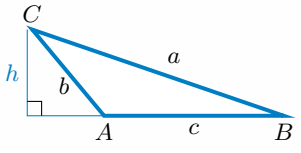


Sine Law

SL **Sine Law**

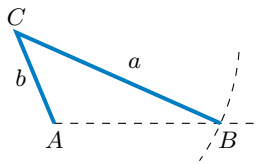
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



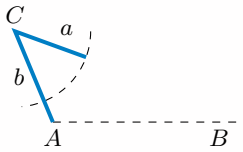


Summary of the Ambiguous Case

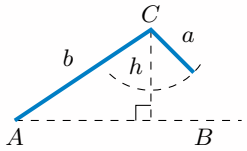
$90° \leq A < 180°$, $a > b$: One solution



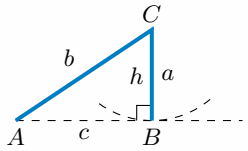
$90° \leq A < 180°$, $a \leq b$: No solution



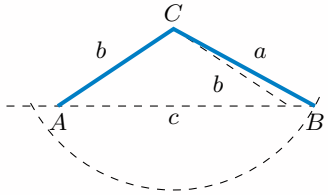
$0° < A < 90°$, $a < b$ sin A : No solution



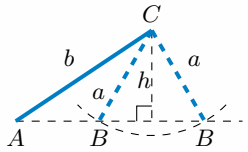
$0° < A < 90°$, $a = b$ sin A : One solution



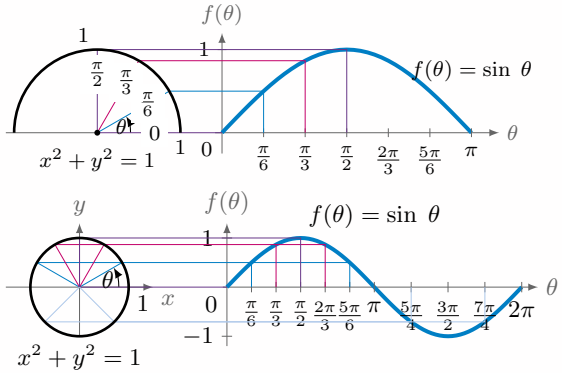
$0° < A < 90°$, $a \geq b$: One solution



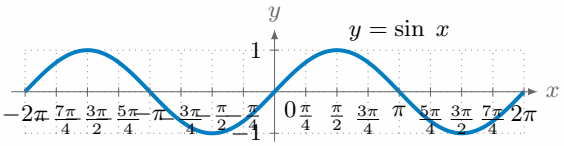
$0° < A < 90°$, b sin $A < a < b$: Two solutions



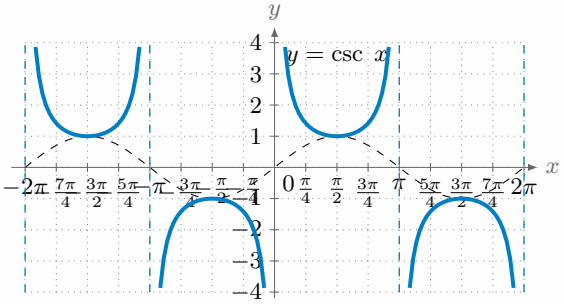
Graphs Trigonometric Functions



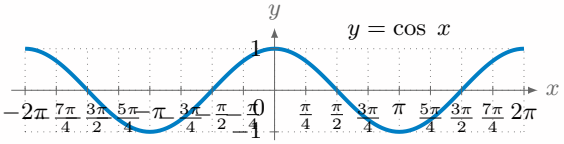
Sine Function



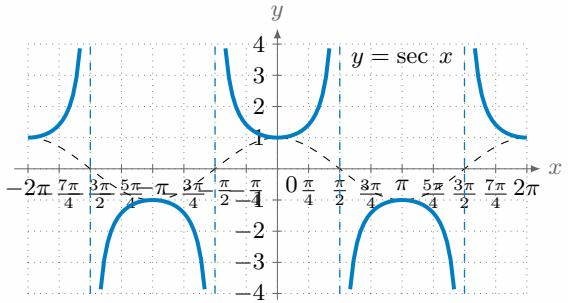
Cosecant Function



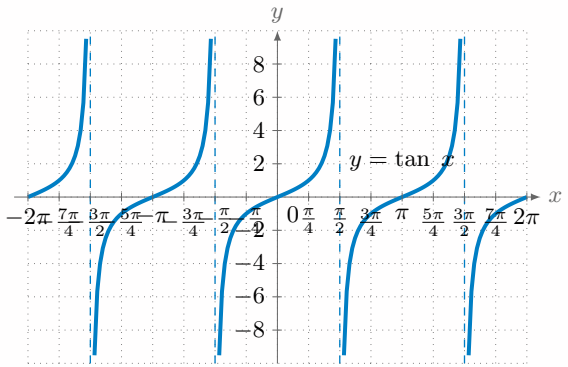
Cosine Function



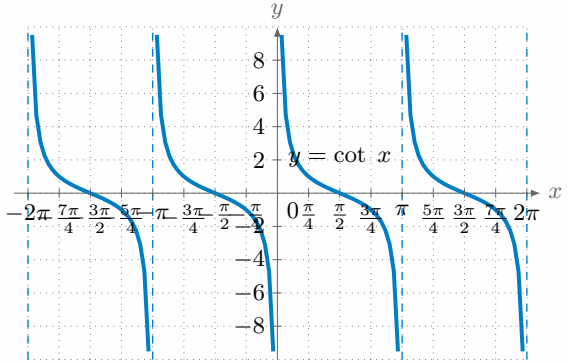
Secant Function



Tangent Function



Cotangent Function



Differentiation by First Principles

DFP Differentiation by first principles

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Notations

Leibiz's first derivative

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{d\left[f(x)\right]}{\mathrm{d}x} = \frac{d}{\mathrm{d}x}\left[f(x)\right]$$

Leibiz's second derivative

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$

Leibiz's nth derivative

$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$$

Leibiz's evaluate derivative at $x = a$

$$\frac{\mathrm{d}y}{\mathrm{d}x}\Big|_{x=a} = \frac{\mathrm{d}y}{\mathrm{d}x}(a)$$

LaGrange's first derivative

$$f'(x)$$

LaGrange's second derivative

$$f''(x)$$

LaGrange's nth derivative

$$f^{(n)}(x)$$

LaGrange's evaluate derivative at $x = a$

$$f'(a)$$

Euler's first derivative

$$Df = D_x f$$

Euler's second derivative

$$D^2 f = D_x^2 f$$

Euler's nth derivative

$$D^n f = D_x^n$$

Differentiation Structural Rules

DS Derivative of a sum

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x) + g(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\right] + \frac{\mathrm{d}}{\mathrm{d}x}\left[g(x)\right]$$

DD Derivative of a difference

$$\left[f(x) - g(x)\right]' = f'(x) - g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x) - g(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\right] - \frac{\mathrm{d}}{\mathrm{d}x}\left[g(x)\right]$$

DPr Derivative of a product "Product Rule"

$$\left[f(x)g(x)\right]' = f'(x)g(x) + f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)g(x)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\right]g(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}\left[g(x)\right]$$

DQ Derivative of a quotient "Quotient Rule"

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\right]g(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}\left[g(x)\right]}{\left[g(x)\right]^2}$$

DCF Derivative of a composite function

$$\left[f\left(g(x)\right)\right]' = \left[g(x)\right]'\left[f\left(g(x)\right)\right]'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[f\left(g(x)\right)\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[g(x)\right]\frac{\mathrm{d}}{\mathrm{d}x}\left[f\left(g(x)\right)\right]$$

Differentiation Monomial Rules

DC Derivative of a constant

$$[c]' = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[c] = 0$$

DCM Derivative of a constant multiple

$$\left[cf(x)\right]' = c\left[f(x)\right]'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[cf(x)\right] = c\frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)\right]$$

DPo Derivative of a power "Power Rule"

$$[x^n]' = nx^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^n] = nx^{n-1}$$

Differentiation Exponential and Logarithmic Function Rules

DExp Derivative of an exponential function

$$\frac{\mathrm{d}}{\mathrm{d}x}[a^x] = a^x \ln a$$

DNExp Derivative of a natural exponential function

$$\frac{\mathrm{d}}{\mathrm{d}x}[e^x] = e^x$$

DL Derivative of a logarithmic function

$$\frac{\mathrm{d}}{\mathrm{d}x}[\log_a x] = \frac{1}{x \ln a}$$

DNL Derivative of a natural logarithmic function

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln x] = \frac{1}{x}$$

Differentiation Trigonometric Function Rules

DSin Derivative of a sine function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$$

DCos Derivative of a cosine function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$$

DTan Derivative of a tangent function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$$

DCsc Derivative of a cosecant function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\csc x) = -\csc x \cot x$$

DSec Derivative of a secant function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$$

DCot Derivative of a cotangent function

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\csc^2 x$$

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