

# Math Reference Sheet

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## Number Systems

<b>N</b>	Natural numbers	$\mathbb{N} = \{1, 2, 3, \dots\}$
<b>Z</b>	Integers	$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$
<b>Q</b>	Rational	$\mathbb{Q} = \left\{ \frac{m}{n} \mid m \in \mathbb{Z}, n \in \mathbb{Z}, n \neq 0 \right\}$
<b>R</b>	Real numbers	
<b>C</b>	Complex numbers	$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$

## Prime Numbers 2-997

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743
751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997

## Prime Divisor rules

- 2** the 1's digit is even
- 3** sum of digits is divisible by 3
- 5** the 1's digit is 0 or 5

## Reducing Fractions Process - RF

Reduce the fraction  $\frac{m}{n}$

1. Simplify by factoring  $m$
2. Simplify by factoring  $n$
3. Find the  $\gcd(m, n)$
4. If the  $\gcd(m, n) = 1$  the fraction is reduced.
5.  $\gcd(m, n)$  is the MID

## Operations

<b>DELIM</b>	Delimiters
<b>DO</b>	Dyadic Operations
<b>OOA</b>	Operation of Addition
<b>OOD</b>	Operation of Division
<b>OOE</b>	Operation of Exponentiation
<b>OON</b>	Operation of Negation
<b>OOS</b>	Operation of Subtraction
<b>OOO</b>	Order of Operations
<b>UO</b>	Unary Operations

## Order Operations

1. DELIM
2. DO (OOE, OOM, OOD, OOA, OOS)
3. UO (OON)

## Operation of Negation

**ONeg** Operation of Negation Notation  
 $-a = \neg a$

## Operation of Subtraction

**DOS** Definition of Subtraction  
 $a + \neg b = a - b$

## Operation of Addition

<b>APA</b>	Associative Property of Addition $(a + b) + c = a + (b + c)$
<b>CPA</b>	Commutative Property of Addition $a + b = b + a$
<b>DPF</b>	Distributive Property Factoring $a \cdot b + a \cdot c = a(b + c)$ $b \cdot a + c \cdot a = (b + c)a$
<b>CD</b>	Common Denominator $\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$

## Operation of Multiplication

<b>APM</b>	Associative Property of Multiplication $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
<b>CPM</b>	Commutative Property of Multiplication $a \cdot b = b \cdot a$
<b>CTJ</b>	Center-Dot to Juxtaposition $a \cdot b = ab$
<b>DPE</b>	Distributive Property Expanding $a(b + c) = a \cdot b + a \cdot c$ $(b + c)a = b \cdot a + c \cdot a$
<b>JTC</b>	Juxtaposition to Center-Dot $ab = a \cdot b$
<b>MC</b>	Center-Dot Notation $a \cdot b$
<b>MJ</b>	Juxtaposition Notation $ab, a(b), (a)b, (a)(b), a[b], [a]b, [a][b]$
<b>MT</b>	Times Notation $a \times b$

## Operation of Division

<b>DOD</b>	Definition of Division $a \div b = \frac{a}{b} = a \cdot b^{-1}, b \neq 0$
<b>FN</b>	Fraction Numerator (upstairs)
<b>FD</b>	Fraction Denominator (downstairs)
<b>RF</b>	Reduce Fraction

## Powers

<b>FTPo</b>	Factor to Power $a_n \cdot a_{n-1} \cdot \dots \cdot a_2 \cdot a_1 = a^n$
<b>PoNegE</b>	Power Negative Exponent $b^{-k} = \frac{1}{b^k}$
<b>PoPo</b>	Power of a Power $(b^m)^k = b^{m \cdot k}$
<b>PoQ</b>	Power of a Quotient $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}, b \neq 0$
<b>PoPr</b>	Power of a Product $(a \cdot b)^k = a^k \cdot b^k$
<b>PoQPo</b>	Power of a Quotient of Powers $\left(\frac{a^m}{b^n}\right)^k = \frac{a^{m \cdot k}}{b^{n \cdot k}}, b \neq 0$
<b>PoPrPo</b>	Power of a Product of Powers $(a^m b^n)^k = a^{m \cdot k} b^{n \cdot k}$
<b>PoTR</b>	Power to Radical $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
<b>PoTL</b>	Power to Logarithm $y = b^x \Rightarrow x = \log_b y$
<b>PoTF</b>	Power to Factor $(a)^n = a_1 \cdot a_2 \cdot \dots \cdot a_n$
<b>PrCBPo</b>	Product of Common Base Powers $b^m \cdot b^n = b^{m+n}$
<b>QCBPo</b>	Quotient of Common Base Powers $\frac{b^m}{b^n} = b^{m-n}$
<b>RTPo</b>	Radical to Power $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

## Identities

<b>AId</b>	Additive Identity $a + 0 = a$
<b>MId</b>	Multiplicative Identity $a \cdot 1 = a$
<b>PoId</b>	Power Identity $b^0 = 1$ , given $b > 0$

## Inverses

<b>ArcCos</b>	Cosine Inverse $\cos^{-1}(\cos \theta) = \theta$
<b>ArcSin</b>	Sine Inverse $\sin^{-1}(\sin \theta) = \theta$
<b>ArcTan</b>	Tangent Inverse $\tan^{-1}(\tan \theta) = \theta$
<b>AI</b>	Additive Inverse $a + (-a) = 0$
<b>EI</b>	Exponential Inverse $\log_a(a^x) = x$
<b>LI</b>	Logarithmic Inverse $a^{\log_a x} = x$
<b>MI</b>	Multiplicative Inverse $a \cdot \frac{1}{a} = 1 = a \cdot a^{-1}, a \neq 0$
<b>Pol</b>	Power Inverse $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = x$

## Equality & Inequality

<b>RPE</b>	Reflexive Property of Equality $a = a$
<b>SPE</b>	Substitution Property of Equality $a = b$ then $F(a) = F(b)$
<b>SPIIn</b>	Substitution Property of Inequality $a < b$ , then $a + c < b + c$ $a < b$ and $c > 0$ , then $ca < cb$ $a < b$ and $c < 0$ , then $ca > cb$
<b>SyPE</b>	Symmetric Property of Equality $a = b$ then $b = a$
<b>TPE</b>	Transitive Property of Equality if $a = b$ and $b = c$ , then $a = c$
<b>TPIn</b>	Transitive Property of Inequality if $a < b$ and $b < c$ , then $a < c$
<b>ZPr</b>	Zero Product Property if $a \cdot b = 0$ , then $a = 0$ or $b = 0$

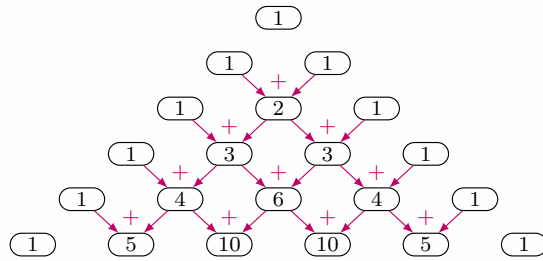
## Simplify Expressions Workflow

- |                             |                          |
|-----------------------------|--------------------------|
| 1. <b>MId</b>               | 21. <b>PoId</b>          |
| 2. <b>ONeg</b>              | 22. <b>PoTF</b>          |
| 3. <b>DOS</b>               | 23. <b>RTPo</b>          |
| 4. <b>DELIM</b> Goto 36, 21 | 24. <b>PoNegE</b> Goto 4 |
| 5. <b>DPE</b>               | 25. <b>PoPr</b>          |
| 6. <b>JTC</b>               | 26. <b>PoQ</b>           |
| 7. <b>CPM</b> Goto 25       | 27. <b>PoPrPo</b>        |
| 8. <b>APM</b>               | 28. <b>PoQPo</b>         |
| 9. <b>OOM</b>               | 29. <b>PrCBPo</b>        |
| 10. <b>RF</b>               | 30. <b>QCBPo</b>         |
| 11. <b>CTJ</b>              | 31. <b>PoPo</b>          |
| 12. <b>CPA</b>              | 32. <b>PoNegE</b>        |
| 13. <b>DPF</b>              | 33. <b>OOE</b>           |
| 14. <b>APA</b>              | 34. <b>PoTR</b>          |
| 15. <b>RF</b>               | 35. <b>PoId</b> Goto 8   |
| 16. <b>OOA</b>              | 36. <b>LPoPo</b>         |
| 17. <b>AId</b> Goto 4       | 37. <b>LPrCBPo</b>       |
| 18. <b>DOS</b>              | 38. <b>LQCBPo</b>        |
| 19. <b>ONeg</b>             | 39. <b>LEF</b> Goto 4    |
| 20. <b>MId</b> <b>DONE!</b> |                          |

## Logarithms

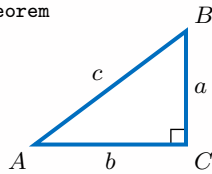
<b>LEV</b>	Logarithm Exponent Visible $\log_b y \Rightarrow \log_b y = x$
<b>LPoPo</b>	Logarithm Power of a Power $\log_b x^n = n \log_b x$
<b>LPrCBPo</b>	Logarithm Product of Common Base Powers $\log_b(mn) = \log_b m + \log_b n$
<b>LQCBPo</b>	Logarithm Quotient of Common Base Powers $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
<b>LTPo</b>	Logarithm to Power $x = \log_b y \Rightarrow y = b^x$

## Pascal's Triangle

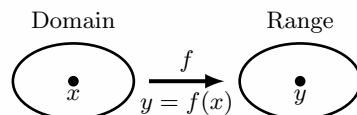


## Pythagorean Theorem

**PyThm** Pythagorean Theorem  
 $a^2 + b^2 = c^2$

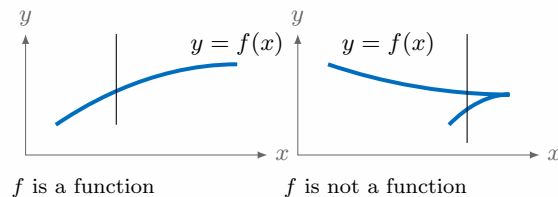


## Function



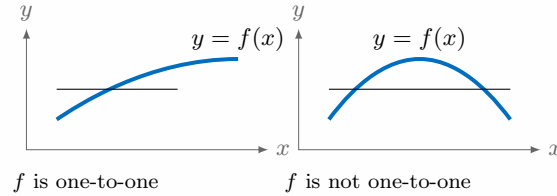
## Function Vertical Line Test

**FVLT** Function Vertical Line Test



## Horizontal Line Test

**FHLT** Function Horizontal Line Test



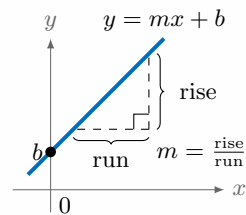
## Quadratic Functions

If  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.  $b^2 - 4ac > 0$  Two distinct real solutions
2.  $b^2 - 4ac = 0$  Two repeated real solutions
3.  $b^2 - 4ac < 0$  Two distinct complex solutions

## Linear Functions



**DBP** Distance between  $P_1 = (x_1, y_1)$   
 &  $P_2 = (x_2, y_2)$   
 $d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

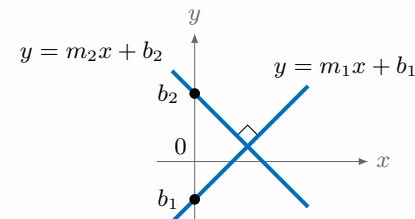
**MBP** Midpoint between  $P_1 = (x_1, y_1)$   
 &  $P_2 = (x_2, y_2)$   
 Midpoint of  $P_1 P_2 = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

**Line Slope** Line Slope through  $P_1 = (x_1, y_1)$   
 &  $P_2 = (x_2, y_2)$   
 $m = \frac{y_2 - y_1}{x_2 - x_1}$

**PSE** Point slope equation through  $P(x_1, y_1)$   
 $y - y_1 = m(x - x_1)$

**SIE** Slope-intercept equation  
 $y = mx + b$

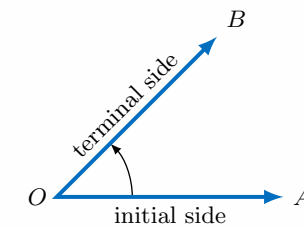
**PrSPL** Product of slopes - Perpendicular Lines  
 $m_1 m_2 = -1$



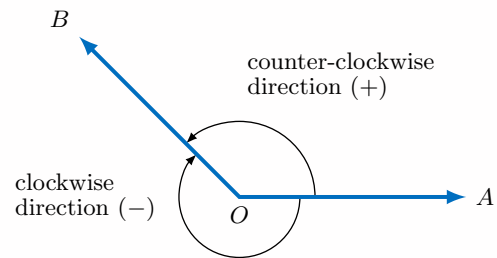
## Greek Alphabet

Letters	Name	Letters	Name
A	$\alpha$ alpha	N	$\nu$ nu
B	$\beta$ beta	$\Xi$	$\xi$ xi
$\Gamma$	$\gamma$ gamma	O	$o$ omicron
$\Delta$	$\delta$ delta	$\Pi$	$\pi$ pi
E	$\epsilon$ epsilon	P	$\rho$ rho
Z	$\zeta$ zeta	$\Sigma$	$\sigma$ sigma
H	$\eta$ eta	T	$\tau$ tau
$\Theta$	$\theta$ theta	$\Upsilon$	$\upsilon$ upsilon
I	$\iota$ iota	$\Phi$	$\phi$ phi
K	$\kappa$ kappa	X	$\chi$ chi
$\Lambda$	$\lambda$ lambda	$\Psi$	$\psi$ psi
M	$\mu$ mu	$\Omega$	$\omega$ omega

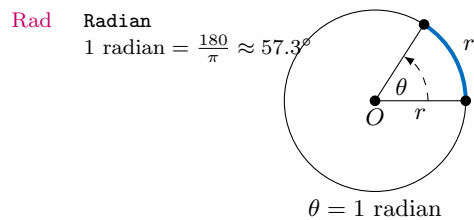
## Angles: Components



## Angle Direction & Magnitude



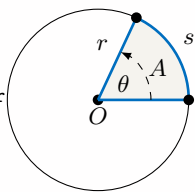
## Radians



## Arc Length and Sector Area

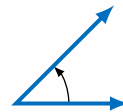
**AL** Arc Length  
 $s = r\theta$

**ASect** Area of a Sector  
 $A = \frac{1}{2}r^2\theta$

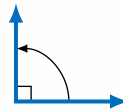


## Classification of Angles

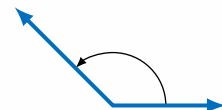
**AA** Acute Angle



**RA** Right Angle



**OA** Obtuse Angle

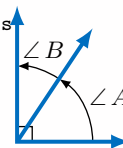


**SA** Straight Angle

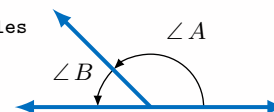


## Angle Pairings

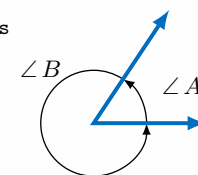
**ComA** Complimentary Angles



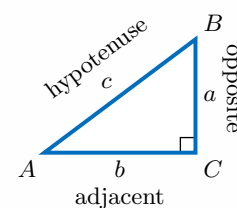
**SA** Supplementary Angles



**ConA** Conjugate Angles



## Trigonometric Function Definitions

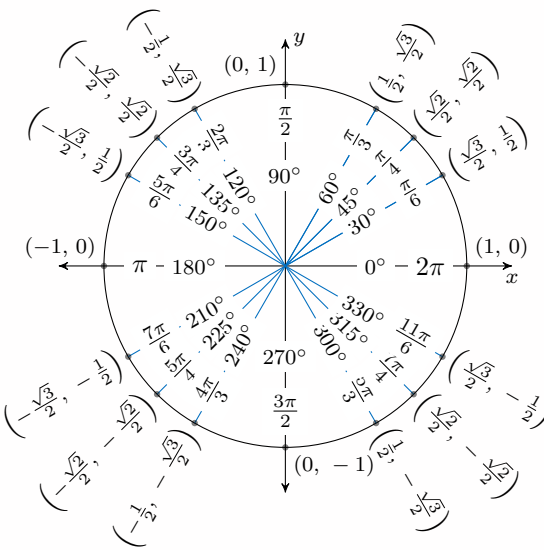
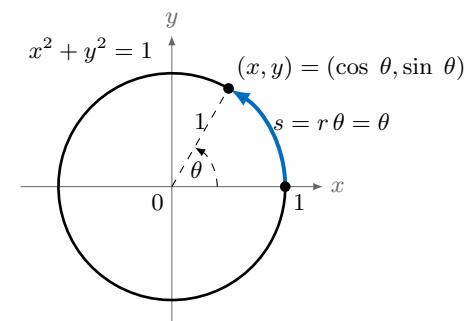


$$\sin A = \frac{\text{opp}}{\text{hyp}} \quad \csc A = \frac{\text{hyp}}{\text{opp}}$$

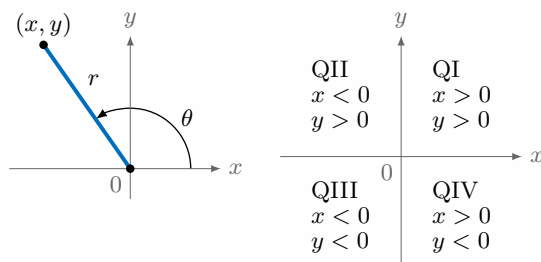
$$\cos A = \frac{\text{adj}}{\text{hyp}} \quad \sec A = \frac{\text{hyp}}{\text{adj}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} \quad \cot A = \frac{\text{adj}}{\text{hyp}}$$

## Unit Circle



## Quadrants



QII	QI
sin +	sin +
cos -	cos +
tan -	tan +
csc +	csc +
sec -	sec +
cot -	cot +
QIII	QIV
sin -	sin -
cos -	cos +
tan +	tan -
csc -	csc -
sec -	sec +
cot +	cot -

## Trigonometric Identities

<b>EOId</b>	<b>Trigonometric Even/Odd Identities</b> $\sin -\theta = -\sin \theta$ $\cos -\theta = \cos \theta$ $\csc -\theta = -\csc \theta$ $\sec -\theta = \sec \theta$ $\tan -\theta = -\tan \theta$ $\cot -\theta = -\cot \theta$
<b>RId</b>	<b>Trigonometric Reciprocal Identities</b> $\sin \theta = \frac{1}{\csc \theta}$ $\cos \theta = \frac{1}{\sec \theta}$ $\cot \theta = \frac{1}{\tan \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\tan \theta = \frac{1}{\cot \theta}$
<b>PyId</b>	<b>Trigonometric Pythagorean Identities</b> $\sin^2 \theta + \cos^2 \theta = 1$ $\sec^2 \theta = \tan^2 \theta + 1$ $\csc^2 \theta = 1 + \cot^2 \theta$
<b>TanId</b>	<b>Tangent Identity</b> $\tan \theta = \frac{\sin \theta}{\cos \theta}$
<b>CotId</b>	<b>Cotangent Identity</b> $\cot \theta = \frac{\cos \theta}{\sin \theta}$
<b>SinDAId</b>	<b>Sine Double Angle Identity</b> $\sin 2\theta = 2 \sin \theta \cos \theta$
<b>CosDAId</b>	<b>Cosine Double Angle Identity</b> $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 1 - 2 \sin^2 \theta$ $= 2 \cos^2 \theta - 1$
<b>TanDAId</b>	<b>Tangent Double Angle Identity</b> $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
<b>SinSAId</b>	<b>Sine Sum of Angles Identity</b> $\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$
<b>SinDiffAId</b>	<b>Sine Difference of Angles Identity</b> $\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$
<b>CosSAId</b>	<b>Cosine Sum of Angles Identity</b> $\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$
<b>CosDAId</b>	<b>Cosine Difference of Angles Identity</b> $\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$
<b>TanSAId</b>	<b>Tangent Sum of Angles Identity</b> $\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$
<b>TanDiffAId</b>	<b>Tangent Difference of Angles Identity</b> $\tan(\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$

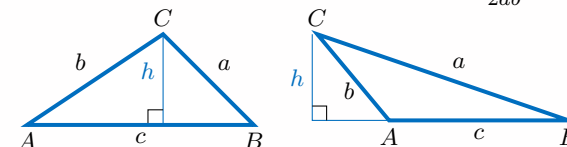
## Cosine Law

**CL** **Cosine Law**

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



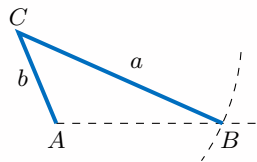
## Sine Law

**SL** **Sine Law**

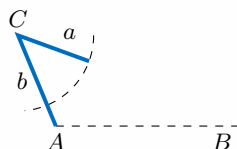
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## Summary of the Ambiguous Case

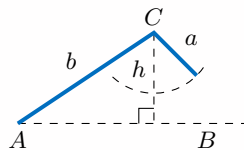
$90^\circ \leq A < 180^\circ, a > b$  : One solution



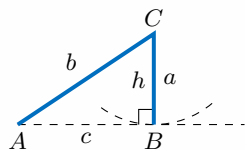
$90^\circ \leq A < 180^\circ, a \leq b$  : No solution



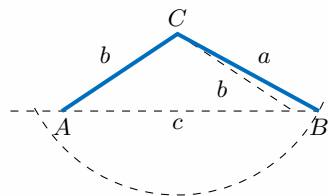
$0^\circ < A < 90^\circ, a < b \sin A$  : No solution



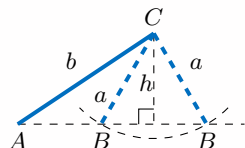
$0^\circ < A < 90^\circ, a = b \sin A$  : One solution



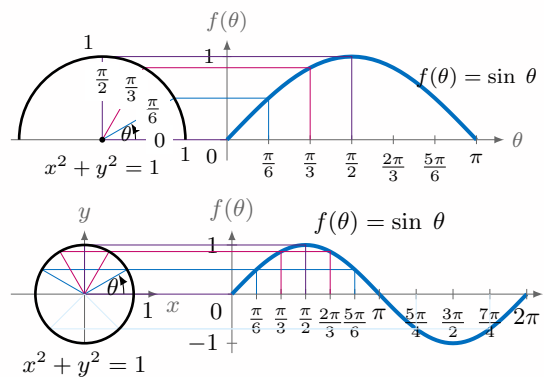
$0^\circ < A < 90^\circ, a \geq b$  : One solution



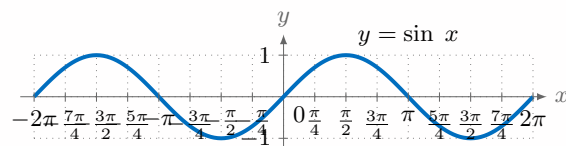
$0^\circ < A < 90^\circ, b \sin A < a < b$  : Two solutions



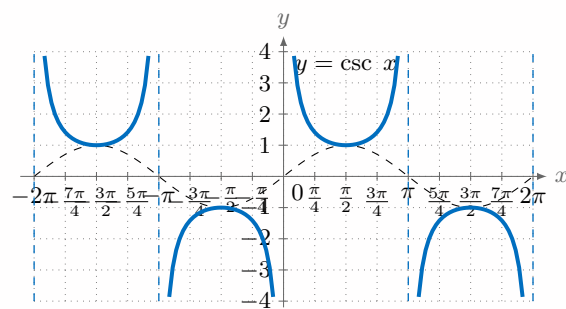
## Graphs Trigonometric Functions



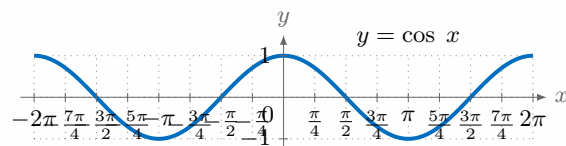
## Sine Function



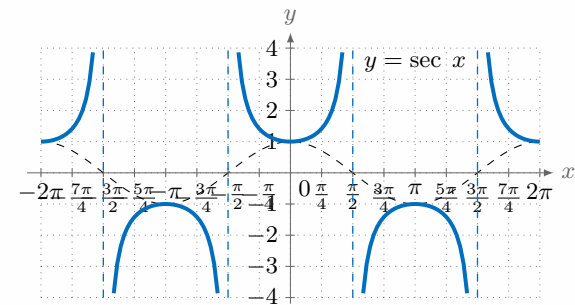
## Cosecant Function



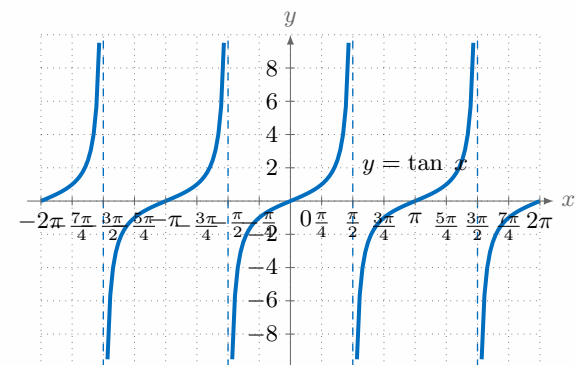
## Cosine Function



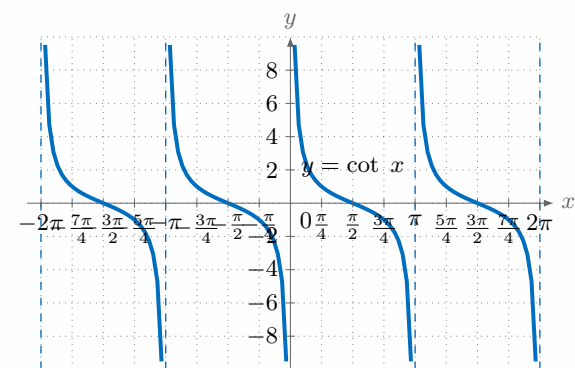
## Secant Function



## Tangent Function



## Cotangent Function



## Differentiation by First Principles

**DFP** Differentiation by first principles

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

## Notations

Leibniz's first derivative

$$\frac{dy}{dx} = \frac{d[f(x)]}{dx} = \frac{d}{dx} [f(x)]$$

Leibniz's second derivative

$$\frac{d^2 y}{dx^2}$$

Leibniz's nth derivative

$$\frac{d^n y}{dx^n}$$

Leibniz's evaluate derivative at  $x = a$

$$\left. \frac{dy}{dx} \right|_{x=a} = \frac{dy}{dx}(a)$$

LaGrange's first derivative

$$f'(x)$$

LaGrange's second derivative

$$f''(x)$$

LaGrange's nth derivative

$$f^{(n)}(x)$$

LaGrange's evaluate derivative at  $x = a$

$$f'(a)$$

Euler's first derivative

$$Df = D_x f$$

Euler's second derivative

$$D^2 f = D_x^2 f$$

Euler's nth derivative

$$D^n f = D_x^n$$

## Differentiation Structural Rules

**DS** Derivative of a sum

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$\frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} [f(x)] + \frac{d}{dx} [g(x)]$$

**DD** Derivative of a difference

$$[f(x) - g(x)]' = f'(x) - g'(x)$$

$$\frac{d}{dx} [f(x) - g(x)] = \frac{d}{dx} [f(x)] - \frac{d}{dx} [g(x)]$$

**DP<sub>r</sub>** Derivative of a product "Product Rule"

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} [f(x)g(x)] = \frac{d}{dx} [f(x)]g(x) + f(x)\frac{d}{dx} [g(x)]$$

**DQ** Derivative of a quotient "Quotient Rule"

$$\left[ \frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{\frac{d}{dx} [f(x)]g(x) - f(x)\frac{d}{dx} [g(x)]}{[g(x)]^2}$$

**DCF** Derivative of a composite function

$$[f(g(x))]' = [g(x)]' [f(g(x))]'$$

$$\frac{d}{dx} [f(g(x))] = \frac{d}{dx} [g(x)] \frac{d}{dx} [f(g(x))]$$

## Differentiation Monomial Rules

**DC** Derivative of a constant

$$[c]' = 0$$

$$\frac{d}{dx} [c] = 0$$

**DCM** Derivative of a constant multiple

$$[cf(x)]' = c[f(x)]'$$

$$\frac{d}{dx} [cf(x)] = c \frac{d}{dx} [f(x)]$$

**DPo** Derivative of a power "Power Rule"

$$[x^n]' = nx^{n-1}$$

$$\frac{d}{dx} [x^n] = nx^{n-1}$$

## Differentiation Exponential and Logarithmic Function Rules

**DE<sub>Exp</sub>** Derivative of an exponential function

$$\frac{d}{dx} [a^x] = a^x \ln a$$

**DNE<sub>Exp</sub>** Derivative of a natural exponential function

$$\frac{d}{dx} [e^x] = e^x$$

**DL** Derivative of a logarithmic function

$$\frac{d}{dx} [\log_a x] = \frac{1}{x \ln a}$$

**DNL** Derivative of a natural logarithmic function

$$\frac{d}{dx} [\ln x] = \frac{1}{x}$$

## Differentiation Trigonometric Function Rules

**DSin** Derivative of a sine function

$$\frac{d}{dx} (\sin x) = \cos x$$

**DCos** Derivative of a cosine function

$$\frac{d}{dx} (\cos x) = -\sin x$$

**DTan** Derivative of a tangent function

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

**DCsc** Derivative of a cosecant function

$$\frac{d}{dx} (\csc x) = -\csc x \cot x$$

**DSec** Derivative of a secant function

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

**DCot** Derivative of a cotangent function

$$\frac{d}{dx} (\cot x) = -\csc^2 x$$