1. Field Properties

1.1 Summary of Field Properties

Name	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive	$a(b + c) = ab + ac$	$(a + b)c = ac + bc$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	$a \cdot a^{-1} = 1 = a^{-1} \cdot a$

Table 1.1: Summary of the Field Properties

1.2 Properties of Addition

Definition 1.2.1 – Commutative Property of Addition (CPA).

$$ab = ba \tag{1.1}$$

Definition 1.2.2 – Associative Property of Addition (APA).

$$a + b + c = (a + b) + c \tag{1.2a}$$
$$a + b + c = a + (b + c) \tag{1.2b}$$

Definition 1.2.3 – Distributive Property Factoring (DPF).

$$ba + ca = (b + c)a \tag{1.3a}$$
$$ab + ac = a(b + c) \tag{1.3b}$$

Definition 1.2.4 – Additive Identity (AId).

$$a + 0 = a \tag{1.4a}$$
$$a = a + 0 \tag{1.4b}$$

Definition 1.2.5 – Additive Inverse (AI).

$$a + (-a) = 0 \tag{1.5a}$$

1.3 Properties of Multiplication

Definition 1.3.1 – Commutative Property of Multiplication (CPM).

$$a \cdot b = b \cdot a \tag{1.6}$$

Definition 1.3.2 – Associative Property of Multiplication (APM).

$$a \cdot b \cdot c = (a \cdot b) \cdot c \tag{1.7a}$$
$$a \cdot b \cdot c = a \cdot (b \cdot c) \tag{1.7b}$$

Definition 1.3.3 – Distributive Property Expanding (DPE).

$$a(b + c) = ab + ac \tag{1.8a}$$
$$(b + c)a = ba + ca \tag{1.8b}$$

Definition 1.3.4 – Multiplicative Identity (MId).

$$1a = a \tag{1.9a}$$
$$a = 1a \tag{1.9b}$$

R If the coefficient of a univariate monomial is the multiplicative identity 1.9a, 1, then it is not shown in it's canonical form.

$$\begin{aligned} C_k x^k &= C_k x^k \\ &= 1x^k \\ &= x^k \end{aligned}$$

Definition 1.3.5 – Multiplicative Inverse (MI).

$$a \cdot \frac{1}{a} = 1 \tag{1.10a}$$
$$a \cdot a^{-1} = 1 \tag{1.10b}$$

1.4 Properties of Subtraction

Definition 1.4.1 – Definition of Subtraction (DOS).

$$a - b = a + \neg b \tag{1.11a}$$
$$a + \neg b = a - b \tag{1.11b}$$

Definition 1.4.2 – Natural Numbers.

$$\mathbb{N} = \{0, 1, 2, 3 \ldots\}$$

R It is not uncommon for zero to be excluded from the natural numbers. In fact, some exclude zero from the natural numbers and then describe the set of natural numbers that include zero the whole numbers.

$$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$$

For the purposes of these notes, zero will be included within the set of natural numbers.

Definition 1.4.3 – Operation of Addition (OOA).

$$\underbrace{\underbrace{a}_{\text{Augend}} + \underbrace{b}_{\text{Addend}}}_{\text{Sum}} \tag{1.12}$$

More generally,

$$\underbrace{\underbrace{a}_{\text{Summand}} + \underbrace{b}_{\text{Summand}}}_{\text{Sum}} \tag{1.13}$$

Definition 1.4.4 – Operation of Multiplication (OOM).

$$\underbrace{\underbrace{a}_{\text{Multiplicand}} \times \underbrace{b}_{\text{Multiplier}}}_{\text{Product}} \tag{1.14}$$

More generally,

$$\underbrace{\underbrace{a}_{\text{Factor}} \times \underbrace{b}_{\text{Factor}}}_{\text{Product}} \tag{1.15}$$

Definition 1.4.5 – Integers.

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Definition 1.4.6 – Positive Integers.

$$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$$

Definition 1.4.7 – Greatest Common Divisor. Suppose that m and n are positive integers. The greatest common divisor is the largest divisor (factor) common to both m and n.

Definition 1.4.8 – Relatively Prime. Two integers m and n are relatively prime to each other, $m \perp n$, if they share no common positive integer divisors (factors) except 1.

$$m \perp n \text{ if } \gcd(m, n) = 1.$$

Definition 1.4.9 – Rational Numbers.

$$\mathbb{Q} = \{m/n \mid m, n \in \mathbb{Z}, n \neq 0\}$$

Definition 1.4.10 – Proper Fraction. Given $m < n$, then the fraction m/n is called proper.

Definition 1.4.11 – Improper Faction. Given $m > n$, then the fraction m/n is called improper.

Definition 1.4.12 – Common Denominator (CD).

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \tag{1.16a}$$

$$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b} \tag{1.16b}$$

Rule 1.4.1 – Fraction Operation of Addition (FOOA).

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \tag{1.17a}$$

$$\frac{ad+bc}{bd} = \frac{a}{b} + \frac{c}{d} \tag{1.17b}$$