

- Homework 3 is worth 1 Mark.
- You are required to master a set of mathematical equations.
- 1 Mark will be award for 10 correct answers or more.

Factorials

- $5! = 5 \times 4 \times 3 \times 2 \times 1 (= 120)$
- $5! = 5 \times 4!$

The Choose Operator

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

$$\binom{3}{1} = \frac{3!}{1! \times (3-1)!} = \frac{3 \times 2!}{1! \times 2!} = \frac{3}{1} = 3$$

The Choose Operator

- $\binom{3}{0} = 1$
- (Remember $0!$ is always equal to 1)
- $\binom{3}{1} = 3$
- $\binom{3}{2} = 3$
- $\binom{3}{3} = 1$

0.0.1 Choose Operator

For the positive integer n and non-negative integer k (with $k \leq n$), the choose operator is calculated as follows:

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

0.0.2 Choose Operator

Evaluate the following:

$$1 \quad \binom{5}{2}$$

$$2 \quad \binom{5}{0}$$

$$3 \quad \binom{10}{1}$$

$$4 \quad \binom{10}{9}$$

0.1 The factorial function

A factorial is a positive whole number, based on a number n , and which is written as “ $n!$ ”. The factorial $n!$ is defined as follows:

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$$

Remark $n! = n \times (n - 1)!$

The factorial function (symbol: $!$) just means to multiply a series of descending natural numbers.

1. Compute Range

The Range is simply the difference between the maximum value

Determine the sample size.

In this first homework, we will focus on some simple calculations, and simple concepts. The learning outcome is to familiarise yourself with the SULIS homework system

Binomial Coefficients

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

$$\bullet \binom{6}{2} = 15 \qquad \bullet \binom{5}{2} = 10 \qquad \bullet \binom{4}{0} = 1 \qquad \bullet \binom{4}{3} = 4$$

More Exercises

Evaluate the following:

(i) $\binom{5}{2}$

(iv) $\binom{6}{6}$

(ii) $\binom{5}{0}$

(v) $\binom{10}{1}$

(iii) $\binom{6}{3}$

(vi) $\binom{10}{9}$

Formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1},$$

which can be written using factorials as whenever $k \leq n$

Evaluate the following

$$\bullet {}^{10}C_0$$

$$\bullet {}^{10}C_1$$

$$\bullet {}^6C_3$$

Solutions

$$\bullet {}^{10}C_0 = 10!/(10! \times 0!) = 1$$

$$\bullet {}^{10}C_1 = 10!/(9! \times 1!) = 10$$

$$\bullet {}^6C_3 = 120$$

0.1.1 Range

- The range of a set of data is the difference between the highest and lowest values in the data set.
- Consider the following data set

$$\{39, 23, 34, 41, 37, 27, 44\}$$

- The highest value (i.e. the maximum) is 44.
- The lowest value (i.e. the minimum) is 23.
- The range is the difference is between these two numbers:

$$\text{Range} = 44 - 23 = 21.$$

2. The Exponential Function

$$f(x) = e^x$$

For most statistical analyses that you are likely to encounter, the value of x is likely to be negative or less than one.

$$f(x) = e^{-1}$$

$$f(x) = e^{0.5}$$

3. Factorials

Permutations

Choose Operators

Multiplication Rule

Factorial

and

(Interesting to note: $0! = 1$)

$$(n + 1)! = (n + 1) \times n!$$

For example $5! = 5 \times 4!$ $6! = 6 \times 5!$ and so on

Combinations Binomial Coefficients

$$\binom{52}{5} = 2598960$$

Easier example

$$\binom{8}{3} = \frac{8!}{3! \times 5!}$$

We can divide above and below by $(5!)$, leaving us with

$$\frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

There are 56 way to pick 3 objects at random from group of 8 objects.