Contents

0.1 Notation for Data Set

- Suppose that we have a data set with n observations. For each observation, a measure is recorded. Conventionally the measures are denoted x unless a more suitable notation is available.
- A subscript can be used to indicate which observation the measure is for.
- Hence we would write a data set as follows; $(x_1, x_2, x_3, x_1 \ldots x_n)$ (i.e. the first, second, third ... nth observation).

0.2 Relational operators

- $>$ means 'is greater than
- $\geq$ means 'is greater than or equal to
- $<$ means 'is less than

- $\leq$ means 'is less than or equal to
- $\neq$ means 'is not equal to
- $\approx$ or $\simeq$ means 'is approximately equal to

0.3 Factorials Numbers

A factorial is a positive whole number, based on a number n , and which is written as "$n!$". The factorial $n!$ is defined as follows:

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

The factorial function (symbol: !) just means to multiply a series of descending natural numbers.

Remark $n! = n \times (n-1)!$

Example:

- $3! = 3 \times 2 \times 1 = 6$
- $4! = 4 \times 3! = 4 \times 3 \times 2 \times 1 = 24$

Remark $0! = 1$ not 0.

Example:

- $4! = 4 \times 3 \times 2 \times 1 = 24$
- $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$
- $3! = 3 \times 2 \times 1 = 6$
- $4! = 4 \times 3! = 4 \times 3 \times 2 \times 1 = 24$
- $1! = 1$
- $0! = 1$

$$(n+1)! = (n+1) \times n!$$

- 5! = 5 x 4!
- 6! = 6 x 5! and so on

0.3.1 Factorials

Examples:

Importantly

$$n! = n \times (n-1)! = n \times (n-1) \times (n-2)!$$

For Example

$$6! = 6 \times 5! = 6 \times 5 \times 4!$$

- factorials
$$n! = (n) \times (n-1) \times (n-2) \times \ldots \times 1$$
  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
  - $3! = 3 \times 2 \times 1$

Factorials

- $5! = 5 \times 4 \times 3 \times 2 \times 1 (= 120)$
- $5! = 5 \times 4!$

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Remark $0! = 1$ not 0.

0.3.2 Choose Operator

For the positive integer n and non-negative integer k (with $k \leq n$), the choose operater is calculated as follows:

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

Evaluate the following:

1 $\binom{5}{2}$

2 $\binom{5}{0}$

3 $\binom{10}{1}$

4 $\binom{10}{9}$

The Choose Operator

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

$$\binom{3}{1} = \frac{3!}{1! \times (3-1)!} \frac{3 \times 2!}{1! \times 2!} = \frac{3}{1} = 3$$

The Choose Operator

- $\binom{3}{0} = 1$
- (Remember 0! is always equal to 1)
- $\binom{3}{1} = 3$
- $\binom{3}{2} = 3$
- $\binom{3}{3} = 1$

0.4 Binomial Coefficients

- $n!$ and $k!$ are the coefficients of n and k respectively.
- $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$
- For example $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- $n! = n \times (n-1)!$
- Importantly $0! = 1$ not 0.

$$\binom{6}{2} = \frac{6!}{2! \times (6-2)!} = \frac{6!}{2! \times 4!}$$
$$= \frac{6 \times 5 \times 4!}{2! \times 4!} = 30/2 = 15$$

More examples of Binomial coefficients on blackboard.

0.5 Binomial Coefficients

For the positive integer n and non-negative integer k (with $k \leq n$), the choose operater is calculated as follows:

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\binom{6}{3} = \frac{6!}{(6-3)!3!} = \frac{6!}{3! \times 3!}$$

$$\frac{6!}{3! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{120}{6} = 120$$

Evaluate the following:

1. $\binom{5}{2}$
2. $\binom{5}{0}$
3. $\binom{6}{3}$
4. $\binom{6}{6}$
5. $\binom{10}{1}$
6. $\binom{10}{9}$

$$\boxed{\binom{n}{k} = \frac{n!}{k! \times (n-k)!}}$$

- $\binom{6}{2} = 15$
- $\binom{5}{2} = 10$
- $\binom{4}{0} = 1$
- $\binom{4}{3} = 4$

Formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1},$$

which can be written using factorials as whenever $k \leq n$

Binomial Coefficients

In the last class, we came across binomial coefficients. Informally, binomial coefficients are the number of ways k items can be selected from a group of n items. The binomial coefficient indexed by n and k is usually written as ${}^{n}C_{k}$ or

$$\binom{n}{k}$$

. C is colloqially known as the "choose operator".

$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

(We call the operator the choose operator. We will use both notations interchangeably.)

Binomial Coefficients

- $n!$ and $k!$ are the coefficients of n and k respectively.
- $n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$
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More examples of Binomial coefficients on blackboard. **Probability Mass Function** (Formally defining something mentioned previously)

- a probability mass function (pmf) is a ***function*** that gives the probability that a discrete random variable is exactly equal to some value.

$$P(X = k)$$

- The probability mass function is often the primary means of defining a discrete probability distribution
- It is conventional to present the probability mass function in the form of a table.
- The p.m.f of a value k is often denoted $f(k)$.

Binomial Example 1 (Revision from Last Class)
Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

Solution: This is a binomial experiment in which the number of trials is equal to 5, the number of successes is equal to 2, and the probability of success on a single trial is 1/6 or about 0.167.
Therefore, the binomial probability is:

$$P(X = 2) =^5 C_2 \times (1/6)^2 \times (5/6)^3 = 0.161$$

Binomial Example 2 Suppose there is a container that contains 6 items. The probability that any one of these items is defective is 0.3. Suppose all six items are inspected.

- What is the probability of 3 defective components?
- What is the probability of 4 defective components?

$$P(3 \text{ defects}) = f(3) = P(X = 3) = \binom{6}{3} 0.3^3 (1 - 0.3)^{6-3} = 0.1852$$
$$P(4 \text{ defects}) = f(4) = P(X = 4) = \binom{6}{4} 0.3^4 (1 - 0.3)^{6-4} = 0.0595$$

0.5.1 Binomial Coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1},$$

which can be written using factorials as whenever $k \leq n$

Evaluat the following

- ${}^{10}C_0$
- ${}^{10}C_1$
- ${}^{6}C_3$

Solutions

- ${}^{10}C_0 = 10!/(10! \times 0!) = 1$
- ${}^{10}C_1 = 10!/(9! \times 1!) == 1$
- ${}^{6}C_3$

0.5.2 Binomial Coefficients

$$\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5.4.3!}{2!.3!} = \frac{5.4}{2.1} = 10$$

$$\binom{5}{0} = \frac{5!}{0!\,(5-0)!} = \frac{5!}{0!.5!} = \frac{5!}{2!} = 1$$

Recall $0! = 1$

- In the last class, we came across binomial coefficients. Informally, binomial coefficients are the number of ways k items can be selected from a group of n items.
- The binomial coefficient indexed by n and k is usually written as ${}^nC_k$ or
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.
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$$\binom{n}{k} = \frac{n!}{k! \times (n-k)!}$$

- (We call the operator the choose operator. We will use both notations interchangeably.)
- $n!$ and $k!$ are the coefficients of n and k respectively.
- $n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$
- For example $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
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- Importantly $0! = 1$ not 0.

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More examples of Binomial coefficients on blackboard.

$$\binom{52}{5} = 2598960$$

Easier example

$$\binom{8}{3} = \frac{8!}{3! \times 5!}$$

We can divide above and below by (5!), leaving us with

$$\frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

There are 56 way to pick 3 objects at random from group of 8 objects.

More Exercises

Evaluate the following:

(i) $\binom{5}{2}$

(ii) $\binom{5}{0}$

(iii) $\binom{6}{3}$

(iv) $\binom{6}{6}$

(v) $\binom{10}{1}$

(vi) $\binom{10}{9}$

0.6 Maximum, Minimum and Range

0.6.1 Range

- The Range is simply the difference between the maximum value

> - The **range** of a set of data is the difference between the highest and lowest values in the data set.
> - Consider the following data set
> $$\{39, 23, 34, 41, 37, 27, 44\}$$
> - The highest value (i.e. the maximum) is 44.
> - The lowest value (i.e. the minimum) is 23.
> - The range is the difference is between these two numbers:
> $$\text{Range } = 44 - 23 = 21.$$

In this frst homework, we will focus on some simple calculations, and simple concepts. The learning outcome is to familiarise yourself with the SULIS homework system

- Calculate the range of the following data set

$$3, 7, 5, 13, 20, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29$$

- Calculate the range of these numbers:

$$3, -7, 5, 13, -2$$

- For each of these data sets, determine the sample size.

0.7 Summation

The summation sign $\sum$ is commonly used in most areas of statistics. Given $x_1 = 3, x_2 = 1, x_3 = 4, x_4 = 6, x_5 = 8$ find:

$$(i) \sum_{i=1}^{i=n} x_i \qquad\qquad (ii) \sum_{i=3}^{i=4} x_i^2$$

$$\begin{aligned} (i) \sum_{i=1}^{i=n} x_i &= x_1 + x_2 + x_3 + x_4 + x_5 \\ &= 3 + 1 + 4 + 6 + 8 \\ &= \mathbf{22} \end{aligned}$$

$$(ii) \sum_{i=1}^{i=n} x_i^2 = x_3^2 + x_4^2 = 9 + 16 = \mathbf{25}$$

When all elements of a data set are used, a simple version of the summation notation can be used. $\sum_{i=1}^{i=n} x_i$ can simply be written as $\sum x$

Example

Given that $p_1 = 1/4, p_2 = 1/8, p_3 = 1/8, p_4 = 1/3, p_5 = 1/6$ find:

- $\sum_{i=1}^{i=n} p_i \times x_i$
- $\sum_{i=1}^{i=n} p_1 \times x_i^2$

0.8 Summation

The summation sign $\sum$ is commonly used in most areas of statistics. Given $x_1 = 3, x_2 = 1, x_3 = 4, x_4 = 6, x_5 = 8$ find:

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Example

Given that $p_1 = 1/4, p_2 = 1/8, p_3 = 1/8, p_4 = 1/3, p_5 = 1/6$ find:

- $\sum_{i=1}^{i=n} p_i \times x_i$
- $\sum_{i=1}^{i=n} p_1 \times x_i^2$

0.9 The Exponential Function

$$f(x) = e^x$$

For most statistical analyses that you are likely to encounter, the value of x is likely to be negative or less than one.

$$f(x) = e{-}1$$

$$f(x) = e^{0.5}$$

Combinations formula

$$^{n}C_k = \frac{n!}{k! \times (n-k)!}$$

- Remark $n! = n \times (n-1)!$
- $0! = 1$

Show that

$$^{n}C_0 = 1$$

Solution:

$$^{n}C_0 = \frac{n!}{0! \times (n-0)!} = \frac{n!}{n!} = 1$$

Show that

$$^{n}C_1 = n$$

Solution:

$$^{n}C_1 = \frac{n!}{1! \times (n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

Compute $^{7}C_2$

Solution:

$$^{7}C_2 = \frac{7!}{2! \times (7-2)!} = \frac{7 \times 6 \times 5!}{2! \times 5!} = \frac{42}{2} = 21$$

Compute $^{11}C_1$

Solution:

$$^{11}C_1 = \frac{11!}{1! \times 10!} = \frac{11 \times 10!}{1 \times 10!} = 11$$

0.9.1 Example: Factorials

Examples:

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- $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$
- $1! = 1$
- $0! = 1$

Importantly

$$n! = n \times (n-1)! = n \times (n-1) \times (n-2)!$$

For Example

$$6! = 6 \times 5! = 6 \times 5 \times 4!$$

- $3! = 3 \times 2 \times 1 = 6$
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- factorials

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  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
  - $3! = 3 \times 2 \times 1$
- Zero factorial : Remark $0! = 1$ not 0.

$$0! = 1$$

0.9.2 Example 1

$$\binom{5}{2} = \frac{5!}{2!\,(5-2)!} = \frac{5.4.3!}{2!.3!} = \frac{5.4}{2.1} = 10$$

Example 2

$$\binom{5}{0} = \frac{5!}{0!\,(5-0)!} = \frac{5!}{0!.5!} = \frac{5!}{2!} = 1$$

Recall $0! = 1$