PHIL 377 Final

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1 For each item below, provide a Fitch-style proof of the claim.

a $\vdash \forall x \forall y(x = y \leftrightarrow y = x)$

> > 1. $a = b$
> >
> > 2. $a = a$ — $=$ **Elim:** 1, 1
> > 3. $b = a$ — $=$ **Elim:** 1, 2
>
> > 4. $b = a$
> >
> > 5. $b = b$ — $=$ **Elim:** 4, 4
> > 6. $a = b$ — $=$ **Elim:** 4, 5
>
> 7. $a = b \leftrightarrow b = a$ — $\leftrightarrow$ **Intro:** 1–3, 4–6
> 8. $\forall y(a = y \leftrightarrow y = a)$ — $\forall$ **Intro:** 7
> 9. $\forall x \forall y(x = y \leftrightarrow y = x)$ — $\forall$ **Intro:** 8

b $\forall x \forall y \neg x = y \vdash K(a) \land \neg K(b)$

> 1. $\forall x \forall y \neg x = y$
>
> 2. $\forall y \neg a = y$ — $\forall$ **Elim:** 1
> 3. $\neg a = a$ — $\forall$ **Elim:** 2
> 4. $a = a$ — $=$ **Intro**
> 5. $\bot$ — $\neg$ **Elim:** 3, 4
> 6. $K(a) \land \neg K(b)$ — **X:** 5

c $\exists y \neg K(y), \forall x(\neg A(x) \to K(x)) \vdash \exists w(A(w) \lor \neg L(w, a))$

> 1. $\exists y \neg K(y)$
> 2. $\forall x(\neg A(x) \to K(x))$
>
> 3. $\neg A(b) \to K(b)$ — $\forall$ **Elim:** 2
>
> > 4. $\neg K(b)$
> >
> > > 5. $\neg A(b)$
> > >
> > > 6. $K(b)$ — $\to$ **Elim:** 3, 5
> > > 7. $\bot$ — $\neg$ **Elim:** 4, 6
> >
> > 8. $A(b)$ — **IP:** 5–7
> > 9. $A(b) \lor L(b, a)$ — $\lor$ **Intro:** 8
> > 10. $\exists w(A(w) \lor L(w, a))$ — $\exists$ **Intro:** 9
>
> 11. $\exists w(A(w) \lor L(w, a))$ — $\exists$ **Elim:** 1, 4–10

d $[\neg L(c, i, b) \lor \neg(H(c) \lor H(c))], \forall x \forall y(G(x, y) \to H(c)), \exists x G(i, x) \land \forall x \forall y \forall z L(x, y, z) \vdash \bot$

> 1. $\neg L(c, i, b) \lor \neg(H(c) \lor H(c))$
> 2. $\forall x \forall y(G(x, y) \to H(c))$
> 3. $\exists x G(i, x) \land \forall x \forall y \forall z L(x, y, z)$
>
> 4. $\forall x \forall y \forall z L(x, y, z)$ — $\land$ **Elim:** 3
> 5. $\forall y \forall z L(c, y, z)$ — $\forall$ **Elim:** 4
> 6. $\forall z L(c, i, z)$ — $\forall$ **Elim:** 5
> 7. $L(c, i, b)$ — $\forall$ **Elim:** 6
>
> > 8. $\neg L(c, i, b)$
> >
> > 9. $\bot$ — $\neg$ **Elim:** 7, 8
>
> 10. $\forall y(G(i, y) \to H(c))$ — $\forall$ **Elim:** 2
> 11. $G(i, j) \to H(c)$ — $\forall$ **Elim:** 10
> 12. $\exists x G(i, x)$ — $\land$ **Elim:** 3
>
> > 13. $G(i, j)$
> >
> > 14. $H(c)$ — $\to$ **Elim:** 11, 13
>
> 15. $H(c)$ — $\exists$ **Elim:** 12, 13–14
> 16. $H(c) \lor H(c)$ — $\lor$ **Intro:** 15
>
> > 17. $\neg(H(c) \lor H(c))$
> >
> > 18. $\bot$ — $\neg$ **Elim:** 16, 17
>
> 19. $\bot$ — $\lor$ **Elim:** 1, 8–9, 17–18

2 Some philosophers are travelling to a conference together. Among them are Fitch, Gödel , and Hobbes. Consider the following dictionary and domain:

- **Domain: The six philosophers going to the conference.**
- $D(x, y)$_____x **is driving** _____y
- $A(x)$_____x **is in car** A
- $B(x)$_____x **is in car** B
- f : **Fitch;** g : **Gödel ;** h : **Hobbes**

Using first order logic, formalize the scenarios described in a-d below.

(Note: Assume 2a is always the case.)

a Everyone is in exactly one of the two cars. (Each person is in at least one of car A or car B, and no one is in both cars.)

$\forall x[(A(x) \vee B(x)) \wedge \neg(A(x) \wedge B(x))]$

b Drivers are driving everyone in their car. (A driver drives all of those who happen to be in the car they are driving.)

(Note: The clearer version implies theres exists a driver.)
$\forall x \forall y \exists z[(D(x, z) \wedge [(A(x) \wedge A(y)) \vee (B(x) \wedge B(y))]) \rightarrow D(x, y)] \wedge \exists x \exists y D(x, y)$

c Either Fitch or Gödel is driving Hobbes.

$D(f, h) \vee D(g, h)$

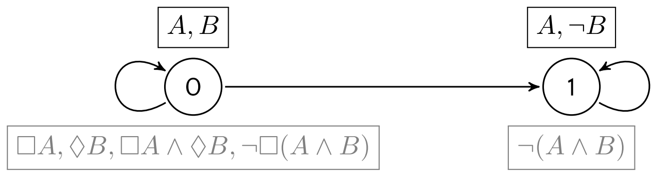
d Fitch is travelling with only two of the others.

(Note: 'travelling with' means 'being in the same car as'.)
(Note 2: 'only' is defined as an adverb as 'no more than' by https://www.dictionary.com/.)
(Note 3: 'only' is listed as a synonym of 'at most' by https://www.thesaurus.com/.)
(Note 4: The following formula is not concise, but it's algorithmic and simple.)
$\exists x_0 \exists x_1 \exists x_2 \exists x_3 \exists x_4[[((((((((((A(f) \wedge A(x_0)) \wedge A(x_1)) \wedge \neg A(x_2)) \wedge \neg A(x_3)) \wedge \neg A(x_4)) \vee (((((B(f) \wedge B(x_0)) \wedge B(x_1)) \wedge \neg B(x_2)) \wedge \neg B(x_3)) \wedge \neg B(x_4))) \vee (((((A(f) \wedge A(x_0)) \wedge \neg A(x_1)) \wedge \neg A(x_2)) \wedge \neg A(x_3)) \wedge \neg A(x_4))) \vee (((((B(f) \wedge B(x_0)) \wedge \neg B(x_1)) \wedge \neg B(x_2)) \wedge \neg B(x_3)) \wedge \neg B(x_4))) \vee (((((A(f) \wedge \neg A(x_0)) \wedge \neg A(x_1)) \wedge \neg A(x_2)) \wedge \neg A(x_3)) \wedge \neg A(x_4))) \vee (((((B(f) \wedge \neg B(x_0)) \wedge \neg B(x_1)) \wedge \neg B(x_2)) \wedge \neg B(x_3)) \wedge \neg B(x_4))] \wedge [(((((((((((((\neg f = x_0 \wedge \neg f = x_1) \wedge \neg f = x_2) \wedge \neg f = x_3) \wedge \neg f = x_4) \wedge \neg x_0 = x_1) \wedge \neg x_0 = x_2) \wedge \neg x_0 = x_3) \wedge \neg x_0 = x_4) \wedge \neg x_1 = x_2) \wedge \neg x_1 = x_3) \wedge \neg x_1 = x_4) \wedge \neg x_2 = x_3) \wedge \neg x_2 = x_4) \wedge \neg x_3 = x_4]]$

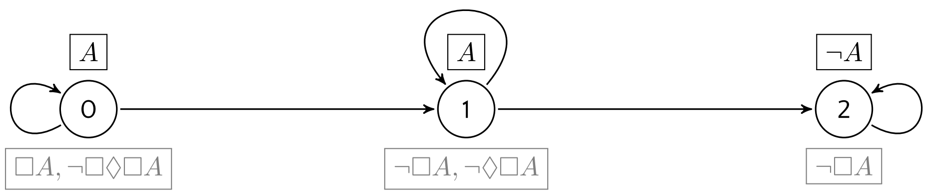
3 For each of the following modal arguments provide a model that is a counterexample. Diagrams that indicate which formulas are true and which are false at which worlds is fine. No need for further explanation, if you use a diagram. If you do not use a diagram, please provide some explanation of which formulas are true at which worlds.

The counterexamples for all of the following is world 0. Gray text is explanations.

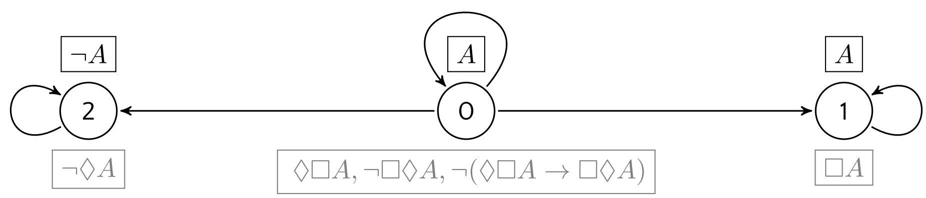
a $\Box A \wedge \Diamond B \vDash_T \Box(A \wedge B)$



b $\Box A \vDash_T \Box\Diamond\Box A$



c $\vDash_{S4} \Diamond\Box A \rightarrow \Box\Diamond A$



d $\Box A \rightarrow \Diamond A \vDash_{K4} \Diamond A \rightarrow \Box A$