PHIL 377 Assignment 1

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1 Which of the following sentences is appropriate for logical analysis using our methods? In each case explain why or why not.

a Will the next president be a republican?

Not a valid sentence, it's a question. Doesn't have a truth value.

b The president is a republican!

Is a valid sentence, as it's either true or false.

c Make the next president a republican!

Not a valid sentence, it's a proclamation. Doesn't have a truth value.

d Beware of the president.

Not a valid sentence, it's a proclamation. Doesn't have a truth value.

2 Organize the following arguments into premise-conclusion form.

a Having cancer is good, for whatever is required by something that is good is itself a good. Being cured of cancer is good, and being cured of cancer requires having cancer.

Whatever is required by something that is good is itself a good.
Being cured of cancer is good.
Being cured of cancer requires having cancer.
∴ Having cancer is good.

b Let's assume Justice Betty is the Judge for the case, then after a long trial Peter will not be given a suspended sentence. I can tell you why I think that. First, the trial will be long unless the Crown prosecutor is brief, but he isn't. Furthermore, if Fred is the defense attorney, as good as he is, Peter will be found guilty. But Fred is the defense attorney. Justice Betty doesn't give out suspended sentences. So, finally, if Peter is found guilty, he will be sentenced.

If Fred is the defense attorney, then Peter will be found guilty.
Fred is the defense attorney.
∴ Peter will be found guilty.
If Peter is found guilty, then Peter will be sentenced.
∴ Peter will be sentenced.
The trial will be long unless the Crown prosecutor is brief.
The Crown prosecutor is not brief.
∴ The trial will be long.
Justice Betty is the Judge for the case.
Justice Betty doesn't give out suspended sentences.
∴ Peter will not be given a suspended sentence.
∴ Peter will not be given a suspended sentence, and the trial will be long.

3 Each of the following questions has to be with the logical concepts concepts discussed in class.

a Give a valid argument with a true conclusion and all false premises.

Nuclear weapons contains the souls of dead physicists.
Dead physicist ghosts contains a great amount of potential energy.
$\therefore$ Nuclear weapons contains a great amount of potential energy.

b Given an invalid argument with a true conclusion.

If 'true' means 'necessarily true'.
Then there's no such thing. The only way for an argument to be invalid is for all the premises to be true, and the conclusion to be false. If the conclusion is necessarily true, then the argument is necessarily valid. Might not be sound, but still valid.
If 'true' means 'contigently true'.
Then the following argument would be invalid:
$1 = 1$.
$\therefore 1 + 1 = 2$.
Even though common sense tells us that the conclusion is necessary true, it is only contigently true.
In binary, $1 + 1 = 10$.
In modular arithmetic mod 2, $1 + 1 = 0$.
Even still, if in the future, the mathematics community decides to adopt a new numeral system. Where the symbol '2' repensents the concept of 'three', then the conclusion would again be false.
Therefore any nessary truth or falsehood is a contigency if you are allowed change the established/assumed definitions, and all arguments would be invalid.

c Give a valid argument with a false conclusion and at least one true premise.

Magnets exhibits electromagnetic force.
Electromagnetic force causes attraction.
Magnets are mined from the Earth.
Earth has gravity.
Gravity causes attriction.
$\therefore$ Electromagnetic force are a manifestation of gravity.

d Given a non-mathematical example of a necessary truth.

Rabbi Shlomo Bergblattstein is a jew.

e Give a mathematical example of a necessary falsehood.

$\exists a, b \in \mathbb{R}$, such that $ab \neq ba$.

4 Explain why, generally, we cannot use logic alone to show that a valid argument is sound. Give an example of an argument where logic alone can be used to show that the argument is sound.

For an argument to be sound, the premises have to be true. For an argument to be valid, we assume the premises to be true and prove that the conclusion is true. Deductive reasoning cannot tell us if an axiom/premise is true or not. Therefore cannot tell us if the argument is sound or not.
Example of an argument where logic alone can be used to show that the argument is sound:
Let T be a true statement.
T
$\therefore T$
This is a trivial argument but it is a valid and sound one.
As T is defined to be true.

5 Explain why in each case the sentence doesn't exhibit truth functional structure, or say why it does.

a It's possible that everyone in North America owns a smart phone.

Yes it does. The sentence mean 'The probablity that everyone in North America owns a smart phone is not zero'.
Which is either true or false. The negation of that sentence is 'The probablity that everyone in North America owns a smart phone is zero' or 'It's impossible that everyone in North America owns a smart phone'.

b The coin must come up heads.

The sentence means 'The coin necessarily comes up heads' or 'The probablity that the coin comes up with heads is one'. Which does have a truth value, and is of truth functional structure.

c The defendant relented only after much testimony was discredited.

Yes, this sentence is of the form 'Event A happened after event B'. So it either happened that way or it didn't happen that way. It has a truth value and is an atomic sentence.

d Jeff loves it that Nenshi is mayor, but not that Kenny is Premier.

Yes, the two atomic sentences are 'Jeff loves it that Nenshi is mayor' and 'Jeff does not love it that Kenny is Premier'. These sentence are joined with an 'and'/'but', therefore it's still of truth functional structure.

6 At the world track and field games some people are musing about the performances of the German, Danish, and French teams. Consider the following dictionary:

F **The French team will win a gold medal.**
G **The German team will win a gold medal.**
D **The Danish team will win a gold medal.**
S **The star German Runner is disqualified.**
R **It rains during most of the games.**

Provide formulas that formalize each of the following statements:

a Exactly one of the teams will win a gold medal.

Formula:

$$([(F \wedge \neg G) \wedge \neg D] \vee [(\neg F \wedge \neg G) \wedge D]) \vee [(\neg F \wedge G) \wedge \neg D]$$

b At least two of the teams will win a gold medal.

Formula:

$$(([(\neg F \wedge G) \wedge D] \vee [(F \wedge \neg G) \wedge D]) \vee [(F \wedge G) \wedge \neg D]) \vee [(F \wedge G) \wedge D]$$

c If the star German runner is disqualified, the Germans will win gold only if neither of the other teams win gold.

Formula:

$$(S \wedge G) \rightarrow (\neg F \wedge \neg D)$$

d The Danes will win gold unless it rains for most of the games, in which case they won't, but the other two teams will win gold.

Formula:

$$(\neg R \wedge D) \vee (R \wedge (\neg D \wedge (F \wedge G)))$$

7 Provide dictionaries for the following arguments, and then provide formulas that formalize the arguments —in standard form—using those dictionaries.

a Having cancer is good, for whatever is required by something that is good is itself a good. Being cured of cancer is good, and being cured of cancer requires having cancer. (2 points)

'A' is 'Having cancer is good.'
'B' is 'Being cured of cancer is good.'
'C' is 'Being cured of cancer.'
'D' is 'Having cancer.'
'E' is 'Whatever is required by something that is good is itself a good.'

$$C \to D$$
$$B$$
$$E$$
$$((C \to D) \wedge (B \wedge E)) \to A$$
$$\therefore A$$

b Let's assume Justice Betty is the Judge for the case, then after a long trial Peter will not be given a suspended sentence. I can tell you why I think that. First, the trial will be long unless the Crown prosecutor is brief, but he isn't. Furthermore, if Fred is the defense attorney, as good as he is, Peter will be found guilty. But Fred is the defense attorney. Justice Betty doesn't give out suspended sentences. So, finally, if Peter is found guilty, he will be sentenced. (3 points)

'A' is 'The trial will be long.'
'B' is 'The Crown prosecutor is brief.'
'C' is 'Peter will be be given a suspended sentence.'
'D' is 'Justice Betty is the Judge for this case.'
'E' is 'Justice Betty doesn't give out suspended sentences.'
'G' is 'Fred is the defense attorney.'
'H' is 'Peter will be found guilty.'
'I' is 'Peter will be sentenced.'

$$G \to H$$
$$G$$
$$\therefore H$$
$$H \to I$$
$$\therefore I$$
$$A \vee B$$
$$\neg B$$
$$\therefore A$$
$$D \wedge E$$
$$(D \wedge E) \to \neg C$$
$$\therefore \neg C$$
$$\therefore A \wedge \neg C$$

c Look, if neither Jenny nor Fred play the lawyer, then Morris, the director, will not be upset. Moreover, if Morris isn't upset, the play will be successful. Thus, the play will get good reviews. Remember, both Jenny and Fred won't play the part of the lawyer, and plays get good reviews when and only when they are successful. (2 points)

'A' is 'Jenny play the lawyer.'
'B' is 'Fred play the lawyer'
'C' is 'Morris will be upset.'
'D' is 'The play will be successful.'
'E' is 'The play will get good reviews.'

$$\begin{aligned}
&\neg(A \land B)\\
&(A \lor B) \rightarrow \neg C\\
&\neg C \rightarrow D\\
&\therefore (A \lor B) \rightarrow D\\
&E \rightarrow D\\
&\therefore E
\end{aligned}$$

This is an invalid argument because the question is an invalid argument.

8 Which of the following expressions is a formula of our formal language? Indicate those that are, and an explanation for those that are not.

a $B \land Z$

Formula.

b $\land H$

Not formula. 'and H'. What 'and H'?

c $\neg O$

Formula.

d $M \neg N$

Not formula. Missing $\land, \lor, \rightarrow$, etc.

e $J \rightarrow (K \rightarrow (A \lor N))$

Formula. It's convention to ommit the outter most brakets.

f $\mathscr{A} \lor B$

Not formula. $\mathscr{A}$ is a metavariable.

g $(I \lor (T \land E))$

Formula.

h $(A \land B \land \neg C)$

Not formula. Every possible way to bracket this yeilds the same logical formula. However it is not convention to ommit brackets for concatenated 'and' statements, Therefore not a formula.

i $(F \leftrightarrow K) \rightarrow \{M \lor K\}$

Not formula. Wrong brackets. Convention is to use only [] or ().

j $((G \lor E) \rightarrow (\neg H \land (K \lor B))$

Not formula. Mismatched brackets, compile error.

9 For each of the following formulas indicate which has the form $\neg\mathscr{A} \to \mathscr{B}$. If it is of that form provide the substitutions for the metavariables $\mathscr{A}, \mathscr{B}$.

a $A \to B$

Let $\mathscr{A} = \neg A, \mathscr{B} = B$.

b $\neg A \to B$

Let $\mathscr{A} = A, \mathscr{B} = B$.

c $\neg A \to \neg B$

Let $\mathscr{A} = A, \mathscr{B} = \neg B$.

d $\neg\neg A \to B$

Let $\mathscr{A} = \neg A, \mathscr{B} = B$.

e $\neg(A \to B)$

$\neg(A \to B)$
$\therefore \neg(\neg(A \wedge \neg B))$
$\therefore (A \wedge \neg B)$
$\therefore \neg(A \wedge \neg B) \to$ (False)
Let $\mathscr{A} = A \wedge \neg B, \mathscr{B} = B \wedge \neg B =$ False.

f $\neg\neg A \to \neg B$

Let $\mathscr{A} = \neg A, \mathscr{B} = \neg B$.

g $\neg(\neg A \to B)$

$\neg(\neg A \to B)$
$\therefore \neg(\neg(\neg A \wedge \neg B))$
$\therefore (\neg A \wedge \neg B)$
$\therefore \neg(\neg A \wedge \neg B) \to$ (False)
Let $\mathscr{A} = \neg A \wedge \neg B, \mathscr{B} = B \wedge \neg B =$ False.

h $\neg\neg(A \to B) \to (C \to D)$

Let $\mathscr{A} = \neg(A \to B), \mathscr{B} = C \to D$.

i $\neg(A \vee \neg B) \to \neg(C \wedge \neg D)$

Let $\mathscr{A} = A \vee \neg B, \mathscr{B} = \neg(C \wedge \neg D)$.

j $\neg(A \leftrightarrow B) \to (\neg C \to D)$

Let $\mathscr{A} = A \leftrightarrow B, \mathscr{B} = \neg C \to D$.

Note: Every formula $\mathscr{C}$, can be written in the form of $\neg\mathscr{A} \to \mathscr{B}$, by setting $\mathscr{A} = \mathscr{C}, \mathscr{B} =$ False.
Note 2: After I've completed the question, I was informed by a peer that the question should be interpreted in the context of the main logical operator instead of logical equivalence. So I've completed the question again under the new assumption. However I've left in my original answer in aswell, due to the ambiguity of the question.

9 (Again) In context of the primary connector.

a $A \to B$

No.

b $\neg A \to B$

Yes. Let $\mathscr{A} = A, \mathscr{B} = B$.

c $\neg A \to \neg B$

Yes. Let $\mathscr{A} = A, \mathscr{B} = \neg B$.

d $\neg\neg A \to B$

Yes. Let $\mathscr{A} = \neg A, \mathscr{B} = B$.

e $\neg(A \to B)$

No.

f $\neg\neg A \to \neg B$

Yes. Let $\mathscr{A} = \neg A, \mathscr{B} = \neg B$.

g $\neg(\neg A \to B)$

No.

h $\neg\neg(A \to B) \to (C \to D)$

Yes. Let $\mathscr{A} = \neg(A \to B), \mathscr{B} = C \to D$.

i $\neg(A \vee \neg B) \to \neg(C \wedge \neg D)$

Yes. Let $\mathscr{A} = A \vee \neg B, \mathscr{B} = \neg(C \wedge \neg D)$.

j $\neg(A \leftrightarrow B) \to (\neg C \to D)$

Yes. Let $\mathscr{A} = A \leftrightarrow B, \mathscr{B} = \neg C \to D$.

10 Consider the following truth value assignment:

$$v(A) = T, v(B) = F, v(C) = T, v(D) = T$$

For each of the following $\mathscr{A}$ compute $v(\mathscr{A})$.

a $(A \wedge C) \to (B \vee D)$

True.

b $(\neg A \vee D) \to B$

False.

c $\neg A \to \neg(B \wedge D)$

True.

d $\neg\neg A \to B$

False.

e $\neg(A \to (B \to (D \wedge \neg C)))$

False

f $\neg\neg A \to \neg B$

True.

g $\neg(\neg(A \vee (A \wedge D)) \to B)$

False.

h $\neg\neg(A \to B) \to (C \to D)$

True.

i $\neg(A \vee \neg B) \to \neg(C \wedge \neg D)$

True.

j $\neg(A \leftrightarrow B) \to (\neg C \to D)$

True.