PHIL 377 Final

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1 For each item below, provide a Fitch-style proof of the claim.

a $\vdash \forall x \forall y (x = y \leftrightarrow y = x)$

1. $a = b$	
2. $a = a$	$=$ **Elim:** 1, 1
3. $b = a$	$=$ **Elim:** 1, 2
4. $b = a$	
5. $b = b$	$=$ **Elim:** 4, 4
6. $a = b$	$=$ **Elim:** 4, 5
7. $a = b \leftrightarrow b = a$	$\leftrightarrow$ **Intro:** 1–3, 4–6
8. $\forall y(a = y \leftrightarrow y = a)$	$\forall$ **Intro:** 7
9. $\forall x \forall y(x = y \leftrightarrow y = x)$	$\forall$ **Intro:** 8

b $\forall x \forall y \neg x = y \vdash K(a) \wedge \neg K(b)$

1. $\forall x \forall y \neg x = y$	
2. $\forall y \neg a = y$	$\forall$ **Elim:** 1
3. $\neg a = a$	$\forall$ **Elim:** 2
4. $a = a$	$=$ **Intro**
5. $\perp$	$\neg$ **Elim:** 3, 4
6. $K(a) \wedge \neg K(b)$	**X:** 5

c $\exists y \neg K(y), \forall x(\neg A(x) \to K(x)) \vdash \exists w(A(w) \vee \neg L(w, a))$

1. $\exists y \neg K(y)$	
2. $\forall x(\neg A(x) \to K(x))$	
3. $\neg A(b) \to K(b)$	$\forall$ **Elim:** 2
4. $\neg K(b)$	
5. $\neg A(b)$	
6. $K(b)$	$\to$ **Elim:** 3, 5
7. $\perp$	$\neg$ **Elim:** 4, 6
8. $A(b)$	**IP:** 5–7
9. $A(b) \vee L(b, a)$	$\vee$ **Intro:** 8
10. $\exists w(A(w) \vee L(w, a))$	$\exists$ **Intro:** 9
11. $\exists w(A(w) \vee L(w, a))$	$\exists$ **Elim:** 1, 4–10

d $[\neg L(c, i, b) \vee \neg(H(c) \vee H(c))], \forall x \forall y(G(x, y) \to H(c)), \exists x G(i, x) \wedge \forall x \forall y \forall z L(x, y, z) \vdash \perp$

1. $\neg L(c, i, b) \vee \neg(H(c) \vee H(c))$	
2. $\forall x \forall y(G(x, y) \to H(c))$	
3. $\exists x G(i, x) \wedge \forall x \forall y \forall z L(x, y, z)$	
4. $\forall x \forall y \forall z L(x, y, z)$	$\wedge$ **Elim:** 3
5. $\forall y \forall z L(c, y, z)$	$\forall$ **Elim:** 4
6. $\forall z L(c, i, z)$	$\forall$ **Elim:** 5
7. $L(c, i, b)$	$\forall$ **Elim:** 6
8. $\neg L(c, i, b)$	
9. $\perp$	$\neg$ **Elim:** 7, 8
10. $\forall y(G(i, y) \to H(c))$	$\forall$ **Elim:** 2
11. $G(i, j) \to H(c)$	$\forall$ **Elim:** 10
12. $\exists x G(i, x)$	$\wedge$ **Elim:** 3
13. $G(i, j)$	
14. $H(c)$	$\to$ **Elim:** 11, 13
15. $H(c)$	$\exists$ **Elim:** 12, 13–14
16. $H(c) \vee H(c)$	$\vee$ **Intro:** 15
17. $\neg(H(c) \vee H(c))$	
18. $\perp$	$\neg$ **Elim:** 16, 17
19. $\perp$	$\vee$ **Elim:** 1, 8–9, 17–18

2 Some philosophers are travelling to a conference together. Among them are Fitch, Gödel , and Hobbes. Consider the following dictionary and domain:

- **Domain: The six philosophers going to the conference.**
- $D(x, y)$_____x **is driving** _____y
- $A(x)$_____x **is in car** A
- $B(x)$_____x **is in car** B
- f : **Fitch;** g : **Gödel ;** h : **Hobbes**

Using first order logic, formalize the scenarios described in a-d below.

(Note: Assume 2a is always the case.)

a Everyone is in exactly one of the two cars. (Each person is in at least one of car A or car B, and no one is in both cars.)

$\forall x[(A(x) \lor B(x)) \land \neg(A(x) \land B(x))]$

b Drivers are driving everyone in their car. (A driver drives all of those who happen to be in the car they are driving.)

(Note: The clearer version implies theres exists a driver.)
$\forall x \forall y \exists z[(D(x, z) \land [(A(x) \land A(y)) \lor (B(x) \land B(y))]) \to D(x, y)] \land \exists x \exists y D(x, y)$

c Either Fitch or Gödel is driving Hobbes.

$D(f, h) \lor D(g, h)$

d Fitch is travelling with only two of the others.

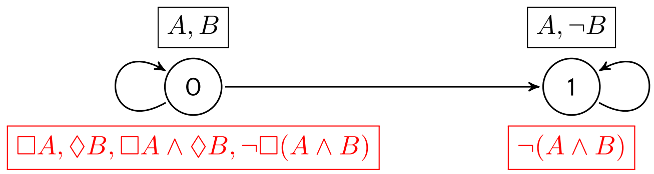
(Note: 'travelling with' means 'being in the same car as'.)
(Note 2: 'only' is defined as an adverb as 'no more than' by https://www.dictionary.com/.)
(Note 3: 'only' is listed as a synonym of 'at most' by https://www.thesaurus.com/.)
(Note 4: The following formula is not concise, but it's algorithmic and simple.)

$$\exists x_0 \exists x_1 \exists x_2 \exists x_3 \exists x_4[[((((((((((A(f) \land A(x_0)) \land A(x_1)) \land \neg A(x_2)) \land \neg A(x_3)) \land \neg A(x_4)) \lor (((((B(f) \land B(x_0)) \land B(x_1)) \land \neg B(x_2)) \land \neg B(x_3)) \land \neg B(x_4))) \lor (((((A(f) \land A(x_0)) \land \neg A(x_1)) \land \neg A(x_2)) \land \neg A(x_3)) \land \neg A(x_4))) \lor (((((B(f) \land B(x_0)) \land \neg B(x_1)) \land \neg B(x_2)) \land \neg B(x_3)) \land \neg B(x_4))) \lor (((((A(f) \land \neg A(x_0)) \land \neg A(x_1)) \land \neg A(x_2)) \land \neg A(x_3)) \land \neg A(x_4))) \lor (((((B(f) \land \neg B(x_0)) \land \neg B(x_1)) \land \neg B(x_2)) \land \neg B(x_3)) \land \neg B(x_4))] \land [(((((((((((((\neg f = x_0 \land \neg f = x_1) \land \neg f = x_2) \land \neg f = x_3) \land \neg f = x_4) \land \neg x_0 = x_1) \land \neg x_0 = x_2) \land \neg x_0 = x_3) \land \neg x_0 = x_4) \land \neg x_1 = x_2) \land \neg x_1 = x_3) \land \neg x_1 = x_4) \land \neg x_2 = x_3) \land \neg x_2 = x_4) \land \neg x_3 = x_4]]$$

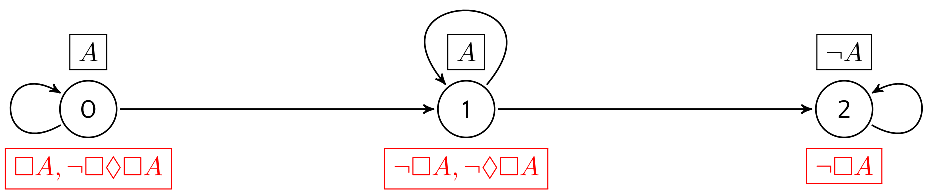
3 For each of the following modal arguments provide a model that is a counterexample. Diagrams that indicate which formulas are true and which are false at which worlds is fine. No need for further explanation, if you use a diagram. If you do not use a diagram, please provide some explanation of which formulas are true at which worlds.

The counterexamples for all of the following is world 0. Red text is explanations.

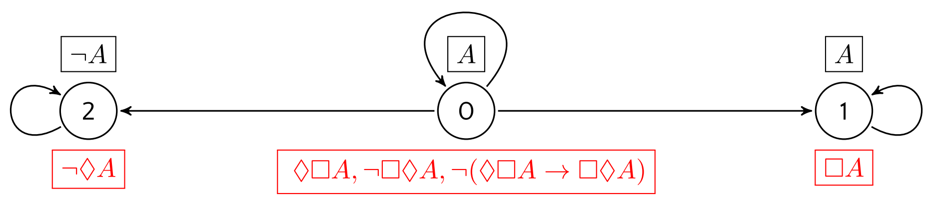
a $\Box A \wedge \Diamond B \vDash_T \Box(A \wedge B)$



b $\Box A \vDash_T \Box\Diamond\Box A$



c $\vDash_{S4} \Diamond\Box A \to \Box\Diamond A$



d $\Box A \to \Diamond A \vDash_{K4} \Diamond A \to \Box A$