CEGEP Linear Algebra Problems
(Supplementary Problems for 201-105-DW)

An open source collection of
CEGEP level linear algebra problems

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November 2, 2016

http://obeymath.org/CEGEPLinearAlgebraProblems

Contents

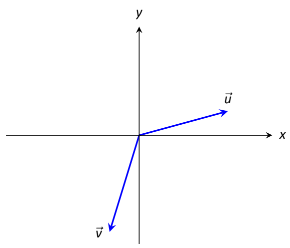
Chapter 1

Vector Geometry

1.1 Introduction to Vectors

1.1.1 [**GHC**] Sketch $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ on the same axes.

a.



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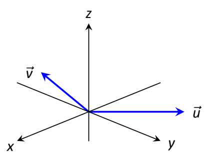

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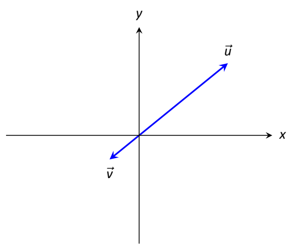

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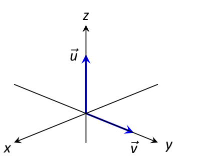


1.1.2 [**MC**] Let $\vec{v} = (-1, 5, -2)$ and $\vec{w} = (3, 1, 1)$.

- **a.** $\vec{v} - \vec{w}$
- **b.** $\vec{v} + \vec{w}$
- **c.** $\frac{\vec{v}}{\|\vec{v}\|}$
- **d.** $\|\frac{1}{2}(\vec{v} - \vec{w})\|$
- **e.** $\|\frac{1}{2}(\vec{v} + \vec{w})\|$
- **f.** $-2\,\vec{v} + 4\,\vec{w}$
- **g.** $\vec{v} - 2\,\vec{w}$.
- **h.** Find the vector $\vec{u}$ such that $\vec{u} + \vec{v} + \vec{w} = \vec{i}$.
- **i.** Find the vector $\vec{u}$ such that $\vec{u} + \vec{v} + \vec{w} = 2\,\vec{j} + \vec{k}$.
- **j.** Is there a scalar m such that $m(\vec{v} + 2\,\vec{w}) = \vec{k}$? If so, find it.

1.1.3 [**GHC**] Under what conditions is $\|\vec{u}\| + \|\vec{v}\| = \|\vec{u}+\vec{v}\|$?

1.1.4 [**JH**] Decide if the two vectors are equal.

- **a.** the vector from $(5, 3)$ to $(6, 2)$ and the vector from $(1, -2)$ to $(1, 1)$
- **b.** the vector from $(2, 1, 1)$ to $(3, 0, 4)$ and the vector from $(5, 1, 4)$ to $(6, 0, 7)$

1.1.5 [**SM**] Find the coordinates of the point which is one third of the way from the point $(1, 2)$ to the point $(3, -2)$.

1.1.6 [**SM**] Given the points P and Q, and the real number r, determine the point R such that $\overrightarrow{PR} = r\overrightarrow{PQ}$. Make a rough sketch to illustrate the idea.

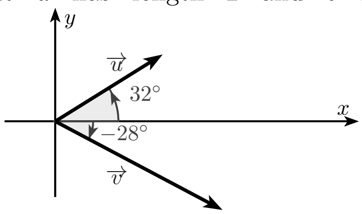
- **a.** $P(1, 4, -5)$ and $Q(-3, 1, 4)$; $r = \frac{1}{4}$
- **b.** $P(2, 1, 1, 6)$ and $Q(8, 7, 6, 0)$; $r = -\frac{1}{3}$
- **c.** $P(2, 1, -2)$ and $Q(-3, 1, 4)$; $r = \frac{4}{3}$

1.2 Dot Product and Projections

1.2.1 [**SM**] Determine all values of k for which the vectors are orthogonal.

a. $(3,-1)$, $(2,k)$
b. $(3,-1)$, (k,k^2)
c. $(1,2,3)$, $(3,-k,k)$
d. $(1,2,3)$, $(k,k,-k)$

1.2.2 [**MH**] Suppose that $\vec{u}$ and $\vec{v}$ are two vectors in the xy-plane with directions as given in the diagram and such that $\vec{u}$ has length 2 and $\vec{v}$ has length 3.



a. Find $\vec{u}\cdot\vec{v}$.
b. Find $\|\vec{u}+\vec{v}\|$.

1.2.3 [**YL**] Given $\vec{u},\vec{v}\in\mathbb{R}^2$ where $\vec{u}=(1,1)$, $||\vec{v}||=1$ and the angle between $\vec{u}$ and $\vec{v}$ is $\pi/4$.

a. Determine $\vec{u}\cdot\vec{v}$.
b. Determine $\vec{v}$.

1.2.4 [**SM**] Consider the following statement: "If $\vec{a}\bullet\vec{b}=\vec{a}\bullet\vec{c}$ then $\vec{b}=\vec{c}$."

a. If the statement is true, prove it. If the statement is false, provide a counterexample.
b. If we specify $\vec{a}\neq\vec{0}$, does that change the result?

1.2.5 [**JH**] Is any vector orthogonal to itself?

1.3 Cross Product

1.3.1 [**GHC**] The magnitudes of vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ are given, along with the angle θ between them. Use this information to find the magnitude of $\vec{u}\times\vec{v}$.

a. $\|\vec{u}\|=2,\quad \|\vec{v}\|=5,\quad \theta=30^\circ$
b. $\|\vec{u}\|=3,\quad \|\vec{v}\|=7,\quad \theta=\pi/2$
c. $\|\vec{u}\|=3,\quad \|\vec{v}\|=4,\quad \theta=\pi$
d. $\|\vec{u}\|=2,\quad \|\vec{v}\|=5,\quad \theta=5\pi/6$

1.3.2 [**GHC**] Show, using the definition of the cross product, that $\vec{u}\times\vec{u}=\vec{0}$.

1.3.3 [**MH**] Consider the three vectors $\vec{u}=(4,-1,-5)$, $\vec{v}=(1,-4,1)$ and $\vec{w}=(1,1,-2)$.

a. Compute the scalar triple product of $\vec{u}$, $\vec{v}$ and $\vec{w}$.
b. What can you deduce about the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, supposing they have the same initial point?

1.3.4 [**MH**] Let $A=\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$ such that $\det A=10$. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be the columns of A. Find $\vec{u}\cdot(\vec{v}\times\vec{w})$.

1.3.5 [**YL**] Prove or disprove: If $\vec{u},\vec{n}_1,\vec{n}_2\in\mathbb{R}^3$ such that $\vec{n}_1$ is not a scalar multiple of $\vec{n}_2$ then

$$\mathrm{proj}_{\vec{n}_1}(\mathrm{proj}_{\vec{n}_1\times\vec{n}_2}(\vec{u}))=\vec{0}$$

1.4 Lines

1.4.1 [**SM**] Write a vector equation for the line that passes through the given points.

a. $(-1,2)$ and $(2,-3)$

b. $(4,1)$ and $(-2,-1)$

c. $(1,3,-5)$ and $(-2,-1,0)$

d. $\left(\frac{1}{2},\frac{1}{4},1\right)$ and $\left(-1,1,\frac{1}{3}\right)$

e. $(1,0,-2,-5)$ and $(-3,2,-1,2)$

1.4.2 [**SM**] Find the equations of the lines through $P(3,-4,7)$ which are parallel to the coordinate axes.

1.4.3 [**SM**] Find the parametric equations of the line through the given point and parallel to the line with given equations

a. $A(0,0,2)$, $\vec{x}=(1,2,-1)+t(2,3,-3)$, $t\in\mathbb{R}$

b. $B(1,0,0)$, $\begin{cases} x = -1+2t \\ y = -1+3t \\ z = 1-2t \end{cases}$, $t\in\mathbb{R}$

1.4.4 [**JH**] Does $(1,0,2,1)$ lie on the line through $(-2,1,1,0)$ and $(5,10,-1,4)$?

1.4.5 [**SM**] Do the lines $\vec{x}=(1,0,1)+t(1,1,-1)$, $t\in\mathbb{R}$ and $\vec{x}=(2,3,4)+s(0,-1,2)$, $s\in\mathbb{R}$ have a point of intersection?

1.4.6 [**SM**] Determine the point of intersections (if any) for each pair of lines.

a. $\vec{x}=(1,2)+t(3,5)$, $t\in\mathbb{R}$ and $\vec{x}=(3,-1)+s(4,1)$, $s\in\mathbb{R}$

b. $\vec{x}=(2,3,4)+t(1,1,1)$, $t\in\mathbb{R}$ and $\vec{x}=(3,2,1)+s(3,1,-1)$, $s\in\mathbb{R}$

c. $\vec{x}=(3,4,5)+t(1,1,1)$, $t\in\mathbb{R}$ and $\vec{x}=(2,4,1)+s(2,3,-2)$, $s\in\mathbb{R}$

d. $\vec{x}=(1,0,1)+t(3,-1,2)$, $t\in\mathbb{R}$ and $\vec{x}=(5,0,7)+s(-2,2,2)$, $s\in\mathbb{R}$

1.4.7 [**SM**] For the given point and line, find by projection the point on the line that is closest to the given point, and use perp to find the distance from the point to the line.

a. point $(0,0)$, line $\vec{x}=(1,4)+t(-2,2)$, $t\in\mathbb{R}$

b. point $(2,5)$, line $\vec{x}=(3,7)+t(1,-4)$, $t\in\mathbb{R}$

c. point $(1,0,1)$, line $\vec{x}=(2,2,-1)+t(1,-2,1)$, $t\in\mathbb{R}$

d. point $(2,3,2)$, line $\vec{x}=(1,1,-1)+t(1,4,1)$, $t\in\mathbb{R}$

1.4.8 [**YL**] Given $\mathcal{E}:\ x-2y=3$ and $P=(-1,\ 1)$.

a. Find the closest point on $\mathcal{E}$ from P.

b. Find the shortest distance from P to $\mathcal{E}$.

1.5 Planes

1.5.1 [**GHC**] Give the equation of the described plane in standard and general forms.

a. Passes through $(2,3,4)$ and has normal vector $\vec{n}=(3,-1,7)$.

b. Passes through $(1,3,5)$ and has normal vector $\vec{n}=(0,2,4)$.

c. Passes through the points $(1,2,3)$, $(3,-1,4)$ and $(1,0,1)$.

d. Passes through the points $(5,3,8)$, $(6,4,9)$ and $(3,3,3)$.

e. Contains the intersecting lines $\vec{x}=(2,1,2)+t(1,2,3)$ and $\vec{x}=(2,1,2)+t(2,5,4)$.

f. Contains the intersecting lines $\vec{x}=(5,0,3)+t(-1,1,1)$ and $\vec{x}=(1,4,7)+t(3,0,-3)$.

g. Contains the parallel lines $\vec{x}=(1,1,1)+t(1,2,3)$ and $\vec{x}=(1,1,2)+t(1,2,3)$.

h. Contains the parallel lines $\vec{x}=(1,1,1)+t(4,1,3)$ and $\vec{x}=(2,2,2)+t(4,1,3)$.

i. Contains the point $(2,-6,1)$ and the line $\vec{x}=\begin{cases} x=2+5t \\ y=2+2t \\ z=-1+2t \end{cases}$

j. Contains the point $(5,7,3)$ and the line $\vec{x}=\begin{cases} x=t \\ y=t \\ z=t \end{cases}$

k. Contains the point $(5,7,3)$ and is orthogonal to the line $\vec{x}=(4,5,6)+t(1,1,1)$.

l. Contains the point $(4,1,1)$ and is orthogonal to the line $\vec{x}=\begin{cases} x=4+4t \\ y=1+1t \\ z=1+1t \end{cases}$

m. Contains the point $(-4,7,2)$ and is parallel to the plane $3(x-2)+8(y+1)-10z=0$.

n. Contains the point $(1,2,3)$ and is parallel to the plane $x=5$.

1.5.2 [**SM**] Determine the scalar equation of the plane with the given vector equation.

a. $\vec{x}=(1,4,7)+s(2,3,-1)+t(4,1,0)$, $s,t\in\mathbb{R}$

b. $\vec{x}=(2,3,-1)+s(1,1,0)+t(-2,1,2)$, $s,t\in\mathbb{R}$

c. $\vec{x}=(1,-1,3)+s(2,-2,1)+t(0,3,1)$, $s,t\in\mathbb{R}$

1.5.3 [**SM**] Determine the point of intersection of the given line and plane.

a. $\vec{x}=(2,3,1)+t(1,-2,-4)$, $t\in\mathbb{R}$, and $3x_1-2x_2+5x_3=11$

b. $\vec{x}=(1,1,2)+t(1,-1,-2)$, $t\in\mathbb{R}$, and $2x_1+x_2-x_3=5$

1.5.4 [**GHC**] Find the point of intersection between the line and the plane.

a. line: $(1,2,3)+t(3,5,-1)$, plane: $3x-2y-z=4$

b. line: $(1,2,3)+t(3,5,-1)$, plane: $3x-2y-z=-4$

c. line: $(5,1,-1)+t(2,2,1)$, plane: $5x-y-z=-3$

d. line: $(4,1,0)+t(1,0,-1)$, plane: $3x+y-2z=8$

1.5.5 [**SM**] Given the plane $2x_1-x_2+3x_3=5$, for each of the following lines, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If the answer is "neither", determine the angle between the direction vector of the line and the normal vector of the plane.

a. $\vec{x}=(3,0,4)+t(-1,1,1)$, $t\in\mathbb{R}$

b. $\vec{x}=(1,1,2)+t(-2,1,-3)$, $t\in\mathbb{R}$

c. $\vec{x}=(3,0,0)+t(1,1,2)$, $t\in\mathbb{R}$

d. $\vec{x}=(-1,-1,2)+t(4,-2,6)$, $t\in\mathbb{R}$

e. $\vec{x}=t(0,3,1)$, $t\in\mathbb{R}$

1.5.6 [**GHC**] Find the distances.

a. The distance from the point $(1,2,3)$ to the plane $3(x-1)+(y-2)+5(z-2)=0$.

b. The distance from the point $(2,6,2)$ to the plane $2(x-1)-y+4(z+1)=0$.

c. The distance between the parallel planes $x+y+z=0$ and $(x-2)+(y-3)+(z+4)=0$

d. The distance between the parallel planes $2(x-1)+2(y+1)+(z-2)=0$ and $2(x-3)+2(y-1)+(z-3)=0$

1.5.7 [**SM**] Use a projection (onto or perpendicular to) to find the distance from the point to the plane.

a. point $(2,3,1)$, plane $3x_1-x_2+4x_3=5$

b. point $(-2,3,-1)$, plane $2x_1-3x_2-5x_3=5$

c. point $(0,2,-1)$, plane $2x_1-x_3=5$

d. point $(-1,-1,1)$, plane $2x_1-x_2-x_3=4$

1.5.8 [**GHC**] Give the equation of the line that is the intersection of the given planes.

a. $p1:\ 3(x-2)+(y-1)+4z=0$, and
$p2:\ 2(x-1)-2(y+3)+6(z-1)=0$.

b. $p1:\ 5(x-5)+2(y+2)+4(z-1)=0$, and
$p2:\ 3x-4(y-1)+2(z-1)=0$.

1.5.9 [**SM**] Determine a vector equation of the line of intersection of the given planes.

a. $x+3y-z=5$ and $2x-5y+z=7$

b. $2x-3z=7$ and $y+2z=4$

1.5.10 [**YL**] Given $\mathcal{E}:\ x-2y=3$ and $P=(-1,\ 1,\ -1)$.

a. Find the equation of a line orthogonal to $\mathcal{E}$ that passes through P.

b. Find the closest point on $\mathcal{E}$ from P.

c. Find the shortest distance from P to $\mathcal{E}$.

Chapter 2

Applications

2.1 The Simplex Method

2.1.1 [**YL**]

a. Maximize $Z = 3x + y$ subject to the constraints

$$\begin{aligned} 2x - y &\leq 60 \\ x + y &\leq 50. \end{aligned}$$

b. Maximize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} 2x - y + z &\leq 100 \\ x + y + 2z &\leq 70. \end{aligned}$$

c. Maximize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} 2x - y - z &\leq 10 \\ x + y + 2z &\leq 60 \\ x - y - z &\leq 10. \end{aligned}$$

d. Maximize $Z = 4x + 3y + 3z$ subject to the constraints

$$\begin{aligned} y - z &\leq 10 \\ 2x - y + 2z &\leq 60 \\ 2x - y - z &\leq 10 \\ x + y + 2z &\leq 30. \end{aligned}$$

e. Maximize $Z = 2x - y + 4z + 2w$ subject to the constraints

$$\begin{aligned} y + z + w &\leq 30 \\ 2x - y + 2z - w &\leq 60 \\ 2x - y - z - w &\leq 10. \end{aligned}$$

2.1.2 [**YL**]

a. Minimize $Z = x + y$ subject to the constraints

$$\begin{aligned} x + y &\geq 2 \\ 3x + y &\geq 4. \end{aligned}$$

b. Minimize $Z = 2x + y + z$ subject to the constraints

$$\begin{aligned} x + z &\geq 2 \\ 2x + y + z &\geq 3. \end{aligned}$$

c. Minimize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} x + 4z &\geq 2 \\ 2x + y + z &\geq 3 \\ x + y + z &\geq 1. \end{aligned}$$

d. Minimize $Z = x + y + z$ subject to the constraints

$$\begin{aligned} x + y &\geq 50 \\ y + z &\geq 50 \\ x + z &\geq 10. \end{aligned}$$

e. Minimize $Z = 3x + y - 2z$ subject to the constraints

$$\begin{aligned} 2x - y &\geq 60 \\ x + y - 2z &\geq 50 \\ x - y - z &\geq 10. \end{aligned}$$

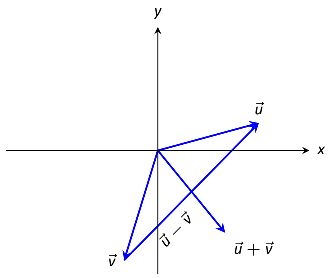
Appendix A

Answers to Exercises

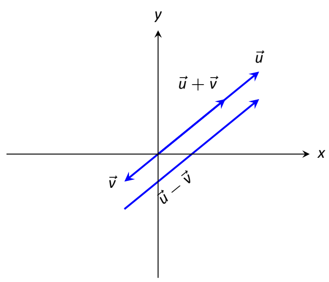
Note that either a hint, a final answer or a complete solution is provided.

1.1.1

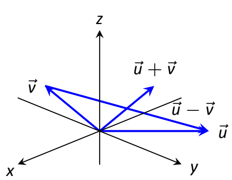
a.



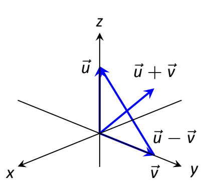
b.



c.



d.



1.1.2

a. $(-4,4,-3)$

b. $(2,6,-1)$

c. $\left(\frac{-1}{\sqrt{30}},\frac{5}{\sqrt{30}},\frac{-2}{\sqrt{30}}\right)$

d. $\frac{\sqrt{41}}{2}$

e. $\frac{\sqrt{41}}{2}$

f. $(14,-6,8)$

g. $(-7,3,-4)$
(h) $(-1,-6,1)$

h. $(-2,-4,2)$

i. No.

1.1.3 When $\vec{u}$ and $\vec{v}$ have the same direction. (Note: parallel is not enough.)

1.1.4

a. No, they are different.

$$\begin{pmatrix}1\\-1\end{pmatrix}\qquad\begin{pmatrix}0\\3\end{pmatrix}$$

b. Yes, they are the same.

$$\begin{pmatrix}1\\-1\\3\end{pmatrix}$$

1.1.5 $\left(\frac{5}{3},\frac{2}{3}\right)$

1.1.6

a. $\left(0,\frac{13}{4},-\frac{11}{4}\right)$

b. $\left(0,-1,-\frac{2}{3},8\right)$

c. $\left(-\frac{14}{3},1,6\right)$

1.2.1

a. $k=6$

b. $k=0$ or $k=3$

c. $k=-3$

d. any $k\in\mathbb{R}$

1.2.2

a. 3

b. $\sqrt{19}$

1.2.3

a. 1.

b. $(0,1)$ and $(1,0)$.

1.2.4

a. The statement is false: Let $\vec{a}$ be any non-zero vector, let $\vec{b}$ be any non-zero vector that is orthogonal to $\vec{a}$, and let $\vec{c} = -\vec{b}$. Then the antecedent of the statement is true, since both sides are equal to 0, while the consequent is false.

b. No.

1.2.5 Clearly $u_1u_1 + \cdots + u_nu_n$ is zero if and only if each u_i is zero. So only $\vec{0} \in \mathbb{R}^n$ is orthogonal to itself.

1.3.1

a. 5

b. 21

c. 0

d. 5

1.3.2 With $\vec{u} = (u_1, u_2, u_3)$, we have

$$\begin{aligned}\vec{u} \times \vec{u} &= (u_2u_3 - u_3u_2, -(u_1u_3 - u_3u_1), u_1u_2 - u_2u_1) \\ &= (0,0,0) \\ &= \vec{0}.\end{aligned}$$

1.3.3

a. 0

b. The vectors are coplanar.

1.3.4 10

1.3.5 Prove. Hint: $(\vec{n}_1 \times \vec{n}_2) \bullet \vec{n}_1$.

1.4.1 Note that alternative correct answers are possible.

a. $\vec{x} = (-1,2) + t(3,-5),\ t \in \mathbb{R}$

b. $\vec{x} = (4,1) + t(-6,-2),\ t \in \mathbb{R}$

c. $\vec{x} = (1,3,-5) + t(-3,-4,5),\ t \in \mathbb{R}$

d. $\vec{x} = \left(\frac{1}{2}, \frac{1}{4}, 1\right) + t\left(-\frac{3}{2}, \frac{3}{4}, -\frac{2}{3}\right),\ t \in \mathbb{R}$

e. $\vec{x} = (1,0,-2,-5) + t(-4,2,1,7),\ t \in \mathbb{R}$

1.4.2 To the x-axis: $\vec{x} = (3,-4,7) + t(1,0,0),\ t \in \mathbb{R}$. To the y-axis: $\vec{x} = (3,-4,7) + t(0,1,0),\ t \in \mathbb{R}$. To the z-axis: $\vec{x} = (3,-4,7) + t(0,0,1),\ t \in \mathbb{R}$.

1.4.3

a. $\begin{cases} x = 2t \\ y = 3t \\ z = 2 - 3t \end{cases},\ t \in \mathbb{R}.$

b. $\begin{cases} x = 1 + 2t \\ y = 3t \\ z = -2t \end{cases},\ t \in \mathbb{R}.$

1.4.4 That line is this set.

$$\left\{ \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 7 \\ 9 \\ -2 \\ 4 \end{pmatrix} t \;\middle|\; t \in \mathbb{R} \right\}$$

Note that this system

$$\begin{aligned} -2 + 7t &= 1 \\ 1 + 9t &= 0 \\ 1 - 2t &= 2 \\ 0 + 4t &= 1 \end{aligned}$$

has no solution. Thus the given point is not in the line.

1.4.5 No, they are skew lines.

1.4.6

a. $\left(-\frac{25}{17}, -\frac{36}{17}\right)$

b. $(0,1,2)$

c. no point of intersection

d. $(7,-2,5)$

1.4.7

a. $\left(\frac{5}{2}, \frac{5}{2}\right),\ \frac{5}{\sqrt{2}}$

b. $\left(\frac{58}{17}, \frac{91}{17}\right),\ \frac{6}{\sqrt{17}}$

c. $\left(\frac{17}{6}, \frac{1}{3}, -\frac{1}{6}\right),\ \sqrt{\frac{29}{6}}$

d. $\left(\frac{5}{3}, \frac{11}{3}, -\frac{1}{3}\right),\ \sqrt{6}$

1.4.8

a. $\left(\frac{1}{5}, -\frac{7}{5}\right)$

b. $\frac{6\sqrt{5}}{5}$

1.5.1

a. Standard form: $3(x-2) - (y-3) + 7(z-4) = 0$
general form: $3x - y + 7z = 31$

b. Standard form: $2(y-3) + 4(z-5) = 0$
general form: $2y + 4z = 26$

c. Answers may vary;
Standard form: $8(x-1) + 4(y-2) - 4(z-3) = 0$
general form: $8x + 4y - 4z = 4$

d. Answers may vary;
Standard form: $-5(x-5) + 3(y-3) + 2(z-8) = 0$
general form: $-5x + 3y + 2z = 0$

e. Answers may vary;
Standard form: $-7(x-2) + 2(y-1) + (z-2) = 0$
general form: $-7x + 2y + z = -10$

f. Answers may vary;
Standard form: $3(x-5) + 3(z-3) = 0$
general form: $3x + 3z = 24$

g. Answers may vary;
Standard form: $2(x-1) - (y-1) = 0$
general form: $2x - y = 1$

h. Answers may vary;
Standard form: $2(x-1)+(y-1)-3(z-1)=0$
general form: $2x+y-3z=0$

i. Answers may vary;
Standard form: $2(x-2)-(y+6)-4(z-1)=0$
general form: $2x-y-4z=6$

j. Answers may vary;
Standard form: $4(x-5)-2(y-7)-2(z-3)=0$
general form: $4x-2y-2z=0$

k. Answers may vary;
Standard form: $(x-5)+(y-7)+(z-3)=0$
general form: $x+y+z=15$

l. Answers may vary;
Standard form: $4(x-4)+(y-1)+(z-1)=0$
general form: $4x+y+z=18$

m. Answers may vary;
Standard form: $3(x+4)+8(y-7)-10(z-2)=0$
general form: $3x+8y-10z=24$

n. Standard form: $x-1=0$
general form: $x=1$

1.5.2

a. $x-4y-10z=-85$

b. $2x-2y+3z=-5$

c. $-5x-2y+6z=15$

1.5.3

a. $\left(\frac{20}{13},\frac{51}{13},\frac{37}{13}\right)$

b. $\left(\frac{7}{3},-\frac{1}{3},-\frac{2}{3}\right)$

1.5.4

a. No point of intersection; the plane and line are parallel.

b. The plane contains the line, so every point on the line is a "point of intersection."

c. $(-3,-7,-5)$

d. $(3,1,1)$

1.5.5

a. The line is parallel to the plane.

b. The line is orthogonal to the plane.

c. The line is neither parallel nor orthogonal to the plane, $\theta\approx 0.702$ radians.

d. The line is orthogonal to the plane.

e. The line is parallel to the plane.

1.5.6

a. $\sqrt{5/7}$

b. $8/\sqrt{21}$

c. $1/\sqrt{3}$

d. 3

1.5.7

a. $\frac{2}{\sqrt{26}}$

b. $\frac{13}{\sqrt{38}}$

c. $\frac{4}{\sqrt{5}}$

d. $\sqrt{6}$

1.5.8

a. Answers may vary:
$$\vec{x}=\begin{cases} x=14t \\ y=-1-10t \\ z=2-8t \end{cases}$$

b. Answers may vary:
$$\vec{x}=\begin{cases} x=1+20t \\ y=3+2t \\ z=3.5-26t \end{cases}$$

1.5.9

a. $\vec{x}=\left(\frac{46}{11},\frac{3}{11},0\right)+t(-2,-3,-11)$, $t\in\mathbb{R}$

b. $\vec{x}=\left(\frac{7}{2},4,0\right)+t(3,-4,2)$, $t\in\mathbb{R}$

1.5.10

a. $\vec{x}=(-1,1,-1)+t(1,-2,0)\quad t\in\mathbb{R}$

b. $\left(\frac{1}{5},-\frac{7}{5},-1\right)$

c. $\frac{6\sqrt{5}}{5}$

2.1.1

a. $Z=370/3$; $x=110/3$, $y=40/3$ from the final tableau

$$\begin{bmatrix} x & y & s_1 & s_2 & Z & \\ 1 & 0 & 1/3 & 1/3 & 0 & 110/3 \\ 0 & 1 & -1/3 & 2/3 & 0 & 40/3 \\ 0 & 0 & 2/3 & 5/3 & 1 & 370/3 \end{bmatrix}$$

b. $Z=380/3$; $x=130/3$, $y=0$, $z=40/3$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & Z & \\ 1 & -1 & 0 & 2/3 & -1/3 & 0 & 130/3 \\ 0 & 1 & 1 & -1/3 & 2/3 & 0 & 40/3 \\ 0 & 0 & 0 & 1/3 & 4/3 & 1 & 380/3 \end{bmatrix}$$

c. $Z=98$; $x=16$, $y=0$, $z=22$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & Z & \\ 1 & -1/5 & 0 & 2/5 & 1/5 & 0 & 0 & 16 \\ 0 & 3/5 & 1 & -1/5 & 2/5 & 0 & 0 & 22 \\ 0 & -1/5 & 0 & -3/5 & 1/5 & 1 & 0 & 16 \\ 0 & 2/5 & 0 & 1/5 & 8/5 & 0 & 1 & 98 \end{bmatrix}$$

d. $Z=95$; $x=25/2$, $y=25/2$, $z=5/2$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & s_4 & Z & \\ 0 & 1 & 0 & 5/8 & 0 & -1/8 & 1/4 & 0 & 25/2 \\ 0 & 0 & 0 & 9/8 & 1 & -5/8 & -3/4 & 0 & 85/2 \\ 1 & 0 & 0 & 1/8 & 0 & 3/8 & 1/4 & 0 & 25/2 \\ 0 & 0 & 1 & -3/8 & 0 & -1/8 & 1/4 & 0 & 5/2 \\ 0 & 0 & 0 & 5/4 & 0 & 3/4 & 5/2 & 1 & 95 \end{bmatrix}$$

e. $Z = 400/3$; $x = 20$, $y = 0$, $z = 50/3$, $w = 40/3$ from the final tableau

$$\begin{bmatrix} x & y & z & w & s_1 & s_2 & s_3 & Z & \\ 0 & 0 & 1 & 0 & 0 & 1/3 & -1/3 & 0 & 50/3 \\ 1 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 & 20 \\ 0 & 1 & 0 & 1 & 1 & -1/3 & 1/3 & 0 & 40/3 \\ 0 & 3 & 0 & 0 & 3 & 2/3 & 1/3 & 1 & 400/3 \end{bmatrix}$$

2.1.2

a. $Z = 2$; $x = 1$, $y = 1$ from the final tableau

$$\begin{bmatrix} x & y & s_1 & s_2 & -Z & \\ 0 & 1 & -3/2 & 1/2 & 0 & 1 \\ 1 & 0 & 1/2 & -1/2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

b. $Z = 3$; $x = 1$, $y = 0$, $z = 1$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & -Z & \\ 0 & -1 & 1 & -2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

c. $Z = 23/7$; $x = 10/7$, $y = 0$, $z = 1/7$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & -1/7 & 1 & -2/7 & 1/7 & 0 & 0 & 1/7 \\ 0 & -4/7 & 0 & -1/7 & -3/7 & 1 & 0 & 4/7 \\ 1 & 4/7 & 0 & 1/7 & -4/7 & 0 & 0 & 10/7 \\ 0 & 2/7 & 0 & 4/7 & 5/7 & 0 & 1 & -23/7 \end{bmatrix}$$

d. $Z = 55$; $x = 5$, $y = 45$, $z = 5$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/2 & 0 & 45 \\ 0 & 0 & 1 & 1/2 & -1/2 & -1/2 & 0 & 5 \\ 1 & 0 & 0 & -1/2 & 1/2 & -1/2 & 0 & 5 \\ 0 & 0 & 0 & 1/2 & 1/2 & 1/2 & 1 & -55 \end{bmatrix}$$

e. $Z = 370/3$; $x = 110/3$, $y = 40/3$, $z = 0$ from the final tableau

$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & 0 & 5/3 & -2/3 & 1/3 & 1 & 0 & 40/3 \\ 0 & 1 & -4/3 & 1/3 & -2/3 & 0 & 0 & 40/3 \\ 1 & 0 & -2/3 & -1/3 & -1/3 & 0 & 0 & 110/3 \\ 0 & 0 & 4/3 & 2/3 & 5/3 & 0 & 1 & -370/3 \end{bmatrix}$$

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