CEGEP Linear Algebra Problems
(Supplementary Problems for 201-105-DW)

An open source collection of
CEGEP level linear algebra problems

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http://obeymath.org/CEGEPLinearAlgebraProblems

Contents

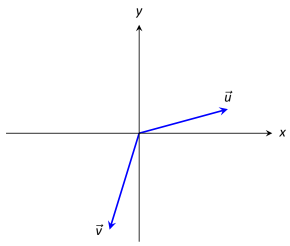
Chapter 1

Vector Geometry

1.1 Introduction to Vectors

1.1.1 [**GHC**] Sketch $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ on the same axes.

a.



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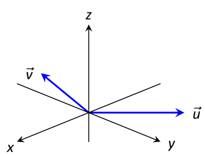

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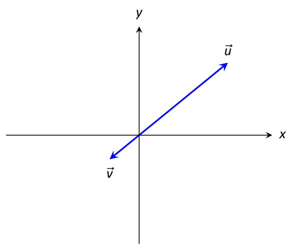

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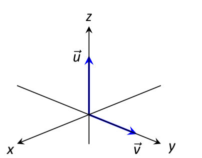


1.1.2 [**MC**] Let $\vec{v} = (-1, 5, -2)$ and $\vec{w} = (3, 1, 1)$.

- **a.** $\vec{v} - \vec{w}$
- **b.** $\vec{v} + \vec{w}$
- **c.** $\frac{\vec{v}}{\|\vec{v}\|}$
- **d.** $\|\frac{1}{2}(\vec{v} - \vec{w})\|$
- **e.** $\|\frac{1}{2}(\vec{v} + \vec{w})\|$
- **f.** $-2\,\vec{v} + 4\,\vec{w}$
- **g.** $\vec{v} - 2\,\vec{w}$.
- **h.** Find the vector $\vec{u}$ such that $\vec{u} + \vec{v} + \vec{w} = \vec{i}$.
- **i.** Find the vector $\vec{u}$ such that $\vec{u} + \vec{v} + \vec{w} = 2\,\vec{j} + \vec{k}$.
- **j.** Is there a scalar m such that $m(\vec{v} + 2\,\vec{w}) = \vec{k}$? If so, find it.

1.1.3 [**GHC**] Under what conditions is $\|\vec{u}\| + \|\vec{v}\| = \|\vec{u}+\vec{v}\|$?

1.1.4 [**GHC**] Find the unit vector $\vec{u}$ in the direction of $\vec{v}$.

- **a.** $\vec{v} = (3, 7)$
- **b.** $\vec{v}$ in the first quadrant of $\mathbb{R}^2$ that makes a $50°$ angle with the x-axis.
- **c.** $\vec{v}$ in the second quadrant of $\mathbb{R}^2$ that makes a $30°$ angle with the y-axis.

1.1.5 [**JH**] Decide if the two vectors are equal.

- **a.** the vector from $(5, 3)$ to $(6, 2)$ and the vector from $(1, -2)$ to $(1, 1)$
- **b.** the vector from $(2, 1, 1)$ to $(3, 0, 4)$ and the vector from $(5, 1, 4)$ to $(6, 0, 7)$

1.1.6 [**GHC**] Let $\vec{u} = (1, -2)$ and $\vec{v} = (1, 1)$.

- **a.** Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u} - 3\vec{v}$.
- **b.** Find $\vec{x}$ where $\vec{u} + \vec{x} = 2\vec{v} - \vec{x}$.
- **c.** Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.

1.1.7 [**GHC**] Let $\vec{u} = (1, 1, -1)$ and $\vec{v} = (2, 1, 2)$.

- **a.** Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $\pi\vec{u} - \sqrt{2}\vec{v}$.
- **b.** Find $\vec{x}$ where $\vec{u} + \vec{x} = \vec{v} + 2\vec{x}$.
- **c.** Sketch the above vectors on the same axes, along with $\vec{u}$ and $\vec{v}$.

1.1.8 [**SM**] Let $\vec{x} = (1, 2, -2)$ and $\vec{x} = (2, -1, 3)$. Determine

- **a.** $2\vec{x} - 3\vec{y}$
- **b.** $-3(\vec{x} + 2\vec{y}) + 5\vec{x}$
- **c.** $\vec{z}$ such that $\vec{y} - 2\vec{z} = 3\vec{x}$
- **d.** $\vec{z}$ such that $\vec{z} - 3\vec{x} = 2\vec{z}$

1.1.9 [**SM**] Find the coordinates of the point which is one third of the way from the point $(1, 2)$ to the point $(3, -2)$.

1.1.10 [**SM**] Given the points P and Q, and the real number r, determine the point R such that $\overrightarrow{PR} = r\overrightarrow{PQ}$. Make a rough sketch to illustrate the idea.

- **a.** $P(1, 4, -5)$ and $Q(-3, 1, 4)$; $r = \frac{1}{4}$
- **b.** $P(2, 1, 1, 6)$ and $Q(8, 7, 6, 0)$; $r = -\frac{1}{3}$
- **c.** $P(2, 1, -2)$ and $Q(-3, 1, 4)$; $r = \frac{4}{3}$

1.2 Dot Product and Projections

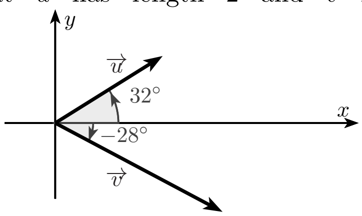
1.2.1 [**GHC**] A vector $\vec{v}$ is given. Give two vectors that are orthogonal to $\vec{v}$.

a. $\vec{v} = (4, 7)$
b. $\vec{v} = (-3, 5)$
c. $\vec{v} = (1, 1, 1)$
d. $\vec{v} = (1, -2, 3)$

1.2.2 [**SM**] Determine all values of k for which the vectors are orthogonal.

a. $(3, -1)$, $(2, k)$
b. $(3, -1)$, (k, k^2)
c. $(1, 2, 3)$, $(3, -k, k)$
d. $(1, 2, 3)$, $(k, k, -k)$

1.2.3 [**MH**] Suppose that $\vec{u}$ and $\vec{v}$ are two vectors in the xy-plane with directions as given in the diagram and such that $\vec{u}$ has length 2 and $\vec{v}$ has length 3.



a. Find $\vec{u} \cdot \vec{v}$.
b. Find $\|\vec{u} + \vec{v}\|$.

1.2.4 [**YL**] Given $\vec{u}, \vec{v} \in \mathbb{R}^2$ where $\vec{u} = (1, 1)$, $||\vec{v}|| = 1$ and the angle between $\vec{u}$ and $\vec{v}$ is $\pi/4$.

a. Determine $\vec{u} \cdot \vec{v}$.
b. Determine $\vec{v}$.

1.2.5 [**SM**] For each of the given pairs of vectors $\vec{a}$, $\vec{b}$, check that $\vec{a}$ is a unit vector, determine $\text{proj}_{\vec{a}}(\vec{b})$ and $\text{perp}_{\vec{a}}(\vec{b})$, and check your results by verifying that $\text{proj}_{\vec{a}}(\vec{b}) + \text{perp}_{\vec{a}}(\vec{b}) = \vec{b}$ and $\vec{a} \bullet \text{perp}_{\vec{a}}(\vec{b}) = 0$ in each case.

a. $\vec{a} = (0, 1)$ and $\vec{b} = (3, -5)$
b. $\vec{a} = \left(\frac{3}{5}, \frac{4}{5}\right)$ and $\vec{b} = (-4, 6)$
c. $\vec{a} = (0, 1, 0)$ and $\vec{b} = (-3, 5, 2)$
d. $\vec{a} = \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ and $\vec{b} = (4, 1, -3)$

1.2.6 [**SM**] Consider the following statement: "If $\vec{a} \bullet \vec{b} = \vec{a} \bullet \vec{c}$ then $\vec{b} = \vec{c}$."

a. If the statement is true, prove it. If the statement is false, provide a counterexample.
b. If we specify $\vec{a} \neq \vec{0}$, does that change the result?

1.2.7 [**SM**] Prove the parallelogram law for the norm:

$$\|\vec{a} + \vec{b}\|^2 + \|\vec{a} - \vec{b}\|^2 = 2\|\vec{a}\|^2 + 2\|\vec{b}\|^2$$

for all vectors in $\mathbb{R}^n$.

1.2.8 [**JH**] Is any vector orthogonal to itself?

1.3 Cross Product

1.3.1 [GHC] The magnitudes of vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$ are given, along with the angle θ between them. Use this information to find the magnitude of $\vec{u} \times \vec{v}$.

a. $\|\vec{u}\| = 2, \quad \|\vec{v}\| = 5, \quad \theta = 30°$

b. $\|\vec{u}\| = 3, \quad \|\vec{v}\| = 7, \quad \theta = \pi/2$

c. $\|\vec{u}\| = 3, \quad \|\vec{v}\| = 4, \quad \theta = \pi$

d. $\|\vec{u}\| = 2, \quad \|\vec{v}\| = 5, \quad \theta = 5\pi/6$

1.3.2 [GHC] Find a unit vector orthogonal to both $\vec{u}$ and $\vec{v}$.

a. $\vec{u} = (1,1,1), \quad \vec{v} = (2,0,1)$

b. $\vec{u} = (1,-2,1), \quad \vec{v} = (3,2,1)$

c. $\vec{u} = (5,0,2), \quad \vec{v} = (-3,0,7)$

d. $\vec{u} = (1,-2,1), \quad \vec{v} = (-2,4,-2)$

1.3.3 [GHC] Show, using the definition of the cross product, that $\vec{u} \times \vec{u} = \vec{0}$.

1.3.4 [MH] Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ such that $\det A = 10$. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be the columns of A. Find $\vec{u} \cdot (\vec{v} \times \vec{w})$.

1.3.5 [YL] Prove or disprove: If $\vec{u}, \vec{n}_1, \vec{n}_2 \in \mathbb{R}^3$ such that $\vec{n}_1$ is not a scalar multiple of $\vec{n}_2$ then

$$\operatorname{proj}_{\vec{n}_1}(\operatorname{proj}_{\vec{n}_1 \times \vec{n}_2}(\vec{u})) = \vec{0}$$

1.4 Lines

1.4.1 [GHC] Write the vector, parametric and symmetric equations of the lines described.

a. Passes through $P = (2,-4,1)$, parallel to $\vec{d} = (9,2,5)$.

b. Passes through $P = (6,1,7)$, parallel to $\vec{d} = (-3,2,5)$.

c. Passes through $P = (2,1,5)$ and $Q = (7,-2,4)$.

d. Passes through $P = (1,-2,3)$ and $Q = (5,5,5)$.

e. Passes through $P = (0,1,2)$ and orthogonal to both $\vec{d}_1 = (2,-1,7)$ and $\vec{d}_2 = (7,1,3)$.

f. Passes through $P = (5,1,9)$ and orthogonal to both $\vec{d}_1 = (1,0,1)$ and $\vec{d}_2 = (2,0,3)$.

g. Passes through the point of intersection and orthogonal of both lines, where $\vec{x} = (2,1,1) + t(5,1,-2)$ and $\vec{x} = (-2,-1,2) + t(3,1,-1)$.

h. Passes through the point of intersection and orthogonal to both lines, where

$$\vec{x} = \begin{cases} x = t \\ y = -2 + 2t \\ z = 1 + t \end{cases} \quad \text{and} \quad \vec{x} = \begin{cases} x = 2 + t \\ y = 2 - t \\ z = 3 + 2t \end{cases}.$$

i. Passes through $P = (1,1)$, parallel to $\vec{d} = (2,3)$.

j. Passes through $P = (-2,5)$, parallel to $\vec{d} = (0,1)$.

1.4.2 [SM] Find the equations of the lines through $P(3,-4,7)$ which are parallel to the coordinate axes.

1.4.3 [SM] Find the parametric equations of the line through the given point and parallel to the line with given equations

a. $A(0,0,2)$, $\vec{x} = (1,2,-1) + t(2,3,-3)$, $t \in \mathbb{R}$

b. $B(1,0,0)$, $\begin{cases} x = -1 + 2t \\ y = -1 + 3t \\ z = 1 - 2t \end{cases}$, $t \in \mathbb{R}$

1.4.4 [SM] Do the lines $\vec{x} = (1,0,1) + t(1,1,-1)$, $t \in \mathbb{R}$ and $\vec{x} = (2,3,4) + s(0,-1,2)$, $s \in \mathbb{R}$ have a point of intersection?

1.4.5 [SM] Determine the point of intersections (if any) for each pair of lines.

a. $\vec{x} = (1,2) + t(3,5)$, $t \in \mathbb{R}$ and
$\vec{x} = (3,-1) + s(4,1)$, $s \in \mathbb{R}$

b. $\vec{x} = (2,3,4) + t(1,1,1)$, $t \in \mathbb{R}$ and
$\vec{x} = (3,2,1) + s(3,1,-1)$, $s \in \mathbb{R}$

c. $\vec{x} = (3,4,5) + t(1,1,1)$, $t \in \mathbb{R}$ and
$\vec{x} = (2,4,1) + s(2,3,-2)$, $s \in \mathbb{R}$

d. $\vec{x} = (1,0,1) + t(3,-1,2)$, $t \in \mathbb{R}$ and
$\vec{x} = (5,0,7) + s(-2,2,2)$, $s \in \mathbb{R}$

1.5 Planes

1.5.1 [SM] Find an equation for the plane through the given point and parallel to the given plane.

a. point $(1, -3, -1)$, plane $2x_1 - 3x_2 + 5x_3 = 17$

b. point $(0, -2, 4)$, plane $x_2 = 0$

1.5.2 [GHC] Give any two points in the given plane.

a. $2x - 4y + 7z = 2$

b. $3(x+2)+5(y-9)-4z = 0$

c. $x = 2$

d. $4(y + 2) - (z - 6) = 0$

1.5.3 [GHC] Give the equation of the described plane in standard and general forms.

a. Passes through $(2, 3, 4)$ and has normal vector $\vec{n} = (3, -1, 7)$.

b. Passes through $(1, 3, 5)$ and has normal vector $\vec{n} = (0, 2, 4)$.

c. Passes through the points $(1, 2, 3)$, $(3, -1, 4)$ and $(1, 0, 1)$.

d. Passes through the points $(5, 3, 8)$, $(6, 4, 9)$ and $(3, 3, 3)$.

e. Contains the intersecting lines $\vec{x} = (2, 1, 2) + t(1, 2, 3)$ and $\vec{x} = (2, 1, 2) + t(2, 5, 4)$.

f. Contains the intersecting lines $\vec{x} = (5, 0, 3) + t(-1, 1, 1)$ and $\vec{x} = (1, 4, 7) + t(3, 0, -3)$.

g. Contains the parallel lines $\vec{x} = (1, 1, 1) + t(1, 2, 3)$ and $\vec{x} = (1, 1, 2) + t(1, 2, 3)$.

h. Contains the parallel lines $\vec{x} = (1, 1, 1) + t(4, 1, 3)$ and $\vec{x} = (2, 2, 2) + t(4, 1, 3)$.

i. Contains the point $(2, -6, 1)$ and the line
$$\vec{x} = \begin{cases} x = 2 + 5t \\ y = 2 + 2t \\ z = -1 + 2t \end{cases}$$

j. Contains the point $(5, 7, 3)$ and the line
$$\vec{x} = \begin{cases} x = t \\ y = t \\ z = t \end{cases}$$

k. Contains the point $(5, 7, 3)$ and is orthogonal to the line $\vec{x} = (4, 5, 6) + t(1, 1, 1)$.

l. Contains the point $(4, 1, 1)$ and is orthogonal to the line
$$\vec{x} = \begin{cases} x = 4 + 4t \\ y = 1 + 1t \\ z = 1 + 1t \end{cases}$$

m. Contains the point $(-4, 7, 2)$ and is parallel to the plane $3(x - 2) + 8(y + 1) - 10z = 0$.

n. Contains the point $(1, 2, 3)$ and is parallel to the plane $x = 5$.

1.5.4 [SM] Determine the scalar equation of the plane that contains the following points.

a. $(2, 1, 5), (4, -3, 2), (2, 6, -1)$

b. $(3, 1, 4), (-2, 0, 2), (1, 4, -1)$

c. $(-1, 4, 2), (3, 1, -1), (2, -3, -1)$

1.5.5 [SM] Determine the scalar equation of the plane with the given vector equation.

a. $\vec{x} = (1, 4, 7) + s(2, 3, -1) + t(4, 1, 0),\ s, t \in \mathbb{R}$

b. $\vec{x} = (2, 3, -1) + s(1, 1, 0) + t(-2, 1, 2),\ s, t \in \mathbb{R}$

c. $\vec{x} = (1, -1, 3) + s(2, -2, 1) + t(0, 3, 1),\ s, t \in \mathbb{R}$

1.5.6 [SM] Determine the point of intersection of the given line and plane.

a. $\vec{x} = (2, 3, 1)+t(1, -2, -4)$, $t \in \mathbb{R}$, and $3x_1-2x_2+5x_3 = 11$

b. $\vec{x} = (1, 1, 2) + t(1, -1, -2)$, $t \in \mathbb{R}$, and $2x_1 + x_2 - x_3 = 5$

1.5.7 [GHC] Find the point of intersection between the line and the plane.

a. line: $(1, 2, 3) + t(3, 5, -1)$, plane: $3x - 2y - z = 4$

b. line: $(1, 2, 3) + t(3, 5, -1)$, plane: $3x - 2y - z = -4$

c. line: $(5, 1, -1) + t(2, 2, 1)$, plane: $5x - y - z = -3$

d. line: $(4, 1, 0) + t(1, 0, -1)$, plane: $3x + y - 2z = 8$

1.5.8 [SM] Given the plane $2x_1-x_2+3x_3 = 5$, for each of the following lines, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If the answer is “neither”, determine the angle between the direction vector of the line and the normal vector of the plane.

a. $\vec{x} = (3, 0, 4) + t(-1, 1, 1)$, $t \in \mathbb{R}$

b. $\vec{x} = (1, 1, 2) + t(-2, 1, -3)$, $t \in \mathbb{R}$

c. $\vec{x} = (3, 0, 0) + t(1, 1, 2)$, $t \in \mathbb{R}$

d. $\vec{x} = (-1, -1, 2) + t(4, -2, 6)$, $t \in \mathbb{R}$

e. $\vec{x} = t(0, 3, 1)$, $t \in \mathbb{R}$

1.5.9 [GHC] Find the distances.

a. The distance from the point $(1, 2, 3)$ to the plane $3(x - 1) + (y - 2) + 5(z - 2) = 0$.

b. The distance from the point $(2, 6, 2)$ to the plane $2(x - 1) - y + 4(z + 1) = 0$.

c. The distance between the parallel planes $x + y + z = 0$ and $(x - 2) + (y - 3) + (z + 4) = 0$

d. The distance between the parallel planes $2(x - 1) + 2(y + 1) + (z - 2) = 0$ and $2(x - 3) + 2(y - 1) + (z - 3) = 0$

1.5.10 [SM] Use a projection (onto or perpendicular to) to find the distance from the point to the plane.

a. point $(2, 3, 1)$, plane $3x_1 - x_2 + 4x_3 = 5$

b. point $(-2, 3, -1)$, plane $2x_1 - 3x_2 - 5x_3 = 5$

c. point $(0, 2, -1)$, plane $2x_1 - x_3 = 5$

d. point $(-1,-1,1)$, plane $2x_1 - x_2 - x_3 = 4$

1.5.11 [**GHC**] Give the equation of the line that is the intersection of the given planes.

a. $p1: \ 3(x-2)+(y-1)+4z=0$, and
$p2: \ 2(x-1)-2(y+3)+6(z-1)=0$.

b. $p1: \ 5(x-5)+2(y+2)+4(z-1)=0$, and
$p2: \ 3x-4(y-1)+2(z-1)=0$.

1.5.12 [**SM**] Determine a vector equation of the line of intersection of the given planes.

a. $x+3y-z=5$ and $2x-5y+z=7$

b. $2x-3z=7$ and $y+2z=4$

1.5.13 [**SM**] In each case, determine whether the given pair of lines has a point of intersection; if so, determine the scalar equation of the plane containing the lines, and if not, determine the distance between the lines.

a. $\vec{x}=(1,3,1)+s(-2,-1,1)$ and $\vec{x}=(0,1,4)+t(3,0,1)$, $s,t\in\mathbb{R}$

b. $\vec{x}=(1,3,1)+s(-2,-1,1)$ and $\vec{x}=(0,1,7)+t(3,0,1)$, $s,t\in\mathbb{R}$

c. $\vec{x}=(2,1,4)+s(2,1,-2)$ and $\vec{x}=(-2,1,5)+t(1,3,1)$, $s,t\in\mathbb{R}$

d. $\vec{x}=(0,1,3)+s(1,-1,4)$ and $\vec{x}=(0,-1,5)+t(1,1,2)$, $s,t\in\mathbb{R}$

Chapter 2

Applications

2.1 The Simplex Method

2.1.1 [**YL**]

a. Maximize $Z = 3x + y$ subject to the constraints

$$\begin{aligned} 2x - y &\leq 60 \\ x + y &\leq 50. \end{aligned}$$

b. Maximize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} 2x - y + z &\leq 100 \\ x + y + 2z &\leq 70. \end{aligned}$$

c. Maximize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} 2x - y - z &\leq 10 \\ x + y + 2z &\leq 60 \\ x - y - z &\leq 10. \end{aligned}$$

d. Maximize $Z = 4x + 3y + 3z$ subject to the constraints

$$\begin{aligned} y - z &\leq 10 \\ 2x - y + 2z &\leq 60 \\ 2x - y - z &\leq 10 \\ x + y + 2z &\leq 30. \end{aligned}$$

e. Maximize $Z = 2x - y + 4z + 2w$ subject to the constraints

$$\begin{aligned} y + z + w &\leq 30 \\ 2x - y + 2z - w &\leq 60 \\ 2x - y - z - w &\leq 10. \end{aligned}$$

2.1.2 [**YL**]

a. Minimize $Z = x + y$ subject to the constraints

$$\begin{aligned} x + y &\geq 2 \\ 3x + y &\geq 4. \end{aligned}$$

b. Minimize $Z = 2x + y + z$ subject to the constraints

$$\begin{aligned} x + z &\geq 2 \\ 2x + y + z &\geq 3. \end{aligned}$$

c. Minimize $Z = 2x + y + 3z$ subject to the constraints

$$\begin{aligned} x + 4z &\geq 2 \\ 2x + y + z &\geq 3 \\ x + y + z &\geq 1. \end{aligned}$$

d. Minimize $Z = x + y + z$ subject to the constraints

$$\begin{aligned} x + y &\geq 50 \\ y + z &\geq 50 \\ x + z &\geq 10. \end{aligned}$$

e. Minimize $Z = 3x + y - 2z$ subject to the constraints

$$\begin{aligned} 2x - y &\geq 60 \\ x + y - 2z &\geq 50 \\ x - y - z &\geq 10. \end{aligned}$$

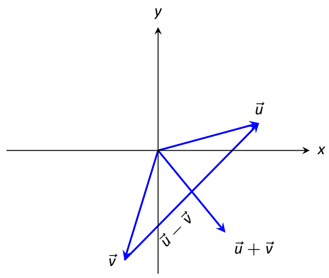
Appendix A

Answers to Exercises

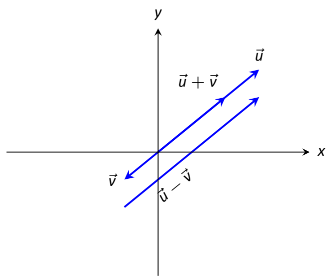
Note that either a hint, a final answer or a complete solution is provided.

1.1.1

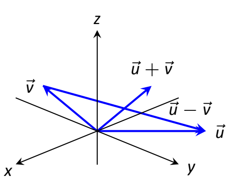
a.



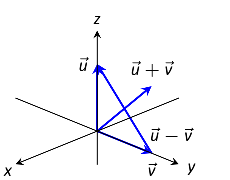
b.



c.



d.



1.1.2

a. $(-4, 4, -3)$

b. $(2, 6, -1)$

c. $\left(\frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}}, \frac{-2}{\sqrt{30}}\right)$

d. $\frac{\sqrt{41}}{2}$

e. $\frac{\sqrt{41}}{2}$

f. $(14, -6, 8)$

g. $(-7, 3, -4)$
(h) $(-1, -6, 1)$

h. $(-2, -4, 2)$

i. No.

1.1.3 When $\vec{u}$ and $\vec{v}$ have the same direction. (Note: parallel is not enough.)

1.1.4

a. $\vec{u} = (3/\sqrt{30}, 7/\sqrt{30})$

b. $\vec{u} = (\cos 50°, \sin 50°) \approx (0.643, 0.766)$

c. $\vec{u} = (\cos 120°, \sin 120°) = (-1/2, \sqrt{3}/2)$.

1.1.5

a. No, they are different.

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

b. Yes, they are the same.

$$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

1.1.6

a. $\vec{u}+\vec{v} = (2, -1)$; $\vec{u}-\vec{v} = (0, -3)$; $2\vec{u} - 3\vec{v} = (-1, -7)$.

b. $\vec{x} = (1/2, 2)$.

1.1.7

a. $\vec{u}+\vec{v} = (3, 2, 1)$; $\vec{u}-\vec{v} = (-1, 0, -3)$; $\pi\vec{u} - \sqrt{2}\vec{v} = (\pi - 2\sqrt{2}, \pi - \sqrt{2}, -\pi - 2\sqrt{2})$.

b. $\vec{x} = (-1, 0, -3)$.

1.1.8

a. $(-4, 7, -13)$

b. $(-10, 10, -22)$
c. $(-1/2, 7/2, 9/2)$
d. $(-3, -6, 6)$

1.1.9 $\left(\frac{5}{3}, \frac{2}{3}\right)$

1.1.10
a. $\left(0, \frac{13}{4}, -\frac{11}{4}\right)$
b. $\left(0, -1, -\frac{2}{3}, 8\right)$
c. $\left(-\frac{14}{3}, 1, 6\right)$

1.2.1
a. Answers will vary; two possible answers are $(-7, 4)$ and $(14, -8)$.
b. Answers will vary; two possible answers are $(5, 3)$ and $(-15, -9)$.
c. Answers will vary; two possible answers are $(1, 0, -1)$ and $(4, 5, -9)$.
d. Answers will vary; two possible answers are $(2, 1, 0)$ and $(1, 1, 1/3)$.

1.2.2
a. $k = 6$
b. $k = 0$ or $k = 3$
c. $k = -3$
d. any $k \in \mathbb{R}$

1.2.3
a. 3
b. $\sqrt{19}$

1.2.4
a. 1.
b. $(0, 1)$ and $(1, 0)$.

1.2.5
a. $\text{proj}_{\vec{a}}(\vec{b}) = (0, -5)$, $\text{perp}_{\vec{a}}(\vec{b}) = (3, 0)$
b. $\text{proj}_{\vec{a}}(\vec{b}) = \left(\frac{36}{25}, \frac{48}{25}\right)$, $\text{perp}_{\vec{a}}(\vec{b}) = \left(-\frac{136}{25}, \frac{102}{25}\right)$
c. $\text{proj}_{\vec{a}}(\vec{b}) = (0, 5, 0)$, $\text{perp}_{\vec{a}}(\vec{b}) = (-3, 0, 2)$
d. $\text{proj}_{\vec{a}}(\vec{b}) = \left(-\frac{4}{9}, \frac{8}{9}, -\frac{8}{9}\right)$, $\text{perp}_{\vec{a}}(\vec{b}) = \left(\frac{40}{9}, \frac{1}{9}, -\frac{19}{9}\right)$

1.2.6
a. The statement is false: Let $\vec{a}$ be any non-zero vector, let $\vec{b}$ be any non-zero vector that is orthogonal to $\vec{a}$, and let $\vec{c} = -\vec{b}$. Then the antecedent of the statement is true, since both sides are equal to 0, while the consequent is false.
b. No.

1.2.7 Expand the left side of the equation by using the fact that $\|\vec{v}\|^2 = \vec{v} \bullet \vec{v}$ for any vector $\vec{v}$ to get to the right side.

1.2.8 Clearly $u_1u_1 + \cdots + u_nu_n$ is zero if and only if each u_i is zero. So only $\vec{0} \in \mathbb{R}^n$ is orthogonal to itself.

1.3.1
a. 5
b. 21
c. 0
d. 5

1.3.2
a. $\pm\frac{1}{\sqrt{6}}(1, 1, -2)$
b. $\pm\frac{1}{\sqrt{21}}(-2, 1, 4)$
c. $(0, \pm 1, 0)$
d. Any vector orthogonal to $\vec{u}$ works (such as $\frac{1}{\sqrt{2}}(1, 0, -1)$).

1.3.3 With $\vec{u} = (u_1, u_2, u_3)$, we have

$$\begin{aligned} \vec{u} \times \vec{u} &= (u_2u_3 - u_3u_2, -(u_1u_3 - u_3u_1), u_1u_2 - u_2u_1) \\ &= (0, 0, 0) \\ &= \vec{0}. \end{aligned}$$

1.3.4 10

1.3.5 Prove. Hint: $(\vec{n}_1 \times \vec{n}_2) \bullet \vec{n}_1$.

1.4.1
a. vector: $\vec{x} = (2, -4, 1) + t(9, 2, 5)$
parametric: $x = 2 + 9t$, $y = -4 + 2t$, $z = 1 + 5t$
symmetric: $(x - 2)/9 = (y + 4)/2 = (z - 1)/5$
b. vector: $\vec{x} = (6, 1, 7) + t(-3, 2, 5)$
parametric: $x = 6 - 3t$, $y = 1 + 2t$, $z = 7 + 5t$
symmetric: $-(x - 6)/3 = (y - 1)/2 = (z - 7)/5$
c. Answers can vary: vector: $\vec{x} = (2, 1, 5) + t(5, -3, -1)$
parametric: $x = 2 + 5t$, $y = 1 - 3t$, $z = 5 - t$
symmetric: $(x - 2)/5 = -(y - 1)/3 = -(z - 5)$
d. Answers can vary: vector: $\vec{x} = (1, -2, 3) + t(4, 7, 2)$
parametric: $x = 1 + 4t$, $y = -2 + 7t$, $z = 3 + 2t$
symmetric: $(x - 1)/4 = (y + 2)/7 = (z - 3)/2$
e. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\vec{x} = (0, 1, 2) + t(-10, 43, 9)$
parametric: $x = -10t$, $y = 1 + 43t$, $z = 2 + 9t$
symmetric: $-x/10 = (y - 1)/43 = (z - 2)/9$
f. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\vec{x} = (5, 1, 9) + t(0, -1, 0)$
parametric: $x = 5$, $y = 1 - t$, $z = 9$
symmetric: not defined, as some components of the direction are 0.
g. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\vec{x} = (7, 2, -1) + t(1, -1, 2)$
parametric: $x = 7 + t$, $y = 2 - t$, $z = -1 + 2t$
symmetric: $x - 7 = 2 - y = (z + 1)/2$
h. Answers can vary; here the direction is given by $\vec{d}_1 \times \vec{d}_2$:
vector: $\vec{x} = (2, 2, 3) + t(5, -1, -3)$

parametric: $x = 2 + 5t$, $y = 2 - t$, $z = 3 - 3t$
symmetric: $(x - 2)/5 = -(y - 2) = -(z - 3)/3$

i. vector: $\vec{x} = (1, 1) + t(2, 3)$
parametric: $x = 1 + 2t$, $y = 1 + 3t$
symmetric: $(x - 1)/2 = (y - 1)/3$

j. vector: $\vec{x} = (-2, 5) + t(0, 1)$
parametric: $x = -2$, $y = 5 + t$
symmetric: not defined

1.4.2 To the x-axis: $\vec{x} = (3, -4, 7) + t(1, 0, 0)$, $t \in \mathbb{R}$. To the y-axis: $\vec{x} = (3, -4, 7) + t(0, 1, 0)$, $t \in \mathbb{R}$. To the z-axis: $\vec{x} = (3, -4, 7) + t(0, 0, 1)$, $t \in \mathbb{R}$.

1.4.3

a. $\begin{cases} x = 2t \\ y = 3t \\ z = 2 - 3t \end{cases}, t \in \mathbb{R}.$

b. $\begin{cases} x = 1 + 2t \\ y = 3t \\ z = -2t \end{cases}, t \in \mathbb{R}.$

1.4.4 No, they are skew lines.

1.4.5

a. $\left(-\frac{25}{17}, -\frac{36}{17}\right)$

b. $(0, 1, 2)$

c. no point of intersection

d. $(7, -2, 5)$

1.5.1

a. $2x_1 - 3x_2 + 5x_3 = 6$

b. $x_2 = -2$

1.5.2 Answers will vary.

1.5.3

a. Standard form: $3(x - 2) - (y - 3) + 7(z - 4) = 0$
general form: $3x - y + 7z = 31$

b. Standard form: $2(y - 3) + 4(z - 5) = 0$
general form: $2y + 4z = 26$

c. Answers may vary;
Standard form: $8(x - 1) + 4(y - 2) - 4(z - 3) = 0$
general form: $8x + 4y - 4z = 4$

d. Answers may vary;
Standard form: $-5(x - 5) + 3(y - 3) + 2(z - 8) = 0$
general form: $-5x + 3y + 2z = 0$

e. Answers may vary;
Standard form: $-7(x - 2) + 2(y - 1) + (z - 2) = 0$
general form: $-7x + 2y + z = -10$

f. Answers may vary;
Standard form: $3(x - 5) + 3(z - 3) = 0$
general form: $3x + 3z = 24$

g. Answers may vary;
Standard form: $2(x - 1) - (y - 1) = 0$
general form: $2x - y = 1$

h. Answers may vary;
Standard form: $2(x - 1) + (y - 1) - 3(z - 1) = 0$
general form: $2x + y - 3z = 0$

i. Answers may vary;
Standard form: $2(x - 2) - (y + 6) - 4(z - 1) = 0$
general form: $2x - y - 4z = 6$

j. Answers may vary;
Standard form: $4(x - 5) - 2(y - 7) - 2(z - 3) = 0$
general form: $4x - 2y - 2z = 0$

k. Answers may vary;
Standard form: $(x - 5) + (y - 7) + (z - 3) = 0$
general form: $x + y + z = 15$

l. Answers may vary;
Standard form: $4(x - 4) + (y - 1) + (z - 1) = 0$
general form: $4x + y + z = 18$

m. Answers may vary;
Standard form: $3(x + 4) + 8(y - 7) - 10(z - 2) = 0$
general form: $3x + 8y - 10z = 24$

n. Standard form: $x - 1 = 0$
general form: $x = 1$

1.5.4

a. $39x + 12y + 10z = 140$

b. $11x - 21y - 17z = -56$

c. $-12x + 3y - 19z = -14$

1.5.5

a. $x - 4y - 10z = -85$

b. $2x - 2y + 3z = -5$

c. $-5x - 2y + 6z = 15$

1.5.6

a. $\left(\frac{20}{13}, \frac{51}{13}, \frac{37}{13}\right)$

b. $\left(\frac{7}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

1.5.7

a. No point of intersection; the plane and line are parallel.

b. The plane contains the line, so every point on the line is a "point of intersection."

c. $(-3, -7, -5)$

d. $(3, 1, 1)$

1.5.8

a. The line is parallel to the plane.

b. The line is orthogonal to the plane.

c. The line is neither parallel nor orthogonal to the plane, $\theta \approx 0.702$ radians.

d. The line is orthogonal to the plane.

e. The line is parallel to the plane.

1.5.9

a. $\sqrt{5/7}$

b. $8/\sqrt{21}$

c. $1/\sqrt{3}$

d. 3

1.5.10

a. $\frac{2}{\sqrt{26}}$

b. $\frac{13}{\sqrt{38}}$

c. $\frac{4}{\sqrt{5}}$

d. $\sqrt{6}$

1.5.11

a. Answers may vary:
$$\vec{x} = \begin{cases} x = 14t \\ y = -1 - 10t \\ z = 2 - 8t \end{cases}$$

b. Answers may vary:
$$\vec{x} = \begin{cases} x = 1 + 20t \\ y = 3 + 2t \\ z = 3.5 - 26t \end{cases}$$

1.5.12

a. $\vec{x} = \left(\frac{46}{11}, \frac{3}{11}, 0\right) + t(-2, -3, -11)$, $t \in \mathbb{R}$

b. $\vec{x} = \left(\frac{7}{2}, 4, 0\right) + t(3, -4, 2)$, $t \in \mathbb{R}$

1.5.13

a. Point of intersection: $(-3, 1, 3)$, $-x + 5y + 3z = 17$

b. No point of intersection, $\frac{9}{\sqrt{35}}$

c. No point of intersection, $\frac{23}{3\sqrt{10}}$

d. Point of intersection $(1, 0, 7)$, $3x - y - z = -4$

2.1.1

a. $Z = 370/3$; $x = 110/3$, $y = 40/3$ from the final tableau
$$\begin{bmatrix} x & y & s_1 & s_2 & Z & \\ 1 & 0 & 1/3 & 1/3 & 0 & 110/3 \\ 0 & 1 & -1/3 & 2/3 & 0 & 40/3 \\ 0 & 0 & 2/3 & 5/3 & 1 & 370/3 \end{bmatrix}$$

b. $Z = 380/3$; $x = 130/3$, $y = 0$, $z = 40/3$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & Z & \\ 1 & -1 & 0 & 2/3 & -1/3 & 0 & 130/3 \\ 0 & 1 & 1 & -1/3 & 2/3 & 0 & 40/3 \\ 0 & 0 & 0 & 1/3 & 4/3 & 1 & 380/3 \end{bmatrix}$$

c. $Z = 98$; $x = 16$, $y = 0$, $z = 22$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & Z & \\ 1 & -1/5 & 0 & 2/5 & 1/5 & 0 & 0 & 16 \\ 0 & 3/5 & 1 & -1/5 & 2/5 & 0 & 0 & 22 \\ 0 & -1/5 & 0 & -3/5 & 1/5 & 1 & 0 & 16 \\ 0 & 2/5 & 0 & 1/5 & 8/5 & 0 & 1 & 98 \end{bmatrix}$$

d. $Z = 95$; $x = 25/2$, $y = 25/2$, $z = 5/2$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & s_4 & Z & \\ 0 & 1 & 0 & 5/8 & 0 & -1/8 & 1/4 & 0 & 25/2 \\ 0 & 0 & 0 & 9/8 & 1 & -5/8 & -3/4 & 0 & 85/2 \\ 1 & 0 & 0 & 1/8 & 0 & 3/8 & 1/4 & 0 & 25/2 \\ 0 & 0 & 1 & -3/8 & 0 & -1/8 & 1/4 & 0 & 5/2 \\ 0 & 0 & 0 & 5/4 & 0 & 3/4 & 5/2 & 1 & 95 \end{bmatrix}$$

e. $Z = 400/3$; $x = 20$, $y = 0$, $z = 50/3$, $w = 40/3$ from the final tableau
$$\begin{bmatrix} x & y & z & w & s_1 & s_2 & s_3 & Z & \\ 0 & 0 & 1 & 0 & 0 & 1/3 & -1/3 & 0 & 50/3 \\ 1 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 & 20 \\ 0 & 1 & 0 & 1 & 1 & -1/3 & 1/3 & 0 & 40/3 \\ 0 & 3 & 0 & 0 & 3 & 2/3 & 1/3 & 1 & 400/3 \end{bmatrix}$$

2.1.2

a. $Z = 2$; $x = 1$, $y = 1$ from the final tableau
$$\begin{bmatrix} x & y & s_1 & s_2 & -Z & \\ 0 & 1 & -3/2 & 1/2 & 0 & 1 \\ 1 & 0 & 1/2 & -1/2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

b. $Z = 3$; $x = 1$, $y = 0$, $z = 1$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & -Z & \\ 0 & -1 & 1 & -2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -3 \end{bmatrix}$$

c. $Z = 23/7$; $x = 10/7$, $y = 0$, $z = 1/7$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & -1/7 & 1 & -2/7 & 1/7 & 0 & 0 & 1/7 \\ 0 & -4/7 & 0 & -1/7 & -3/7 & 1 & 0 & 4/7 \\ 1 & 4/7 & 0 & 1/7 & -4/7 & 0 & 0 & 10/7 \\ 0 & 2/7 & 0 & 4/7 & 5/7 & 0 & 1 & -23/7 \end{bmatrix}$$

d. $Z = 55$; $x = 5$, $y = 45$, $z = 5$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/2 & 0 & 45 \\ 0 & 0 & 1 & 1/2 & -1/2 & -1/2 & 0 & 5 \\ 1 & 0 & 0 & -1/2 & 1/2 & -1/2 & 0 & 5 \\ 0 & 0 & 0 & 1/2 & 1/2 & 1/2 & 1 & -55 \end{bmatrix}$$

e. $Z = 370/3$; $x = 110/3$, $y = 40/3$, $z = 0$ from the final tableau
$$\begin{bmatrix} x & y & z & s_1 & s_2 & s_3 & -Z & \\ 0 & 0 & 5/3 & -2/3 & 1/3 & 1 & 0 & 40/3 \\ 0 & 1 & -4/3 & 1/3 & -2/3 & 0 & 0 & 40/3 \\ 1 & 0 & -2/3 & -1/3 & -1/3 & 0 & 0 & 110/3 \\ 0 & 0 & 4/3 & 2/3 & 5/3 & 0 & 1 & -370/3 \end{bmatrix}$$

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