Name:

Note: There are questions on both sides of the page.

Let $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ and $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$. For each function below, determine whether it is an injection or a surjection (or neither, or both):

[3] 1. $f : \mathbb{Z}_5 \to \mathbb{Z}_5$, given by $f(x) = 3x + 2 \pmod 5$.

We compute the values of f via the following table:

x	$3x+2$	$f(x)$
0	2	2
1	5	0
2	8	3
3	11	1
4	14	4

We see that f attains every value in $\mathbb{Z}_5$, and never takes the same value twice; therefore, f is both an injection and a surjection.

[3] 2. $g : \mathbb{Z}_6 \to \mathbb{Z}_6$, given by $g(x) = 3x + 2 \pmod 6$.

We again compute the values of g according to the table below:

x	$3x+2$	$g(x)$
0	2	2
1	5	5
2	8	2
3	11	5
4	14	2
5	17	5

Since $\operatorname{range} g = \{2, 5\} \neq \mathbb{Z}_6$, g is not sujective, and since $g(0) = g(2) = 2$, g is not injective.

Total: 10 points

[3] 3. $h : \mathbb{Z}_5 \to \mathbb{Z}_5$, given by $h(x) = x^3 + 4 \pmod 5$.

(Note: I've corrected the typo – h is defined mod 5, not mod 6.) The table of values for h is given by

x	$x^3 + 4$	$h(x)$
0	4	4
1	5	0
2	12	2
3	31	1
4	68	3

We see that every element of $\mathbb{Z}_5$ appears as $h(x)$ for some x, so h is a surjection, and no value of $h(x)$ appears twice, so h is an injection.

[1] 4. $H : \mathbb{Z}_5 \to \mathbb{Z}_6$, given by $H(x) = x^3 + 4 \pmod 6$.

This problem proceeds as above, except that we compute remainders modulo 6. The table of values is

x	$x^3 + 4$	$H(x)$
0	4	4
1	5	5
2	12	0
3	31	1
4	68	2

From the table, we see that H is injective, since no value appears twice, but H is not surjective, since $3 \notin \operatorname{range} H$.