1. If $z = 3 - 2i$ and $w = -5 + 4i$, compute:
   (a) $3z$
   (b) $z - 2w$
   (c) $2w - 3z$
   (d) zw
   (e) $\overline{z}$ (The complex conjugate is defined by $\overline{x + iy} = x - iy$.)
   (f) $|w|$ (The complex modulus (norm) is defined by $|w| = \sqrt{w\overline{w}}$.)
   (g) $\dfrac{z^2}{w}$

2. Solve for z in the following equations:
   (a) $z + (2 - 3i) = -5 + 4i$
   (b) $3z - 2i = (2 - i)(3 + 4i)$
   (c) $2iz = 1 + i$
   (d) $(3 + 2i)z - 1 + 3i = 4 + i$

3. Find the eigenvalues of the following matrices:

$$A = \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2+i \\ 2-i & 7 \end{bmatrix}$$

4. Verify that $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} i \\ 1 \end{bmatrix}$ are eigenvectors for the matrix A in the previous problem, and that $\begin{bmatrix} 2+i \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2-i \end{bmatrix}$ are eigenvectors for the matrix B in the previous problem.

5. (Bonus superfun challenge problem) Let $Z = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.
   (a) Verify that Z has eigenvalues $\pm i$ and eigenvectors $\vec{v} = \begin{bmatrix} i \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ i \end{bmatrix}$.
   (b) Show that $\langle \vec{v}, \vec{w} \rangle = 0$, where $\langle \vec{v}, \vec{w} \rangle = \vec{v} \cdot \overline{\vec{w}}$ is the complex version of the dot product. (The notation $\overline{\vec{w}}$ means take the complex conjugate of each entry in $\vec{w}$.)
   (c) A matrix U is called **unitary** if $U^*U = I$, where $U^* = (\overline{U})^T$ is the *Hermitian conjugate* of U, formed by taking the transpose of the complex conjugate of U.
   Let $U = \dfrac{1}{\sqrt{2}} \begin{bmatrix} i & -1 \\ -1 & i \end{bmatrix}$. (Note that the columns of U are eigenvectors of Z.) Show that U is unitary and that $U^*ZU = \begin{bmatrix} i & 0 \\ 0 & -1 \end{bmatrix}$.
   (d) Compute Z^{423}.

Name:

Tutorial time:

Please submit **one** *completed* solution from the worksheet for feedback.

Note: Your solution needs to contain enough detail for it to be clear what problem you're trying to solve!