Math 1565 Tutorial #4 Solutions

1. Using **the definition of the derivative**, determine the equation of the line tangent to the curve $y = \dfrac{1}{\sqrt{x}}$ at the point $(4, \frac{1}{2})$.

   By definition, we have

   $$\begin{aligned}
   f'(4) &= \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} \\
   &= \lim_{h \to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{2}}{h} \\
   &= \lim_{h \to 0} \frac{2 - \sqrt{4+h}}{2h\sqrt{4+h}} \\
   &= \lim_{h \to 0} \left(\frac{2 - \sqrt{4+h}}{2h\sqrt{4+h}} \right) \left(\frac{2 + \sqrt{4+h}}{2 + \sqrt{4+h}} \right) \\
   &= \lim_{h \to 0} \frac{4 - (4+h)}{2h\sqrt{4+h}(2 + \sqrt{4+h})} \\
   &= \lim_{h \to 0} \frac{-1}{2\sqrt{4+h}(2 + \sqrt{4+h})} \\
   &= \frac{-1}{2\sqrt{2}(2 + \sqrt{2})} = -\frac{1}{16}.
   \end{aligned}$$

   Since $f'(4)$ gives the slope of the tangent line, and it passes through the point $(4, 1/2)$, we have the equation

   $$y - \frac{1}{2} = -\frac{1}{16}(x - 4)$$

   for the line.