MATH 1565 - Tutorial #5 Solutions

Determine $f'(x)$ for the following functions:

1. $f(x) = x^{38} + 4\ln(x) - 3e^x + \pi^{3000}$.

   Using the sum and constant rules,

   $$f'(x) = 38x^{37} + 4 \cdot \frac{1}{x} - 3e^x.$$

2. $f(x) = x^5 e^x \tan(x)$.

   Using the product rule (twice),

   $$f'(x) = 5x^4 e^x \tan(x) + x^5 e^x \tan(x) + x^5 e^x \sec^2(x).$$

3. $f(x) = \cot(x)$. (Write $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$ and use the quotient rule.)

   Using the quotient rule,

   $$\begin{aligned} f'(x) &= \frac{\left(\frac{d}{dx}(\cos(x))\right)\sin(x) - \cos(x)\frac{d}{dx}(\sin(x))}{\sin^2(x)} \\ &= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} \\ &= \frac{-1}{\sin^2(x)} = -\csc^2(x). \end{aligned}$$

4. $f(x) = \cos(e^x + 13x^{11} - \sin(x))$.

   Using the chain rule,

   $$\begin{aligned} f'(x) &= -\sin(e^x + 13x^{11} - \sin(x))\frac{d}{dx}(e^x + 13x^{11} - \sin(x)) \\ &= -\sin(e^x + 13x^{11} - \sin(x))(e^x + 13(11x^{10}) - \cos(x)) \end{aligned}$$