Math 1410 Assignment #4
University of Lethbridge, Spring 2017

Sean Fitzpatrick

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Due date: Thursday, March 23rd, by 4 pm.

Please review the **Guidelines for preparing your assignments** before submitting your work. You can find these guidelines, along with the required cover page, in the Assignments section on our Moodle site.

Assigned problems

1. Determine the null space and column space of the matrix $A = \begin{bmatrix} 2 & -3 & 1 & 4 \\ -1 & 2 & 2 & -3 \\ 1 & 0 & 8 & -1 \end{bmatrix}$.

2. Factor the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -3 & 1 \\ -1 & 2 & 4 \end{bmatrix}$ as a product of elementary matrices.

3. For each statement below, either prove the statement or give a counterexample showing that it is false.

    (a) If A and B are both invertible, then $A + B$ is invertible.

    (b) If $AB = I$, then $AB = BA$.

    (c) If $AB = B$ for some matrix $B \neq 0$, then A is invertible.

    (d) If A^3 is invertible, then A is invertible.

4. Let A be a non-zero $n \times n$ matrix, and let I be the $n \times n$ identity matrix.

    (a) Show that if $A^2 = 0$, then $(I - A)^{-1} = I + A$.

    (b) Show that if $A^3 = 0$, then $(I - A)^{-1} = I + A + A^2$.

    (c) Find the inverse of $B = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

    (d) Given that $A \neq 0, A^2 \neq 0, \ldots, A^{n-1} \neq 0$ but $A^n = 0$, determine a formula for $(I - A)^{-1}$, and show that your answer is correct.