*University of Lethbridge*
Department of Mathematics and Computer Science
19th March, 2015, 10:55 - 11:45 am
**MATH 1410B - Test #2**

Last Name:________________________________________

First Name:________________________________________

Student Number:____________________________________

Tutorial Section: ____________________________________

**Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.**

**Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.**

**No external aids are allowed, with the exception of a 5-function calculator.**

For grader's use only:

| Page | Grade |
|---|---|
| 2 | /10 |
| 3 | /10 |
| 4 | /10 |
| 5 | /5 |
| 6 | /5 |
| Total | /40 |

A

1. SHORT ANSWER: For each of the questions below, please provide a short (one line) answer.

[2] (a) Suppose $\det A = 4$ and the matrix $B$ is obtained from $A$ by first multiplying the first row of $A$ by 5, and then exchanging rows 1 and 3. What is $\det B$?

[2] (b) Let $A$ and $B$ be $3 \times 3$ matrices. If $\det A = 2$ and $\det B = -3$, what is the value of $\det(2A^2B^TA^{-1})$?

[2] (c) Calculate the dot product of the vectors $\vec{u} = \begin{bmatrix} 1 & -2 & 4 \end{bmatrix}^T$ and $\vec{v} = \begin{bmatrix} 4 & -2 & 3 \end{bmatrix}^T$

[2] (d) Let $\vec{v}$ be a vector in $\mathbb{R}^n$. If $\|\vec{v}\| = 3$ and $\vec{w} = -4\vec{v}$, what is $\|\vec{w}\|$?

[2] (e) Find $\vec{x}$, given that $\vec{u} = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}^T$ and $\vec{v} = \begin{bmatrix} -4 & 7 & 5 \end{bmatrix}^T$, and $3\vec{u} - 2\vec{x} = \vec{v}$.

2. Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 4 \\ -3 & 0 & 1 \end{bmatrix}$.

[5] (a) Compute $\det A$.

(b) Given the system of equations $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 4 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, use Cramer's rule to find

[5] the value of $y$, if possible.

3. (a) Find a vector equation of the line in $\mathbb{R}^3$ that passes through the points $P = (2, -3, 1)$
[4] and $Q = (4, 1, -2)$.

(b) Let $L_1$ be the line through $P_1 = (2, 0, -1)$ with direction vector $\vec{d_1} = \begin{bmatrix} -1 & 3 & 2 \end{bmatrix}^T$, and let $L_2$ be the line through $P_2 = (8, 6, 7)$ with direction vector $\vec{d_2} = \begin{bmatrix} -4 & 0 & -2 \end{bmatrix}^T$.
[6] Determine the point of intersection of $L_1$ and $L_2$, if any.

4. Find the shortest distance from the point $P = (3, 2, -1)$ to the line $L$ given by the vector
[5] equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

5. Consider the triangle in $\mathbb{R}^3$ with vertices $P = (2, 0, -3)$, $Q = (5, -2, 1)$, and $R = (7, 5, 3)$.

[3] (a) Show that the triangle is a right-angled triangle.

*Hint:* Recall that if the dot product of two non-zero vectors is zero, then those vectors meet at a right angle.

[2] (b) Compute the lengths of the three sides of the triangle and verify that the Pythagorean theorem $(a^2 + b^2 = c^2)$ holds.



**Total Points: 40**