

# Math 1410 Assignment #5

## University of Lethbridge, Spring 2017

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**Due date:** Friday, March 31st, by 12 pm.

Please review the **Guidelines for preparing your assignments** before submitting your work. You can find these guidelines, along with the required cover page, in the Assignments section on our Moodle site.

### Assigned problems

1. Suppose  $A$  and  $B$  are  $4 \times 4$  matrices such that  $\det(A) = 3$  and  $\det(B) = -4$ . Determine the values of
  - (a)  $\det(A^2B)$
  - (b)  $\det(B^T BAB^{-1})$
  - (c)  $\det(2AB^{-1})$
2. Suppose  $\det(AB) = 0$ . Must it be the case that  $\det(A) = 0$  or  $\det(B) = 0$ ? Prove this, or give a counterexample.
3. We say that an  $n \times n$  matrix  $B$  is **similar** to an  $n \times n$  matrix  $A$  if  $B = P^{-1}AP$  for some invertible matrix  $P$ , and write  $B \sim A$ .
  - (a) Show that if  $B \sim A$ , then  $\text{tr}(B) = \text{tr}(A)$ .
  - (b) Show that if  $B \sim A$ , then  $\det(B) = \det(A)$ .
  - (c) Suppose  $A$  is similar to a matrix  $D = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & y \end{bmatrix}$ , and we know that  $\text{tr}(A) = 0$ , and  $\det(A) = 16$ . What are the values of  $x$  and  $y$ ?
4. Let  $\text{adj}(A)$  denote the adjugate matrix of an  $n \times n$  matrix  $A$ . Show that

$$\det(\text{adj}(A)) = (\det(A))^{n-1}.$$