University of Lethbridge
Department of Mathematics and Computer Science
10^{th} November 2014, 2:00-2:50 pm
MATH 3500 - Test #2

Last Name:____

First Name:____

Student Number:_____________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Page	Grade
2	/8
3	/10
4	/10
5	/12
Total	/40

1. Let (a_n) be the sequence defined by $a_n = \cos\left(\frac{n\pi}{3}\right)$ for $n = 1, 2, 3, \ldots$.

   [2] (a) Recall that $|\cos x| \leq 1$ for all $x \in \mathbb{R}$. Explain why this guarantees that (a_n) has a covergent subsequence.

   [4] (b) Find the set of subsequential limits of (a_n). (That is find the set of limits of convergent subsequences. It might help to recall that $\cos(\pi/3) = 1/2$.)

   [2] (c) What are the values of $\limsup a_n$ and $\liminf a_n$?

[5] 2. Use the $\epsilon - \delta$ definition of the limit to prove that $\lim_{x \to 2} \frac{x+1}{2x-1} = 1$.

3. Use the $\epsilon - \delta$ definition of uniform continuity to prove that $f(x) = \frac{x}{x+1}$ is continuous on
[5] $[0, 2]$.

4. Decide whether or not the given function f is uniformly continuous on its domain D. Justify your answer in each case with a suitable theorem.

[2] (a) $f(x) = x^2 + 2x$ on $D = [0, 3]$

[2] (b) $f(x) = 1/x^2$ on $D = (0, 1]$

[2] (c) $f(x) = \dfrac{1}{x}\sin^2 x$ on $D = (0, \pi)$.

5. Suppose $f : [a, b] \to \mathbb{R}$ is a continuous function.

[2] (a) Is it possible for the range of f to equal $[0, 1] \cup [2, 3]$? Why or why not?

[2] (b) Is it possible for the range of f to equal either $(0, 1)$ or $[1, \infty)$? Why or why not?

6. Let $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$.

[4] (a) Show that f is discontinuous at every point $a \neq 0$. (Hint: if a is rational/irrational, consider a sequence of irrational/rational numbers converging to a.)

[4] (b) Prove that f is differentiable (and hence continuous) at $x = 0$.

[4] 7. Prove that if $f'(x) \neq 0$ for all $x \in (a, b)$, then f is either strictly increasing or strictly decreasing on (a, b). (Caution: f' is not guaranteed to be continuous.)

 Total Points: 40