1. Compute the derivatives of the following functions:

[2] (a) $f(x) = \arctan(x)$

$$f'(x) = \frac{1}{1+x^2}$$

[2] (b) $g(x) = \arcsin(x^3)$

$$g'(x) = \frac{1}{\sqrt{1-(x^3)^2}}\frac{d}{dx}(x^3) = \frac{3x^2}{\sqrt{1-x^6}}.$$

[2] (c) $h(x) = \sec(\arccos(x))$ (Hint: simplify first!)

We note that $\sec(\arccos(x)) = \dfrac{1}{\cos(\arccos(x))} = \dfrac{1}{x}$. Therefore,

$$h'(x) = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -x^{-2} = -\frac{1}{x^2}.$$

[3] 2. Find $\dfrac{dy}{dx}$ in terms of x and y given that $\sin(y) = x^2y^3$.

Differentiating both sides of $\sin(y) = x^2y^3$ with respect to x, we find

$$\cos(y)\frac{dy}{dx} = 2xy^3 + 3x^2y^2\frac{dy}{dx}.$$

Solving for $\frac{dy}{dx}$, we find

$$\frac{dy}{dx} = \frac{2xy^3}{\cos(y) - 3x^2y^2}.$$

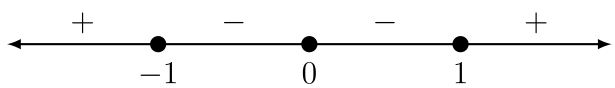
3. Consider the function $f(x) = 3x^5 - 5x^3$.

[1] (a) Compute $f'(x)$

$$f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1) = 15x^2(x-1)(x+1).$$

[1] (b) Construct a sign diagram for $f'(x)$.

From part (a), the zeros of f' are at $x = -1$, $x = 0$, and $x = 1$. Our sign diagram is given by



[1] (c) Classify each critical point of $f(x)$ as a local minimum, local maximum, or neither.

Using the first derivative test, we see that f has a local maximum at $(-1, f(-1)) = (-1, 2)$, and a local minimum at $(1, f(1)) = (1, -2)$. The critical point at $x = 0$ is neither a maximum nor a minimum.

[2] (d) Determine the intervals where $f(x)$ is

- Increasing:

  $f(x)$ is increasing on $(-\infty, -1) \cup (1, \infty)$

- Decreasing:

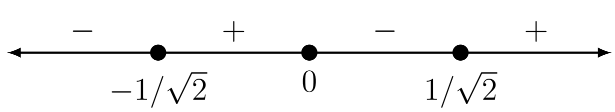
  $f(x)$ is decreasing on $(-1, 1)$.

[1] (e) Compute $f''(x)$.

$$f''(x) = \frac{d}{dx}(15x^4 - 15x^2) = 60x^3 - 30x = 30x(2x^2 - 1) = 30x(\sqrt{2}x - 1)(\sqrt{2}x + 1).$$

[1] (f) Construct a sign diagram for $f''(x)$.

From part (e), the zeros of f'' are at $x = -1/\sqrt{2}$, $x = 0$, and $x = 1/\sqrt{2}$. The sign diagram is given by



[2] (g) Determine the intervals on which the graph of f is

- Concave up:

  $f(x)$ is concave up on $(-1/\sqrt{2}, 0) \cup (1/\sqrt{2}, \infty)$.

- Concave down:

  $f(x)$ is concave down on $(-\infty, -1/\sqrt{2}) \cup (0, 1/\sqrt{2})$.