MATH 1410 ASSIGNMENT #5
UNIVERSITY OF LETHBRIDGE, FALL 2016

SEAN FITZPATRICK

Due date: Tuesday, November 29th, by 4:00 pm.

Please review the **Guidelines for preparing your assignments** before submitting your work. You can find these guidelines, along with the required cover page, in the Assignments section on our Moodle site.

Assigned problems.

(1) Recall that an $n \times n$ matrix A is **idempotent** if $A^2 = A$. Show that:

- (a) The identity matrix I is the only invertible idempotent matrix.
- (b) A matrix A is idempotent if and only if $I - 2A$ is self-inverse.
- (c) If A is idempotent, then $I - kA$ is invertible for any $k \neq 1$, and

$$(I - kA)^{-1} = I + \left(\frac{k}{1-k}\right)A.$$

(2) Recall that an $n \times n$ matrix A is **symmetric** if $A^T = A$, and **antisymmetric** if $A^T = -A$.

- (a) Show that $B + B^T$ is symmetric for **any** $n \times n$ matrix B.
- (b) Show that $B - B^T$ is antisymmetric for **any** $n \times n$ matrix B.
- (c) Given any $n \times n$ matrix B, find a symmetric matrix U and an antisymmetric matrix V such that $B = U + V$.

(3) What can be said about the determinant of A if:

- (a) A is idempotent.
- (b) A is self-inverse.
- (c) A is antisymmetric.

(4) Determine all values of k such that the following matrices are invertible:

$$A = \begin{bmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{bmatrix} \quad B = \begin{bmatrix} k & k & 0 \\ k^2 & 4 & k^2 \\ 0 & k & k \end{bmatrix}$$

See over for hints and suggestions.

Hints and suggestions:

- For 1(b), to prove an "A if and only if B" statement you must prove two things: A implies B, and B implies A. In this case that means you must prove (i) if A is idempotent, then $I-2A$ is self-inverse, and (ii) if $I-2A$ is self-inverse, then A is idempotent.
- For 1(c), remember that showing $B=A^{-1}$ is the same thing as showing $AB=I$.
- For 2(a) and 2(b) you should be giving a *general* argument, relying only on the properties of the transpose. Your results here are intended as a hint for 2(c).
- For problem 3(c), assume A is an $n\times n$ matrix. There are two cases, depending on whether n is even or odd.
- For problem 4, use determinants!