University of Lethbridge
Department of Mathematics and Computer Science
21st December, 2016, 9:00 am - 12:00 pm

MATH 1410 - FINAL EXAM (DEFERRED)

Last Name:____

First Name:____

Student Number:____________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page. Additional scrap paper may be requested if needed.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

No external aids are permitted, with the exception of a basic (non-scientific and non-graphing) calculator.

For grader's use only:

Page	Grade
2	/10
3	/10
4	/12
5	/11
6	/9
7	/8
8	/9
9	/9
10	/8
11	/7
12	/7
Total	/100

1. DEFINITIONS. (2 points each) Give the definition of the term in bold. Write your answer as a complete sentence.

   (a) What is the **modulus** of a complex number?

   (b) What is a **unit vector**?

   (c) What does it mean for an $n \times n$ matrix A to be **anti-symmetric**?

   (d) What is the **null space** of a matrix?

   (e) What is an **eigenvalue** of an $n \times n$ matrix A?

2. SHORT ANSWER – Calculations (2 points each): You do not have to show your work.

   (a) Compute the magnitude (length) of the vector $\vec{v} = \langle -3, 4, 1 \rangle$

   (b) Compute the product zw of the complex numbers $z = -2 + 3i$ and $w = 5 + 4i$.

   (c) Given $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, determine an elementary matrix E such that $EA = \begin{bmatrix} 3 & -5 \\ 2 & -8 \end{bmatrix}$.

   (d) Compute the trace of the matrix $A = \begin{bmatrix} 2 & -3 & 7 \\ -4 & 6 & 0 \\ 3 & -5 & -1 \end{bmatrix}$.

   (e) Compute the determinant of $A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$.

3. SHORT ANSWER – More calculations (3 points each): You don't have to show work, but it can earn you part marks if you do.

   (a) The augmented matrix for a system of equations in the variables x, y, z can be put into the row-echelon form $\left[\begin{array}{ccc|c} 1 & -2 & -4 & 2 \\ 0 & 1 & 3 & 4 \end{array}\right]$. What is the solution of the system?

   (b) Suppose A and B are 2×2 matrices such that $\det(A) = 3$ and $\det(B) = -11$. What is the value of $\det(2A^2B^T(AB)^{-1})$?

   (c) Calculate the projection of the vector $\vec{w} = \langle 4, 2, -1 \rangle$ onto the vector $\vec{v} = \langle 2, -1, 2 \rangle$.

   (d) What is the reduced row-echelon form of the matrix $A = \begin{bmatrix} 3 & -6 \\ 2 & -3 \end{bmatrix}$?

4. SHORT ANSWER – Yet more calculations (points as indicated):

[3] (a) Write the complex number $\dfrac{5-2i}{3+4i}$ in the form $x+iy$.

[3] (b) Determine the matrix X such that $4X + \begin{bmatrix} 2 & 3 \\ -2 & 7 \end{bmatrix} = 3\begin{bmatrix} 5 & 0 \\ 1 & -2 \end{bmatrix}$.

[3] (c) Compute the matrix product $\begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$.

[2] (d) Compute $T\left(\begin{bmatrix} -3 \\ 2 \end{bmatrix}\right)$, where $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a matrix transformation such that $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$.

[9] 5. Compute the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

[8] 6. Compute the determinant of the matrix

$$A = \begin{bmatrix} 12 & 0 & 1 & -3 \\ 1 & -2 & 7 & -4 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & -4 & 2 \end{bmatrix}$$

7. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 6 \end{bmatrix}$.

[6] (a) Compute the inverse of A.

[3] (b) Solve the system $A\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$.

8. Consider the vectors $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 6 \\ 3 \\ -4 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -2 \\ -5 \\ 14 \end{bmatrix}$.

[6] (a) Determine whether or not the vector $\begin{bmatrix} 3 \\ -1 \\ 7 \end{bmatrix}$ belongs to the span of $\vec{u}, \vec{v}$, and $\vec{w}$.

[3] (b) Are the vectors $\vec{u}, \vec{v}, \vec{w}$ linearly independent? Explain.

9. Consider the lines ℓ_1 and ℓ_2 given by

$$\begin{aligned}\langle x, y, z\rangle &= \langle 2, -1, 3\rangle + s\langle 3, 0, -4\rangle \quad \text{for } \ell_1\text{, and}\\ \langle x, y, z\rangle &= \langle 1, 3, -5\rangle + t\langle 1, 2, -6\rangle \quad \text{for } \ell_2.\end{aligned}$$

[4] (a) Show that the lines intersect.

[4] (b) Determine the equation of the plane containing ℓ_1 and ℓ_2.

10. Let A be an $n \times n$ matrix and let I denote the $n \times n$ identity matrix.

[2] (a) Show that if $A^2 = 0$, then $(I - A)^{-1} = I + A$. (Hint: $Y = X^{-1}$ if and only if $XY = I$.)

[2] (b) Show that if $A^3 = 0$, then $(I - A)^{-1} = I + A + A^2$.

[3] (c) Suppose that $A^n = 0$ for some integer $n \geq 3$. Propose a formula for $(I - A)^{-1}$ and show that your formula is correct.

[3] 11. Verify that $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} i \\ 1 \end{bmatrix}$ are eigenvectors of the matrix $Z = \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix}$.

[4] 12. Prove that if A is an invertible matrix and λ is an eigenvalue of A, then $\dfrac{1}{\lambda}$ is an eigenvalue of A^{-1}. (For a 2 point bonus: why does the invertibility of A guarantee that $\lambda \neq 0$?)

 Total Points: 100