

University of Lethbridge
Department of Mathematics and Computer Science
MATH 2565 - Tutorial #4
Thursday, February 1

First Name: _____

Last Name: _____

Print your name clearly in the space above.

Complete the problems on the back of this page to the best of your ability. If there is a problem you especially desire feedback on, please indicate this.

It is recommended that you work out the details on scrap paper before writing your solutions on the worksheet.

Additional practice (don't include your solutions here):

1. Evaluate the improper integral, if possible:

(a) $\int_0^{\infty} x^2 e^{-2x} dx$

(b) $\int_0^1 \frac{\sqrt{x} + 1}{x} dx$

2. Determine if the improper integral converges or diverges:

(a) $\int_0^{\infty} \frac{1}{\sqrt{x^3 + 2x^2 + 5}} dx$

(b) $\int_1^{\infty} e^{-x} \ln(x) dx$

3. Determine the area bounded by the given curves:

(a) $y = \sqrt{x+2}$, $y = \frac{1}{x+1}$, $x = 0$ and $x = 2$. (c) $y = x$ and $y = x^3$.

(b) $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x - 1$. (d) $y = x^2 + 1$, $y = \frac{1}{4}(x-3)^2 + 1$, and $y = 1$.

1. Evaluate the improper integral, if possible:

(a) $\int_1^{\infty} \frac{\ln(x)}{x^2} dx$

(b) $\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx$

2. Determine whether or not the following improper integrals converge or diverge. Use either direct comparison or limit comparison, as appropriate.

(a) $\int_1^{\infty} \frac{1}{\sqrt{x^2 + x}} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x^2 - x}} dx$

3. (For in-class discussion): how do you show that $\int_0^\infty \frac{x^n}{e^x} dx$ exists for any positive integer n ?
4. Let $p(x)$ be any polynomial function. Does $\int_0^\infty \frac{p(x)}{e^x} dx$ converge or diverge? Justify your answer.
5. Find the area between the given curves:
- (a) $y = \cos x$ and $y = \sin 2x$, between $x = 0$ and $x = \pi/2$.
- (b) $y = x$, $y = 5x$, and $y = 6 - x^2$, in the first quadrant.