University of Lethbridge
Department of Mathematics and Computer Science

MATH 1565 - Tutorial #3

Print your name and student number clearly in the space above.

Complete the problems on the back of this page to the best of your ability. If there is a problem you especially desire feedback on, please indicate this.

It is recommended that you work out the details on scrap paper before writing your solutions on this page.

1. For this problem you may use the fact (proven in class) that $g(x) = |x|$ is continuous on $\mathbb{R}$.

   (a) Prove that if f is continuous, then $|f|$ is continuous.

   (b) Give a counterexample showing that the converse to part (a) is false. That is, find a function f such that $|f|$ is continuous everywhere, but f is not.

2. On what intervals is $x^2 < x^4$? On what intervals is $x^4 < x^2$? If the graph of $f(x)$ always lies between the graphs $y = x^2$ and $y = x^4$, for what real numbers a can you determine the value of $\lim_{x \to a} f(x)$ using the Squeeze Theorem? Find these values.

3. Consider the function $f(x) = x^3 + 2x^2 - 1$.

   (a) Prove that f has a root (x-intercept) in $[0, 1]$.

   (b) Determine an interval of length $1/16$ containing the root from part (a).