University of Lethbridge
Department of Mathematics and Computer Science
MATH 2565 - Tutorial #2
Thursday, January 18

First Name:____

Last Name:____

Print your name clearly in the space above.

Complete the problems on the back of this page to the best of your ability. If there is a problem you especially desire feedback on, please indicate this.

It is recommended that you work out the details on scrap paper before writing your solutions on the worksheet.

Additional practice (don't include your solutions here):

1. $\displaystyle\int \sin^5(x)\cos^6(x)\,dx$
2. $\displaystyle\int \sin(x)\sin(2x)\,dx$
3. $\displaystyle\int \sqrt{9-x^2}\,dx$
4. $\displaystyle\int \frac{8}{\sqrt{x^2+2}}\,dx$
5. $\displaystyle\int \frac{1-\tan^2(x)}{\sec^2(x)}\,dx$
6. $\displaystyle\int \frac{dx}{\cos(x)-1}\,dx$

Evaluate the following integrals.

1. $\displaystyle\int \tan^4(x)\sec^6(x)\,dx$

2. $\displaystyle\int \tan^3(x)\sec^5(x)\,dx$

3. $\displaystyle\int \sin(8x)\cos(5x)\,dx$

4. $\displaystyle\int_0^{\pi/6} \sqrt{1+\cos(2x)}\,dx$

5. $\displaystyle\int \frac{5x^2}{\sqrt{x^2-10}}\,dx$, using a secant substitution.

6. $\displaystyle\int \frac{5x^2}{\sqrt{x^2-10}}\,dx$, using a hyperbolic substitution.

Discussion problem (no submission required):
Prove the following formulas, where m and n are integers:

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\sin(mx)\cos(nx)\,dx = 0$$
$$\frac{1}{\pi}\int_{-\pi}^{\pi}\sin(mx)\sin(nx)\,dx = \begin{cases}0, & \text{if } m \neq n\\ 1, & \text{if } m = n\end{cases}$$
$$\frac{1}{\pi}\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx = \begin{cases}0, & \text{if } m \neq n\\ 1, & \text{if } m = n\end{cases}$$

Suppose a function f can be written as a *finite Fourier series*

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{N} a_n \sin(nx) + \sum_{m=1}^{M} b_m \cos(mx).$$

Show that the coefficients a_n $(n = 0, \ldots, N)$ and b_m $(m = 1, \ldots, M)$ are given by

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx,\, a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx,\, (i = 1, \ldots, N),\, b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx.$$