SEAN FITZPATRICK

Due date: Thursday, October 27th, by 4:30 pm.

Please review the **Guidelines for preparing your assignments** before submitting your work. You can find these guidelines, along with the required cover page, in the Assignments section on our Moodle site.

Assigned problems.
  (1) An $n \times n$ matrix A is called **idempotent** if $A^2 = A$, where $A^2 = AA$.

   (a) Show that the following matrices are idempotent:
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

   (b) Let I denote the $n \times n$ identity matrix. Show that if A is idempotent, then so is $I - A$, and that $A(I - A) = 0$.

   (c) Show that if A is an $n \times n$ idempotent matirx and B is any other $n \times n$ matrix, then
$$C = A + BA - ABA$$
   is an idempotent matrix.

  (2) Determine the matrix A such the matrix transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \end{bmatrix}$ perfoms the following transformations of the Cartesian plane, in order:
   • First, a vertical reflection across the x-axis.
   • Second, a horizontal reflection across the y-axis.
   • Third, a counter-clockwise rotation through an angle of 90°.

  (3) In each of the following, either explain why the statement is true, or give an example showing that it is false:
   (a) If A is an $m \times n$ matrix where $m < n$, then $AX = B$ has a solution for every column B.
   (b) If $AX = B$ has a solution for some column B, then it has a solution for every column B.
   (c) If X_1 and X_2 are solutions to $AX = B$, then $X_1 - X_2$ is a solution to $AX = 0$.
   (d) If $AB = AC$ and $A \neq 0$, then $B = C$.
   (e) If $A \neq 0$, then $A^2 \neq 0$.