List of potentially useful facts and definitions (you may remove this page)

Properties of $\mathbb{R}$

Completeness axiom: $A \subseteq \mathbb{R}$ bounded above $\Rightarrow \sup A$ exists.
Archimedean property: $\forall x > 0, \exists n \in \mathbb{N}$ such that $1/n < x$.
Neighbourhood: $N_\epsilon(x) = \{y \in \mathbb{R} : |x - y| < \epsilon\}$.
A point $x \in A$ is an **interior point** of A if $\exists \epsilon > 0$ such that $N_\epsilon(x) \subseteq A$.
A set A is **open** if every point $a \in A$ is an interior point. (A is equal to its interior A°)
A point $x \in \mathbb{R}$ is a **boundary point** of $A \subseteq \mathbb{R}$ if $\forall \epsilon > 0$, $N_\epsilon(x) \cap A \neq \emptyset$ and $N_\epsilon(x) \cap (\mathbb{R} \setminus A) \neq \emptyset$.
A point $x \in \mathbb{R}$ is a **limit point** of $A \subseteq \mathbb{R}$ if $\forall \epsilon > 0$, $(N_\epsilon(x) \setminus \{x\}) \cap A \neq \emptyset$.
A point $x \in A$ is an **isolated point** of A if it is not a limit point.
The **closure** $\overline{A}$ of $A \subseteq \mathbb{R}$ is the union of A and its limit points.
A set $A \subseteq \mathbb{R}$ is **closed** iff $X \setminus A$ is open, iff $A = \overline{A}$.
A set $A \subseteq \mathbb{R}$ is **bounded** if $A \subseteq [a, b]$ for some $a, b \in \mathbb{R}$. A set $A \subseteq \mathbb{R}$ is **compact** if every open cover of A admits a finite subcover.
A set $A \subseteq \mathbb{R}$ is compact iff it is **closed and bounded**.
The **union** of any collection of **open** sets is open.
The **intersection** of any collection of **closed** sets is closed.

Sequences

A sequence (a_n) **converges** to $a \in \mathbb{R}$ if $\forall \epsilon > 0, \exists N \in \mathbb{N}$ such that $|a_n - a| < \epsilon$ for all $n \geq N$.
If a is a limit point of A, there is a sequence (a_n) in A converging to a.
A set A is closed iff the limit of every sequence (a_n) in A that converges belongs to A.
A sequence is **monotone** if it is either increasing or decreasing.
A bounded monotone sequence converges.
A sequence (a_n) is **Cauchy** if $\forall \epsilon > 0 \exists N \in \mathbb{N}$ such that if $m, n > N$, then $|a_m - a_n| < \epsilon$.
A sequence is Cauchy if and only if it converges.

Continuity

$\lim_{x \to a} f(x) = L$ iff $\forall \epsilon > 0, \exists \delta > 0$ such that $0 < |x - a| < \delta \Rightarrow |f(x) - L| <]\epsilon$.
$\lim_{x \to a} f(x) = L$ iff for every sequence (a_n) with $a_n \to a$, $f(a_n) \to L$.
f is **continuous** at a if $\forall \epsilon > 0, \exists \delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon$.
If f is continuous on D and D is compact, then $f(D)$ is compact.
If f is continuous on $[a, b]$ then f has the intermediate value property on $[a, b]$.
f is **uniformly continuous** on $D \subseteq \mathbb{R}$ if $\forall \epsilon > 0, \exists \delta > 0$ such that $\forall x, y \in D$, if $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$.
If f is continuous on D and D is compact, then f is uniformly continuous.
If f is uniformly continuous on D then f is bounded on D.
f is uniformly continuous on (a, b) iff f can be extended to a continuous function on $[a, b]$.

Derivatives

f is **differentiable** at a if $f'(a) = \lim_{x \to a}(f(x) - f(a))/(x - a)$ exists.
If f is differentiable at a then f is continuous at a.
$f'(x)$ always has the intermediate value property.
Mean Value Theorem: if f is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that $f'(c)(b - a) = f(b) - f(a)$.
l'Hospital's rule is a thing, but not a thing you'll be asked about.
The **remainder** in Taylor's Theorem is given, for some c between a and x, by $R_{n,a,f}(x) = f^{(n+1)}(c)(x - a)^{n+1}/(n + 1)!$.

Integration

Lower sum: $L(f, P) = \sum_{i=1}^n m_i(x_i - x_{i-1})$, $m_i = \inf\{f(x) : x_{i-1} \leq x \leq x_i\}$
Upper sum: $U(f, P) = \sum_{i=1}^n M_i(x_i - x_{i-1})$, $M_i = \sup\{f(x) : x_{i-1} \leq x \leq x_i\}$
If $P_1 \subseteq P_2$, $L(f, P_1) \leq L(f, P_2) \leq U(f, P_2) \leq U(f, P_1)$
f is **intergrable** on $[a, b]$ iff $\forall \epsilon > 0, \exists P_\epsilon$ such that $U(f, P_\epsilon) - L(f, P_\epsilon) < \epsilon$.
Every continuous function is integrable.
FTC I: If $F(x) = \int_a^x f(t)\, dt$, f continuous on $[a, b]$, then $F'(x) = f(x)$.
FTC II: If $F'(x) = f(x)$ on $[a, b]$, then $\int_a^b f(x)\, dx = F(b) - F(a)$.