University of Lethbridge
Department of Mathematics and Computer Science
19th October, 2015, 4:00 - 4:50 pm
MATH 1010A - Test #1

Last Name: **Solutions**

First Name: **The**

Student Number: ____________________

Tutorial Section: ____________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

No external aids are allowed, with the exception of a 5-function calculator.

For grader's use only:

Page	Grade
2	/10
3	/10
4	/10
5	/10
Total	/40

A

1. Simplify the following expressions. Your final answer should be in the form $\dfrac{A}{B}$, where A and B are integers.

[3] (a) $\dfrac{1-\left(\frac{4}{3}\right)\left(\frac{3}{8}\right)}{1+\left(\frac{4}{3}\right)\left(\frac{3}{8}\right)}$.

$$\frac{1-\left(\dfrac{4}{\cancel{3}}\right)\left(\dfrac{\cancel{3}}{8}\right)}{1-\left(\dfrac{4}{\cancel{3}}\right)\left(\dfrac{\cancel{3}}{8}\right)} = \frac{1-\dfrac{4}{8}}{1+\dfrac{4}{8}} = \frac{1-\dfrac{1}{2}}{1+\dfrac{1}{2}} = \frac{\dfrac{1}{2}}{\dfrac{3}{2}} = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}.$$

[3] (b) $\left(\dfrac{81}{16}\right)^{-3/4}$.

$$\left(\frac{81}{16}\right)^{-3/4} = \left(\frac{16}{81}\right)^{3/4} = \left(\frac{2^4}{3^4}\right)^{3/4} = \frac{(2^4)^{\frac{3}{4}}}{(3^4)^{\frac{3}{4}}} = \frac{2^3}{3^3} = \frac{8}{27}$$

2. Simplify the following expression. Your final answer should be in the form $\dfrac{A}{B}$, where A
[4] and B are polynomials.

$$\frac{3x}{1+x} + \frac{2x-3}{1-x} - \frac{2x-4}{1-x^2}$$

$$\begin{aligned}
\frac{3x}{1+x} + \frac{2x-3}{1-x} - \frac{2x-4}{1-x^2} &= \frac{3x(1-x)}{(1+x)(1-x)} + \frac{(2x-3)(1+x)}{(1-x)(1+x)} - \frac{2x-4}{1-x^2} \\
&= \frac{3x(1-x)+(2x-3)(1+x)-(2x-4)}{1-x^2} \\
&= \frac{3x-3x^2+2x^2-x-3-2x+4}{1-x^2} \\
&= \frac{1-x^2}{1-x^2} = 1
\end{aligned}$$

3. Solve the following inequalities. Express your answer in **interval notation**.

[3] (a) $\dfrac{2}{3} - 2x \leq \dfrac{4x-3}{2}$.

Dividing by 2 on the right-hand side, we have $\dfrac{2}{3} - 2x \leq 2x - \dfrac{3}{2}$. Adding $2x$ to both sides to bring all the terms involving x to the right, and adding $\dfrac{3}{2}$ to both sides to bring all constant terms to the left, we obtain

$$\frac{2}{3} + \frac{3}{2} \leq 4x.$$

Since $\dfrac{2}{3} + \dfrac{3}{2} = \dfrac{4}{6} + \dfrac{9}{6} = \dfrac{13}{6}$, we have $\dfrac{13}{6} \leq 4x$, and dividing both sides by 4 (noting that $4 > 0$), we have

$$\frac{13}{24} \leq x \quad \text{or} \quad x \geq \frac{13}{24}. \quad \text{In interval notation,} \quad x \in \left[\frac{13}{24}, \infty\right).$$

[4] (b) $7x \geq 4 - x \geq 3$

We are given a pair of inequalities. The first one is $7x \geq 4 - x$; adding x to both sides gives us $8x \geq 4$, and dividing by 8, we find that $x \geq \dfrac{1}{2}$.
The second inequality is $4 - x \geq 3$, which simplifies to $1 \geq x$, or $x \leq 1$. Since we must satisfy **both** inequalities, we must have $\dfrac{1}{2} \leq x$ **and** $x \leq 1$, so $\dfrac{1}{2} \leq x \leq 1$. In interval notation, our solution is $x \in [\frac{1}{2}, 1]$.

[3] (c) $|3x - 5| \leq 4$

We recall that the statement $|a| \leq b$ is equivalent to $-b \leq a \leq b$ for any $a \in \mathbb{R}$ and any $b > 0$. For the given inequality this gives us

$$-4 \leq 3x - 5 \leq 4.$$

Adding 5 to all three parts of this compound inequality yields $1 \leq 3x \leq 9$, and if we divide throughout by $3 > 0$, we obtain $\frac{1}{3} \leq x \leq 3$. Thus we must have $x \in [\frac{1}{3}, 3]$.

4. Consider the function $f(x) = x|x|$.

[3] (a) Evaluate the following: $f(1), f(1) + f(-2), f(2) \cdot f(-3)$.

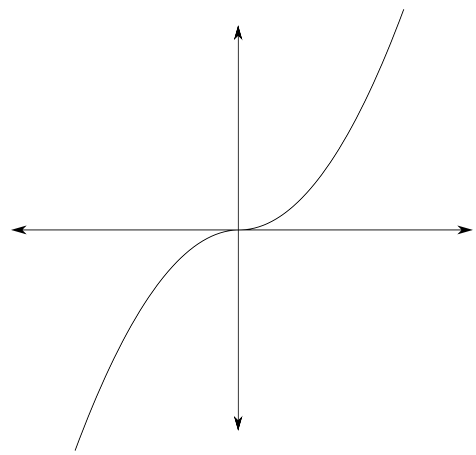
We recall that $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$, so that, for example, $|1| = 1$, since $1 > 0$, while $|-2| = -(-2) = 2$, since $2 < 0$. (Note that $|x| \geq 0$ for all $x \in \mathbb{R}$.) Thus, we can compute as follows:

$f(1) = 1|1| = 1(1) = 1$. $\quad f(1) + f(-2) = 1|1| + (-2)|-2| = 1(1) - 2(2) = 1 - 4 = -3.$

$$f(2) \cdot f(-3) = (2|2|)(-3|-3|) = 2(2)(-3)(3) = -36.$$

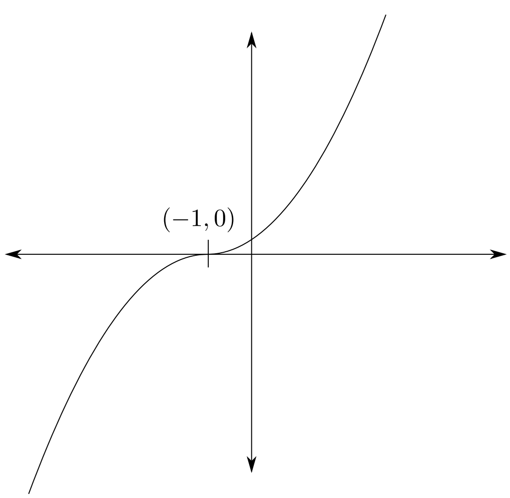
[3] (b) Sketch the graph of f. (Hint: consider the cases $x \geq 0$ and $x < 0$.)

For $x \geq 0$, we have $|x| = x$, so $f(x) = x|x| = x(x) = x^2$. For $x < 0$, $|x| = -x$, so $f(x) = x(-x) = -x^2$. Thus our graph should consists of the part of the parabola $y = x^2$ for $x \geq 0$, and the parabola $y = -x^2$, for $x < 0$, giving us the graph on the right:



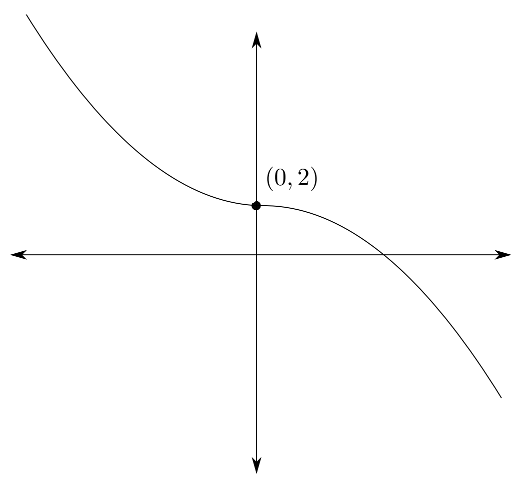
[4] (c) Sketch the graphs $y = f(x+1)$ and $y = -\frac{1}{2}f(x) + 2$.

Note: if your graph in part (b) is incorrect, credit will be awarded for correct transformations of an incorrect graph.



$y = f(x+1)$

This is a shift one unit to the left.



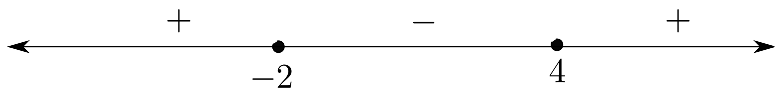
$y = -\frac{1}{2}f(x) + 2$

This is a reflecion across the x-axis, followed by a vertical scaling by 1/2, and a shift two units up.

5. Consider the function $f(x) = x^2 - 2x - 8$.

[3] (a) Determine the sign diagram for f.

We have $f(x) = x^2 - 2x - 8 = (x+2)(x-4)$, so f has zeros at $x = -2$ and $x = 4$. We check that for $x > 4$, $x + 2 > 0$ and $x - 4 > 0$, so $f(x) > 0$. For $-2 < x < 4$, $x + 2 > 0$ but $x - 4 < 0$, so $f(x) < 0$. For $x < -2$, $x + 2 < 0$ and $x - 4 < 0$, so $f(x) > 0$. This gives us the sign diagram as follows:



(b) Using your answer from part (a), solve the following:

[1] i. $f(x) = 0$.

From the sign diagram, we immediately see that $x^2 - 2x - 8 = 0$ when $x = -2$ or $x = 4$.

[1] ii. $f(x) \geq 0$.

Since the regions where $f(x) \geq 0$ correspond to the zeros and the intervals on the sign diagram with + signs, we immediately see that $f(x) \geq 0$ for $x \in (-\infty, -2] \cup [4, \infty)$.

[4] (c) Since f is a quadratic function, its graph is a parabola. Determine the location of the vertex of the parabola, as well as any x or y-intercepts. (You do not have to sketch the graph.)

We already have from part (b) that the x-intercepts are at $(-2, 0)$ and $(4, 0)$. Setting $x = 0$ gives us $f(0) = -8$, so the y-intercept is at $(0, -8)$. To find the vertex, we complete the square to put f in the "vertex form" as follows:

$$f(x) = x^2 - 2x - 8 = (x^2 - 2x + 1) - 1 - 8 = (x-1)^2 - 9.$$

From this, we can immediately conclude that the location of the vertex is the point $(1, -9)$.

[1] (d) What is the domain of the function $g(x) = \dfrac{1}{\sqrt{8 + 2x - x^2}}$?

In order for $g(x)$ to be defined, the denominator must be nonzero, and the polynomial under the square root cannot be negative. This leads to the condition $8 + 2x - x^2 > 0$, which is equivalent to $x^2 - 2x - 8 < 0$, or $f(x) < 0$, where f is our function from part (a). From the sign diagram, we can immediately conclude that $f(x) < 0$ for $x \in (-2, 4)$, so the domain of g is the open interval $(-2, 4)$.

Total Points: 40