University of Lethbridge

Department of Mathematics and Computer Science

15 December, 2017

MATH 1560 - Final Exam

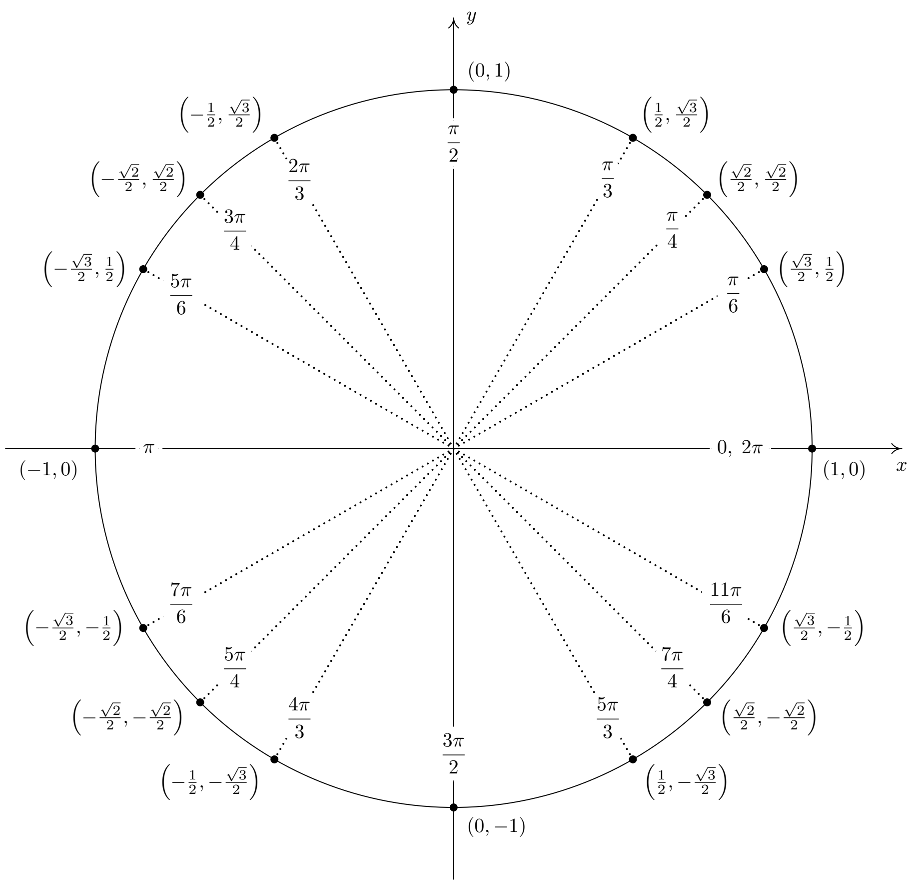
Examiner: Sean Fitzpatrick

Print your name and student number clearly in the space above. You may remove this cover page, and use the back for scrap paper. If you want any work on the back of this page to be graded, you must clearly indicate this on the page containing the corresponding question.

Answer the questions in the space provided. Show all work and necessary justification. Partial credit may be awarded for partially correct work.

No outside aids are permitted, with the exception of a basic calculator.

Extra space for rough work. And also, a unit circle.



 Total Points: 100

1. Evaluate the following limits:

[2] (a) $\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1}$

[2] (b) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

[2] (c) $\lim_{x\to 0} \frac{x}{\sin(3x)}$

[2] (d) $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$

[2] (e) $\lim_{x\to \infty} \frac{3x^2-x+2}{x^3+x+4}$

[5] 2. Show that

$$f(x) = \begin{cases} 2x^2 - 5, & \text{if } x \geq 2 \\ x + 1, & \text{if } x < 2 \end{cases}$$

is continuous at $x = 2$.

[5] 3. Using the **limit definition of the derivative**, find the slope of the tangent line to the graph $y = \dfrac{1}{x-2}$ at the point $(0, -1/2)$.

 Total Points: 100

4. Compute the derivative $f'(x)$ for each of the following functions:

[2] (a) $f(x) = 2x^4 - 5x^3 + 3x - \sqrt{x}$.

[2] (b) $f(x) = \dfrac{1 - x^5}{x^2}$.

[2] (c) $f(x) = x^3 \ln(x)$

[2] (d) $f(x) = \sin^3(x) + \sin(x^3)$

[2] (e) $f(x) = \arcsin(e^x)$

[6] 5. Using implicit differentiation, determine the equation of the tangent line to the curve

$$x^3y - 3xy^2 = x$$

at the point $(2, 1)$.

[4] 6. Compute the derivative of $f(x) = \ln\left(\dfrac{(2x+1)^6 e^{4x}}{\sqrt{1+x^4}}\right)$

[4] 7. Determine the absolute maximum and minimum values of $f(x) = x^3 - 3x$ on $[0, 2]$.

8. Find all critical numbers of $f(x) = x(x-5)^{2/3}$, and classify each one as a local maximum,
[4] local minimum, or neither.

9. Give an example of a continuous function f with a critical number c such that $f'(c)$ does not
[2] exist.

10. A function and its first and second derivatives are given by:

$$f(x) = \frac{1}{9}x(x-4)^3, \quad f'(x) = \frac{4}{9}(x-1)(x-4)^2, \quad f''(x) = \frac{4}{3}(x-2)(x-4).$$

[1] (a) What is the domain of f?

[1] (b) Give any intercepts or asymptotes in the graph of f.

[1] (c) What are the critical points of f?

[1] (d) On what intervals is f increasing/decreasing?

[1] (e) What are the inflection points of f?

[1] (f) On what intervals is f concave up/down?

[4] (g) Sketch the graph of f.

11. A rectangular rabbit enclosure needs to be built with an area of 200 m^2. The enclosure is being built next to a crocodile compound, which is already fenced. What is the minimum amount of fencing needed for the remaining three sides?

[5]

12. The height of a triangle is increasing at a rate of 3 cm/min. At some point in time, the base of the triangle is 12 cm long, and the height is 9 cm. At what rate must the base of the triangle be changing for the area to remain constant?

[5]

[3] 13. Use a linear approximation to estimate the value of $(1.2)^5$

14. Calculate the following Taylor polynomials:

[3] (a) The degree 5 Maclaurin polynomial for $f(x) = \sin(x)$.

[4] (b) The degree 3 Taylor polynomial for $f(x) = \dfrac{1}{x^2}$ about $a = 1$.

15. Compute the following indefinite integrals:

[2] (a) $\displaystyle\int \left(8x^3 - \frac{4}{x^2} + 2\right) dx$

[2] (b) $\displaystyle\int \left(\sec^2(x) + \frac{1}{x}\right) dx$

16. Evaluate the following definite integrals:

[3] (a) $\displaystyle\int_0^2 (x^2 - 2x)\, dx$

[3] (b) $\displaystyle\int_0^{\sqrt{3}} \frac{x}{\sqrt{x^2+1}}\, dx$

[4] 17. Use a Riemann sum with 4 rectangles and right endpoints to approximate the value of the integral $\int_0^2 (2x^2 - x)\,dx$.

18. Evaluate the indefinite integrals:

[3] (a) $\int x\sin(x^2)\,dx$

[3] (b) $\int \frac{x^3}{x^4+1}\,dx$

An extra problem for your entertainment and/or enlightenment, worth 5 bonus points.

19. Consider the function $F(x) = \int_1^x \frac{1}{t}\,dt$.

(a) Confirm that $F(1) = 0$ and $F'(x) = \frac{1}{x}$. (What well-known function satisfies these conditions?)

(b) Show that $\int_a^{ab} \frac{1}{t}\,dt = \int_1^b \frac{1}{t}\,dt$. (Hint: use the substitution $u = t/a$).

(c) Show that $F(ab) = F(a) + F(b)$ for $a, b > 0$.
(Use the definition above, not properties of well-known functions like $\ln(x)$. You will need a property of definite integrals and your result from part (b).)

(d) Show that for $a > 0$, $F(a^b) = bF(a)$. (Hint: try the substitution $u = t^{1/b}$, so $t = u^b$.)

Extra space for rough work.