Practice for Quiz 20
Math 2580
Spring 2016

Sean Fitzpatrick

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If you can answer the following problems, you should be well-prepared for Quiz 20:

1. Use Green's Theorem to evaluate the given line integral. Assume the orientation of the curve is positive, unless otherwise indicated.

   (a) $\int_C x^2y^2\,dx + 4xy^3\,dy$, where C is the triangle with vertices $(0,0)$, $(1,3)$, and $(0,3)$.

   (b) $\int_C xe^{-2x}\,dx + (x^4 + 2x^2y^2)\,dy$, where C is the boundary of the region between the circles $x^2+y^2=1$ and $x^2+y^2=4$.

   (c) $\int_C y^3\,dx - x^3\,dy$, where C is the circle $x^2+y^2=4$.

   (d) $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle y^2\cos x, x^2 + 2y\sin x\rangle$ and C is the trianglular path from $(0,0)$ to $(2,6)$ to $(2,0)$, and back to $(0,0)$.

2. Find a normal vector to the given parameterized surface at the given point:

   (a) $x = 2u$, $y = u^2+v$, $z = v^2$, at the point $(0,1,1)$.

   (b) $x = u^2 - v^2$, $y = u+v$, $z = u^2+4v$, at the point $(-\frac{1}{4}, \frac{1}{2}, 2)$.

   Note: a surface in $\mathbb{R}^3$ is parameterized by a map $\Phi(u,v) = (x(u,v), y(u,v), z(u,v))$. The derivative of this map is $D_{(u,v)}\Phi = \begin{bmatrix} x_u & x_v \\ y_u & y_v \\ z_u & z_v \end{bmatrix} = [\Phi_u|\Phi_v]$, where

$$\Phi_u(u,v) = \langle x_u(u,v), y_u(u,v), z_u(u,v)\rangle$$
$$\Phi_v(u,v) = \langle x_v(u,v), y_v(u,v), z_v(u,v)\rangle$$

   are the partial derivatives of Φ with respect to u and v, viewed as vectors. At a given point $\Phi(u_0,v_0)$ on the surface, the vectors $\Phi_u(u_0,v_0)$ and $\Phi_v(u_0,v_0)$ are both tangent to the surface. (Make sure you understand why!) Given two tangent vectors, you can use the cross product to get a normal vector.