Name:

Solve **one** of the following two questions:

1. Let $U = \{p \in \mathcal{P}_4(\mathbb{R}) : p(6) = 0\}$.

[2] (a) Find a basis for U. (Hint: note that $p(x) = x - 6$ is an element of U.)

[4] (b) Extend the basis in part (a) to a basis of $\mathcal{P}_4(\mathbb{R})$.

[4] (c) Find a subspace W of $\mathcal{P}_4(\mathbb{R})$ such that $U \oplus W = \mathcal{P}_4(\mathbb{R})$.

2. Suppose U and W are subspaces of V such that $V = U \oplus W$. Show that if $\{u_1, \ldots, u_m\}$ is
[10] a basis for U, and $\{w_1, \ldots, w_k\}$ is a basis for W, then

$$\{u_1, \ldots, u_m, w_1, \ldots, w_k\}$$

is a basis for V.

Total: 10 points