Reminders

The **ratio test** tells us that a series $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, diverges if this limit is greater than 1, and is inconclusive if the limit equals 1. For a power series $\sum a_n x_n$, this tells us that we need

$$\lim_{n\to\infty}\left|\frac{a_{n+1}x^{n+1}}{a_n x^n}\right| = |x| \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1,$$

which we can use to get the **radius of convergence**. To get the corresponding **interval of convergence**, you also need to test the endpoints of the interval to see if they need to be included. Often at one of the two endpoints you need to use the **alternating series test**: if $\sum_{n=1}^{\infty}(-1)^n a_n$ is an alternating series where $\{a_n\}$ is a positive **decreasing** sequence and $\lim_{n\to\infty} a_n = 0$, then this alternating series converges. Note that this tells us that while the harmonic series $\sum \frac{1}{n}$ diverges, the alternating series $\sum \frac{(-1)^n}{n}$ converges.

Another useful fact for alternating series is the **alternating series approximation theorem**: we have

$$\left|\sum_{n=1}^{\infty}(-1)^n a_n - \sum_{n=1}^{N}(-1)^n a_n\right| < a_{N+1}.$$

This can often be used to estimate the error in truncating an alternating power series, like we see for $\sin x$ or $\cos x$. Another approximation result is Taylor's Theorem. If we can recognize our power series as a Taylor series for some function $f(x)$, then we have

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n - \sum_{n=1}^{N}\frac{f^{(n)}(a)}{n!}(x-a)^n = R_N(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-c)^{n+1},$$

where c is some number in the interval of convergence for the Taylor series for $f(x)$.

Problems (from the textbook)

- Section 3.5, 25-28.
- Section 3.6, 6-18 (even), 26, 30, 32, 34.
- Section 3.8, 8, 10, 12, 18, 20, 22, 26, 28, 32.