Name: **Tutorial time:**

Problem you want feedback on:

Please complete all problems below.

1. Let $A = \begin{bmatrix} 1 & 5 & -3 \\ 2 & 3 & 4 \\ -2 & -7 & 3 \end{bmatrix}$

   (a) Compute $\det A$ using cofactor (Laplace) expansion along the row or column of your choice.

   (b) Compute $\det A$ by first using row operations to reduce A to triangular form. (Keep in mind that some row operations effect the value of $\det A$.)

(c) Use Cramer's rule to find the value of x in the solution to the following system of equations:

$$\begin{array}{rcrcrcr} x & + & 5y & - & 3z & = & 2 \\ 2x & + & 2y & + & 4z & = & -1 \\ -2x & - & 7y & + & 3z & = & 0 \end{array}$$

2. Let A be a 3×3 matrix such that $\det A = 4$. Compute the determinant of the following matrices:

(a) $B = EA$, where E is the elementary matrix $E = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) The matrix C obtained by switching rows 2 and 3 of A.

(c) The matrix $2A$.

3. With the help of your classmates, come up with as many answers as possible to fill in the blank below:

An $n \times n$ matrix A is invertible if and only if _____.

4. In each case, either prove the statement, or give an example showing that it is false:

   (a) $\det(A + B) = \det A + \det B$.

   (b) If $\det A = 0$, then A has two equal rows.

   (c) For any 2×2 matrix A, $\det(A^T) = \det A$.

   (d) $\det(-A) = -\det A$

   (e) If $\det A \neq 0$ and $AB = AC$, then $B = C$.

5. What can be said about $\det A$ if:

   (a) $A^2 = A$

   (b) $A^2 = I$

   (c) $PA = P$, where P is invertible.