Vectors, directional derivatives and the chain rule

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$$\begin{tikzcd} \mathbb{R}^n \arrow[r, bend left, "g"] \arrow[rr, bend right, "f\circ g"'] & \mathbb{R}^m \arrow[r, bend left, "f"] & \mathbb{R}^p \end{tikzcd},$$

we have the corresponding composition

$$\begin{tikzcd} T_{\mathbf{x}}\mathbb{R}^n \arrow[r, bend left, "\mathbf{D}g(\mathbf{x})"] \arrow[rr, bend right, "\mathbf{D}(f\circ g)(\mathbf{x})"'] & T_{\mathbf{y}}\mathbb{R}^m \arrow[r, bend left, "\mathbf{D}f(\mathbf{y})"] & T_{\mathbf{z}}\mathbb{R}^p \end{tikzcd}.$$

$$\begin{tikzcd} \mathbf{x} \in \mathbb{R}^n \arrow[r, "F"] \arrow[d, "g"] & \mathbf{y} \in \mathbb{R}^n \arrow[d, "f"] \\ g(\mathbf{x}) \in \mathbb{R} \arrow[r, equal] & f(\mathbf{y}) \in \mathbb{R} \end{tikzcd}.$$