University of Lethbridge
Department of Mathematics and Computer Science
MATH 1410 - Tutorial #10
Wednesday, March 28

1. Let $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 4 & 6 \\ -1 & 2 & 0 \end{bmatrix}$. (**Note:** you may need to do some work for this problem on scrap.)

(a) Compute $\det(A)$ by doing a cofactor expansion along a row or column (your choice).

(b) Perform the row operation $R_1 + 3R_3 \to R_1$ and compute the determinant of the resulting matrix by cofactor expansion along the first column.

(c) Perform the row operation $R_2 + 3R_1 \to R_2$ and compute the determinant of the resulting matrix by cofactor expansion along the third column.

(d) (Try but don't include your solution) Perform the *column operation* $C_2 + 2C_1 \to C_2$ and compute the determinant of the resulting matrix by cofactor expansion along the third row.

2. Compute the determinant of the matrix $A = \begin{bmatrix} -1 & 0 & 3 & 4 & 2 \\ 0 & 1 & 4 & -1 & 2 \\ 2 & 0 & -1 & 3 & 0 \\ 1 & 0 & -3 & -5 & -2 \\ 0 & 2 & 0 & 3 & 1 \end{bmatrix}$.

Hint: Row operations of the form $R_i + kR_j \to R_i$ do not change the value of the determinant. Once you have enough zeros in a column, expand.
(You might try creating two more zeros in column 1, or one more zero in column 2.)

3. Let A and B be 3×3 matrices, with $\det(A) = 2$ and $\det(B) = -3$. What is the value of:

   (a) $\det(AB^2)$

   (b) $\det(B^{-1}A^3B)$

   (c) $\det(2A^{-1}B)$

4. Consider the system

$$\begin{aligned} 2x \ + ay &= s \\ 3ax + 6y &= t \end{aligned}$$

   (a) For which values of a will the system have a unique solution? (Hint: use a determinant.)

   (b) Use Cramer's rule to solve the system (in terms of a, s, t) when possible, as determined by part (a).