University of Lethbridge
Department of Mathematics and Computer Science
16th November, 2015, 4:00 - 4:50 pm

MATH 1010A - Test #2

Last Name: **Solutions**

First Name: **The**

Student Number: ____________________

Tutorial Section: ____________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

No external aids are allowed, with the exception of a 5-function calculator.

For grader's use only:

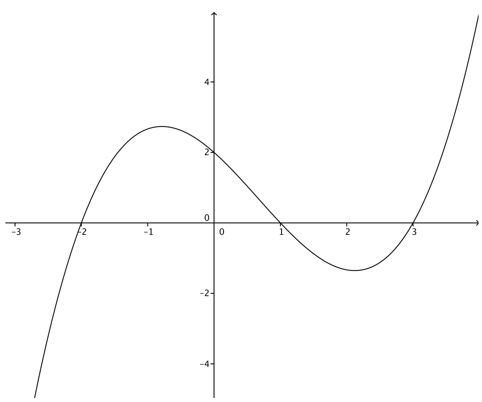
Problem	Grade
1	/8
2	/12
4	/10
5	/10
Total	/40

B

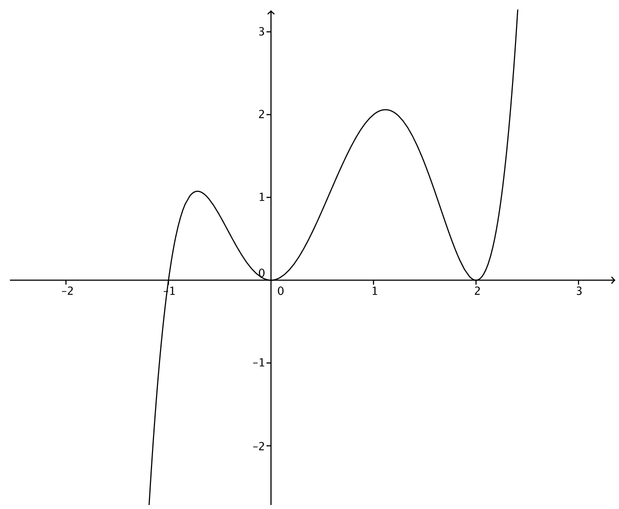
[8] 1. Match the following functions with their graphs below:

$$f(x) = x^2(x+1)(x-2)^2, \qquad h(x) = \frac{(x+2)(x-1)}{(x+1)(x-2)},$$

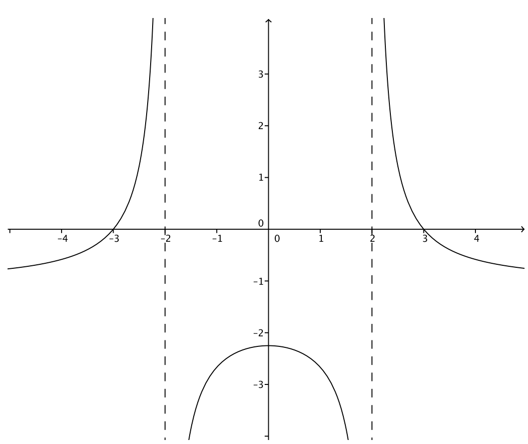
$$g(x) = \frac{1}{3}(x+2)(x-1)(x-3), \qquad k(x) = \frac{9-x^2}{x^2-4}$$



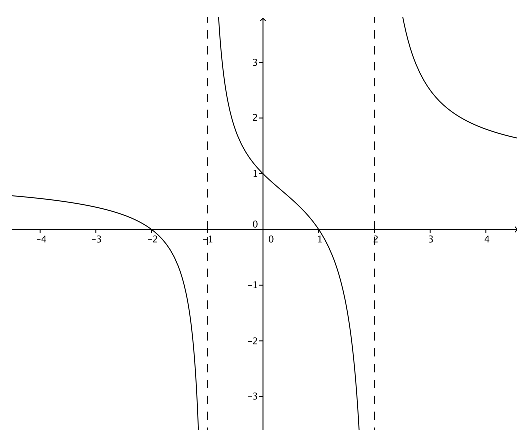
$g(x)$



$f(x)$



$k(x)$



$h(x)$

2. Let $f(x) = x^3 - 3x^2 - 6x + 8$.

[1] (a) Show that $x - 1$ is a factor of $f(x)$.

Since $f(1) = 1^3 - 3(1)^2 - 6(1) + 8 = 1 - 3 - 6 + 8 = 0$, we know that $x - 1$ is a factor of $f(x)$, by the factor theorem.

[4] (b) Find $a, b \in \mathbb{R}$ such that $f(x) = (x - 1)(x - a)(x - b)$.

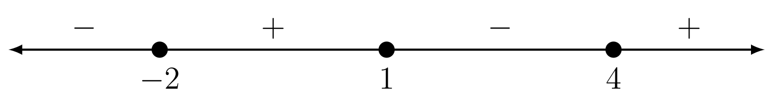
From (a), we know that $f(x) = (x-1)g(x)$ for some quadratic function $g(x)$. We can find $g(x)$ using long division, as follows:

$$
\begin{array}{r|rrrr}
 & x^2 & -2x & -8 & \\
\hline
x-1 & x^3 & -3x^2 & -6x & +8 \\
 & -x^3 & +x^2 & & \\
\hline
 & & -2x^2 & -6x & \\
 & & 2x^2 & -2x & \\
\hline
 & & & -8x & +8 \\
 & & & 8x & -8 \\
\hline
 & & & & 0
\end{array}
$$

It follows that $f(x) = (x-1)(x^2 - 2x - 8) = (x-1)(x-4)(x+2)$, so we can take $a = 4$ and $b = -2$.

[2] (c) Construct the sign diagram for $f(x)$.

From part (b), we see that f changes sign at $x = 4$, $x = 1$, and $x = -2$. Since $f(x) > 0$ for $x > 4$, we obtain:



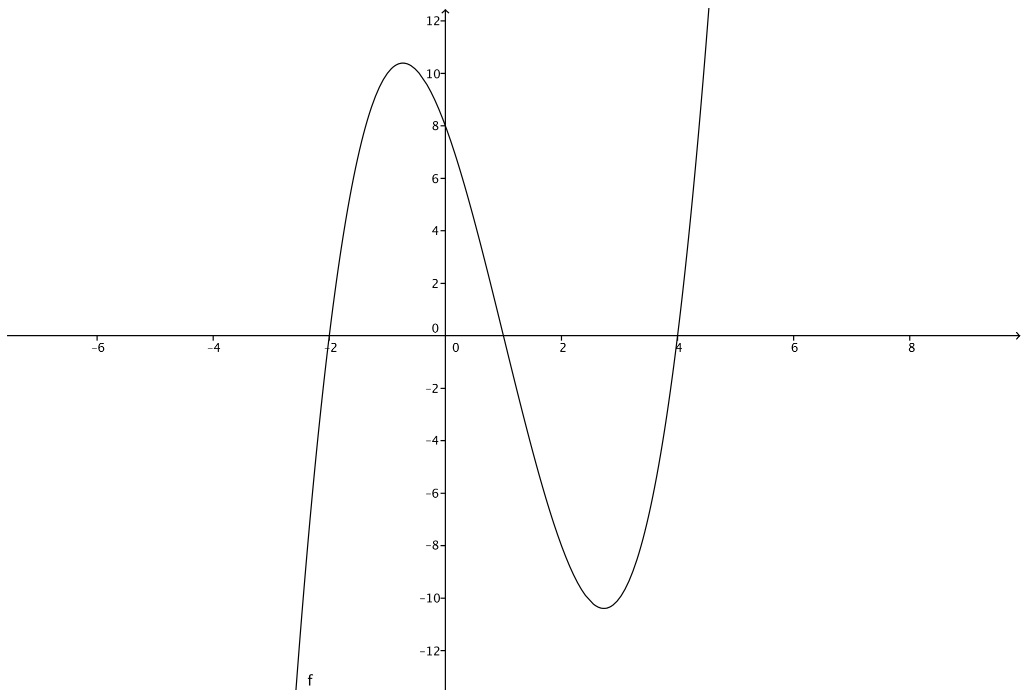
[2] (d) Solve the inequality $x^3 + 8 \geq 3x^2 + 6x$.

Rearranging the inequality gives us $x^3 - 3x^2 - 6x + 8 \geq 0$, so the solution to the inequality is given by the set of all x such that $f(x) \geq 0$. From the sign diagram in part (c), we see that the solution is therefore

$$x \in [-2, -1] \cup [4, \infty).$$

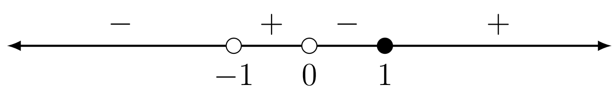
[3] (e) Sketch the graph of the function f from the previous page.

From the sign diagram on the previous page, we see that the graph $y = f(x)$ has x-intercepts $(-2, 0)$, $(1, 0)$, and $(4, 0)$. Since $f(0) = 8$, we see that $(0, 8)$ is the y-intercept. The graph of f is given as follows:

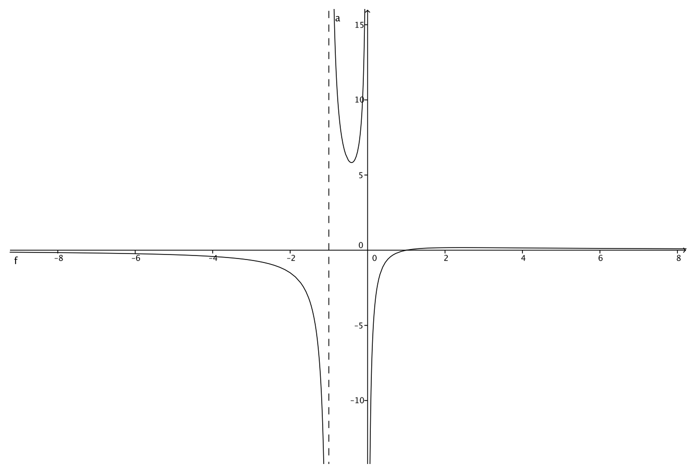


[6] 3. (a) Sketch the graph of $f(x) = \dfrac{x-1}{x^2+x}$.

Since $f(x) = \dfrac{x-1}{x(x+1)}$, we see that $f(0)$ is undefined, so there is no y-intercept, there is an x-intercept at $(1, 0)$ and the graph of f has vertical asymptotes $x = -1$ and $x = 0$. Since the degree of the denominator is less than the degree of the numerator, the graph $y = f(x)$ has $y = 0$ as a horizontal asymptote. The sign diagram of f is given by



From the sign diagram we can determine the behaviour of $f(x)$ near the vertical asymptotes, giving us the graph

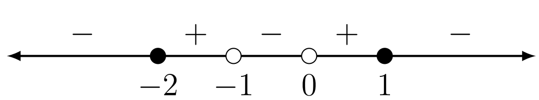


[4] (b) Solve the rational inequality $\dfrac{2}{x} - \dfrac{2}{x+1} \geq 1$.

(Note: this problem was taken directly off one of your WeBWorK assignments.)
To solve the inequality, we move everything to the left-hand side and get a common denominator. We have

$$\begin{aligned}\frac{2}{x} - \frac{2}{x+1} - 1 &= \frac{2(x+1) - 2x - 1(x)(x+1)}{x(x+1)} = \frac{2x+2-2x-x^2-x}{x(x+1)}\\ &= \frac{-x^2-x+2}{x(x+1)} = \frac{-(x+2)(x-1)}{x(x+1)},\end{aligned}$$

so the original inequality is equivalent to $g(x) \geq 0$, where $g(x) = \dfrac{-(x+2)(x-1)}{x(x+1)}$. The sign diagram for $g(x)$ is:



From the sign diagram, we see that $g(x) \geq 0$ for $x \in [-2, -1) \cup (0, 1]$.

[2] 4. (a) What are the values of $\sin\left(\frac{17\pi}{4}\right)$ and $\cos\left(\frac{-25\pi}{6}\right)$?

Since $\frac{17\pi}{4} = \frac{16\pi}{4} + \frac{\pi}{4} = 4\pi + \frac{\pi}{4}$ is in Quadrant I, we have $\sin\left(\frac{17\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

Since $-\frac{25\pi}{6} = -\frac{24\pi}{6} - \frac{\pi}{6} = -4\pi - \frac{\pi}{6}$ is in Quadrant IV, we have $\cos\left(-\frac{29\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

[4] (b) What is the value of $\cos\left(\frac{5\pi}{12}\right)$? (Hint: $5 = 9 - 4$)

Using the angle addition formula for cos, we have

$$\begin{aligned}
\cos\left(\frac{5\pi}{12}\right) &= \cos\left(\frac{9\pi}{12} - \frac{4\pi}{12}\right) = \cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) \\
&= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\
&= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{2}+\sqrt{6}}{4}.
\end{aligned}$$

Alternatively, one can use the half-angle formula for cos to obtain

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{1}{2}\cdot\frac{5\pi}{6}\right) = \sqrt{\frac{1+\cos(5\pi/6)}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}.$$

[4] (c) Verify the identity $\frac{1}{1-\sin\theta} = \sec^2\theta + \sec\theta\tan\theta$.

Beginning with the right-hand side (and taking care not to set the two sides equal to each other before we've verified that they are, in fact, equal), we have

$$\begin{aligned}
\sec^2\theta + \sec\theta\tan\theta &= \frac{1}{\cos^2\theta} + \frac{1}{\cos\theta}\left(\frac{\sin\theta}{\cos\theta}\right) \\
&= \frac{1}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta} \\
&= \frac{1+\sin\theta}{\cos^2\theta} \\
&= \frac{1+\sin\theta}{1-\sin^2\theta} \\
&= \frac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} \\
&= \frac{1}{1-\sin\theta},
\end{aligned}$$

giving us the left-hand side.

 Total Points: 40