1. Find an equation for the line tangent to the graph of the given function f at the point $(a, f(a))$:

   (a) $f(x) = (3x^2 + 2)\tan(x)$, $a = 0$.

   The prodcut rule gives us

   $$f'(x) = 6x\tan(x) + (3x^2 + 2)\sec^2(x).$$

   Thus,

   $$f'(0) = 6(0)\tan(0) + (3(0^2) + 2)\sec^2(0) = 0 + 2(1) = 2$$

   is the slope of the tangent line at the point $(0, f(0)) = (0, 0)$. The equation of the tangent line is therefore $y = 2x$.

   (b) $f(x) = \dfrac{x^2 - 2x + 3}{x^2 + 4}$, $a = 1$.

   Using the quotient rule, we have

   $$f'(x) = \frac{(2x - 2)(x^2 + 4) - (x^2 - 2x + 3)(2x)}{(x^2 + 4)^2}.$$

   When $x = 1$, this gives us the slope

   $$f'(1) = \frac{(2 - 2)(5) - (1 - 2 + 3)(2)}{5^2} = \frac{4}{25}.$$

   Since $f(1) = \frac{2}{5}$, we get the equation

   $$y - \frac{2}{5} = \frac{4}{25}(x - 1)$$

   for the tangent line. (Equivalently, this can be written as $4x - 25y = -6$.)

   (c) $f(x) = (x^4 + 2x)^5$, $a = -1$

   Using the Chain rule, we find

   $$f'(x) = 5(x^4 + 2x)^4 \frac{d}{dx}(x^4 + 2x) = 5(x^4 + 2x)^4(4x^3 + 2),$$

   so

   $$f'(-1) = 5(1 - 2)^4(-4 + 2) = 5(1)(-2) = -10$$

   is the slope of the tangent line, and when $x = -1$, $y = f(-1) = -1$, so our equation is $y + 1 = -10(x + 1)$, or $y = -10x - 11$.

2. Compute the derivative of $f(x) = \sin(2x)$:

   (a) Using the Chain Rule.

   We find $f'(x) = \cos(2x)\dfrac{d}{dx}(2x) = 2\cos(2x)$.

   (b) Using the identity $\sin(2x) = 2\sin(x)\cos(x)$.

   Using the constant and product rules,

$$f'(x) = 2\left(\frac{d}{dx}(\sin(x))(\cos(x)) + \sin(x)\frac{d}{dx}(\cos(x))\right) = 2(\cos^2(x) - \sin^2(x)).$$

   Do your answers in parts (a) and (b) agree?

   Yes, because $\cos(2x) = \cos^2(x) - \sin^2(x)$.

3. Given $f(x) = \tan(x)$, compute $f''(x)$ (also denoted $\dfrac{d^2}{dx^2}(\tan(x))$).

   The first derivative is given by $f'(x) = \sec^2(x) = (\sec(x))^2$. Applying the Chain Rule to this expression, we find

$$\begin{aligned} f''(x) &= \frac{d}{dx}(\sec(x))^2 \\ &= 2\sec(x)\frac{d}{dx}(\sec(x)) \\ &= 2\sec(x)(\sec(x)\tan(x)) \\ &= 2\sec^2(x)\tan(x). \end{aligned}$$

4. Discuss with your classmates, but don't hand in:

   Determine values of A and B such that the derivative of

$$f(x) = \begin{cases} Ax^2 + Bx + 2, & \text{if } x \le 2, \\ Bx^2 - A, & \text{if } x > 2 \end{cases}$$

   is everywhere continuous. (Hint: note that if $f'(x)$ exists, $f(x)$ itself must be continuous.)

   Note that continuity of $f(x)$ requires that $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$, which implies that $4A + 2B + 2 = 4B - A$.

   For $x < 2$, we have $f'(x) = 2Ax + B$, and for $x > 2$, we have $f'(x) = 2Bx$. If we want $f'(x)$ to be continuous at 2, we need $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$, so we must have $4A + B = 4B$. This second equation suggests $4A = 3B$, or $B = \frac{4}{3}A$. Plugging this into our earlier equation, we find

$$4A + 2\left(\frac{4}{3}A\right) + 2 = 4\left(\frac{4}{3}A\right) - A,$$

   which simplifies to $A = -\frac{6}{7}$, and thus $B = -\frac{8}{7}$.