Solutions to Quiz 10 Practice Problems
Math 2580
Spring 2016

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1. Find the following antiderivatives:

   (a) $\displaystyle\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$ (letting $u = 3x$)

   (b) $\displaystyle\int \cos(x)\,dx = \sin(x) + C$ (immediate)

   (c) $\displaystyle\int \frac{2x}{1+x^2}\,dx = \ln(1+x^2) + C$ (letting $u = 1 + x^2$)

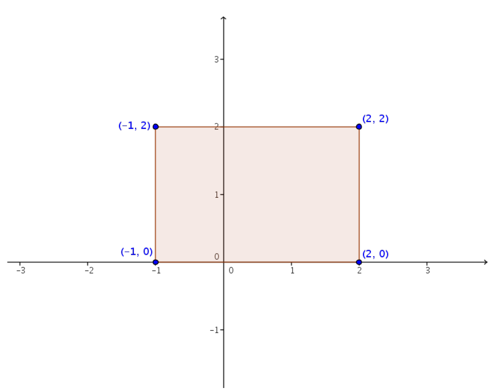
   (d) $\displaystyle\int x\sin(x)\,dx = -x\cos(x) + \sin(x) + C$ (by parts)

   (e) $\displaystyle\int \sin^2(x)\,dx = \int \frac{1}{2}(1-\cos(2x))\,dx = \frac{x}{2} - \frac{1}{4}\sin(2x) + C$

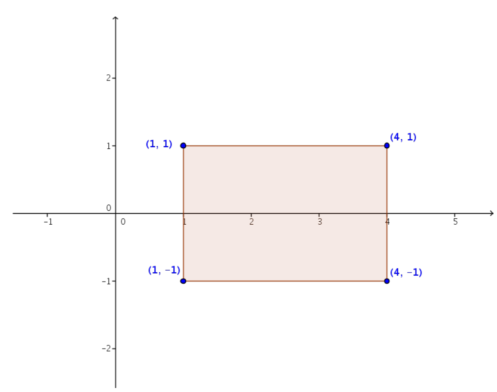
   (f) $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right) + C$

2. Sketch the following rectangles in $\mathbb{R}^2$:

   (a) $[-1,2]\times[0,2]$

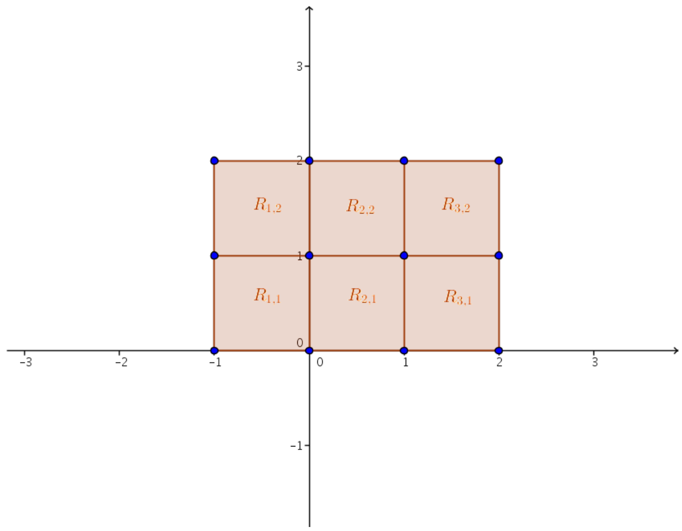


   (b) $[1,4]\times[-1,1]$

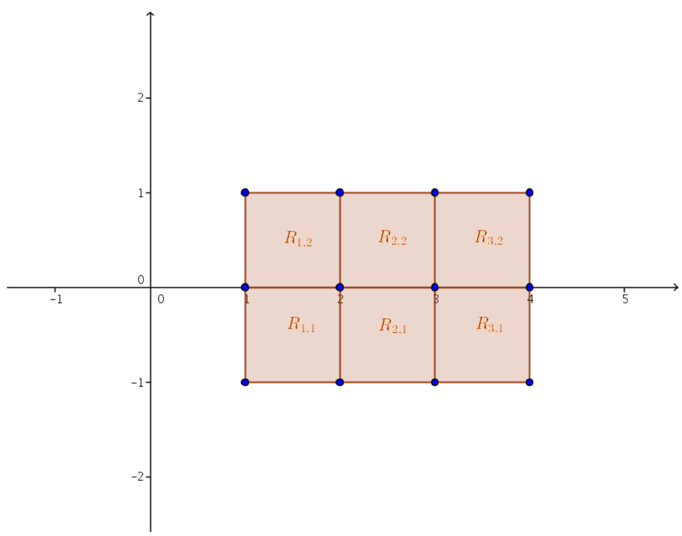


3. For each of the rectangles from Problem 2, determine uniform partitions $x_0 < x_1 < x_2 < x_3$ of the x-interval into three sub-intervals and $y_0 < y_1 < y_2$ of the y-interval into two sub-intervals. Use these partitions to divide the given rectangle into six sub-rectangles R_{ij}, with $1 \leq i \leq 3$ and $1 \leq j \leq 2$.

   (a) We let $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2$, and $y_0 = 0, y_1 = 1, y_2 = 2$. The corresponding rectangles are $R_{1,1} = [-1,0] \times [0,1], R_{1,2} = [-1,0] \times [1,2], R_{2,1} = [0,1] \times [0,1], R_{2,2} = [0,1] \times [1,2], R_{3,1} = [1,2] \times [0,1], R_{3,2} = [1,2] \times [1,2]$.

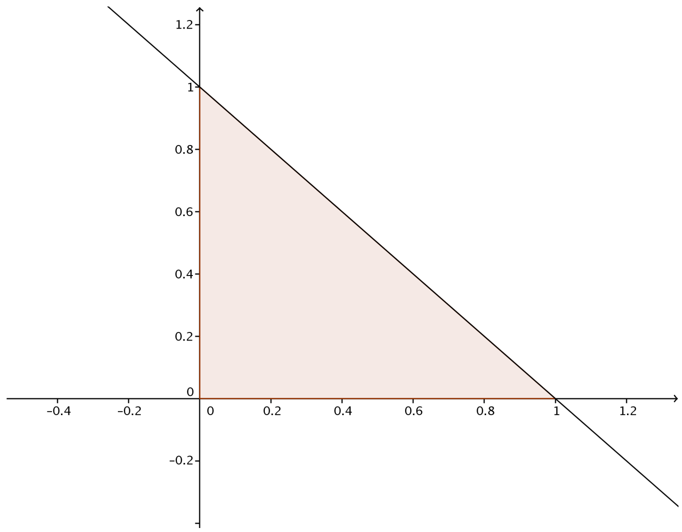


   (b) We let $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4$ and $y_0 = -1, y_1 = 0, y_1 = 1$. The corresponding rectangles are $R_{1,1} = [1,2] \times [-1,0], R_{1,2} = [1,2] \times [0,1], R_{2,1} = [2,3] \times [-1,0], R_{2,2} = [2,3] \times [-1,0], R_{3,1} = [3,4] \times [-1,0], R_{3,3} = [3,4] \times [0,1]$.



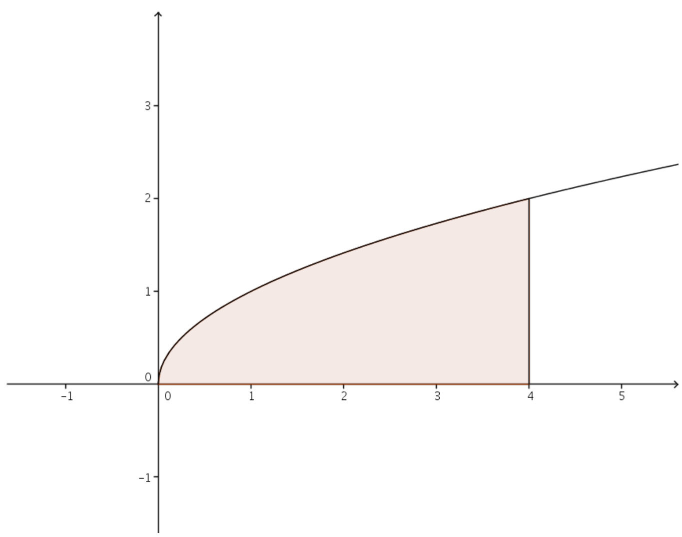
4. Sketch each of the subsets of $\mathbb{R}^2$ below and express them as both a Type 1 region and a Type 2 region:

   (a) The region bounded by the coordinate axes and the line $x + y = 1$.



   As a Type 1 region, it is given by the inequalities $0 \leq y \leq 1 - x$; $0 \leq x \leq 1$. As a Type 2 region, it is given by the inequalities $0 \leq x \leq 1 - y$; $0 \leq y \leq 1$.

   (b) The region bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 4$.



   As a Type 1 region, it is given by $0 \leq y \leq \sqrt{x}$; $0 \leq x \leq 4$. As a Type 2 region, it is given by $y^2 \leq x \leq 4$; $0 \leq y \leq 2$.