Name and student number: Solutions

1. Let $A = \{a, b, c\}$ and let $R = \{(a, a), (a, c), (b, b), (b, c), (c, a), (c, b)\}$ define a relation on A. Determine whether the following statements are true or false. Explain your answer.

[1] (a) For each $x \in A$, $x\,R\,x$.

This is false, since $c \in A$ but $(c, c) \notin R$.

[2] (b) For every $x, y \in A$, if $x\,R\,y$, then $y\,R\,x$.

This is true: we have both (a, c) and (c, a) as well as (b, c) and (c, b). The other two elements of R are (a, a) and (b, b), which are unchanged if we swap the the order. (If $a\,R\,a$, then $a\,R\,a$, etc. which is trivially true. It was enough to take note of the pairs where the two terms were different.)

[2] (c) For every $x, y, z \in A$, if $x\,R\,y$ and $y\,R\,z$, then $x\,R\,z$.

This is false, since (a, c) and (c, b) belong to R, but $(a, b) \notin R$.

[1] (d) The relation R defines a function from A to A.

This is false, since for example both (a, a) and (a, c) are elements of R, and for a function, a cannot be related to two different elements.

2. Let $A = \{a, b\}$, and consider the relations $R_1 = \{(a, a), (b, b)\}$ and $R_2 = \{(a, a), (a, b)\}$.
[4] Show that R_1 is an equivalence relation but R_2 is not. Is R_2 transitive?

R_1 is clearly reflexive, and it's trivially symmetric and transitive: there are no ordered pairs containing different elements of A, and it's a tautology that if $a\,R_1\,a$ then $a\,R_1\,a$, etc.

R_2 is not a equivalence relation since it's neither reflexive ($b \in A$ but $(b, b) \notin R$) nor symmetric ($(a, b) \in R_2$ but $(b, a) \notin R_2$). It's enough to point out just one of these two.

However, R_2 is transitive: we need to show that for all $x, y, z \in A$, if $(x, y) \in R_2$ and $(y, z) \in R_2$, then $(x, z) \in R_2$. The only possibility here for the "if" part is $x = a, y = a, z = b$ (since we need the second coordinate of the first pair to match the first coordinate of the second pair), and it's certainly true that if $(a, a) \in R_2$ and $(a, b) \in R_2$, then $(a, b) \in R_2$.

(Note that transitivity for R_1 follows by taking $x = y = z = a$ or $x = y = z = b$.)

Total: 10 points