List of potentially useful facts and definitions (you may remove this page)

Complex numbers

- Addition: $(a+ib)+(c+id)=(a+c)+i(b+d)$
- Multiplication: $(a+ib)(c+id)=(ac-db)+i(ad+bc)$
- Conjugate: $\overline{a+ib}=a-ib$
- Modulus: $|a+ib|=\sqrt{a^2+b^2}$. (Note $|z|^2=z\overline{z}$.)
- Reciprocals (for division): $\dfrac{1}{z}=\dfrac{\overline{z}}{z\overline{z}}=\dfrac{1}{|z|^2}\overline{z}$.
- Euler's formula: $\cos\theta+i\sin\theta=e^{i\theta}$. (Also written as $\operatorname{cis}\theta$.)
- Polar form: $a+ib=re^{i\theta}$, where $r=|a+ib|$ and $\tan\theta=\dfrac{b}{a}$.

Vectors

- Vector from $A=(a_1,a_2,a_3)$ to $B=(b_1,b_2,b_3)$:
$$\vec{v}=\overrightarrow{AB}=\langle b_1-a_1,b_2-a_2,b_3-a_3\rangle.$$
- Position vector for $P=(x,y,z)$: $\vec{p}=\overrightarrow{OP}=\langle x,y,z\rangle$.
- Vector addition:
$$\vec{v}+\vec{w}=\langle v_1,v_2,v_3\rangle+\langle w_1,w_2,w_3\rangle=\langle v_1+w_1,v_2+w_2,v_3+w_3\rangle.$$
- Scalar multiplication: $c\vec{v}=c\langle v_1,v_2,v_3\rangle=\langle cv_1,cv_2,cv_3\rangle$.
- Magnitude (length): $\|\vec{v}\|=\|\langle v_1,v_2,v_3\rangle\|=\sqrt{v_1^2+v_2^2+v_3^2}$.
- Dot product: $\langle v_1,v_2,v_3\rangle\cdot\langle w_1,w_2,w_3\rangle=v_1w_1+v_2w_2+v_3w_3$.
- Cross product: using the notation $\begin{vmatrix} a & b \\ c & d\end{vmatrix}=ad-bc$,
$$\langle v_1,v_2,v_3\rangle\times\langle w_1,w_2,w_3\rangle=\begin{vmatrix} v_2 & v_3 \\ w_2 & w_3\end{vmatrix}\hat{\imath}-\begin{vmatrix} v_1 & v_3 \\ w_1 & w_3\end{vmatrix}\hat{\jmath}+\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2\end{vmatrix}\hat{k}.$$
- Projection: $\operatorname{proj}_{\vec{u}}\vec{v}=\left(\dfrac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$.

Lines and Planes

- Line through $P_0=(x_0,y_0,z_0)$ in the direction of $\vec{v}=\langle a,b,c\rangle$:
$$\langle x,y,z\rangle=\langle x_0,y_0,z_0\rangle+t\langle a,b,c\rangle=\vec{p}_0+t\vec{v}.$$
- Plane through $P_0=(x_0,y_0,z_0)$ with normal vector $\vec{n}=\langle A,B,C\rangle$: $\vec{n}\cdot(\vec{p}-\vec{p}_0)=A(x-x_0)+B(y-y_0)+C(z-z_0)=0$.

The Unit Circle