1. Compute the following indefinite integrals (antiderivatives):

(a)

$$\begin{aligned}\int \frac{1-\sin^2 x}{\cos x}\,dx &= \int \frac{\cos^2(x)}{\cos(x)}\,dx \\ &= \int \cos(x)\,dx = \sin(x) + C.\end{aligned}$$

(b)

$$\begin{aligned}\int \frac{e^x+1}{e^x}\,dx &= \int \left(\frac{e^x}{e^x} + \frac{1}{e^x}\right)\,dx \\ &= \int (1+e^{-x})\,dx \\ &= x - e^{-x} + C.\end{aligned}$$

Note: for the second term, we relied on the observation that the derivative of e^{-x} is $-e^{-x}$. Without that observation, one can make the substitution $u=-x$, so $dx=-du$, and $\int e^{-x}\,dx = \int e^u(-du) = -e^u + C = e^{-x} + C$.

(c) $\displaystyle\int xe^{x^2}\,dx$

Here, we make the substitution $u = x^2$, so $du = 2x\,dx$, or $x\,dx = \frac{1}{2}\,du$. Thus,

$$\int xe^{x^2}\,dx = \int \frac{1}{2}e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C.$$

(d) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

If we make the substitution $u=\sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}\,dx$, so $\frac{1}{\sqrt{x}}\,dx = 2\,du$. Thus,

$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \int \sin(u)(2\,du) = -2\cos(u) + C = -2\cos(\sqrt{x}) + C.$$

2. Evaluate the following definite integrals. (In some cases there may be shortcuts...)

   (a) $\int_3^3 x^3 e^{\sin(x)}\sqrt{1+x^2}\,dx = 0,$
   since the upper and lower limits of integration are the same.

   (b) $\int_{-3}^3 x\cos(x^4+1)\,dx = 0,$
   since $f(x) = x\cos(x^4+1)$ satisfies

   $$f(-x) = (-x)\cos((-x)^4+1) = -x\cos(x^4+1) = -f(x),$$

   meaning that $f(x)$ is an odd function. Since the limits of integration with respect to x are symmetric, the integral is zero.

   (c) $\int_0^3 (x^2-3x+1)\,dx$

   Using the power rule for antiderivatives, we have

   $$\int_0^3 (x^2-3x+1)\,dx = \left(\frac{x^3}{3} - 3\frac{x^2}{2} + x\right)\Bigg|_0^3 = \frac{27}{3} - \frac{27}{2} + 3 = 0 = -\frac{3}{2}.$$

   (d) $\int_1^9 x\sqrt{x^2+1}\,dx$

   If we let $u = x^2+1$, then $du = 2x\,dx$, so $x\,dx = \frac{1}{2}\,du$. When $x=1$, $u = 1^2+1 = 2$, and when $x = 9$, $u = 9^1+1 = 82$. Thus, we have

   $$\int_1^9 x\sqrt{x^2+1}\,dx = \int_2^{82} \sqrt{u}\frac{1}{2}\,du = \frac{1}{3}u^{3/2}\Bigg|_2^{82} = \frac{82^{3/2}-2^{3/2}}{3}.$$

   (e) $\int_0^1 e^x\sin(e^x)\,dx$

   With $u = e^x$, we have $du = e^x\,dx$, and when $x = 0$, $u = 1$, and when $x = 1$, $u = e$. Thus,

   $$\int_0^1 e^x\sin(e^x)\,dx = \int_1^e \sin(u)\,du = (-\cos(u))|_1^e = \cos(1) - \cos(e).$$