Math 1565 Tutorial #8 Solutions

[4] 1. Find the absolute maximum and minimum of $f(x) = 3x^{2/3} - 2x$ on $[-1, 2]$.

Solution: We first check the endpoints:

$$f(-1) = 3(1) - 2(-1) = 5, \text{ and } f(2) = 3(2^{2/3}) - 2(2) \approx 0.762$$

Next, we find

$$f'(x) = 2x^{-1/3} - 2 = \frac{2(1 - x^{1/3})}{x^{1/3}},$$

so $f'(x) = 0$ when $x = 1$, and $f'(x)$ is undefined when $x = 0$. Both points are in the domain of f (and the given interval), so both are critical numbers. The critical values are

$$f(0) = 0 \text{ and } f(1) = 1.$$

Comparing values, we see that the absolute minimum is $f(0) = 0$, and the absolute maximum is $f(-1) = 5$.

[2] 2. Use the Mean Value Theorem to show that for any $a, b \in \mathbb{R}$,

$$|\sin(b) - \sin(a)| \leq |b - a|.$$

Solution: Consider $f(x) = \sin(x)$, and choose any two real numbers a and b. We can assume $a < b$ since the result holds when $a = b$ (since $0 \leq 0$), and if $a > b$ we can simply reverse the roles of a and b. We know that f is continuous on $[a, b]$ and differentiable on (a, b), so by the Mean Value Theorem, there exists some $c \in (a, b)$ such that

$$\sin(b) - \sin(a) = \cos(c)(b - a),$$

using the fact that $f'(x) = \cos(x)$. If we take the absolute value of both sides of this equation, we get

$$|\sin(b) - \sin(a)| = |\cos(c)||b - a| \leq 1|b - a|,$$

since $|\cos(c)| \leq 1$, which is what we needed to show.

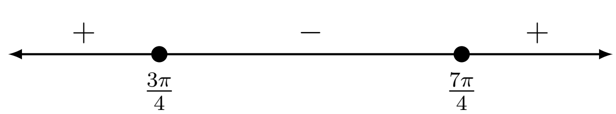
[4] 3. Find and classify the critical points of $f(x) = e^x \sin(x)$ for $x \in [0, 2\pi]$

Solution: Using the product rule, we find

$$f'(x) = e^x \sin(x) + e^x \cos(x) = e^x(\sin(x) + \cos(x)).$$

Since $e^x \neq 0$ for all $x \in \mathbb{R}$, the critical points must occur when $\sin(x) + \cos(x) = 0$, or equivalently, when $\tan(x) = -1$.

For $x \in [0, 2\pi]$, this is satisfied when $x = 3\pi/4$ and $x = 7\pi/4$. Choosing appropriate test values (0, π, and 2π are good choices), we find that the sign diagram is given by



Using the First Derivative Test, we see that $\left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{3\pi/4}\right)$ is a local maximum, and $\left(\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}e^{7\pi/4}\right)$ is a local minimum.