Name: Solutions

Choose **one** of the following two problems:

1. Prove the following statement: For all nonzero real numbers x and y, if x is a rational number and y is an irrational number, then $\dfrac{x}{y}$ is an irrational number.

   Proof. Suppose to the contrary that x is rational and y is irrational, but $\dfrac{x}{y}$ is rational. Since $x \neq 0$, $\dfrac{y}{x}$ is defined, and is rational, since it's equal to $\dfrac{1}{x/y}$. But if x and $\dfrac{y}{x}$ are both rational, then
   $$y = x\left(\frac{y}{x}\right)$$
   is rational, contradicting the assumption that y is irrational. Thus, it must be the case that $\dfrac{x}{y}$ is rational. $\square$

2. Prove the following statement: For all real numbers x, either $x+\sqrt{2}$ is irrational, or $-x+\sqrt{2}$ is irrational.

   Proof. Suppose to the contrary that $x+\sqrt{2}$ and $-x+\sqrt{2}$ are both rational. If this is the case, then it's also true that $a = \frac{1}{2}(x+\sqrt{2})$ and $b = \frac{1}{2}(-x+\sqrt{2})$ are rational.

   Since the sum of two rational numbers is rational, this implies that
   $$a+b = \frac{x}{2}+\frac{\sqrt{2}}{2}-\frac{x}{2}+\frac{\sqrt{2}}{2} = \sqrt{2}$$
   is rational, which contradicts the fact that $\sqrt{2}$ is irrational. Since we arrived at our contradiction by assuming that both $x+\sqrt{2}$ and $-x+\sqrt{2}$ are rational, it must be the case that at least one of them is irrational. $\square$

   Note: for #2, if $P(x)$ is the predicate "$x+\sqrt{2}$ is irrational" and $Q(x)$ is the predicate "$-x+\sqrt{2}$ is irrational", then we are trying to prove $P(x) \vee Q(x)$, and we start our proof by contradiction by assuming the negation, which is $\neg P(x) \wedge \neg Q(x)$, by one of de Morgan's laws.

Total: 10 points