1. Compute the derivatives of the following functions:

[2] (a) $f(x) = e^x \cos(x)$

$$\begin{aligned} f'(x) &= \left(\frac{d}{dx}e^x\right)\cos(x) + e^x\left(\frac{(}{d}dx\cos(x)\right) \\ &= e^x\cos(x) - e^x\sin(x). \end{aligned}$$

[2] (b) $g(x) = \dfrac{\sin(x)}{x^2+1}$

$$\begin{aligned} g'(x) &= \frac{\left(\frac{d}{dx}\sin(x)\right)(x^2+1) - \sin(x)\left(\frac{d}{dx}(x^2+1)\right)}{(x^2+1)^2} \\ &= \frac{\cos(x)(x^2+1) - 2x\sin(x)}{(x^2+1)^2}. \end{aligned}$$

[2] (c) $h(x) = \tan^3(x)$

$$\begin{aligned} h'(x) &= 3\tan^2(x)\frac{d}{dx}(\tan(x)) \\ &= 3\tan^2(x)\sec^2(x). \end{aligned}$$

[2] (d) $r(x) = (x^2+1)^x$

Using the fact that $e^{\ln y} = y$ for any $y$, we write $r(x) = e^{\ln(x^2+1)^x} = e^{x\ln(x^2+1)}$. The Chain Rule then gives us

$$\begin{aligned} r'(x) &= e^{x\ln(x^2+1)}\frac{d}{dx}(x\ln(x^2+1)) \\ &= (x^2+1)^x\left((1)\ln(x^2+1) + x\left(\frac{2x}{x^2+1}\right)\right). \end{aligned}$$

[3] 2. Compute the derivative of $f(x) = \ln\left(\sqrt[3]{\dfrac{x^2(x-3)^3}{(x^4+4x)(2x-1)^4}}\right)$.

(Hint: there is an easy way and a hard way.)

Using properties of logarithms, we have

$$f(x) = \frac{1}{3}\left(2\ln(x) + 3\ln(x-3) - \ln(x^4+4x) - 4\ln(2x-1)\right).$$

Thus,

$$f'(x) = \frac{1}{3}\left(\frac{2}{x} + \frac{3}{x-3} - \frac{4x^3+4}{x^4+4x} - \frac{4(2)}{2x-1}\right).$$

[3] 3. Find the equation of the tangent line to the curve $(x^2+y^2)^2 = 4xy$ at the point $(1,1)$. (Suggestion: use implicit differentiation.)

Computing the derivative of both sides of the given equation with respect to $x$ (and assuming that $y$ is defined implicitly as a function of $x$), we have

$$\begin{aligned}
\frac{d}{dx}((x^2+y^2)^2) &= \frac{d}{dx}(4xy)\\
2(x^2+y^2)\frac{d}{dx}(x^2+y^2) &= 4(1)y + 4x\frac{dy}{dx}\\
2(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) &= 4y + 4x\frac{dy}{dx}\\
\frac{dy}{dx}(4y(x^2+y^2) - 4x) &= 4y - 4x(x^2+y^2)\\
\frac{dy}{dx} &= \frac{4y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - 4x}.
\end{aligned}$$

When $x = 1$ and $y = 1$, we find $\frac{dy}{dx} = \frac{4-4-4}{4+4-4} = -1$. The equation of the tangent line is therefore

$$y - 1 = -1(x-1).$$

[1] 4. Write down an example of a function that is continuous everywhere, but not differentiable everywhere. (Just give the funciton. You don't have to show that it's a valid example.)

There are many examples, but the standard one is $f(x) = |x|$. This function is continuous at all points, including $x = 0$, but $f'(0)$ is undefined. (Note that $f'(x) = -1$ for $x < 0$ and $f'(x) = +1$ for $x > 0$.)