Math 4310 Assignment #8
University of Lethbridge, Fall 2014

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Due date: Friday, October 31st, by 5 pm.

Please submit solutions to the following problems (except where indicated). As usual, all the textbook exercises are recommended as practice problems if they don't appear as an assigned problem below.

1. Prove that any finite subset of a topological space is compact.

2. Let X be a set and let $\mathcal{T}_1, \mathcal{T}_2$ be two topologies on X, such that $\mathcal{T}_1 \subseteq \mathcal{T}_2$.

   (a) Prove that if $(X, \mathcal{T}_2)$ is compact, then $(X, \mathcal{T}_1)$ is compact.

   (b) Prove that if $(X, \mathcal{T}_1)$ is Hausdorff and $(X, \mathcal{T}_2)$ is compact, then $\mathcal{T}_1 = \mathcal{T}_2$.

3. Prove that if $\{A_\alpha\}$ is any collection of compact subsets of a Hausdorff space X, then $\bigcap_\alpha A_\alpha$ is compact.

4. Prove that if Y is compact, then the projection $\pi_X : X \times Y \to X$ is a closed map.

5. Prove the following theorem: Let Y be a compact Hausdorff space, and let $f : X \to Y$ be a map. Then f is continuous if and only if the graph of f, $\Gamma_f = \{(x, f(x)) : x \in X\}$ is closed in $X \times Y$.

   Hint: If Γ_f is closed and V is a neighbourhood of $f(x_0)$ in Y, then the intersection of Γ_f and $X \times (Y \setminus V)$ is closed. Now apply the previous problem.