Practice for Quiz 3
Math 2580
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Not on the quiz, but have a look at Figure 14.R.1 on p. 761 (Ch. 14 Review Exercises) in the Marsden-Weinstein text. It's a sketch of several level surfaces for the function $f(x, y, z) = x^2 + y^2 - z^2$. (This is what I was hoping to animate but it turns out this causes my software to choke.)

If you can answer the following problems, you should be well-prepared for Quiz 3:

1. Express the partial derivative $f_y(x, y, z)$ as a limit.
2. Calculate all four second-order partial derivatives for the function $f(x, y) = \cos(xy^2)$. Verify that the mixed second-order partial derivatives are equal.
3. Give a convincing (but not rigorous – no $\epsilon - \delta$) argument that
$$\lim_{(x,y)\to(0,0)} \frac{x^2y + y^3}{x^2 + y^2} = 0.$$
4. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$ does not exist.

   (Hint: find a curve through the origin such that the limit is not zero if you approach the origin along that curve.)
5. Find the equation of the tangent plane to the graph $z = x^3 + y^3 - 6xy$ at the point $(1, 2, -3)$.

6. In one variable we define the derivative by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Suppose $f : D \subseteq \mathbb{R}^n \to \mathbb{R}$ is a function of n variables. To simplify notation, let us use the vector $\mathbf{x}$ to stand for the point $(x_1, x_2, \ldots, x_n)$, and let the vector $\mathbf{h}$ represent the point $(h_1, h_2, \ldots, h_n)$. (The added bonus of using vectors is that $\mathbf{x} + \mathbf{h}$ makes sense!)

We might be tempted to generalize the definition of the derivative to n variables by writing a limit of the form

$$f'(\mathbf{x}) = \lim_{\mathbf{h} \to \mathbf{0}} \frac{f(\mathbf{x}+\mathbf{h}) - f(\mathbf{x})}{\mathbf{h}}.$$

What are two problems with attempting such a definition?

Hint: one problem is with the denominator – does that even make sense? For the other problem, think about taking partial derivatives. How often does the derivative with respect to x equal the derivative with respect to y? With that in mind, how likely is it that the above limit exists?