Math 2000 Tutorial Worksheet

September 15, 2015

For this week's tutorial, begin by discussing what it means for a sentence to be a "statement" in mathematics, and make sure that you can identify the hypothesis and conclusion in a conditional statement. Under what conditions is a conditional statement true? When is it false?

If you're confident that you understand the above, then you should try some of the following problems with your classmates:

1. (Section 1.1 #5) Let P be the statement "Student X passed every assignment in Calculus I", and let Q be the statement "Student X received a grade of C or better in Calculus I".

   (a) What does it mean for P to be true? What does it mean for Q to be true?

   (b) Suppose that Student X passed every assignment in Calculus I and received a grade of B-, and that the instructor made the statement $P \rightarrow Q$. Would you say that the instructor lied, or told the truth?

   (c) Suppose that Student X passed every assignment in Calculus I and received a grade of C-, and that the instructor made the statement $P \rightarrow Q$. Would you say that the instructor lied, or told the truth?

   (d) Now suppose that Student X failed two assignments in Calculus I and received a grade of D, and that the instructor made the statement $P \rightarrow Q$. Would you say that the instructor lied, or told the truth?

2. (Section 1.1 #7) The following is the statement of a theorem that can be proved using the quadratic formula. (You do not need to know how the theorem is proved.)

   Theorem: Let a, b, and c be real numbers. If f is a quadratic function of the form $f(x) = ax^2 + bx + c$ and $ac < 0$, then the graph of f has two x-intercepts.

   Using **only** this theorem, what can you conclude about the graphs of the following functions?

   (a) $g(x) = -8x^2 + 5x - 2$ (c) $k(x) = 8x^2 - 5x - 7$ (e) $f(x) = -4x^2 - 3x + 7$

   (b) $h(x) = -\frac{1}{3}x^2 + 3x$ (d) $j(x) = -\frac{77}{91}x^2 + 210$ (f) $F(x) = -x^4 + x^3 + 9$

3. (Section 1.2 #2) For each of the following statements, first construct a "know-show table" (two-column proof) and then write a formal proof in paragraph form:

   (a) If x is an even integer and y is an even integer, then $x + y$ is an even integer.

   (b) If x is an even integer and y is an odd integer, then $x + y$ is an odd integer.

   (c) If x is an odd integer and y is an odd integer, then $x + y$ is an odd integer.

4. (Section 1.2 #8) Is the following statement true or false?

$$\text{If } a \text{ and } b \text{ are nonnegative real numbers and } a + b = 0, \text{ then } a = 0.$$

   Either give a counterexample to show that it is false, or give the outline of a proof to argue that it is true.