University of Lethbridge
Department of Mathematics and Computer Science
MATH 1410 - Tutorial #6
Wednesday, February 28

Student #1 :_____

Student #2 :_____

Student #3 :_____

Student #4 :_____

(Moodle ID not required.)

Additional practice: (**do not submit**).

1. Use Gaussian elimination to find the reduced row-echelon form of the matrix:

(a) $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 & 2 & -1 \\ -2 & 1 & 3 & 4 \\ -1 & 4 & 5 & 3 \end{bmatrix}$

2. Solve the system of equations:

(a) $\begin{aligned} 2x - 3y &= 7 \\ -x + 2y &= 2 \end{aligned}$

(b) $\begin{aligned} x - 2y + 4z &= 2 \\ 2x - 3y + z &= -2 \\ -x + 2y - 2z &= 6 \end{aligned}$

1. For each system of equations below, write down the corresponding augmented matrix.

(a)
$$\begin{aligned} 2x - 3y + z &= 2 \\ 2y - 5z &= -3 \\ -3x + 2z &= 7 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 4x_2 - 7x_4 &= 0 \\ -3x_1 - x_2 + 4x_3 &= 2 \\ 2x_2 - 4x_3 + x_4 &= -5 \end{aligned}$$

2. For each augmented matrix below, write down a corresponding system of equations using whatever variables you prefer.

(a) $\left[\begin{array}{ccc|c} 2 & -1 & 0 & 4 \\ -3 & 4 & 1 & -2 \\ 0 & 2 & 3 & -7 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 3 & 2 & 0 & 1 & -5 \\ 0 & 4 & 2 & -7 & 2 \end{array}\right]$

3. State whether or not the given augmented matrix is in reduced row-echelon form (RREF), and if not, why.

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $\left[\begin{array}{cccc|c} 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

4. Suppose you want to perform Gaussian elimination on the augmented matrices below. For each matrix, what are the first two row operations you would perform, and why?

(a) $\left[\begin{array}{ccc|c} 1 & -4 & 2 & 0 \\ -2 & 4 & 1 & 6 \\ 3 & 2 & -1 & 1 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 2 & 4 & -8 & 10 \\ -1 & 2 & 4 & -5 \\ 0 & 1 & 5 & 2 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 3 & 2 & -7 & 4 \\ 1 & 2 & -4 & 0 \\ 0 & -1 & 3 & 2 \end{array}\right]$

5. For each matrix A and B below, write down the row operation that transforms A into B.

(a) $A = \begin{bmatrix} 3 & -2 & 5 \\ 2 & 8 & -4 \\ 1 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 7 & -3 \\ 6 & 8 & 1 \\ 1 & 12 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 7 & -3 \\ 0 & -13 & 10 \\ 1 & 12 & -6 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & 4 \\ -5 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 6 & 0 \\ 1 & 3 & 4 \\ 4 & -2 & 3 \end{bmatrix}$

6. Write down the augmented matrix of the following system, and then use Gaussian elimination to solve the system.

$$\begin{aligned} x \ &+ 2y - \ z \ = \ 4 \\ -x + \ y \ &- 2z = -1 \\ 2x \ &+ 6y - 3z = \ 5 \end{aligned}$$

7. A system in variables x, y, z has an augmented matrix with RREF $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 4 \\ 0 & 1 & 2 & 6 \end{array}\right]$.

   Write down the system of equations corresponding to this matrix. How would you describe the solution to the system?

   (Hint: what geometric problem corresponds to a system of two equations in three variables?)