University of Lethbridge
Department of Mathematics and Computer Science
Today's Date, 2015, Current Time

MATH 1010 - PRACTICE EXAM

Last Name:____

First Name:____

Student Number:____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

1. Evaluate the following limits:

[2] (a) $\lim_{x \to 0} \dfrac{x^2 + x}{x^3 - 4x^2 + 5x + 43}$

[2] (b) $\lim_{x \to 1} \dfrac{x^2 - 2x + 1}{x - 1}$

[3] (c) $\lim_{x \to 0} \dfrac{\tan 5x}{x}$

[3] (d) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + 4x}}{3x + 2}$

[4] 2. Determine the value of a such that the function

$$f(x) = \begin{cases} 4x - 3, & \text{if } x \le a \\ x^2 + 2, & \text{if } x > a \end{cases}$$

is continuous.

[4] 3. Using only the **definition of the derivative**, compute $f'(2)$ if $f(x) = x^2 + 3$.

4. Compute the derivatives of the following functions:

[2] (a) $f(x) = 4x^5 - 7x^3 - 4x^2 + 2015$

[2] (b) $g(x) = x^3 \sin(x)$

[3] (c) $h(x) = \dfrac{x^3 + x}{\sqrt{x}}$

[2] (d) $k(x) = \sin(2x)$

[3] (e) $F(x) = \dfrac{\tan x}{x \sec x}$

[5] 5. Find the global (absolute) maximum and minumum values of $f(x) = x + \dfrac{1}{x}$ on the interval $[1, 5]$.

[5] 6. Find and classify the critical points of the function $g(x) = x^3 + 5x^2 - 11$.

[4] 7. Determine the function $f(x)$ such that $f'(x) = x^3 - \sqrt{x}$ for all $x \geq 0$, and $f(0) = 3$.

[3] 8. (a) Verify the trigonometric identity $\dfrac{1}{1+\sin(x)} = \sec^2(x) - \sec(x)\tan(x)$

[3] (b) Find a function $f(x)$ such that $f'(x) = \dfrac{1}{1+\sin(x)}$

9. Consider the function $f(x) = x^3 - 3x^2 + 4$.

[4] (a) Given that $a = 2$ is a zero of multiplicity 2, find the remaining real zero of f.

[2] (b) Construct the sign diagram for f.

[2] (c) Solve the inequality $x^3 \geq 3x^2 - 4$.

[2] (d) Give a rough sketch of the graph of f based on your sign diagram from part (b).

10. Consider again the function $f(x) = x^3 - 3x^2 + 4$ from the previous page.

[2] (a) Calculate $f'(x)$.

[2] (b) Construct the sign diagram for $f'(x)$.

[3] (c) Calculate $f''(x)$, and construct the sign diagram for $f''(x)$.

[5] (d) Use the information in parts (b) and (c) to determine the location of any local maxima, local minima, and inflection points in the graph of $f(x)$. Then use this information to produce a more accurate version of your sketch from the previous page.

11. Consider the function $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$.

[3] (a) Construct the sign diagram for $f(x)$.

[3] (b) Determine any vertical and horizontal asymptotes for the graph $y = f(x)$.

[4] (c) Draw a rough sketch of the graph of f using the information in parts (a) and (b).

12. Consider again the function $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$.

[3] (a) Calculate $f'(x)$.

[2] (b) The function f has one critical point. Using the sign diagram for $f'(x)$, find and classify this critical point, and find the corresponding critical value.

[3] (c) Using your answer in (b), redraw your sketch from the previous page so that it accurately reflects the information above.

13. Solve the following inequalities:

[3] (a) $|2x - 7| \leq 8$.

[5] (b) $\dfrac{2x+17}{x+1} > x + 5$.

14. Find the exact values of the trigonometric functions below at the given angles:

[2] (a) $\cos(9\pi/4)$

[2] (b) $\sin(41\pi/6)$

[3] (c) $\cot(-23\pi/3)$

[3] (d) $\sin(\theta)$, if the angle θ lies in Quadrant IV and $\cos(\theta) = 12/13$.

[4] 15. Suppose that θ is a Quadrant I angle. Show that $\cos(\theta/2) = \sqrt{\dfrac{1+\cos\theta}{2}}$.

Hint: $\cos(\theta) = \cos(\theta/2 + \theta/2)$.

[2] 16. Use your result in part (a) to evaluate $\cos(\pi/8)$.

[4] 17. Show that for any angles α and β,

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin(\alpha)\cos(\beta).$$

Some true stuff from the course that you possibly didn't remember

- We say $x = a$ is a zero of **multiplicity** k for a polynomial $p(x)$ if $(x-a)^k$ is a factor of $p(x)$, but $(x-a)^{k+1}$ is not.
- The **Factor Theorem** states that for a polynomial function $p(x)$, $p(a) = 0$ if and only if $(x-a)$ is a factor of p.
- Values of $\sin\theta$ and $\cos\theta$ in the first quadrant:

$$
\begin{aligned}
\sin 0 &= 0 & \cos 0 &= 1 \\
\sin \pi/6 &= 1/2 & \cos \pi/6 &= \sqrt{3}/2 \\
\sin \pi/4 &= \sqrt{2}/2 & \cos \pi/4 &= \sqrt{2}/2 \\
\sin \pi/3 &= \sqrt{3}/2 & \cos \pi/3 &= 1/2 \\
\sin \pi/2 &= 1 & \cos \pi/2 &= 0
\end{aligned}
$$

- Fundamental identities:
  1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\csc\theta = \dfrac{1}{\sin\theta}$
  2. $\cos^2\theta + \sin^2\theta = 1$
  3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
  4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
  5. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
  6. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- Obvious but occasionally forgotten facts that are sometimes useful in conjunction with some of the identities above:
  1. $2\theta = \theta + \theta$
  2. $\theta = \dfrac{\theta}{2} + \dfrac{\theta}{2}$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$.
- $f'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$.
- $\dfrac{d}{dx}(x^n) = nx^{n-1}$, $\dfrac{d}{dx}(\sin x) = \cos x$, and $\dfrac{d}{dx}(\cos x) = -\sin x$.
- $(fg)' = f'g + fg'$ and $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$.
- A **critical point** for a function f is a number c such that $f'(c) = 0$ (or doesn't exist); $f(c)$ is the corresponding critical value.
- If you spend all your time reading this page, you won't have time to complete the exam.