University of Lethbridge
Department of Mathematics and Computer Science
6^{th} October 2014, 2:00-2:50 pm
MATH 3500 - Test #1

Last Name:________________________________

First Name:________________________________

Student Number:________________________________

For full credit you must answer three of the four problems on this test. If you include work for all four problems, make sure that you clearly indicate which three problems you wish to have graded. If you solve all four problems and do not indicate which problems should be graded, only the first three problems will be graded.

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Problem	Grade
1	/10
2	/10
3	/10
4	/10
Total	/30

1. (a) Find the supremum and infimum of each of the following sets, or explain why they do not exist.

[2] i. $A = \{x \in \mathbb{R} : |x - 2| < 1\}$

[2] ii. $B = \left\{(-1)^n \left(1 + \frac{1}{n}\right) : n \in \mathbb{N}\right\}$

[2] iii. $C = \{x \in \mathbb{R} : x^2 < 0\}$

[4] (b) Let $x \in \mathbb{R}$ and set $A = \{q \in \mathbb{Q} : q < x\}$. Prove that $x = \sup A$.

2. Let $B = \left\{ \dfrac{(-1)^n n}{n+1} : n \in \mathbb{N} \right\}$.

   [3] (a) What are the limit points of B? (You should explain your answer but a formal proof is not required.)

   [2] (b) Is B open, closed, both, or neither? Explain.

[2] (c) Which points of B are isolated points? Why?

[3] (d) What are the closure, interior, and boundary of$B$?

3. (a) Define what it means for a set $x \in \mathbb{R}$ to be a *limit point* of a set $A \subseteq \mathbb{R}$. (A limit
[3] point is also known as an accumulation point.)

(b) Prove that if a set $K \subseteq \mathbb{R}$ is compact, than any subset $E \subseteq K$ with infinitely many
[7] elements has a limit point that belongs to K.

[4] 4. (a) Prove that $\lim_{n\to\infty} \dfrac{n}{n+1} = 1$ using the definition of convergence.

(b) Let (a_n) be the sequence defined by $a_1 = 1$ and $a_{n+1} = \sqrt{2a_n + 2}$ for $n \geq 1$. Using induction, one can prove that for all $n \in \mathbb{N}$ we have $a_n \leq a_{n+1} < 3$. Explain why we
[3] can conclude that (a_n) converges, and find the limit of the sequence.

[3] (c) Prove that if (a_n) and (b_n) are Cauchy sequences, then so is $c_n = |a_n - b_n|$.
Hint: triangle inequality.

 Total Points: 30