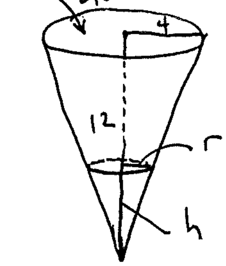
Tutorial 10 Solutions.

1. A water container in the shape of an inverted cylindrical cone is being filled with water at a rate of 10 ml per minute. If the height of the cone is 12 cm and the base radius is 4 cm, at what rate is the level of water rising when the water is 3 cm deep? [5]

$\frac{dV}{dt} = 10 \text{ cm}^3/\text{min}$



Volume of water given by

$V = \frac{1}{3}\pi r^2 h$.

Similar triangles (similar cones?) give us

$$\frac{h}{r} = \frac{12}{4} = 3 \Rightarrow h = 3r \text{ or } r = \frac{1}{3}h$$

$$\therefore V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi}{27}h^3$$

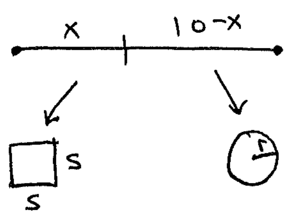
$$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}.$$

$$\Rightarrow \frac{dh}{dt} = \frac{9}{\pi h^2}\frac{dV}{dt}$$

when $h = 3$,

$$\frac{dh}{dt} = \frac{9}{\pi(3)^2}(10) = \frac{10}{\pi} \text{ cm/min.}$$

2. A piece of wire 10 cm long is cut into two pieces. The first piece is bent into a square, and the second into a circle. At what point (if any) should the wire be cut so that the combined area of the square and circle is a minimum? At what point (if any) should it be cut so that the area is a maximum? [5]



$2\pi r = 10 - x$

$\Rightarrow r = \frac{10-x}{2\pi}$

$4s = x \Rightarrow s = \frac{x}{4}$

$$A = s^2 + \pi r^2$$

$$\Rightarrow A(x) = \frac{x^2}{16} + \frac{(10-x)^2}{4\pi}$$

where $0 \le x \le 10$.

End point values:

$x = 0$ (all circle)

$$A(0) = \frac{10^2}{4\pi} = \frac{25}{\pi}$$

$x = 10$ (all square)

$$A(10) = \frac{10^2}{16} = \frac{25}{4}$$

Critical values:

$$A'(x) = \frac{x}{8} + \frac{10-x}{2\pi}(-1) = \frac{\pi x + 4x - 40}{8\pi}$$

$A'(x) = 0$ if $x = \frac{40}{\pi+4}$

$$A\left(\frac{40}{\pi+4}\right) = \frac{25}{\pi+4}.$$

after some arithmetic

$\frac{25}{\pi} > \frac{25}{4} > \frac{25}{\pi+4}$

so $A(0) = 25/\pi$ IS MAX

$A\left(\frac{40}{\pi+4}\right) = 25/(\pi+4)$ IS MIN