1. Evaluate the following limits:

[3] (a) $\lim_{x\to 3} \dfrac{x^2-5x+6}{x^2-4x+3}$

$$\begin{aligned}\lim_{x\to 3} \frac{x^2-5x+6}{x^2-4x+3} &= \lim_{x\to 3} \frac{(x-2)(x-3)}{(x-1)(x-3)}\\ &= \lim_{x\to 3} \frac{x-2}{x-1}\\ &= \frac{3-2}{3-1} = \frac{1}{2}.\end{aligned}$$

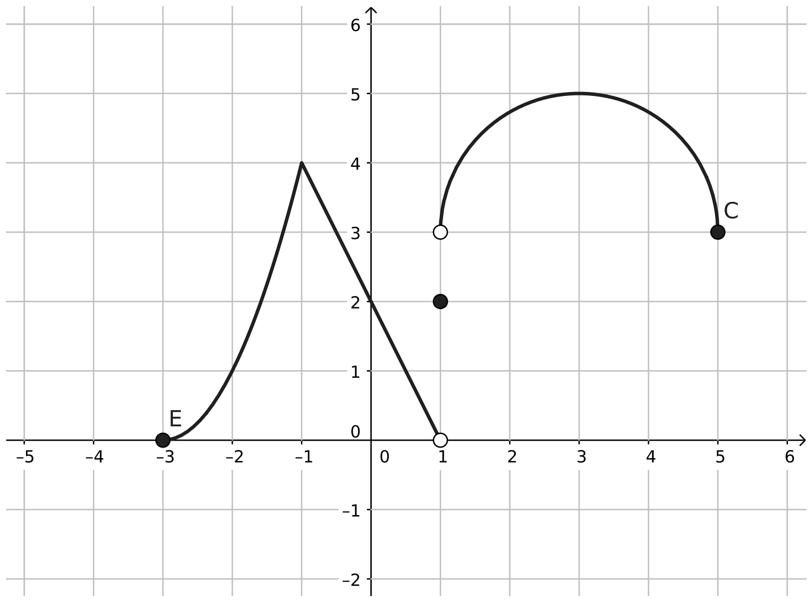
[3] (b) $\lim_{x\to 0} \dfrac{\sin(5x)}{x}$

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(5x)}{x} &= \lim_{x\to 0} 5\left(\frac{\sin(5x)}{5x}\right)\\ &= 5\lim_{x\to 0} \frac{\sin(5x)}{5x}\\ &= 5(1) = 1.\end{aligned}$$

[3] (c) $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{x^2+x}\right)$

$$\begin{aligned}\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{x^2+x}\right) &= \lim_{x\to 0} \left(\frac{(x+1)-1}{x(x+1)}\right)\\ &= \lim_{x\to 0} \frac{x}{x(x+1)}\\ &= \lim_{x\to 0} \frac{1}{x+1}\\ &= \frac{1}{0+1} = 1.\end{aligned}$$

2. The graph of a function f is given below:



[1] (a) What is the domain of f? $\text{dom}(f) = [-3, 5]$.

[1] (b) $\lim_{x \to -1^-} f(x) = \underline{\quad 4 \quad}$ and $\lim_{x \to -1^+} f(x) = \underline{\quad 4 \quad}$

[1] (c) $\lim_{x \to 1^-} f(x) = \underline{\quad 0 \quad}$ and $\lim_{x \to 1^+} f(x) = \underline{\quad 3 \quad}$

[1] (d) On what interval(s) is f continuous? $[-3, 1)$ and $(1, 5]$.

[2] 3. What are the horizontal and vertical asymptotes (if any) of $f(x) = \dfrac{\sqrt{x^2+1}}{x-1}$?

There is a vertical asymptote at $x = 1$ since $1 - 1 = 0$ but $\sqrt{1^2 + 1} = \sqrt{2} \neq 0$.
We note that

$$\frac{\sqrt{x^2+1}}{x-1} = \frac{\sqrt{x^2(1+1/x^2)}}{x(1-1/x)} = \frac{|x|\sqrt{1+1/x^2}}{x(1-1/x)}.$$

For $x > 0$, we have $|x| = x$ and $f(x) = \dfrac{\sqrt{1+1/x^2}}{1-1/x} \to 1$ as $x \to \infty$.

For $x < 0$, we have $|x| = -x$ and $f(x) = -\dfrac{\sqrt{1+1/x^2}}{1-1/x} \to -1$ as $x \to \infty$.

Thus, $y = 1$ and $y = -1$ are both horizontal asymptotes.