Name and student number: Solutions

[5] 1. Let A and B be subsets of some universal set U. Prove that if $A \subseteq B$, then $B^c \subseteq A^c$.

Solution: By definition, $A \subseteq B$ if and only if for all $x \in U$, $x \in A \to x \in B$. But this is if and only if for all all $x \in U$, $x \notin B \to x \notin A$, by taking the contrapositive. This in turn tells us that for all $x \in U$, $x \in B^c \to x \in A^c$ by definition of the complement of a set, and therefore $B^c \subseteq A^c$, by definition of the subset relation.

Total: 10 points

[5] 2. Prove the following assertion, or give a counterexample to show that it is false:

For any subsets A, B, C, and D of some universal set U, if $A \subseteq C$ and $B \subseteq D$, and $A \cap B = \emptyset$, then $C \cap D = \emptyset$. (Here, $\emptyset$ denotes the empty set.)

Solution: Consider the case where $A = \{1\}$, $B = \{2\}$, and $C = D = \{1, 2\}$. Then $A \subseteq C$, $B \subseteq D$, and $A \cap B = \emptyset$, but $C \cap D = \{1, 2\} \neq \emptyset$.

Total: 10 points