1. Calculate the four 4th roots of the complex number $z = -2\sqrt{3} + 2i$.

   Writing z in polar form, we have $z = 4\left(-\dfrac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 4e^{i(5\pi/6)}$. If w is a 4th root of z, then $w^4 = z$. Writing $w = re^{i\theta}$, we have $w^4 = r^4 e^{i(4\theta)} = 4e^{i(5\pi/6)} = z$.
   Comparing these two numbers, we have $r^4 = 4$, so $r = \sqrt[4]{4} = \sqrt{2}$, and (since we can add any multiple of 2π to the argument $5\pi/6$ without changing the value of z) $4\theta = \dfrac{5\pi}{6}, \dfrac{17\pi}{6}, \dfrac{29\pi}{6}, \dfrac{41\pi}{6}, \ldots$
   Dividing by 4, we get the values $\theta = \dfrac{5\pi}{24}, \dfrac{17\pi}{24}, \dfrac{29\pi}{24}, \dfrac{41\pi}{24}$, (the next value is $\dfrac{53\pi}{24} = \dfrac{5\pi}{24} + 2\pi$, and things continue repeat from there) so the four roots are

   $$w_0 = \sqrt{2}e^{i(5\pi/24)}, w_1 = \sqrt{2}e^{i(17\pi/24)}, w_2 = \sqrt{2}e^{i(29\pi/24)}, \text{ and } w_3 = \sqrt{2}e^{i(41\pi/24)}.$$

2. Let $P = (1, 0, -2)$, $Q = (-3, 2, 4)$, and $R = (0, 5, -1)$ be points in $\mathbb{R}^3$.

   (a) Calculate the vectors $\vec{u} = \overrightarrow{PQ}$, $\vec{v} = \overrightarrow{QR}$, and $\vec{w} = \overrightarrow{PR}$.

   We have

   $$\begin{aligned} \vec{u} &= \langle -3-1, 2-0, 4-(-2)\rangle = \langle -4, 2, 6\rangle \\ \vec{v} &= \langle 0-(-3), 5-2, -1-4\rangle = \langle 3, 3, -5\rangle \\ \vec{w} &= \langle 0-1, 5-0, -1-(-2)\rangle = \langle -1, 5, 1\rangle. \end{aligned}$$

   (b) Check that $\vec{u} + \vec{v} = \vec{w}$.

   $$\vec{u} + \vec{v} = \langle -4, 2, 6\rangle + \langle 3, 3, -5\rangle = \langle -1, 5, 1\rangle = \vec{w}.$$

   (c) Explain, with a diagram, why your result in part (b) makes sense. (You do not have to accurately plot the points P, Q, R.)

   Any diagram showing the three points, labelled P, Q, R, and the vectors between them, will do. The point is to notice that the vector from P to R is the same as the vector obtained by applying the "tip-to-tail" rule for adding vectors. The vector $\vec{u} + \vec{v}$ also gets us from P to R, but takes a detour through the point Q along the way.

3. Let $\vec{a} = \langle 2, -4, 3\rangle$, $\vec{b} = \langle -5, 2, 7\rangle$, and $\vec{c} = \langle 1, 0, -3\rangle$. Calculate the following:

   (a) $4\vec{a} - 3\vec{b}$

$$4\vec{a} - 3\vec{b} = 4\langle 2, -4, 3\rangle - 3\langle -5, 2, 7\rangle = \langle 8, -16, 12\rangle + \langle 15, -6, -21\rangle = \langle 23, -22, -9\rangle.$$

   (b) $\|3\vec{c}\|$

   We have $3\vec{c} = 3\langle 1, 0, -3\rangle = \langle 3, 0, -9\rangle$, so

$$\|3\vec{c}\| = \|\langle 3, 0, -9\rangle\| = \sqrt{3^2 + 0^2 + (-9)^2} = \sqrt{90}.$$

   (c) $3\|\vec{c}\|$

   We have $\|\vec{c}\| = \sqrt{1^2 + 0^2 + (-3)^3} = \sqrt{10}$, so $3\|\vec{c}\| = 3\sqrt{10}$. Note that this is the same value as the previous answer: $\sqrt{90} = \sqrt{9 \cdot 10} = 3\sqrt{10}$.

   (d) $\vec{a} \cdot (2\vec{b} - \vec{c})$

   Since $2\vec{b} - \vec{c} = \langle -10, 4, 14\rangle - \langle 1, 0, -3\rangle = \langle -11, 4, 17\rangle$, we have

$$\vec{a} \cdot (2\vec{b} - \vec{c}) = \langle 2, -4, 3\rangle \cdot \langle -11, 4, 11\rangle = 2(-11) - 4(4) + 3(17) = 13.$$

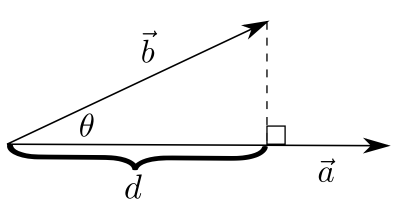
   (e) $2(\vec{a} \cdot \vec{b}) - \vec{a} \cdot \vec{c}$

   We have

$$\begin{aligned} \vec{a} \cdot \vec{b} &= 2(-5) - 4(2) + 3(7) = 3 \text{ and} \\ \vec{a} \cdot \vec{c} &= 2(1) - 4(0) + 3(-3) = -7, \end{aligned}$$

   so $2(\vec{a} \cdot \vec{b}) - \vec{a} \cdot \vec{c} = 2(3) - (-7) = 13$, which is the same as the previous value.

4. Referring to the diagram below, argue that the indicated distance d is given by $d = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|}$.



Since $\|\vec{b}\|$ is the length of the hypotenuse of the right-angled triangle shown, we have $\cos\theta = \dfrac{d}{\|b\|}$, and thus $d = \|\vec{b}\|\cos\theta$. We also know that $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$, so $\cos\theta = (\vec{a} \cdot \vec{b})/(\|\vec{a}\|\|\vec{b}\|)$. Thus,

$$d = \|\vec{b}\|\cos\theta = \|\vec{b}\|\left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}\right) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|},$$

as required.