University of Lethbridge
Department of Mathematics and Computer Science
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MATH 2560A - Practice Exam

Last Name:______________________________

First Name:______________________________

Student Number:______________________________

Tutorial Section: ______________________________

All problems on this exam are taken from the course text, with textbook references provided for each problem. (A text reference of the form (2.3.15) means Problem 15 from Section 2.3.) I've chosen odd-numbered problems so that you can look up the answers in the back. If you want help with a solution, you can use the online forum or drop by the help session on Sunday.

Keep in mind that this is my version of what a Math 2560 exam might look like based on the topics provided by Dr. Connolloy: it may bear no resemblance to the real thing. (This is also what a Math 2560 exam might look like if assembled in 30 minutes by someone with far less time than they'd like to have.)

1. Evaluate the following immediate integrals:

   (a) (1.1.5) $\displaystyle\int x(x^2+1)^8\,dx =$

   (b) (1.1.7) $\displaystyle\int \frac{1}{2x+7}\,dx =$

   (c) (1.1.11) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

   (d) (1.1.15) $\displaystyle\int \sin^2(x)\cos(x)\,dx =$

   (e) (1.1.19) $\displaystyle\int \tan^2(x)\sec^2(x)\,dx =$

   (f) (1.1.25) $\displaystyle\int e^{x^3}x^2\,dx =$

   (g) (1.1.27) $\displaystyle\int \frac{e^x+1}{e^x}\,dx =$

   (h) (1.1.33) $\displaystyle\int \frac{\ln(x^3)}{x}\,dx =$

   (i) (1.1.77) $\displaystyle\int_2^6 x\sqrt{x-2}\,dx =$

   (j) (1.1.81) $\displaystyle\int_{-1}^1 \frac{1}{1+x^2}\,dx =$

2. (2.2. 13) Compute the volume of the solid of revolution obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 0$ about:

   (a) The x-axis.

   (b) The line $x = 2$.

3. (2.4.31) Find the area of the surface generated by revolving $y = x^3$, for $0 \leq x \leq 1$, about the x-axis.

4. State whether the given series converges or diverges. If possible, find its sum.

(a) (3.2.21) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{5^n}$

(b) (3.2.23) $\displaystyle\sum_{n=1}^{\infty} n^{-4}$.

(c) (3.2.25) $\displaystyle\sum_{n=1}^{\infty} \frac{10}{n!}$

(d) (3.2.29) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$.

(e) (3.2.37) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$.

5. Determine whether the series converges or diverges:

   (a) (3.3.17) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$

   (b) (3.3.7) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

   (c) (3.3.31) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^2+17}$

   (d) (3.3.37) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n+n}.$

6. Determine the convergence of the following series:

   (a) (3.4.7) $\displaystyle\sum_{n=1}^{\infty} \frac{n!10^n}{(2n)!}$

   (b) (3.4.11) $\displaystyle\sum_{n=1}^{\infty} \frac{10 \cdot 5^n}{7^n - 3}$.

   (c) (3.5.19) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

7. Determine the radius and interval of convergence for the following power series:

   (a) (3.6.11) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n}$

   (b) (3.6.19) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}(x-5)^n$

   (c) (3.6.31) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

8. Find a formula for the requested Taylor series:

   (a) (3.8.7) $f(x) = \cos x$, expanded about $a = \pi/2$.

   (b) (3.8.11) $f(x) = \dfrac{x}{x+1}$, expanded about $a = 1$.

9. (3.8.25) Using known Taylor series, determine the Taylor series of the function $f(x) = \cos(x^2)$.

10. (3.8.31) Use the first 4 terms of a Taylor series to approximate the value of the integral $\displaystyle\int_0^{\sqrt{\pi}} \sin(x^2)\, dx$.

11. Solve the following differential equations:

    (a) (4.4.7) $\dfrac{dy}{dx} = \dfrac{y^2 + 1}{x^2 + 1}$, where $y(0) = 1$.

    (b) (4.4.9) $y' = xe^{-y}$, where $y(0) = 1$.

    (c) (4.5.3) $y' + 3x^2y = \sin(x)e^{-x^2}$, where $y(0) = 1$.