# Practice for Quiz 5
# Math 2580
# Spring 2016

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If you can answer the following problems, you should be well-prepared for Quiz 5:

1. Let $f(x,y) = xy$ and suppose $x = g(t)$ and $y = h(t)$. Show that applying the chain rule to the derivative $\dfrac{d}{dt}f(g(t),h(t)) = \dfrac{d}{dt}(g(t)h(t))$ produces the product rule for derivatives in one variable.

2. Suppose an insect is flying through a room along the path
$$r(t) = (e^t, t^2, \sin t),$$
and that the temperature in the room is given by $T(x,y,z) = \sin(x)\cos(y)\sqrt{z}$. Find the rate $\dfrac{dT}{dt}$ at which the temperature experienced by the insect changes as it flies through the room.

3. Consider the function $F : D \subseteq \mathbb{R}^2 \to \mathbb{R}^2$, where $D = \{(u,v) | v \geq 1\}$, given by
$$F(u,v) = \left(\sqrt[3]{uv}, \sqrt[3]{\frac{u}{v^2}}\right).$$
Calculate the derivative matrix $D_{(u,v)}f$ at a general point $(u,v) \in D$.

   *Suggestion:* When computing partial derivatives, you could go ahead and work with the functions as given, using the chain rule and product/quotient rules, or you could write, for example, $\sqrt[3]{uv} = u^{1/3}v^{1/3}$. (Always choose the most convenient possible form for your functions before differentiating!)

4. Let $g(x,y) = xy^2\cos(xy)$, where $x = \sqrt[3]{uv}$ and $y = \sqrt[3]{\dfrac{u}{v^2}}$. Compute $\dfrac{\partial g}{\partial u}$ and $\dfrac{\partial g}{\partial v}$ using the Chain Rule.

   (Note that $(x,y) = F(u,v)$, where $F$ is the function from the previous problem.)

5. What is the **gradient** of a continuously differentiable function $f : \mathbb{R}^3 \to \mathbb{R}$? How is the gradient of $f$ related to the derivative $D_{(x,y,z)}f$?

6. Calculate the gradient of the function $f(x,y) = 3x^2 - 4xy$ at the point $(1,2)$.