The problems on this worksheet are for in-class practice during tutorial. You are free to collaborate and to ask for help. They don't count for course credit, but it's a good idea to make sure you know how to do everything before you leave tutorial – similar problems may show up on a test or assignment.

Some integration formulas you may need:

- Volume of a solid of revolution, washer method, rotation about a horizontal axis:

$$V = \pi \int_a^b (r_{out}(x)^2 - r_{in}(x)^2)\, dx,$$

  where r_{out} gives the outer radius (distance from the far side of the region being rotated to the axis) and r_{in} gives the inner radius. If the axis of rotation is vertical, reverse the roles of x and y.

- Volume of a solid of revolution, shell method, rotation about a vertical axis:

$$V = 2\pi \int_a^b r(x)h(x)\, dx,$$

  where $r(x)$ is the radius of the shell (distance to axis of rotation) and $h(x)$ is the height of the shell. If the region lies between $y = g(x)$ (above) and $y = f(x)$ (below), and the axis of rotation is the y-axis we get the formula

$$V = 2\pi \int_a^b x(g(x) - f(x))\, dx$$

  as a special case. If the axis of rotation is horizontal, reverse the roles of x and y.

- Arc length of a curve $y = f(x)$, for $a \le x \le b$:

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

- Surface area generated by revolving $y = f(x)$, $a \le x \le b$ about the x-axis:

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\, dx.$$

- Surface area generated by revolving $y = f(x)$, $a \le x \le b$ about the y-axis:

$$S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\, dx.$$

Exercises

1. Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $x = 1$, and $y = 0$ about the x-axis.
2. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 1$, and $x = 0$ about the x-axis.
3. Repeat Problem 2, but revolving about the y-axis.
4. Find the volume of the solid generated by revolving the region bounded by $x = y - y^2$ and $x = 0$ about the y-axis.
5. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = 6x - 2x^2$ and $y = 0$, about the y-axis.
6. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = \sqrt{x}$ about the x-axis.
7. Find the length of the curve $y = \dfrac{1}{12}x^3 + \dfrac{1}{x}$, for $x \in [1, 4]$.
8. Find the area of the surface obtained by revolving $y = \sqrt{x}$, for $x \in [0, 1]$, about the x-axis.
9. Find the area of the surface obtained by revolving $y = x^2$, for $x \in [0, 1]$, about the y-axis.
10. Find the area of the surface obtained by revolving $x = 1 + 2y^2$, $1 \leq y \leq 2$, about the x-axis.