University of Lethbridge
Department of Mathematics and Computer Science
MATH 1560 - Tutorial #10
Monday, March 26

Intermediate Value Theorem (zero version): Suppose a function f is continuous on $[a, b]$, and either (a) $f(a) < 0$ and $f(b) > 0$, or (b) $f(a) > 0$ and $f(b) < 0$. Then there exists some real number $c \in (a, b)$ such that $f(c) = 0$.

Extra fun: Apply Newton's method to the equation $x^2 - a = 0$ (where $a > 0$) to derive the formula

$$a_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

This formula represents the algorithm used by ancient Babylonians to compute $\sqrt{a}$.

1. Consider the *Intermediate Value Theorem* (IVT), which is stated on the reverse of this page.

   (a) Use the IVT to show that the equation $3x^4 - 8x^3 + 2 = 0$ has a solution on the interval $[2, 3]$.

   (b) Use Newton's method to find the solution, correct to six decimal places.

2. Explain why Newton's Method doesn't work for finding a solution to the equation $x^3 - 3x + 6 - 0$ if the initial approximation is $x_1 = 1$.

3. Apply Newton's Method to the equation $1/x - a = 0$ to derive the reciprocal algorithm $x_{n+1} = 2x_n - ax_n^2$.

   (This algorithm is used by computers to compute reciprocals without dividing.)