Practice for Quizzes 23 and 24
Math 2580
Spring 2016

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For Quiz 23 on Tuesday, you should make sure you can do the problems from Section 2 of the handout on surface integrals. For Quiz 24 on Thursday, you should be able to do the following problems:

1. Use Stokes' theorem to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$, and S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, oriented upward.

2. Evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, directly, where $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$, and S is the disk $x^2 + y^2 \leq 4$ in the plane $z = 5$.

3. How are Problems 1 and 2 related?

4. Use Stokes' theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle xy, 2x, 3y \rangle$, and C is the curve of intersection of the plane $x + z = 5$ and the cylinder $x^2 + y^2 = 9$.

5. Use Stokes' theorem to show that if $\mathbf{F}$ is C^1 vector field defined on all of $\mathbb{R}^3$ such that $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is conservative.

6. Use the Divergence theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = \langle x^4, -x^3 z^2, 4xy^2 z \rangle$, where S is the boundary of the region bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = x + 2$.

7. Use the Divergence theorem to evaluate $\iint_S (2x + 2y + z^2)\, dS$, where S is the sphere $x^2 + y^2 + z^2 = 1$.

   Hint: This is the surface integral of a scalar field, but you can re-write it as a the integral of a vector field. (What is the unit normal vector for the given sphere?)