Math 2580 Assignment #1
University of Lethbridge, Spring 2016

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Due date: Thursday, January 21, by 3 pm.

Please provide solutions to the problems below, using the following guidelines:

- Your submitted assignment should be a **good copy** – figure out the problems first, and then write down organized solutions to each problem.
- You should include a **cover page** with the following information: the course number and title, the assignment number, your name, and a list of any resources you used or people you worked with.
- Since you have plenty of time to work on the problems, assignment solutions will be held to a higher standard than on a test. Your explanations should be clear enough that any of your classmates can understand your solutions.
- Group work is permitted, but copying is not. If you're not sure what the difference is, feel free to ask. If you get help solving a problem, you should (a) make sure you completely understand the solution, and (b) re-write the solution for your good copy by yourself, in your own words.
- Assignments can be submitted in class, or in the designated drop box on the 5th floor of University Hall, across from the Math Department office.
- Late assignments will not be accepted without prior permission.

Assigned problems

1. Each of the equations below describes a quadric surface. Identify (as an ellipsoid, hyperboloid, etc.) and sketch each surface.

   (a) $\dfrac{x^2}{4} + \dfrac{y^2}{9} + z^2 = 1.$

   (b) $x^2 + z^2 = 1 - 2y^2$

   (c) $z + y^2 = 2x^2.$

   Note: For each sketch, you will probably find it helpful to compute a few sections of each surface in planes parallel to the three coordinate planes. (That is, try sketching a few of the curves obtained by setting one of the variables equal to a constant.)

2. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Compute f_x and f_y for $(x,y) \neq (0,0)$.

   (b) Show that $f_x(0,0) = f_y(0,0) = 0$. (Hint: use the definitions. What is the value of $f(x,0)$ and $f(0,y)$?

   (c) Show that $f_x(0,y) = -y$ when $y \neq 0$.

   (d) What is $f_y(x,0)$ when $x \neq 0$?

   (e) Show that $f_{yx}(0,0) = 1$ and $f_{xy}(0,0) = -1$. (You'll need to use limits again.)

   (f) Why does this not contradict the theorem about equality of mixed partials?