Name and student number: Solutions

1. Let $A = \{0, 3, 7\}$ and $B = \{1, 2, 3, 5\}$ be subsets of the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

[1] (a) What is $A \cup B$?

Solution: Since $A \cup B$ is the set of all elements of U that belong to either A or B, we have

$$A \cup B = \{0, 3, 7, 1, 2, 5\}.$$

[1] (b) What are the complements A^c and B^c?

Solution: The complement of a set $A \subseteq U$ consists of all elements of U that do not belong to A. Thus, we have

$$A^c = \{1, 2, 4, 5, 6, 8\} \quad \text{and} \quad B^c = \{0, 4, 6, 7, 8\}.$$

[1] (c) What is the intersection $A^c \cap B^c$?

Solution: Since $x \in A^c \cap B^c$ if and only if $x \in A^c$ and $x \in B^c$, $A^c \cap B^c$ consists of all elements of U that are common to the two sets A^c and B^c from part (b). Thus, we have

$$A^c \cap B^c = \{4, 6, 8\}.$$

[1] (d) What is the relationship between your answers in (a) and (c)?

Solution: We have that $A \cup B = \{0, 1, 2, 3, 5, 7\}$ and $A^c \cap B^c = \{4, 6, 8\}$. Thus, $A^c \cap B^c$ consists of all the elements of U that do not belong to $A \cup B$. In other words, $A^c \cap B^c = (A \cup B)^c$.

[1] (e) Give **two** subsets of U that are subsets of both A and B.

Solution: We note that $A \cap B = \{3\}$, and thus $\{3\}$ is a subset of both A and B. Another subset that is common to both is the empty set $\emptyset$, which is a subset of every set.

2. Recall that one of de Morgan's laws for logic is that $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.

[1] (a) What is the corresponding de Morgan's law for sets?

Solution: Let A and B be subsets of some universal set U, and let x be some element of U. If we let P represent the assertion $x \in A$, and Q represent $x \in B$, then $P \vee Q$ is the assertion that $x \in A$ or $x \in B$, or in other words, $x \in A \cup B$. Thus, $\neg(P \vee Q)$ is the assertion $x \notin (A \cup B)$, or equivalently, $x \in (A \cup B)^c$.

On the other hand, $\neg P \wedge \neg Q$ means $x \notin A$ and $x \notin B$, so $x \in A^c$ and $x \in B^c$, and thus $x \in A^c \cap B^c$ by definition of intersection. Since we are claiming that these two statements are equivalent, we must have $x \in (A \cup B)^c \leftrightarrow x \in A^c \cap B^c$, or $(A \cup B)^c = A^c \cap B^c$.

[3] (b) Prove that $(A \cup B)^c \subseteq A^c \cup B^c$ ($\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ in the textbook notation).

Reminder: to prove $U \subseteq V$, assume that $x \in U$ and deduce $x \in V$. A two-column proof is acceptable but not required.

Solution: Suppose that $x \in (A \cup B)^c$. Then $x \notin A \cup B$, so it is not the case that $x \in A$ or $x \in B$; that is, x belongs to neither A nor B, so $x \notin A$ and $x \notin B$. But this means that $x \in A^c$ and $x \in B^c$, so $x \in A^c \cap B^c$ as required.

[1] (c) What remains to be proved in order to establish your claim in part (a)?

Solution: Since we've proved $(A \cup B)^c \subseteq A^c \cap B^c$, and two sets are equal if and only if each is a subset of the other, we still have to prove that $A^c \cap B^c$ is a subset of $(A \cup B)^c$.

Total: 10 points