Practice for Quiz 4
Math 2580
Spring 2016

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If you can answer the following problems, you should be well-prepared for Quiz 4:

1. Find the equation of the tangent plane to the graph of f at the point $(2, -1, f(2, -1))$, if $f(x, y) = x^2 + 4y^2$.

2. Let $f(x, y) = x^2y + xy^3$. Find a normal vector to the graph $z = f(x, y)$ at the point $(1, 1, 2)$.

3. Use a linear approximation to the function $f(x, y) = x^3 + y^3 - 6xy$ to give an approximate value for
$$(0.99)^3 + (2.01)^3 - 6(0.99)(2.01).$$

4. Verify[1] the chain rule for the function $f(x, y, z) = x + y^2 + z^3$ and curve $\mathbf{r}(t) = (\cos t, \sin t, t)$.

5. Express your chain rule formula from the previous problem as a product of two derivative matrices. (One will be a row vector, and one will be a column vector.)

6. Find the derivative matrix for the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(u, v) = (u \sin v, e^{uv})$, and evaluate it at the point $(0, 1)$.

   (That is, compute $\dfrac{\partial(x, y)}{\partial(u, v)}$ if $x = u \sin v$ and $y = e^{uv}$.)

[1] That is, calculate $\dfrac{d}{dt}(f(\mathbf{r}(t))$ first by using the chain rule, and then by explicitly substituting in the parameterization and differentiating with respect to t, and verify that the two answers are the same