1. In terms of the spherical coordinates ρ, φ, θ, we have:

$$x = \rho \sin\varphi \cos\theta$$
$$y = \rho \sin\varphi \sin\theta$$
$$z = \rho \cos\varphi$$

2. Evaluate $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sin(x^2+y^2)\,dy\,dx$ by converting to polar coordinates.

The bounds $-1 \leq x \leq 1$, $-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$ describe the entire disc $x^2+y^2 \leq 1$. In polar coordinates, we have

$$\int_0^{2\pi}\int_0^1 \sin(r^2) r\,dr\,d\theta = \int_0^{2\pi} \left(-\frac{1}{2}\cos(r^2)\Big|_0^1\right) d\theta = \pi(1-\cos(1)).$$

3. Describe the surface given in spherical coordinates by $\varphi = \pi/4$.

This surface is the upper half ($z \geq 0$) of the cone $x^2+y^2 = z^2$. To see this, note that when $\varphi = \pi/4$, we have $x = \rho\cos\theta/\sqrt{2}$ and $y = \rho\sin\theta/\sqrt{2}$, so $x^2+y^2 = \dfrac{\rho^2}{2}$, and $z = \rho/\sqrt{2}$, so $z^2 = x^2+y^2$. Since φ is measured from the positive z-axis, if $\varphi \in [0, \pi/2]$, we must be above the xy-plane.

4. Convert the following integral to polar coordinates:

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_1^{\sqrt{2}}\int_0^x xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_0^{\sqrt{4-x^2}} xy\,dy\,dx$$

The trick here is to notice the three variable limits: $y = \sqrt{1-x^2}$, $y = \sqrt{4-x^2}$, and $y = x$. The first two describe cicles, of radius 1 and 2, respectively. If we plot these circles in the first quadrant, along with the line $y = x$, we see that $y = x$ intersects $y = \sqrt{1-x^2}$ when $x = 1/\sqrt{2}$. The first integral thus involves the region above the circle $x^2+y^2 = 1$, but below the line $y = x$. The line $y = x$ intersects $y = \sqrt{4-x^2}$ when $x = \sqrt{2}$. The second integral involves the region between the x-axis and $y = x$, for $1 \leq x \leq \sqrt{2}$. The third is the region above the x-axis and below the circle $x^2+y^2 = 4$.

In polar coordinates, this becomes $1 \leq r \leq 2$, where $0 \leq \theta \leq \pi/4$, so our integral is simply $\int_0^{\pi/4}\int_1^2 (r\cos\theta)(r\sin\theta) r\,dr\,d\theta$.