University of Toronto at Mississauga

Mid-Term Make-up Exam

MAT232HF
Calculus of Several Variables

Instructor: Sean Fitzpatrick
Duration: 110 minutes

NO AIDS ALLOWED. **Total: 60 marks**

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

Student ID Number: ______________________________

You may not use calculators, cell phones, or PDAs during the exam. Partial credit will be given for partially correct work. Please read through the entire test before starting, and take note of how many points each question is worth. Please put a box around your solutions so that the grader may find them easily.

FOR MARKER'S USE ONLY	
Problem 1:	/10
Problem 2:	/10
Problem 3:	/10
Problem 4:	/15
Problem 5:	/8
Problem 6:	/7
TOTAL:	/60

1. (a) Obtain the equation of the ellipse with foci at $(\pm 3, 0)$ and major axis of length
[5] 10, and then sketch the graph.

(b) Eliminate the parameter to sketch the parametric curve $x(t) = \cos 2t$, $y(t) = \sin t$, $t \in [-\pi, \pi]$. Then, describe the motion of a particle moving according to these
[5] equations as t changes.

2. Recall the identity $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$.

[4] (a) Show that

$$(\vec{u} \times \vec{v}) \times \vec{w} = (\vec{w} \cdot \vec{u})\vec{v} - (\vec{v} \cdot \vec{w})\vec{u}$$

Hint: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

(b) Let $\vec{a}$ be a given non-zero vector, and $\hat{u}$ a unit vector. Show that $\vec{a}$ can be written as $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$, where $\vec{a}_{\parallel} = (\vec{a} \cdot \hat{u})\hat{u}$ is parallel to $\hat{u}$ and $\vec{a}_{\perp} = (\hat{u} \times \vec{a}) \times \hat{u}$
[6] is perpendicular to $\vec{a}$.

3. In each case, determine whether or not the given line L and plane P are parallel, or intersect. If they intersect, find the point of intersection.

[5] (a) $L: \ x = 7 - 4t, \ y = 3 + 6t, \ z = 9 + 5t$, and $P: \ 4x + y + 2z = 17$.

[5] (b) $L: \ x = 3 + 3t, \ y = 6 - 5t, \ z = 2 + 3t$, and $P: \ 3x + 2y - 4z = 1$.

[8] 4. (a) Find all first-order partial derivatives of the following functions:

(i) $f(x, y) = \dfrac{x \sin y}{y \cos x}$.

(ii) $g(x, y, z) = \ln(\frac{x}{y}) - ye^{xz}$.

(iii) $h(x, y, z) = xy + yz + zx$.

[7] (b) Given $u(x,y) = \dfrac{xy}{x+y}$, show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

5. Consider the elliptic paraboloid $x^2 + \frac{y^2}{b^2} = z$.

[2] (a) Describe the trace of this paraboloid in the plane $z = 1$.

[3] (b) What happens to this trace as $b \to \infty$?

[3] (c) What happens to the paraboloid as $b \to \infty$?

[7] 6. Show that the limit

$$\lim_{(x,y)\to(1,1)} \frac{x - y^4}{x^3 - y^4}$$

does not exist.

Extra space for rough work. Do **not** tear out this page.