Name: **Tutorial day and time:**

Select one *completed* problem for feedback:

1. Consider the vectors $\vec{w} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \vec{v}_1 = \begin{bmatrix} -4 \\ 0 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

   Recall that the question "Does $\vec{w}$ belong to the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?" is the same as the question "Are there scalars x_1, x_2, x_3 such that $\vec{w} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3$?"

   Show that this question is, in turn, equivalent to the question of whether or not there is a solution to the following system of equations:

$$\begin{array}{rcrcrcr} -4x_1 & & & + & 2x_3 & = & 2 \\ & & x_2 & - & x_3 & = & -3 \\ 3x_1 & + & 3x_2 & + & x_3 & = & 1 \end{array}$$

   (You do not have to solve the system.)

2. Let $U = \left\{ \begin{bmatrix} 2x - y \\ x + 3y \\ 4y - x \end{bmatrix} \middle| \, x, y \in \mathbb{R} \right\}$. Find vectors $\vec{a}$ and $\vec{b}$ such that $U = \text{span}\{\vec{a}, \vec{b}\}$.

3. Determine if the following subsets of $\mathbb{R}^2$ are subspaces. Explain your answer.

   (a) $U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, 3x - 2y = 0 \right\}$

   (b) $V = \left\{ \begin{bmatrix} 2x - 1 \\ x + 2 \end{bmatrix} \middle| \, x \in \mathbb{R} \right\}$

4. Using only the vector space properties of $\mathbb{R}^n$ (Theorem 19 in Section 4.2), show the following:

   (a) $0\vec{v} = \vec{0}$ for any vector $\vec{v} \in \mathbb{R}^n$. (Hint: use property 10 and the fact that $0 + 0 = 0$.)

   (b) If $c\vec{v} = \vec{0}$ for some scalar c and vector $\vec{v}$, then either $c = 0$ or $\vec{v} = \vec{0}$.
   (Hint: there are two cases – either c equals zero, or it doesn't.)