University of Lethbridge
Department of Mathematics and Computer Science
MATH 1565 - Tutorial #2 Solutions

1. Evaluate the limit $\lim_{x\to 0} \dfrac{1-\cos(5t)}{\sin^2(3t)}$.

   We note that by direct substitution, the limit is of $0/0$ form. Since $1-\cos^2\theta = \sin^2\theta$, we have

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos(5t)}{\sin^2(3t)} &= \lim_{x\to 0} \frac{1-\cos(5t)}{\sin^2(3t)} \cdot \frac{1+\cos(5t)}{1+\cos(5t)} = \lim_{x\to 0} \frac{1-\cos^2(5t)}{\sin^2(3t)(1+\cos(5t))} \\
&= \lim_{x\to 0} \frac{1}{1+\cos(5t)} \cdot \left(\frac{\sin(5t)}{\sin(3t)}\right)^2 = \lim_{x\to 0} \frac{1}{1+\cos(5t)} \left(\frac{5}{3} \cdot \frac{\sin(5t)}{5t} \cdot \frac{3t}{\sin(3t)}\right)^2 \\
&= \frac{1}{1+1}\left(\frac{5}{3}(1)(1)\right)^2 = \frac{25}{18}.
\end{aligned}$$

2. Evaluate the limit $\lim_{x\to 2} \dfrac{x^3-8}{x^4-16}$.

   Trying direct substitution, we see that the limit is of form $0/0$, so we evaluate by factoring:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^3-8}{x^4-16} &= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)(x^2+4)} \\
&= \lim_{x\to 2} \frac{x^2+2x+4}{(x+2)(x^2+4)} = \frac{2^2+2^2+4}{(2+2)(2^2+4)} = \frac{12}{32} = \frac{3}{8}.
\end{aligned}$$

3. Evaluate the limit $\lim_{x\to -\infty} \dfrac{\sqrt{x^2+2}}{3x-4}$.

   This time, we see that we're in an "∞/∞" situation, which also requires further investigation. We find:

$$\begin{aligned}
\lim_{x\to -\infty} \frac{\sqrt{x^2+2}}{3x-4} &= \lim_{x\to -\infty} \frac{\sqrt{x^2(1+2/x^2)}}{x(3-4/x)} = \lim_{x\to -\infty} \frac{\sqrt{x^2}\sqrt{1+2/x^2}}{x(3-4/x)} \\
&= \lim_{x\to -\infty} \frac{-x\sqrt{1+2/x^2}}{x(3-4/x)} = \lim_{x\to -\infty} \frac{-\sqrt{1+2/x^2}}{3-4/x} \\
&= \frac{-\sqrt{1+0}}{3-0} = -\frac{1}{3}.
\end{aligned}$$

   Note: Recall that $\sqrt{x^2} = |x|$. Since $x \to -\infty$, we can assume that $x < 0$, and thus $\sqrt{x^2} = -x$, which is why we have $-x$ in the numerator in the second line above.