Name: Solutions

1. Find a number $\lambda \in \mathbb{C}$ such that

$$\lambda(1+i, 2-3i, 3+4i) = (3+i, 5-10i, 6+7i),$$

or explain why no such number can exist.

(Note: the numbers above are different from the original quiz, because (a) I'm working from home and don't have a hard copy of the quiz with me, and (b) I accidentally overwrote the file while making the second quiz.)

Solution: Using the rule for scalar multiplication in $\mathbb{C}^3$, the vector on the left must be $(\lambda(1+i), \lambda(2-3i), \lambda(3+4i))$, and this must equal $(3+i, 5-10i, 6+7i)$. Equating the first entries, we have

$$\lambda(1+i) = 3+i.$$

If we multiply both sides by $1-i$, we get $2\lambda = 4-2i$, so $\lambda = 2-i$. Thus, looking at the second entry, on the left we have $\lambda(2-3i) = (2-i)(2-3i) = 1-8i$, which does not equal $5-10i$. Therefore, no such λ can exist.

2. Let V be a vector space over a field $\mathbb{F}$. Prove that for any $a \in \mathbb{F}$ and $v \in V$, if $av = 0$, then $a = 0$ or $v = 0$.

Proof: Given $a \in \mathbb{F}$, either $a = 0$ or $a \neq 0$. If $a = 0$, then we're done. If $a \neq 0$, then from $av = 0$ we have

$$v = 1v = \left(\frac{1}{a} \cdot a\right) v = \frac{1}{a}(av) = \frac{1}{a}(0) = 0.$$

Total: 10 points