University of Lethbridge
Department of Mathematics and Computer Science
13^{th} February, 2015, 3:00 - 3:50 pm
MATH 3410 - Test #1

Last Name:____

First Name:____

Student Number:____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Page	Grade
2	/12
3	/8
4	/10
Total	/30

1. True/False: For each of the statements below, state whether it is true or false, and give a **brief** explanation supporting your choice.

[3] (a) The set $U = \{(x, y, xy) \,|\, x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

[3] (b) If a vector space V can be written as a direct sum $V = U \oplus W$, and for some $v \in V$ we have $v \notin U$, then $v \in W$.

[3] (c) For any subspace $U \subseteq V$, where V is finite-dimensional, there exists a subspace $W \subseteq V$ such that $V = U \oplus W$.

[3] (d) If $T : V \to W$ is a linear transformation, and we know $\dim V = 4$ and $\dim W = 3$, then T cannot be one-to-one.

Please provide a solution to **one** of the two problems on this page:

[8] 2. Suppose that the vectors v_1, v_2, v_3, v_4 form a basis for V. Prove that the vectors

$$v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4$$

also form a basis for V.

[8] 3. Determine whether or not the vector $v = (1, 3, -4)$ belongs to the span of the vectors $(2, 0, 1), (0, 3, -4)$, and $(4, -3, 9)$.

Please provide a solution to **one** of the two problems on this page:

4. Suppose $T : V \to W$ is injective, and the vectors $v_1, \dots, v_n$ are linearly independent in V.
[10] Prove that the vectors $Tv_1, \dots, Tv_n$ are linearly independent in W.

5. Let $V = \mathbb{R}^{3,1} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y, z \in \mathbb{R} \right\}$, and let $T : V \to V$ be the linear transformation
[10] given by
$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 0 & 4 \\ 4 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x - y + 3z \\ -x + 4z \\ 4x - y - 5z \end{bmatrix}.$$
Determine the null space and range of T.

Extra space for rough work or to complete a problem, as needed. Please do not remove this page. If there is work to be graded on this page, please indicate this next to the corresponding question.