*University of Lethbridge*
Department of Mathematics and Computer Science
20th June, 2017, 9:00 am - 12:00 pm
**MATH 1560 - FINAL EXAM**
Examiner: Sean Fitzpatrick

Last Name:____________________________________________

First Name:____________________________________________

Student Number:________________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page. Additional scrap paper may be requested if needed.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

A basic (non-graphing, non-programmable) calculator is permitted. All other outside aids, including, but not limited to, mobile phones, textbooks, computers, drones, elves, and scrap paper, are prohibited.

For grader's use only:

| Page | Grade |
|---|---|
| 2 | /10 |
| 3 | /10 |
| 4 | /10 |
| 5 | /10 |
| 6 | /10 |
| 7 | /10 |
| 8 | /10 |
| 9 | /10 |
| 10 | /10 |
| 11 | /10 |
| Total | /100 |

1. For each limit below, evaluate the limit, or explain why it does not exist. (If a limit is infinite, indicate whether the value is $+\infty$ or $-\infty$.)

[2] (a) $\lim\limits_{x\to 2} \dfrac{x+3}{x^2+1}$.

[2] (b) $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4}$.

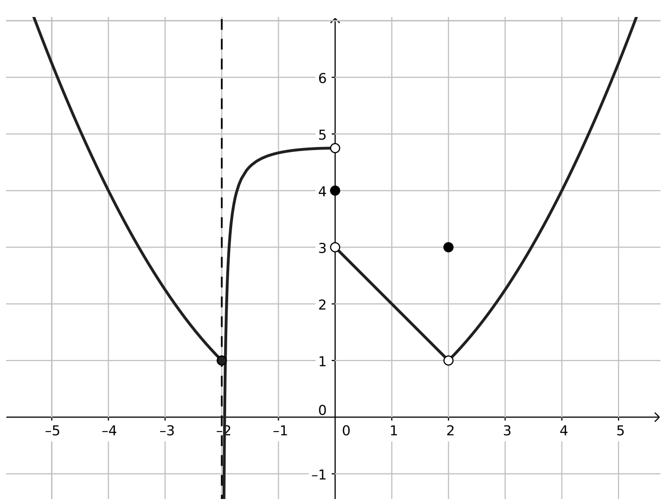
[2] (c) $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{x}$.

[2] (d) $\lim\limits_{x\to 0} \dfrac{x\sin(x)}{1-\cos(x)}$

[2] (e) $\lim\limits_{x\to 1} \dfrac{x^2-5x+6}{x^2-1}$

[4] 2. Let $f(x) = \dfrac{1}{x^2}$. **Using the definition of the derivative**, compute $f'(1)$.

[6] 3. The graph of a function $f$ is shown below. Give the values of the indicated limits:




$\lim_{x\to -2^-} f(x) =$ ______________

$\lim_{x\to -2^+} f(x) =$ ______________

$\lim_{x\to 0^-} f(x) =$ ______________

$\lim_{x\to 0^+} f(x) =$ ______________

$\lim_{x\to 2^-} f(x) =$ ______________

$\lim_{x\to 2^+} f(x) =$ ______________

For $a = -2$, $a = 0$, and $a = 2$, use your results above to determine whether or not $f$ is continuous at $x = a$. If not, indicate the type of discontinuity.

At $a = -2$:

At $a = 0$:

At $a = 2$:

4. Compute the derivatives of the following functions:

[2] (a) $f(x) = 3x^4 - 5x^2 + \cos(x) + 2\pi^3$.

[3] (b) $g(x) = e^x \tan(x)$

[3] (c) $h(x) = \dfrac{x^2 + 2x}{\sqrt{x}}$

[2] (d) $r(x) = \arcsin(x^3)$

[4] 5. Use implicit differentiation to find $\dfrac{dy}{dx}$ given that

$$x^3y^2 = 4x - 2y.$$

6. Compute the derivative of the following functions:

[3] (a) $f(x) = \ln\left(\dfrac{x^3e^{x^2}}{\sqrt{x+1}}\right)$.

[3] (b) $g(x) = x^x$

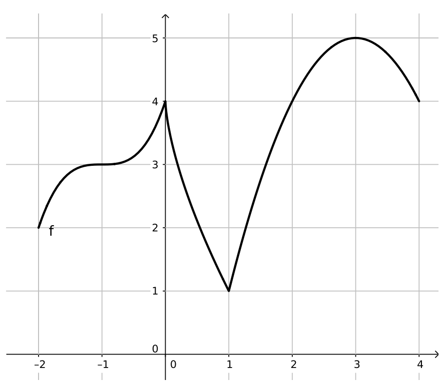
7. Let $f(x) = 6x^{1/2} - x^{3/2}$, for $x \in [1, 4]$.

[4] (a) Find and classify all critical points of $f$.

[1] (b) Determine the absolute maximum and minimum of $f$ on the given interval.

[1] (c) Determine the intervals on which $f$ is increasing and decreasing.

[4] 8. Repeat steps (a) - (c) in Question 7 for the function $g$ whose graph is given below.



[10] 9. Sketch the graph of the function $f(x) = \dfrac{x}{(x+3)^2}$.

Your sketch doesn't have to be artistically perfect, but it should be large enough for me to see, and any intercepts, critical points, inflection points, and asymptotes must be clearly labelled.

In addition to your sketch, you should give the domain of $f$, as well as the intervals on which $f$ is increasing and decreasing, and concave up and concave down.

10. The length of a rectangle is increasing at a rate of 8 cm/s, while its width is increasing at a rate of 3 cm/s. At what rate is the area of the rectangle increasing at the moment when
[5] its length is 20 cm and its width is 10 cm?

11. Find the dimensions of the rectangle of *largest area* that can be drawn on the plane, if the base of the rectangle must lie on the $x$-axis, and the other two corners of the rectangle
[5] (above the $x$-axis) must lie on the parabola $y = 8 - x^2$.

12. Compute the degree three Taylor polynomial, centred at $a = 0$, for the following functions:

[4] (a) $f(x) = \sqrt{x+1}$

[4] (b) $g(x) = \displaystyle\int_0^x e^{-t^2}\,dt$

[2] 13. Determine a function $f(x)$ such that $f'(x) = 3x^2 - \dfrac{3}{x^2}$ and $f(1) = 3$.

14. Evaluate the following indefinite integrals:

[2] (a) $\displaystyle\int (2x^3 - \sqrt{x} + \cos(x))\,dx$

[2] (b) $\displaystyle\int \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^2+1}\right) dx$

[3] (c) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

[3] (d) $\displaystyle\int \frac{\sin(x)}{\cos^2(x)}\,dx$

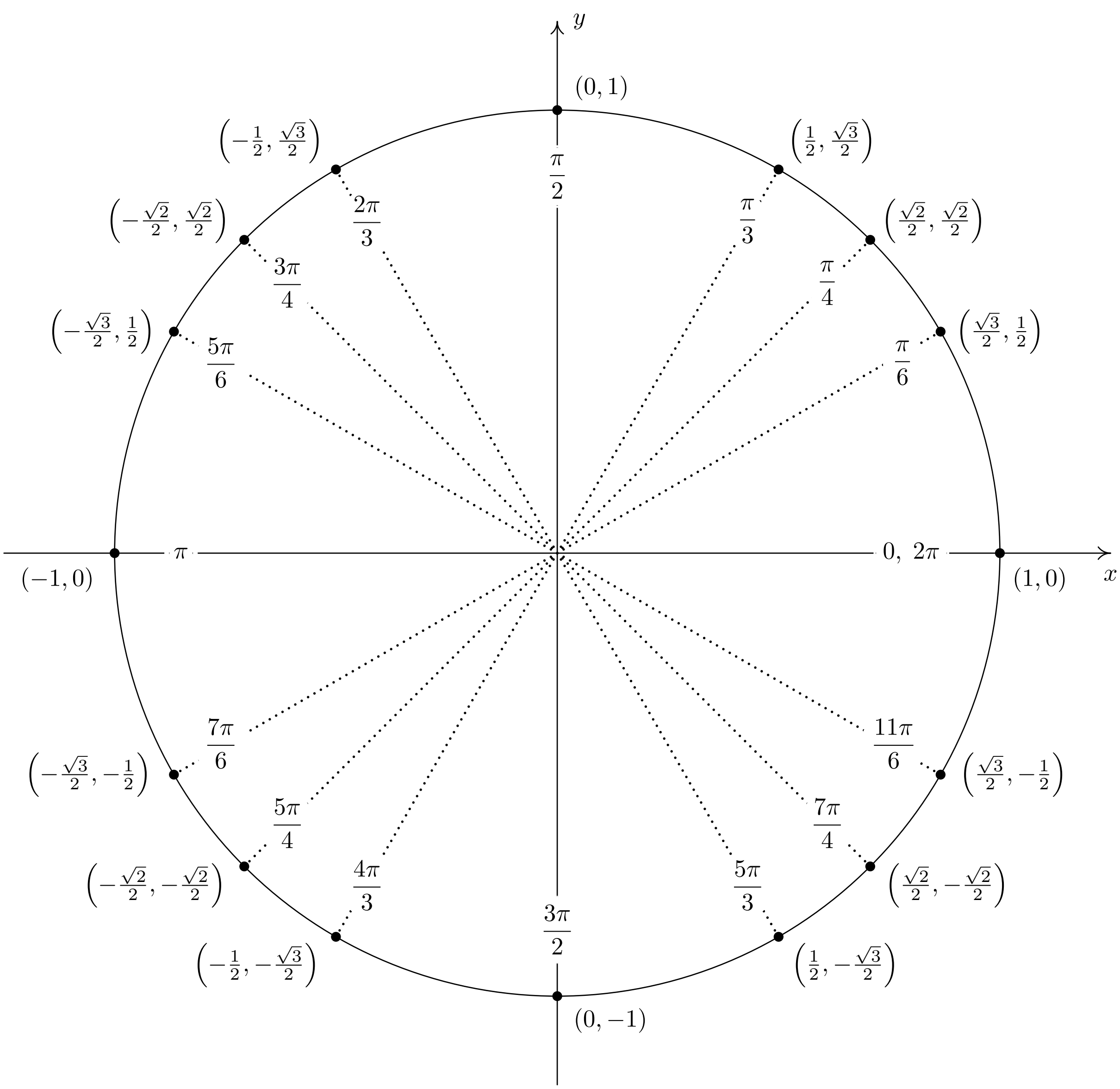
15. Evaluate the following definite integrals:

[3] (a) $\displaystyle\int_1^e \frac{(\ln(x))^2}{x}\,dx$

[3] (b) $\displaystyle\int_2^6 x\sqrt{x-2}\,dx$ (Hint: if $u = x - 2$, then $x = u + 2$...)

16. Express the integral $\displaystyle\int_0^4 x^2 e^x\,dx$ as a limit of Riemann sums. You **do not** have to evaluate
[4] the limit.

Extra space for rough work. And also, a unit circle.




 **Total Points: 100**