University of California, Berkeley
Department of Mathematics
5th October, 2012, 12:10-12:55 pm
MATH 53 - Test #1

Last Name:_____

First Name:_____

Discussion Section: _____

Name of GSI: _____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Page	Grade
1	/12
2	/12
3	/12
Total	/36

C

[4] 1. Find the equation of the tangent line to the curve C represented by the vector-valued function $\mathbf{r}(t) = \langle t^3, \sin(\pi t), 2t + 1\rangle$ at the point $(1, 0, 3)$.

[5] 2. Find the area of the cardioid $r = 1 + \cos\theta$.

[3] 3. Evaluate $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^3 - y^3}{x - y}$, or explain why it does not exist.

4. Consider the two lines in $\mathbb{R}^3$ given by

$$\mathbf{r}_1(t) = \langle 1, 6, 1\rangle + t\langle 0, 4, -2\rangle$$
$$\mathbf{r}_2(s) = \langle 0, 3, 0\rangle + s\langle 1, -1, 3\rangle.$$

[2] (a) Verify that the two lines intersect at the point $(1, 2, 3)$.

[3] (b) Find the cosine of the angle between the two lines.

[4] (c) Find the equation of the plane that contains the two lines.

[3] (d) Find the distance between the point $P(3, 4, -2)$ and the plane from part (c).

5. (a) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + 2y^2 + z^2}$ at the point
[5] $(4, 5, 6)$. (Note: $4^2 + 2(5^2) + 6^2 = 100 = 10^2$.)

(b) Use your result from part (a) to approximate the value of $\sqrt{(4.1)^2 + 2(4.95)^2 + (6.03)^2}$
[2]

6. Use the chain rule to compute $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ if $f(x, y, z) = xe^{y^2 z}$, where $x = 2uv$, $y = u^2 - v^2$,
[5] and $z = u$.