

University of Toronto at Mississauga

Mid-Term Exam

MAT232HF

Calculus of Several Variables

Instructor: Sean Fitzpatrick

Duration: 110 minutes

NO AIDS ALLOWED.

Total: 60 marks

Family Name: _____
(Please Print)

Given Name(s): _____
(Please Print)

Please sign here: _____

Student ID Number: _____

You may not use calculators, cell phones, or PDAs during the exam. Partial credit will be given for partially correct work. Please read through the entire test before starting, and take note of how many points each question is worth. Please put a box around your solutions so that the grader may find them easily.

FOR MARKER'S USE ONLY	
Problem 1:	/11
Problem 2:	/10
Problem 3:	/8
Problem 4:	/15
Problem 5:	/10
Problem 6:	/6
TOTAL:	/60

- [3] 1. (a) Sketch the conic section $4y^2 - 9x^2 - 18x - 8y = 41$

- [4] (b) Sketch the parametric curve $x(t) = 3 + 2 \cos t$, $y(t) = 5 - 3 \sin t$, $t \in [0, 2\pi]$.

[4]

- (c) Identify the traces of the surface $9x^2 + 4z^2 - 36y^2 - 36 = 0$ in each of the co-ordinate planes. Then, sketch the surface.

2. Let \vec{u} and \vec{v} be non-zero vectors such that $|\vec{u} + \vec{v}| = |\vec{u} - \vec{v}|$.

[2] (a) What can you conclude about the parallelogram spanned by \vec{u} and \vec{v} ?

[3] (b) Show that $\vec{u} \cdot \vec{v} = 0$.

[5] (c) Prove the parallelogram law: $|\vec{u} - \vec{v}|^2 + |\vec{u} + \vec{v}|^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2$.

3. Determine whether or not the following sets of points lie on the same line. If they do, give the vector equation of the line. If not, give the equation of the plane containing them.

[4] (a) $P = (0, -2, 4), Q = (1, -3, 5), R = (4, -6, 8)$

[4] (b) $P = (1, 1, 1), Q = (3, -2, 3), R = (3, 4, 6).$

[8]

4. (a) Find all first-order partial derivatives of the following functions:

(i) $f(x, y) = e^2 e^{xy}$.

(ii) $h(x, y, z) = x^2 y^3 z^4$.

(iii) $k(x, y, z) = z \sin(x - y)$.

[4] (b) Verify that $f_{xy} = f_{yx}$ for $f(x, y) = xye^{-xy}$.

[3] (c) Can there exist a continuous function $f(x, y)$ such that $f_x(x, y) = \cos^2(xy)$ and $f_y(x, y) = \sin^2(xy)$? Why?

- [6] 5. Find and classify the critical points of the function

$$f(x, y) = 2x^3 + y^3 - 3x^2 - 12x - 3y$$

6. Evaluate the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{(x^2 + y^2)^{3/2}}$$

as (x, y) approaches the origin along:

[1]

(a) the x -axis.

[2]

(b) the line $y = mx$.

[3]

(c) the path $\vec{r}(t) = \frac{1}{t}\hat{i} + \frac{\sin t}{t}\hat{j}$, $t > 0$.

Extra space for rough work. Do **not** tear out this page.