University of California, Berkeley
Department of Mathematics
15^{th} February, 2013, 12:10-12:55 pm

MATH 53 - Test #1

Last Name:____

First Name:____

Student Number:____

Discussion Section: ______________________________________

Name of GSI: ____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

There is a list of potentially useful formulas available on the last page of the exam.

For grader's use only:

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Total	/40

B

1. (a) Describe the motion of a particle whose position $(x(t), y(t))$ at time $t \in [-\pi, \pi]$ is given by $x = \cos^2 t$, $y = \sin t$. (In particular, what is the Cartesian equation of the curve?) [6]

   (b) Set up, but do not evaluate, the integral which computes the length of the curve from part (a). How does this compare to the distance travelled by the particle? [4]

2. Find the equation of the tangent line to the curve represented by the vector-valued function $\mathbf{r}(t) = \langle t^3 + t, t^2 + 1, 5 - t^4 \rangle$ at the point $(2, 2, 4)$ [4]

[8] 3. (a) Find the equation of the line of intersection of the planes given by the equations

$$\begin{aligned} 3x - y + 4z &= 4 \\ x + 2y - z &= -1. \end{aligned}$$

[2] (b) What is the cosine of the angle of intersection of the two planes in part (a)?

[3] (c) What is the distance from the plane $3x - y + 4z = 4$ to the point $(-2, 3, 1)$?

[7] 4. Find the area of the triangle ΔPQR, for points $P(0,0,0)$, $Q(0,-1,1)$, $R(2,4,-2)$.

[6] 5. Let $\mathbf{a}$, $\mathbf{b}, \mathbf{c}$ be nonzero vectors in $\mathbb{R}^3$. For each of the following, prove the statement, or give an example showing that the statement is false:

(a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

(b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

(c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

List of potentially useful formulas and facts:

In the following, assume $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ are constant vectors in $\mathbb{R}^3$, and $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ is a vector-valued function with domain $[a, b]$.

- Length of a vector: $\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- Dot product: $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$
- Cross product: $\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_3, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$; $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin\theta$.
- Projections: $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|} \left(\dfrac{\mathbf{a}}{\|\mathbf{a}\|} \right)$, $\operatorname{comp}_{\mathbf{a}} \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|}$.
- Planes: $ax + by + cz = d$, or $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$.
- Lines: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$, or $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$.
- Quadric surfaces: there aren't any on the midterm, so you can relax and play with some vectors.
- Parametric area: $A = -\displaystyle\int_a^b y(t) x'(t) \, dt$ for a positively-oriented curve.
- Tangent vectors: For each $t_0 \in [a, b]$, $\mathbf{r}(t)$ has tangent vector $\mathbf{r}'(t_0) = \langle x'(t_0), y'(t_0), z'(t_0) \rangle$.
- Parametric length: $L = \displaystyle\int_a^b \|\mathbf{r}'(t)\| \, dt$.

List of basic facts I hope were in fact entirely unnecessary to include:

- $\sin^2\theta + \cos^2\theta = 1$
- $\dfrac{d}{dt}(t^n) = nt^{n-1}$, $\dfrac{d}{dt}\sin t = \cos t$, $\dfrac{d}{dt}\cos t = -\sin t$
- $\dfrac{d}{dt}(f(t)g(t)) = f'(t)g(t) + f(t)g'(t)$