FACULTY OF APPLIED SCIENCE AND ENGINEERING
University of Toronto

MAT294H1Y
Calculus and Differential Equations

Term Test #1 - Early sitting
Duration: 110 minutes

NO AIDS ALLOWED. **Total: 50 marks**

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

Student ID Number: ______________________________

You may not use calculators, cell phones, or PDAs during the test. Partial credit will be given for partially correct work. Please read through the entire test before starting, and take note of how many points each question is worth.

FOR MARKER'S USE ONLY	
Problem 1:	/10
Problem 2:	/15
Problem 3:	/8
Problem 4:	/9
Problem 5:	/8
TOTAL:	/50

1. (a) Calculate the first-order partial derivatives of the following functions:

[2] (i) $f(x, y, z) = z \sin(x - y)$

[2] (ii) $f(x, y) = x^2 y e^{xy}$

[2] (iii) $f(x, y, z) = \dfrac{\ln z}{xy}$

[4] (b) Find all second-order partial derivatives of the function

$$f(x, y) = x^2 \tan y + y \ln x.$$

2. Let $f(x, y) = x^3 - 3xy - y^3$.

[3] (a) Locate all critical points of f.

(b) Classify any critcal points found in part (a) as local maxima, local minima, or
[4] saddle points.

[5] (c) Find the maximum and minimum of $f(x,y)$ subject to the constraint $x+2y-1=0$.

[3] (d) Find the absolute maximum and minumum of $f(x,y)$ on the region bounded by the coordinate axes and the line $x+2y=1$.

Hint: The only thing you still have to check is the value of $f(x,y)$ along the axes.

3. Let $f(x, y) = \sqrt{x^2 + y^2}$.

[2] (a) Find $\nabla f(x, y)$.

[3] (b) Find the equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at the point $(3, 4)$.

[3] (c) Use the differential df to approximate the value of $\sqrt{(2.97)^2 + (3.04)^2}$.
Note: $3/5 = 0.6$ and $4/5 = 0.8$.

4. Let $f(x, y, z) = \ln(x + y + z)$ and let $\vec{r}(t) = \cos^2 t\,\hat{\imath} + \sin^2 t\,\hat{\jmath} + t^2\,\hat{k}$.

[2] (a) Write the chain rule formula for the derivative $\dfrac{d}{dt}(f(\vec{r}(t)))$.

[2] (b) Use your formula from (a) to evaluate $\dfrac{d}{dt}(f(\vec{r}(t)))$ when $t = \pi/4$.

[2] (c) Verify your calculation in part (b) by first making the substitutions $x = \cos^2 t$, $y = \sin^2 t$ and $z = t^2$ in f and then differentiating with respect to t.

[3] (d) Find the derivative of f in the direction of the curve $\vec{r}(t)$ at the point $(1/2, 1/2, \pi^2/16)$.

Hint: You've done most of the work for this problem already!

[3] 5. (a) Show that the limit

$$f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

does not exist by making the substitution $y = mx$, where m can be any real number.

(b) Recall that $f(\vec{x})$ is differentiable at $\vec{a}$ if and only if

$$\lim_{\vec{h}\to\vec{0}} \frac{f(\vec{a}+\vec{h}) - f(\vec{a}) - \nabla f(\vec{a})\cdot\vec{h}}{||\vec{h}||} = 0.$$

Show that the statement, "f is continuous at any point at which it is differentiable" holds for functions of more than one variable.
Hint: Note that $\lim_{\vec{x}\to\vec{a}} f(\vec{x}) = \lim_{\vec{h}\to\vec{0}} f(\vec{a}+\vec{h})$.

Extra space for rough work. Do **not** tear out this page.