Math 3410 Assignment #2
University of Lethbridge, Spring 2015

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Due date: Wednesday, February 11, by 5 pm.

Please provide solutions to the problems below, using the same guidelines as for Assignment #1:

1. Let U be a subspace of a vector space V, and let $S : U \to W$ be a non-zero linear transformation. That is, we assume that there exists some $u \in U$ such that $Su \neq 0$. Prove that the function $T : V \to W$ given by

$$Tv = \begin{cases} Sv, & \text{if } v \in U \\ 0, & \text{if } v \notin U \end{cases}$$

   is **not** a linear transformation.

   Hint: if $u \in U$ and $v \notin U$, can $u + v$ be an element of U? What about $-v$?

2. Suppose V is a finite-dimensional vector space, and let $U \subseteq V$ be a subspace. Prove that any linear transformation $S : U \to W$ can be extended to a linear transformation $T : V \to W$.

   Hint: any basis of U can be extended to a basis for V.

3. Suppose that V is a finite-dimensional vector space, and $T : V \to W$ is a linear transformation. Prove that there exists a subspace $U \subseteq V$ such that:

   (a) $U \cap \operatorname{null} T = \{0\}$, and

   (b) $\operatorname{range} T = \{Tu : u \in U\}$.

4. Suppose V and W are finite-dimensional vector spaces.

   (a) Prove that there exists an injective (one-to-one) linear transformation $T : V \to W$ if and only if $\dim V \leq \dim W$.

   (b) Prove that there exists a surjective (onto) linear transformation $T : V \to W$ if and only if $\dim V \geq \dim W$.