University of Lethbridge
Department of Mathematics and Computer Science
MATH 1410 - Tutorial #9
Wednesday, March 7

Additional practice: (**do not submit**).

1. Determine the inverse of the following matrices, if possible:

(a) $\begin{bmatrix} 4 & -3 \\ -2 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -1 & 4 \\ 0 & 3 & -5 \\ 4 & 1 & 3 \end{bmatrix}$

1. Let $A = \begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & -5 \\ 1 & 1 & 9 \end{bmatrix}$.

(a) Compute A^{-1}.

(b) Solve for X, if $AX + \begin{bmatrix} 2 & -3 \\ -1 & -5 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$

2. For each equation given below, explain why A is invertible, and determine an expression (in terms of A) for A^{-1}.

   Hint: Recall that A is invertible provided there exists a matrix B such that $AB = I$, in which case $B = A^{-1}$.

   (a) $A^5 = I$

   (b) $A^4 = 9I$

   (c) $A^2 - 5A + 6I = 0$

3. Show that the matrix $A = \begin{bmatrix} -3 & -3 \\ 10 & 8 \end{bmatrix}$ satisfies the equation $A^2 - 5A + 6I = 0$. Using your result from problem 2(c), determine A^{-1}.

4. For each problem below, assume T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.

   (a) Given vectors $\vec{u}, \vec{v} \in \mathbb{R}^2$ such that $T(\vec{u}) = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ and $T(\vec{v}) = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$, determine the value of $T(4\vec{u} - 3\vec{v})$.

   (b) Determine the matrix of T, if $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$.

   (c) *For fun*: find the matrix of the transformation T in part(a), if $\vec{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

   Hint: First determine how to write $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in terms of $\vec{u}$ and $\vec{v}$.