

*University of Lethbridge*  
Department of Mathematics and Computer Science  
**MATH 2565 - Tutorial #10**  
Thursday, March 22

Name: \_\_\_\_\_

**Note:** You may do this assignment as a group, if you wish, by listing additional names under the space above, up to a maximum of 3 students per group.

1. Find the radius and interval of convergence for the following power series:

(a) 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n5^n} x^n$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$$

(c) 
$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}$$

2. Let  $p$  and  $q$  be real numbers with  $p < q$ . Find a power series whose radius of convergence is:

(a)  $[p, q]$

(b)  $(p, q)$

(c)  $[p, q)$

(d)  $(p, q]$

3. Given that  $\sum_{n=0}^{\infty} c_n 4^n$  is convergent, can we conclude that each of the following series is convergent?

(a)  $\sum_{n=0}^{\infty} c_n (-2)^n$

(b)  $\sum_{n=0}^{\infty} c_n (-4)^n$

4. Suppose  $\sum_{n=0}^{\infty} c_n x^n$  converges when  $x = -4$  and diverges when  $x = 6$ . What can be said about the convergence or divergence of the following series?

(a)  $\sum_{n=0}^{\infty} c_n$

(b)  $\sum_{n=0}^{\infty} c_n 8^n$

(c)  $\sum_{n=0}^{\infty} c_n (-3)^n$

5. Recall that  $f(x) = \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ , for  $|x| < 1$ .

- (a) Find a power series representation for  $g(x) = (1+x)^{-2}$ . What is the radius of convergence?

- (b) Find a power series representation for  $h(x) = \frac{x^2}{(1+x)^3}$ .

6. Find a power series representation for the function:

(a)  $f(x) = x^2 \arctan(x^3)$

(b)  $g(x) = \left(\frac{x}{2-x}\right)^3$

7. Express the antiderivative as a power series;

(a)  $\int \frac{t}{1+t^3} dt$

(b)  $\int \frac{\arctan x}{x} dx$