

Name:

Solve the following **two** questions. (Question #2 is on the back of the page.)

- [5] 1. Suppose $T \in \mathcal{L}(V)$ and $(T - 2I)(T - 3I)(T - 4I) = 0$. Suppose λ is an eigenvalue of T .
Prove that $\lambda = 2$ or $\lambda = 3$ or $\lambda = 4$.

Hint: Compute $(T - 2I)(T - 3I)(T - 4I)v$, where v is an eigenvector with eigenvalue λ .

2. Suppose $T \in \mathcal{L}(V)$ is invertible. Prove that $E(\lambda, T) = E(\frac{1}{\lambda}, T^{-1})$ for every $\lambda \in \mathbb{F}$ with $\lambda \neq 0$.

Reminder: the eigenspace $E(\lambda, T)$ is defined to be $\text{null}(T - \lambda I)$.