

Please complete all problems below.

1. Let $\vec{a} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$.

(a) Calculate $\vec{a} \times \vec{b}$

(b) Find the area of the parallelogram spanned by the vectors \vec{a} and \vec{b} .

(c) Calculate the volume of the parallelepiped spanned by the vectors $\vec{a}, \vec{b}, \vec{c}$.

2. Find the equation of the plane that passes through the points $(2, 1, 3)$, $(3, -1, 5)$, and $(1, 2, -3)$.

3. Find the equation of the plane that contains the lines

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

4. Show that the shortest distance from a point P to the line L through P_0 with direction vector \vec{d} is $\frac{\|\vec{P_0P} \times \vec{d}\|}{\|\vec{d}\|}$.

5. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ -2 \end{bmatrix}.$$

(a) Find a matrix A such that $T(\vec{x}) = A\vec{x}$ for all vectors \vec{x} in \mathbb{R}^2 .

(b) Describe the effect of T on the square $0 \leq x, y \leq 1$. What is the resulting region, and what is its area?

Name:

Tutorial time:

Please submit **one** *completed* solution from the worksheet for feedback.

Note: Please recopy the question you are solving so we know which solution you're submitting.