Practice for Quiz 17
Math 2580
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Problems from Quiz 17 will be a review of material from Math 2570, and some basic problems from 18.1.

If you can answer the following problems, you should be well-prepared for Quiz 17:

1. Calculate the derivative of the following vector-valued functions:

   (a) $\mathbf{r}(t) = \langle t^2, t^3, t^4 \rangle$

   (b) $\mathbf{r}(t) = \langle \sin(t), e^{3t}, \cos(2t) \rangle$.

   (c) $\mathbf{r}(t) = \langle \sin(t^2), \ln(t^2 + 1) \rangle$.

2. Calculate $\|\mathbf{r}'(t)\|$ for the vector-valued functions in problem 1. (Note that if $\mathbf{r}(t)$ is interpreted as position with respect to time, then $\mathbf{r}'(t)$ is velocity, and $\|\mathbf{r}'(t)\|$ is speed.

3. Show that $\dfrac{d}{dt} \|\mathbf{r}(t)\|^2 = 2\mathbf{r}(t) \cdot r'(t)$.

4. Determine a vector-valued function $\mathbf{r}(t)$ and an interval $[a, b]$ that parameterize the line segment from $(1, 2, 0)$ to $(4, -3, 2)$.

5. Evaluate $\displaystyle\int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot r'(t) \, dt$ for the vector field $\mathbf{F}$ and curve $\mathbf{r}$ given by

   (a) $\mathbf{F}(x, y) = x^2\mathbf{i} - xy\mathbf{j}$, and $\mathbf{r}(t) = \sin(t)\mathbf{i} + \cos(t)\mathbf{j}$, $a = 0$, $b = \pi$.

   (b) $\mathbf{F}(x, y, z) = \langle xy^2, xyz, yz^2 \rangle$, $\mathbf{r}(t) = \langle t, t^2, 4t \rangle$, $a = 0$, $b = 1$.