University of Lethbridge
Department of Mathematics and Computer Science
20th March, 2015, 10:00 - 10:50 am
MATH 1410A - Test #2

Last Name:_____

First Name:_____

Student Number:_____

Tutorial Section: _____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

No external aids are allowed, with the exception of a 5-function calculator.

For grader's use only:

Page	Grade
2	/10
3	/10
4	/10
5	/5
6	/5
Total	/40

A

1. SHORT ANSWER: For each of the questions below, please provide a short (one line) answer.

[2] (a) Suppose $\det A = 4$ and the matrix B is obtained from A by first multiplying the first row of A by 5, and then exchanging rows 1 and 3. What is $\det B$?

[2] (b) Let A and B be 3×3 matrices. If $\det A = 2$ and $\det B = -3$, what is the value of $\det(2A^2B^TA^{-1})$?

[2] (c) Calculate the dot product of the vectors $\vec{u} = \begin{bmatrix} 1 & -2 & 4 \end{bmatrix}^T$ and $\vec{v} = \begin{bmatrix} 4 & -2 & 3 \end{bmatrix}^T$

[2] (d) Let $\vec{v}$ be a vector in $\mathbb{R}^n$. If $\|\vec{v}\| = 3$ and $\vec{w} = -4\vec{v}$, what is $\|\vec{w}\|$?

[2] (e) Find $\vec{x}$, given that $\vec{u} = \begin{bmatrix} 2 & -1 & 3 \end{bmatrix}^T$ and $\vec{v} = \begin{bmatrix} -4 & 7 & 5 \end{bmatrix}^T$, and $3\vec{u} - 2\vec{x} = \vec{v}$.

2. Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 4 \\ -3 & 0 & 1 \end{bmatrix}$.

[5] (a) Compute $\det A$.

(b) Given the system of equations $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & 4 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, use Cramer's rule to find

[5] the value of y, if possible.

3. (a) Find a vector equation of the line in $\mathbb{R}^3$ that passes through the points $P = (2, -3, 1)$ and $Q = (4, 1, -2)$.
[4]

(b) Let L_1 be the line through $P_1 = (2, 0, -1)$ with direction vector $\vec{d_1} = \begin{bmatrix} -1 & 3 & 2 \end{bmatrix}^T$, and let L_2 be the line through $P_2 = (8, 6, 7)$ with direction vector $\vec{d_2} = \begin{bmatrix} -4 & 0 & -2 \end{bmatrix}^T$. Determine the point of intersection of L_1 and L_2, if any.
[6]

4. Find the shortest distance from the point $P = (3, 2, -1)$ to the line L given by the vector equation [5]

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

5. Consider the triangle in $\mathbb{R}^3$ with vertices $P = (2, 0, -3)$, $Q = (5, -2, 1)$, and $R = (7, 5, 3)$.

[3] (a) Show that the triangle is a right-angled triangle.
Hint: Recall that if the dot product of two non-zero vectors is zero, then those vectors meet at a right angle.

(b) Compute the lengths of the three sides of the triangle and verify that the Pythagorean
[2] theorem ($a^2 + b^2 = c^2$) holds.