University of Lethbridge
Department of Mathematics and Computer Science
MATH 2565 - Tutorial #8
Thursday, March 8

First Name:_____

Last Name:_____

Additional practice (don't include your solutions here):

1. Solve by separation of variables:

   (a) $x^5y' + y^5 = 0$

   (b) $xy' = (1 - 4x^2)\tan y$

   (c) $xy^2 - y'x^2 = 0$

   (d) $(1 + x^2)\,dy + (1 + y^2)\,dx = 0$

2. Solve the linear differential equation:

   (a) $y' + y = \dfrac{1}{1 + e^{2x}}$

   (b) $xy' - 3y = x^4$

   (c) $y' + y\cot x = 2x\csc x$

   (d) $(1 + x)y' + 4y = 3x$

3. Determine if the series converges or diverges:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{6^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{10}{n!}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n!} + \frac{1}{n}\right)$

1. Solve the initial value problem:

   (a) $\dfrac{y'}{1+x^2} = \dfrac{x}{y}$, $y(1) = 3$.

   (b) $y' - 2xy = 6xe^{x^2}$, $y(0) = 1$.

   (c) $x^2y' + xy = 2x$, $y(1) = 4$.

2. Initially 5 grams of salt are dissolved in 20 litres of water. Brine with concentration of salt 2 grams of salt per litre is added at a rate of 3 litres a minute. The tank is mixed well and is drained at 3 litres a minute. How long does the process have to continue until there are 20 grams of salt in the tank?

3. Find the limit of the sequence, or explain why it does not converge:

(a) $a_n = \dfrac{n-1}{n} - \dfrac{n}{n-1}$

(b) The sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ for all $n \geq 1$. (You may assume that the sequence converges.)

4. Determine if the series converges or diverges:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$. (Geometric?)

(d) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ (Telescoping?)