Math 3410 Assignment #6
University of Lethbridge, Spring 2015

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April 8, 2015

Due date: Thursday, April 16th, by 5 pm.

Suggested homework

The textbook lacks decent computational problems on characteristic and minimal polynomials, and Jordan Canonical Form. For some extra practice, you may want to take a look at the section on JCF from the Linear Algebra "wikibook", available here: `http://en.wikibooks.org/wiki/Linear_Algebra/Jordan_Canonical_Form`. (This is essentially Jim Hefferon's free linear algebra textbook.) At the bottom of the page, you find exercises. Suggested exercises are: 2, 3, 5 (do maybe the first three), 6-9, and 15. At the very bottom of the page you'll find a link to solutions. (The solutions occasionally refer to a "string basis", by which they appear to mean a basis corresponding to a nilpotent operator, as described in our text.)

Assigned problems

1. Suppose $T \in \mathcal{L}(V)$ is normal. Prove that $\operatorname{null} T^k = \operatorname{null} T$ for every positive integer k.

   Hint: The inclusion $\operatorname{null} T \subseteq \operatorname{null} T^k$ is easy. Recall that for normal operators, $\|Tu\| = \|T^*u\|$ for all u. From this, deduce that if $v \in \operatorname{null} T^k$, then $T^*T^{k-1}v = 0$, and then show $v \in \operatorname{null} T^{k-1}$.

2. Suppose V is a complex inner product space and $T \in \mathcal{L}(V)$ is a normal operator such that $T^9 = T^8$. Prove that T is self-adjoint and $T^2 = T$.

   Hint: Use the complex spectral theorem and show that the only possible eigenvalues of T are 0 or 1.

3. Suppose $T \in \mathcal{L}(V)$, m is a positive integer, and $v \in V$ is such that $T^{m-1}v \neq 0$ but $T^m v = 0$. Prove that the vectors $v, Tv, T^2v, \ldots, T^{m-1}v$ are linearly independent.

4. Determine all possible Jordan Canonical Forms for a linear transformation with characteristic polynomial $(x-2)^3(x-3)^2$. Find the corresponding minimal polynomial for each JCF.