University of Lethbridge
Department of Mathematics and Computer Science
3^{rd} March, 2015, 3:05-4:20 pm
Math 2000B - Midterm

Last Name:____

First Name:____

Student Number:______________________________________

Record your answers below each question in the space provided. **Left-hand pages may be used as scrap paper for rough work.** If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

The value of each problem is indicated in the left-hand margins. The value of a problem does not always indicate the amount of work required to do the problem.

Outside aids, including, but not limited to, cheat sheets, smart phones, laptops, spy cameras, drones, and telepathic communication, are not permitted. You can keep a calculator with you if it makes you feel better.

For grader's use only:

Page	Grade
2	/10
3	/10
4	/12
5	/12
6	/10
7	/6
Total	/60

1. For each of the following sentences, decide whether or not it is an assertion. If it is, indicate whether it is true or false, and why.

[2] (a) The number 4 is an even integer.

[2] (b) For each integer n, $n^2 - 1$ is a prime number.

[2] (c) There exists some $x \in \mathbb{R}$ such that $x + y = 3$.

2. For each conditional statement below, identify the hypothesis and conclusion, and indicate whether or not the statement is true or false.

[2] (a) If $3 + 5 = 10$, then $8 < -3$.

[2] (b) The fact that $14 = 2(7)$ implies that 14 is an odd integer.

3. For the following problems, you do **not** need to show your work.

[2] (a) List four integers n such that $n \equiv 3 \pmod{7}$.

[2] (b) Give an example of a tautology and an example of a contradiction. Your example can be specific or symbolic (using P, Q, etc.)

[2] (c) What is the contrapositive of the statement "If p is a prime number, then $p = 2$ or p is an odd number"?

[2] (d) What does it mean to say that a subset of the integers is *inductive*?

[2] (e) What is the negation of the statement "For all $m \in \mathbb{Z}$, there exists some $n \in \mathbb{Z}$ such that $2m - 5n = 7$"?

[6] 4. Prove the following logical equivalence: $(P \wedge Q) \to (R \vee S) \equiv (\neg R \wedge \neg S) \to (\neg P \vee \neg Q)$.

[6] 5. Show that the syllogism $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautology.

6. Are the following propositions true or false? Justify your conclusion with a proof[1] or counterexample.

[4] (a) For $a, b, c \in Z$, with $a \neq 0$, if $a|(bc)$, then $a|b$ or $a|c$.

[4] (b) For any $n \in \mathbb{N}$, if n^3 is even, then n is even.

[4] (c) For any $a, b \in \mathbb{Z}$, if $a \equiv 5 \pmod 9$ and $b \equiv 7 \pmod 9$, then $(a + b) \equiv 3 \pmod 9$.

[1]Any true statements on this page can be proved using a direct proof of either the original statement or its contrapositive.

[5] 7. Use proof by cases to prove the following proposition: For each $n \in \mathbb{Z}$, if $n \not\equiv 0 \pmod 3$, then $n^2 \equiv 1 \pmod 3$.
Hint: The division algorithm gives three possible remainders when n is divided by 3.

[5] 8. Use proof by contradiction to prove the following proposition: For each $n \in \mathbb{N}$, $\sqrt{3n+2}$ is not a natural number.
Hint: The two problems on this page are related.

[6] 9. Use mathematical induction to prove that for each natural number n,

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Total Points: 60