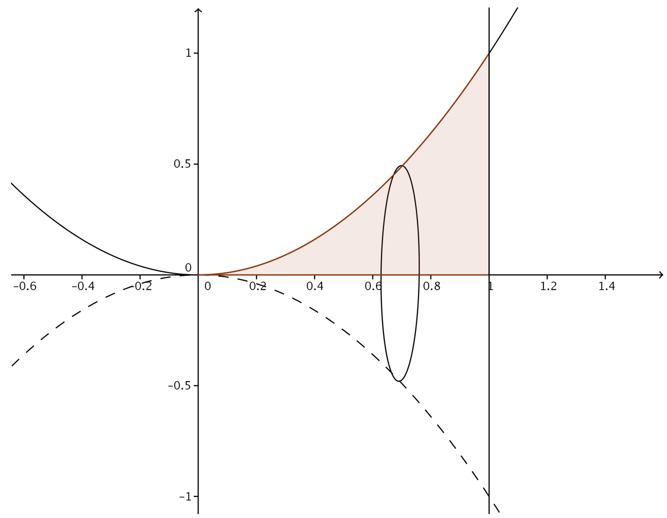
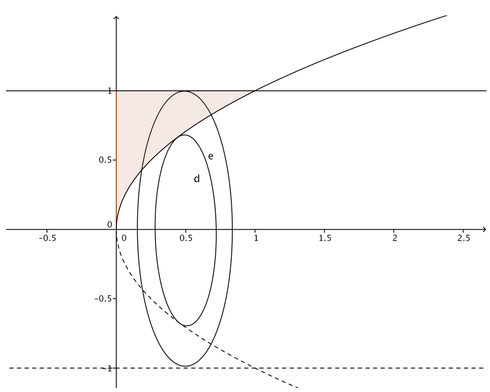
1. Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $x = 1$, and $y = 0$ about the x-axis.

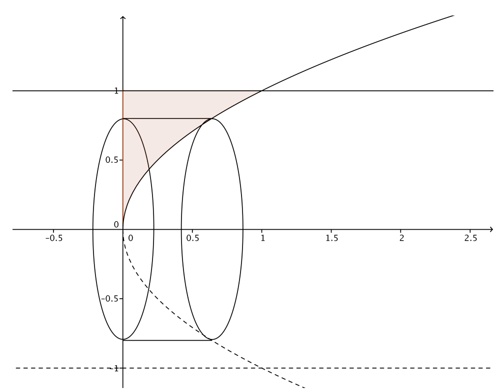


Using the disc method, our cross-sectional area is $A(x) = \pi y^2 = \pi(x^2)^2 = \pi x^4$. The volume is therefore

$$V = \int_0^1 \pi x^4 \, dx = \frac{\pi}{5}.$$

2. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 1$, and $x = 0$ about the x-axis.





Using washers, our cross-sectional area is $A(x) = \pi(1)^2 - \pi(\sqrt{x})^2 = \pi(1-x)$, so the volume is

$$V = \int_0^1 \pi(1 - x) \, dx = \frac{\pi}{2}.$$

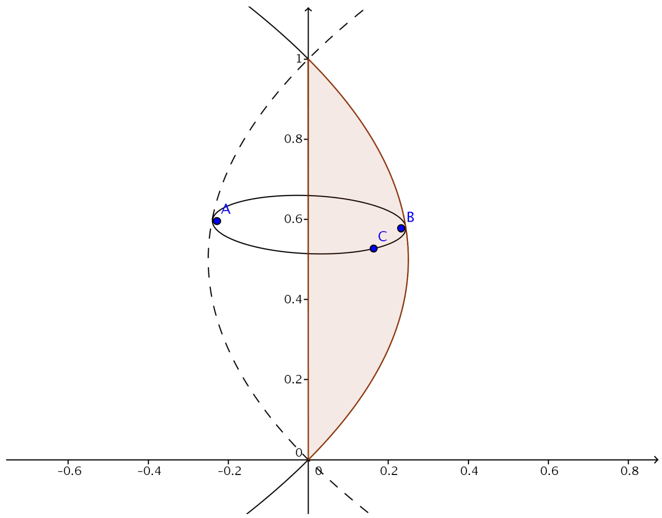
If we use cylindrical shells instead, each shell has surface area $A(y) = 2\pi r(y)h(y) = 2\pi y(y^2) = 2\pi y^3$, so the volume is

$$V = \int_0^1 2\pi y^3 \, dy = \frac{\pi}{2}.$$

3. Repeat Problem 2, but revolving about the y-axis.

   The resulting solid is identical to the one in problem 1, except that it's revolved around the y-axis instead of the x-axis, and the volume is again $\pi/5$.

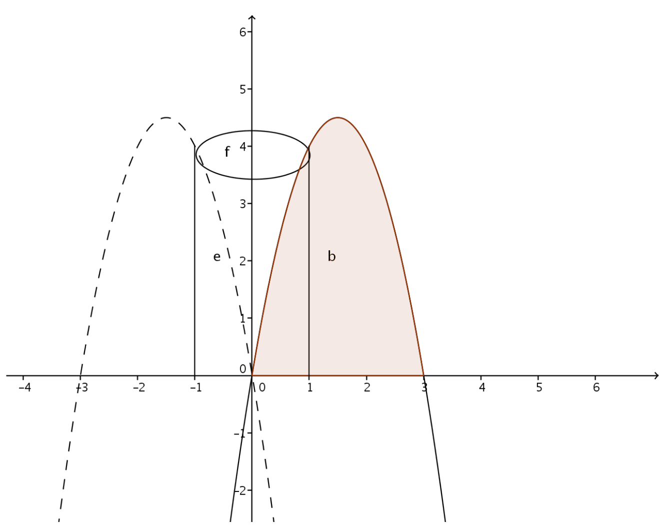
4. Find the volume of the solid generated by revolving the region bounded by $x = y - y^2$ and $x = 0$ about the y-axis.



   Using discs, we have cross-sectional area $A(y) = \pi(y - y^2)^2$, so the volume is

$$V = \int_0^1 \pi(y^2 - 2y^3 + y^4)\, dy = \frac{\pi}{30}.$$

5. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = 6x - 2x^2$ and $y = 0$, about the y-axis.

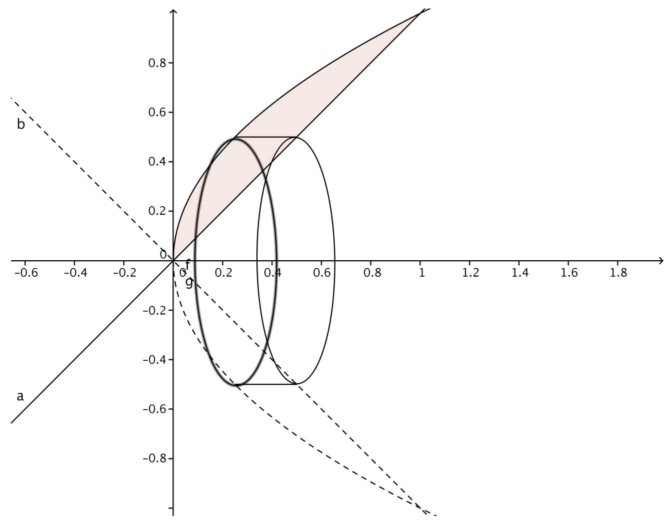


   Note that $6x - 2x^2 = 2x(3 - x)$, so the graph $y = 6x - 2x^2$ intersects the x-axis when $x = 0$ and $x = 3$. Using shells, the height of each cylinder is $h = y = 6x - 2x^2$, and the radius is

$r = x$, so the surface area of each shell is $2\pi x(6x - 2x^2)$, and the volume is

$$V = \int_0^3 2\pi x(6x - 2x^2)\,dx = 27\pi.$$

6. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = \sqrt{x}$ about the x-axis.



The graphs $y = x$ and $y = \sqrt{x}$ can be rewritten as $x = y^2$ and $x = y$. Using cylindrical shells, the radius of each cylinder is $r = y$, and the height is $h = y - y^2$, so the surface area of each shell is $2\pi y(y - y^2)$, and the volume is

$$V = \int_0^1 2\pi(y^2 - y^3)\,dy = \frac{\pi}{6}.$$

7. Find the length of the curve $y = \dfrac{1}{12}x^3 + \dfrac{1}{x}$, for $x \in [1, 4]$.

Arc length is given by $L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$, so we first compute

$$\begin{aligned}
1 + (y')^2 &= 1 + \left(\frac{1}{4}x^2 - \frac{1}{x^2}\right)^2 = 1 + \left(\frac{x^4 - 4}{4x^2}\right)^2 \\
&= 1 + \frac{x^8 - 8x^4 + 16}{16x^4} = \frac{16x^4 + x^8 - 8x^4 + 16}{16x^4} \\
&= \left(\frac{x^4 + 4}{4x^4}\right)^2.
\end{aligned}$$

Thus,

$$L = \int_0^1 \sqrt{\left(\frac{x^4 + 4}{4x^4}\right)^2}\,dx = \int_0^1 \left(\frac{x^4}{4} + \frac{1}{x^4}\right)\,dx = 6.$$

8. Find the area of the surface obtained by revolving $y = \sqrt{x}$, for $x \in [0, 1]$, about the x-axis.

   Since we're revolving about the x-axis and y is given as a function of x, we use the formula $S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\, dx$. With $f(x) = \sqrt{x}$, we have

$$1 + f'(x)^2 = 1 + \left(\frac{1}{2\sqrt{x}}\right)^2 = \frac{4x+1}{4x}.$$

   The surface area is thus

$$S = 2\pi \int_0^1 \sqrt{x}\sqrt{\frac{4x+1}{4x}}\, dx = \pi \int_0^1 \sqrt{4x+1}\, dx = \frac{\pi}{4} \cdot \frac{2}{3}(4x+1)^{3/2}\bigg|_0^1 = \frac{\pi}{6}(5^{3/2} - 1).$$

9. Find the area of the surface obtained by revolving $y = x^2$, for $x \in [0, 1]$, about the y-axis.

   This time we're revolving a function of x about the y-axis, so we use the formula $S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\, dx$. With $f(x) = x^2$ we have $f'(x) = 2x$, so

$$S = 2\pi \int_0^1 x\sqrt{1 + 4x^2}\, dx = \frac{\pi}{4} \int_1^5 \sqrt{u}\, du = \frac{\pi}{6}(5^{3/2} - 1),$$

   using the substitution $u = 1 + 4x^2$, so $du = 8x\, dx$, and when $x = 0$, $u = 1$, and when $x = 4$, $u = 5$. Notice that the answer is the same as the previous problem. Draw a picture for both surfaces and you'll see that this is not a coincidence.

10. Find the area of the surface obtained by revolving $x = 1 + 2y^2$, $1 \le y \le 2$, about the x-axis.

    Since we have x given as a function of y and we're revolving about the x-axis, we use the formula $S = \int_c^d y\sqrt{1 + g'(y)^2}\, dy$. Here, $g(y) = 1 + 2y^2$, so $g'(y) = 4y$. Thus,

$$S = 2\pi \int_1^2 y\sqrt{1 + 16y^2}\, dy = \frac{\pi}{16} \int_{17}^{65} \sqrt{u}\, dy = \frac{\pi}{24}(65^{3/2} - 17^{3/2}).$$