University of Lethbridge
Department of Mathematics and Computer Science
15^{th} October, 2014, 5:00-5:50 pm
Math 4310 - Term Test I

Last Name:____

First Name:____

Student Number:____________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

There is a list of potentially useful formulas available on the last page of the exam.

For grader's use only:

Page	Grade
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3	/8
4	/9
5	/6
Total	/35

1. For each of the following, give an example, or explain why no such example exists:

[3] (a) A subset of a topological space that is both open and closed.

[3] (b) A continuous function $f : X \to Y$, if X is equipped with the indiscrete topology.

[3] (c) An interior point that is not a limit point.

[3] (d) A metric space that is not Hausdorff.

[8] 2. Let $X = l^1(\mathbb{R}) = \left\{ \sum_{n=1}^{\infty} a_n \;\middle|\; \sum_{n=1}^{\infty} |a_n| < \infty \right\}$ be the space of absolutely convergent sequences of real numbers. Prove that the function $d : X \times X \to \mathbb{R}$ given by

$$d\left(\sum a_n, \sum b_n\right) = \sum_{n=1}^{\infty} |a_n - b_n|$$

is well-defined (i.e. that $d(x, y)$ is finite for all $x, y \in X$) and makes X into a metric space.

3. (a) Define what it means for a set $\mathcal{B}$ of subsets of a set X to be a **basis** for a topology on
[3] X.
(Either of the two definitions we discussed is acceptable.)

(b) Let X and Y be topological spaces, and let $\mathcal{B}$ be a basis for the topology on Y. Prove that a function $f : X \to Y$ is continuous if and only if $f^{-1}(U)$ is open in X for every
[6] $U \in \mathcal{B}$.

[6] 4. Solve **one** of the following two problems:

(a) Let X, Y, and Z be topological spaces, and equip $X \times Y$ with the product topology. Show that a map $f : Z \to X \times Y$ is continuous if and only if the maps $\pi_X \circ f : Z \to X$ and $\pi_Y \circ f : Z \to Y$ are continuous.
(Hint: one direction is easy. For the other, use 3(b).)

(b) Given a topological space X, let X_0 denote the space with the same underlying set as X, but with the cofinite topology. Show that the identity map $I : X \to X_0$ (given by $I(x) = x$) is continuous if and only if X is a T_1 space.

Hint: X is T_1 if and only if finite point sets are closed.

 Total Points: 35