

1. The limit $\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$ represents the derivative of a function f at some point a .

(a) Identify the function and the point.

Comparing to $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$, we have $f(x) = \frac{1}{x}$ and $a = 2$.

(b) Evaluate the limit.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} &= \lim_{h \rightarrow 0} \frac{\frac{2 - (2+h)}{(2+h)(2)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{-h}{2(2+h)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{2(2+h)} = -\frac{1}{4}. \end{aligned}$$

2. (**Bonus**) Using the definition of the derivative, show that for any differentiable function f and constant c , we have $(c \cdot f)'(x) = c \cdot f'(x)$

Using the definition, we have:

$$\begin{aligned} (c \cdot f)'(x) &= \lim_{h \rightarrow 0} \frac{(cf)(x+h) - (cf)(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c(f(x+h)) - c(f(x))}{h} \\ &= \lim_{h \rightarrow 0} \frac{c(f(x+h) - f(x))}{h} \\ &= c \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = c(f'(x)). \end{aligned}$$

3. Compute the derivatives of the following functions:

(a) $f(x) = 3x^3 - 2x^2 + \sqrt{2}$

Using sum, constant, and power rules, we have

$$f'(x) = 3(3x^2) - 2(2x) + 0 = 9x^2 - 4x.$$

(b) $g(x) = x^2 \sin(x)$

Using the product rule (and the derivatives for x^2 and $\sin(x)$), we have

$$\begin{aligned} g'(x) &= \left(\frac{d}{dx}(x^2) \right) \sin(x) + x^2 \left(\frac{d}{dx}(\sin(x)) \right) \\ &= 2x \sin(x) + x^2 \cos(x). \end{aligned}$$

(c) $h(x) = \frac{x^2 + x}{2 - 3x}$

Using the quotient rule, we have

$$\begin{aligned} h'(x) &= \frac{\left(\frac{d}{dx}(x^2 + x) \right) (2 - 3x) - (x^2 + x) \left(\frac{d}{dx}(2 - 3x) \right)}{(2 - 3x)^2} \\ &= \frac{(2x + 1)(2 - 3x) + 3(x^2 + x)}{(2 - 3x)^2}. \end{aligned}$$