Chapter 1: Systems of Linear Equations
Math 1410
University of Lethbridge
Spring 2015

Linear equations

Definition

A linear equation in n variables $x_1, x_2, \ldots, x_n$ is an equation of the form

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b,$$

where $a_1, a_2, \ldots, a_n, b$ are constants (real numbers).

Examples:

Systems of equations

A system of equations (linear or otherwise) is a collection of one or more equations for which we want to find all common solutions (if any).

Example (A non-linear system)

Solve the system

$$x^2 + y^2 = 5$$
$$x^2 - y^2 = 1$$

Solutions to non-linear systems can be very complicated (and even impossible to solve exactly). For linear systems (which we will study) there are systematic methods for solving them.

A "biological" example

Example

A biologist wants to feed rats a diet consisting of fish and meal so that the rats get 30 grams of protein and 20 grams of carbohydrate every day. If fish consists of 70% protien and 10% carbohydrate, while meal consists of 30% protein and 60% carbohydrate, how much of each food is needed every day?

Geometric solutions

Linear equation in two dimensions:

$$ax + by = c$$

Linear equation in three dimensions:

$$ax + by + cz = d$$

Algebraic solutions

A visual approach only works in two or three dimensions. (Realistically, it doesn't work that well in 3D either.)

Example

Solve the system:

$$2x - 3y = 7$$
$$-x + 4y = 2$$

Some applied situations (economics, air traffic control) involve hundreds or even thousands of variables. In these cases, only algebraic (or numerical) methods will work.