Name and student number:

[2] 1. (a) Suppose $k \leq l$ and let $\mathbb{N}_k = \{1, 2, \ldots, k\}$ and let $\mathbb{N}_l = \{1, 2, \ldots, l\}$. Show that the function $f : \mathbb{N}_k \to \mathbb{N}_l$ given by $f(n) = n$ for $n = 1, 2, \ldots, k$ is one-to-one.

[1] (b) Is the function f in part (a) necessarily onto? Explain.

[2] (c) Suppose $k \leq l$, and suppose A and B are sets, with $|A| = k$ and $|B| = l$. Let $g : A \to \mathbb{N}_k$ and $h : B \to \mathbb{N}_l$ be bijections. (The bijections g and h exist, by the definition of the cardinality of finite sets.) Use part (a) and the bijections g and h to construct a one-to-one function from A to B.

[5] 2. Suppose every student at a university has three initials, say F.M.L. for First, Middle, Last. How many students must the university have to guarantee that two students have the same initials?

Hint: Let A be the set of university students, and let B be the set of letters in the alphabet, so $|B| = 26$. The set of all possible initials can be identified with $B \times B \times B$, since an ordered triple of letters (b_1, b_2, b_3) corresponds to a set of initials. If $f : A \to B \times B \times B$ is the function that assigns each student to their initials, how big must the cardinality of A be to guarantee that f cannot be one-to-one?

Total: 10 points