UNIVERSITY OF TORONTO AT MISSISSAUGA

June 2007 Examination

MAT232HF

Instructor: Sean Fitzpatrick

Duration: 3 Hours

NO AIDS ALLOWED. **Total: 100 marks**

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

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Students may be charged with an academic offence for possessing the following items during the writing of an exam: any unauthorized aid, cell phones, pagers, wristwatch computers, personal digital assistants (PDAs), IPODS, MP3 players, or any other electronic device. If any of these items are in your possession, please turn them off and put them with your belongings at the front of the room before the examination begins and no penalty will be imposed. A penalty MAY BE imposed if any of these items are kept with you during the writing of your exam.

Please note, students are **NOT** *allowed to petition to RE-WRITE a final examination.*

FOR MARKER'S USE ONLY			
Problem 1:	/22	Problem 4:	/15
Problem 2:	/16	Problem 5:	/15
Problem 3:	/12	Problem 6:	/20
		TOTAL:	/100

Instructions: There are 6 problems in this exam, sorted by topic (not necessarily by difficulty). Please read over the entire exam before beginning, and make sure that there are no missing pages.

Each of the six problems has several sub-parts. The value of each sub-part is indicated in the left-hand margin in square brackets. In the case that one sub-part depends on a part preceeding it, the correct use of an incorrect previous answer will receive full credit.

Please write your solutions in the space provided. If there is not enough space you may continue your solution on the back of the *previous* page. There are also two pages at the end of the exam for rough work.

Try to solve as many problems as possible. Partial credit will be given for partially correct work.

To aid in evaluating some of the integrals which you may encounter, a list of indefinite integrals is given below:

$$\int u^a du = \frac{u^{a+1}}{a+1}, \text{ for } a \neq -1$$

$$\int \sin\theta d\theta = -\cos\theta$$

$$\int \cos\theta d\theta = \sin\theta$$

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta$$

$$\int \sin^3\theta d\theta = \frac{1}{3}\cos^3\theta - \cos\theta$$

$$\int \sin^4\theta d\theta = \frac{3\theta}{8} - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta$$

Total Marks: 100

[4] 1. (a) For the function $f(x,y) = \sin(x^2 \ln y)$, find $f_x(x,y)$ and $f_y(x,y)$.

(b) Let $w = \sqrt{u^2 + v^2 + z^2}$, where $u = 3e^t \sin s$, $v = 3e^t \cos s$, and $z = 4e^t$.

[2] (i) Write out the chain rule formulas for $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$.

[4] (ii) Calculate $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ using the formulas in (i).

[4] (iii) Calculate $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ by explicitly making the given substitutions. Confirm that your answers agree with those in (ii).

Total Marks: 100

(c) Let $h(x, y, z) = x^3 + y^3 + z^3 - 5xyz$.

[3] (i) Calculate $\nabla h(2, 1, 1)$.

[2] (ii) Calculate the directional derivative of $h(x, y, z)$ at the point $(2, 1, 1)$ in the direction of $\vec{a} =< 1, 0, -1 >$.

[3] (iii) Find the equation for the tangent plane to the surface $h(x, y, z) = 0$ at the point $(2, 1, 1)$.

Total Marks: 100

2. Let D be the region contained within the curve given in polar coordinates by $r = 2\sin\theta$.

[4] (a) Sketch the region D, and find its area.

(b) Suppose that D bounds a plane lamina of mass density $\delta(x, y) = \sqrt{x^2 + y^2}$. Find
[3] the mass of the lamina.

Total Marks: 100

[3] (c) Find the centroid of the region D, with respect to the density $\delta(r, \theta) = 1$.

[2] (d) Find the volume of the solid of revolution obtained by revolving D about the x-axis.

Hint: $V = A \cdot d$, by the First Theorem of Pappus.

Total Marks: 100

[4] (e) Find the volume of the solid which lies above D and below the surface $z = 4 - \sqrt{(x^2 + y^2)}$.

Total Marks: 100

3. Consider the parametric surface S given by the equations

$$x = a \sin u \cos v,\ y = b \sin u \sin v,\ z = c \cos u,$$

for $0 \le u \le \pi$ and $0 \le v \le 2\pi$.

[4] (a) Use trigonometric identities (such as $\sin^2 t + \cos^2 t = 1$) to eliminiate the parameters u and v, and obtain the equation of a quadric surface.

[3] (b) Identfiy and sketch this surface, if $a = 1$, $b = 1$, and $c = 2$.

Total Marks: 100

[5] (c) Set up the integral $A(S) = \iint\limits_R \left| \vec{N}(u,v) \right| dudv$, in terms of a, b and c, which gives the surface area of S (where $\vec{N}(u,v) = \vec{r}_u \times \vec{r}_v$ is the normal to the surface S at $(u,v) \in R$).

Evaluate the integral in the case $a = b = c$. (Your answer should look familiar.)

Total Marks: 100

4. Let D be the region in the xy-plane bounded by the curves $y = x$, $y = 2x$, $xy = 1$ and $xy = 2$.

[3] (a) Sketch the region, noting all points of intersection of the given curves.

[2] (b) Find a change of variables $u = f(x, y)$, $v = g(x, y)$ that transforms the region D into the rectangle $1 \leq u \leq 2$, $1 \leq v \leq 2$.

[2] (c) Solve for x and y in terms of u and v.

Total Marks: 100

[3] (d) Compute the Jacobian $J = \dfrac{\partial(x,y)}{\partial(u,v)}$.

[5] (e) Evaluate $\displaystyle\iint_D \left(\frac{(x-y)^2}{x^2} - 1 \right) dA$.

Total Marks: 100

5. Consider the function $f(x, y) = 4xy - 2x^4 - y^2$.

[4] (a) Find the critical points of $f(x, y)$.

[6] (b) Classify all critical points found in (a).

Total Marks: 100

[5] (c) Find the absolute maximum and minimum (if any) of $f(x,y)$ subject to the constraint $2x-y=1$.

Total Marks: 100

6. Consider the vector field $\vec{F}(x, y) = (3x^2 + 2y^2)\hat{\imath} + (4xy + 6y^2)\hat{\jmath}$.

[3] (a) Show that $\vec{F}(x, y)$ is conservative.

[5] (b) Find a function $f(x, y)$ such that $\nabla f(x, y) = \vec{F}(x, y)$.

Total Marks: 100

(c) If C is the segment of the parabola $x = 2y^2$ from $(0,0)$ to $(2,1)$, compute the line integral $\int_C \vec{F} \cdot d\vec{r}$:

[5] (i) Directly.

Total Marks: 100

[3] (ii) Using the Fundamental Theorem of Calculus for line integrals.

[4] (d) State Green's Theorem in the plane.

Total Marks: 100

Extra space for rough work. Do **not** tear out this page.

Total Marks: 100

Extra space for rough work. Do **not** tear out this page.

Total Marks: 100