Name:

The questions below are worth 5 points each, and the quiz is out of 10. You can either choose two, or solve all 3 for a maximum score of 15/10. Feel free to use the back of the page for extra space.

1. Suppose $T \in \mathcal{L}(V)$ and $\lambda \in \mathbb{F}$. Prove that λ is an eigenvalue of T if and only if $\overline{\lambda}$ is an eigenvalue of T^*.

2. Suppopse $S, T \in \mathcal{L}(V)$ are self-adjoint. Prove that ST is self-adjoint if and only if $ST = TS$.

3. Suppose that T is a normal operator on V and that 3 and 4 are eigenvalues of T. Prove that there exists a vector $v \in V$ such that $\|v\| = \sqrt{2}$ and $\|Tv\| = 5$.

   Hint for #3: Eigenvectors corresponding to distinct eigenvalues of a normal operator are orthogonal. (Hint hint: Pythagorus.)

Total: 10 points