Name: **Tutorial time:**

Problem you want feedback on:

Please complete all problems below.

1. In each case, give an elementary matrix E such that $EA = B$:

   (a) $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

   (e) $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

2. Find an invertible matrix U such that UA is in row-echelon form, where $A = \begin{bmatrix} 3 & 5 & 0 \\ 3 & 7 & 1 \\ 1 & 2 & 1 \end{bmatrix}$. How do you know U is invertible?

3. Write the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 6 \end{bmatrix}$ as a product of elementary matrices.

4. Prove that if A is invertible, so is A^T, and $(A^T)^{-1} = (A^{-1})^T$.

   Hint: You will need the fact that for any matrices M and N such that the product MN is defined, $(MN)^T = N^T M^T$. Use the fact that the inverse is unique.

5. Prove that if A is invertible and $a \in \mathbb{R}$ is a scalar, then aA is invertible.

   Hint: Find an expression for $(aA)^{-1}$.