MATH 1560 - Tutorial #1 Solutions

Additional practice:

1. Given $f(x) = \sqrt{x}$, $g(x) = x^2$, and $h(x) = e^{2x}$, compute:

   (a) $f(f(x)) = \sqrt{\sqrt{x}} = (x^{1/2})^{1/2} = x^{1/4} = \sqrt[4]{x}$.

   (b) $f(g(x)) = \sqrt{x^2} = |x|$.
   Note that it is only true that $\sqrt{x^2} = x$ when $x \geq 0$. The function $f(x)$ is the *positive* square root function. If $x < 0$ we have $\sqrt{x^2} = -x$. For example, if $x = -2$, then $x^2 = 4$, and $\sqrt{x^2} = \sqrt{4} = 2 = -(-2) = -x$.

   (c) $g(f(x)) = (\sqrt{x})^2 = x$.
   Here we do get back x. Note, however, that the domain of this function includes only $x \geq 0$, since the square root of a negative number is not a real number.

   (d) $g(h(x)) = (e^{2x})^2 = e^{2(2x)} = e^{4x}$.

   (e) $h(f(x)) = e^{2\sqrt{x}}$.

2. Determine if the function is even, odd, or neither:

   (a) $f(x) = \dfrac{x^3}{x^2+1}$
   Since $f(-x) = \dfrac{(-x)^3}{(-x)^2+1} = \dfrac{-x^3}{x^2+1} = -f(x)$, the function is odd.

   (b) $g(x) = x|x|$
   Since $g(-x) = (-x)|-x| = -x|x| = -g(x)$, the function is odd. (It is a general property of the absolute value function that $|-x| = |x|$ for any x, since the absolute value removes any negative sign.)

   (c) $h(x) = \cos(x^5)$
   We have $h(-x) = \cos((-x)^5) = \cos(-x^5) = \cos(x^5) = h(x)$, since cosine is an even function.

Assigned problems:

1. Find the domain of the following functions. Write your answer in interval notation.

   (a) $f(x) = \dfrac{x+1}{x^2+5x+6} = \dfrac{x+1}{(x+2)(x+3)}$ has domain

$$\{x \in \mathbb{R} \,|\, x \neq -2, -3\} = (-\infty, -3) \cup (-3, -2) \cup (-2, \infty),$$

   since we cannot divide by zero.

(b) For $g(x) = \sqrt{x+2} - \sqrt{3-x}$, we need $x + 2 \geq 0$ for the first square root to be defined, **and** we need $3 - x \geq 0$ for the second square root to be defined. (Both must be defined for $g(x)$ to be defined.) Rearranging these inequalities, we need $x \geq -2$ (equivalently, $-2 \leq x$) and $3 \geq x$ (equivalently, $x \leq 3$). The domain is therefore

$$\{x \in \mathbb{R} \mid -2 \leq x \text{ and } x \leq 3\} = \{x \in \mathbb{R} \mid -2 \leq x \leq 3\} = [-2, 3].$$

(c) For $h(x) = \ln(x^2 - 1)$, we need $x^2 - 1 > 0$, since the domain of the logarithm function is $(0, \infty)$. To ensure that $x^2 - 1 = (x-1)(x+1)$ is positive, we must have either $x > 1$, so that both factors are positive, or $x < -1$, so that both factors are negative. The domain is therefore

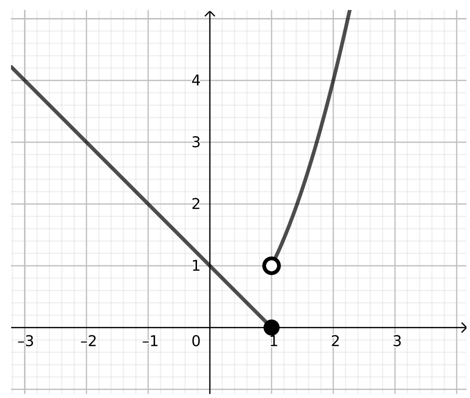
$$\{x \in \mathbb{R} \mid x < -1 \text{ or } x > 1\} = (-\infty, -1) \cup (1, \infty).$$

2. Consider the function $f(x) = \begin{cases} 1 - x, & \text{if } 1 \leq 1 \\ x^2, & \text{if } x > 1. \end{cases}$. Evaluate $f(-1)$, $f(0)$, $f(1)$, and $f(2)$. Then, sketch a rough graph.

Note that for $x = -1, 0, 1$ we have $x \leq 1$, so we use the expression $1 - x$ to evaluate f: $f(-1) = 1 - (-1) = 2$, $f(0) = 1 - 0 = 1$, and $f(1) = 1 - 1 = 0$.

Since $2 > 1$, we use the expression x^2 to evaluate f, giving us $f(2) = x^2 = 4$.

The sketch is given as follows:



3. Rewrite $f(x) = |x^2 - x - 2|$ as a piecewise function (without the absolute value).

With $g(x) = x^2 - x - 2$, we note that

$$f(x) = \begin{cases} g(x), & \text{if } g(x) \geq 0 \\ -g(x), & \text{if } g(x) < 0 \end{cases}$$

We have $x^2 - x - 2 = (x+1)(x-2) \geq 0$ when either $x \leq -1$ (both factors are negative) or $x \geq 2$ (both factors are positive), and $x^2 - x - 2 < 0$ when $-1 < x < 2$ (when $x + 1 > 0$ but $x - 2 < 0$). Thus,

$$f(x) = \begin{cases} x^2 - x - 2, & \text{if } x \leq -1 \\ -x^2 + x + 2, & \text{if } -1 < x < 2 \\ x^2 - x - 2, & \text{if } x \geq 2 \end{cases}$$