MATH 1410 - Tutorial #3 Supplement

Wednesday, January 31

1. Suppose you are given vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$ and asked to compute vectors $\vec{w}_\parallel$ and $\vec{w}_\perp$ such that $\vec{w}_\parallel$ is parallel to $\vec{v}$, $\vec{w}_\perp$ is orthogonal to $\vec{v}$, and

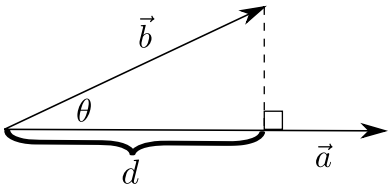
$$\vec{w}_\parallel + \vec{w}_\perp = \vec{w}.$$

It is a good idea, in such problems, to construct a diagram to illustrate the problem. Which of the following are true statements about that diagram?

- It should be drawn as tiny as possible so it doesn't take up too much room.
- It should be drawn reasonably large, so that it's easy to read.
- The vectors should be plotted in three dimensions with accurate magnitudes and directions.
- Magnitudes and directions are unimportant.
- Vectors shouldn't be drawn at right angles unless you know they're orthogonal.
- The diagram should be a simple two-dimensional schematic.
- All vectors should be clearly labelled.
- The diagram can be used to establish any calculations that are required.

Having decided on which of the above are important, find the vectors $\vec{w}_\parallel$ and $\vec{w}_\perp$, given $\vec{v} = \langle 2, -1, 2\rangle$ and $\vec{w} = \langle 4, -3, 1\rangle$.

2. Explain, using the diagram below, why the distance d as indicated is given by $d = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|}$.



3. Show that

$$(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = \|\vec{u}\|^2 - \|\vec{v}\|^2$$

for any vectors $\vec{u}$, $\vec{v}$ in $\mathbb{R}^3$.