Practice Problems for Quiz 3
Math 2000A

Quiz #3 will take place in class on Thursday, September 25.

1. I've just posted a handout to our Moodle page entitled "Logical Equivalences". It contains a list of some of the rules from propositional logic we've encountered. Work your way through this list as follows: let A, B, C be sets. Let P be the statement $x \in A$, Q the statement $x \in B$, and R the statement $x \in C$. Each of the definitions (e.g. of $P \vee Q$, etc.) and each of the equivalences corresponds to a definition or rule involving sets. Work out what each of these are.

   (For example, $P \vee Q$ becomes $(x \in A) \vee (x \in B)$, which by definition means that $x \in (A \cup B)$, the equivalence $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ corresponds to the set equality $A \cup (B \cup C) = (A \cup B) \cup C$, and so on.) If you assume that A, B, C are subsets of some universal set U, then you can consider a tautology T to be equivalent to the statement $x \in U$, and a contradiction F to mean the same thing as $x \in \emptyset$.

   Once you've done all of those, try the following:

   Two equivalences missing from the list that we frequently use are $P \to Q \equiv \neg P \vee Q$ and $P \wedge \neg Q \equiv \neg(P \to Q)$. Translate these into statements about sets.

2. Determine which of the following assertions are true and which are false (give a reason for each one):

   (a) $\emptyset \in \emptyset$

   (b) $1 \in \{1\}$

   (c) $\{1, 2\} = \{2, 1\}$

   (d) $\emptyset = \{\emptyset\}$

   (e) $\emptyset \subseteq \{\emptyset\}$

   (f) $1 \subseteq \{1\}$

   (g) $\{1\} \subseteq \{1, 2\}$

   (h) $\emptyset \in \{\emptyset\}$

3. For each set below, determine (i) the cardinality of the set, and (ii) the power set of the set.

   (a) $\emptyset$

   (b) $\{1\}$

   (c) $\{1, 2\}$

   (d) $\{1, 2, 3\}$

   (e) $\{\emptyset, \{1\}\}$

   (f) $\mathcal{P}(\mathcal{P}(\emptyset))$

   (For part (f), $\mathcal{P}(A)$ denotes the power set of A.)

4. Assume that the universal set is the set $\mathbb{R}$ of real numbers. Let A, B, C, D be subsets of $\mathbb{R}$, where

$$\begin{array}{ll} A = \{-3,-2,2,3\} & B = \{x \in \mathbb{R} \,|\, x^2 = 4 \text{ or } x^2 = 9\} \\ C = \{x \in \mathbb{R} \,|\, x^2 + 2 = 0\} & D = \{x \in \mathbb{R} \,|\, x > 0\} \end{array}$$

(a) Is the set A equal to the set B?

(b) Is the set A a subset of the set B?

(c) Is the set C equal to the set D?

(d) Is the set C a subset of the set D?

For the next two problems, the "roster method" refers to the method of describing a set by listing its elements, e.g. finite sets such as $\{1,2,3,4\}$ or infinite sets such as $\{1,2,3,\ldots\}$, and S^c denotes the compliment of a set S.

5. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$ be the universal set, and let

$$\begin{array}{ll} A = \{3,4,5,6,7\} & B = \{1,5,7,9\} \\ C = \{3,6,9\} & D = \{2,4,6,8\} \end{array}$$

Use the roster method to list all of the elements of each of the following sets.

(a) $A \cap B$

(b) $A \cup B$

(c) $(A \cup B)^c$

(d) $A^c \cap B^c$

(e) $(A \cup B) \cap C$

(f) $A \cap C$

(g) $B \cap C$

(h) $(A \cap C) \cup (B \cap C)$

(i) $B \cap D$

(j) $(B \cap D)^c$

(k) $A \setminus D$

(l) $B \setminus D$

(m) $(A \setminus D) \cup (B \setminus D)$

(n) $(A \cup B) \setminus D$

6. Repeat problems (a)-(n) of the previous question, if the sets A, B, C, D are given by

$$\begin{array}{ll} A = \{n \in \mathbb{N} \,|\, n \geq 7\} & B = \{n \in \mathbb{N} \,|\, n \text{ is odd}\} \\ C = \{n \in \mathbb{N} \,|\, n \text{ is a multiple of } 3\} & D = \{n \in \mathbb{N} \,|\, n \text{ is even}\} \end{array}$$

7. Consider the following assertion: If $A \subseteq B$, then $B^c \subseteq A^c$.

(a) Write the negation of this statement.

(b) Write the contrapositive of this statement.

(c) Identify three different conditional statements contained within this statement.

8. Let A and B be subsets of some universal set U. Prove each of the following:

(a) $A \cap B \subseteq A$

(b) $A \cap A = A$

(c) $A \subseteq A \cup B$

(d) $A \cup A = A$

(e) $A \cap \emptyset = \emptyset$

(f) $A \cup \emptyset = A$