University of Lethbridge
Department of Mathematics and Computer Science
31st October, 2016, 9:00 - 9:50 am

MATH 1410A - Test #2

Last Name:____

First Name:____

Student Number:____

Tutorial Section: ____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, including intermediate steps. (You should show your work if you want to earn part marks.) Unless otherwise indicated, failure to justify your work may result in loss of marks, even for a correct answer.

No external aids are allowed, with the exception of a 5-function calculator.

For grader's use only:

Page	Grade
2	/12
3	/12
4	/10
5	/8
6	/8
Total	/50

A

1. Assume that A, B, and X are matrices of the same size.

[3] (a) Solve for X in terms of A and B, given that $2A - 3X = B$.

[3] (b) Determine the entries of the matrix X from part (a) if $A = \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -4 & 5 \end{bmatrix}$.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a matrix transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 4 \end{bmatrix}.$$

[3] (a) Determine the matrix of T. (That is, find the matrix A such that $T(\vec{x}) = A\vec{x}$ for any vector $\vec{x}$ in $\mathbb{R}^3$.)

[3] (b) Compute $T\left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\right)$.

[4] 3. Determine vectors $\vec{u}$ and $\vec{v}$ such that $U = \text{span}\{\vec{u}, \vec{v}\}$, where U is the subspace

$$U = \left\{ \begin{bmatrix} 3a - 2b \\ b - 3a \\ 5a + 4b \end{bmatrix} \middle| \; a, b \text{ are real numbers} \right\}.$$

(Recall that $\text{span}\{\vec{u}, \vec{v}\} = \{a\vec{u} + b\vec{v} \,|\, a, b \in \mathbb{R}\}$.)

[4] 4. Verify that $x = -\dfrac{7}{2}, y = 2, z = \dfrac{1}{2}$ is a solution to the system
$$\begin{aligned} 2x + y \phantom{{}+3z} &= -5 \\ x \phantom{{}+y} + 3z &= -2 \\ -y + 6z &= 1 \end{aligned}$$

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(\vec{u}) = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ and $T(\vec{v}) = \begin{bmatrix} 0 \\ 6 \\ -3 \end{bmatrix}$ for

[4] some vectors $\vec{u}, \vec{v}$ in $\mathbb{R}^2$. What is the value of $T(5\vec{u} - 3\vec{v})$?

 Total Points: 50

6. Each of the matrices below is in row-echelon form, and represents a system of linear equations in the variables x, y, and z. If the system has no solution, explain why. If it does, determine the solution using either back substitution, or by finding the reduced row-echelon form of the matrix.

[10]

(a) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 5 & -4 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

[8] 7. Compute the matrix products AB and BA, where $A = \begin{bmatrix} 3 & 1 & -2 \\ -4 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 3 & 2 \\ 5 & -3 \end{bmatrix}$.

[8] 8. Solve the system

$$\begin{aligned} 3x - 4y - 5z &= 2 \\ x - 2y - z &= 4 \\ -2x + 2y + 4z &= 2 \end{aligned}$$

Total Points: 50