Math 2000 Tutorial Worksheet

October 2, 2015

This week's tutorial will focus on proofs of conditional statements. Please discuss the following problems with your classmates:

1. (Section 3.1 #1) Prove each of the following statements:

   (a) For all integers a, b, and c with $a \neq 0$, if $a \mid b$ and $b \mid c$, then $a \mid (b - c)$.

   (b) For each $n \in \mathbb{Z}$, if n is odd, then n^3 is odd.

   (c) For each integer a, if 4 divides $a - 1$, then 4 divides $a^2 - 1$.

   (Hint: all three can be proved using previously established results.)

2. (Section 3.1 #3 (partial)) Determine if each of the following statements is true or false. If a statement is true, then write a formal proof of that statement, and if the statement is false, then provide a counterexample that shows it is false.

   (a) For all integers a, b, and c, with $a \neq 0$, if $a \mid b$, then $a \mid (bc)$.

   (b) For all integers a and b, with $a \neq 0$, if $6 \mid (ab)$, then $6 \mid a$ or $6 \mid b$.

3. (Section 3.1 #8) Let a and b be integers. Prove that if $a \equiv 7 \pmod 8$ and $b \equiv 3 \pmod 8$, then

   (a) $a + b \equiv 2 \pmod 8$ (b) $a \cdot b \equiv 5 \pmod 8$.

   (You should try proving these in two ways: (i) directly, and (ii) using a known result from class.)

4. Prove that for all integers n, n is odd if and only if n^3 is odd.

5. Prove that for all integers x, if $7x + 9$ is even, then x is odd. (Hint: use proof by contrapositive.)

6. Prove that there exist integers x and y such that $2x - 3y = 2$. (Hint: try a constructive proof.)

7. If you've done all the above and there's still time, use proof by contradiction to prove that $\sqrt{18}$ is irrational. (See Section 3.3 #4.)
   Hint: You may assume that $\sqrt{2}$ is irrational.