University of Lethbridge
Department of Mathematics and Computer Science
16th October, 2014, 12:15-1:30 pm
Math 2000A - Midterm

Last Name:____

First Name:____

Student Number:______________________________________

Record your answers below each question in the space provided. **Left-hand pages may be used as scrap paper for rough work.** If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

The value of each problem is indicated in the left-hand margins. The value of a problem does not always indicate the amount of work required to do the problem.

For grader's use only:

Page	Grade
2	/10
3	/10
4	/11
5	/11
6	/8
Total	/50

1. For each of the following sentences, decide whether or not it is an assertion. If it is, indicate whether it is true or false, and why. If it is not, explain why.

[2] (a) The number 4 is an even integer.

[2] (b) For each integer n, $n^2 - 1$ is a prime number.

[2] (c) There exists some $x \in \mathbb{R}$ such that $x + y = 3$.

[2] 2. (a) Define the **union** of two sets A and B.

[2] (b) What is the Law of the Excluded Middle?

3. For the following problems, you do **not** need to show your work.

[2] (a) If $A = \{n \in \mathbb{Z} : n = 2k \text{ for some } k \in \mathbb{Z}\}$ and $B = \{n \in \mathbb{Z} : n = 3k \text{ for some } k \in \mathbb{Z}\}$, what is $A \cap B$?

[2] (b) If $A = \mathbb{Z}$ and $B = \mathbb{N}$, what is $A \setminus B$?

[2] (c) What is the contrapositive of the statement "If p is a prime number, then $p = 2$ or p is an odd number." ?

[2] (d) If $A_n = \{1, n, 2n\}$ for $n = 1, 2, 3, \ldots$, what is $\displaystyle\bigcup_{n=2}^{4} A_n$?

[2] (e) What is the negation of the statement "For all $n \in \mathbb{Z}$, there exists some $m \in \mathbb{Z}$ such that $m > n$."?

[4] 4. Prove the following logical equivalence: $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$.
Formal justification of each step is not required.

[7] 5. Give a two-column proof of the following deduction: $(P \vee \neg Q) \rightarrow \neg R, Q \rightarrow P; \therefore \neg R$

6. Are the following propositions true or false? Justify your conclusion with a proof or counterexample.

[3] (a) For $a, b, c \in Z$, if $a|bc$, then $a|b$ or $a|c$.

[4] (b) For any subsets A and B of some universal set U, $(A \cup B) \setminus A = B \setminus A$.

[4] (c) For any subsets A, B, and C of some universal set U, if $A \cup C \subseteq B \cup C$, then $A \subseteq B$.

[4] 7. Prove the following assertion: For any integer n, if $n^2 - 1$ is even, then $n^2 - 1$ is divisible by 4.

[4] 8. Let I be a nonempty index set, and $\mathcal{A} = \{A_\beta : \beta \in I\}$ be an indexed family of sets. Prove that if $A_\beta \subseteq B$ for all $\beta \in I$, then $\bigcup_{\beta \in I} A_\beta \subseteq B$.

 Total Points: 50