University of California, Berkeley
Department of Mathematics
12^{th} April, 2013, 12:10-12:55 pm

MATH 53 - Test #3

Last Name:____

First Name:____

Student Number:____

What is your discussion section number (201-215)? ____________________ [1]

What is the name of your GSI? ______________________________ [1]

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

There is a list of potentially useful formulas available on the last page of the exam.

For grader's use only:

Page	Grade
1	/2
2	/14
3	/12
4	/12
Total	/40

A

1. Evaluate the integral $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dy\,dx$. [6]

   (You can do it without reversing the order of integration, but it's not recommended.)

2. The integral below computes the area of a region. Sketch the area, and compute it by converting to polar coordinates. [6]

$$\int_{-2}^{-1}\int_{-x}^{\sqrt{8-x^2}} dy\,dx + \int_{-1}^{1}\int_{\sqrt{2-x^2}}^{\sqrt{8-x^2}} dy\,dx + \int_{1}^{2}\int_{x}^{\sqrt{8-x^2}} dy\,dx$$

3. Evaluate the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ by converting to polar coordinates. [6]

4. Find the centroid (geometric center) of the triangle with vertices $(0, 0)$, $(-4, -2)$, and $(4, 2)$. [8]

5. Set up, but do not evaluate, the integral $\iiint_E x^2 \cos(yz)\, dV$, where E is the tetrahedron with vertices $(2,0,0)$, $(0,4,0)$, and $(0,0,1)$. [6]

6. Let $E \subseteq \mathbb{R}^3$ be the region bounded below by the cone $z = \sqrt{x^2+y^2}$ and above by the sphere $x^2+y^2+z^2 = 2z$. Express the volume of E as a triple integral in **both** cylindrical and spherical coordinates. You do not have to compute the volume. [6]

List of potentially useful facts and formulas

- Fubini's Theorem: if f is continuous on the rectangle $R = [a, b] \times [c, d]$ then
$$\iint_R f(x, y)\, dA = \int_c^d \int_a^b f(x, y)\, dx\, dy = \int_a^b \int_c^d f(x, y)\, dy\, dx.$$

- For a Type I region D given by $a \le x \le b$, $g(x) \le y \le h(x)$,
$$\iint_D f(x, y)\, dA = \int_a^b \int_{g(x)}^{h(x)} f(x, y)\, dy\, dx.$$

- For a Type II region D given by $g(y) \le x \le h(y)$, $c \le y \le d$,
$$\iint_D f(x, y)\, dA = \int_c^d \int_{g(y)}^{h(y)} f(x, y)\, dx\, dy.$$

- If $D = D_1 \cup D_2$, where D_1, D_2 intersect along a continuous curve, then
$$\iint_D f(x, y)\, dA = \iint_{D_1} f(x, y)\, dA + \iint_{D_2} f(x, y)\, dA.$$

- Polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$, and
$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta) r\, dr\, d\theta.$$

- Center of mass: for lamina occupying a region D with a density $\rho(x, y)$,
$$m = \iint_D \rho(x, y)\, dA, \quad \overline{x} = \frac{1}{m}\iint_D x\rho(x, y)\, dA, \quad \overline{y} = \frac{1}{m}\iint_D y\rho(x, y)\, dA.$$

- Triple integrals: like double integrals, but with one more variable. (Fubini still applies.)

- Cylindrical coordinates: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $dV = r\, dz\, dr\, d\theta$.

- Spherical coordinates: $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$, $z = \rho\cos\phi$, $dV = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$.

- Average value: $f_{\text{av}} = \dfrac{1}{A(D)}\iint_D f(x, y)\, dA$ or $f_{\text{av}} = \dfrac{1}{V(E)}\iiint_E f(x, y, z)\, dV$, where $A(D)$ and $V(E)$ denote the area of D and volume of E, respectively.

- $\sin^2\theta + \cos^2\theta = 1$.

 Total Points: 40