University of Lethbridge
Department of Mathematics and Computer Science
17th November, 2014, 5:00-5:50 pm
Math 4310 - Term Test II

Last Name:____

First Name:____

Student Number:____

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Page	Grade
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3	/8
4	/10
5	/10
Total	/40

[6] 1. Let X be a topological space. List as many conditions as possible equivalent to the statement "X is not connected." You don't need to justify your responses.

(Note: It is possible to earn bonus points on this problem. I will give 2 points per correct response, up to 6 points, plus 1 point for any additional correct responses.)

[6] 2. Let X be a subspace of $\mathbb{R}^n$ (with the Euclidean topology). List as many conditions as possible equivalent to the statement "X is compact." The same scoring rules apply as in Problem #1.

3. Let X and Y be topological spaces, where X is compact and Y is Hausdorff, and let $f : X \to Y$ be continuous.

[5] (a) Prove that if $f : X \to Y$ is a surjection, then f is a quotient map.

[3] (b) Prove that if $f : X \to Y$ is a bijection, then f is a homeomorphism.

[5] 4. Prove that if $f : X \to Y$ is a continuous surjection and X is path-connected, then Y is path-connected.

[5] 5. Suppose that $X = A \cup B$, with A, B nonempty, disjoint, open subsets of X. (That is, $\{A, B\}$ is a separation of X.) Prove that a map $f : X \to Y$ is continuous if and only if the restrictions $f|_A : A \to Y$ and $f|_B : B \to Y$ are continuous.

[6] 6. Suppose $\{F_n | n \in \mathbb{N}\}$ is a family of nonempty closed subsets of a topological space X, such that $F_{n+1} \subseteq F_n$ for each $n \in \mathbb{N}$. Prove that if X is compact, then $\bigcap_{n\in\mathbb{N}} F_n$ is nonempty.

Hint: To get a contradiction, suppose $\bigcap_{n\in\mathbb{N}} F_n = \emptyset$, and take complements.

[4] 7. Consider the cylinder $X = S^1 \times [-1, 1]$, and let $\sim$ be the equivalence relation on X defined by the partition $\mathcal{P}$ consisting of the sets $S^1 \times \{-1\}$, $S^1 \times \{1\}$, and the single point sets $\{(x, t)\}$, where $x \in S^1$ and $t \in (-1, 1)$. Describe the quotient space $X/\sim$. (A formal proof is not required.)