Please complete all problems below.

1. Let S be the subspace of $\mathbb{R}^3$ spanned by the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix}.$$

   (a) Determine if the vectors $\vec{a} = \begin{bmatrix} 1 \\ -8 \\ -4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix}$ belong to S. If either vector does belong to S, write it as a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

   (b) Are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly independent? If not, determine a basis for S.

2. Let $\vec{b}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ -1 \end{bmatrix}$, and let $\vec{u} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 3 \\ -2 \\ 5 \\ 1 \end{bmatrix}, \vec{w} = \begin{bmatrix} 1 \\ -1 \\ 4 \\ 3 \end{bmatrix}.$

   (a) Show that $B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ is an orthogonal set of vectors.

   (b) Write each the vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ above as a linear combination of the vectors in B, or explain why it is impossible to do so.

   Hint: For any vector $\vec{a} \in S = \operatorname{span} B$, we have the "Fourier decomposition"

$$\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}_1}{\|\vec{b}_1\|^2} \right) \vec{b}_1 + \left(\frac{\vec{a} \cdot \vec{b}_2}{\|\vec{b}_2\|^2} \right) \vec{b}_2 + \left(\frac{\vec{a} \cdot \vec{b}_3}{\|\vec{b}_3\|^2} \right) \vec{b}_3.$$

   Compute the right-hand side of the above equation for each of the given vector. If the result is equal to the original vector, then you've got you're linear combination. If it isn't then that vector isn't in the span of B. (What you've found instead is the *orthogonal projection* of that vector onto the subspace S.)

3. Let L_1, L_2 be the lines through the origin with direction vectors $\vec{v} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, respectively.

   (a) Verify that L_1 and L_2 are perpendicular.

   (b) Let S be the plane through the origin containing the lines L_1 and L_2. (Note that planes through the origin are subspaces.) Find the point in S that is closest to the point $P = (3, 3, 1)$ using orthogonal projection.

   Question for reflection: your other option for solving 3(b) is to use the cross product to find the normal vector, and then use the method for finding the shortest distance you learned in ealier in the course. Which do you prefer?

   Challenge problem: what if the plane isn't through the origin? Can you still use orthogonal projection? If not, how can you modify the method?

Name:

Tutorial time:

Please submit **one** *completed* solution from the worksheet for feedback.

Note: Your solution needs to contain enough detail for it to be clear what problem you're trying to solve!