University of Lethbridge
Department of Mathematics and Computer Science
14^{th} March, 2017, 1:40 - 2:55 pm
MATH 1410A - Test #2

Last Name:____

First Name:____

Student Number:______________________________________

Tutorial Time: ______________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

To earn partial credit, you must show your work. Correct answers without adequate justification in most cases do not receive full marks.

No external aids are allowed, with the exception of a 5-function calculator.

For grader's use only:

Problem	Grade
1	/8
2	/9
3	/9
4	/8
5	/5
6	/6
7	/5
Total	/50

1. Complete the following definitions:

[2] (a) The **null space** of an $m \times n$ matrix A is the set null(A) defined by

null$(A) =$

[2] (b) A set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is **linearly dependent** if:

[2] (c) A set $S \subseteq \mathbb{R}^n$ is a **subspace** if:

[2] (d) The **span** of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ is the set:

span$\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\} =$

2. Perform the computations as indicated:

[3] (a) Simplify the following linear combination (write it as a single vector):

$$4\begin{bmatrix}2\\-1\\3\end{bmatrix} - 2\begin{bmatrix}1\\0\\3\end{bmatrix} + 3\begin{bmatrix}0\\5\\-2\end{bmatrix} =$$

[3] (b) Compute $T\left(\begin{bmatrix}2\\-3\end{bmatrix}\right)$ for the matrix transformation $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}4 & -2\\-3 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$.

[3] (c) Verify that $x = 2, y = -3, z = 1$ is a solution to the system $\begin{aligned}2x - y + 3z &= 10\\ -x + 2y + 5z &= -3\\ 5x + 2y - 4z &= 0\end{aligned}$

[9] 3. Each of the matrices below is in row-echelon form, and represents a system of linear equations in the variables x, y, and z. If the system has no solution, explain why. If it does, determine the solution using either back substitution, or by finding the reduced row-echelon form of the matrix.

(a) $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 5 & -4 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

4. Given the matrices $A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}$, compute:

[4] (a) AB

[4] (b) BA

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a matrix transformation such that

$$T\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 9 \\ -1 \\ -4 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} -9 \\ 1 \\ 7 \end{bmatrix}.$$

[3] (a) What is the value of $T\left(3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}\right)$?

[2] (b) Given that $T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$, and $T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, determine a matrix A such that $T(\vec{x}) = A\vec{x}$ for any vector $\vec{x} \in \mathbb{R}^3$.

 Total Points: 50

[6] 6. Solve the following system of linear equations, if possible:

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 3 \\ -3x_1 - 6x_2 + 2x_3 - x_4 &= -7 \\ 2x_1 + 4x_2 - x_3 &= 4 \end{aligned}$$

[5] 7. Solve **one** of the following two problems.

(a) Let A be an $m \times n$ matrix and consider the set $\text{null}(A) = \{\vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0}\}$. Prove that $\text{null}(A)$ is a subspace of $\mathbb{R}^n$.

(b) Determine whether or not the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}$ are linearly independent.

Total Points: 50