

The problems on this worksheet are for in-class practice during tutorial. You are free to collaborate and to ask for help. They don't count for course credit, but it's a good idea to make sure you know how to do everything before you leave tutorial – similar problems may show up on a test or assignment.

1. Solve the following separable differential equations:

(a) $xy' = y + 2x^2y$, where $y(1) = 1$.

(b) $\frac{dy}{dx} = \frac{x^2 + 1}{y^2 + 1}$, where $y(0) = 1$. (Give an implicit solution.)

(c) $(4y + yx^2) dy - (2x + xy^2) dx = 0$

2. Solve the following linear differential equations. State an interval on which the general solution is defined.

(a) $\frac{dy}{dx} + y = e^{3x}$

(b) $(1 + x^2) dy + (xy + x^3 + x) dx = 0$

(c) $(1 - x^3) \frac{dy}{dx} = 3x^2y$

(d) $(x^2 + x) dy + (xy + x^3 + x) dx = 0$

(e) $\cos x \frac{dy}{dx} + y \sin x = 1$

3. Determine whether the sequence converges or diverges. If it converges, give the limit.

(a) $a_n = (-1)^n \frac{n}{n^2 + 1}$

(b) $a_n = (-1)^n \frac{2n + 1}{3n + 4}$

(c) $a_n = \frac{n - 1}{n} - \frac{n}{n - 1}$

(d) $a_n = \frac{4n}{\sqrt{9n^2 + 4}}$

(e) $a_n = 1 - \frac{1}{n}$

(f) The sequence $\{a_n\}$ defined by $a_1 = 1$ and $a_{n+1} = 3 - \frac{1}{a_n}$ for all $n \geq 1$. (You may assume that the sequence converges. If you want to actually *show* that it converges, feel free to ask me how.)

(g) The sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for all $n \geq 1$. (Again, you may assume that the sequence converges.)