Math 1410 Assignment #2
University of Lethbridge, Spring 2015

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January 28, 2015

Due date: Wednesday, February 4th, by 5 pm.

For instructions on completing this assignment, please see Assignment #1.

Assigned problems

1. Recall that an $n \times n$ matrix A is **symmetric** if $A^T = A$, and **antisymmetric** if $A^T = -A$.
   (a) Show that $B + B^T$ is symmetric for **any** $n \times n$ matrix B.
   (b) Show that $B - B^T$ is antisymmetric for **any** $n \times n$ matrix B.
   (c) Given an arbirary $n \times n$ matrix B, find a symmetric matrix U and an antisymmetric matrix V such that $B = U + V$.

2. For each of the following statements, either explain why it is true, or give an example showing that it is false:
   (a) If $A \neq 0$ is a square matrix, then A is invertible.
   (b) If A and B are both invertible, then $A + B$ is invertible.
   (c) If A and B are both invertible, then $(A^{-1}B)^T$ is invertible.
   (d) If $A^4 = 3I_n$, then A is invertible. (Hint: can you find a matrix B such that $AB = I_n$?)

3. Simplify the following matrix product:
$$B^{-1}(AB^T)^T(BA^{-1})A$$

4. Let A and B be $n \times n$ invertible matrices.
   (a) Show that $A^{-1} + B^{-1} = A^{-1}(A + B)B^{-1}$.
   (b) Show that **if** $A + B$ is invertible, then $A^{-1} + B^{-1}$ is also invertible, and find a formula for $(A^{-1} + B^{-1})^{-1}$.