[1] 1. (**Bonus**) Write down your name (first and last.)

The Solutions

Solve **one** of the following two problems: (10 points for either)

[10] 2. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, and define an equivalence relation on A by

$$a \sim b \text{ if and only if } a^2 \equiv b^2 \pmod 9.$$

Prove that $\sim$ is an equivalence relation on A, and determine the distinct equivalence classes of $\sim$.

We see that $\sim$ is reflexive, since $a^2 \equiv a^2 \pmod 9$ for all $a \in A$. Whenever $a^2 \equiv b^2 \pmod 9$, we know that $b^2 \equiv a^2 \pmod 9$, and thus $a \sim b \to b \sim a$ for all $a, b \in A$, so $\sim$ is symmetric. Finally, if $a^2 \equiv b^2 \pmod 9$ and $b^2 \equiv c^2 \pmod 9$, then $a^2 \equiv c^2 \pmod 9$, so if $a \sim b$ and $b \sim c$, then $a \sim c$, so $\sim$ is transitive. We conclude that $\sim$ is an equivalence relation.

Since $3^2 = 9 \equiv 0 \pmod 9$ and $6^2 = 36 \equiv 0 \pmod 9$, we have $[0] = \{0, 3, 6\}$. Similarly, $1^2 = 1 = 0(9) + 1$ and $8^2 = 64 = 9(9) + 1$, so $1 \sim 8$ and $[1] = \{1, 8\}$. Since $2^2 = 4$ and $7^2 = 49 = 5(9)+4$, we have $[2] = \{2, 7\}$, and since $4^2 = 16 = (1)9+7$ and $5^2 = 25 = 2(9)+7$, we have $[4] = \{4, 5\}$. Thus,

$$A = [0] \cup [1] \cup [2] \cup [4] = \{0, 3, 6\} \cup \{1, 8\} \cup \{2, 7\} \cup \{4, 5\}$$

gives a partition of A into equivalence classes with respect to the relation $\sim$.

[10] 3. Write down the modular arithmetic addition and multiplication tables for $\mathbb{Z}_7$.

We will write the elements of $\mathbb{Z}_7 = \{[0], [1], [2], [3], [4], [5], [6]\}$ without the square brackets for convenience. The tables are as follows:

$\oplus$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

$\odot$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Total: 10 points