University of California, Berkeley
Department of Mathematics
15^{th} March, 2013, 12:10-12:55 pm
MATH 53 - Test #2

Last Name:________________________________

First Name:________________________________

Student Number:____________________________

Discussion Section: ___________________________

Name of GSI: _______________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

There is a list of potentially useful formulas available on the last page of the exam.

For grader's use only:

Page	Grade
1	/15
2	/15
3	/10
Total	/40

A

1. Let $f(x, y) = x^2 e^{xy}$.

[4] (a) Find the linearization of f at the point $(1, 0)$.

[3] (b) Find the derivative of f in the direction of $\mathbf{v} = \langle -3, 4\rangle$ at the point $(1, 0)$.

[5] (c) If $x(t) = t^2$ and $y(t) = 2t - 2$, use the chain rule to find the tangent vector to the curve $\mathbf{r}(t) = \langle x(t), y(t), f(x(t), y(t))\rangle$ when $t = 1$.

[3] (d) Verify that the tangent vector found in part (c) is tangent to the surface $z = f(x, y)$ at the point $(1, 0, 1)$.

2. Let $f(x, y) = 3xy - x^3 - y^3$.

[8] (a) Find and classify the critical points of f.

(b) Find the absolute maximum and minimum of f on the set D given by the triangular region with vertices at $(0, 0)$, $(0, 1)$, and $(1, 1)$. [7]

[2] 3. (a) Define what it means for a function $f(x, y, z)$ to be *continuous* at a point (a, b, c) in its domain.

[3] (b) Define what it means for a function $f(x, y, z)$ to be *differentiable* at a point (a, b, c) in its domain.

[5] (c) Show that if f is differentiable at a point (a, b, c), then it is continuous at (a, b, c).
Hint: You can show this using only the above two definitions and the limit laws.

List of potentially useful formulas and facts:

- The limit of $f(x, y)$ as $(x, y) \to (a, b)$ is L if for every $\epsilon > 0$ there exists a $\delta > 0$ such that
$$0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \quad \Rightarrow \quad |f(x, y) - L| < \epsilon.$$
- Clairaut's Theorem: If f is defined on a disk D and f_{xy}, f_{yx} are continuous on D, then $f_{xy} = f_{yx}$ on D.
- The linearization of a function $f(x, y)$ at a point (a, b) is given by
$$L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b),$$
and the equation of the tangent plane to $z = f(x, y)$ at the point $(a, b, f(a, b))$ is $z = L(x, y)$.
- If the first order partial derivatives of a function f exist in a neighborhood of a point, and are continuous at that point, then f is differentiable at that point.
- If $w = f(x, y, z)$ and $x = x(t)$, $y = y(t)$, $z = z(t)$, then
$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}.$$
If $x = x(u, v)$, $y = (u, v)$, $z = z(u, v)$, then
$$\frac{\partial w}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u},$$
with a similar formula for $\partial w/\partial v$, and variations on the above Chain Rule formulas for other numbers of variables.
- The derivative of f in the direction of a vector $\mathbf{v}$ at a point $\mathbf{x}_0$ can be computed according to $D_{\mathbf{v}} f(\mathbf{x}_0) = \dfrac{\nabla f(\mathbf{x}_0) \cdot \mathbf{v}}{\|\mathbf{v}\|}$.
- The tangent plane to a level surface $f(x, y, z) = k$ at a point (a, b, c) is given by $\nabla f(a, b, c) \cdot \langle x - a, y - b, z - c\rangle = 0$.
- The tangent vector to a curve $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ is $\mathbf{r}'(t) = \langle x'(t), y'(t), z'(t)\rangle$.
- Suppose f has a critical point at (a, b), and the second derivatives of f are continuous on a neighborhood of (a, b). Let $D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}(a, b)^2$. If
  (a) $D > 0$ and $f_{xx}(a, b) > 0$, then f has a local minimum at (a, b).
  (b) $D > 0$ and $f_{xx}(a, b) < 0$, then f has a local maximum at (a, b).
  (c) $D < 0$, then f has a saddle point at (a, b).
- If $f(x, y)$ has a maximum or minimum subject to the constraint $g(x, y) = c$ at (a, b), then $\nabla f(a, b) = \lambda \nabla g(a, b)$.

 Total Points: 40