University of Lethbridge
Department of Mathematics and Computer Science
MATH 2565 - Tutorial #5
Thursday, February 8

First Name:____

Last Name:____

Additional practice (don't include your solutions here):

1. Find the volume of a right circular cone whose height is 12 and base radius is 4.
2. Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $x = 1$, and $y = 0$ about the x-axis.
3. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 1$, and $x = 0$ about

   (a) the x-axis (b) the y-axis

4. Find the volume of the solid generated by the region bounded by $y = 4 - x^2$ and $y = 0$, when rotated about

   (a) the x-axis. (b) the line $y = -1$. (c) the line $x = 2$.

5. Find the volume of the solid generated by revolving the region bounded by $x = y - y^2$ and $x = 0$ about the y-axis.
6. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = 6x - 2x^2$ and $y = 0$, about the y-axis.

1. Find the volume of the solid S whose base is the region of the xy-plane bounded by $y = x^2$ and $y = 2 - x^2$, and whose cross-sections parallel to the y-axis are squares.

2. Find the volume of the solid S generated when the region bounded by the curves $y^2 = x$ and $x = 2y$ is revolved around the y-axis.

3. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = \sqrt{x}$ about the x-axis.

4. Find the length of the curve $y = \dfrac{1}{12}x^3 + \dfrac{1}{x}$, for $x \in [1, 4]$.

Some integration formulas you may need:

- Volume of a solid of revolution, washer method, rotation about a horizontal axis:

$$V = \pi \int_a^b (r_{out}(x)^2 - r_{in}(x)^2)\, dx,$$

  where r_{out} gives the outer radius (distance from the far side of the region being rotated to the axis) and r_{in} gives the inner radius. If the axis of rotation is vertical, reverse the roles of x and y.

- Volume of a solid of revolution, shell method, rotation about a vertical axis:

$$V = 2\pi \int_a^b r(x)h(x)\, dx,$$

  where $r(x)$ is the radius of the shell (distance to axis of rotation) and $h(x)$ is the height of the shell. If the region lies between $y = g(x)$ (above) and $y = f(x)$ (below), and the axis of rotation is the y-axis we get the formula

$$V = 2\pi \int_a^b x(g(x) - f(x))\, dx$$

  as a special case. If the axis of rotation is horizontal, reverse the roles of x and y.

- Arc length of a curve $y = f(x)$, for $a \leq x \leq b$:

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

- Surface area generated by revolving $y = f(x)$, $a \leq x \leq b$ about the x-axis:

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\, dx.$$

- Surface area generated by revolving $y = f(x)$, $a \leq x \leq b$ about the y-axis:

$$S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\, dx.$$