University of California, Berkeley
Department of Mathematics
5^{th} October, 2012, 12:10-12:55 pm

MATH 53 - Test #1

Last Name:____

First Name:____

Discussion Section: ____________________________________

Name of GSI: _______________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

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3	/12
Total	/36

B

[4] 1. Find the equation of the tangent line to the curve C represented by the vector-valued function $\mathbf{r}(t) = \langle 4 - 3t, e^{t^2}, \ln(1+t) \rangle$ at the point $(4, 1, 0)$.

[5] 2. Find the area of the circle $r = 4\cos\theta$ using an integral in polar coordinates.

[3] 3. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

4. Consider the two lines in $\mathbb{R}^3$ given by

$$\mathbf{r}_1(t) = \langle 3, 2, 3\rangle + t\langle 3, 0, 2\rangle$$
$$\mathbf{r}_2(s) = \langle 0, 1, 2\rangle + s\langle 0, 1, -1\rangle.$$

[2] (a) Verify that the two lines intersect at the point $(0, 2, 1)$.

[3] (b) Find the cosine of the angle between the two lines.

[4] (c) Find the equation of the plane that contains the two lines.

[3] (d) Find the distance between the point $P(1, -1, 2)$ and the plane from part (c).

[5] 5. (a) Find the equation of the tangent plane to the surface $z = \sin(3 + x^2 - y^2)$ at the point $(1, 2, 0)$.

[2] (b) Show that the surface from part (a) has the horizontal tangent plane $z = 1$ at every point on the hyperbola $y^2 - x^2 = 3 - \frac{\pi}{2}$.

[5] 6. Use the chain rule to compute $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ if $f(x, y, z) = x^3 y z^2$, where $x = 2u + v$, $y = u - 3v$, and $z = uv$.

 Total Points: 36