University of Lethbridge
Department of Mathematics and Computer Science
MATH 1565 - Quiz #3 Solution

1. Determine the equation of the tangent line to the curve

$$(x^2 + y^2 + y)^2 = x^2 + y^2$$

at the point $(1, 0)$.

Taking the derivative of both sides of the equation with respect to x, we have

$$2(x^2 + y^2 + y)\left(2x + 2y\frac{dy}{dx} + \frac{dy}{dx}\right) = 2x + 2y\frac{dy}{dx}.$$

Having taken the derivative, we can put in $x = 1$, $y = 0$, giving us

$$2(1 + 0 + 0)\left(2 + 0 + \frac{dy}{dx}\right) = 2(1) + 0,$$

so $2(2 + dy/dx) = 2$, giving us $2 + dy/dx = 1$, so $\dfrac{dy}{dx} = -1$.

With the slope $m = -1$ and point $(1, 0)$ we obtain the equation

$$y - 0 = -1(x - 1) \text{ or } y = -x + 1$$

for the tangent line.

Note: it was not necessary to solve for $\dfrac{dy}{dx}$ in terms of x and y before putting in the given values, but if you did, you should get:

$$4y(x^2 + y^2 + y)\frac{dy}{dx} + 2(x^2 + y^2 + y)\frac{dy}{dx} + 4x(x^2 + y^2 + y) = 2x + 2y\frac{dy}{dx},$$

so

$$\frac{dy}{dx} = \frac{2x - 4x^3 - 4xy^2 - 4xy}{4x^2y + 4y^3 + 6y^2 + 2x^2}.$$