Name: Solutions

1. Determine the domain and range of each of the following relations on $\mathbb{R}$, and sketchthe graph of each relation:

   (a) $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \,|\, x^2 + y^2 = 10\}$

   Since $x^2 \geq 0$ and $y^2 \geq 0$, we note that $x^2 = 10 - y^2 \leq 10$, so $x^2 \leq 10$, which means that $-\sqrt{10} \leq x \leq \sqrt{10}$. Similarly $-\sqrt{10}] \leq y \leq \sqrt{10}$. Thus

   $$\operatorname{dom} R = \{x \in \mathbb{R} \,|\, -\sqrt{10} \leq x \leq \sqrt{10}\} = [-\sqrt{10}, \sqrt{10}]$$
   $$\operatorname{range} R = \{y \in \mathbb{R} \,|\, -\sqrt{10} \leq y \leq \sqrt{10}\} = [-\sqrt{10}, \sqrt{10}].$$

   The graph (omitted) is that of a circle of radius $\sqrt{10}$ centred at the origin.

   (b) $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \,|\, x^2 = y^2\}$

   We have

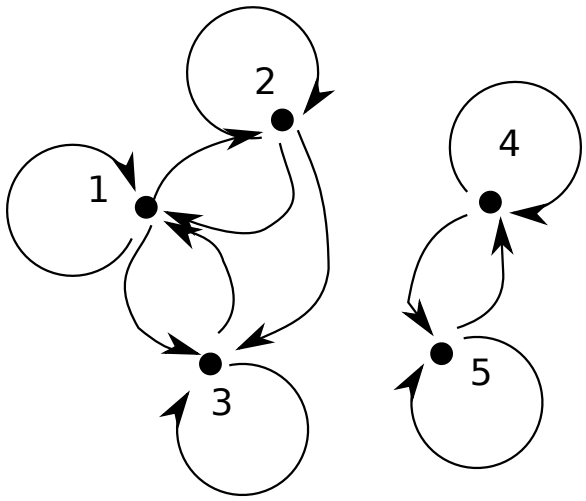
   $$x^2 = y^2 \leftrightarrow x^2 - y^2 = 0 \leftrightarrow (x - y)(x + y) = 0 \leftrightarrow x = y \text{ or } x = -y,$$

   and we see immediately that $\operatorname{dom} S = \operatorname{range} S = \mathbb{R}$, since $(x, x) \in S$ for all $x \in \mathbb{R}$. The graph (omitted) is the pair of lines $y = x$ and $y = -x$. (It should look like a big X.)

2. Let $A = \{1, 2, 3, 4, 5\}$. Draw a directed graph that represents the relation R on A defined by

   $$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,3), (4,4), (4,5), (5,4), (5,5)\}.$$

   A (somewhat poorly rendered) directed graph of the relation is given as follows:



   Bonus: Is R an equivalence relation? Why or why not?

   It is not an equivalence relation because it is not symmetric: we have $(2, 3) \in R$ but $(3, 2) \notin R$.

Total: 10 points