Practice for Quiz 19
Math 2580
Spring 2016

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If you can answer the following problems, you should be well-prepared for Quiz 19:

1. Find a function f such that $\nabla f = \mathbf{F}$, and use this to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the given curve:
   (a) $\mathbf{F}(x, y) = \dfrac{y^2}{1+x^2}\mathbf{i} + 2y \arctan x\mathbf{j}$, C parameterized by $\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j}$, $0 \leq t \leq 1$.
   (b) $\mathbf{F}(x, y, z) = (2xz + y^2)\mathbf{i} + 2xy\mathbf{j} + (x^2 + 3z^2)\mathbf{k}$, C parameterized by $x = t^2, y = t + 1, z = 2t - 1$, $0 \leq t \leq 2$.
   (c) $\mathbf{F}(x, y, z) = \langle e^y, xe^y, (z + 1)e^z\rangle$, C parameterized by $\mathbf{r}(t) = \langle t, t^2, t^3\rangle$, $t \in [0, 1]$.
2. Calculate the curl of the given vector field:
   (a) $\mathbf{F}(x, y, z) = \langle 2xy, xz, y^2z\rangle$
   (b) $\mathbf{F}(x, y, z) = \langle y \cos xy, x \cos xy, -\sin z\rangle$
3. Verify that Green's Theorem holds for the following line integrals in the plane:
   (a) $\int_C xy^2\, dx + x^3\, dy$, where C is the rectangle with corners at $(0, 0), (2, 0), (2, 3)$, and $(0, 3)$.
   (b) $\int_C y\, dx - x\, dy$, where C is the unit circle.
   (c) $\int_C x\, dx + y\, dy$, where C consists of the line segments from $(0, 1)$ to $(0, 0)$, and from $(0, 0)$ to $(1, 0)$, and the portion of the parabola $y = 1 - x^2$ from $(1, 0)$ to $(0, 1)$.