University of Lethbridge
Department of Mathematics and Computer Science
MATH 1565 - Quiz #3 Solution

1. Determine the equation of the tangent line to the curve

$$(x^2 + y^2 + x)^2 = x^2 + y^2$$

at the point $(0, -1)$.

Taking the derivative of both sides with respect to x, we have

$$2(x^2 + y^2 + x)\left(2x + 2y\frac{dy}{dx} + 1\right) = 2x + 2y\frac{dy}{dx}.$$

Since the derivative is complete, we can substitute $x = 0$, $y = -1$, giving us

$$2(0 + 1 + 0)\left(0 - 2\frac{dy}{dx} + 1\right) = 0 - 2\frac{dy}{dx},$$

or $-4\dfrac{dy}{dx} + 2 = -2\dfrac{dy}{dx}$. Solving for $\dfrac{dy}{dx}$, we have

$$\frac{dy}{dx} = 1.$$

Thus we have the point $(0, -1)$ and the slope $m = 1$, so our line is

$$y - (-1) = 1(x - 0), \text{ or } y = x - 1.$$

Note: it was not necessary to solve for $\dfrac{dy}{dx}$ in terms of x and y before putting in the given values, but if you did, you should get:

$$4x(x^2 + y^2 + x) + 4y(x^2 + y^2 + x)\frac{dy}{dx} + 2(x^2 + y^2 + x) = 2x + 2y\frac{dy}{dx}$$

so

$$\frac{dy}{dx} = \frac{-6x^2 - 4xy^2 - 4x^3 - 2y^2}{4x^2y + 4y^3 + 4xy - 2y}.$$