

*University of California, Berkeley*  
Department of Mathematics  
5<sup>th</sup> October, 2012, 12:10-12:55 pm  
**MATH 53 - Test #1**

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

Name of GSI: \_\_\_\_\_

**Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.**

**Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.**

For grader's use only:

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Total	/36

A

- [4] 1. Find the equation of the tangent line to the curve  $C$  represented by the vector-valued function  $\mathbf{r}(t) = \langle e^{2t}, t^2, \sin t \rangle$  at the point  $(1, 0, 0)$ .

- [5] 2. Find the area of one loop of the 4-leaved rose  $r = \cos 2\theta$ .

- [3] 3. Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$  does not exist.

4. Consider the two lines in  $\mathbb{R}^3$  given by

$$\begin{aligned}\mathbf{r}_1(t) &= \langle 2, 0, -2 \rangle + t\langle 1, 0, -2 \rangle \\ \mathbf{r}_2(s) &= \langle -2, 1, 0 \rangle + s\langle 3, -1, 0 \rangle.\end{aligned}$$

[2] (a) Verify that the two lines intersect at the point  $(1, 0, 0)$ .

[3] (b) Find the cosine of the angle between the two lines.

[4] (c) Find the equation of the plane that contains the two lines.

[3] (d) Find the distance between the point  $P(2, -1, 3)$  and the plane from part (c).

- [5] 5. (a) Find the equation of the tangent plane to the surface  $z = \sqrt{x^2 + y^3}$  at the point  $(1, 2, 3)$ .

- [2] (b) Use the result from part (a) to approximate the value of  $\sqrt{(1.03)^2 + (2.05)^3}$ .

- [5] 6. Use the chain rule to compute  $\frac{\partial f}{\partial u}$  and  $\frac{\partial f}{\partial v}$  if  $f(x, y, z) = x^2y + y^2z^3$ , where  $x = v^2$ ,  $y = u^2$ , and  $z = u^2v^2$ .