

Name:**Tutorial time:**

1. For each of the functions below, use the sign diagram of its derivative to find and classify any critical points.

(a) $f(x) = 2x^4 - 4x^2 + 6$

(b) $g(x) = \frac{x^2}{1+x}$

(c) $h(x) = x^{7/3} - 7x^{1/3}$

2. For each function below, determine the intervals on which it is increasing/decreasing and concave up/concave down.

(Note: these are the same functions as in the previous problem.)

(a) $f(x) = 2x^4 - 4x^2 + 6$

(b) $g(x) = \frac{x^2}{1+x}$

(c) $h(x) = x^{7/3} - 7x^{1/3}$

Bonus challenge fun problem:

3. Assume all functions below are differentiable on the interval $(-c, c)$ Using the Mean Value Theorem, show that:

(a) If $f(0) \geq 0$ and $f'(x) > 0$ for all $x \in (0, c)$, then $f(x) > 0$ for all $x \in (0, c)$.

(b) If $f(0) > g(0)$ and $f'(x) > g'(x)$ for all $x \in (0, c)$, then $f(x) > g(x)$ for all $x \in (0, c)$.
(Hint: apply part (a) to the function $h = f - g$.)

(c) Let $n > 1$ be an integer. Prove that $(1 + x)^n > 1 + nx$ for all $x \in (0, \infty)$.

(Note: the bonus is the fun you're having with this challenge!)