FACULTY OF APPLIED SCIENCE AND ENGINEERING
University of Toronto

MAT294H1Y
Calculus and Differential Equations

Term Test #2
Duration: 110 minutes

NO AIDS ALLOWED. **Total: 50 marks**

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

Student ID Number: ______________________________

You may not use calculators, cell phones, or PDAs during the test. Partial credit will be given for partially correct work. Please read through the entire test before starting, and take note of how many points each question is worth.

FOR MARKER'S USE ONLY	
Problem 1:	/10
Problem 2:	/10
Problem 3:	/10
Problem 4:	/10
Problem 5:	/10
TOTAL:	/50

[5] 1. (a) Evaluate the double integral

$$\int_0^1 \int_y^{\sqrt{y}} (x+y)\,dx\,dy.$$

[5] (b) Evaluate the double integral

$$\int_0^1 \int_y^1 e^{y/x}\, dx\, dy.$$

Hint: Change the order of integration.

[5] 2. (a) Find the volume of the solid bounded by the coordinate planes, and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1,$$

where $a, b, c > 0$.

[5] (b) Find the volume of the solid bounded by the cone $z^2 = x^2 + y^2$ and the cylinder $x^2 + y^2 = 2y$.

Hint #1: Use polar coordinates.

Hint #2: $\dfrac{d}{d\theta}\left(\dfrac{\cos^3\theta}{3} - \cos\theta\right) = \sin^3\theta.$

3. A plane lamina has mass density $\delta(x, y) = xy$, and occupies the region R given by the inequalities

$$0 \leq x \leq 1 \quad \text{and} \quad x^2 \leq y \leq 1.$$

[4] (a) Find the mass of the lamina.

[6] (b) Find the centre of mass of the lamina.

4. Let T be the region in space bounded by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$. (You may want to sketch the region.)

   [5] (a) Using **cylindrical coordinates**, find the mass of the solid bounded by T whose mass density is given by $\delta(x, y, z) = \lambda z$, where λ is a positive constant.

[5] (b) Using **spherical coordinates**, find the volume of the solid bounded by T.

5. Evaluate the integral

$$\iint_R xy\,dx\,dy,$$

where R is the region in the 1$^{\text{st}}$ quadrant bounded by the curves $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$ and $x^2-y^2=4$.