Math 2000 Tutorial Worksheet

December 2nd, 2015

1. (Section 7.1 #2) Let $A = \{a, b, c\}$ and let $R = \{(a, a), (a, c), (b, b), (b, c), (c, a), (c, b)\}$ define a relation on A. Are the following statements true or false? Explain.
   (a) For each $x \in A$, $x\,R\,x$.
   (b) For every $x, y \in A$, if $x\,R\,y$, then $y\,R\,x$.
   (c) For every $x, y, z \in A$, if $x\,R\,y$ and $y\,R\,z$, then $x\,R\,z$.
   (d) R is a function from A to A.

2. (Section 7.1 #4) Let U be a nonempty set, and let R be the "subset relation" on $\mathcal{P}(U)$. That is,
$$R = \{(S, T) \in \mathcal{P}(U) \times \mathcal{P}(U) | S \subseteq T\}.$$
   (a) Write the open sentence $(S, T) \in R$ using standard subset notation.
   (b) What is the domain of the relation R?
   (c) What is the range of the relation R?
   (d) Is R a function from $\mathcal{P}(U)$ to $\mathcal{P}(U)$? Explain.

3. (Section 7.1#5) Repeat parts (b)-(d) of Problem 2 for the "element of" relation
$$R = \{(x, S) \in U \times \mathcal{P}(U) | x \in S\}.$$

4. (Section 7.1 #6 and 7) Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 + y^2 = 100\}$.
   (a) Determine the set of all values of x such that $(x, 6) \in S$, and determine the set of all values of x such that $(x, 9) \in S$.
   (b) Determine the domain and range of the relation S, and write each set using set builder notation.
   (c) Is the relation on S a function from $\mathbb{R}$ to $\mathbb{R}$? Explain.
   (d) Since S is a relation on $\mathbb{R}$, its elements can be graphed in the coordinate plane. Describe the graph of the relation S. Is the graph consistent with your answers in parts (a) - (c)? Explain.
   (e) Repeat parts (a) - (d) for the relation $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} | y = \sqrt{100 - x^2}\}$. What is the connection between the relations S and T?

5. (Section 7.1 #8) Determine the domain and range of each of the following relations on $\mathbb{R}$ and sketch the graph of each relation.

   (a) $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 + y^2 = 100\}$
   (b) $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} | y^2 = x + 10\}$
   (c) $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} || x| + |y| = 10\}$
   (d) $W = \{(x, y) \in \mathbb{R} \times \mathbb{R} | x^2 = y^2\}$.

6. (Section 7.2 #2) Let $A = \{a, b, c\}$. For each of the following, draw a directed graph that represents a relation on A with the specified properties:

   (a) A relation on A that is symmetric but not transitive.
   (b) A relation on A that is transitive but not symmetric.
   (c) A relation on A that is symmetric and transitive but not reflexive.
   (d) A relation on A that is not reflexive on A, is not symmetric, and is not transitive.
   (e) A relation on A, other than the identity relation, that is an equivalence relation on A.

7. (Section 7.2 #5) A relation R is defined on $\mathbb{Z}$ as follows: For all $a, b \in \mathbb{Z}$, $a\,R\,b$ if and only if $|a-b| \leq 3$. Is R an equivalence relation on $\mathbb{Z}$? If not, is R reflexive? Symmetric? Transitive? Justify all conclusions.

8. (Section 7.2 #9) Define the relation $\sim$ on $\mathbb{Q}$ as follows: For $a, b \in \mathbb{Q}$, $a \sim b$ if and only if $a - b \in \mathbb{Z}$.

   (a) Show that $\sim$ is an equivalence relation. (See Progress Check 7.9.)
   (b) List four different elements of the set $C = \left\{ x \in \mathbb{Q} \,\middle|\, x \sim \frac{5}{7} \right\}$.
   (c) Use set builder notation, without using the symbol $\sim$, to specify the set C.
   (d) use the roster method to specify the set C.

9. (Section 7.2 #13) Let $\sim$ and $\approx$ be relations on $\mathbb{Z}$ defined as follows;

   - For $a, b \in \mathbb{Z}$, $a \sim b$ if and only if $2a + 3b \equiv 0 \pmod 5$.
   - For $a, b \in \mathbb{Z}$, $a \approx b$ if and only if $a + 3b \equiv 0 \pmod 5$.

   (a) Is $\sim$ an equivalence relation on $\mathbb{Z}$? If not, is this relation reflexive? Symmetric? Transitive?
   (b) Is $\approx$ an equivalence relation on $\mathbb{Z}$? If not, is this relation reflexive? Symmetric? Transitive?