

Name:**Tutorial time:**

Please complete all problems below.

1. Solve the system of linear equations below by (a) forming the corresponding augmented matrix, and (b) using Gaussian elimination (row operations) to reduce it to row-echelon form.

$$\begin{array}{rrcrcl} x & - & 2y & + & z & = & 4 \\ -x & + & y & - & 2z & = & -2 \\ 2x & - & 4y & + & 3z & = & 9 \end{array}$$

2. Consider the following system of two equations in three variables:

$$\begin{array}{rrcr} x & + & 2y & - & 3z = 6 \\ 2x & + & 5y & - & 4z = -3 \end{array}$$

- (a) What geometric object in \mathbb{R}^3 (3-dimensional space) is defined by each of the individual equations? (A point? A line? A plane? Something else?)
- (b) Find a one-parameter family of solutions to the system of equations above. (It should only take one row operation.)
- (c) Write your solution to part (b) in vector form (as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dots$). What geometric object in \mathbb{R}^3 does your solution represent? Does this seem reasonable? (Note that any point (x, y, z) that satisfies **both** of the original equations belongs to the **intersection** of the objects defined by those equations.)
- (d) Suppose it had turned out that the system of equations had no solutions. How would you explain this visually?

Exam preparation: Do not hand in this page. Take it home with you, and make sure you can confidently answer every question below. (You will have to be able to do some – perhaps all – of these things on the test, so anything left unanswered represents marks left unearned on your test.)

How do I ...

1. Compute the distance between two points? (In \mathbb{R}^2 ? In \mathbb{R}^3 ? In \mathbb{R}^n ?)
 2. Determine the vector \overrightarrow{PQ} whose tail is at the point P and whose head is at the point Q ?
 3. Determine the point Q , if given P and \overrightarrow{PQ} , or the point P , if given Q and \overrightarrow{PQ} ?
 4. Calculate the length of a vector?
 5. Add two vectors together, or multiply a vector by a scalar?
 6. Calculate the dot product of two vectors? And use it to find the angle between two vectors?
 7. Define a line in \mathbb{R}^3 ? (What are the parametric equations? What is the vector equation?)
 8. Find the equation(s) of a line, given a point on the line and a vector parallel to the line (direction vector)? What if I'm given two points on the line? Or a point on the line and the equation(s) of a parallel line?
 9. Find either a point on a line, or a vector in the direction of the line, if given the equations of the line?
 10. Determine the equation of a plane, if given a point on the plane and the normal vector?
 11. Determine a point on a plane, or the normal vector to the plane, if given the equation of the plane?
 12. Compute a projection? And describe a projection using a diagram? (If I'm projecting a vector \vec{a} onto a vector \vec{b} , and drawing each vector as an arrow, where do I locate the tails of \vec{a} , \vec{b} , and $\text{proj}_{\vec{b}} \vec{a}$? Where do I locate the heads?)
 13. Take a vector \vec{a} , and, with respect to a second vector \vec{b} , write it as $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$ where \vec{a}_{\parallel} is parallel to \vec{b} , and \vec{a}_{\perp} is orthogonal to \vec{b} ?
 14. Find the shortest distance from a point to a line? Or a plane?
 15. Find the intersection of a line and a plane?
 16. Find the intersection of two lines?
 17. Find the intersection of two planes?
 18. Tell if a matrix is in (reduced) row-echelon form?
 19. Determine the solution of a system from the row-echelon form of its augmented matrix?
 20. Tell if system of equations has one solution, no solution, or infinitely many solutions?
- (Note that the last two items require you to solve a system of equations.)