Math 4310 Assignment #6
University of Lethbridge, Fall 2014

Sean Fitzpatrick

October 14, 2014

Due date: Friday, October 17th, by 5 pm.

Please submit solutions to the following problems (except where indicated). As usual, all the textbook exercises are recommended as practice problems if they don't appear as an assigned problem below.

1. Prove that any normed vector space is a metric space.
2. Let $X = l^\infty$ denote the set of all *bounded* sequences of real numbers. (Thus, $x = (x_1, x_2, x_3, \ldots) \in X$ if there exists some $M \geq 0$ such that $|x_i| \leq M$ for all $i = 1, 2, 3, \ldots$. Prove that $\|x\| = \sup\{|x_n| : n \in \mathbb{N}\}$ defines a norm on X.
3. Let A be a subset of a metric space X. An element $a \in A$ is called an *isolated point* of A if there exists an $\epsilon > 0$ such that $N_\epsilon(a) \cap A = \{a\}$. Prove that the closure of A is equal to the disjoint union of the limit points of A and the isolated points of A.
4. Let X be a topological space and $A \subseteq X$. Prove that $\overline{A} = X \setminus (X \setminus A)^\circ$. That is, the closure of A is the complement of the interior of the complement of A.
5. Let S be a subset of a topological space X, and let S be given the subspace topology. Show that if A is a relatively open subset of S, then $A \cap T$ is a relatively open subset of $S \cap T$ for any subset $T \subseteq X$.
6. Under what condition is a space X with the cofinite topology a Hausdorff space?