

University of California, Berkeley
Department of Mathematics
12th April, 2013, 12:10-12:55 pm
MATH 53 - Test #3

Last Name: _____

First Name: _____

Student Number: _____

What is your discussion section number (201-215)? _____

[1]

What is the name of your GSI? _____

[1]

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

There is a list of potentially useful formulas available on the last page of the exam.

For grader's use only:

Page	Grade
1	/2
2	/14
3	/12
4	/12
Total	/40

B

1. Evaluate the integral $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \sqrt{4-x^2} dx dy$.

[6]

(You can do it without reversing the order of integration, but it's not recommended.)

2. The integral below computes the area of a region. Sketch the area, and compute it by converting to polar coordinates.

[6]

$$\int_{-2}^{-1} \int_{-y}^{\sqrt{8-y^2}} dx dy + \int_{-1}^{\sqrt{2}} \int_{\sqrt{2-y^2}}^{\sqrt{8-y^2}} dx dy + \int_{\sqrt{2}}^{2\sqrt{2}} \int_0^{\sqrt{8-y^2}} dx dy$$

3. Evaluate the integral $\int_0^4 \int_{-\sqrt{4x-x^2}}^0 \sqrt{x^2 + y^2} dy dx$ by converting to polar coordinates. [6]

4. Find the centroid (geometric center) of the triangle with vertices $(0, 0)$, $(-1, -3)$, and $(1, 3)$. [8]

5. Set up, but do not evaluate, the integral $\iiint_E x^2 \cos(yz) dV$, where E is the tetrahedron with vertices $(1, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$.

[6]

6. Let $E \subseteq \mathbb{R}^3$ be the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 2z$. Express the volume of E as a triple integral in **both** cylindrical and spherical coordinates. You do not have to compute the volume.

[6]

List of potentially useful facts and formulas

- Fubini's Theorem: if f is continuous on the rectangle $R = [a, b] \times [c, d]$ then

$$\iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dx dy = \int_a^b \int_c^d f(x, y) dy dx.$$

- For a Type I region D given by $a \leq x \leq b$, $g(x) \leq y \leq h(x)$,

$$\iint_D f(x, y) dA = \int_a^b \int_{g(x)}^{h(x)} f(x, y) dy dx.$$

- For a Type II region D given by $g(y) \leq x \leq h(y)$, $c \leq y \leq d$,

$$\iint_D f(x, y) dA = \int_c^d \int_{g(y)}^{h(y)} f(x, y) dx dy.$$

- If $D = D_1 \cup D_2$, where D_1, D_2 intersect along a continuous curve, then

$$\iint_D f(x, y) dA = \iint_{D_1} f(x, y) dA + \iint_{D_2} f(x, y) dA.$$

- Polar coordinates: $x = r \cos \theta$, $y = r \sin \theta$, and

$$\iint_D f(x, y) dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r \cos \theta, r \sin \theta) r dr d\theta.$$

- Center of mass: for lamina occupying a region D with a density $\rho(x, y)$,

$$m = \iint_D \rho(x, y) dA, \quad \bar{x} = \frac{1}{m} \iint_D x \rho(x, y) dA, \quad \bar{y} = \frac{1}{m} \iint_D y \rho(x, y) dA.$$

- Triple integrals: like double integrals, but with one more variable. (Fubini still applies.)
- Cylindrical coordinates: $x = r \cos \theta$, $y = r \sin \theta$, $z = z$, $dV = r dz dr d\theta$.
- Spherical coordinates: $x = \rho \cos \theta \sin \phi$, $y = \rho \sin \theta \sin \phi$, $z = \rho \cos \phi$, $dV = \rho^2 \sin \phi d\rho d\phi d\theta$.
- Average value: $f_{\text{av}} = \frac{1}{A(D)} \iint_D f(x, y) dA$ or $f_{\text{av}} = \frac{1}{V(E)} \iiint_E f(x, y, z) dV$, where $A(D)$ and $V(E)$ denote the area of D and volume of E , respectively.
- $\sin^2 \theta + \cos^2 \theta = 1$.