University of Lethbridge
Department of Mathematics and Computer Science
December 18th, 2015, 6:00 - 9:00 pm
MATH 1010 - FINAL EXAM

Last Name:____

First Name:____

Student Number:______________________________________

Record your answers below each question in the space provided. Left-hand pages may be used as scrap paper for rough work. If you want any work on the left-hand pages to be graded, please indicate so on the right-hand page.

Partial credit will be awarded for partially correct work, so be sure to show your work, and include all necessary justifications needed to support your arguments.

For grader's use only:

Page	Grade
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1. Evaluate the following limits:

[2] (a) $\lim_{x\to 0} \dfrac{99x^{11} - 23x^6 + 1010}{5x^4 - 7x^2 + 141x + 1}$

[2] (b) $\lim_{x\to 2} \dfrac{x^2 - 2x}{x^2 - 5x + 6}$

[3] (c) $\lim_{x\to 0} \dfrac{\sin 3x}{x}$

[3] (d) $\lim_{x\to -\infty} \dfrac{\sqrt{3x^2 + 4}}{x}$

[5] 2. Determine the set of all x such that the function

$$f(x) = \begin{cases} 2\sin x, & \text{if } x \leq 0 \\ x^2 - 5, & \text{if } 0 < x \leq 2 \\ 3x - 7, & \text{if } x > 2 \end{cases}$$

is continuous. Express your answer in interval notation.

[5] 3. Using only the **definition of the derivative**, compute $f'(2)$ if $f(x) = \dfrac{1}{x-1}$.

4. Compute the derivatives of the following functions:

[2] (a) $f(x) = 2x^7 - 3x^3 + \dfrac{4}{x} + \sqrt{2}$

[2] (b) $g(x) = \tan(x)\cos(x)$

[3] (c) $h(x) = \dfrac{2x^4 - 3x}{x^2}$

5. Suppose you know that $f(3) = 4$, $g(3) = -2$, $f'(3) = \frac{4}{3}$, and $g'(3) = \frac{1}{2}$ for two functions f and g.
If $h(x) = f(x)g(x)$, what is the equation of the tangent line to the curve $y = h(x)$ when
[3] $x = 3$?

[5] 6. Find the global (absolute) maximum and minimum values of $f(x) = x^3 - 3x$ on the interval $[0, 2]$.

[5] 7. Find and classify the critical points of the function $g(x) = x^{5/3} - 5x^{2/3}$.

8. Solve the following inequalities. Express your answers in interval notation.

[3] (a) $|3x - 5| \leq 7$.

[3] (b) $x^2 < 12 - x$.

[4] (c) $1 \leq \dfrac{6}{x^2 + x}$.

9. Find the exact values of the trigonometric functions below at the given angles:

[2] (a) $\cos(11\pi/3)$

[2] (b) $\sin(43\pi/4)$

[3] (c) $\cot(-23\pi/6)$

[3] (d) $\cos(\theta)$, if the angle θ lies in Quadrant II and $\sin(\theta) = 4/5$.

10. (a) Show that $\frac{d}{dx}(\tan(x)) = \sec^2(x)$, or that $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$.
(Do **only one** of the two. The derivatives of $\sin(x)$ and $\cos(x)$ are given on the last
[3] page.)

[4] (b) Show that $\frac{1}{1-\sin(x)} = \sec^2(x) + \sec(x)\tan(x)$.

[3] (c) Determine the function $f(x)$ such that $f'(x) = \frac{1}{1-\sin(x)}$ and $f(\pi) = 3$.

11. Consider the function $f(x) = x^3 - 12x + 16$.

[3] (a) Given that $a = 2$ is a zero of multiplicity 2, find the remaining real zero of f.

[1] (b) Construct the sign diagram for f.

[2] (c) Calculate $f'(x)$.

[2] (d) Construct the sign diagram for f'.

[2] (e) Calculate $f''(x)$, and construct the sign diagram for f''.

(f) Sketch the graph of $f(x)$. Your sketch should be clearly labelled with the following information:

[5] x and y-intercepts, any local maxima or minima, and any inflection points.

Note: Your vertical scale does not need to match your horizontal scale.

12. Consider the function $f(x) = \dfrac{x}{x^2 - 2x + 1}$.

[1] (a) What is the domain of f?

[1] (b) Construct a sign diagram for f.

[1] (c) Determine any x or y-intercepts for the graph of f.

[2] (d) Determine any horizontal or vertical asymptotes for the graph of f.

[3] (e) Calculate $f'(x)$.

[2] (f) Construct the sign diagram for f', and determine any local maxima or minima.

[5] (g) Sketch the graph of f.

Some true stuff from the course that you possibly didn't remember

- We say $x = a$ is a zero of **multiplicity** k for a polynomial $p(x)$ if $(x-a)^k$ is a factor of $p(x)$, but $(x-a)^{k+1}$ is not.
- The **Factor Theorem** states that for a polynomial function $p(x)$, $p(a) = 0$ if and only if $(x-a)$ is a factor of p.
- Values of $\sin\theta$ and $\cos\theta$ in the first quadrant:

$$\begin{aligned} \sin 0 &= 0 & \cos 0 &= 1 \\ \sin \pi/6 &= 1/2 & \cos \pi/6 &= \sqrt{3}/2 \\ \sin \pi/4 &= \sqrt{2}/2 & \cos \pi/4 &= \sqrt{2}/2 \\ \sin \pi/3 &= \sqrt{3}/2 & \cos \pi/3 &= 1/2 \\ \sin \pi/2 &= 1 & \cos \pi/2 &= 0 \end{aligned}$$

- Fundamental identities:
  1. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\csc\theta = \dfrac{1}{\sin\theta}$
  2. $\cos^2\theta + \sin^2\theta = 1$
  3. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
  4. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
  5. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
  6. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- Obvious but occasionally forgotten facts that are sometimes useful in conjunction with some of the identities above:
  1. $2\theta = \theta + \theta$
  2. $\theta = \dfrac{\theta}{2} + \dfrac{\theta}{2}$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$.
- $f'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$.
- $\dfrac{d}{dx}(x^n) = nx^{n-1}$, $\dfrac{d}{dx}(\sin x) = \cos x$, and $\dfrac{d}{dx}(\cos x) = -\sin x$.
- $(fg)' = f'g + fg'$ and $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$.
- A **critical point** for a function f is a number c such that $f'(c) = 0$ (or doesn't exist); $f(c)$ is the corresponding critical value.
- If you spend all your time reading this page, you won't have time to complete the exam.