Practice Problems for Quiz 5
Math 2000A

Quiz #5 will take place in class on Thursday, October 9th. (This is the last quiz before the midterm!)

1. Determine the negations of the definitions for the union, intersection, and difference of sets, in order to complete the following sentences:

   (a) $x \notin A \cup B$ if and only if ...

   (b) $x \notin A \cap B$ if and only if ...

   (c) $x \notin A \setminus B$ if and only if ...

2. Let P, Q, R and S be subsets of some universal set U. Assume that $(P \setminus Q) \subseteq (R \cap S)$.

   (a) Complete the following sentence: For all $x \in U$, if $x \in (P \setminus Q)$, then ...

   (b) Write a useful negation of the sentence in part (a).

   (c) Write the contrapositive of the sentence in part (a).

3. Let $A = \{x \in \mathbb{R} : x^2 < 4\}$ and $B = \{x \in \mathbb{R} : x < 2\}$.

   (a) Is $A \subseteq B$? Justify your conclusion with a suitable proof or counterexample.

   (b) Is $B \subseteq A$? Justify your conclusion with a suitable proof or counterexample.

4. Prove the following proposition: For all subsets A and B of some universal set U, $A \subseteq B$, if and only if $B^c \subseteq A^c$, where A^c, B^c denote the complements of A and B, respectively.

5. Let A, B, C, and D be subsets of some universal set U. For each of the following propositions, either prove that it is true, or show that it is false by giving a counterexample:

   (a) If $A \subseteq B$ and $C \subseteq D$, and A and C are disjoint, then B and D are disjoint.

   (b) If $A \subseteq B$ and $C \subseteq D$, and B and D are disjoint, then A and C are disjoint.

   (Recall that two sets U and V are disjoint if $U \cap V = \emptyset$.)

6. Determine whether the following biconditional statements are true or false. If a statement is found to be false, indicate whether one direction or the other (either the "if" part, or the "only if" part) is true.

   (a) For all subsets A and B of some universal set U, $A \subseteq B$ if and only if $A \cap B^c = \emptyset$.

(b) For all subsets A and B of some universal set U, $A \subseteq B$ if and only if $A \cup B = B$.

(c) For all subsets A, B, C of some universal set U, $A \subseteq B \cup C$ if and only if $A \subseteq B$ or $A \subseteq C$.

7. Prove the following set equalities:

$$A \setminus \emptyset = A \quad (A^c)^c = A \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad (A \cap B)^c = A^c \cup B^c$$

8. Prove or disprove (via counterexample) the following set equalities:

   (a) $A \setminus (A \cap B^c) = A \cap B$

   (b) $(A^c \cup B)^c \cap A = A \setminus B$

   (c) $(A \cup B) \setminus A = B \setminus A$

   (d) $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$

9. For each natural number n, let $A_n = \{n, n+1, n+2, n+3\}$. Determine the elements of the following sets:

$$\bigcap_{j=1}^{3} A_j \quad \bigcup_{k=3}^{7} A_k \quad A_9 \cap \left(\bigcup_{n=3}^{7} A_n \right) \quad \bigcup_{i=3}^{7} (A_9 \cap A_i)$$

10. Let I be a nonempty indexing set, and let $\mathcal{A} = \{A_\beta : \beta \in I\}$ be an indexed family of sets.

   (a) Prove that for each $\beta \in I$, $A_\beta \subseteq \bigcup_{\alpha \in I} A_\alpha$.

   (b) Prove that $\left(\bigcup_{\gamma \in I} A_\gamma \right)^c = \bigcap_{\gamma \in I} A_\gamma^c$.

   (c) Prove that for any set B, $B \cup \left(\bigcap_{\beta \in I} A_\beta \right) = \bigcap_{\beta \in I} (B \cup A_\beta)$.