1. If $z = 5 - 3i$ and $w = -2 + 4i$, compute the following:

   (a) $z + w = (5 - 3i) + (-2 + 4i) = (5 - 2) + i(-3 + 4) = 3 + i$

   (b) $zw = (5 - 3i)(-2 + 4i) = -10 + 20i + 6i - 12i^2 = -10 + 26i - 12(-1) = 2 + 26i$

   (c) $z + \overline{z} = (5 - 3i) + \overline{(5 - 3i)} = (5 - 3i) + (5 + 3i) = 10$

   (d) $\dfrac{z}{w^2}$

   First, we compute $w^2 = (-2 + 4i)^2 = (-2 + 4i)(-2 + 4i) = 4 - 8i - 8i + 16i^2 = 4 + 16(-1) - 16i = -12 - 16i$. Thus

$$\begin{aligned}\frac{z}{w^2} &= \frac{5-3i}{-12-16i} = \frac{5-3i}{-12-16i}\cdot\frac{-12+16i}{-12+16i}\\ &= \frac{-60+80i+36i-48i^2}{(-12)^2-12(16i)+12(16i)-(16i)^2} = \frac{-60+48+116i}{144+256} = \frac{-12+116i}{400}\\ &= -\frac{12}{400} + i\frac{116}{400} = -\frac{3}{100} + i\frac{29}{100}i.\end{aligned}$$

2. Find all solutions (real or complex) to the following:

   (a) $z^2 + z + 2 = 0$

   Using the quadratic equation, we have

$$z = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)} = \frac{-1 \pm \sqrt{-7}}{2} = -\frac{1}{2} + i\frac{\sqrt{7}}{2}.$$

   (b) $z^4 - 16 = 0$

   Factoring, we have

$$z^4 - 16 = (z^2 + 4)(z^2 - 4) = (z + 2i)(z - 2i)(z + 2)(z - 2) = 0,$$

   so the solutions are $z = 2, -2, 2i, -2i$.

3. Convert the points $\left(3, \dfrac{2\pi}{3}\right)$, $\left(-4, \dfrac{-3\pi}{4}\right)$, and $\left(2, \dfrac{7\pi}{6}\right)$ from polar to rectangular coordinates.

   Cartesian coordinates are related to polar by $x = r\cos\theta, y = r\sin\theta$. For $r = 3, \theta = \dfrac{2\pi}{3}$, we have

   $$x = 3\cos\left(\frac{2\pi}{3}\right) = 3\left(-\frac{1}{2}\right) = -\frac{3}{2}, y = 3\sin\left(\frac{2\pi}{3}\right) = 3\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2},$$

   so our point is $\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$. For $r = -4$, $\theta = -\dfrac{3\pi}{4}$, we have

   $$x = -4\cos\left(-\frac{3\pi}{4}\right) = -4\left(-\frac{1}{\sqrt{2}}\right) = 2\sqrt{2}, y = -4\sin\left(-\frac{3\pi}{4}\right) = -4\left(-\frac{1}{\sqrt{2}}\right) = 2\sqrt{2},$$

   so the second point is $(2\sqrt{2}, 2\sqrt{2})$ in rectangular coordinates. For $r = 2, \theta = \dfrac{7\pi}{6}$, we have

   $$x = 2\cos\left(\frac{7\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}, y = 2\sin\left(\frac{7\pi}{6}\right) = 2\left(-\frac{1}{2}\right) = -1,$$

   so the last point is given in rectangular coordinates by $(-\sqrt{3}, -1)$.

4. Convert the points $(2, -2)$ and $(-3, \sqrt{3})$ from rectangular to polar coordinates.

   For the point $(2, -2)$, we have $r = \sqrt{2^2 + (-2)^2} = \sqrt{8}$, and $\tan\theta = \dfrac{-2}{2} = -1$, which gives us a related angle of $\pi/4$. Since the point $(2, -2)$ is in the fourth quadrant, we take $\theta = -\frac{\pi}{4}$. (If you want to use θ between 0 and 2π instead, then take $\theta = \dfrac{7\pi}{4}$.) Thus, we have the point $\left(\sqrt{8}, -\dfrac{\pi}{4}\right)$.

   For the point $(3, -\sqrt{3})$, we have $r = \sqrt{(-3)^2 + \sqrt{3}^2} = \sqrt{12}$, and $\tan\theta = \dfrac{-\sqrt{3}}{3}$, so the related angle is $\pi/6$. Since the point $(3, -\sqrt{3})$ is in the second quadrant, we take $\theta = \dfrac{5\pi}{6}$, and our point is given in polar coordinates by $\left(\sqrt{12}, \frac{5\pi}{6}\right)$.

5. Let $z = 1 + i\sqrt{3}$ and let $w = \sqrt{2} - i\sqrt{2}$. Compute the following:

   (a) The polar forms of z and w.

   We have $|z| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$, and $\tan(\arg z) = \sqrt{3}$. Since z is in the first quadrant, $\arg z = \dfrac{\pi}{3}$.
   Similarly, $|w| = \sqrt{\sqrt{2}^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$, and $\tan(\arg w) = -1$. Since w is in the fourth quadrant, $\arg w = -\dfrac{\pi}{4}$.
   Thus, $z = 2e^{i\pi/3}$ and $w = 2e^{-i\pi/4}$. (Or $z = 2\,\text{cis}(\pi/3)$ and $w = 2\,\text{cis}(-\pi/4)$ if you prefer this notation.)

(b) z^2w

We have
$$z^2w = (2e^{i\pi/3})^2(2e^{-i\pi/4}) = 4e^{2i\pi/3}(2e^{-i\pi/4}) = 8e^{5i\pi/12},$$
where we used the fact that $\frac{2\pi}{3} - \frac{\pi}{4} = \frac{8\pi}{12} - \frac{3\pi}{12} = \frac{5\pi}{2}$. In Cartesian coordinates,
$$z^2w = 8\cos\left(\frac{5\pi}{12}\right) + i \cdot 8\sin\left(\frac{5\pi}{12}\right).$$

It is possible to work out the values above and verify that the result agrees with doing things the long way in Cartesian coordinates, but the above answer is sufficient.

(c) $\frac{z^4}{w}$

Here, we have
$$\frac{z^4}{w} = (2e^{i\pi/3})^4(2e^{-i\pi/4})^{-1} = 16e^{4i\pi/3}\left(\frac{1}{2}e^{i\pi/4}\right) = 8e^{19i\pi/12}.$$