Practice Problems for Quiz 2
Math 2000A

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Quiz #2 will take place in class on Thursday, September 18. The rules for the quiz are the same as for Quiz #1.

1. Give a two-column proof of the following deductions:
   (a) $A \leftrightarrow (B \wedge C)$, $A \wedge (\neg B \vee D)$, $\therefore (B \wedge D) \vee (A \leftrightarrow C)$
   (b) $A \vee B$, $A \to C$, $B \to (\neg C \vee D)$, $\therefore C \vee D$
   (c) $(P \vee \neg R) \to R$, $\therefore P \to R$
2. From the textbook, Exercise 3.12 (parts 1-6).
3. Recall that an integer is defined to be *even* if it is a multiple of 2; i.e., n is an even integer if $n = 2k$ for some integer k. Give a two-column proof of the following assertion:

   If n is even and m is any integer, then nm is also even.

   You do not need to use formal logic; instead, each of your justifications should be a hypothesis, a definition, or a known property of integer arithmetic. (For example, you may use the fact that multiplication of integers is *associative*: for any integers a, b, c we have $a(bc) = (ab)c$.)
4. Use proof by contradiction to establish the following:
   (a) Deduction: $A \to C$, $B \to \neg C$, $\therefore \neg(A \wedge B)$
   (b) Claim: there is no smallest rational number.
   (Rational numbers are those that can be written as fractions a/b. You may use the following "theorem" as one of your justifications: If r is a rational number, then $r/2$ is rational as well.)
5. Two well-known (and yet far too common) logical fallacies are *affirming the consequent* ($P \to Q$, Q, $\therefore P$) and *denying the antecedent* ($P \to Q$, $\neg P$, $\therefore \neg Q$). Show that these deductions are invalid by means of a counterexample.