University of California, Berkeley

FINAL EXAMINATION, Spring 2012
DURATION: 3 hours

Department of Mathematics

MATH 49 Multivariable Calculus Equivalency

Examiner: Sean Fitzpatrick

Total: 60 points

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Please sign here: ______________________________

Student ID Number: ______________________________

No aids, electronic or otherwise, are permitted, with the exception of the formula sheet provided with your exam. Partial credit will be given for partially correct work. Please read through the entire exam before beginning, and take note of how many points each question is worth.

Good Luck!

FOR GRADER'S USE ONLY	
Problem 1:	/8
Problem 2:	/10
Problem 3:	/12
Problem 4:	/8
Problem 5:	/12
Problem 6:	/10
TOTAL:	/60

[8] 1. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle 2x, x + y, y + 3z \rangle$, and C is the straight line segment from $(1, 0, 2)$ to $(3, -4, 4)$.

2. Consider the vector field $\mathbf{F}(x, y, z) = \langle y^2, axy + z^3, byz^2 \rangle$, where a and b are constants.

   [6] (a) Find values of a and b such that $\mathbf{F}$ is a conservative vector field, and then find a potential function $f(x, y, z)$ such that $\nabla f(x, y, z) = \mathbf{F}(x, y, z)$.

   [4] (b) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line segment between the points $(1, 2, 0)$ and $(2, -1, 3)$ using the values of a and b found in part (a).

3. Let S be the parametric surface given by $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, v\rangle$, for $u \in [0,2]$ and $v \in [-\pi,\pi]$. (S is a "spiral ramp".)

[6] (a) Find an equation for the tangent plane to S at the point $(0,1,\pi/2)$.

[6] (b) Find the surface area of S.

4. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle -y, x, -2 \rangle$, and S is the surface of the cone $z = \sqrt{x^2 + y^2}$, for $1 \leq z \leq 4$, oriented towards the xy-plane.

[8]

[12] 5. Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = \langle x^2, xy, z\rangle$ and the solid E bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

6. Prove *Gauss' Law*: Let $\mathbf{F}$ be the vector field $\mathbf{F} = \dfrac{\mathbf{r}}{\|\mathbf{r}\|^3}$, where $\mathbf{r}(x, y, z) = \langle x, y, z \rangle$. Let E be any closed, bounded region in $\mathbb{R}^3$ with piecewise-smooth boundary S, oriented by the outward-pointing unit normal vector, such that S does not pass through the origin. Then [10]

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \begin{cases} 4\pi, & \text{if } 0 \in E \\ 0, & \text{if } 0 \neq E \end{cases},$$

Hint: Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ directly for the sphere $x^2 + y^2 + z^2 = a^2$. Then use the Divergence Theorem.