
Submit your homework to Gradescope (through Canvas) as a PDF file containing your name and UCSC ID number. Feel free to discuss solutions with your colleagues (other students), explore the internet, and consult textbooks for inspiration. Make use of discussion sessions and office hours. Do not copy solutions. Doing so will hinder your learning. You must type or write your solutions on your own.

1. (25 points) Recall the following deterministic select algorithm:
 - (a) Divide the n elements into groups of size 5. So $n/5$ groups.
 - (b) Find the median of each group.
 - (c) Find the median x of the $n/5$ medians by a recursive call to Select.
 - (d) Call Partition with x as the pivot.
 - (e) Make a recursive call to Select either on the smaller elements or larger elements (depending on the situation); if the pivot is the answer we are done.

Then, the recurrence for the running time was $T(n) = T(\frac{1}{5}n) + T(\frac{7}{10}n) + O(n)$.

Suppose we modify the algorithm as follows. If $T(n)$ denotes the running time of the new recursive algorithm, what is the running time of each step?

- (a) Divide the n elements into groups of size 11. So $n/11$ groups.
- (b) Find the median of each group.
- (c) Find the median x of the $n/11$ medians by a recursive call to the new Select algorithm.
- (d) Call Partition with x as the pivot.
- (e) Make a recursive call to the new Select algorithm either on the smaller elements or larger elements (depending on the situation); if the pivot is the answer we are done.

What recurrence relation for the running time do you obtain?

2. (15 points) You are given an array $A[1 \dots n]$ of n integers, an integer k (where $1 \leq k \leq n$), and a range specified by a lower bound L and an upper bound U . Design an $O(n)$ time algorithm to find the k largest elements in the array that fall within the range $[L, U]$. Assume all the integers are distinct for convenience.