The Expected Wage Premium and Models of Random Job Search*

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Abstract

Models of random search on the job make clear predictions about the *expected wage premium*: the premium or discount in pay that workers should anticipate in their next job offer relative to their current wage. Using survey data, I document empirical facts about the expected wage premium and I show that, as predicted by classic models of random job search: (a) the average expected wage premium is negative (a discount), and (b) it decreases with job tenure. However, these models cannot reconcile two observed empirical patterns: the substantial dispersion in expected wage premium, which suggests a sizable ladder of job opportunities workers can climb, and the small magnitude of the average expected discount, which indicates that wage gains from climbing this ladder are modest. I propose a model that can reconcile these facts, featuring: (i) productivity gains that are not immediately reflected in the wages of employed workers, and (ii) reallocation events in which workers move jobs for reasons other than wage gains. When calibrated to match these new empirical facts, the proposed model predicts less wage growth and lower wage inequality from job search and search frictions than classic random search models.

Keywords: Expected wage premium, Job search, Job ladder, Wage inequality, Wage growth

JEL Codes: E24, J33

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1 Introduction

Models of random search on the job are the main structural tool for studying wage inequality and wage growth. The theory of random search on the job is characterized by a *job ladder* mechanism: there is a heterogeneity of job opportunities in the market which are ranked based on their wage paying potential; workers randomly meet and positively select better job opportunities throughout their career. A crucial prediction arising from this mechanism is how the wage of new offers received by employed workers should compare to their current wage. For instance, in the canonical Burdett and Mortensen (1998) model of wage posting: (i) the average worker should expect a discount (i.e., a lower wage) relative to their current wage on the next offer received from the market; (ii) the expected discount should be increasing in job tenure. Every model of random search on the job yields similar predictions, and they reflect the importance of job search in determining wages: if climbing the job ladder is important - there is a substantial dispersion of job opportunities in the market - these expected discounts should be sizable, especially for high-tenure workers who are likely to be at the top of the ladder.

This paper leverages these predictions and available information on workers' expectations regarding new job offers to revisit the theory of random job search and to shed new light on the role of job search for wage growth and inequality. Using data from the New York Fed's Survey of Consumer Expectations, I compute the *expected wage premium* (W_p) ratio, defined as the premium (or discount) workers anticipate in wages from new job offers relative to their current wages. For instance, a worker with a W_p of -10% expects that new offers, on average, will pay 10% less than their current job. This measure enables the evaluation of the theory's core predictions and is informative about the average distribution of available job opportunities (the job ladder) in the market.

The first contribution of this paper is to verify how well empirical observations on the *expected wage premium* align with the random search theory. I first validate the use of data on workers' subjective expectations through a rationality test using actual job offers received by them, finding no evidence of systematic bias in workers' forecasts. Then, I document a set of empirical facts about the *expected wage premium* and compare them to the predictions of two classic models of random search on the job: the Burdett and Mortensen (1998) (BM) wage-posting model and the Postel-Vinay and Robin (2002) (PVR) wage-bargaining model. Notably, as predicted by the theory of random job search: (i) the average W_p is negative—workers, on average, expect a discount in the wages of new job offers; and (ii) there is a clear negative covariance between W_p and job tenure—high-tenure workers anticipate larger discounts.

Interestingly, the magnitudes of average W_p — both unconditional and conditional on job tenure — are small in the data, indicating modest wage gains from job search for most workers, but the variance of W_p is sizable, indicating a substantial dispersion of job opportunities in the market (a sizable job ladder to be climbed). Neither benchmark model can jointly account for both these empirical facts: If calibrated with enough dispersion in job opportunities so they can match the observed variance in W_p, the BM model predicts an average W_p of –32% and the PVR model –15.3%, while in the data the average is only –2.3%. For workers with ten or more years of tenure, the models predict even larger discounts: –48.3% for BM and –33.2% for PVR, compared to only –6.8% in the data.

These observations raise important questions: why can't classic models of random job search account for the relationship between the mean and variance of W_p? Are workers not climbing the job ladder much, as suggested by the small magnitudes of the mean W_p? Or perhaps the W_p is not reflecting how much of the job ladder they have climbed?

Further investigation into the number of job offers received by employed workers suggests that, indeed, not all job transitions observed can be explained by upward moves on the job ladder. This data also casts doubt on the hypothesis that workers can significantly *direct their job search* toward better job positions, which could otherwise explain why W_p does not decline as expected when workers move to better jobs. An alternative hypotheses explored in this paper is that job offers improve over time relative to workers wages, due to productivity gains not incorporated into workers' pay. This can explain why W_p remains higher than what classic models predict as workers advance on the job ladder.

The two aforementioned hypotheses—(A) limited ladder climbing, and (B) gradual offer improvement—have different implications for the role of job search in generating wage inequality and wage growth. This paper's second contribution, then, is to propose a random search model (RSM) that incorporates these two mechanisms and assess their relative importance in explaining the empirical patterns of W_p.

The proposed model extends the PVR model of wage bargaining with the following features: productivity gains steaming from human capital accumulation and aggregate productivity growth; fixed-wage contracts; and reallocation events. The combination of human capital and productivity growth with fixed-wage contracts are key to the model, they generate a mechanism by which new offers workers receive are frequently improving relative to their wages, increasing W_p. The other feature, reallocation toward worst jobs, slows down the effect of positive selection in the economy, reducing average wages, thus increasing W_p as well. Since these two mechanics have distinct implications for the importance of job search in driving wages, I discipline them using data on cross sectional wage differences between workers of different experience level, the aggregate growth rate of wages over time, and the survey data on contact rates between workers and job opportunities. I then calibrate the model to match moments of the W_p distribution and I show that it is able to account very well for all documented empirical regularities.

The last contribution of this paper is to assess the role of search for wage growth and wage inequality according to the data on job offers. Through the lens of the proposed model, characteristics of the average job ladder are inferred from data on the W_p. This provides a novel take on the issue of assessing the role of job search for wage growth and wage inequality, as prior research have relied on data from observed wage changes.

According to the proposed model, job search accounts for 38.4% of total wage growth over 20 years for college-educated workers, similar to what Bagger et al. (2014), obtained using Danish matched employer-employee data. In their best attempt to fit the data, the PVR and BM would instead attribute close to 60% of the estimated wage growth to job search. With respect to wage inequality, both the proposed model and PVR suggest that job search explains about 37% of the wage dispersion not explained by workers observable characteristics. However, the BM model attributes approximately 57% of wage dispersion to job search. I conclude by discussing how the different mechanisms in each model affect how job search influence wages.

This paper directly contributes to the literature employing structural search models to examine how search frictions and job search impact wage dispersion and wage growth. It relates to works such as Burdett and Mortensen (1998), Bontemps et al. (2000), Postel-Vinay and Robin (2002), Mortensen (2005), Jolivet et al. (2006), and Tjaden and Wellschmied (2014) on wage inequality, and Bagger et al. (2014) on wage growth. This paper provides direct evidence of the job ladder mechanism central to all these models and offers the first assessment of the importance of job search for wage inequality and wage growth based on data from job offers made to workers.

Additionally, the paper examines core assumptions in job search theory that are important for aligning search models with the new empirical findings. It speaks to questions such as whether wages are bargained or posted (Hall and Krueger (2012)) and whether wage contracts are better characterized by piece rates or fixed values (Herkenhoff et al. (2018)). Findings suggest that (i) non-wage-motivated job transitions, (ii) bargaining, and (iii) fixed-wage contracts are essential for explaining observed empirical patterns of the expected wage premium. The approach here echoes Hornstein et al. (2011), in which assumptions typically incorporated in a random search model are evaluated based on how well theoretical implications from the model align with the data.

The paper is organized as follows: Section 2 discusses the theoretical relationship between the job ladder mechanism and the expected wage premium in models of random job search. Section 3 documents empirical facts about the expected wage premium and compares these findings to predictions from the canonical BM and PVR random search frameworks. It also presents data on the number of job offers workers receive, providing evidence that workers are not climbing the job ladder as quickly as suggested by job-to-job transition data. Additionally, this section discusses how the data on job offers fails to support the hypothesis that workers can direct their search toward better job opportunities.

Section 4 introduces a random search model (RSM) featuring productivity gains from human capital accumulation and aggregate productivity, fixed but renegotiable wages, and reallocation shocks. Section 5 details the calibration strategy used to discipline the job ladder and productivity gains within the model, and shows that the proposed model accounts well for the data. Section 6 evaluates how much of wage inequality and wage growth can be attributed to job search according to the proposed model in light of the data on expected wage offers and discusses how classic models of random job search overestimate the importance of job search in determining wages. Section 7 concludes

2 The Job Ladder and Expected Wage Premium

The job ladder is a central feature in models of random job search with search on the job. In such a framework, there exists a diverse set of job opportunities in the market. A job opportunity x is typically characterized either by a wage, w — in models with wage posting — or by the productivity p of a job vacancy — in models with wage bargaining. Workers randomly meet job opportunities throughout their careers and move to better jobs as they arise.

In this setting, job opportunities, represented by x, are ranked by a strict preference structure established by the worker's utility function, the production function, and the wage setting protocol. This structure usually implies that if $x' > x$, then x' is strictly preferred over x, and so workers move upward the job ladder as they encounter and accept better ranked jobs.

Define **job tenure** (t_n) as the length of time a worker has been continuously employed in their current position. Higher tenure indicates a longer period without changing jobs. Then, assuming that the distribution of job opportunities follows the cumulative density function (cdf) $F(x)$, the solution to this class of models is characterized by an endogenous distribution of workers over job opportunities, $G(x)$, and the general results:

$$F(x) > G(x) \qquad \forall x \tag{1}$$

$$G(x|t_n) > G(x|t'_n) \quad \forall x, \text{ for } t'_n > t_n \tag{2}$$

On average, the jobs that workers hold are better than a randomly selected job from the overall distribution of available opportunities, so $G(x)$ first-order stochastically dominates (FSD) $F(x)$. Moreover, tenure on the job (t_n) and job quality are positively correlated, so $G(x|t'_n)$ FSD $G(x|t_n)$ for $t'_n > t_n$. Intuition for the results is straightforward: (1) because workers move to better jobs when they meet them, they tend to hold positions that are better than the average position on the job ladder; (2) high tenure workers are those who haven't met an opportunity better than their current job in a long while, so they are more likely to be holding a good position.

How does these result relate to wages? In a setting with wage posting such as the BM, the implications for wages are direct:

$$F(w) > G(w) \qquad \forall w \tag{3}$$

$$G(w|t_n) > G(w|t'_n) \quad \forall w, \text{ for } t'_n > t_n \tag{4}$$

On average, workers hold a better wage than the average wage offered. Moreover, wages are increasing in tenure. Now, let's define the *expected wage premium*, for worker i, as:

$$W_{p,i} = \frac{E_F[w]}{w_i} - 1$$

Assuming $g(w)$ has the support: $[\underline{w}, \overline{w}]$. From the (3) and (4), it follows that[1]:

$$E[W_{p,i}] = \int_w \Big(\frac{E_F[w]}{w}\Big) g(w) dw - 1 \quad < 0 \tag{5}$$

and also

$$E[W_{p,i}|t'_n] \quad < \quad E[W_{p,i}|t_n] \quad \text{for} \quad t'_n > t_n \tag{6}$$

On average, worker expected wage premium should be negative. Moreover, the expected wage premium should be decreasing in tenure. The expected wage premium statistic provides predictions of the mechanism that can be verified in the data.

In models of wage bargaining, predictions about the W_p are not so straightforward to be derived. With wage bargaining, job opportunities are drawn from a distribution of productivity, $F(p)$, and the mapping from opportunities to wages will give rise to an aggregate distribution of wage offers, $H(w)$, which might not be stochastically dominated by $G(w)$, like in a wage posting model, so (3) and (5) might not hold in general - even though it will be true in typical parametrization as shown throughout this paper. Nonetheless, in these models, under standard preferences, $Cov(p, w) > 0$, so (4) and by consequence (6) will, in fact, hold generally.

Quantitatively, magnitudes of W_p reflect the importance of climbing the ladder for determining wages. If the ladder is narrow (low dispersion of opportunities), even workers on the top of the ladder shouldn't expect new offers to differ much from their current wage, and W_p should be small in magnitude. On the other hand if the job ladder is sizable, W_p will be too.

[1] In rigor, the statements should require some lower bound on $\underline{w}$ so it is sufficiently distant from zero; $\underline{w} > 1$ is sufficient

3 Empirical Analysis

This section introduces, documents and analysis the empirical regularities of the expected wage premium through the lens of random search theory. It focus on the data obtained from the New York Fed's Survey of Consumer Expectation. This paper also uses CPS data, with details on its handling provided in the appendix.

3.1 Data and measurement

The New York Fed's Survey of Consumer Expectation (NY-SCE) interviews a nationally representative sample of households, following a rotating panel structure. The core module of the survey is conducted monthly and contains mostly questions that assess households' expectations about the macro-economy on top of collecting basic socioeconomic information. In addition to the core module, other thematic special modules are conducted every 4 months with the goal of collecting detailed information about different aspect of households economic reality. This work relies mostly on information from the 'Labor Market Survey' special module, which collects information about households' recent experiences and activities in the labor market, such as the number job offers they received in the last 4 months, the wage of those offers, whether they were actively looking for a job or not, what is the minimum wage they would require to accept a job (or change jobs), etc. It also collects information about workers' (head of households who report to be in the labor market) expectations regarding their future labor market outcomes, including their believes about future job offers they might receive. Around 1300 households receive questionnaires every month, and each household stays in the panel for twelve months. Since the Labor Market submodule is conducted every 4 months, I observe the same household 3 times at most for the purposes of this study. This work uses data from July 2014 to November 2021. With the available data, I am able to compute, for each worker-period observed, the expected wage premium (W_p), defined in the data as:

$$W_p = \frac{\text{Expected Wage of Future Job Offers}}{\text{Current Wage}} - 1$$

The data on the expected wage of future job offers is collected by the question [oo2a] from the Labor Market Survey Module, which is reproduced below:

Question OO2a - Expected Average Offer *Think about the job offers that you may receive within the coming four months. Roughly speaking, what do you think the average annual salary for these offers will be for the first year?*

In the Appendix, I list the questions associated with all other variables used in this paper: current wages, number of offers received, wage of offers received, tenure on the current job, and others. Information on wages is asked in annual terms. Expectations about labor market outcomes are usually asked 'over the next four months,' and past experiences are asked with respect to 'the previous four months.' The main dataset used is composed of workers over 24 years old, not self-declared students, employed in a single full-time job, with a wage range between $15,000 and a third of a million dollars per year. I also discard observations of workers who declared having received or expect to receive job offers outside the aforementioned wage range. Lastly, I discard around 120 observations for which the computed expected wage premium exceeds 100%—more than double the current wage—as a large proportion of those appear to result from respondents' typos. Table 1 provides a summary of the main dataset.

College-educated workers represent the largest demographic in the sample. Therefore, this paper will primarily focus on this group.

	Obs.	Av. Wage	Av. Age	% Male
Main Sample	9,481	75,147	45	0.58
College Degree	6,355	94,115	41	0.59
Some College	2,463	64,075	46	0.53
High School	663	50.464	52	0.61

Table 1: Summary Main Dataset

3.2 Expected and Realized Wage Offers

The empirical strategy relies on workers' subjective expectations as unbiased predictors of the wages for offers they might receive from new job opportunities. Ideally, data on actual job offers would be used for the empirical analysis. While the NY-SCE captures information on the wages of actual job offers, there are naturally fewer observations of these realized offers. Thus, I use the data on realized offer wages to validate the assumption that workers form rational, unbiased expectations about the wages of offers they might receive.

To validate this assumption, I run a linear regression of the average wage of job offers that workers reported receiving (see question NL2 in the Appendix) in a particular survey wave on their expected average wage of new offers, as reported in the prior survey wave, while controlling for their current wage from the first wave.

The estimated regression coefficient of 0.951 (SE: 0.035) – regression table in the Appendix – is not statistically distinguishable from unity, indicating consistency with the unbiased expectation hypothesis.

The absence of systematic bias in workers' wage expectations for new job offers has been documented by Conlon et al. (2018), while Caplin et al. (2023) also find that workers are generally accurate in predicting future income changes.

3.3 Empirical Facts on the Expected Wage Premium

Table 2 presents summary statistics for the distribution of the W_p, alongside a visualization of the distribution. All information in this section is presented for two broad categories of educational level: 'College' and 'Non-college' where the latter category aggregates workers with some college and high School degree only. Table 2 also presents a summary for the whole sample, and the plotted distribution is also a distribution for the whole main sample.

The average expected wage premium is negative for the entire sample, as well as within each educational group. On average, workers anticipate a 2.3% discount in their next job offer relative to their current wage, with college-educated workers expecting a smaller reduction of just 1%. It is worth noting that the distribution of W_p suggests that this expected discount may be understated in the survey data. A notable number of workers report a W_p of zero, and there seem to be a corresponding absence of slightly negative W_p values. This suggests that workers may be "rounding" their expected wage offers to match their current wage, especially when they foresee only a slight decrease. A smoothing kernel estimation could address the issue, but it might introduce other sources of imprecision.

The variance of W_p for the main sample is 0.053, a standard deviation of around 0.25 on the discount or premium workers expect to get in the next offer received. This suggest a sizable dispersion of opportunities in the job ladder. On that note, nearly one-third of workers anticipate a discount of 10% or more in their next offer, while 15% expect these discounts to exceed 20%. Skewness shows the distribution to be symmetric, even though overall the left tail is slightly thicker.

Moments	**Sample**	**College**	**NoCollege**
Mean W_p	-0.023	-0.009	-0.035
Var W_p	0.053	0.049	0.057
Skewness	0.00	0.00	0.00
Share $< -10\%$	0.32	0.28	0.35
Share $> 10\%$	0.22	0.23	0.22
Share $< -20\%$	0.17	0.14	0.20
Share $> 20\%$	0.11	0.11	0.11
Obs.	9,578	6,355	3,223

Table 2: Summary W_p

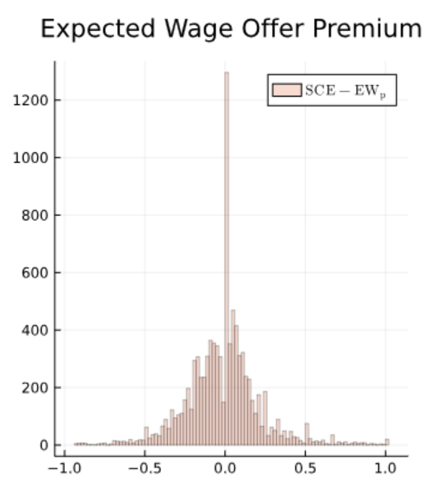
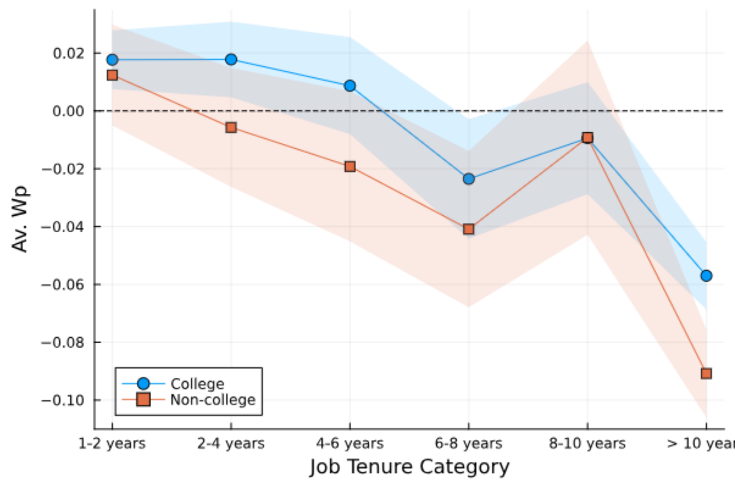


Figure 1 shows the tenure profile of W_p for the two sub-groups of the sample, according to tenure categories. The expected wage premium is decreasing in job tenure. In fact, workers with low job tenure expect an actual premium ($W_p > 0$) on the next job offer they receive from the market. The average expected premium becomes negative for the group of non-college-educated workers after the second year of tenure, but it remains positive for college educated workers until around 6 years of job tenure. Workers with 10 or more years of job tenure, in the college educated group, expect, on average, a discount of 5.7% on the next offers they receive, while the other group expect this discount to be 9.1%.



Tenure	**1-2 years**	**2-4 years**	**4-6 years**	**6-8 years**	**8-10 years**	10^{+} **years**
College	0.017	0.017	0.008	-0.023	-0.009	-0.057
Non-College	0.012	-0.006	-0.019	-0.041	-0.001	-0.091

Figure 1: Mean W_p by Job Tenure

Overall, the qualitative predictions of the job ladder mechanism are supported by the data. Notably, the prediction that the expected wage premium W_p decreases with job tenure, shared by all models of random job search, is well observed. Furthermore, the variance of W_p, which reflects the dispersion of job opportunities, is higher among non college-educated workers, which is consistent with their lower average W_p according to the mechanism, reinforcing the conclusion that job search is particularly crucial for this demographic, as highlighted by prior research.

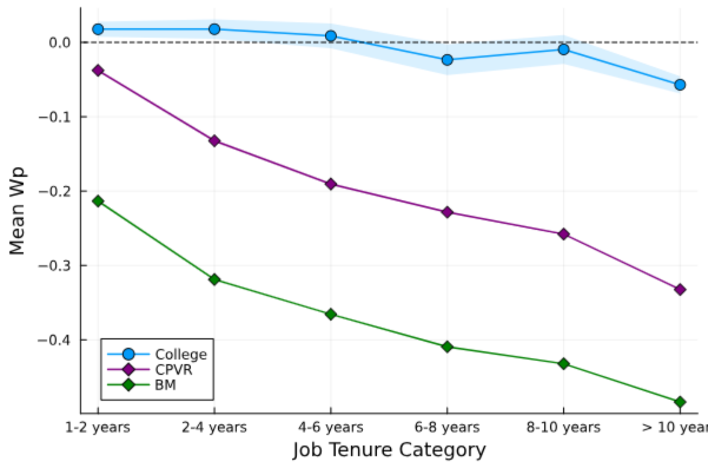
Two additional points are noteworthy: First, the variance of W_p is higher among non college-educated workers, consistently, according to the ladder mechanism, with their lower average W_p and suggesting a more relevant role for job search within this demographic, as previously highlighted in the literature. Second, the observed positive average W_p can be parsimoniously explained through a wage bargaining protocol based on opportunity costs - as will be shown later - which typically results in back-loaded wage structures.

Quantitatively, how well can two reference frameworks of random job search - the BM model of wage posting and the PVR framework of wage bargaining - account for the empirical facts on the expected wage premium? I parametrize the BM and the Cahuc et al. (2006) version of PVR (CPVR) to feature a job ladder disperse enough so they both replicate the variance of W_p observed in the data for the group of college-educated workers. I also calibrate the contact rate in both models to match the JtJ transition rate of 21.3% a year, observed for this demographic in the CPS[2]. Table 8, in the appendix, details other parameter choices, which are standard in the literature.

The resulting job tenure profile of mean W_p from these classic models are plotted in figure 2. Table 9, in the appendix, list other moments for comparison. Clearly, both models predict the W_p to be much lower than what is observed in the data. The unconditional mean W_p is -32% in BM and -15.3% in CPVR. The CPVR model of wage bargaining approximates the data better than the BM model of wage posting, but the difference is still substantial. As discussed previously, the job ladder mechanism establishes a relation between the dispersion of the ladder and the W_p. According to these two classic models of random search, if the ladder is sizable, as implied by the dispersion of W_p, then worker - especially hight-tenure workers who should be closer to the top of the ladder - should expect a big discount on new wage offers received from the ladder, a fact not seemingly supported by the data.

A natural explanation for this disconnection is: (A) workers do not climb the wage ladder as much as implied by the observed JtJ transition rate and the mechanism of positive selection embedded in standard search models. Another potential line of explanation is

[2]Own estimation based on Fujita et al. (2024)



Tenure	**1-2 years**	**2-4 years**	**4-6 years**	**6-8 years**	**8-10 years**	10^{+} **years**
College	0.017	0.017	0.008	-0.023	-0.009	-0.057
BM	-0.213	-0.318	-0.365	-0.409	-0.432	-0.483
CPVR	-0.037	-0.132	-0.190	-0.228	-0.257	-0.332

Figure 2: Mean W_p by Job Tenure - BM and CPVR

(B): offers improve, becoming more competitive - relative to workers' wage - at the same time that workers are climbing the ladder. These two explanations have implications for the importance of job search in promoting wage growth. According to A, job search is less important for wage growth, as some job transitions may not be motivated by wage gains. According to B, W_p might not reflect the effects of positive selection and the size of the job ladder so directly as implied from standard random search models. Determining the relative influence of each possibility is thus essential for assessing what the data on job offer can truly reveal about the role of job search in wage determination.

3.4 Evidences from the Frequency of Offers

The NY-SCE provides additional information that can be used to study how exactly workers are climbing the job ladder. In particular, the survey collects data on the number of job offers received by workers, which can be used to evaluate hypothesis (A) about their job ladder progression. In Question 'NL1' of the survey, workers report the number of job offers they received in the previous four months. College-educated workers receive an average of

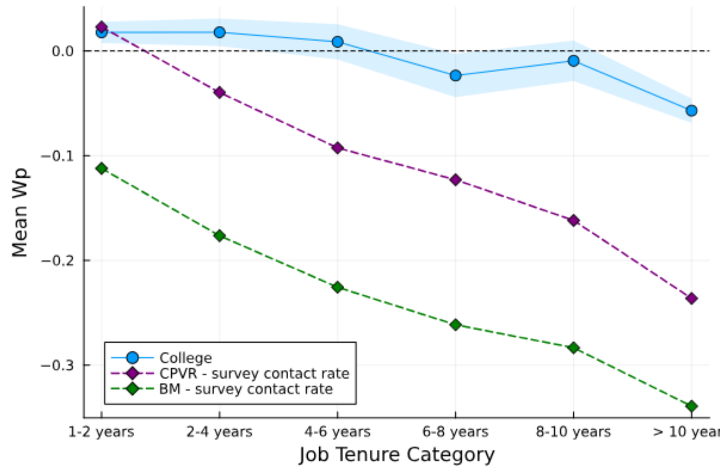


Figure 3: Mean W_p by Tenure - Survey Contact Rate

0.3 job offers in this period, while non-college-educated workers receive 0.38. If instead of averaging, I estimate the contact rate by fitting a Poisson distribution on the data, the result is similar.

For college-educated workers, this yields an annual contact rate of 0.9, which is significantly lower than the 2.58 contact rate required in standard random job search models to match the observed JtJ transition data for this group. If workers only switch jobs to pursue better opportunities, a 0.9 contact rate translates to a JtJ transition rate of just 14.2%, substantially lower than what is observed in the data.

Setting the contact rate in the benchmark models to what is measured from the survey data improves the fit of the (conditional) mean W_p to the data substantially, as shown in figure 3. Naturally, the lower contact rate implies that workers encounter fewer opportunities to move up the job ladder, which in turn lowers mean wages and raises mean W_p.

The lower contact rate still cannot fully explain the high mean W_p in the data. A substantial gap remains, specially for workers with high tenure on the job - those in the top of the ladder. Moreover, now job to job transitions are counterfactually low. A potential solution to the latter is considering reallocation events, typically employed in the literature to explain the fraction of observed JtJ transitions that result in a wage cut for the worker. As will be shown later, a model incorporating reallocation events and the contact rates directly measure from the survey will account well for both the JtJ transitions rates and the share of transitions that happen with a wage cut.

What else can we learn from the data on job offers? One potential explanation for why job offers might improve as workers are climbing the job ladder - hypotheses (B) - is some form of *directed search* by workers. In models of directed search - Moen (1997) - workers can direct their search efforts toward finding job positions that are better than their current one. Therefore, as workers climb the job ladder, job offers they expect to receive also improve since they are able to redirect search effort towards increasingly better positions.

Perfectly directed search, as is typically modeled, is readily falsified by the data: more than half of the offers workers receive are a discount over their current wage. One will naturally entertain that perhaps allowing for some kind of imperfectly directed search mechanism can help to conciliate the low magnitudes of W_p, with some dispersion in W_p. However, data on the frequency of job offers does not seem to support this hypothesis.

The directed search mechanism is characterized by the trade-off between contact rate (or probability of receiving an offer) and the quality of the job opportunity workers are aiming for. As shown in Menzio et al. (2016), this results in a negative correlation between contact rates and tenure. Data on the number of offers received by workers, however, reveals very little tenure effect on contact rates, especially for college-educated workers.

Figure 4 shows estimated contact rates by tenure category, controlling for age. Contact rates don't decrease much in the first 5 years of job tenure. For college educated workers, contact rates is very close to one across every tenure category besides 10 or more years of tenure. Contact rates for non-college educated workers decrease considerably at some point between 4 and 8 years of job tenure, remaining constant thereafter. In contrast, as shown in Menzio et al. (2016), directed search predicts a sharp decline in contact rates on the first years of job tenure and continuous decrease thereafter.

Even if job search is imperfectly directed, the trade offs between better and fewer offers embedded in such mechanism would predict decreasing contact rates to some extend. Maybe

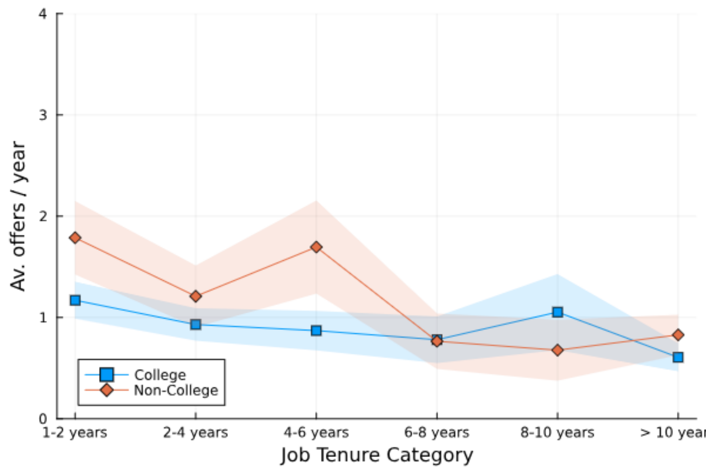


Figure 4: Contact Rates by Job Tenure

an some alternative modeling approach to how workers direct their search which can rationalize both observations on W_p and on contact rates. This paper, instead, will show that the observed empirical regularities can be rationalized well in a labor market model with random job search where job offers improve over time due to productivity gains generated by human capital accumulation and growing aggregate productivity factor.

4 A Labor Market Model With Productivity Growth

The proposed model builds on the Cahuc et al. (2006) - CPVR - framework of wage bargaining based on opportunity cost, which, compared to a wage-posting framework, accomplishes both A and B: (A) It is a known feature of this setting that wages are back-loaded - wage gains are initially modest when workers move to more productive firms, gradually increasing through the process of renegotiation - which slows down the effect of positive selection on wages; (B) As workers climb the ladder, the market responds by offering higher wages in attempts to poach them, thereby improving job offers as workers progress up the job ladder.

I extend the CPVR model by incorporating human capital accumulation and aggregate productivity growth. Also, in the proposed model, wage contracts will feature a **fixed** wage, rather than a piece rate based on joint production — an approach often seen in labor market models with productivity gains.[3]. The combination of the fixed wage structure with growth dynamics will be key: job offers will increase over time relative to workers current wages as workers accumulate human capital and benefit from aggregate growth, delivering B. More specifically: employed workers will receive some fixed wage w, which remains unchanged until a wage renegotiation is triggered. A renegotiation occurs only when an outside offer is good enough so the employer is forced into renegotiating wages, in order to retain the worker. Meanwhile, human capital and aggregate productivity grow will raise outside offers and the worker's expected wage premium W_p over time. This effect is especially pronounced for high-tenure workers, who often go extended periods without a strong enough outside offer that trigger a wage renegotiation.

Lastly, the model will feature reallocation events, when workers will choose to accept an offer from job opportunity of lower rank (lower productivity) than that of his current employer. These events can be related to workers sometimes deciding between jobs for reasons other than potential wage gains (due to amenities, for example), obviously slowing down the effect of positive section on wages (A).

4.1 Workers and Firms

Time is continuous. There is unit mass of workers who operate in the same particular market. Workers differ in their ability level which is represented by the amount ε units of labor efficiency they can provide. Workers can be either unemployed or matched with a firm. Workers exit the market at the rate μ and are replaced by new unemployed workers who begin they career with some ability level: ε_0. There is a distribution of initial ability in the population of workers, with cdf Γ over the interval $[\varepsilon_{0,min}, \varepsilon_{0,max}]$. Throughout their career, workers increase their ability following a human capital accumulation process: at the Poisson rate α, an employed worker with ability level $\varepsilon = \varepsilon_h$ will increase her ability to ε_{h+1},

[3] Bagger et al. (2014),Gregory (2023) for example

according to:

$$\varepsilon_{h+1} = \varepsilon_h + \rho_\varepsilon(\varepsilon_{max} - \varepsilon_h)$$

where $\varepsilon_{max} = \kappa\varepsilon_0$ is the maximum level of human capital a worker with initial ability ε_0 can achieve. In this specification, human capital trajectory is concave and the parameter ρ_ε controls the concavity of the trajectory. Workers ability can differ either due to initial conditions or because they had a different history of human capital shocks.

In principle, the worker's problem varies with initial ability, but in a standard setting with linear preferences and production, the problem for workers with different initial levels of ability are merely a rescaling of the same structure, leading to identical predictions for W_p. For simplicity, I will omit the state variable ε_0 during the exposition.

At any point in time workers can meet a job vacancy with job specific productivity p distributed over $[p_{min}, p_{max}]$ according to a cdf F(p), this represents the job ladder workers can climb. If worker and job vacancy form a match, the job-worker pair produce according to:

$$y_t(\varepsilon, p) = A_t p \varepsilon$$

where A_t is an aggregate productivity factor which growths through time at the constant rate g_a. Production is the current income for the employer who owns the job vacancy and whose current cost is the flow wage, w, it agreed to pay the worker. An employed worker current flow income then is the current wage she receives from her employer. An unemployed worker will generate income from home production: $A_t b\varepsilon$, where b represents efficiency of home production.

Workers flow utility is linear in their income $U(x) = x$, they discount the future at a rate r and maximize expected lifetime utility by deciding whether to accept or reject job offers made by job vacancies they meet. A worker can only be matched with one job at a time.

Employers discount the future at the same rate, r, and maximize the vacancy's discounted flow profit.

4.2 Market Flows and Wage Setting

Unemployed workers meet new job vacancies at a rate λ_0 while those employed meet new vacancies at the rate λ_1. Upon meeting, worker's and vacancy's decision to form a match will depend on gains from trade, unless a **reallocation** happens. Gains from trade are defined as the difference between the joint value of the match and the outside options of the job vacancy and the worker. The outside option of every job vacancy is zero, as they leave the market if not matched with any worker. The outside option of an unemployed worker is the value of unemployment and that of an employed worker is the maximum value it can get in her current match, which is the total joint value of the current match.

Formally, define $V_{t,0}(\varepsilon)$ the value function, at time t of an unemployed worker with ability level ε; $V_t(\varepsilon, w, p)$ the value function at time t of a workers with ability ε, receiving the wage w and working at a job of productivity p; and $J_t(\varepsilon, w, p)$ the value derived by an employer who employs the worker type ε at the job with productivity p and pays the wage w. The total value of the match is then defined as:

$$\Omega_t(\varepsilon, p) = V_t(\varepsilon, w, p) + J_t(\varepsilon, w, p)$$

Since the employers' opportunity cost of a job is zero, the maximum wage that it would agree to pay the worker would be such that $J = 0$, so that $V = \Omega$.

When an unemployed worker with ability ε meets a firm with productivity p and $\Omega_t(\varepsilon, p) > V_{t,0}(\varepsilon)$, there are gains from trade if the worker is employed by p. Worker and employer are going to negotiate but since there are gains from trade, they will be able to reach an agreement on how to split the gains from trade and will form a match. I assume the outcome of the negotiation follows a generalized Nash-bargaining solution in such an way that workers receives a constant share β of the gains from trade on top of his opportunity cost. Therefore the worker will be employed receiving a wage w, such that:

$$V_t(\varepsilon, w, p) = V_{0,t}(\varepsilon) + \beta[\Omega_t(\varepsilon, p) - V_{0,t}(\varepsilon)]$$

When a worker employed at a job with productivity p, earning the wage w, meets a new job vacancy with productivity p':

In case $p' > p$, there are gains from trade in moving to job p'. Current employer and new job vacancy are going to bid for the worker but the more productive new job will ultimately be able to poach the worker and will negotiate with her the new wage w' that will also be determined by the Nash-bargaining solution:

$$V_t(\varepsilon, w', p') = \Omega_t(\varepsilon, p) + \beta[\Omega_t(\varepsilon, p') - \Omega_t(\varepsilon, p)]$$

If $p' \leq p$, there is no gains from trade for moving to the new job vacancy. Workers current employer and new job vacancy will bid for the worker and with probability γ, the worker will be reallocated at the end of the bidding process. That means that after the least productive job ,p', makes its final offer to the worker, which delivers the worker the total value of the match with that job, the worker will accept it, even though her current employer could have delivered her a higher value (in terms of lifetime expected wage). These reallocation events can be interpreted as the worker deciding to move for reasons other than increasing lifetime income, reasons exogenous to the model. The worker then moves to p' earning the wage that delivers her the total value of the match:

$$V_t(\varepsilon, w', p') = \Omega_t(\varepsilon, p')$$

Lastly, in case $p' \leq p$ and there is no reallocation, the worker stays in the more productive job.

The new job vacancy still bids for the worker (an offer is always made) and if $\Omega_t(\varepsilon, p') > V_t(\varepsilon, w, p)$, i.e. the new vacancy is able to offer the worker a higher value than what she obtains with her current wage, the current employer is forced to renegotiate the flow wage paid to the worker in order to retain the worker. In this case the re-bargained wage w' has to be sufficient to cover the value of the outside offer:

$$V_t(\varepsilon, w', p) = \Omega_t(\varepsilon, p')$$

Naturally, there is a threshold productivity q such that meeting a vacancy with productivity $p' > q$ will allow the workers to at least renegotiate their current wage and increase expected lifetime utility, this threshold is a function of the workers' current ability, wage, and employer productivity:

$$\Omega_t(\varepsilon, q(\varepsilon, w, p)) = V_t(\varepsilon, w, p)$$

Matches will also be exogenously separated at the rate δ, in which case the worker will fall into unemployment and the job vacancy extinguished.

4.3 Bellman Equations

The value function of the employed worker can be written as:

$$
\begin{aligned}
(r+u+\delta+\alpha+\lambda_1)V_t(\varepsilon,w,p) = \quad & w \quad + \\
\text{No change:} \quad & +\lambda_1(1-\gamma)F(q(\varepsilon,w,p))V_t(\varepsilon_i,w,p)+ \\
\text{Renegotiation:} \quad & +\lambda_1(1-\gamma)\int_{q_t(\varepsilon_i,w,p)}^{p} \Omega_t(\varepsilon,p')f_t(p')dp'+ \\
\text{Poaching:} \quad & +\lambda_1\int_{p}^{\bar{p}} \left\{\Omega_t(\varepsilon,p)+\beta[\Omega_t(\varepsilon,p')-\Omega_t(\varepsilon,p)]\right\}f_t(p')dp'+ \\
\text{Reallocation:} \quad & +\lambda_1\gamma\int_{\underline{p}_t}^{p} \Omega_t(\varepsilon,p')f_t(p')dp'+ \\
& +\delta V_{0,t}(\varepsilon)+\alpha V_t(\varepsilon',w,p)+\frac{d}{dt}V_t(\varepsilon,w,p)
\end{aligned}
$$

The last term captures the fact that the value of every agent in this economy is improving through time due to productivity growth. The value function of unemployed worker can be written as:

$$
\begin{aligned}
(r+u+\lambda_0)V_{0,t}(\varepsilon) = b_t\varepsilon \quad & + \\
\text{Employment:} \quad & +\lambda_1\int_{\underline{p}}^{\bar{p}} V_{0,t}(\varepsilon)+\beta[\Omega_t(\varepsilon,p')-V_t(\varepsilon)]f_t(p')dp'+ \\
& +\frac{d}{dt}V_{0,t}(\varepsilon)
\end{aligned}
$$

And the value function of an employer:

$$
\begin{aligned}
(r+u+\delta+\alpha+\lambda_1)J_t(\varepsilon,w,p) = A_tp\varepsilon - w \quad & + \\
\text{No Change:} \quad & +\lambda_1(1-\gamma)F(q_t)J_t(\varepsilon,w,p) \quad + \\
\text{Renegotiation:} \quad & +\lambda_1(1-\gamma)\int_{q_t(\varepsilon_i,w,p)}^{p} \left\{\Omega(\varepsilon,p)-\Omega(\varepsilon,p')\right\}f(p')dp'+ \\
& +\alpha J_t(\varepsilon',w,p)+\frac{d}{dt}J_t(\varepsilon,w,p)
\end{aligned}
$$

Note that the value an employer derives from a match will be driven to zero in the event of a reallocation or if the worker in that match meets a job vacancy with productivity higher than his.

Using the definition: $\Omega = V + J$,we get a recursion for the total value of the match:

$$
\begin{aligned}
(r+u+\delta+\alpha+\lambda_1)\Omega_t(\varepsilon,p) &= A_t p\varepsilon \quad + \\
\text{No change:} \quad & + \lambda_1(1-\gamma)F_t(p)\Omega_t(\varepsilon,p) \quad + \\
\text{Poaching:} \quad & + \lambda_1 \int_p^{\bar{p}_t} \{\Omega_t(\varepsilon,p) + \beta[\Omega_t(\varepsilon,X) - \Omega_t(\varepsilon,p)]\} f_t(X)dX + \\
\text{Reallocation:} \quad & + \lambda_1\gamma \int_{\underline{p}_t}^{p} \Omega_t(\varepsilon,X) f_t(X)dX + \\
& + \delta V_{0,t}(\varepsilon) + \alpha\Omega_t(\varepsilon',p) + \frac{d}{dt}\Omega_t(\varepsilon,p)
\end{aligned}
$$

Next I define an equilibrium concept for this labor market economy with growth.

4.4 A Labor Market Equilibrium With Growth

Define the wage setting function: $w(\varepsilon, p, p', t')$ as the wage bargained by a worker with productivity ε, with firm p', using the offer from firm p as an outside option, at time t'.

Define the re-scaled wage function: $w^*(\varepsilon, p, p', t, t') = \frac{w(\varepsilon,p,p',t')}{A_t}$. which follows a law of motion based on the wage setting protocol and the growth rate of A_t, g_a.

Then, given a distribution $F(p)$, a balanced growth path for this labor market economy is defined as:

a. A value function for the unemployed worker, $V_0(\varepsilon)$; a value function for the employed worker: $V(\varepsilon, w, p)$; a value function for the employer: $J(\varepsilon, w, p)$; a value function for the total value of the match $\Omega(\varepsilon, p)$,

b. A wage setting function: $w(\varepsilon, p, p', t')$,

c. A stationary distribution of workers over (ε, p, w^*, b), b denoting the unemployment state.

such that:

1. the value functions are the solutions to the Bellman equations,
2. $w(\varepsilon, p, p', t')$ follows the wage setting rules,
3. the distributions evolve according to the re-scaled wage law of motion, the transitions determined by gains from trade and reallocation rules,
4. inflow of workers in (ε, p, w^*, b) = outflow of workers in (ε, p, w^*, b).

5 Calibration and Fit

The model is parametrized with one unit of time representing one year. The calibration focuses on replicating empirical regularities for workers with a college degree; thus, unless otherwise stated, information used for the calibration pertains to this demographic. The $f(p)$ distribution of vacancies' productivity is set as a generalized Pareto distribution, with location $k_p = 1$ (p_{min} =1), scale σ_p and shape Σ_p.

Table 3 list the parameters with assigned values. I set standard values for the discount and mortality rates (exit rates). For the aggregate growth rate of labor productivity in the economy, g_A, I assign the growth rate of real wages between January of 1995 and January of 2020, 1%, for all employed workers, obtained from the FRED. The unemployment search rate is obtained from the EU rates estimated form the CPS. The contact rate for employed workers is obtained from the SCE. The job separation rate is estimated from EU transition in the CPS.

As presented before, the model admits the existence of a plurality of initial ability levels: ε_0. Nevertheless I focus on the problem of a mass of workers with the same initial ability,

Parameter	Description	Value	Source/Target
r	Discount rate (annual)	0.03	Standard
u	Mortality rate (avg. 40 years)	0.025	Standard
g_A	Growth rate of productivity	0.01	Wage growth (1995-2020)
λ_0	Unemployed search rate	4.7	Unemployment rate: 0.39/month
λ_1	Employed search rate	0.9	From SCE
δ	Job separation rate	0.045	EU rate (per year)
ε_0	Initial ability	1	Normalization

Table 3: Assigned Parameters for the Proposed Model

which will be normalized to 1. Due to the linearity in the functional form of workers flow utility and production technology, the problem of workers with another initial ability is just a re-scaled version of the problem of a worker with $\varepsilon_0 = 1$, and so workers heterogeneity in initial ability level is inconsequential for predictions regarding the W_p.

Table 4 shows the parameters that are calibrated internally. Target moments for the calibration are:

1. **Job to Job transition rates**, observed from the CPS.
2. **Moments from the W_p distribution:** average W_p by tenure category (6); the variance of W_p; the skewness of W_p.
3. **Cross-sectional wage differences by experience (proxied by age)**: average wage of a workers in their 20th, 10th and 5th year in the market vs that of workers on their first year in the market (23 years old) - these are also obtained from the CPS.

Only the contact rate of employed worker and the reallocation probability, γ, affect the JtJ rate in the economy. Since contact rate is assigned from the survey data, the JtJ rate pins down the reallocation probability in my framework. In the literature, reallocation shocks are commonly set by targeting the percentage of observed job to job transitions that involved a wage cut. The model does a good job in reproducing this observation even thought, it is not targeted, as shown below.

Parameter	Description	Value	Source/Target
γ	Reallocation Probability	0.054	21.3% JtJ rate
β_1	Bargaining	0.238	Target Moments
σ_p	Scale - $f(p)$	0.138	Target Moments
Σ_p	Shape - $f(p)$	0.4	Target Moments
κ	H.C. potential	1.92	Target Moments
ρ_ε	H.C. step size	0.008	Target Moments
α	rate of H.C. increase	2.58	Target Moments

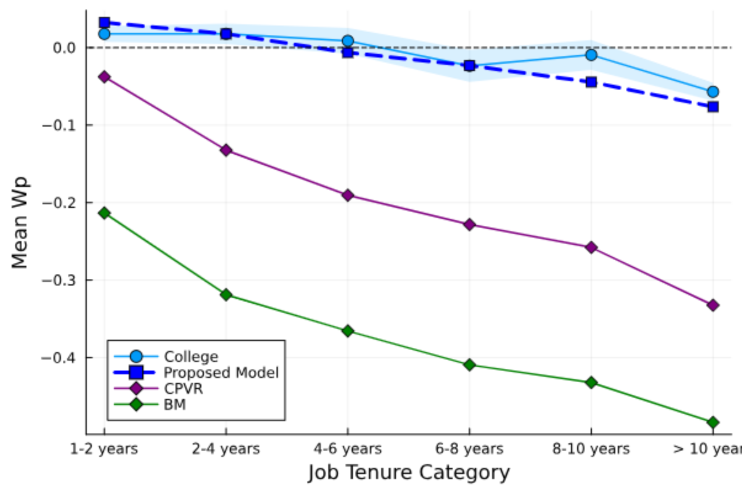
Table 4: Calibrated Parameters for the Proposed Model

Aside from JtJ transitions, every other moment is affected by all calibrated parameters. But some set of moments are more informative about some set of parameters. In particular, moments from the W_p distribution are more informative about the bargaining power and the parameters of the productivity distribution, while moments of the cross sectional wage difference are mostly informative about the human capital process.

Figure 5 shows the model fit of the mean W_p conditional on job tenure. The red line with square markers W_p in the figure represents the predicted values of the proposed model. The proposed model is able to fit tenure profile of W_p very well. For instance, a worker with 10 year or more of tenure in the model expects a discount of 7.6% in the next wage offer received, very similar to what is observed in the data. Moreover, the model is able to reproduce the fact that workers with low tenure on the job do expect a premium in the next job offer they receive from the market. For comparison, I reproduce again predictions from BM and CPVR.

In the model, the positive mean W_p for low tenure workers happens due to the fact that in models of bargaining based on opportunity cost, wages are back loaded. Therefore, when moving to more productive jobs, workers are willing to accept a modest (or even negative) increase to wages as they hope that renegotiation will allow them to bring their wage up in the future. Thus, during the first years of employment, workers might expect wage offer, even from less productive jobs, to be better than their current wage.

The top section of 5 shows the model fit for the remainder of the targeted moments and



Tenure	**1-2 years**	**2-4 years**	**4-6 years**	**6-8 years**	**8-10 years**	10^+ **years**
College	0.017	0.017	0.008	-0.023	-0.009	-0.057
Proposed	0.032	0.018	-0.0064	-0.023	-0.044	-0.076
CPVR	-0.038	-0.132	-0.191	-0.228	-0.258	-0.332
BM	-0.213	-0.318	-0.365	-0.409	-0.432	-0.483

Figure 5: Model Fit - Mean W_p by Job Tenure

for some selected non-targeted moments. Again, for comparison I show the fit obtained from the two benchmark models. I require the calibration process to match the variance of W_p, as usual. Skewness is the moment of the W_p distribution in which the proposed model does worse, although a skewness of -0.09 is still associated with very symmetrical distribution. As mentioned before, JtJ transitions can be matched using the reallocation shock. Finally, the proposed model tracks observed wage differences by experience very well.

Why is skewness included as a target? Skewness plays a big role in disciplining bargaining power. In principle, a lower level of bargaining power would help to bring mean W_p up for low tenure workers due to back loaded wages. With very low bargaining power, some workers would accept huge pay cuts in order to move to more productive firms and therefore they would have very high W_p. This mass of workers would bring the average W_p up, helping the model to fit the mean W_p, but they would also make the W_p distribution more positively skewed. Fitting skewness is, thus, important so the low average W_p is not explained in the model by a small mass of workers with very high W_p but rather by increasing W_p for the whole spectrum of workers.

The bottom part of table 5 shows selected non-targeted moments. Mean W_p delivered by the model is slightly positive as the majority of workers are low tenure and the model predicts a higher expected wage premium for those than what is in the data. Median W_p reflects the symmetry of W_p distribution in the model. The model predicts that 28% of JtJ transitions, well within the range documented by previous works.[4]

Targeted	**College**	**Proposed**	**BM**	**CPVR**
Var W_p	0.049	0.049	0.049	0.049
Skewness	0.00	-0.09	-0.04	0.267
JtJ - year	0.213	0.213	0.213	0.213
Log Wage diff. (1-5)	0.223	0.222	0.295	0.226
Log Wage diff. (1-10)	0.377	0.382	0.416	0.362
Log Wage diff. (1-20)	0.551	0.551	0.468	0.442
Not Targeted				
Mean W_p	-0.009	0.006	-0.320	-0.153
Median W_p	0.00	0.003	-0.319	-0.159
% JtJ w/ cut	0.34	0.28	0	0.38

Table 5: Model Fit - Targeted and Non-Targeted moments

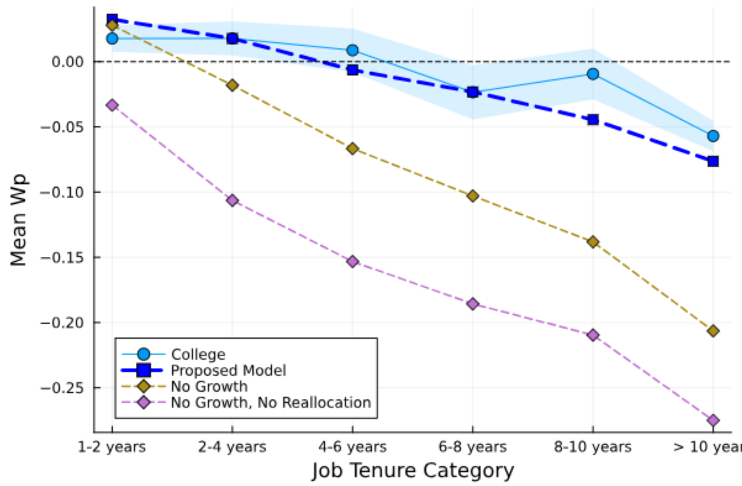
Results show that, under a reasonable calibration, the proposed model can account remarkably well for the tenure profile of mean W_p and other moments of the W_p distribution. I discuss next what is the role played by each of the features added to the CPVR original framework in accounting for the data.

Figure 6 again compares the tenure profile of the average W_P for workers in both the dataset and the proposed model, it also includes the fit for two variations of the proposed model: one that excludes the processes of human capital accumulation and aggregate productivity growth (represented by the golden dashed line with square markers) and another one that additionally omits the reallocation shock, alongside the growth in ability and productivity[5]. For each variant of the proposed model, parameters are re calibrated to fit the

[4]Jolivet et al. (2006) documents that the fraction of such transition was 23% in the US, using the PSID. Tjaden and Wellschmied (2014) find that this fraction is 34% in the PSID data from the 1990s

[5]This second variant is very similar to the original CPVR, the only difference is that in CPVR the

same set of target moments but excluding moments on log wage differentials by experience, as these variants are not suited to track those moments.



Tenure	1-2 years	2-4 years	4-6 years	6-8 years	8-10 years	10^+ years
College	0.017	0.017	0.008	-0.023	-0.009	-0.057
Proposed	0.032	0.018	-0.0064	-0.023	-0.044	-0.076
No Growth...	0.0279	-0.0181	-0.0667	-0.1029	-0.1382	-0.2064
...(+)No Reallocation	-0.0333	-0.1064	-0.1532	-0.1857	-0.2096	-0.2750

Figure 6: Comparison of W_p by Tenure Across Different Models

The combination of fixed wages with growth in human capital and productivity delivers wage offers that improve through time relative to worker's wages, while the reallocation shock slows down the effect of positive selection. From the plot, it is clear that improving offers are most important for explaining the observed low expected wage discount of workers with high tenure on the job. Without these features, mean W_p for workers with 10 or more years of tenure would fall to -20.6%. On the other hand, the mean W_p for workers with one to two years of tenure is barely affected by this mechanism. The effect of reallocation on W_p is more uniform across the tenure distribution, if anything, it is less important for high tenure worker.

Increases in W_p due to productivity growth and human capital accumulation—which

bargaining over the gains from trade is applied when the worker move to a better job and also when she renegotiates, in my set up workers only extract a share of gains from trade when moving to a different employers enhance the offers workers receive from new job opportunities— mostly affect high-tenure workers, as they are more likely to have spent a long time without renegotiating their wages. When a worker initially moves to a highly productive firm, frequent wage renegotiation is possible within the first few years. Over time, however, as the worker's wage approaches the maximum that their current employer is willing to pay, renegotiation opportunities become less common. At this stage, further renegotiation often requires very good outside offers, and so new offers might substantially improve relative to the worker's wage before she has an opportunity to renegotiate.

6 Wage Growth and Wage Dispersion

The proposed model can account well for the empirical regularities of the expected wage premium. Using the models' structure, the expected wage premium data is sufficient to infer characteristics of the job ladder workers are climbing throughout their careers. This enables an assessment of the role of search frictions in determining wages.

A large body of econometric and structural research on the labor market is dedicated to estimating the determinants of wage inequality and wage growth. The main issue all this body of research try to address in some way or another is: the job ladder is not observed. Estimating how much of workers' pay is determined by their characteristics, and how much can be attributed to their access to different job opportunities throughout their career depends crucially on estimating the degree on which these opportunities in fact differ from each other, given workers' set of (potentially unobserved) characteristics.

Previous structural efforts to study wage inequality and growth estimated the job ladder using firm-level data and stratifying job opportunities based on observable characteristics, such as education level and occupation. However, this approach can overestimate the range of opportunities truly available to workers, as individuals with similar observable characteristics may still face distinct job ladders due to unobserved factors. For example, two workers in the same occupation might have differing job prospects based on the prestige of their degree institution or on how they ranked among their colleagues. More recent research, following Low et al. (2010) tries to estimate the job ladder by comparing the volatility of wage changes for workers who are switching jobs versus that of those who stay in the same job, but this approach can be sensitive to factors like productivity or preference shocks, which may also inflate the estimated dispersion of job opportunities.

A discussion of the methods used so far in the literature to estimate the job ladder is beyond the scope of this paper. Instead, this paper brings a novel perspective on the issue as it estimates an average job ladder directly from the data on expected wage of job offers reported by workers. Through the lens of the proposed model, the data on job offers allow us to map the range of job opportunities workers have available in the labor market, circumventing many challenges faced by those previous attempts which relied solely on information about workers current wages.

This section then examines the role of job search in wage growth and wage inequality implied by the expected wage premium. I also show that the results are substantially different if one does not account for all features that allow the proposed model to account well for the W_p data. In particular, the proposed model delivers a reduced importance of search when compared to the classic benchmark models.

Throughout this section, I will use the following decomposition of log wage, $\hat{w}$:

$$\hat{w_{ij}} = \hat{s}_i + \hat{A}_t + \hat{p_j} + \hat{\varepsilon}_i$$

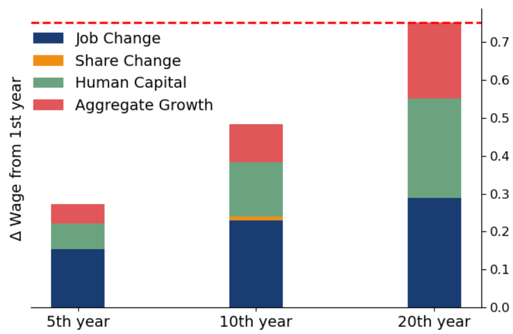
The log wage of a worker i employed at firm j can be decomposed into the share of total production that is paid to the worker, $\hat{s}_i$, and the components of the total production of the match. Since wages are fixed, this share changes as A_t increases, when the worker receives a human capital shock, re-bargains the contract, or moves to another firm.

For this analysis, I define the role of search in wage inequality and growth as all the inequality and growth that would otherwise not exist without search frictions. The comparison point is therefore a competitive economy without such frictions.

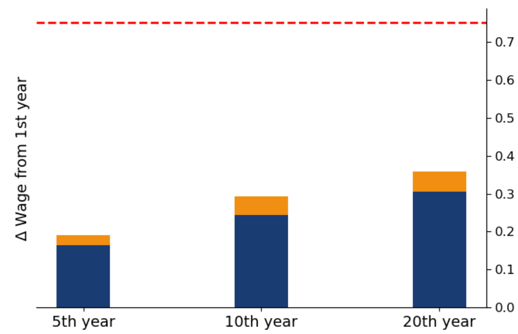
6.1 Wage Growth

Figure 7 presents the profile of log wage growth estimated by the proposed model, a variant of the proposed model with no growth in human capital and productivity, CPVR and BM. Each plots shows the expected log wage increase of a worker on her first year of employment for when she is on her 5th, 10th and 20th year in the labor market, conditional on being employed on those periods. Total expected log wage growth is decomposed in the growth due to the workers moving to more productive job opportunities (job change, $\Delta\hat{p}$), workers bargaining a higher share of production as wage payment (share change, $\Delta\hat{s}$), production increasing due to human capital growth (human capital, $\Delta\hat{\varepsilon}$) and due to growth in aggregate productivity (aggregate growth, $\Delta\hat{A}_t$).

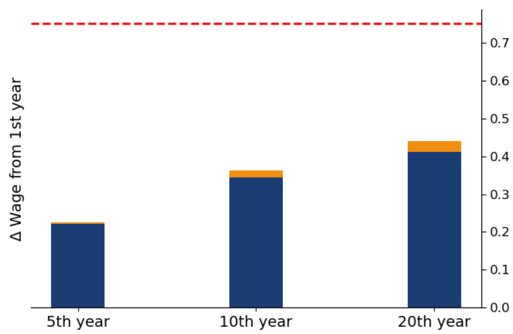
According to the proposed model, estimated wage growth in 20 year is 0.751 log points (marked by the red line), very similar to what was obtained by Bagger et al. (2014) using Danish matched employer-employee data. The other models cannot replicate this wage growth but what we are actually interested in is the wage growth that each model attributes to job search, which is composed of productivity change plus share changes (the two sources of wage growth that would be absent in a world with an efficient labor markets). The proposed model delivers the least amount of wage growth due to search: 0.288 (38.4% of total estimated wage growth), in 20 years of market experience. Most of the gains from search happens in the first 10 years of professional life, all again in line with Bagger et al. (2014). In the model with no growth in productivity or human capital, gains from search increase to 0.360 (47,9%). Most of the difference between these two models comes from the role of changes in the productivity of the employer and changes in the labor share paid to workers. In the proposed model, changes in the share play almost no role for wage growth. In fact, the contribution is slightly negative for wage growth in 20 years, whereas in the



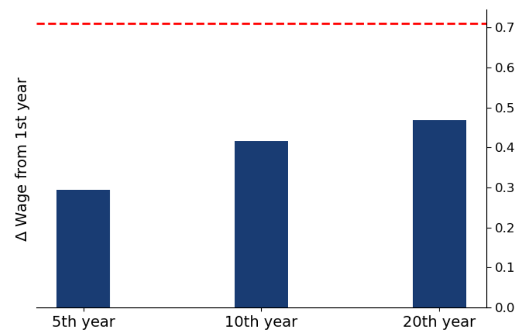
(a) Proposed model



(b) Proposed model with no growth



(c) Cahuc, Postel-Vinay and Robin



(b) Burdett and Mortensen

Figure 7: Wage Growth Comparison

model without growth, changes in share account for 0.055 log points (7.3%) of wage growth.

Why in the proposed model the share change plays almost no role for wage increase in comparison to a variant with no growth? That happens because of the combination between fixed wages and growth. In anticipation for the fact that they will spend some time with a fixed wage, during a period of (fast) human capital and productivity growth, young workers bargain a high initial share of production as payment during their initial years of employment, higher than what they would have bargained otherwise in the absence of growth. As time passes, human capital and productivity growth will decrease the share (as they increase total production) at the same time that workers will be looking for opportunities to renegotiate their wages to increase the share. The net effect is an almost zero change in average share paid to workers during their career. In comparison, with no growth, the initial share paid to workers is lower and gains from renegotiation more substantial. The exact same difference would exist between the proposed model and model with growth in productivity and human capital accumulation but with wage contracts characterized by a piece rate of total production. In this case, workers would not have their share of production decreasing with time and so gains in share during their career would be more significant.

Wage gains from job search estimated with the CPVR's framework are even more substantial, 0.44 in 20 years (or 58.7% of total estimated wage growth). The main difference between CPVR and the proposed model is the absence of a reallocation shock. With no reallocation, every job transition is a step up in the productivity ladder, therefore workers experience higher gains from moving to more productive firms through their careers. The fact that wage growth from the change in share is noticeable lower in CPVR when compared to the proposed model with no growth mostly reflects the effect of the reallocation shock as well. Workers that are reallocated move to a less productive job but are able to bargain a high share of production as wage payment[6] to work to less productive firms. That is why in the model with reallocation we observe a higher growth in the share of production paid to worker together with lower gains from moving to more productive firms.

Finally, the BM model of wage posting delivers the highest returns to search: 0.47 over 20 years. The reason why the wage posting framework delivers higher returns to job search is that it requires a higher dispersion of job opportunities (a higher σ_p) in order to generate the variance of W_p observed in the data (see table 8). The higher dispersion of job offers implies higher gains from climbing the job rank as the job ladder becomes more disperse.

In the BM model of wage posting, changing jobs amounts to the same as changing wages (there is no negotiation, therefore no change in the share of production). This model delivers the highest returns to jo search: 0.47 over 20 years. The reason why the wage posting framework delivers higher returns to job search is that it requires a higher dispersion of job opportunities (a higher σ_p) in order to generate the variance of W_p observed in the data (see

[6] the share of some young workers can initially exceed 1 as firms are expecting the share to go down in the future.

Var Component	Proposed	No Growth	CPR	BM
(1) $Var(\hat{p})$	0.122	0.134	0.120	0.143
(2) $Var(\hat{s})$	0.050	0.053	0.045	0
(3) $Var(\hat{\varepsilon}\|\varepsilon_0)$	0.015	0	0	0
(4) $2Cov(\hat{p}, \hat{s})$	-0.097	-0.107	-0.079	0
(5) $2Cov(\hat{p}, \hat{\varepsilon}\|\varepsilon_0)$	0.018	0	0	0
(6) $2Cov(\hat{s}, \hat{\varepsilon}\|\varepsilon_0)$	-0.001	0	0	0
1+2+4	0.075	0.080	0.086	0.143
$Var(\hat{w}) - Var(\hat{\varepsilon}\|\varepsilon_0)$	0.092	0.080	0.086	0.143
$Var(\hat{w})$	0.106	0.080	0.086	0.143

Table 6: Decomposition of Variance of $\hat{w}$

table 8). The higher dispersion of job offers implies higher gains from climbing the job rank as the job ladder becomes more disperse.

6.2 Wage Dispersion

Table 6 shows the decomposition of the cross sectional variance of wages obtained by simulating the same 4 models. A measure of total wage dispersion related to job search is shown in the second to last row and is represented by the sum of all components except pure dispersion in human capital accumulation, $Var(\hat{\varepsilon}|\varepsilon_0)$[7]. The third to last row shows wage dispersion of a counterfactual scenario in which there is no human capital process, which is an alternative measure of dispersion generated by job search. Wage dispersion in the other models all happen because of job search.

In the proposed model, search related log wage dispersion is 0.075-0.092. The residual wage dispersion from a mincerian wage regression estimated from the CPS data is 0.25 log points (Appendix for details). Therefore, search related wage dispersion accounts for 30%-36.8% of the wage dispersion not explained by workers observables in this model, which

[7] $Var(\hat{\varepsilon}|\varepsilon_0)$ represents the variance in ability level between workers with the same initial level of ability: ε_0. In other words, the variance in ability that happens due to the fact that workers with the same initial level of ability might have different trajectories of human capital accumulation. In principle, the model could generate as much dispersion in wages as desired with the proper distribution of initial ability level, but that would not change the amount of dispersion generated by job search since initial ability would not correlate to anything - it is a purely person effect

is on the lower side of what has been estimated previously[8]. The remainder should be largely explained by workers unobserved person effect. The proposed model can, in principle, generate any level of dispersion in unobserved person effect if the distribution of initial ability ε_0 is set appropriately. In fact, Kline et al. (2020) estimated, using data from Italy, that unobserved person effect can accounts for 56.3% of measured wage dispersion, close to what would be required to close the gap between the presented simulation and the residual dispersion estimated from the data.

Interestingly, all models with wage bargaining deliver very similar measures of wage dispersion due to search, even though they are very different economies. As shown in table 8 (in the Appendix), the job ladder is much more dispersed (σ_p) in the proposed model (0.138) than in the CPVR (0.083). Still, they generate very similar levels of wage dispersion from job search. That happens because without reallocation events, CPVR requires a higher contact rate to match the JtJ transition rate. The higher contact rate generates more wage dispersion which then requires the dispersion of job productiveness to be lowered, otherwise the model would overshoot the variance of W_p (a combination of the variance of wages and the variance of expected wage offers). In the case of the proposed model without growth, the lack of growth reduces the variance of W_p generated by the model, requiring a higher dispersion of job productiveness to bring the dispersion of W_p up to what is observed in the data, but the higher dispersion is mostly offset by the more negative covariance between productivity and the share of production paid as wages. It seems like that matching the variance of W_p demands some level of wage dispersion, in the class of wage bargaining models, irrespectively of the presence of reallocation shocks, the level of contact rates or other features.

In contrast, the BM model generates much higher wage dispersion driven by search dynamics than the proposed model. The reason is straightforward: in wage-posting models, all the dispersion in W_p stems from wage variation alone, as there is no variation in expected

[8] Postel-Vinay and Robin (2002) and Jolivet et al. (2006) estimated that 50 % to 60% of wage dispersion for high skill workers should be attributed to search frictions. Tjaden and Wellschmied (2014) estimated this value to be much lower: 15%-20%.

wage offers between workers - the distribution of wage offers is invariant to their state. Consequently, wage dispersion must be high to match the variance of W_p. This would be also true in models where piece-rate contracts are *posted* instead of bargained, or in models where wages are bargained exclusively with unemployment as the outside option for workers.

7 Conclusion

Data on expected wages of job offers workers receive provides valuable insight into the distribution of job opportunities — the job ladder — that workers face in the market. This paper utilizes this data to (1) evaluate the theory in random search models of the labor market and (2) investigate the importance of job search for wages, as inferred from the job ladder recovered from expected wage offers.

I document that the observed relationship between job offers workers expect and their current wages aligns qualitatively with random search theory. However, it also challenges standard random search models. According to such models, given the wide range of job opportunities implied by the data, workers — particularly those with high tenure — should have wages substantially higher than the average job offer they might receive, which is not observed. This discrepancy can be addressed in a model where not all job transitions are motivated by wage gains and where wages do not immediately adjust to reflect productivity increases.

Through the lens of this model, the data on the expected wage premium is sufficient to assess the role of job search in wage inequality and wage growth. Using this data, I estimate that job search (search frictions) explains 37% of wage dispersion in the data. This figure is substantially lower than estimates from earlier research on the matter. Moreover, job search accounts for 38.4% of the wage growth observed over a 20-year period, a number again substantially lower than what is obtained from classic models of random job search, but more closely aligned to more recent research on the topic.

In addition to these contributions, the process of aligning a random search model with the data has provided several insights that warrant further exploration: (i) The data can be well explained by models where workers (or firms) do not direct their search; (ii) the data fits better if wages are bargained rather than posted; (iii) the data is better explained if wages are not frequently readjusted to account for changes in productivity.

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Appendix A - Support Tables

Table 7 shows the summary of the regression used to verify whether there is a systematic bias in workers' expectations about the wage of the next job offer they might receive. The regression coefficient of the average wage of the job offers that workers received, reported in the survey period (wave) $t+1$, on workers' expectations of the wage of new offers, collected during the survey period (wave) t, is not statistically different from 1. The table also shoes that workers current wage has no predictive power on the wage of offers they might receive, when controlling for their expectations.

Table 7: Rationality Test

	Realized Average Wage Offer (t+1)
Expected Wage of New Offers (t)	0.952
	(0.036)
Current Wage (t)	0.027
	(0.029)
Observations	692
Adjusted R^2	0.767
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 8 presents the parameters for the main models presented in the paper. The $f(x)$ distribution of job opportunities is set as a generalized Pareto distribution, with location $k_p = 1$ (p_{min} =1), scale σ_p and shape Σ_p.

For the CPVR, the bargaining power is calibrated by targeting the tenure profile of W_p and skewness of the W_p distribution.

Table 9 shows moments from benchmark models of random search for comparison with empirical patterns of W_p.

Parameter	Description	BM	CPVR	Proposed	No Growth
r	Discount rate (annual)	0.03	0.03	0.03	0.03
u	Mortality rate (avg. 40 years)	0.025	0.025	0.025	0.025
g_A	growth rate of Productivity	-	-	0.01	0
λ_0	Unemployed Search Rate	4.7	4.7	4.7	4.7
δ	Job Separation Rate	0.045	0.045	0.045	0.045
ε_0	Initial ability	1	1	1	1
λ_1	Contact Rate	2.58	2.58	0.90	0.90
γ	Reallocation Probability	-	-	0.054	0.054
β	Bargaining power	-	0.246	0.238	0.229
σ_p	Scale	0.105	0.083	0.138	0.1507
κ	H.C. potential	-	-	1.92	-
ρ_ε	H.C. step size	-	-	0.008	-
α	rate of H.C. increase	-	-	2.58	-

Table 8: Parameters of Main Models Presented

Moments	College	BM	CPVR
Mean W_p	-0.009	-0.320	-0.153
Var W_p	0.049	0.049	0.049
Skewness	0.00	-0.05	0.26
Share < -0.10	0.28	0.82	0.61
Share > 0.10	0.23	0.03	0.11

Table 9: Summary Comparison of Models

Appendix B - Data

Questions from the Survey of Consumer Expectations

Below questions from the SCE used to build the variables employed in the empirical section. Basic questions about demographics are left out.

Question OO2a - Expected Average Offer *Think about the job offers that you may receive within the coming four months. Roughly speaking, what do you think the average annual salary for these offers will be for the first year?*

NL1 (Added November 2014) *How many job offers did you receive in the last 4 months* *(since July 2016)? Remember a job offer is not necessarily a job that you accepted.*

NL2 (Added November 2014) *Thinking about the 3 best job offers that you received in the last 4 months, What was their annual salary? And were they for a full-time or a part-time job? Note the best offer is the offer you would be most likely to accept.*

OO2new (Added March 2015) *Over the next 4 months, how many job offers do you expect to receive? Remember that a job offer is not necessarily a job you will accept.*

OO2u (shown if Q12new =1 and Q10 is code 1,2, or 5) (Added November 2014) *What do you think is the percent chance that within the coming four months, you will receive at least one job offer? Remember that a job offer is not necessarily a job you will accept.*

OO2e - OO2e (shown if Q10 codes 3,4,7,8 or 9 and not codes 1,2 or 5) (Added November 2014) *What do you think is the percent chance that within the coming four months, you will receive at least one job offer from another employer? Remember that a job offer is not necessarily a job you will accept.*

Q10 *What is your current employment situation?*

☐ *Working full-time (for someone or self-employed) (1)*

☐ *Working part-time (for someone or self-employed) (2)*

☐ *Not working, but would like to work (3)*

☐ *Temporarily laid off (4)*

☐ *On sick or other leave (5)*

☐ *Permanently disabled or unable to work (6)*

☐ *Retiree or early retiree (7)*

☐ *Student, at school or in training (8)*

☐ *Homemaker (9)*

☐ *Other (please specify)(10)*

CPS and SCE details - [to be added. Please inquire if needed]